THEORY OF ZONE RADIOMETRY

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FOREWORD

This document is one of two documents that constitute the final report for Contract NAS8-28089, "Study of Viscous Mixing Plume Flow Field." This study was performed by the Lockheed-Huntsville Research & Engineering Center, Inc., for the National Aeronautics & Space Administration, George C. Marshall Space Flight Center, Alabama.


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Section 1
INTRODUCTION AND SUMMARY

A spectroscopic instrumentation system was developed by Rocketdyne Division of North American Rockwell which was used to measure temperature and concentration distributions in (hopefully) axisymmetric and two-dimensional combusting flows (Ref. 1). This measurement technique has become known as zone radiometry.

The success of the method depends both on how accurately the detected radiation can be converted by analysis into the desired temperatures and concentrations and on how closely the flow meets the dimensional limitations of this measurement scheme. Since the technology of radiative transfer was being very actively researched during the same time that the zone radiometry experiments were being performed, a critique of the Rocketdyne data reduction procedures is in order to determine whether or not application of the present state-of-the-art radiation analyses can yield more accurate flowfield information.

Theoretically, a temperature and a partial pressure distribution for a given species can be determined from a set of measurements made at one particular spectral level. If sets of measurements at more than one spectral level are made, partial pressures of several species and some average temperature can be determined. Practically, such multiple determinations have not yet been possible. At best, one set of measurements has been used to establish temperature and the partial pressure of one species, and another set used to determine the partial pressure of a second species. Therefore, an analysis at one particular wavelength is all that is required of available zone radiometry data. Such an analysis is described in this report.

The final goal of this report is to present a recommended data reduction scheme for the zone radiometry system. The limitations in this scheme will be clearly stated and quantitatively evaluated when possible.
To appreciate the utility of zone radiometry methods, one should realize that the technique was developed and used extensively to measure "axisymmetric" rocket motor plumes. All propellant systems cannot be measured with satisfactory accuracy by this method. Plumes with carbon particles may become too "optically thick" for the transmission part of the measurement to be made, and plumes with only water vapor as the optically active species may be too "thin" for accurate measurements. Furthermore, all motors lack axial symmetry to some degree; no good measure of this feature has yet been devised.

Despite the fact that much experimental data are available, no definitive comparison with calculated flows exists. Experiments with carefully designed burners are the primary source of radiation property data, but when a real rocket motor is studied both analysis and experiment become more difficult.

Zone radiometry has also been used on two-dimensional mixing studies. These studies have not been as extensive as those on motors. Again definitive comparisons with calculations have not been made, nor have error analysis for reducing data been previously reported.

This report serves as a prelude to a more complete data comparison study which will be forthcoming. A detailed treatment of the radiation analysis and synopsis of the zone radiometry method is reported herein, so that questions regarding the accuracy of reported data and experiments can be determined.
Section 2
RADIATION ANALYSIS

The zone radiometry system is an instrument used to determine temperature and composition of optically active species in either an axisymmetric or planar gas flow field. The radiative energy transfer analysis which must be used to relate the radiation measurements to the flow properties is described in this section.

The basic radiative exchange process is sketched in Fig. 1. Radiation from the hot zones is seen by the detector at all times. When the chopper is open, radiation from the source which is even hotter than the zones plus radiation from the hot zones is seen. Particular frequency (or wave length) intervals are measured with the detector by selectively filtering away the unwanted radiation. The radiative exchange process is essentially one-dimensional, as the angle $\beta$ is quite small; hence, one line of sight is viewed. The dimensions of the gases on the immediate sides of $\beta$ are assumed to be such that radiative equilibrium with the adjacent, lateral-gas zones is maintained, so there is no net radiative exchange in the lateral direction.

To quantitatively describe radiative exchange, the concept of intensity must be used. At a point, $P$, consider the monochromatic intensity,

$$I_{\nu} = \lim_{d\sigma, d\Omega, d\nu \rightarrow 0} \left( \frac{dE_{\nu}}{d\sigma \cos \theta d\Omega d\nu d\Omega d\nu dt} \right)$$  \hspace{1cm} (2.1)

The term $E_{\nu}$ represents the radiant energy in $(\nu, \nu + d\nu)$, where $\nu$ is the frequency of the radiation. The term $t$ is time; the geometric factors $\sigma$, $\theta$, $\Omega$ are defined by Fig. 2. The fact that the above limit exists is an
Fig. 1 - Zone Radiometer System

Fig. 2 - Definition of Geometric Terms Used to Define Intensity

\[ d\sigma = \text{incremental surface} \]
\[ \bar{N} = \text{surface normal} \]
\[ \bar{N}_1 = \text{parallel to surface normal} \]
\[ d\Omega = \text{increment of solid angle} \]
\[ \theta = \text{angle between solid angle direction and surface normal} \]
\[ \beta = \text{solid angle of all } d\Omega \text{ over } d\sigma \]

(\(d\sigma\) is located at point \(R\))
experimentally observed fact (Milne, Ref. 2, p. 84). Radiation is emitted from each point on dσ; therefore, an integration in Ω is required to calculate the radiation flux through dσ. In optics, this is not the case as intensity is defined at a point with dσ missing in the limit expression.

$I_\nu$ is independent of $S$ unless it is modified by the transmitting medium, whereas $E_\nu$ is not. There are other intensities which could have been defined; they are $I_\omega$ and $I_\lambda$. These are defined on the basis of a unit of wave number, $\omega$, or wave length, $\lambda$, in the limiting expression. If the transporting medium has a unit or known index of refraction, conversions between these intensities can be easily made. $I_\nu$ is somewhat more basic because it is independent of the index of refraction. However, the overriding criteria to use in selecting the intensity to use is the availability of property data. These data are available in select wave number increments; therefore, $I_\omega$ will be used.

Radiation in the absence of emission is attenuated according to

$$I_\omega \{S_i\} = I_\omega \{S_{i+1}\} \exp \left(- \int_{S_i}^{S_{i+1}} K_\omega \rho \, dS \right)$$

(2.2)

where here and henceforth brackets indicate functionality and where $K_\omega$ is mass absorption coefficient and $\rho$ is the density of the absorbing medium. If $K_\omega$ and $\rho$ are independent of $S$ this relationship is called the Beer-Lambert law. In general, optical thickness $= \int_{S_1}^{S_2} K_\omega \rho \, dS$. A spectral absorption coefficient may be defined by the equation shown on the following page.
\[ a_\omega = \frac{I_\omega(S_{i+1}) - I_\omega(S_i)}{I_\omega(S_{i+1})} = \frac{I_\omega(\text{absorbed})}{I_\omega(\text{incident})} \quad (2.3) \]

In general, radiation may be absorbed, reflected or transmitted; or, fractionally,

\[ a_\omega + \rho_\omega + \tau_\omega = 1 \quad (2.4) \]

Thus

\[ \tau_\omega = 1 - a_\omega = 1 - \frac{I_\omega(S_i)}{I_\omega(S_{i+1})}, \text{ if } \rho_\omega = 0 \quad (2.5) \]

or

\[ \tau_\omega = \exp \left( \int - K_\omega \rho \, dS \right) \quad (2.6) \]

The two absorption coefficients are related by:

\[ a_\omega = 1 - \exp \left( \int - K_\omega \rho \, dS \right) \quad (2.7) \]

Elements along the solid angle will not only absorb radiation but will emit at a rate of

\[ \frac{dE_\omega}{dt} = J_\omega (\rho \, d\sigma \, dS) \, d\omega \, d\Omega. \quad (2.8) \]

where \( J_\omega \) is the emission coefficient. \( J_\omega \) will be isotropic.
The geometry is such that the detector is normal to the view angle through the plume; hence, \( \cos \theta = 1 \).

2.1 THE EQUATION OF TRANSFER

Now a radiation heat balance on a control volume consisting of the solid angle \( \beta \) between \( S_i \) and \( S_{i+1} \) can be made. Consider three cross sections of \( \beta \), those at \( S_i, S_{i+1} \), \( S_{i+1/2} \); call them \( A_i, A_{i+1}, A_{i+1/2} \). Since we wish to calculate the radiation to the detector, let the radiation at \( S_{i+1} \) be \( I_\omega \) and that at \( S_i \) be \( I_\omega + dI_\omega \), i.e. \( I_\omega \) is positive in the negative \( S \) direction. The heat balance becomes:

\[
\int_{\Omega} I_\omega \, d\Omega \, A_i \, dt - \int_{\Omega} (I_\omega + dI_\omega) \, d\Omega \, A_{i+1} \, dt = \\
- \int_{\Omega} K_\omega \rho (S_{i+1} - S_i) I_\omega \, d\Omega \, dt \, A_{i+1/2} \\
+ \int_{\Omega} J_\omega \rho A_{i+1/2} (S_{i+1} - S_i) \, d\Omega \, dt
\]

(2.9)

The limits on the \( \Omega \) integration are over the solid angle \( \beta \); all variables are constant with respect to this integration. The integration converts intensity to flux (Milne, Ref. 2, p. 85). Since \( \beta \) is small and \( S_i \rightarrow S_{i+1} \), Eq. (2.9) becomes,

\[
I_\omega A_i \, dt \int_0^\beta d\Omega - (I_\omega + dI_\omega) A_{i+1} \, dt \int_0^\beta d\Omega = \\
- K_\omega \rho \, dS \omega \, dt \, A_{i+1/2} \int_0^\beta d\Omega \\
+ J_\omega \rho A_{i+1/2} \, dS \, dt \int_0^\beta d\Omega
\]

(2.10)
Let

\[ A_{i+1/2} = A_i + \frac{dA}{2} = A_{i+1} - \frac{dA}{2} \]  

(2.11)

\[ I_\omega \left( A_{i+1/2} - \frac{dA}{2} \right) dt \beta - \left( I_\omega + dI_\omega \right) \left( A_{i+1/2} + \frac{dA}{2} \right) dt \beta \]

\[ = - K_\omega \rho \ dS \ I_\omega \ dt \ A_{i+1/2} \ \beta \]

\[ + J_\omega \rho A_{i+1/2} \ dS \ dt \ \beta \]  

(2.12)

Dividing by \( (A_{i+1/2}) dt \beta \ dS \) and neglecting products of differentials.

\[ - \frac{dI_\omega}{dS} = - K_\omega \rho I_\omega + \rho J_\omega \]  

(2.13)

Assume the flow to be in local thermodynamic equilibrium, such that Kirchoff's theory can be used to give

\[ J_\omega = K_\omega I_\omega \]  

(2.14)

where \( I_{\omega b} \) is Planck's blackbody intensity

\[ I_{\omega b} = \frac{2 C^2 \hbar \omega^3}{\exp \left( \frac{h \omega}{kT} \right) - 1} \]  

(2.15)

Similar intensities based on other measures of spectral interval may also be defined. For convenience, several of these are tabulated in Table 1. Let

\[ dx = \rho \ dS \]  

(2.16)
Table 1

RADIATION RELATIONSHIPS WITH RESPECT TO FREQUENCY, WAVELENGTH AND WAVE NUMBER

I. INTENSITIES (SIEGEL AND HOWELL, REF. 3, PP. 20 AND 31 – FOR THE INDEX OF REFRACTION EQUAL TO ONE)

\[
I_{\nu b} = \frac{2 \hbar \nu^3}{C^2 \left( \exp \left( \frac{\hbar \nu}{kT} \right) - 1 \right)} = \frac{N_{\nu b}}{\pi}
\]

\[
I_{\lambda b} = \frac{2 \hbar C^2}{\lambda^5 \left( \exp \left( \frac{\hbar C}{k \lambda T} \right) - 1 \right)} = \frac{N_{\lambda b}}{\pi}
\]

\[
I_{\omega b} = \frac{2 \hbar C^2 \omega^3}{\left( \exp \left( \frac{\hbar C \omega}{k T} \right) - 1 \right)} = \frac{N_{\omega b}}{\pi}
\]

where \( \nu \) is frequency in \( \text{time}^{-1} \), \( \lambda \) is wavelength in \( \text{length} \), and \( \omega \) is wave number in \( \text{length}^{-1} \). \( C \) is the speed of light in a vacuum, \( \hbar \) is Planck's constant, and \( k \) is Boltzmann's constant.
II. ILLUSTRATION OF $\nu$, $\lambda$ AND $\omega$ RELATIONSHIPS (SIEGEL AND HOWELL, REF. 3, p. 20)

<table>
<thead>
<tr>
<th>For Carbon Dioxide ($CO_2$)</th>
<th>For Water ($H_2O$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda \nu = C$ (Speed of light in a vacuum)</td>
<td>$\lambda \nu = C$ (Speed of light in a vacuum)</td>
</tr>
<tr>
<td>for $\lambda_o = 4.45 \mu$ for $CO_2$</td>
<td>for $\lambda_o = 2.49 \mu$ for $H_2O$</td>
</tr>
<tr>
<td>$\nu_o = \frac{C}{\lambda_o} = 6.736 \times 10^{13} \text{ sec}^{-1}$</td>
<td>$\nu_o = \frac{C}{\lambda_o} = 1.2039 \times 10^{14} \text{ sec}^{-1}$</td>
</tr>
<tr>
<td>$\lambda_o \omega_o = 1$</td>
<td>$\lambda_o \omega_o = 1$</td>
</tr>
<tr>
<td>$\omega_o = \frac{1}{\lambda_o} = 2247 \text{ cm}^{-1}$</td>
<td>$\omega_o = \frac{1}{\lambda_o} = 4016 \text{ cm}^{-1}$</td>
</tr>
<tr>
<td>for $d\omega = +25 \text{ cm}^{-1}$</td>
<td>for $d\omega = +25 \text{ cm}^{-1}$</td>
</tr>
<tr>
<td>$d\lambda = -\frac{1}{\eta_o} d\omega = -1.98 \times 10^{-7} d\omega$</td>
<td>$d\lambda = -\frac{1}{\omega_o} d\omega = -6.20 \times 10^{-8} d\omega$</td>
</tr>
<tr>
<td>$d\lambda = -0.465 \times 10^{-5} \mu$</td>
<td>$d\lambda = -1.550 \times 10^{-6} \mu$</td>
</tr>
<tr>
<td>$d\nu = -\frac{\nu_o^2}{C} d\lambda = -1.514 \times 10^{-16} d\lambda$</td>
<td>$d\nu = -\frac{\nu_o^2}{C} d\lambda = -4.834 \times 10^{17} d\lambda$</td>
</tr>
<tr>
<td>$d\nu = +0.705 \times 10^{11} \text{ sec}^{-1}$</td>
<td>$d\nu = +0.748 \times 10^{12} \text{ sec}^{-1}$</td>
</tr>
</tbody>
</table>
Then the energy balance becomes

\[
- \left( \frac{d I_\omega}{K_\omega dx} \right) + I_\omega = I_{\omega b} \tag{2.17}
\]

Equation (2.17) is called the equation of transfer and is of fundamental importance in radiative transfer. The derivation given is consistent with Milne (Ref. 2). Other discussions of this equation are given by Viskanta (Ref. 4) and Kourganoff (Ref. 5).

