THE PREDICTION OF THREE-DIMENSIONAL LIQUID–PROPELLANT ROCKET NOZZLE ADMITTANCES

by

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**Abstract**

Crocco's three-dimensional nozzle admittance theory is extended to be applicable when the amplitudes of the combustor and nozzle oscillations increase or decrease with time. An analytical procedure and a computer program for determining nozzle admittance values from the extended theory are presented and used to compute the admittances of a family of liquid-propellant rocket nozzles. The theoretical predictions are shown to be in good agreement with available experimental data.

**Key Words (Suggested by Author(s))**

- Combustion instability
- Liquid rockets
- Nozzles

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INTRODUCTION

The interaction between the pressure oscillations inside an unstable rocket combustion chamber and the wave motion in the convergent section of the exhaust nozzle can have a significant effect on the stability characteristics of the rocket motor and is an important consideration in analytical studies concerned with the prediction of the stability of liquid-propellant rocket engines. This report is concerned with the investigation of this interaction.

To determine the stability of a liquid-propellant rocket engine, the equations describing the behavior of the oscillatory flow field throughout the rocket motor must be solved. To simplify the problem, it is convenient to analyze the oscillations in the combustion chamber and the nozzle separately. For such an analysis, the combustion chamber extends from the injector face to the nozzle entrance as shown in Fig. 1. All the combustion is assumed to take place in the combustion chamber where the mean flow Mach number is generally assumed to be low. On the other hand, no combustion is assumed to take place in the nozzle and its mean flow Mach number increases from a low value at the nozzle entrance to unity at the throat. Downstream of the throat the flow is supersonic and disturbances in this region cannot propagate upstream and affect the chamber conditions. Therefore, in combustion instability studies it is only necessary to consider the behavior of the oscillations in the converging section of the nozzle since only these oscillations can influence the conditions in the combustion chamber.

The nozzle admittance \(^1,^2\) is the boundary condition that must be satisfied by the combustor flow oscillations at the nozzle entrance. Defined as the ratio of the axial velocity perturbation to the pressure perturbation at the nozzle entrance, the nozzle admittance can also be used to determine whether wave motion in the nozzle under consideration adds or removes energy from the combustor oscillations. Furthermore, this boundary condition influences the structures and resonant frequencies of the natural modes of the combustor under investigation.

To theoretically determine the nozzle admittance, the equations which describe the behavior of the waves in the convergent section of the exhaust nozzle must be solved. These equations have been developed by
Crocco\(^2\) and were solved numerically to obtain admittance values for one- and three-dimensional oscillations. These values were tabulated over a wide range of frequencies and entrance Mach numbers for a specific nozzle geometry. By applying the scaling technique developed in Ref. 2, the admittances of related nozzles can be determined. It was pointed out\(^2\), however, that interpolation of the tabulated values can result in large errors in the predicted nozzle admittances; furthermore, the accuracy of the scaling procedure is open to question. In addition, Crocco's theory is only applicable to constant amplitude periodic wave motions, and in its present form it cannot be applied to cases where the amplitude of the oscillations varies in time.

In this report, the equations needed for computing the nozzle admittance are presented and their solutions are outlined. Crocco's theory is extended to account for wave-amplitude variation with time. Typical theoretical predictions are shown and compared with available experimental data. The effects of the nozzle geometry and chamber Mach number on the nozzle admittance are presented in plots showing frequency dependence of the real and imaginary parts of the nozzle admittance. The effects of the decay coefficient are also assessed. A manual describing the use of the computer program which calculates nozzle admittance values along with a program listing is presented in the appendix.

**SYMBOLS**

\begin{align*}
A, B, C & \quad \text{variable coefficients defined below Eq. (14)} \\
c & \quad \text{nondimensional speed of sound, } c/\bar{c}_o \\
\hat{e}_\psi, \hat{e}_\theta, \hat{e}_n & \quad \text{unit vectors} \\
i & \quad \sqrt{-1} \\
J_m & \quad \text{Bessel function of the first kind of order } m \\
K(\psi,\theta,t) & \quad \text{a function having the following space and time dependence:} \\
M & \quad \text{Mach number at the nozzle entrance}
\end{align*}
m  number of mode diametral nodal lines
n  number of mode tangential nodal lines
p  nondimensional pressure, $p^*/p_0^*$
q  nondimensional velocity, $q^*/c_0^*$
r  nondimensional radius, $r^*/r_0^*$
$r_{cc}$ nondimensional radius of curvature at the nozzle entrance, $r_{cc}^*/r_0^*$
$r_{ct}$ nondimensional radius of curvature at the nozzle throat, $r_{ct}^*/r_0^*$
S  nondimensional frequency, $\omega^*r_0^*/c_0^*$
$S_{mn}$ the nth root of the equation
\[
\frac{dJ_m(x)}{dx} = 0
\]
t  nondimensional time, $t^*r_0^*/c_0^*$
u  nondimensional axial velocity component, $u^*/c_0^*$
v  nondimensional radial velocity component, $v^*/c_0^*$
w  nondimensional tangential velocity component, $w^*/c_0^*$
y  irrotational specific nozzle admittance defined in Eq. (13)
y = $\tilde{\rho}c_0^* u' \tilde{p}^*$ = $\gamma \tilde{\rho}c_0^* u' \tilde{p}^*$
z  nondimensional axial coordinate, $z^*/r_0^*$
$\gamma$ ratio of specific heats
$\zeta$ a function used to compute the nozzle admittance; defined below Eq. (13)
$\theta$ tangential coordinate, radians
$\theta_0$ nozzle half-angle, degrees
$\lambda$ nondimensional temporal decay coefficient, $\lambda^*r_0^*/c_0^*$
$\rho$ nondimensional density, $\rho^*/\rho_0^*$
$\tau$ a function used to compute the nozzle admittance; $\tau = 1/\zeta$
$\phi$ nondimensional steady state velocity potential, $\phi^*/c_0^*r_0^*$
$\psi$ a function describing the $\phi$-dependence of the radial velocity perturbation
$\psi$ nondimensional steady state stream function, $\frac{1}{2\tilde{\rho}(\psi)q(\psi)}r^2$
$\omega$ nondimensional frequency, $\omega^*r_0^*/c_0^*$
\textbf{ANALYSIS}

\textbf{Derivation of the Wave Equations}

The equations used by Crocco\textsuperscript{2} to compute the nozzle admittance will be developed from the conservation equations. To keep the problem mathematically tractable and yet physically meaningful, the following assumptions were employed.

(1) The nozzle flow is a calorically perfect gas consisting of a single species.

(2) Viscosity and heat conduction are negligible.

(3) The steady state flow is one-dimensional; this assumption implies that the nozzle is slowly converging.

(4) The amplitudes of the waves are small so that only linear terms in the perturbed quantities need to be retained in the conservation equations.

(5) The oscillations are assumed to be irrotational.

Using these assumptions, the equations of motion in nondimensional form become

\textbf{Continuity}

\[ \frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]  

(1)
Momentum

\[ \frac{\partial q}{\partial t} + \frac{1}{2} \nabla q^2 = - \frac{1}{\rho} \nabla p \]

(2)

and, from the isentropic conditions, \( c^2 = p/\rho \) and \( p = \rho' \).