The detector in the zone radiometer system does not indicate \( I_\omega \), but rather the product \( (\beta I_\omega) \). Since \( \beta \) is a constant, the signal is proportional to \( I_\omega \). \( \beta \) is a definite number, the view angle of the radiometer. The important point is that Eq. (2.17) is valid for any constant value of \( \beta \). More will be said of these choices in subsequent pages.

To solve the equation of transfer, an integrating factor is introduced so that the two terms on the LHS of Eq. (2.17) may be combined.

\[
\frac{d}{K_\omega dx} \left[ - I_\omega \exp \left( - \int_o^x K_\omega dx' \right) \right] = I_{\omega b} \exp \int_o^x - K_\omega dx' \tag{2.18}
\]

Primes denote dummy variables.

\[
I_\omega \exp \left( \int_{x_1}^{x_2} - K_\omega dx' \right) \bigg|_o^x = \int_o^x - I_{\omega b} \exp \left( \int_o^{x'} - K_\omega dx'' \right) K_\omega dx' \tag{2.19}
\]

\[
I_\omega \{0\} = I_\omega \{x\} \exp \left( \int_o^x - K_\omega dx' \right) + \int_o^x I_{\omega b} \exp \left( \int_o^{x'} - K_\omega dx'' \right) K_\omega dx' \tag{2.20}
\]
This equation affirms that the intensity which arrives at 0 comes from x and is attenuated between x and 0 or from emission plus self-absorption along x to 0, thus the two terms on the RHS of Eq. (2.20).

Note the first term on the RHS of Eq. (2.20) is often omitted with the understanding that x becomes so large that the path becomes "optically thick." This point is discussed in Goody (Ref. 6) p. 456. When this omission is used, the term is recovered by using a boundary condition on the RHS, when the integration is performed, that equals the omitted term. Such a ploy will be used here.

$I_\omega$ is a monochromatic radiation intensity. Experimentally, a specific wave number cannot be isolated for study, so a wave number interval is used. An appropriately averaged intensity, called radiance, is obtained by

$$ \overline{I}_\omega = \frac{1}{\Delta \omega} \int_{\omega_1}^{\omega_2} \int_{0}^{x} I_\omega \omega \left[ \exp \left( \int_{0}^{x} K_\omega \, dx' \right) \right] K_\omega \, dx' \, d\omega \quad (2.21) $$

Recall

$$ \tau_\omega = \exp \left( \int_{0}^{x} K_\omega \, dx' \right) \quad (2.22) $$

Therefore

$$ \frac{d \tau_\omega}{dx} = \left[ \exp \left( \int_{0}^{x} K_\omega \, dx' \right) \right] (-K_\omega) \quad (2.23) $$

$$ \overline{I}_\omega = \frac{1}{\Delta \omega} \int_{\omega_1}^{\omega_2} \int_{0}^{x} -I_\omega b \left( \frac{d \tau_\omega}{dx} \right) \, dx' \, d\omega \quad (2.24) $$

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To exchange the order of integration, define

\[
\bar{T} = \frac{1}{\Delta \omega} \int_{\omega_1}^{\omega_2} \tau_\omega \, d\omega \tag{2.25}
\]

and

\[
\bar{I}_{\omega b} = \frac{1}{\Delta \omega} \int_{\omega_1}^{\omega_2} I_{\omega b} \, d\omega \tag{2.26}
\]

Then

\[
\bar{I}_\omega = - \int_{0}^{\bar{T}} \bar{I}_{\omega b} \, d\bar{T} \tag{2.27}
\]

To be consistent \(\bar{T}\) should have a subscript \(\omega\), but this is not convenient.

Golden (Ref. 7) strongly contested the possibility of this inversion of order. Simmons (Ref. 8) presented several other ways of accomplishing the averaging and an alternate derivation.

Equation (2.27) may be approximated with finite-differences as:

\[
\bar{I}_\omega = \sum_{i=1}^{N} \bar{I}_{\omega b} \{ \bar{T}_i \} \left[ \bar{T}_{i-1} - \bar{T}_i \right] \tag{2.28}
\]

or

\[
\bar{I}_\omega = \bar{I}_{\omega b1} (1 - \bar{T}_1) + \bar{I}_{\omega b2} (\bar{T}_1 - \bar{T}_2) + \cdots \bar{I}_{\omega bN} (\bar{T}_{N-1} - \bar{T}_N) \tag{2.29}
\]

since

\[
\bar{\sigma}_0 = 0, \quad \bar{T}_0 = 1.
\]
It is crucial to the understanding of non-grey, radiation analysis to appreciate that the quantities in Eqs. (2.27), (2.28) and (2.29) are spectrally averaged; however, from this point on the overbars will not be shown because only averaged quantities are of interest.

2.2 THE EVALUATION OF $\tau$ AVERAGE

Equation (2.28) represents the intensity which is proportional to that detected in the zone radiometry experiments. The summation can be directly evaluated, but even though the blackbody intensities are well behaved the transmittances are not. This is the reason that Krakow et al. (Ref. 9) developed the Curtis-Godson approximation to properly average the $\tau$'s. The development of this approximation is given below.

Two problems must be simultaneously solved. First, an average over a certain wave number interval must be established, because radiative transitions occur in a discontinuous manner even with respect to the narrow acceptance interval of the detecting device. For isothermal, homogenous gases, this wave number averaging has been accomplished with "band models." The Random Band model with constant line widths was chosen for this purpose. It is stated:

$$- \ln \tau = 2\pi \left( \frac{X}{d} \right) f \{X\}$$  \hspace{1cm} (2.30)

where $\{ \}$ denotes functionality and

$$X = \frac{\left( \frac{s}{d} \right) S}{2\pi \left( \frac{\gamma}{d} \right)}$$  \hspace{1cm} (2.31)

$\left( \frac{s}{d} \right)$ and $\left( \frac{\gamma}{d} \right)$ are band model parameters and $f$ is the probability distribution for line strengths. $S$ is still distance. Specific values of the band model parameters will be subsequently quoted.
The second problem which must be overcome is to devise a way that the band model representation of the transmittance may be used for a non-homogeneous, nonisothermal path. The Curtis-Godson approximation may be used to calculate an average for such paths. Let

\[
\left(\frac{s}{d}\right)_h S = \sum_h \left(\frac{s}{d}\right)_h S_h
\]  

(2.32)

and

\[
\left(\frac{s}{d}\right)_h S \left(\frac{\gamma}{d}\right)_h = \sum_h \left(\frac{s}{d}\right)_h S_h \left(\frac{\gamma}{d}\right)_h
\]  

(2.33)

where \( h \) represents a zonal increment of constant temperature and composition. By combining Eqs. (2.31) and (2.33)

\[
X = \left[ \sum_h \left(\frac{s}{d}\right)_h S_h \right]^{2/2\pi} \sum_h \left(\frac{s}{d}\right)_h S_h \left(\frac{\gamma}{d}\right)_h
\]

(2.34)

Within each zone

\[
X^*_h = \left(\frac{s}{d}\right)_h S_h / 2\pi \left(\frac{\gamma}{d}\right)_h
\]

(2.35)

\[
\ln \tau^*_h = 2\pi \left(\frac{\gamma}{d}\right)_h \{X^*_h\}
\]

(2.36)

where stars emphasize zonal properties.
Therefore, Eq. (2.30) becomes

\[
-ln \tau = \frac{\sum_h x_h^* \left(-\ln \tau_h^* \right) \left\{ x_h^* \right\}^2}{\sum_h x_h^* \left(-\ln \tau_h^* \right) \left\{ x_h^* \right\}^2} \tag{2.37}
\]

Equation (2.37) was derived and substantiated by experiments in Krakow et al., (Ref. 9). Two limits for this expression exist.

If \( X \rightarrow 0 \), i.e., is less than 0.1,

\[
-ln \tau \approx \sum_h \left(-\ln \tau_h^* \right) \tag{2.38}
\]

If \( X \rightarrow \infty \), i.e., is greater than 3,

\[
-ln \tau \approx \left[ \sum_h \left(-\ln \tau_h^* \right)^2 \right]^{1/2} \tag{2.39}
\]

Now the equations developed in the two previous paragraphs may be used, if appropriate band model data are available. The General Dynamics experiments (Refs. 10, 11 and 12) provide such data. An exponential probability distribution was used and values of \((s/d)\) and \((\gamma/d)\) were determined.

\[
f\{X\} = X \left[1 + \pi X/2 \right]^{-1/2} \tag{2.40}
\]
\[-\ln \tau = \frac{(s/d) S}{\left[ 1 + \frac{(s/d)}{4} \frac{S}{(\gamma/d)} \right]^{1/2}} \]  

(2.41)

Furthermore, let $(s/d) = k$ and $(\gamma/d) = a$

\[-\ln \tau = \frac{kS}{\left[ 1 + \frac{kS}{4a} \right]^{1/2}} = \frac{\sum h k_h S_h}{\left[ 1 + \frac{\left( \sum h k_h S_h \right)^2}{4 \sum h k_h S_h a_h} \right]^{1/2}} \]  

(2.42)

This is the same relationship that is used by Reardon and Huffaker (Ref. 13) to calculate radiation from a line of sight. For a single isothermal, constant-composition zone:

\[-\ln \tau^* = \frac{k S^*}{\left[ 1 + \frac{k S^*}{4a} \right]^{1/2}} \]  

(2.43)

\[k = \left( \frac{k_o}{P_o} \right) \left( \frac{273}{T^\circ K} \right) P_i \]  

(2.44)

\[P_o = \text{reference state of 1 atmosphere} \]

$k_o$ is in (cm\(^{-1}\)) and is tabulated for H\(_2\)O, CO and CO\(_2\) in the General Dynamics reports (Refs. 10, 11 and 12). The term $P_i$ represents the partial pressure of the radiating species in atmospheres.
Unless the pressure is much lower than one atmosphere, Doppler broadening is negligible with respect to collision broadening. Assuming such a case, "a" can be calculated from the tabulated data in Ref. 12, pp. 22-23 or from Reardon and Huffaker (Ref. 13) pp. 141-144.

For continuous radiators, grey gases (throughout the spectral range of interest), "a" → ∞ and

\[-\ln \tau^* = k S^*\]  

(2.45)

This correctly implies that the optically thin limit Eq. (2.38), can be used to calculate integrated values of τ. Thus non-grey gases, which are optically thin because of geometry and density distributions in addition to grey gases obey Eq. (2.38).

In general, "a" is the fine structure parameter which is the ratio of the line width, γ, to line spacing, d. Width of a radiating line is broadened by collisions between the atoms and molecules of the gas. The general form of the line width with collisional broadening terms included according to Reardon and Huffaker (Ref. 13) and Reardon et al. (Ref. 14) is

\[
\gamma_{c_i} = \sum_j \left( \gamma_{i,j}\right)_{at} P_j \left( \frac{273}{T} \right)^{n_{i,j}} + \left( \gamma_{i,i}\right)_{at} P_i \left( \frac{273}{T} \right)^{n_{i,i}}
\]

(2.46)

The term \(i\) is the species being considered, \(P_j\) are the species partial pressures in atmospheres. The exponent values of \(n_{i,j} = 1/2\) and \(n_{i,i} = 1\) were recommended by General Dynamics/Convair (Ref. 12). The summation, \(j\), runs through the number of species in the gas. A representative set of the constants needed to calculate the line width with collisional broadening for, in this example, the water molecule is presented in Table 2.
Table 2
LINE WIDTH WITH COLLISIONAL BROADENING FOR WATER

\[
\gamma_{c_i} = \sum_j^{N} (\gamma_{i,j})_{at} P_j \left( \frac{273}{T} \right)^{n_{i,j}} + (\gamma_{i,i})_{at} P_i \left( \frac{273}{T} \right)^{n_{i,i}}
\]

\[
T = 273^\circ K
\]

\[
n_{i,j} = 1/2 \quad \text{and} \quad n_{i,i} = 1
\]

for \( i = H_2O \)

for \( j \)

\( \gamma_{ij} \) becomes

\( \gamma_{ij} \) (nonresonating)

\( j = H_2O \)

\( \gamma_{ij} = 0.09 \)

\( = N_2 \)

\( = 0.09 \)

\( = O_2 \)

\( = 0.04 \)

\( = H_2 \)

\( = 0.05 \)

\( = CO_2 \)

\( = 0.12 \)

\( = CO \)

\( = 0.10 \)

\( \gamma_{i,i} \) for water (resonating)

\( = 0.44 \)
The evaluation of the $1/d$ value to complete the calculation of $a$, is defined by Reardon and Huffaker (Ref. 13) as

\[
\frac{1}{d} = \frac{a^0}{\gamma^0}
\]  

(2.47)

where

\[
a^0 = 10^{(b_\nu + c_\nu T^2)}
\]  

(2.48)

The $b_\nu$ and $c_\nu$ are constant over spectral regions and for water are listed as a function of temperature in Ref. 13.