To obtain the linearized wave equations, the dependent variables are expressed in the following form:

\[ q = \tilde{q} + q', \quad p = \tilde{p} + p', \quad \rho = \tilde{\rho} + \rho' \]

(3)

Substituting these expressions into Eqs. (1) and (2), neglecting all non-linear terms involving primed quantities, and separating the resulting system of equations into a set of steady state equations and a set of unsteady equations yield the system of steady state equations:

\[ \nabla \cdot (\tilde{p} \tilde{q}) = 0; \quad \tilde{c}^2 = \tilde{\rho}' - 1 = 1 - \frac{\gamma - 1}{2} \tilde{q}^2; \quad \tilde{\rho} = \tilde{\rho}' \]

(4)

and the following system of unsteady linear equations that describe the wave motion:

\[ \frac{\partial p'}{\partial t} + \nabla \cdot (\tilde{q} \tilde{p}' + \tilde{p} \tilde{q}') = 0 \]

(5)

\[ \frac{\partial q'}{\partial t} + \nabla (\tilde{q} \cdot q') = - \nabla \left( \frac{\tilde{p}'}{\gamma \tilde{\rho}} \right) \]

(6)

\[ p' = \tilde{c}^2 \rho' \]

(7)

To simplify the application of the boundary conditions at the nozzle walls, these wave equations are solved in the orthogonal coordinate system shown in Fig. 1. In this coordinate system the steady state velocity potential \( \varphi \) replaces the axial coordinate \( z \), the steady state stream function \( \psi \) replaces the radial coordinate \( r \) and the angle \( \theta \) is used to denote azimuthal variations. Using this coordinate system the velocity vectors can be expressed as follows:
Using the definitions of the steady state velocity potential and stream function for a one-dimensional mean flow, it can be shown that

\[ \varphi = \frac{\partial \varphi}{\partial z} \]

\[ \psi = \frac{1}{2} \varphi^2 \varphi \psi (\varphi)^2 \]

Rewriting Eqs. (5) and (6) in the \((\varphi, \psi, \theta)\) coordinate system yields the following system of equations:

**Continuity**

\[ \frac{\partial (\rho^\prime)}{\partial t} + q^2 \frac{\partial (\rho^\prime + u^\prime)}{\partial \varphi} + 2q \frac{\partial (\varphi^\prime)}{\partial \psi} + \frac{\partial (\rho^\prime)}{\partial \theta} = 0 \]  

\[ \text{Equation 8} \]

**Momentum**

\[ \varphi\text{-component} \]

\[ \frac{\partial (u^\prime)}{\partial t} + \frac{\partial (q^2 u^\prime)}{\partial \varphi} + \frac{\partial (p^\prime)}{\partial \psi} = 0 \]  

\[ \text{Equation 9} \]

\[ \psi\text{-component} \]

\[ \frac{\partial (v^\prime)}{\partial t} + q^2 \frac{\partial (v^\prime)}{\partial \varphi} + \frac{\partial (p^\prime)}{\partial \psi} = 0 \]  

\[ \text{Equation 10} \]

\[ \theta\text{-component} \]

\[ \frac{\partial (rw^\prime)}{\partial t} + q^2 \frac{\partial (rw^\prime)}{\partial \varphi} + \frac{\partial (p^\prime)}{\partial \psi} = 0 \]  

\[ \text{Equation 11} \]

Equations (7) through (11) constitute a system of five equations in the five unknowns -- \( \rho^\prime/\rho, u^\prime/\bar{u}, v^\prime/r\bar{u}, rw^\prime, \) and \( p^\prime/\bar{p}. \) These equations are solved by the method of separation of variables and the solutions are
\[
\frac{u'}{q} = \frac{d\hat{\phi}(\varphi)}{d\varphi} K(\psi, \theta, t)
\]

\[
\frac{v'}{r^2 p q} = \hat{\phi}(\varphi) \frac{\partial}{\partial \varphi} K(\psi, \theta, t)
\]

\[
rw' = \hat{\phi}(\varphi) \frac{\partial}{\partial \theta} K(\psi, \theta, t)
\]

\[
\frac{P'}{p} = -\left[ i(w - i\lambda) \hat{\phi}(\varphi) + \frac{q^2(\varphi)}{c} \frac{d\hat{\phi}(\varphi)}{d\varphi} \right] K(\psi, \theta, t)
\]

\[
\frac{\rho'}{\rho} = -\left[ \frac{i}{c}(w - i\lambda) \hat{\phi}(\varphi) + \frac{q^2(\varphi)}{c} \frac{d\hat{\phi}(\varphi)}{d\varphi} \right] K(\psi, \theta, t)
\]

where

\[
K(\psi, \theta, t) = \begin{cases} 
J_m \left[ \frac{1}{\sqrt{\psi}} \right] \cos m\theta e^{i(w - i\lambda)t} & \text{for standing waves} \\
J_m \left[ \frac{1}{\sqrt{\psi}} \right] e^{i(w - i\lambda)t} & \text{for spinning waves}
\end{cases}
\]

These solutions identically satisfy the momentum and energy equations. Substituting these solutions into Eq. (8) and eliminating variables give the following differential equation for the function \(\hat{\phi}\):

\[
\frac{-q^2(c^2 - q^2)}{\rho} \frac{d^2 \hat{\phi}}{d\varphi^2} - \frac{q^2}{c^2} \frac{d^2 q}{d\varphi^2} + \frac{2i(w - i\lambda)}{c} \frac{d\hat{\phi}}{d\varphi} + \left[ (w - i\lambda)^2 - \frac{\gamma - \frac{1}{2}}{2} i(w - i\lambda) \frac{d^2 q}{d\varphi^2} - \frac{q^2}{\rho} \frac{d^2 q}{d\varphi^2} - \frac{S_{mn}}{r} \right] \hat{\phi} = 0
\]

(12)

The function \(\hat{\phi}\) can be related to the specific acoustic admittance by the formula

\[
y = \gamma pc \frac{u'}{p'} = -\frac{\gamma p c \zeta}{q^2 \zeta + i(w - i\lambda)}
\]

(13)
where $\zeta = \frac{1}{\xi} \frac{d\xi}{d\phi}$. Using the definition of $\zeta$ and Eq. (12), the following differential equation for $\zeta$ is derived:

$$\frac{d\zeta}{d\phi} - \frac{B}{A} \zeta + \zeta^2 = -\frac{C}{A} \tag{14}$$

where

$$A = \bar{q}^2(\bar{c}^2 - \bar{q}^2)$$

$$B = \bar{q}^{\frac{1}{2}} \left[ \frac{1}{2} \frac{d\bar{q}^2}{d\phi} + 2i(w - i\lambda) \right]$$

$$C = \left[ (w - i\lambda)^2 - \frac{S_{mn}^2 - c^2}{c^2} - i(w - i\lambda) \frac{\gamma - 1}{2} \frac{\bar{q}^2}{c^2} \frac{d\bar{q}^2}{d\phi} \right]$$

Equation (14) is a complex Riccati equation which must be solved numerically to obtain $\zeta$. Once the value of $\zeta$ is determined at the nozzle entrance, the nozzle admittance can be computed directly from Eq. (13).

Inspection of Eq. (14) shows that the value of $\zeta$ depends upon its coefficients $A$, $B$, and $C$ which in turn depend upon $w$, $\lambda$, $S_{mn}$, and the space dependence of $\bar{q}$ and $\bar{c}$ in the nozzle. The behavior of $\bar{q}$ and $\bar{c}$ in the nozzle can be computed once the value of $\gamma$ and the nozzle contour are specified.

To determine $\zeta$ for given values of $w$, $\lambda$, $S_{mn}$ and $\gamma$ and a specific nozzle contour, Eq. (14) must be integrated numerically. A major difficulty which can occur during this integration is that $\zeta$ becomes unbounded whenever $\bar{\xi}$ approaches zero, which causes numerical difficulties in the integration scheme. Crocco and Sirignano noted that this phenomenon occurred for low Mach numbers and high values of $w/S_{mn}$. At these Mach numbers and frequencies they developed asymptotic solutions for $\zeta$.

Instead of using the asymptotic solution, an exact numerical solution is obtained in this study. The problem is resolved by introducing a new dependent variable

$$\tau = \frac{1}{\zeta} = -\frac{\bar{\xi}}{d\bar{\xi}}$$
As \( \Phi \) approaches zero and the magnitude of \( \zeta \) becomes large, \( \tau \) becomes small. Introducing the definition of \( \tau \) into Eq. (14) gives the following Riccati equation for \( \tau \)

\[
\frac{d\tau}{d\Phi} + \frac{B}{A} \tau - \frac{C}{A} \tau^2 = 1
\]  

(15)

At those regions where \( \zeta \) becomes unbounded, Eq. (15) is integrated instead of Eq. (14).

**Method of Solution**

To obtain the nozzle admittance from Eq. (13), values of \( \zeta \) and \( \tau \) are computed by numerically integrating Eq. (14) or (15). To evaluate the coefficients \( A, B, \) and \( C \), a differential equation that describes the variations of the steady state velocity in the subsonic portion of the nozzle must be derived. Differentiating the continuity equation

\[
\frac{\partial^2 q}{\partial r^2} = \frac{\rho_{th}}{r_{th}^2} q = \text{constant}
\]  

(16)

where \( \frac{\partial^2 q}{\partial r^2} = \frac{c_{th}^2}{c_{th}^2} = 2/(\gamma + 1) \), and using Eq. (4) yield the following differential equation

\[
\frac{dq}{dr} = -\frac{1}{r_{th}(\gamma + 1)} \left[ \frac{1}{q}\left( \frac{2}{q-2} \right) \right] \frac{2\gamma - 1}{2(\gamma - 1)}
\]  

(17)

Using Eq. (17) and the specified nozzle contour in terms of \( r(z) \), the quantity \( dq/d\zeta \) can be obtained from the relationship

\[
\frac{d\zeta^2}{d\Phi} = \frac{dq^2}{dr} \frac{dr}{dz} \frac{dz}{d\Phi} = 2 \frac{dq}{dr} \frac{dr}{dz}
\]  

(18)

Once \( \zeta^2 \) is known the corresponding value of \( c^2(\Phi) \) can be obtained by use of Eq. (4). To evaluate \( dr/dz \) in Eq. (18), the nozzle contour shown in Fig. 2 is used. Starting at the combustion chamber the contour is generated by a circular arc of radius \( r_{cc} \) turned through an angle \( \theta_1 \), the nozzle half-angle. This arc connects smoothly to a straight line which is inclined
at an angle $\theta_1$ to the nozzle axis. This straight line then joins with another circular arc of radius $r_{ct}$ which turns through an angle $\theta_1$ and ends at the throat. Using this nozzle contour, in regions I, II and III of Fig. 2

$$\frac{dr}{dz}\bigg|_I = -\frac{\left[2r_{ct}(r-r_{th}) - (r-r_{th})^2\right]^{\frac{1}{2}}}{r_{ct} + r_{th} - r}$$

$$\frac{dr}{dz}\bigg|_{II} = -\tan \theta_1$$

$$\frac{dr}{dz}\bigg|_{III} = \frac{\left[2r_{cc}(1-r) - (1-r)^2\right]^{\frac{1}{2}}}{1 - r_{cc} - r}$$

Utilizing the appropriate expression for $dr/dz$, Eq. (18) can now be solved simultaneously with Eq. (14) or (15) to determine the nozzle admittance.

The numerical integration of these equations must start at some initial point where the initial conditions are known. Since the equation for $\zeta$ is singular at the throat, the integration is initiated at a point that is located a short distance upstream of the throat. The needed initial conditions are obtained by expanding the dependent variables in a Taylor series about the throat. To obtain this Taylor series, its coefficients $\zeta(0) = \zeta_0$ and $\zeta_1 = \frac{dr}{d\phi}\bigg|_{\phi = 0}$ must be evaluated at the throat where $\phi = 0$. These coefficients are evaluated by substituting the series

$$\zeta = \zeta_0 + \zeta_1 \phi + \ldots$$

into Eq. (14) and taking the limit as $\phi \to 0$. The results are

$$\zeta_0 = \zeta(0) = \frac{C_0}{B_0}$$

$$\zeta_1 = \frac{dr}{d\phi}\bigg|_{\phi = 0} = \left[B_1\left(C_0\right) - A_1\left(C_0\right)^2 - C_1\right]/(A_1 - B_0)$$

where
\[ c_0 = c \bigg| \phi = 0 = \left[ \frac{(w - i\lambda)^2}{(\gamma + 1)(w - i\lambda)} - \frac{2}{r_{th} r_{ct}} \right] \]

\[ b_0 = b \bigg| \phi = 0 = \frac{4}{\gamma + 1} \left[ \frac{1}{r_{th} r_{ct}} + i(w - i\lambda) \right] \]

\[ b_1 = \frac{db}{d\phi} \bigg| \phi = 0 = \frac{4}{\gamma + 1} \left[ \frac{1}{r_{th} r_{ct}} + i(w - i\lambda) \right] \]

\[ a_1 = \frac{da}{d\phi} \bigg| \phi = 0 = \frac{-4}{(\gamma + 1)(r_{th} r_{ct})} \]

\[ c_1 = \frac{dc}{d\phi} \bigg| \phi = 0 = \frac{2}{(\gamma + 1)(w - i\lambda)} \left[ \frac{2}{r_{th} r_{ct}} - \frac{i(w - i\lambda)}{3} \right] (6 + \gamma) \]

The following relations are used in the evaluation of the above quantities:

\[ \frac{d^2}{d\phi^2} \bigg| \phi = 0 = \frac{2}{\gamma + 1} \]

\[ \frac{d^2}{d\phi^2} \bigg| \phi = 0 = \frac{4}{(\gamma + 1)(r_{th} r_{ct})} \]

Once \( \xi_0 \) and \( \xi_1 \) are known, the initial condition at \( \phi = \phi_1 \) is obtained from the expression \( \xi(\phi_1) = \xi_0 + \xi_1 \phi_1 \).

The numerical solution is obtained by use of a modified Adams predictor-corrector scheme, and employing a Runge-Kutta scheme of order four to start the numerical integration. Initially, Eqs. (14) and (18) are integrated to determine \( \xi \); if the magnitude of \( \xi \) exceeds a specified value at which numerical difficulties can occur, the integration of Eq. (14) is terminated. Using the value of \( \xi \) at that point, \( \tau \) is computed and the
integration proceeds using Eq. (15). Similarly, should the magnitude of \( \tau \) become excessively large, the integration of Eq. (15) is terminated, \( \zeta \) is computed from the value of \( \tau \) at that point, and the integration proceeds using Eq. (14). This process is repeated until the nozzle entrance is reached. A computer program utilizing this procedure has been written in FORTRAN V for use on the UNIVAC 1108 computer and it is presented in the Appendix.