While

\[
\gamma^0 = \left[ 0.44 \left( \frac{273}{T} \right) + 0.09 \left( \frac{273}{T} \right)^{1/2} \right] C^0 + 0.044 \left( \frac{273}{T} \right)^{1/2} (1 - C^0)
\]  

(2.49)

where

\[
C^0 = -0.1002 + 0.2802 \times 10^{-3} T - 0.1089 \times 10^{-6} T^2 + 0.0291 \times 10^{-9} T^3
\]  

(2.50)

Values of $1/d$ shall be tabulated as a function of $\lambda$ and $T$ in Ref. 14. These tabulated values of $1/d$ provide an alternate method of obtaining the fine structure parameter, $a$.

If more than one species is optically active in a given spectral interval, Eq. (2.42) is modified and used thusly:
\[-\ln(\tau_{\text{MS}}) = \sum_i \left( \frac{\sum_h k_h S_h}{1 + \frac{\sum_h k_h S_h^2}{4 \sum_h k_h S_h a_h}} \right)^{1/2} \]  \hspace{1cm} (2.51)

where the \( i \) summation is on all active species. Remember \([-\ln(\tau_{\text{MS}})]\) is called optical depth and the MS indicates multi-species. The type of summation indicated in Eq. (2.51) is not obvious, but is what is used.

The origin of the relationships necessary to determine temperature and concentrations from zone radiometry experiments has now been established. These relationships will now be applied to the specific experiments which have been performed by Rocketdyne.
Section 3
DATA REDUCTION PROCEDURE

The solution to the equation of transfer — Eq. (2.29), the Curtis-Godson approximation as stated in Eq. (2.37), and specified band model parameters may now be used to reduce zone radiometry data. Brewer (Ref. 16) describes a computer program to perform such a calculation; unfortunately, he uses a distribution function, \( f \), which is not compatible with the reported General Dynamics/Convair k's and a's. This may not introduce a significant error, but it prevents one from using the reported program directly.

Rocketdyne chose not to reduce their data in this manner. They approximated Eq. (2.37) with its grey gas limit Eq. (2.38) and then reduced the radiometry data, arguing that since \( a \rightarrow \infty \), \( X \rightarrow 0 \) for CO\(_2\) and that experiments with water first were optically thin and second that they used water vapor radiation data which were taken with the same resolution as their spectrometer (Ref. 15). The first two of these arguments may well be valid, and their validity can be determined with analysis. The third is highly improbable because not only isothermal property data must be available (which may be) but data for the same temperature and compositions as those in the measured plumes must be also. However, if the first two arguments are valid, the third is unnecessary. Herget (Ref. 15) contends because of these arguments that his studies are not limited to optically thin cases. Let us reserve judgment on this contention until some subsequent calculations and experimental observations are made.

Specifically, Eq. (2.38) may be written as

\[
\tau_k = (\tau_1^*) (\tau_2^*) \ldots (\tau_h^*)
\]  

where the subscript \( k \) represents the \( k^{th} \) row of zones and \( h \) represents the number of zones. Eq. (2.29) may be multiplied by \( \tau \), so that \( N_{\omega bi} \)'s appear on
the RHS where \( N_{\omega b} \) is emissive power. In fact, a summation of any of the terms listed in Table 1 could have been used. Actually neither intensity nor emissive power is measured but the radiation from the solid angle \( \beta \) which intersects the detector surface. This angle is not measured, but the radiation from an internal blackbody source along \( \beta \) is. Since any of the terms in Table 1 may be calculated from the internal blackbody temperature, the LHS of Eq. (2.29) is simply calibrated. Eq. (2.29) is used as stated below.

\[
I_\lambda = I_{\lambda b_1} (1 - \tau_1^*) + I_{\lambda b_2} (\tau_1^* - \tau_1^* \tau_2^*) + \ldots
\]

\[
= I_{\lambda b_1} (\epsilon_1^*) + I_{\lambda b_2} (\tau_1^* [1 - \tau_2^*]) + \ldots
\]

\[
= I_{\lambda b_1} (\epsilon_1^*) + I_{\lambda b_2} (\tau_1^* \tau_2^*) + I_{\lambda b_3} \frac{(\tau_1^* \tau_2^* - \tau_1^* \tau_2^* \tau_3^*)}{\tau_2^* \epsilon_3^*}
\]

\[
= \sum_i I_{\lambda b_i} \epsilon_i^* \tau_{i-1}^*
\]  

\( \epsilon^* \) is defined as \( 1 - \tau^* \), and \( \tau^* \) is defined by Eq. (2.43). Since \( I_\lambda \) is an averaged intensity over some small spectral interval and is measurable, it will be called radiance.

Before considering the Rocketdyne experiments in more detail, consider the following definition of a new \( \tau^* \), namely,

\[
\ln (\tau_h^*) = \frac{k_h S_h}{\left[ \left( \sum_{n=1}^{h} k_n S_n \right)^2 \right]^{1/2} + \left( \sum_{n=1}^{h} k_n S_n a_n \right)}
\]  

(3.3)
This $\tau_h^{**}$ has the property that if it is used in Eqs. (3.1) and (3.2), Eq. (2.42) will result. This means that the assumption on optical depth which was used by Rocketdyne will be removed. Other means could be used to eliminate this assumption, but, as will be confirmed in subsequent discussion, this means would require the least amount of revision to the existing Rocketdyne data reduction program. An additional benefit is that a convenient check of the deviation from optical thinness can also be made with this parameter.

In summary, the following equation must be solved, either exactly or approximately, to reduce zone radiometry data for one component and wavelength. Functionality is emphasized for clarity.

$$I_{\lambda,\Delta\omega} = \sum_i I_{\lambda\theta_i} \{\lambda, T_i\} \left[ \tau_{i-1} \{\lambda, \Delta\omega, T_{i-1}\} - \tau_i \{\lambda, \Delta\omega, T_i\} \right]$$ (3.4)

Now the specific geometry of the zone radiometry experiments can be considered.

3.1 ONE-DIMENSIONAL ISOTHERMAL FLOWS

The one-dimensional test situation (Fig. 3) such as the Rocketdyne Composite Engine Study (Ref. 17) will be used to demonstrate the procedure used to convert measured values of emissive power into temperature and composition values. Recalling Fig. 1, the basic geometry of this one-dimensional flow contains all of the features shown except that there is a single zone. Two radiance readings are made using the zone radiometer. One radiance reading is made with the chopper closed giving the radiance of only the zone while the other reading made with the chopper open provides a radiance value containing the zone radiation and the grey body source radiation. Using the finite difference form of Eq. (3.2) with these measured...
Fig. 3 - Rocketdyne Composite Engine Study (Ref. 17)
radiances, $I_{\lambda A}$ (with the chopper closed) and $I_{\lambda B}$ (with the chopper open), the transmittance and blackbody radiance of the zone can be calculated from:

$$I_{\lambda A} = I_{\lambda b} (1 - \tau) \quad (3.5)$$

$$I_{\lambda B} = I_{\lambda b} (1 - \tau) + I_{\lambda b} \text{source} \tau \quad (3.6)$$

Note that the temperature of the radiating gas is assumed not to be change when it is impinged upon by the radiating source.

The zone radiometry measurement data presented by Rocketdyne are in terms of radiance with units of $W \, cm^{-2} \, sr^{-1} \, \mu m^{-1}$. The form of Planck's law applicable for relating the blackbody radiance at a particular wavelength to the temperature was used to evaluate the temperature of the zone (see Table 1).

$$I_{\lambda b} = \frac{2C^2}{\lambda^5 \left( \exp \left( \frac{hC}{k\lambda T} \right) - 1 \right)} \quad (3.7)$$

The evaluation of the partial pressure of the radiating species makes use of the representation of transmittance. When the species can be treated as a continuum radiator the transmittance can be calculated using Eq. (2.45)

$$- \ln \tau^* = kS$$

where the $S$ is distance and the $k$ value contains the partial pressure of the species as a correction factor to the absorption coefficient at standard conditions, $k_0$. Repeating Eq. (2.44)
The tabulated $k_o$ value corresponding to the calculated zone temperature makes the solution for the partial pressure straightforward.

Whenever band models must be used to represent the radiation process, the transmittance is given by Eq. (2.42). The solution for the partial pressure is no longer a simple process. The line width parameter, $a$, is dependent on temperature, the local pressure and partial pressure of all the constituent species as described previously. An iterative method is used in which an estimate of the partial pressure of the radiating species is made using the simpler continuum radiation transmittance (Eq. (2.45)). With this estimated pressure and the calculated temperature, a corresponding "$a$" value is evaluated using Eqs. (2.46-2.50). The first iteration on the partial pressure can then be made using Eq. (2.43). The iteration procedure is continued until the desired degree of agreement is obtained between succeeding pressure values.

Summarizing the procedure for evaluating the temperature and partial pressure from zone radiometry measurements of a one-dimensional flow:

1. Obtain the zone radiance and transmittance using Eqs. (3.5) and (3.6).

2. Solve for the zone temperature using Eq. (3.7).

3. Use Eq. (2.45) for continuum radiation to calculate the partial pressure to complete the solution, or

4. Use an iterative procedure to calculate the partial pressures for non-continuum radiation requiring a band model representation of transmittances as follows:

   a. Estimate a partial pressure value using the continuum radiation representation for the transmittance, Eq. (2.45).
b. Evaluate the fine structure parameter, \( a \), with Eqs. (2.46) through (2.50) using the temperature and the estimated pressure value.

c. Calculate the partial pressure using Eq. (2.43).

d. Compare the newly calculated partial pressure with that used in Step 4b.
   - "Poor" agreement: Repeat from Step 4b using new partial pressure.
   - "Good" agreement: Consider solution complete.

3.2 AXISYMMETRIC NONISOTHERMAL FLOWS

Application of zone radiometry measurement techniques to axisymmetric flows uses the same principles as for the one-dimensional situation but the reduction of the measured intensities to temperature and partial pressure becomes more complex.

A schematic of the axisymmetric zone layout is given in Fig. 4.

Fig. 4 - Definition of Zones and Lines of Sight in Axisymmetric Zone Radiometry
The zones consist of concentric circular regions in which the physical properties are assumed uniform. The line of sight (LOS) is defined such that the $n$th LOS passes through the $n$th zone and all zones outside of it. The zones are not necessarily of the same size. (In the Rocketdyne zone radiometry data reduction program (see Appendix and Ref. 18) the zones are nevertheless assumed to all be of the same size.) Whether or not the same size zones are used the path lengths within the zones are variable and dependent upon the location of the zone within the axisymmetric array. The path length is calculated using geometric relationships. For example the path length, $l$, in the fourth zone on the third line of sight is (see Fig. 5) calculated as

$$l = \sqrt{R_3^2 - R_1^2} - \sqrt{R_2^2 - R_1^2}$$

(3.8)

---

Fig. 5 - Path Length Definition in an Axisymmetric Zone Radiometry Array

---

Zone Number

---

Line-of-Sight Number

---

$*$
From Fig. 4 it can be seen that the signal received by the detectors along a respective LOS has in general passed through an inhomogeneous region. The radiances measured for the LOS can be used to calculate the temperatures and partial pressure within the zones using the following procedure.

Again the two radiance readings (one with the chopper and one without it) are made along each line of sight. The zones are maintained at sufficiently small sizes that the line of sight through the concentric zones can be approximated by one-dimensional slabs as in Fig. 6.

* Zone Number.

Fig. 6 - One-Dimensional Approximation of the Lines of Sight for Axisymmetric Zone Radiometry

The radiances values measured for these lines of sight can be mathematically represented as they were in Eqs. (3.5) and (3.6) for the single one-dimensional case.
\[ I_{\lambda A_j} = \sum_{i=1}^{n} I_{\lambda b_i, j} \epsilon_{i, j} \tau_{i-1, j} \quad (3.9) \]

\[ I_{\lambda B_j} = \sum_{i=1}^{n} I_{\lambda b_i, j} \epsilon_{i, j} \tau_{i-1, j} + I_{\lambda b \text{ Source}} \tau_j \quad (3.10) \]

where \( j \) is the line of sight under study and the \( i \) is summed over all the zones to \( n \). Subtracting Eq. (3.9) from Eq. (3.10) provides \( n \) relationships for \( \tau_j \), which is the mean transmittance of the entire \( j \text{th} \) line of sight. The other \( n \) equations needed to solve for the \( 2n \) unknowns, \( T_i \) and \( P_i \), come from Eq. (3.9). The representation of the transmittance now becomes the prime question. Rocketdyne uses the grey gas limit for the transmittance, Eq. (3.1). The \( n \) values of \( \tau_j \) can then be expanded as

\[ \tau_j = \prod_{i=1}^{n} \tau_{i, j} = (\tau_{1, j}) (\tau_{2, j}) \ldots (\tau_{n, j}) \quad (3.11) \]

The \( 2n \) equations consisting of Eqs. (3.9) and (3.11) are solved using matrix algebra. Since Rocketdyne has automated the solution procedure in a data reduction program, an iterative process is used to evaluate the unknowns.

Summarizing the procedure for evaluating the temperatures and partial pressures in the \( n \) zones of an axisymmetric flow using the zone radiometry measurements is:

1. Construct equations for the measured line of sight radiance using Eq. (3.9).
2. Construct equations for the mean transmittance through an entire line of sight using Eq. (3.11).
3. Place the transmittance represented by Eq. (3.1) in matrix form. (Using the measured mean line-of-sight transmittance values, \( \tau_j \), the matrix can be solved for the zone transmittances \( \tau_{i, j}^* \).)
4. Replace the transmittances in the equations constructed in Step 1 with the calculated zone transmittances. (The matrix representing the mean measured radiance is then ready for solution for the zonal blackbody radiance functions.)

5. Solve for the temperatures in the zones using Eq. (3.7). (For continuum radiators this completes the solution procedure since the zone partial pressures can be evaluated using the calculated \( k \)'s, tables of \( k \) versus temperature and Eq. (2.44).)