RESULTS AND DISCUSSION

Using the previously mentioned computer program, theoretical values of the real and imaginary parts of the nozzle admittance have been computed for several nozzle configurations having contours similar to the one presented in Fig. 2. In these computations the radii of curvature, \( r_{cc} \) and \( r_{ct} \), are assumed to be equal. The admittance values are presented as functions of the nondimensional frequency \( S \) in Figs. 3 through 9 where they are compared with available experimental data obtained from Ref. 3. In these figures, the frequency has been nondimensionalized by the ratio of the steady state speed of sound at the nozzle entrance to the chamber radius \( r_c \).

Admittances for Longitudinal Modes

Longitudinal-type instabilities in general occur in the range of \( S \) from 0 to approximately 1.8 which is in the vicinity of the cutoff frequency of the first tangential modes. The cutoff frequency of a particular transverse mode is \( S_{mn} \sqrt{(1 - M^2)} \) where \( S_{mn} \) is the transverse mode eigenvalue and the subscripts \( m \) and \( n \) respectively denote the number of diametral nodal lines and the number of tangential nodal lines. Values of \( S_{mn} \) are given in Table 1 for several values of \( m \) and \( n \).

For longitudinal modes good agreement exists between the experimental and theoretical values of the real and imaginary parts of the admittance as shown in Figs. 3 through 5. The effect of changing the nozzle half-angle is presented in Fig. 3 for a nozzle with an entrance Mach number \( M \) of 0.08 and \( r_{cc}/r_c = 0.44 \). The data indicate that increasing \( \theta_1 \) increases the frequency at which the real and imaginary parts of the admittance attain maximum values. These data also indicate that the assumption of a one-dimensional mean flow
Table 1. Values of Transverse Mode Eigenvalues; $S_{mn}$

<table>
<thead>
<tr>
<th>Transverse Wave Pattern</th>
<th>$m$</th>
<th>$n$</th>
<th>$S_{mn}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>First Tangential (1T)</td>
<td>1</td>
<td>0</td>
<td>1.8413</td>
</tr>
<tr>
<td>Second Tangential (2T)</td>
<td>2</td>
<td>0</td>
<td>3.0543</td>
</tr>
<tr>
<td>First Radial (1R)</td>
<td>0</td>
<td>1</td>
<td>3.8317</td>
</tr>
<tr>
<td>Third Tangential (3T)</td>
<td>3</td>
<td>0</td>
<td>4.2012</td>
</tr>
<tr>
<td>Fourth Tangential (4T)</td>
<td>4</td>
<td>0</td>
<td>5.3175</td>
</tr>
<tr>
<td>First Tangential, First Radial (1T,1R)</td>
<td>1</td>
<td>1</td>
<td>5.3313</td>
</tr>
<tr>
<td>Fifth Tangential (5T)</td>
<td>5</td>
<td>0</td>
<td>6.4154</td>
</tr>
<tr>
<td>Second Tangential, First Radial (2T,1R)</td>
<td>2</td>
<td>1</td>
<td>6.7060</td>
</tr>
<tr>
<td>Second Radial (2R)</td>
<td>0</td>
<td>2</td>
<td>7.0156</td>
</tr>
</tbody>
</table>

used in the development of the theory appears to be valid. Even for nozzles with half-angles as high as 45 degrees, for which it has been shown that the mean flow is two-dimensional, the experimental and theoretical nozzle admittance values are in good agreement.

Examination of Fig. 4 shows that the entrance Mach number $M$ has a significant effect on the admittance values for $\theta_1 = 15$ degrees and $r_{cc}/r_c = 0.44$. However, increasing the nozzle half-angle appears to decrease the influence of the entrance Mach number, and for $\theta_1 = 45$ degrees variations in $M$ has little effect. The dependence of the nozzle admittance upon the radius of curvature for a nozzle with $M = 0.16$ and $\theta_1 = 30$ degrees is shown in Fig. 5.

The data presented in Figs. 3 through 5 show that for longitudinal modes the real part of the nozzle admittance is always positive. As indicated by Crocco, positive values of the real part of the nozzle admittance imply that the nozzle removes acoustic energy from the combustor wave system which implies that the nozzle exerts a stabilizing influence upon the chamber oscillations.

In combustion instability analyses of liquid-propellant rocket motors, it is often assumed that the nozzle is short. This assumption implies that the nozzle length and throat diameter are much smaller than the chamber length and diameter so that the wave travel time in the nozzle is much shorter than the wave travel time in the chamber. For a short nozzle the real and imaginary
parts of the admittance are independent of frequency and are given by the expressions\(^5\)

\[ y_r = \frac{\gamma - 1}{2} M; \quad y_i = 0 \]

These theoretical short nozzle admittance results do not agree with the results obtained for typical liquid rocket nozzles presented in Figs. 3 through 5. The disagreement is especially evident for nozzles with low values of \(\theta_1\), which imply that the nozzle is long, and for high values of \(S\) where the wave length of the oscillation becomes of the same order of magnitude as a characteristic nozzle dimension.

**Admittances for Mixed First Tangential-Longitudinal Modes**

The mixed first tangential-longitudinal modes are those three-dimensional modes which exist between the cutoff frequencies of the first tangential \((S \simeq 1.8)\) and second tangential \((S \simeq 3.0)\) modes. Theoretical and experimental nozzle admittance data for these modes are presented in Figs. 6 through 8.

In Fig. 6 the influence of the nozzle half-angle on the admittance values is shown. The theoretical and experimental results are in good agreement and they indicate that increasing \(\theta_1\) increases the frequency at which the real and imaginary parts of the admittance reach maximum values.

The effect of Mach number on the admittance values is presented in Fig. 7 for \(\theta_1 = 15\) degrees and \(r_{oc}/r_c = 0.44\). Mach number effects are especially significant at the higher frequencies. However, as shown in Ref. 3, increasing the nozzle half-angle decreases the dependence of the admittance values on the Mach number. The effect of changing the radii of curvature on the admittance values is presented in Fig. 8.

The results presented in Figs. 6 through 8 show that for mixed first tangential-longitudinal modes the real part of the nozzle admittance can be negative which means that the nozzle radiates wave energy back into the combustor; this process exerts a destabilizing influence on the oscillations in the chamber.\(^2\) These negative values occur only for three-dimensional modes and, as shown by Crocco,\(^2\) their cause can be traced to the term involving \(S_{mn}\) in Eq. (12). For longitudinal modes, for which \(S_{mn}\)
is zero, the real part of the nozzle admittance is always positive, and for those modes the nozzle always exerts a stabilizing influence upon the combustor oscillations.

Effect of Decay Coefficient upon Admittance Data

The nozzle admittance theory has been modified to include the effects of a temporal decay coefficient, \( \lambda \). Typical results are shown in Figs. 9 and 10 for values of \( \lambda \) of -0.05, 0, and 0.05. These results indicate that varying \( \lambda \) affects both the real and imaginary parts of the admittance. Therefore, the decay coefficient should be included in the nozzle admittance computations when the oscillations are not neutrally stable.

SUMMARY AND CONCLUSIONS

The equations necessary to determine the nozzle admittance for one- and three-dimensional oscillations have been developed. The analytical approach used in solving the nozzle wave equations is outlined and employed to obtain nozzle admittance data for typical nozzle configurations. These data show the dependence of the nozzle admittance values upon nozzle geometry, nozzle Mach number, mode of oscillation, and the temporal damping coefficient.