6. Use an iteration procedure (for noncontinuum radiation, requiring a band model representation of the transmittance) to solve for the temperatures and partial pressures in the zones as follows:
   a. Use Eqs. (2.46) and (2.50) to evaluate the fine structure parameter, \( a \), using the zone temperatures and the zone partial pressures from Step 5.
   b. Reevaluate the zone transmittance values using Eq. (2.43). The modification to the Rocketdyne program to define the transmittance of the zone with Eq. (3.3) would eliminate the grey gas assumption inherent in Eq. (2.43) and make the solution valid for all optical thicknesses.
   c. Return to Step 4 and repeat Steps 4 and 5.
   d. Compare the newly calculated partial pressures and temperatures of the zones with those obtained previously in Step 5.
      • "Poor" agreement: Repeat from Step 6 using new partial pressures and temperatures for the zones.
      • "Good" agreement: Consider solution complete.

3.3 ROCKETDYNE ZONE RADIOMETER DATA REDUCTION PROGRAM

The Rocketdyne automated data reduction program is listed in the Appendix. An input guide and flow chart of the program are also presented. To aid potential users of the data reduction program, a sample case is given along with sample input and output.
Section 4
EXAMPLE PROBLEMS

To illustrate the calculation techniques discussed in this report, several example problems will be solved. The first is one typical of an axisymmetric alcohol-burning Atlas vernier engine; the second represents a planar, hydrogen-oxygen diffusion flame, i.e., the composite engine experiment.

Problem 1 — LOX-Alcohol Engine

Due to the behavior of gaseous radiation properties, it is desirable to choose example problems in which the temperature and composition are specified. Consider first the following thermal path:

<table>
<thead>
<tr>
<th>Detector</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
<th>2000</th>
<th>1500</th>
<th>3500° source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone:</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Zones 1, 2, 4 and 5 are 2 cm long; zone 3 is 4 cm long.

Partial pressure, CO₂: 0.27 atm
H₂O: 0.58 atm

Wave lengths of measurement: 4.45 (μ) or 2247 (1/cm)
2.49 (μ) or 4016 (1/cm)

Solution Procedure

1. Evaluate \( k = k_0 \left( \frac{273°K}{T°K} \right) \left( \frac{P_i \text{ atm}}{1 \text{ atm}} \right) \)
Species: H$_2$O

<table>
<thead>
<tr>
<th>T (°K)</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_0$ (l/cm)</td>
<td>1.22 x 10$^{-2}$</td>
<td>2.33 x 10$^{-2}$</td>
<td>3.05 x 10$^{-2}$</td>
</tr>
<tr>
<td></td>
<td>1.47 x 10$^{-1}$</td>
<td>1.43 x 10$^{-1}$</td>
<td>1.50 x 10$^{-1}$</td>
</tr>
</tbody>
</table>

Species: CO$_2$

<table>
<thead>
<tr>
<th>T (°K)</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_0$ (l/cm)</td>
<td>10.99</td>
<td>13.30</td>
<td>13.55</td>
</tr>
</tbody>
</table>

(Not measured but assumed zero)

- **Evaluate fine structure parameters.**

a. line density, (1/d)

(i) CO$_2$, (1/d) = (1/DLR) (cm) at $\omega = 2247$ (1/cm)

<table>
<thead>
<tr>
<th>(1/DLR)</th>
<th>T (°K)</th>
<th>Ref. 11, p. 59</th>
</tr>
</thead>
<tbody>
<tr>
<td>181.1</td>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>356</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>510</td>
<td>2500</td>
<td></td>
</tr>
</tbody>
</table>
\( H_2O, \frac{1}{d} = \frac{a^*}{\gamma^*} \)  

Ref. 13, p. 194

Evaluation of \( a^* \) is a rather nebulous operation, but Rocketdyne's programmed values will be presented. Reference 14 is supposed to have tabulated values of \( \frac{1}{d} \) when it is published.

\[
a^* = 10^{(b_i + C_i T^2)}
\]

\begin{tabular}{|c|c|c|c|}
\hline
\( b_i \) & \( c_i \) & \( i \) & Ref. 12 \\
\hline
-1.366 & 0.165 \times 10^{-6} & 2247 (1/cm) & p. 134 \\
-1.62 & 0.180 \times 10^{-6} & 4016 (1/cm) & \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\( T(°K) \) & \( C_i T^2 \) & \( b_i + C_i T^2 \) & \( i \) & \( \frac{1}{a^*} \) & \( a^* \) \\
\hline
1500 & 0.370 & -0.996 & 2247 (1/cm) & 9.9 & 0.101 \\
2000 & 0.660 & -0.706 & 5.09 & 0.196 \\
2500 & 1.030 & -0.336 & 2.166 & 0.461 \\
\hline
1500 & 0.405 & -1.215 & 4016 (1/cm) & 16.4 & 0.061 \\
2000 & 0.720 & -0.900 & 7.92 & 0.126 \\
2500 & 1.125 & -0.495 & 3.13 & 0.319 \\
\hline
\end{tabular}

To complete the \( \frac{1}{d} \) calculation for \( H_2O \), define \( \gamma^* = \left[ 0.44 \left( \frac{T_0}{T} \right) + 0.09 \left( \frac{T_0}{T} \right)^{1/2} \right] C^* + 0.044 \left( \frac{T_0}{T} \right)^{1/2} (1 - C^*) \)  

Ref. 13, p. 194

where

\[
C^* = -0.1002 + 0.2802 \times 10^{-3} T - 0.1089 \times 10^{-6} T^2 \\
+ 0.0291 \times 10^{-9} T^3.
\]
Evaluate $C^*$ and $\gamma^*$ for the temperatures in the zones.

For $T = 1500^\circ K$

\[
C^* = \begin{bmatrix}
-0.1002 \\
0.245 \\
-0.345
\end{bmatrix}

\begin{bmatrix}
+0.423 \\
+0.098 \\
+0.521
\end{bmatrix}
= +0.176
\]

$T = 2000^\circ K$

\[
C^* = \begin{bmatrix}
-0.100 \\
-0.436 \\
-0.536
\end{bmatrix}

\begin{bmatrix}
+0.560 \\
+0.233 \\
+0.793
\end{bmatrix}
= +0.257
\]

$T = 2500^\circ K$

\[
C^* = \begin{bmatrix}
-0.100 \\
-0.682 \\
-0.782
\end{bmatrix}

\begin{bmatrix}
+0.701 \\
+0.456 \\
+1.157
\end{bmatrix}
= +0.377
\]

$\gamma^* = \left[0.44 \left(\frac{T_0}{T}\right) + 0.09 \left(\frac{T_0}{T}\right)^{1/2}\right] C^* + 0.044 \left(\frac{T_0}{T}\right)^{1/2} (1 - C^*)$

Use the $C^*$ values to calculate the $\gamma^*$ values.

$\gamma_{1500}^* = \left[0.080 + 0.0383\right] \left(0.176\right) + \left(0.0187\right) \left(0.824\right)$

\[= (0.1183)(0.176) + 0.0154 = 0.0209 + 0.0154 = 0.0363\]

$\gamma_{2000}^* = \left[0.0593 + 0.033\right] \left(0.257\right) + \left(0.0161\right) \left(0.743\right)$

\[= 0.0237 + 0.0120 = 0.0357\]

$\gamma_{2500}^* = \left[0.0481 + 0.0297\right] \left[0.377\right] + \left(0.0145\right) \left(0.623\right)$

\[= 0.0292 + 0.0091 = 0.0393\]
The $(1/d)$ values for $H_2O$ at the temperatures and in the wavelengths of interest are:

<table>
<thead>
<tr>
<th>$(1/d)$ (cm)</th>
<th>$T , (^0K)$</th>
<th>$i (1/cm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.78</td>
<td>1500</td>
<td>2247</td>
</tr>
<tr>
<td>5.49</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>11.78</td>
<td>2500</td>
<td></td>
</tr>
<tr>
<td>1.68</td>
<td>1500</td>
<td>4016</td>
</tr>
<tr>
<td>3.52</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>8.15</td>
<td>2500</td>
<td></td>
</tr>
</tbody>
</table>

b. Obtain the collision half widths, $\gamma'_c$'s, to complete the calculation of the fine structure parameters, $a$'s.

$$\gamma_{c_i} = \sum_j \gamma_{i,j} P_j \left(\frac{273}{T}\right)^{1/2} + \gamma_{i,i} P_i \left(\frac{273}{T}\right)$$  \hspace{1cm} (2.46)

$j = \text{all species}.$

For $CO_2$ at $1500^0K$,

Ref. 14

$$\gamma_{c_{CO_2}} = \frac{(0.07)(0.58)}{2.35} + \frac{(0.09)(0.27)}{2.35} + \frac{(0.01)(0.27)}{5.50}$$

$$= 0.0173 + 0.0103 + 0.0005 = 0.0281 \, (1/cm)$$

At $2000^0K$,

$$\gamma_{c_{CO_2}} = \frac{(0.07)(0.58)}{2.73} + \frac{(0.09)(0.27)}{2.73} + \frac{(0.01)(0.27)}{7.34}$$

$$= 0.0149 + 0.0089 + 0.0004 = 0.0242$$
At $2500^\circ$K

$$\gamma_{cCO_2} = \frac{(0.07)(0.58) + (0.09)(0.27)}{3.03} + \frac{(0.01)(0.27)}{9.15}$$

$$= \frac{(0.0406 + 0.0243)}{3.03} + 0.0003 = 0.0217$$

For $H_2O$: Ref. 14

$$\gamma_{cH_2O} = \frac{(0.09)(0.58) + (0.12)(0.27)}{(T/273)^{1/2}} + \frac{(0.44)(0.58)}{(T/273)}$$

$$\frac{0.0845}{(T/273)^{1/2}} + \frac{0.255}{(T/273)}$$

At $1500^\circ$K

$$\gamma_{cH_2O} = 0.0360 + 0.0465 = 0.0825$$

At $2000^\circ$K

$$\gamma_{cH_2O} = 0.0310 + 0.0347 = 0.0657$$

At $2500^\circ$K

$$\gamma_{cH_2O} = 0.0279 + 0.0279 = 0.0558$$

The radiation parameters for this sample problem are summarized on the following page.
The intensities measured by the detector are the blackbody radiation of the species within the zones attenuated by the zones between that particular zone and the detector.

The blackbody intensity for the zonal temperatures and the wave numbers of interest are:

<table>
<thead>
<tr>
<th>$I_{\lambda b}$ (Ref. 19)</th>
<th>$T$</th>
<th>$i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8942 (W/cm$^2$ - µ-sr)</td>
<td>1500 (°K)</td>
<td>2247 (1/cm)</td>
</tr>
<tr>
<td>1.691</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>2.580</td>
<td>2500</td>
<td></td>
</tr>
<tr>
<td>4.493</td>
<td>3500</td>
<td></td>
</tr>
<tr>
<td>2.687</td>
<td>1500</td>
<td>4016</td>
</tr>
<tr>
<td>7.271</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>13.56</td>
<td>2500</td>
<td></td>
</tr>
<tr>
<td>29.19</td>
<td>3500</td>
<td></td>
</tr>
</tbody>
</table>

The attenuation (or in the converse sense, the transmittance, $\tau$) of the radiation of the zones to the detector is calculated for each of the optical paths. Starting from the detector, the first transmittance, $\tau_1$, involves only zone one. The calculation of the second transmittance, $\tau_2$, includes the first and second zones. These calculations continue until all the transmittances are determined.
For CO$_2$ radiation:

$$K_1 L_1 = (0.540 \text{ cm}^{-1})(2 \text{ cm}) = 1.080$$

$$-\ln \tau_1 = \frac{1.080}{\left(1 + \frac{1.080}{4(5.09)}\right)^{1/2}} = \frac{1.080}{(1.053)^{1/2}} = 1.050$$

$$\tau_1 = 0.350$$

$$K_2 L_2 = (0.485)(2) = 0.970$$

$$-\ln \tau_2 = \frac{\sum K_h L_h}{\left(1 + \frac{\left(\sum K_h L_h\right)^2}{4 \sum (\gamma_c/d)_h K_h L_h}\right)^{1/2}}$$

$$= \frac{2.05}{\left(1 + \frac{4.20}{4 \left[(5.09)(1.08) + (8.61)(0.97)\right]}\right)^{1/2}}$$

$$= \frac{2.05}{\left(1 + \frac{1.05}{5.50 + 8.35}\right)^{1/2}} = \frac{2.05}{1.04} = 1.94$$

$$\tau_2 = 0.144$$

$$K_3 L_3 = (0.400)(4) = 1.60$$
\[-\ln \tau_3 = \frac{2.05 + 1.60}{\left(1 + \frac{(3.65)^2}{4 \left(13.85 + (11.1)(1.6)\right)}\right)^{1/2}} = \frac{3.65}{1.052} = 3.48\]

\[\tau_3 = 0.0310\]

\[K_4 L_4 = K_2 L_2 = 0.970\]

\[-\ln \tau_4 = \frac{3.65 + 0.97}{\left(1 + \frac{(4.62)^2}{4 \left[31.65 + (8.61)(0.97)\right]}\right)^{1/2}} = \frac{4.62}{1.06} = 4.35\]

\[\tau_4 = 0.0130\]

\[K_5 L_5 = K_1 L_1 = 1.080\]

\[-\ln \tau_5 = \frac{4.62 + 1.08}{\left(1 + \frac{(5.70)^2}{4 \left[40.00 + (5.09)(1.08)\right]}\right)^{1/2}} = \frac{5.70}{1.09} = 5.25\]

\[\tau_5 = 0.0053\]

Use the zonal blackbody radiation and transmittance values to evaluate the measured intensity.

\[I_\lambda = I_{\lambda b1} (1 - \tau_1) + I_{\lambda b2} (\tau_1 - \tau_2) + I_{\lambda b3} (\tau_2 - \tau_3) + I_{\lambda b4} (\tau_3 - \tau_4) + I_{\lambda b5} (\tau_4 - \tau_5)\]
\[ I_\lambda = \begin{pmatrix} 0.894(1 - 0.350) &= (0.894)(0.650) = 0.580 \\ 1.691(0.350 - 0.144) &= (1.691)(0.206) = 0.348 \\ 2.580(0.144 - 0.0310) &= (2.580)(0.113) = 0.292 \\ 1.691(0.0310 - 0.0130) &= (1.691)(0.018) = 0.031 \\ 0.894(0.0130 - 0.0053) &= (0.894)(0.008) = 0.007 \end{pmatrix} \]

\[ I_\lambda = 1.258 \text{ W/cm}^2\mu\text{-sr} \]

If a 3500\textdegree K blackbody source has also been transmitting,

\[ I_{\lambda WS} = 1.258 + (0.0053)(4.493) \]
\[ = 1.272 \text{ W/cm}^2\mu\text{-sr} \]

Neglecting non-grey effects introduces errors of 5 to 10\% in intensity; these become a smaller percentage when they are converted to temperature. Such errors do not become unreasonably increased when using the data reduction programs — as evidenced by Appendix D of Ref. 18.