The results can be summarized as follows for longitudinal and mixed first tangential-longitudinal modes. Decreasing the nozzle length by increasing the nozzle half-angle and Mach number or by decreasing the throat and entrance radii of curvature decreases the frequency dependence of the nozzle admittance. Good agreement exists between the theoretical predictions and available experimental data. However, the nozzle admittance values for typical liquid rocket nozzles are not in agreement with the values obtained from short nozzle theory. Including the effects of a temporal damping coefficient in the nozzle admittance computations changes the admittance values. Therefore, when the oscillations are not neutrally stable, the temporal decay coefficient should be accounted for in the computations.
Figure 1. Typical Mathematical Model of a Liquid Rocket Engine
Figure 2. Nozzle Contour
Figure 3. The Effect of Nozzle Half-Angle on the Theoretical and Experimental Nozzle Admittance Values for Longitudinal Modes
Figure 4. The Effect of Entrance Mach Number on the Theoretical and Experimental Nozzle Admittance Values for Longitudinal Modes.
Figure 5. The Effect of the Radii of Curvature on the Theoretical and Experimental Nozzle Admittance Values for Longitudinal Modes
Figure 6. The Effect of the Nozzle Half-Angle on the Theoretical and Experimental Nozzle Admittance Values for Mixed First Tangential-Longitudinal Modes
Figure 7. The Effect of Entrance Mach Number on the Theoretical and Experimental Nozzle Admittance Values for Mixed First Tangential-Longitudinal Modes
Figure 8. The Effect of the Radii of Curvature on the Theoretical and Experimental Nozzle Admittance Values for Mixed First Tangential-Longitudinal Modes
Figure 9. Effect of the Temporal Decay Coefficient on the Theoretical Nozzle Admittance Values for Longitudinal Modes

\[ \lambda = -0.05 \]
\[ \lambda = 0 \]
\[ \lambda = 0.05 \]

\[ M = 0.25, \ \theta_1 = 20^\circ \]
\[ r_{cc}/r_c = 1, \ \frac{r_{ct}}{r_c} = 0.92 \]
Figure 10. Effect of the Temporal Decay Coefficient on the Theoretical Nozzle Admittance Values for Mixed First Tangential-Longitudinal Modes
APPENDIX

COMPUTER PROGRAM USED TO DETERMINE THE IRROTATIONAL NOZZLE ADMITTANCE

The computer program for calculating the irrotational nozzle admittance from Crocco's theory which is extended to account for temporal damping is written in FORTRAN V interpretive language compatible with the UNIVAC 1108 machine language compiler. This program consists of seven routines - the main or control program and six subroutines. The names of the routines are listed in Table A-1 in sequential order. The FORTRAN symbols used in these routines and their definitions are presented in Table A-2 in alphabetical order. The input parameters necessary for the admittance computations must be specified in the main program and are listed in Table A-3. The output parameters and their definitions are listed in Table A-4. A detailed flow chart of the computer program is shown in Fig. A-1, and the program listing and sample output are presented in Tables A-5 and A-6, respectively.

This computer program has been written to predict nozzle admittances for nozzle contours shown in Fig. 2. The run time required depends upon the number of admittance values desired and the nozzle length. To obtain 40 admittance values at different frequencies for the nozzles investigated in this study, one to two minutes of run time on the UNIVAC 1108 computer are required.
Table A-1. List of Subroutines in the Computer Program Used to Determine the Irrotational Nozzle Admittance