For \( \text{H}_2\text{O} \) radiation at 2247 (1/cm), the transmittances are

\[ K_1L_1 = 2.58 \times 10^{-3} \]

\[ -\ln \tau_1 = \frac{2.58 \times 10^{-3}}{(1 + \frac{2.58 \times 10^{-3}}{4(0.229)})^{1/2}} = \frac{2.58 \times 10^{-3}}{(1.0028)^{1/2}} = 2.58 \times 10^{-3} \]

\[ \tau_1 \approx 1 \]

\[ K_2L_2 = 3.68 \times 10^{-3} \]

\[ -\ln \tau_2 = \frac{6.26 \times 10^{-3}}{(1 + \frac{(6.26 \times 10^{-3})^2}{4 [0.59 \times 10^{-3} + 1.32 \times 10^{-3}])}}^{1/2} = \frac{6.26 \times 10^{-3}}{1.0026} = 6.26 \times 10^{-3} \]
\[
\tau_2 \approx 1
\]

\[
K_3 L_3 = 7.76 \times 10^{-3}
\]

\[
-\ln \tau_3 = \left( \frac{14.02 \times 10^{-3}}{1 + \frac{(1.40 \times 10^{-2})^2}{4 \left[ 1.91 \times 10^{-3} + 5.11 \times 10^{-3} \right]}} \right)^{1/2} = \frac{1.40 \times 10^{-2}}{1.005} = 1.40 \times 10^{-2}
\]

\[
\tau_3 = 0.9861
\]

\[
K_4 L_4 = K_2 L_2 = 3.68 \times 10^{-3}
\]

\[
-\ln \tau_4 = \left( \frac{17.70 \times 10^{-3}}{1 + \frac{(1.770 \times 10^{-2})^2}{4 \left[ 7.02 \times 10^{-3} + 1.32 \times 10^{-3} \right]}} \right)^{1/2} = \frac{1.77 \times 10^{-2}}{1.005} = 1.77 \times 10^{-2}
\]

\[
\tau_4 = 0.9825
\]

\[
K_5 L_5 = K_1 L_1 = 2.58 \times 10^{-3}
\]

\[
-\ln \tau_5 = \left( \frac{20.28 \times 10^{-3}}{1 + \frac{400 \times 10^{-6}}{4 \left[ 8.34 \times 10^{-3} + 0.59 \times 10^{-3} \right]}} \right)^{1/2} = \frac{2.03 \times 10^{-2}}{1.005} = 2.03 \times 10^{-2}
\]

\[
\tau_5 = 0.9799
\]

\[
I_{\lambda b 1} (1 - \tau_1) = 0
\]

\[
I_{\lambda b 2} (\tau_1 - \tau_2) = 0
\]

\[
I_{\lambda b 3} (\tau_2 - \tau_3) = (1 - 0.9861) = 0.0139
\]
\[ I_{\lambda 4} (T_3 - T_4) = (0.9861 - 0.9825) = 0.0036 \]
\[ I_{\lambda 5} (T_4 - T_5) = (0.9825 - 0.9799) = 0.0026 \]

\[
\begin{align*}
I_\lambda &= 0.892 (0) + 1.691 (0) + 2.580 (0.0139) + 1.691 (0.0036) + 0.8921 (0.0026) = 0 + 0 + 0.0358 + 0.0061 + 0.0023 = 0.0442 \\
&= 0.0442 \text{ W/cm}^2 \mu \text{-sr}
\end{align*}
\]

The intensity when the 3500°K blackbody source is also transmitting is

\[
I_{\lambda WS} = 0.0442 + (0.9799)(4.493) = 4.434 \text{ W/cm}^2 \mu \text{-sr}
\]

In this example, water vapor radiation is very optically thin.

**Total Radiation at 2247 (1/cm)**

Since both CO\(_2\) and H\(_2\)O are optically active at 2247 (1/cm), both contribute to the total radiation flux. To account for multi-species emission, the natural log of the transmittance of each is calculated and then all such logs are summed.

\[
\begin{align*}
- \ln (\tau_1)_{\text{MS}} &= 1.050 + 0.0026 = 1.053 \\
- \ln (\tau_2)_{\text{MS}} &= 1.940 + 0.0062 = 1.946 \\
- \ln (\tau_3)_{\text{MS}} &= 3.48 + 0.014 = 3.49 \\
- \ln (\tau_4)_{\text{MS}} &= 4.35 + 0.018 = 4.37 \\
- \ln (\tau_5)_{\text{MS}} &= 5.25 + 0.020 = 5.27
\end{align*}
\]
<table>
<thead>
<tr>
<th>(τₜ)</th>
<th>(τₜ₋₁ - τₜ)</th>
<th>Iₓₖ(τₜ₋₁ - τₜ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.348</td>
<td>0.652</td>
<td></td>
</tr>
<tr>
<td>0.143</td>
<td>0.205</td>
<td></td>
</tr>
<tr>
<td>0.0305</td>
<td>0.112</td>
<td></td>
</tr>
<tr>
<td>0.0126</td>
<td>0.0179</td>
<td></td>
</tr>
<tr>
<td>0.0051</td>
<td>0.0075</td>
<td></td>
</tr>
</tbody>
</table>

In this case the results are identical to those for CO₂ alone.

Since the radiance from the water vapor is only 4% of that from CO₂, Rocketdyne assumed it negligible (consistent with the exact calculation).

Repeat the transmittance and intensity calculations for H₂O radiation at 4016 (1/cm).

\[ K_1 L_1 = (1.55 \times 10^{-2})(2) = 3.10 \times 10^{-2} \]

\[ a_1 = 0.139 \]

\[ -\ln \tau_1 = \frac{3.10 \times 10^{-2}}{1 + \frac{3.10 \times 10^{-2}}{4(0.139)}}^{1/2} = \frac{3.10 \times 10^{-2}}{1.025} = 3.02 \times 10^{-2} \]

\[ \tau_1 = \exp(-0.031) = 0.9704 \]

\[ K_2 L_2 = 2.24 \times 10^{-2} \]

\[ -\ln \tau_2 = \frac{5.34 \times 10^{-2}}{1 + \frac{28.4 \times 10^{-4}}{4[4.31 \times 10^{-3} + 5.15 \times 10^{-3}]}}^{1/2} = \frac{5.34 \times 10^{-2}}{1.04} = 5.13 \times 10^{-2} \]

\[ \tau_2 = 0.950 \]

\[ K_3 L_3 = 3.80 \times 10^{-2} \]

\[ -\ln \tau_3 = \frac{9.14 \times 10^{-2}}{1 + \frac{83 \times 10^{-4}}{4[9.46 \times 10^{-3} + 1.73 \times 10^{-2}]}}^{1/2} = \frac{9.14 \times 10^{-2}}{1.04} = 8.80 \times 10^{-2} \]
\[ \tau_3 = 0.916 \]

\[ K_4 L_4 = 2.24 \times 10^{-2} \]

\[ -\ln \tau_4 = \left( \frac{11.38 \times 10^{-2}}{1 + \frac{130 \times 10^{-4}}{4(26.76 \times 10^{-3} + 5.15 \times 10^{-3})}} \right)^{1/2} = \frac{1.138 \times 10^{-1}}{1.05} = 1.08 \times 10^{-1} \]

\[ \tau_4 = 0.898 \]

\[ K_5 L_5 = 3.10 \times 10^{-2} \]

\[ -\ln \tau_5 = \left( \frac{14.48 \times 10^{-2}}{1 + \frac{210 \times 10^{-4}}{4(31.91 \times 10^{-3} + 4.31 \times 10^{-3})}} \right)^{1/2} = \frac{1.448 \times 10^{-1}}{1.07} = 1.35 \times 10^{-1} \]

\[ \tau_5 = 0.874 \]

\[ I_{\lambda b} = I_{\lambda b1} (1 - \tau_1) = (2.687) (0.0296) = 0.080 \]

\[ I_{\lambda b2} (\tau_1 - \tau_2) = (7.271) (0.0204) = 0.150 \]

\[ I_{\lambda b3} (\tau_2 - \tau_3) = (13.56) (0.034) = 0.460 \]

\[ I_{\lambda b4} (\tau_3 - \tau_4) = (7.271) (0.018) = 0.131 \]

\[ I_{\lambda b5} (\tau_4 - \tau_5) = (2.687) (0.024) = 0.065 \]

\[ I_{\lambda} = 0.886 \text{ W/cm}^2\mu\text{-sr} \]

\[ I_{\lambda WS} = 0.886 + (0.874)(29.19) = 0.886 + 25.50 = 26.39 \text{ W/cm}^2\mu\text{-sr} \]
Problem 2 — Composite Engine

This is an example of single-zone radiation. Let the path length be the distance between the side walls less the initial jet widths of coolant gases, $4.46 - 0.80 = 3.66$ in. or $9.30$ cm. Static pressure $15.3$ psia. Mass fractions: $0.9438$, water and $0.0545$, hydrogen.

The mole fraction of water is $0.656$, giving a partial pressure of $0.683$ atm. The wave number of interest is $4016$ $(1/cm)$.

<table>
<thead>
<tr>
<th>$k$ $(1/cm)$</th>
<th>$T$ $(^\circ K)$</th>
<th>$kL$</th>
<th>$(1 + kL/4a)^{1/2}$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.825 \times 10^{-2}$</td>
<td>1500</td>
<td>0.1695</td>
<td>1.145</td>
<td>0.8630</td>
</tr>
<tr>
<td>$1.33 \times 10^{-2}$</td>
<td>2000</td>
<td>0.1235</td>
<td>1.065</td>
<td>0.8910</td>
</tr>
<tr>
<td>$1.12 \times 10^{-2}$</td>
<td>2500</td>
<td>0.1040</td>
<td>1.030</td>
<td>0.9040</td>
</tr>
</tbody>
</table>

The measured intensity for the zone can be represented by $I_\lambda = (1 - \tau)(I_{\lambda b})$. In particular for each zone

$$(0.137)(2.687) = 0.368 \text{ at } 1500^\circ K$$
$$(0.109)(7.271) = 0.792 \text{ at } 2000^\circ K$$
$$(0.096)(13.56) = 1.310 \text{ at } 2500^\circ K$$

When a $3500^\circ K$ blackbody source is also radiating, the measured intensity for each zone is calculated as

$$I_{\lambda WS} = 0.368 + (0.863)(29.19) = 0.368 + 25.05 = 25.418 \text{ at } 1500^\circ K$$
$$= 0.792 + (0.891)(29.19) = 0.792 + 25.95 = 26.74 \text{ at } 2000^\circ K$$
$$= 1.310 + (0.904)(29.19) = 1.310 + 26.20 = 27.51 \text{ at } 2500^\circ K$$
The experiment corresponding to this calculation would indicate:

\[ I_{\lambda WS} = 27.51 \]
\[ I_{\lambda} = 1.310 \]

therefore,

\[ \tau = \frac{27.51 - 1.310}{29.19} = \frac{26.20}{29.19} = 0.900 \]

\[ 1 - \tau = 0.100 \]

\[ I_{\lambda b} = \frac{1.310}{0.100} = 13.10, \quad \therefore T = 2470^\circ K \approx 2500^\circ K. \]

Not using band models would introduce intensity errors of up to 15\% in the variable range presented here; temperature errors would be somewhat less. If \( \epsilon \) is much less than 0.1, serious errors would be introduced into the temperature determination.

The assumption of a grey gas may be used for CO\(_2\) and of an optically thin gas may be used for H\(_2\)O, in the examples presented, without introducing excessive errors. Such errors could be removed by using a more complete data analysis program. The theoretical radiation analysis presented in Section 2 should provide a very adequate basis for an accurate data analysis calculation in ranges of experiments for which the illustrative examples are typical, i.e., no improvement to the Curtis-Godson approximation is necessary.
Section 5
CONCLUSIONS AND RECOMMENDATIONS

This report has demonstrated that sufficient radiation property data exist to study zone radiometry of CO$_2$ and H$_2$O. Such data also exist for soot and CO.

Data reduction procedures currently used are adequate for the studies which Rocketdyne has performed. These work because CO$_2$ is a grey gas and H$_2$O is optically thin in their experiments. Sample problems show this behavior. More accurate data reduction schemes could be devised, but this would not substantially improve existing data. However, to remove that criticism such procedures should be developed.

The only apparent reason for experiments of the alcohol-LOX vernier engine type not yielding accurate temperature and partial pressure data is lack of axial symmetry. The two-dimensional mixing study is probably so optically thin, that accurate transmittances would be very difficult to determine. In any event, all future studies should be preceded by an error analysis of expected data.
REFERENCES


Appendix

ZONE RADIOMETER DATA REDUCTION PROGRAM
Appendix

The data reduction procedure for axisymmetric zone radiometry readings was automated by Rocketdyne in a computer program* called the Axisymmetric Zone Radiometer Data Reduction Program. A description of the program is given here by outlining the operations occurring in the subroutines, a detailed flow chart of the procedure and a listing of the program. An input guide is given, and also a sample input and output are included to aid potential users of the program.