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAIN</td>
<td>Specifies the nozzle geometry and operating conditions in the converging section of the nozzle</td>
</tr>
<tr>
<td>NOZADM</td>
<td>Specifies initial conditions at the throat, computes the final nozzle admittance values, and contains all output formats</td>
</tr>
<tr>
<td>RKZT</td>
<td>Uses the Runge-Kutta of order four to obtain initial values for the modified Adams integration routine</td>
</tr>
<tr>
<td>RKZDIF</td>
<td>Computes the differential element in the converging section of the nozzle used to solve Eq. (14)</td>
</tr>
<tr>
<td>RKTDIF</td>
<td>Computes the differential element in the converging section of the nozzle used to solve Eq. (15)</td>
</tr>
<tr>
<td>ZADAMS</td>
<td>Numerically integrates Eq. (14) using the modified Adams numerical integration scheme</td>
</tr>
<tr>
<td>TADAMS</td>
<td>Numerically integrates Eq. (15) using the modified Adams numerical integration scheme</td>
</tr>
<tr>
<td>Variable</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>A</td>
<td>Real coefficient A of Eqs. (14) and (15)</td>
</tr>
<tr>
<td>A(5)</td>
<td>Coefficients of the Runge-Kutta formulas of order four</td>
</tr>
<tr>
<td>AF</td>
<td>Nondimensional temporal damping coefficient λ</td>
</tr>
<tr>
<td>ANGLE</td>
<td>Nozzle half-angle, degrees</td>
</tr>
<tr>
<td>ALR</td>
<td>Derivative of the coefficient A evaluated at the throat</td>
</tr>
<tr>
<td>BI</td>
<td>Imaginary part of the coefficient B in Eqs. (14) and (15)</td>
</tr>
<tr>
<td>BR</td>
<td>Real part of the coefficient B in Eqs. (14) and (15)</td>
</tr>
<tr>
<td>BOI</td>
<td>Value of BI at the throat</td>
</tr>
<tr>
<td>BOR</td>
<td>Value of BR at the throat</td>
</tr>
<tr>
<td>B1I</td>
<td>Derivative of BI evaluated at the throat</td>
</tr>
<tr>
<td>B1R</td>
<td>Derivative of BR evaluated at the throat</td>
</tr>
<tr>
<td>C</td>
<td>Nondimensional speed of sound squared, $c^2$</td>
</tr>
<tr>
<td>CI</td>
<td>Imaginary part of the coefficient C in Eqs. (14) and (15)</td>
</tr>
<tr>
<td>CM</td>
<td>Mach number at the nozzle entrance</td>
</tr>
<tr>
<td>COR(5)</td>
<td>Formula for the corrector in the modified Adams integration routine</td>
</tr>
<tr>
<td>CR</td>
<td>Real part of the coefficient C in Eqs. (14) and (15)</td>
</tr>
<tr>
<td>COI</td>
<td>Value of CI at the throat</td>
</tr>
<tr>
<td>COR</td>
<td>Value of CR at the throat</td>
</tr>
<tr>
<td>CLI</td>
<td>Derivative of CI evaluated at the throat</td>
</tr>
<tr>
<td>C1R</td>
<td>Derivative of CR evaluated at the throat</td>
</tr>
<tr>
<td>DP</td>
<td>Integration stepsize</td>
</tr>
<tr>
<td>DP(5)</td>
<td>Derivative used in the corrector formula in the modified Adams integration routine</td>
</tr>
<tr>
<td>DR</td>
<td>Derivative of the local wall radius with respect to axial distance</td>
</tr>
<tr>
<td>DU</td>
<td>Derivative of the nondimensional velocity $\bar{q}^2$ with respect to the wall radius r</td>
</tr>
<tr>
<td>DWC</td>
<td>Increment of the nondimensional frequency $\bar{\omega}$</td>
</tr>
<tr>
<td>Variable</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>DY(5,4)</td>
<td>Derivative used in the modified Adams integration scheme</td>
</tr>
<tr>
<td>F</td>
<td>Constant given as $\frac{\bar{q}}{\bar{p}}$ evaluated at the nozzle entrance</td>
</tr>
<tr>
<td>FZ(4,5)</td>
<td>Derivative used in the Runge-Kutta method</td>
</tr>
<tr>
<td>Fl</td>
<td>Lumped parameter determined by the conditions at the throat</td>
</tr>
<tr>
<td>F2</td>
<td>Lumped parameter determined by the conditions at the throat</td>
</tr>
<tr>
<td>GAM</td>
<td>Ratio of specific heats $\gamma$</td>
</tr>
<tr>
<td>G(5)</td>
<td>Dependent variable in the Runge-Kutta integration routine</td>
</tr>
<tr>
<td>H</td>
<td>Integration stepsize</td>
</tr>
<tr>
<td>I</td>
<td>Integer counter</td>
</tr>
<tr>
<td>IP</td>
<td>Integer constant. If IP = 0 the nozzle admittance is output. If IP ≠ 0 the amplitude and phase of the pressure oscillation are output along the length of the nozzle</td>
</tr>
<tr>
<td>IQ</td>
<td>If IQ = 2, the integration of Eq. (15) for $\tau$ is complete</td>
</tr>
<tr>
<td>IQZ</td>
<td>= 1: Eq. (15) for $\tau$ is integrated</td>
</tr>
<tr>
<td></td>
<td>= 2: Eq. (14) for $\zeta$ is integrated</td>
</tr>
<tr>
<td>J</td>
<td>Integer variable</td>
</tr>
<tr>
<td>JOPT</td>
<td>= 1: Eq. (15) for $\tau$ is integrated</td>
</tr>
<tr>
<td></td>
<td>= 2: Eq. (14) for $\zeta$ is integrated</td>
</tr>
<tr>
<td>K</td>
<td>Integer variable</td>
</tr>
<tr>
<td>N</td>
<td>Integer variable</td>
</tr>
<tr>
<td>NU</td>
<td>Number of differential equations to be solved by the Runge-Kutta or the modified Adams integration routine</td>
</tr>
<tr>
<td>NWC</td>
<td>Number of frequency points</td>
</tr>
<tr>
<td>P</td>
<td>Value of the steady state velocity potential</td>
</tr>
<tr>
<td>PARG</td>
<td>Phase of the pressure oscillation in the nozzle</td>
</tr>
<tr>
<td>PHI</td>
<td>Imaginary part of $\phi$</td>
</tr>
<tr>
<td>PHIR</td>
<td>Real part of $\phi$</td>
</tr>
<tr>
<td>Variable</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>PI</td>
<td>Imaginary part of the pressure oscillation</td>
</tr>
<tr>
<td>PMAG</td>
<td>Magnitude of the pressure oscillation</td>
</tr>
<tr>
<td>PR</td>
<td>Real part of the pressure oscillation</td>
</tr>
<tr>
<td>PRED(5)</td>
<td>Predictor formula for the modified Adams integration routine</td>
</tr>
<tr>
<td>Q</td>
<td>Constant given as ((r_{th}/4)(\frac{2}{(\gamma + 1)}))</td>
</tr>
<tr>
<td>QBAR</td>
<td>Nondimensional steady state velocity (\bar{q})</td>
</tr>
<tr>
<td>R</td>
<td>Local wall radius (r)</td>
</tr>
<tr>
<td>RCC</td>
<td>Ratio of the radius of curvature at the nozzle entrance to the radius at the nozzle entrance</td>
</tr>
<tr>
<td>RCT</td>
<td>Ratio of the radius of curvature at the throat to the radius at the nozzle entrance</td>
</tr>
<tr>
<td>RHO</td>
<td>Nondimensional, steady-state density (\bar{\rho})</td>
</tr>
<tr>
<td>RT</td>
<td>Nondimensional throat radius</td>
</tr>
<tr>
<td>R1</td>
<td>Nondimensional radius at the entrance to Section 2 of the converging portion of the nozzle</td>
</tr>
<tr>
<td>R2</td>
<td>Nondimensional radius at the entrance to Section 3 of the converging portion of the nozzle</td>
</tr>
<tr>
<td>SRTR</td>
<td>Constant give as (\sqrt{r_{th}r_{cc}/r_c})</td>
</tr>
<tr>
<td>SVN</td>
<td>(s_{mn})</td>
</tr>
<tr>
<td>SVNFR</td>
<td>(s_{mn}r_c/r_{th})</td>
</tr>
<tr>
<td>SYI</td>
<td>Imaginary part of the specific admittance (\gamma)</td>
</tr>
<tr>
<td>SYR</td>
<td>Real part of the specific admittance (\gamma)</td>
</tr>
<tr>
<td>T</td>
<td>Nozzle half-angle, in radians</td>
</tr>
<tr>
<td>TDN</td>
<td>Inverse of the square of the magnitude of (\zeta)</td>
</tr>
<tr>
<td>TI</td>
<td>Imaginary part of (\tau)</td>
</tr>
<tr>
<td>TMAG</td>
<td>Magnitude of (\tau)</td>
</tr>
<tr>
<td>TPI</td>
<td>Derivative of TI with respect to (\phi)</td>
</tr>
<tr>
<td>Variable</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>TPR</td>
<td>Derivative of TR with respect to ( \varphi )</td>
</tr>
<tr>
<td>TR</td>
<td>Real part of ( \tau )</td>
</tr>
<tr>
<td>TZ</td>
<td>Value of ( \varphi ) at the nth integration point</td>
</tr>
<tr>
<td>T2</td>
<td>Square of the magnitude of ( \tau )</td>
</tr>
<tr>
<td>U</td>
<td>Steady state velocity squared, ( q^2 )</td>
</tr>
<tr>
<td>UZ</td>
<td>Dependent variable in the Runge-Kutta integration scheme</td>
</tr>
<tr>
<td>W</td>
<td>Nondimensional frequency ( S )</td>
</tr>
<tr>
<td>WC</td>
<td>Nondimensional frequency ( \omega )</td>
</tr>
<tr>
<td>X</td>
<td>Value of ( \varphi ) at the nth integration point</td>
</tr>
<tr>
<td>Y(5)</td>
<td>Dependent variable used in the modified Adams integration scheme</td>
</tr>
<tr>
<td>YI</td>
<td>Imaginary part of the irrotational nozzle admittance defined by Crocco in Ref. 2</td>
</tr>
<tr>
<td>YR</td>
<td>Real part of the nozzle admittance defined by Crocco in Ref. 2</td>
</tr>
<tr>
<td>ZDN</td>
<td>Inverse of the square of the magnitude of ( \zeta )</td>
</tr>
<tr>
<td>ZI</td>
<td>Imaginary part of ( \zeta )</td>
</tr>
<tr>
<td>ZMAG</td>
<td>Magnitude of ( \zeta )</td>
</tr>
<tr>
<td>ZPI</td>
<td>Derivative of ( ZI ) with respect to ( \varphi )</td>
</tr>
<tr>
<td>ZPR</td>
<td>Derivative of ( ZR ) with respect to ( \varphi )</td>
</tr>
<tr>
<td>ZR</td>
<td>Real part of ( \zeta )</td>
</tr>
<tr>
<td>ZOI</td>
<td>Value of ( ZI ) at the throat</td>
</tr>
<tr>
<td>ZOR</td>
<td>Value of ( ZR ) at the throat</td>
</tr>
<tr>
<td>ZLI</td>
<td>Value of ( ZPI ) at the throat</td>
</tr>
<tr>
<td>ZLR</td>
<td>Value of ( ZPR ) at the throat</td>
</tr>
<tr>
<td>Z2</td>
<td>Square of the magnitude of ( \zeta )</td>
</tr>
</tbody>
</table>
Table A-3. Input Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAM</td>
<td>Ratio of specific heats, $\gamma$</td>
</tr>
<tr>
<td>CM</td>
<td>Mach number at the nozzle entrance</td>
</tr>
<tr>
<td>SVN</td>
<td>$N$th root of the equation $\frac{dJ_y(x)}{dx} = 0$. Corresponds to $S_m$. Values of $S_m$ are given in Table 1 for various acoustic modes</td>
</tr>
<tr>
<td>WC</td>
<td>Initial value of $\omega$</td>
</tr>
<tr>
<td>DWC</td>
<td>Increment of frequency</td>
</tr>
<tr>
<td>NWC</td>
<td>Number of frequency points desired</td>
</tr>
<tr>
<td>ANGLE</td>
<td>Nozzle half-angle, degrees</td>
</tr>
<tr>
<td>RCT</td>
<td>Radius of curvature at the throat nondimensionalized with respect to the chamber radius</td>
</tr>
<tr>
<td>RCC</td>
<td>Radius of curvature at the nozzle entrance nondimensionalized with respect to the chamber radius</td>
</tr>
<tr>
<td>IP</td>
<td>$= 0$: nozzle admittances are printed $\neq 0$: pressure magnitude and phase are printed at each point along the nozzle</td>
</tr>
<tr>
<td>AF</td>
<td>Temporal damping coefficient $\lambda$</td>
</tr>
<tr>
<td>Variable</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>WC</td>
<td>Nondimensional frequency, $\omega$</td>
</tr>
<tr>
<td>YR</td>
<td>Real part of the admittance as defined by Crocco in Ref. 2</td>
</tr>
<tr>
<td>YI</td>
<td>Imaginary part of the admittance as defined by Crocco in Ref. 2</td>
</tr>
<tr>
<td>W</td>
<td>Nondimensional frequency</td>
</tr>
<tr>
<td>SYR</td>
<td>Real part of the specific admittance $y$</td>
</tr>
<tr>
<td>SYI</td>
<td>Imaginary part of the specific admittance $y$</td>
</tr>
</tbody>
</table>
Table A-5. Listing of the Computer Program Used to Determine the Irrotational Nozzle Admittance (Page 1 of 10)

```
1* COMMON/X1/5AM, SVN, ANGLE, RCT, RCC /X2/T,RT, Q, R1, R2, IP, WC, AF
2* COMMON/X3/Z1R, ZII
3* COMMON/X4/CW
4* GAM = 1.233
5* AF = 0
6* IP=0
7* RCC = 1
8* RCT = 5.457*2/11.82
9* NC = 40
10* D1,C = 0.05
11* ANGLE = 20
12* CM = 0.25
13* DO 1:0 I = 1.2
14* IF(I,.EQ.2) GO TO 5
15* SVN = 0
16* NC = 27
17* GO TO 20
18* 5 SVN = 1.84129
19* NC = 20
20* CONTINUE
21* DO 2:0 J = 1.3
22* AF = 0.05*(J-2)
23* IF (1,.LE.0.2) GO TO 25
24* WC = 0.55
25* GO TO 30
26* 25 WC = 1.55
27* CONTINUE
28* IF(IP,.EQ.0) GO TO 10
29* WRITE(6, 1000) CM, SVN, GAM, ANGLE, RCT, RCC
30* 10 CALL NOZADW(CM, NW, DWC).
31* CONTINUE
32* 200 CONTINUE
33* 1000 FORMT(46X, 28MPRESSURE MAGNITUDE AND PHASE, //, 38X, 1
34* 1 14MACH NUMBER =, F3.2, 7H SVN =, F6.4, 9H GAMMA =, F3.1
35* 2 //, 22X, 15NOZZLE ANGLE =, F4.1, 21H RADIUS OF CURVATURE!
36* 3 //, 22X, 15NOZZLE ANGLE =, F4.1, 21H RADIUS OF CURVATURE!
37* 4 //, 22X, 15NOZZLE ANGLE =, F4.1, 21H RADIUS OF CURVATURE!
38* STOP
39* END
```
Table A-5. Continued (Page 2 of 10)