PROGRAM SUBROUTINES

MAIN Subroutine

The Axisymmetric Zone Radiometer Data Reduction Program main driver is responsible for reading the program input and calculating the local temperature and species concentrations of water or carbon dioxide.

The main program does the following (in sequence):

1. Evaluates the path lengths
2. Reads data
3. Solves for the product of zonal absorption coefficient and zonal partial pressure of radiating species, \((kp)_i\)
4. Constructs attenuation matrix to solve for zonal blackbody radiance
5. Solves for the zonal blackbody radiance
6. Uses Planck's distribution law to solve for zonal temperatures
7. If the radiating species are considered to be continuous radiators, the solution is complete. The local temperatures along with the product of the absorption coefficient and partial pressures for each zone is output.

8. If the radiating species require the use of spectral averaged data (band models), the zonal temperatures and partial pressures are iterated.

9. Evaluate the fine structure parameter, \( a \), for each zone using the zonal temperatures and partial pressures.

10. Calculate new absorption coefficients for each zone.

11. Repeat calculation steps from Step 4 until successive values of the temperatures and partial pressures are within a preset limit.

**QUAD Subroutine**

This subroutine contains three entries: QUAD0, QUAD1 and QUAD2. The QUAD0 entry generates the appropriate weighting function for quadratic distribution of properties. The QUAD1 entry sums the product of path lengths and weighting factors to obtain the coefficient for the average \((k_p)\) values. The QUAD2 entry sets up the average \(k_p\) values using the path lengths and the weighting factors.

**SLIT Subroutine**

A routine to correct for spatial resolution in deflection data. The correction technique is from the University of Tennessee described in AF CRL 465, pages 59-62. These corrections are usually small and occur at the edge of the plume. Rocketdyne has modified the experimental procedure such that radiance and transmission (smoothed) data are available rather than deflection data making it unnecessary to apply the slit corrections in this subroutine. For completeness the capability to read in deflection data and correct it for spatial resolution has been left in the program.

**ISIMEQ Function Subroutine**

This subprogram solves a set of simultaneous linear equations with up to 30 variables. This is a standard matrix solution subprogram. Throughout the zone radiometry data reduction program, this subroutine is used to solve for the variable of interest in each zone and then returns the answers as a column matrix in column one.
PROGRAM FLOW CHART
Path Length Evaluation

Input Data

Deflection Data?  
KDATA < 0

Yes

Call SLITO to correct for spatial resolution and return corrected transmittance and radiance data

Convert Transmittance using 

\[-\ln \tau_j = k_i P_{i,j} \]

Call ISIMEQ to solve matrix equation

\[-\ln \tau_j = (k P_k) \ell_{i,j} \]

for the product of absorption coefficient and partial pressure in the zones, \((k P_k)\)
Call QUAD2 to get matrix of average values of $k_i, P_k, l_i, j$

Apply Beer's Law to get calculated transmittances

$$\tau_{i,j} = ZAT_{ij} = e^{-k_i P_k l_i, j}$$

Reentry Point for Iterative Calculation

Compute Attenuation Matrix $PAL(J,I)$

Calculate Emissivity

$$\epsilon_{ij} = 1 - ZAT_{ij}$$

Set Up Line of Sight Radiance Equations

$$I_{\lambda_j} = \sum I_{\lambda_{b_i}} \epsilon_{i,j} \tau_{i-1,j}$$
Call ISIMEQ to Solve Radiance Matrix Equations for $I_{\lambda b_i}$

Solve for Zonal Temperatures, $T_i$, using Planck Distribution

$$T_i = T_B = \frac{C\kappa}{\lambda\kappa \ln \left( 1 + \frac{2C^2\kappa}{I_{\lambda b_i} \lambda^5} \right)}$$

Spectral Averaged Data Necessary

$|KDATA| > 1$

Solution Complete Write Out

1. Title
2. Zone
3. Average $kP_k$ Product at Each Zone
4. Emissivity for Each Zone
5. Radiation Energy at Each Station
6. Temperature at Each Station
7. Zone Width in cm
Evaluate an Absorption Coefficient for a Standard Condition using the Calculated Temperature

\[ \ln k^0 = \ln k_\omega = F_0 + F_1 T^{-1} + F_2 T^{-2} + F_3 T^{-3} \]

Modify Absorption Coefficient for Conditions Present

\[ k = \left( \frac{k^0}{P_o} \right) \left( \frac{273}{T_0 K} \right)^P_i \]
Evaluate Fine Structure Parameter

$$\log_{10} a_i(T) = A_{o_i} + A_{2i} T^2$$

Modify Previously Calculated Partial Pressures Using Band Model Absorption Coefficients

$$P_{k_i} = \frac{\text{average} (k_i P_{k_i} T_i)}{(\text{zone width}) (\text{band model } k_i)}$$

Evaluate ASTAR

The Average Fine Structure Parameter, $a$

$$a = \frac{a^* Y_o}{Y_o^*}$$

Write Out Calculated Values for Each Zone

1. Number of Iteration
2. Number of Zone
3. Band Model Absorption Coefficient
4. Partial Pressure of Radiating Species
5. Temperature
6. Fine Structure Parameter
Set Up Iteration Procedure

Yes

ITERP < 0

No

Evaluate Attenuation Coefficient ZAT(I, J) Using Statistical Model

\[ ZAT(I, J) = e^{-\frac{kS}{1 + \frac{kS}{4a}}^{1/2}} \]

Go to Begin Iteration Loop and Solve for New Temperature
ITER = 1

Yes

No

ITERP > 0

Yes

No

Store Present $T_i$ and $P_{k_i}$ values

Loop to Obtain Consistent Partial Pressures

Compute Zone Path Lengths

$$B(I, J) = \ell_{i,j} = \left[2(\text{zone width}) DZ(I-1, J-1)\right]$$

Modify for Statistical Representation

$$B(I, J) = \frac{B(I, J)}{(1 + kS/4a)^{1/2}}$$

Establish Matrix Equations

$$-\ln \tau_j = B(I, J) (k P_{k_i})$$
Solve for New \( (k P_k)_i \) Values

Check the Deviation from Previously Calculated \( (k P_k)_i \) Values

Are the \( (k P_k)_i \) Values Behaving?

Yes

Do Up to 10 Iterations on Partial Pressure Values

Set INTERP = -100 Which Allows up to 110 Iterations on Partial Pressures and Absorption Coefficients

\( \text{ITER} > 10 \)

Yes

QUIT

Write Out DIVERGING TOO BAD

No

Return to Compute \( k_i, P_{k_i} \) and \( a_i \)

No
ITERP < 0

Yes

QUIT - Write Out
DIVERGING TOO BAD

No

Return to Evaluate Attenuation Matrix and Solve for Zonal Blackbody Temperatures

ITER > 2

Yes

No

Bypass Deviation Check and Do Another Iteration.

\[
\frac{\sum (\Delta T)^2}{\text{Number of Zones}}_{\text{new}} > \frac{\sum (\Delta T)^2}{\text{Number of Zones}}_{\text{old}}
\]

or

\[
\frac{\sum (\Delta p_k)^2}{\text{Number of Zones}}_{\text{new}} > \frac{\sum (\Delta p_k)^2}{\text{Number of Zones}}_{\text{old}}
\]

Yes

QUIT - Write Out
DIVERGING TOO BAD

No
\[
\frac{\sum (\Delta T)^2}{\text{Number of Zones}_{\text{new}}} < 10.0
\]
and
\[
\frac{\sum (\Delta P_k)^2}{\text{Number of Zones}_{\text{new}}} < 0.1 P_{\text{static}}
\]

Try Another Iteration. Continue until Successful or ITERP > 10 in Pressure Iteration.

Calculation Complete

Print Out Results

Read In New Case
<table>
<thead>
<tr>
<th>SMOOTHED DATA</th>
<th>RADIANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANSMISSION</td>
<td></td>
</tr>
</tbody>
</table>

6 POINTS / CARD

AROSPTION COEFFICIENT PARAMETERS FOR SPECT. AVG. DATA

TO USE PREVIOUS PARAMETERS SET DATA TYPE = 3

REAL * 4  SMN(30,30)  DZ(30,30)  ZAT(30,30)  PAL(30,30)  AM(18)  00000370
1  R(30,30)  T(30,30)  TN(30,30)  TP(30,30)  ENF(30,30)  EFN(30,30)  ENNF(30,30)  00000380
2  FNPF(30)  FPS(30)  BF(30)  AKP(30)  RAD(30)  TR(30)  X(30)  00000390
3  DF(30)  DGP(30)  DGRF(30)  DGPF(30)  DGP(30)  DGRF(30)  DGP(30)  00000400
4  DRPP(30)  DGR(30)  AK(30)  P(30)  POLD(30)  TOLD(30)  00000410
5  ARSTAR(30)  420

INTEGER * 4  IA(30)

COMMON N, A, SMN, DZ, B, AKP, AVKP(30,30)  00 451

1 FORMAT (1RA4)
2 FORMAT (112, 3F12.8, 213, 16, F12.8)  00 490
3 FORMAT (6F12.8)
4 FORMAT (112, 18X, 1RA4 / 1*7, 1K1, 9X, 1TN(K) / 6X, 1TP(K) / 00000510
1 9X, 1T(K) / 7X, 1NF(K) / 6X, 1NF(K) / 9X / 00000520
5 FORMAT (112, 18X, 1RA4 / 1*7, 1K1, 9X, 1NF(K) / 7X, 1T(K) / 00000530
6 FORMAT (112, 18, 2F12.4)
7 FORMAT (112, 18, 6F12.4)
8 FORMAT (112, 18X, 1RA4 / 9X, 1RX, 1RADIAL PROPERTIES / 9X / 9X / 00000560
1 9X, 6X, 1K1 / 7X, 1RAD(K) / 7X, 1T(K) / 00000570
2 3011, 110, 3F12.4, 6X, 6F6.0 / 00 580
9 FORMAT (112, 18 / 150(11, 6F12.8 /))  00 590
10 FORMAT (112, 18 / 150(11, 6F12.8 /))  00 590
11 FORMAT (112, 18X, 1RA4 / 01NPUT DATA 112, 3F12.4, 213, 16, F12.4)  00000620
12 FORMAT (10 ITERATION NO 13, 1 WAS CONSISTENT AT IERP = 13)  00000630
13 FORMAT (1R*, 18X, 13, 1 ITERATIONS WERE REQUIRED)  00000640
14 FORMAT (19X, 1SLT CORRECTION INCLUDED)  00 642
15 FORMAT (19X, 1STEP FUNCTION PROPERTY DISTRIBUTION ASSUMED)  00000644
1A FORMAT (10, 18X, 1ZONE WIDTH = 1, 1CM USED TO COMPUTE EPS)  00000646

C GENERATE SMN AND X 650
DO 104 1 = 1,30
X(1) = 1
DO 104 J = 1,30
104 SMN(I,J) = SQRT(FLOAT(J**2 - I**2))

C REDUCED PATH LENGTH MATRIX, 0TH LOS OMITTED - DZ(LOS,ZONE)=(Y2-Y1)0000701
DO 110 J = 1,29
L = J - 1
DO 108 J = 1,L
108 DZ(I,J) = 0.0
DO 110 J = 1,29
110 DZ(I,J) = SMN(I,J+1) - SMN(I,J)

C GENERATE QUADRATIC FUNCTION WEIGHTING FACTORS 00000715
CALL QUAO

C READ TITLE AND PARAMETER CARDS 0 720
C LOOP TO HERE FOR MULTIPLE PROBLEMS 00 730

120 READ (5,1) AM
READ (5,2) NA,WC,ENRB,QA,ISF,KDATA,ML
WRITE (6,11) AM, N, A, WC, ENRB, QA, ISF, KDATA, ML 00 760
IF (ISF.EQ.1) CALL SLITI 775
IF (N.EQ.0) GO TO 120 777
IF (KDATA*GT.0) GO TO 150 780

C READ DEFLECTION DATA 790
READ (5,3) (DBR(K),K=1,N)
READ (5,3) (DFK(K),K=1,N)
READ (5,3) (DPP(K),K=1,N)
READ (5,3) (DFP(K),K=1,N)
READ (5,3) (DBF(K),K=1,N)
READ (5,3) (DBF(K),K=1,N)
READ (5,3) (DFBP(K),K=1,N)

C GET TRANSMISSION AND RADIANCE FROM DEFLECTIONS 00000890
CAL = WC * ENRB
DO 145 K = 1,N
145 TN(K) = DBF(K) / DGR(K) 900
910
920
930
MULTIPLY BY EMISSIVITY = 1 - ZAT FOR EACH ZONE AFTER ADDITION

I

LMSHREC D306101

00001700
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1950
1960
1970
1980
1990
00002000
00002030
00002040
00002060
A-18
RAD(1) = 0.0
252 CONTINUE
C MAKE CHARTS
N = N + 1
TB(N) = 0.0
RAD(N) = 0.0
FPS(N) = 0.0
C GO GET NEXT DATA SET
GO TO 120
C DATA ERROR
270 WRITE (6,271) WL
271 FORMAT ('0 RAD VALUE FOR WL= ', F12.2)
    IF (IARS(KDATA)*LF*1) GO TO 120
    READ (5,1) DUMMY
    READ (5,1) DUMMY
GO TO 120
C ITERATE - IS THIS FIRST TIME
300 IF (ITER.GT.0) GO TO 350
    IF (IARS(KDATA)*EQ.3) GO TO 340
    READ DATA ON K AND A
C LN K = F0 + F1/T + F2/T**2 + F3/T**3
    LOG ARAR = A0 + A2*T**2
    DATA AND ANALYSIS ARE FROM GD/C REPORT OF DEC. 1966
    READ (5,307) F0, F1, F2, F3, A0, A2
    READ (5,307) PT
307 FORMAT (6E12.7)
C 340 WRITE (6,341) F0,F1,F2,F3,A0,A2,PT
341 FORMAT (19X, 'BAND MODEL DATA USED', / 19X, 6E12.4/ 19X, 'PT =', 1 F12.4)
C 350 ITER = ITER + 1
    ITERP = 0
C COMPUTE K FOR EACH ZONE - ALSO P AND ASTAR
353 DO 355 I = 1,N
   T(I) = TB(I)
   IF (T(I)*GT*4000.0) T(I) = 4000.0
   AK(I) = EXP (F0 + F1/T(I) + F2/T(I)**2 + F3/T(I)**3)
   AK(I) = (273.0/T(I))*AK(I)
   P(I) = (AVKP(1,1) / A) / AK(I)
   TS = T(I) / 273.0
   THETA = 0.044 / TS + 0.09 / SORT(TS)
   CSTAR = -0.1002 + TS*(0.076495 - TS*(0.008116 - TS* 0.000592))
   G = THETA * P(I) + (PT - P(I)) / 0.044 / SORT(TS)
   GS = THETA * CSTAR + (1.0 - CSTAR) * 0.044 / SORT(TS)
355 ASTAR(I) = 10.0 * (AO + A2*T(I)**2) * G / GS
C WRITE (6,10) ITRP, (K, AK(K), P(K), T(K), ASTAR(K), K=1,N)
IF (1TRP*LT*0) GO TO 406
IF (1TRP*EQ*1) GO TO 377
IF (1TRP*GT*0) GO TO 377
ST = ST
SP = SP
ST = 0.0
SP = 0.0
C CALCULATE DEVIATION
DO 365 I = 1,N
   ST = ST + (T(I) - TOLD(I))**2
365 SP = SP + (P(I) - POLD(I))**2
   ST = SORT (ST / N)
   SP = SORT (SP / N)
   WRITE (6,368) ITRP, ST, SP
   IF (1TRP*EQ*2) GO TO 377
368 FORMAT (*I0 ITRP = 'I3, 'ST = ', F12.2, 'SP = ', F12.4)
C C ARE WE DOING BETTER
IF (ST*GT*ST0.OR.SP*GT*SP0) GO TO 425
IF (ST*LT*10.0.AND.SP*LT*0.01*PT) GO TO 250
C C TRY AGAIN
C C STORE PRESENT VALUES
377 DO 379 I = 1,N
   TOLD(I) = T(I)
379
A-21
425 WRITE (6,426) ITER, ST, STO, SP, SPO
426 FORMAT ('1 DIVERGING - TOO BAD - ITER =', I4, 'ST =', F12.2, 
 GO TO 250
 FND
SUBROUTINE QUAD

USED WITH ZR-5 TO REPLACE STEP FUNCTION WITH QUADRATIC
IGA *NE* 0 REVERTS TO STEP FUNCTION
3 ENTRIES QUAD0 GENERATES WEIGHT MATRICES \( \text{WIN} \text{(LOS}+1\text{,ZONE}+1) \)
QUAD1 SUMS \text{WIN} TO GET COEFFICIENT OF AKP(ZONE+1)
QUAD2 DOES AVKP = \text{WIN} * AKP
WIN DOES NOT CONTAIN ZONE LENGTH = A SO NEEDS ONLY TO BE SET ONCE

ROTH QUAD1 AND QUAD2 APPLY LENGTH FACTOR
REAL *8 \text{WIN} (30,30,3) \* C0* C1* C2* DY* Y1* Y2

SETUP WEIGHT MATRIX
ENTRY QUAD0
DO 3 I = 2,30
ZERO EXTRA ELEMENTS FOR THIS LOS+1
II = I - 1
DO 1 J = 1,II
DO 1 K = 1,II
1 \text{WIN} (I,J,K) = 0.0
GET EM FOR EACH ZON+1
C0 = (I-1) * (I-1)
DO 3 J = 1,30
Y1 = DSORT(J-1,J+1 - C0)
Y2 = DSORT(J*J - C0)
DY = Y2 - Y1
C1 = C0 + Y1 * Y2
C2 = 12.0 * (Y2 + C0 * DLOG((Y2 + J) / (Y1 + J - 1))) / DY
DY = DY / 24.0
3 \text{WIN} (I,J) = DY * (1.0 + 4.0 * C1 + J * (1.0 + 4.0 * J)) - J * C2
WIN (I,J,2) = DY * (2.0 + 4.0 * (1-4*J) - 8.0 * C1 + (2*J-1) * C2)
WIN (I,J,3) = DY * (1.0 + (J-1) * (8.0*J - C2) + 4.0 * C1)

DO FIRST ROW = 0TH LOS
DO 4 J = 1,30
WIN (I,J,1) = 0.04167
WIN(1, J+3) = 0.04167
4 WIN(1, J+2) = 0.01666
GO TO 20

C SUM APPROPRIATE WIN ELEMENTS
ENTRY QUAD1(IQA)
DY = 2.0 * A
IF (IQA NE 0) GO TO 7
K = N - 1
DO 6 I = 1:N
R(I+1) = 0*C
DO 5 J = 2*K
5 B(I, J) = DY * (WIN(I, J-1,3) + WIN(I, J+2) + WIN(I, J+1,1))
6 B(I, N) = DY * (WIN(I, J-1,3) + WIN(I, J+2))
B(I+1) = DY
R(2, I) = DY * WIN(2, 2, I)
GO TO 20

C STEP FUNCTION OPTION
7 DO 8 I = 1:N
R(I+1) = 0*C
8 R(I+1) = DY
DO 9 I = 2:N
DO 9 J = 2:N
9 B(I, J) = DY * NZ(I-1, J-1)
GO TO 20

C CALCULATE AVK P
ENTRY QUAD2(IQA)
DY = A
IF (IQA NE 0) GO TO 11
AKP(N+1) = 0*C
AVKP(I+1) = DY * (WIN(I, I+1, 1) + WIN(I, I+2) + AKP(I) + WIN(I, I+3) + AKP(I+1))
K = N - 1
DO 10 I = 2: N
AVKP(I+1) = DY * (WIN(I, I+1) + AKP(I-1) + WIN(I+1, I) + AKP(I))
1 + WIN(I, I+3) + AKP(I+1))
AVKP(I, N) = DY * (WIN(I, N+1) + AKP(N-1) + WIN(I, N+2) + AKP(N))
AVKP(I, 1) = 0*C
DO 10 J = 2:K
10 AVKP(I,J) = DY * (WIN(I,J+1) * AKP(J-1) + WIN(I,J+2) * AKP(J) + WIN(I,J+3) * AKP(J+1))
1
GO TO 20
20 RETURN
END

NON-QUAD AVKP
11 AVKP(I+1) = DY * AKP(I)
DO 12 I = 2,N
AVKP(I+1) = DY * AKP(I)
AVKP(I+1) = 0.0
DO 12 J = 2,N
12 AVKP(I,J) = DY * DZ(I-1,J-1) * AKP(J)
SUBROUTINE SLIT
C ROUTINE TO CORRECT FOR SPATIAL RESOLUTION ELEMENT
C TWO ENTRIES SLIT1 READ A NEW SLIT FUNCTION
C SLIT0 APPLY SLIT FUNCTION
C
COMMON NZONE, A, DUMMY(3630)
REAL *4 SF0(50), SF1(50), X(400), Y(400), Z(400), T(30)
INTEGER JCHAR(2) /55,16/
C
READ SLIT FUNCTION
ENTRY SLIT1
READ (6,1) NS0, AS0, (SF0(I+1), I=1,NS0)
1 FORMAT (112, SF12.6 / (6F12.6))
C     NS0 = NO. OF ELEMENTS TO BE READ IN
C
WRITE (6,10) NS0, AS0, (1, SF0(I+1), I=1,NS0)
10 FORMAT ('SLIT FUNCTION INPUT' / '0', I3, ' POINTS WITH', F7,4, 1
     ' CM SPACING' / '('*, I6, F12.4))
C
SF0(1) = 0.0
SF0(NS0+2) = 0.0
AS1 = 0.0
C
GO TO 99
C
APPLY SLIT FUNCTION
ENTRY SLIT0(I)
C EXPAND BY FACTOR OF 10 USING CUBIC FIT TO 4 POINTS
C USE LINEAR WEIGHTED AVERAGE TO REDUCE DISCONTINUITIES
C USE SYMMETRY TO GET EXTRA POINTS IN CENTER
C
0TH L0S GOES TO X(31)
X(1) = T(4)
X(11) = T(3)
X(21) = T(2)
DO 2 I = 1, NZONE
2 X(10*I + 21) = T(I)
C EXTEND OUTER EDGE
K = 10 * NZONE + 31
X0 = 0.0
IF (T(NZONE) .GE. T(NZONE-1)) X0 = 1.0
IF (T(NZONE) .LE. T(NZONE-1)) X0 = T(NZONE)
\[ X(K) = X0 \]
\[ X(K+10) = X(K-10) \]

**FILL IN GAPS**

\[ DO 20 I = 31*K+10 \]
\[ X0 = X(I) \]
\[ X2 = (X(I+10) + X(I-10) - 2.0 * X0) / 400.0 \]
\[ X3 = X0 - (X(I-10) - (X(I+10) - X(I-20))) / 3.0 \]
\[ X1 = (X(I-10) - X(I+10) - X3) / 40.0 \]
\[ X0 = X0 / 2.0 \]
\[ X3 = X3 / 4000.0 \]

\[ DO 20 J = 1,9 \]
\[ X(I-J) = X0 + J * (X1 + J * (X2 + J * X3)) + X(I-J) \]
\[ C = (10.0 - J) / 10.0 \]
\[ X(I+J) = (X0 - J * (X1 - J * (X2 - J * X3))) * C \]
\[ L = J + 10 \]
\[ 20 X(I-L) = (X0 + L * (X1 + L * (X2 + L * X3))) * C + X(I-L) \]

**ADJUST LAST ZONE USING QUADRATIC CONTRIBUTION**

\[ C = (X(K-10) - X(K)) / 2000.0 \]
\[ X1 = X(K) / 20.0 \]

\[ DO 22 I = 1,9 \]
\[ J = 10 - I \]
\[ X0 = X(K-J) + I * (X1 + C * J * J) \]
\[ IF (X0 LT 0.0) X0 = 0.0 \]
\[ IF (X0 GT 1.0) X0 = 1.0 \]
\[ IF (ABS(X0-X(K)) GE 0.01) ABS(X(K-J-1)-X(K))= X0= X(K) + (J/(J+1,0)) \]

\[ 1 ** 2 * (X(K-J-1) - X(K)) \]

\[ 22 X(K-J) = X0 \]

**FIND FIRST CROSSING POINT**

\[ X0 = X(K) \]
\[ IF(X0 GE 1.0) GO TO 24 \]
\[ DO 23 . I = 62*K \]
\[ 23 IF(X(I) LE 0.0) GO TO 26 \]
\[ 24 DO 25 . I = 62*K \]
\[ 25 IF(X(I) GE 1.0) GO TO 26 \]
\[ 26 DO 27 . J = 1*K \]
\[ 27 X(J) = X0 \]
\[ DO 28 . I = 1,30 \]
\[ X(K+1) = X0 \]
28 X(I) = X(62 - I)

C ADJUST SLIT FUNCTION SPACING TO MATCH DATA

C IF (AS1 .EQ. A/10.) GO TO 55
AS1 = A/10.
AS2 = AS1 / AS0
C EQUALLY SPACED - SYMMETRY NOT ASSUMED - GIVEN CENTER TO EDGE
C NO. OF POINTS TO MATCH DATA SPACING
NS1 = ((NS0 - 1) / AS2 + 1.75)
C MAKE NEW SLIT FUNCTION ODD
C IF (MOD(NS1,2) .EQ. 0) NS1 = NS1 + 1
ICO = NS0 / 2 + 2
IC1 = NS1 / 2 + 1
J = NS1 - IC1
C IF (MOD(NS0+2) .EQ. 0) GO TO 45
C ODD NO. IN ORIGINAL
C ODD - ODD
SF1(IC1) = SF0(ICO)
DO 30 I = 1, J
X0 = I * AS2
N = X0
X0 = X0 - N
M = N + ICO
SF1(IC1 + I) = SF0(M) + X0 * (SF0(M+1) - SF0(M))
M = ICO - N
30 SF1(IC1 - I) = SF0(M) + X0 * (SF0(M-1) - SF0(M))
C
C EVEN IN ORIGINAL
45 SF1(IC1) = 0.5 * (SF0(ICO-1) + SF0(ICO))
C DO 50 I = 1, J
X0 = I * AS2 - 0.5
N = X0
X0 = X0 - N
M = ICO + N
SF1(IC1 + I) = SF0(M) + X0 * (SF0(M+1) - SF0(M))
M = ICO - 1 - N
GO TO 52
C
50 SF1(1C1 - 1) = SF0(M) + X0 * (SF0(M-1) - SF0(M))

C
C
NORMALIZE
C 0 C = 0.0
DO 52 I = 1,NS1
IF (SF1(I) LT 0.0) SF1(I) = 0.0
53 C = C + SF1(I)
DO 54 I = 1,NS1
54 SF1(I) = SF1(I) / C
C
WRITE (6,61) NS1, AS1, (I, SF1(I), I = 1,NS1)
61 FORMAT (*1SLIT FUNCTION USED, /10*, 13, ' POINTS WITH', F7.4, 1
   ' CM SPACING', /50(*1, 16, F7.4/))

C
C
SET LOOP PARAMETERS
C 55 J = NS1 / 2
JO = J + 1
C
FIND LAST OBSERVED DEVIATION
X0 = X(K)
I = K - 1
56 IF (X(I) LT X0) GO TO 57
I = I - 1
GO TO 56
57 KMAX = I - J
C
KMAX IS LAST ALLOWED DEVIATION IN TRUE FUNCTION - 1ST TRY Z = X
DO 58 I = 1,KMAX
58 Z(I) = X(I)
C
CLAMP END OF TRUE FUNCTION
DO 59 I = KMAX,K
59 Z(I) = X0
C
NCY = 0
C
LOOP FOR SLIT CORRECTION
60 DO 66 I = JO,KMAX
   C = SF1(JO) * Z(I)
   DO 67 L = 1,J
   67 C = C + SF1(JO - L) * Z(I - L) + SF1(JO + L) * Z(I + L)
   68 Y(I) = C
C

FUNCTION ISIMEQ( DSM, NF, NC, A, B, DET, C )
SUBPROGRAM TO SOLVE SIMULTANEOUS LINEAR EQUATIONS

DATE- 1/13/67 MODIFIED FOR COMPILATION IN RELEASE 14

DSM     DIMENSIONED SIZE OF COEFFICIENT MATRIX
NF      ACTUAL NUMBER OF EQUATIONS FOR THIS CALL
NC      NUMBER OF COLUMNS IN CONSTANT MATRIX
A       COEFFICIENT MATRIX
B       CONSTANT MATRIX
DET     INPUT - SCALE FACTOR, OUTPUT - FACTOR TIMES DETERMINANT VALUE OF COEFFICIENT MATRIX
C       TEMPORARY STORAGE FOR SUBROUTINE
ISIMEQ  RETURNS 1 IF OK, 2 IF OVFLO, 3 IF SINGULAR
        IF NC IS NEGATIVE, THE INVERSE OF THE COEFFICIENT MATRIX IS REQUIRED, MATRIX B IS SET UP AS IDENTITY.