SUBROUTINE NO2ADM(CM, NW, DWC)

DIMENSION D(5, P, G(5), T(5))

COMMON /X/0, G31, G32, G33, R3, R31, R32, R33, T3, T31, T32, T33

COMMON /X/0, G31, G32, G33, R3, R31, R32, R33, T3, T31, T32, T33

T = 3,1415927 * ANGLE / 180

RITE(6, 1000) CM, SVN, GAM, AF, ANGLE, RCT, RCC

DO 10 N = 1, NWC

20 W = WC + DWC

25 RT = (CM**2/4) * (((CM-1) * CM/2) - ((CM-1)/(4*CM)) * ((CM-1)/(4*CM-1)))

10 CONTINUE

Q = (0.25 * RT) * ((2/(GAM + 1)) - ((2/(GAM + 1)) * (GAM + 1)/(GAM + 1)))

PHIR = 1

P = 0

10 CONTINUE

PHII = 0

15 R1 = RT + RCT * (1 - COS(T))

16 R2 = 1 - RCT * (1 - COS(T))

R = RT

18 P = 0

19 U = 2 / (GAM + 1)

SRTT = (RT * RCT) * 0.5

AIR = 4 / (GAM + 1) * SRTT

BOR = -AIR * 4 * AF / (GAM + 1)

BOI = 4 * WC / (GAM + 1)

SVNR = SVN / RT

COR = WC * WC - (SVNR * SVNR) / 2 / (GAM + 1)

1 AF = -AF * AF - 2 * AF * (GAM - 1) / ((GAM + 1) * SRTT)

COI = -2 * WC * (GAM - 1) / (GAM + 1) * SRTT - 2 * AF * WC

BIR = (24 + 4 * GAM) / (3 * RCT * RT * (GAM + 1)) - 6 * AF / SRTT * (GAM + 1)

BII = 8 * WC / (SRTT * (GAM + 1))

CII = 2 * (GAM - 1) * SVNR * SVNR / (SRTT * (GAM + 1))

CII = AF * (BIR * BAF / (SRTT * (GAM + 1)) * (GAM - 1) / 0.5

CII = -AIR * WC * (GAM - 1) / 0.5

ZOR = (BIR + COI + BOI) / (BOI + BOR + BOI)

ZOI = (BIR * COI - BOI * COR) / (BOR + BOR + BOI)

F1 = BIR + ZOR + BII + ZOI = ZOR + ZOR * AIR + AIR + AIR + ZOI

F2 = BIR + ZOR + BII + ZOI - 2 * AIR + AIR + ZOI + AIR + CII

ZI = (F1 + AIR - BOR) + (F2 + BOI) / ((AIR - BOR) * (AIR + BOR) + BOI)

ZII = (F2 * AIR - BOR) + F1 * BOI) / ((AIR - BOR) * (AIR + BOR) + BOI)

C = U

41 G(1) = U

42 G(2) = ZOR

43 G(3) = ZOI

44 G(4) = PHIR * ZOR + PHII + ZOI

45 G(5) = PHII + ZOR + ZOI + PHIR

47 DTY(1, 1) = -AIR

48 DTY(2, 1) = ZIR

49 DTY(3, 1) = ZII

50 DTY(4, 1) = PHIR

51 DTY(5, 1) = PHII

52 IQZ = 2

DO 30 I = 2, 4

CALL RKT7(5, DP, P, G, DP, IQZ)