LOGICAL DVO
INTEGER DSM, C, T, SUB1, SUB2, R, D
DIMENSION B(1), C(1)

INITIALIZE
N = NF
D = DSM
M = JARS(NC)
ISIMEQ = 1
DVO = .FALSE.

DO 1 I = 1, N
1 C(I) = 1
IF(NC) 5, 15, 15

INVERSE REQUIRED
= SUB2 = 0
DO 10 J = 1, N
   SUB1 = SUB2
   DO 6 I = 1, N
       SUB1 = SUB1 + 1
6   B(SUB1) = 0.0
   SUB1 = SUB2 + J
       B(SUB1) = 1.0
10 SUB2 = SUB2 + D
GO TO 15
ENTRY DETRM(DSM, NF, A, DET)
DIMENSION A(1)
N = NF
D = DSM
DVO = *TRUE*
C START MAIN LOOP
15 DO 1000 L = 1.N
   LP1 = L + 1
   DO 40 I = L*N
      PIVOT = A*0
      SUB1 = (L-1) * D + I
      SUB2 = SUB1
      DO 20 J = L*N
         IF(ABS(PIVOT) .GE. ABS(A(SUB1))) GO TO 20
         PIVOT = A(SUB1)
         JB = J
      20 SUB1 = SUB1 + D
   C COMPUTE DETERMINANT
      DET = DET * PIVOT
      IF(*NOT* DVO) GO TO 24
   C TEST FOR SINGULAR MATRIX
      24 IF(PIVOT .EQ. 0.0) GO TO 2000
      DO 25 J = L*N
         A(SUB2) = A(SUB2) / PIVOT
      25 SUB2 = SUB2 + D
      IF (DVO) GO TO 35
      SUB1 = I
      DO 30 J = I*N
         A(SUB1) = A(SUB1) / PIVOT
      30 SUB1 = SUB1 + D
      IF (I .EQ. L) JP = JB
100 CONTINUE
C INTERCHANGE COLUMNS
100 IF (JP .EQ. L) GO TO 260
   IF (DVO) GO TO 110
      T = C(L)
      C(L) = C(JP)
      C(JP) = T
   110 D = D * L - D
      T = D * JP - D
DO 120 I = 1 + N
SUB1 = R + 1
SUB2 = T + 1
S = A(SUB1)
A(SUB1) = A(SUB2)
120 A(SUB2) = S
DET = -DET
C REDUCE PIVOT COLUMN
260 R = D * L - D
DO 400 I = 1 + N
IP = R + 1
PIVOT = A(IP)
IF (I * EQ. L * OR. PIVOT * EQ. 0.0) GO TO 400
SUB1 = L
SUB2 = I
DO 360 J = 1 + N
IF (J * LT. LP1) GO TO 300
S = PIVOT * A(SUB1)
A(SUB2) = A(SUB2) - S
IF (ABS(A(SUB2)) * LT. ABS(2.0E-6 * S)) A(SUB2) = 0.0
300 IF (DVO * OR. J * GT. M) GO TO 350
R(SUB2) = R(SUB2) - PIVOT * B(SUB1)
350 SUB1 = SUB1 + D
360 SUB2 = SUB2 + D
400 CONTINUE
1000 CONTINUE
IF (DVO) GO TO 1500
C REARRANGE VARIABLES
1100 DO 1201 L = 1 + N
SUB1 = C(L)
SUB2 = L
DO 1200 J = 1 + M
A(SUB1) = B(SUB2)
SUB1 = SUB1 + D
1200 SUB2 = SUB2 + D
1201 CONTINUE
1500 RETURN
C SINGULAR COEFFICIENT MATRIX
2000 IF (DVO) GO TO 3000
ISIMEQ = 3
3000
INPUT GUIDE TO THE DATA REDUCTION PROGRAM
FOR AXISYMMETRIC ZONE RADIOMETRY
## INPUT GUIDE TO THE DATA REDUCTION PROGRAM
### FOR AXISYMMETRIC ZONE RADIOMETRY

<table>
<thead>
<tr>
<th>Card</th>
<th>Col.</th>
<th>Format</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-72</td>
<td>18A4</td>
<td>Title card</td>
</tr>
<tr>
<td>2</td>
<td>1-12</td>
<td>I12</td>
<td>Number of zones</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td>Zone width, cm</td>
</tr>
<tr>
<td></td>
<td>25-36</td>
<td>F12.8</td>
<td>Correction to plume radiance due to window absorption</td>
</tr>
<tr>
<td></td>
<td>37-48</td>
<td>F12.8</td>
<td>Blackbody radiance, W/cm² -μ -sr</td>
</tr>
<tr>
<td></td>
<td>49-51</td>
<td>I3</td>
<td>Property variation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 0 Quadratic property variation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 Step function; constant zonal properties</td>
</tr>
<tr>
<td></td>
<td>52-54</td>
<td>I3</td>
<td>Slit function correction control parameter</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 Apply a new function</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 0 Apply previously used function</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= -1 Do not correct for slit function</td>
</tr>
<tr>
<td></td>
<td>55-60</td>
<td>I6</td>
<td>Data type</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 0 or 1 For spectral data</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 For spectrally averaged data</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>(Positive values imply smoothed input — zero or negative values imply deflection values input)</td>
</tr>
<tr>
<td>2</td>
<td>61-72</td>
<td>F12.8</td>
<td>Wavelength in microns</td>
</tr>
<tr>
<td>3a</td>
<td></td>
<td></td>
<td>Deflection Data (or) Smoothed Data For Deflection Data</td>
</tr>
<tr>
<td></td>
<td>1-12</td>
<td>F12.8</td>
<td>Blackbody Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3b</td>
<td>1-12</td>
<td>F12.8</td>
<td>Flame Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
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<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>Card</td>
<td>Col.</td>
<td>Format</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>3c</td>
<td>1-12</td>
<td>F12.8</td>
<td>Blackbody Prime Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3d</td>
<td>1-12</td>
<td>F12.8</td>
<td>Flame Prime Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3e</td>
<td>1-12</td>
<td>F12.8</td>
<td>Greybody Alone Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3f</td>
<td>1-12</td>
<td>F12.8</td>
<td>Greybody through Flame Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3g</td>
<td>1-12</td>
<td>F12.8</td>
<td>Greybody Alone Prime Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3h</td>
<td>1-12</td>
<td>F12.8</td>
<td>Greybody through Flame Prime Data; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3'a</td>
<td>1-12</td>
<td>F12.8</td>
<td>For Smoothed Data Line of Sight Spectral Radiance Data in W/cm(^2)-micron-ster; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td>3'b</td>
<td>1-12</td>
<td>F12.8</td>
<td>Line of Sight Transmittance Data which is dimensionless; 6 values to a card until all zone data are input</td>
</tr>
<tr>
<td></td>
<td>13-24</td>
<td>F12.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61-72</td>
<td>F12.8</td>
<td></td>
</tr>
</tbody>
</table>

A-36
SAMPLE INPUT TO THE DATA REDUCTION PROGRAM FOR AXISYMMETRIC ZONE RADIOMETRY

<table>
<thead>
<tr>
<th>SAMPLE CASE FOR ZONE RADIOMETRY</th>
<th>CO2 RADIATING</th>
</tr>
</thead>
<tbody>
<tr>
<td>16  0.45</td>
<td>1  -1  1  4.45</td>
</tr>
<tr>
<td>1.229  1.219  1.196  1.163  1.114  1.05</td>
<td></td>
</tr>
<tr>
<td>0.953  0.823  0.647  0.442  0.28  0.167</td>
<td></td>
</tr>
<tr>
<td>0.09  0.033  0.004  0.001</td>
<td></td>
</tr>
<tr>
<td>0.18  0.182  0.184  0.19  0.203  0.227</td>
<td></td>
</tr>
<tr>
<td>0.263  0.31  0.372  0.453  0.649  0.661</td>
<td></td>
</tr>
<tr>
<td>0.784  0.92  0.99  0.999</td>
<td></td>
</tr>
</tbody>
</table>

A-37
OUTPUT OF THE DATA REDUCTION PROGRAM FOR AXISYMMETRIC ZONE RADIOMETRY

<table>
<thead>
<tr>
<th>INPUT DATA</th>
<th>16</th>
<th>0.4500</th>
<th>0.0000</th>
<th>0.0000</th>
<th>1</th>
<th>-1</th>
<th>1</th>
<th>4.4500</th>
</tr>
</thead>
</table>

SAMPLE CASE FOR ZONE RADIOMETRY  CO2 RADIATING
### SAMPLE CASE FOR ZONE RADIOMETRY CO2 RADIATING

<table>
<thead>
<tr>
<th>K</th>
<th>T(K)</th>
<th>NF(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.180</td>
<td>1.2290</td>
</tr>
<tr>
<td>2</td>
<td>1.182</td>
<td>1.2190</td>
</tr>
<tr>
<td>3</td>
<td>1.184</td>
<td>1.1960</td>
</tr>
<tr>
<td>4</td>
<td>1.190</td>
<td>1.1630</td>
</tr>
<tr>
<td>5</td>
<td>1.203</td>
<td>1.1140</td>
</tr>
<tr>
<td>6</td>
<td>1.227</td>
<td>1.0500</td>
</tr>
<tr>
<td>7</td>
<td>1.263</td>
<td>0.9510</td>
</tr>
<tr>
<td>8</td>
<td>1.310</td>
<td>0.8230</td>
</tr>
<tr>
<td>9</td>
<td>1.372</td>
<td>0.6470</td>
</tr>
<tr>
<td>10</td>
<td>1.453</td>
<td>0.4420</td>
</tr>
<tr>
<td>11</td>
<td>1.549</td>
<td>0.2800</td>
</tr>
<tr>
<td>12</td>
<td>1.661</td>
<td>0.1670</td>
</tr>
<tr>
<td>13</td>
<td>1.784</td>
<td>0.0900</td>
</tr>
<tr>
<td>14</td>
<td>1.920</td>
<td>0.0330</td>
</tr>
<tr>
<td>15</td>
<td>0.900</td>
<td>0.0040</td>
</tr>
<tr>
<td>16</td>
<td>0.999</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

### SAMPLE CASE FOR ZONE RADIOMETRY CO2 RADIATING

#### RADIAL PROPERTIES

<table>
<thead>
<tr>
<th>K</th>
<th>KP(K)</th>
<th>EPS(K)</th>
<th>RAD(K)</th>
<th>T(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.180</td>
<td>0.078</td>
<td>1.966</td>
<td>2.159</td>
</tr>
<tr>
<td>2</td>
<td>1.174</td>
<td>0.075</td>
<td>1.943</td>
<td>2.146</td>
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<tr>
<td>3</td>
<td>1.182</td>
<td>0.078</td>
<td>1.843</td>
<td>2.088</td>
</tr>
<tr>
<td>4</td>
<td>1.189</td>
<td>0.081</td>
<td>1.767</td>
<td>2.045</td>
</tr>
<tr>
<td>5</td>
<td>1.192</td>
<td>0.082</td>
<td>1.694</td>
<td>2.002</td>
</tr>
<tr>
<td>6</td>
<td>1.195</td>
<td>0.079</td>
<td>1.673</td>
<td>1.990</td>
</tr>
<tr>
<td>7</td>
<td>1.197</td>
<td>0.079</td>
<td>1.624</td>
<td>1.961</td>
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<tr>
<td>8</td>
<td>1.155</td>
<td>0.067</td>
<td>1.530</td>
<td>1.906</td>
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<td>9</td>
<td>1.136</td>
<td>0.059</td>
<td>1.327</td>
<td>1.781</td>
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<tr>
<td>10</td>
<td>1.140</td>
<td>0.050</td>
<td>0.988</td>
<td>1.664</td>
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<tr>
<td>11</td>
<td>0.917</td>
<td>0.040</td>
<td>0.706</td>
<td>1.567</td>
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<tr>
<td>12</td>
<td>0.048</td>
<td>0.030</td>
<td>0.523</td>
<td>1.224</td>
</tr>
<tr>
<td>13</td>
<td>0.048</td>
<td>0.020</td>
<td>0.417</td>
<td>1.133</td>
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<tr>
<td>14</td>
<td>0.016</td>
<td>0.007</td>
<td>0.412</td>
<td>1.128</td>
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<tr>
<td>15</td>
<td>0.002</td>
<td>0.000</td>
<td>0.373</td>
<td>0.993</td>
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<tr>
<td>16</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.572</td>
</tr>
</tbody>
</table>

ZONE WIDTH = .450 CM USED TO COMPUTE EPS
STEP FUNCTION PROPERTY DISTRIBUTION ASSUMED

A-39

LOCKHEED - HUNTSVILLE RESEARCH & ENGINEERING CENTER