30 P = P + DP

56 U = G(1)

57 ZR = G(2)

58 ZI = G(3)

59 PHIR = G(4)

PHII = G(5)
Table A-5. Continued (Page 3 of 10)

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>61</td>
<td></td>
<td>(\text{DY}(1, 1) = \text{GP}(1))</td>
</tr>
<tr>
<td>62</td>
<td></td>
<td>(\text{DY}(2, 1) = \text{GP}(2))</td>
</tr>
<tr>
<td>63</td>
<td></td>
<td>(\text{DY}(3, 1) = \text{GP}(3))</td>
</tr>
<tr>
<td>64</td>
<td></td>
<td>(\text{DY}(4, 1) = \text{GP}(4))</td>
</tr>
<tr>
<td>65</td>
<td>30</td>
<td>(\text{DY}(5, 1) = \text{GP}(5))</td>
</tr>
<tr>
<td>66</td>
<td></td>
<td>(Y(1) = U)</td>
</tr>
<tr>
<td>67</td>
<td></td>
<td>(Y(2) = ZR)</td>
</tr>
<tr>
<td>68</td>
<td></td>
<td>(Y(3) = ZI)</td>
</tr>
<tr>
<td>69</td>
<td></td>
<td>(Y(4) = P_{II})</td>
</tr>
<tr>
<td>70</td>
<td></td>
<td>(Y(5) = P_{II})</td>
</tr>
<tr>
<td>71</td>
<td></td>
<td>\text{CALL ZIAMS(5, DP, P, Y, DY, IGZ)}</td>
</tr>
<tr>
<td>72</td>
<td></td>
<td>\text{IF IP, EQ. 1} 30 TO 10</td>
</tr>
<tr>
<td>73</td>
<td></td>
<td>(U = Y(1))</td>
</tr>
<tr>
<td>74</td>
<td></td>
<td>(ZR = Y(2))</td>
</tr>
<tr>
<td>75</td>
<td></td>
<td>(ZI = Y(3))</td>
</tr>
<tr>
<td>76</td>
<td></td>
<td>(P_{II} = Y(4))</td>
</tr>
<tr>
<td>77</td>
<td></td>
<td>(P_{III} = Y(5))</td>
</tr>
<tr>
<td>78</td>
<td></td>
<td>(C = 1 - U^{<em>}C_{5}^{</em>}*(\text{GAM} - 1))</td>
</tr>
<tr>
<td>79</td>
<td></td>
<td>(RHO = C_{5}^{<em>}</em>(1/(\text{GAM} - 1)))</td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>(F = \text{GAM} \text{RHO} \text{ / (GAM + RHO)})</td>
</tr>
<tr>
<td>81</td>
<td></td>
<td>\text{IF IP, EQ. 1} 30 TO 35</td>
</tr>
<tr>
<td>82</td>
<td></td>
<td>(ZJ = U^{<em>}Z_{R}^{</em>} + (U^{<em>}Z_{R}^{</em>} + AF) + (\text{WC} + U^{<em>}ZI) + (\text{WC} + U^{</em>}ZI))</td>
</tr>
<tr>
<td>83</td>
<td></td>
<td>(YI = F^{*}(\text{WC} + ZR - AF + ZI) / ZDN)</td>
</tr>
<tr>
<td>84</td>
<td></td>
<td>(\text{GO TO 30})</td>
</tr>
<tr>
<td>85</td>
<td>35</td>
<td>(TR = Y(2))</td>
</tr>
<tr>
<td>86</td>
<td></td>
<td>(TI = Y(3))</td>
</tr>
<tr>
<td>87</td>
<td></td>
<td>(ZC = (U^{<em>}AC + TR - WC + TI) + (U^{</em>}AF + TR + WC + TI) + (U^{*}TR) + (WC + TR))</td>
</tr>
<tr>
<td>88</td>
<td></td>
<td>(YI = F^{<em>}(C^{</em>}TR + AC + TR + TI) / ZDN)</td>
</tr>
<tr>
<td>89</td>
<td></td>
<td>(YI = F^{*} \text{WC} * TR / ZDN)</td>
</tr>
<tr>
<td>90</td>
<td>40</td>
<td>(SYR = \text{SAM} * (C^{<em>}</em>(1 + \text{GAM}^{<em>}1)/(2</em>(\text{GAM} - 1)))) * YR)</td>
</tr>
<tr>
<td>91</td>
<td></td>
<td>(SYI = \text{SAM} * (C^{<em>}</em>(1 + \text{GAM}^{<em>}1)/(2</em>(\text{GAM} - 1)))) * YI)</td>
</tr>
<tr>
<td>92</td>
<td></td>
<td>(n = \text{WC} * (C^{*} + 5))</td>
</tr>
<tr>
<td>93</td>
<td></td>
<td>\text{WHILE} (U, 1005) WC TR YI, WR SYR, SYI</td>
</tr>
<tr>
<td>94</td>
<td></td>
<td>\text{CONTINUE}</td>
</tr>
<tr>
<td>95</td>
<td>1000</td>
<td>\text{FORM I(1, R, 5X), 30H THEORETICAL NOZZLE ADMITTANCES, //, 25X,}</td>
</tr>
<tr>
<td>96</td>
<td></td>
<td>(1 \text{ MC} \text{ NUMBER} = F_{3}, 2, 7, 4, 9 \text{ GAMMA} = F_{3}, 1)</td>
</tr>
<tr>
<td>97</td>
<td></td>
<td>(1 \text{ DECAY COEFFICIENT} = F_{6}, 4, //,)</td>
</tr>
<tr>
<td>98</td>
<td></td>
<td>(2 \text{ TR + NOZZLE ANGLE} = F_{4}, 1, 2X, 21 \text{ RADIUS OF CURVATURE:})</td>
</tr>
<tr>
<td>99</td>
<td></td>
<td>(3 \text{ TR ENTRANCE} = F_{6}, 4, 12 \text{ ENTRANCE} = F_{6}, 4, //, 34X, 2 \text{ HWC,})</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>(4 \text{ R = TR - 0.5, 8X, 2X, TR, AX, 1HW, 8X, 3HSYR, 8X, 3HSYI, //})</td>
</tr>
<tr>
<td>101</td>
<td>1005</td>
<td>\text{FORM I(3X, F6, 4, 5F10, 5)}</td>
</tr>
<tr>
<td>102</td>
<td></td>
<td>\text{RI, MC}</td>
</tr>
<tr>
<td>103</td>
<td></td>
<td>\text{END}</td>
</tr>
</tbody>
</table>
SUBROUTINE RKTZ(NU, H, T1, U, DUM, JOPT)
COMMON/X2/T*R,T,G,R1,R2,IF,WC,AF
DIMENSION U(5), A(5), UZ(5), FZ(4+5), DUM(5)
A(1) = 0
A(2) = 0
A(3) = 0.5
A(4) = 0.5
A(5) = 1.0
T2 = T1
DO 10 J = 1, NU
   UZ(J) = U(J)
   DUM(J) = FZ(1,J)
10     IF(JOPT .EQ. 2) GO TO 15
14    CALL RKTDIF(TZ, UZ, DUM)
15    GO TO 20
16    CALL RKZDIF(TZ, UZ, DUM)
17    DO 25 J = 1, NU
18    FZ(1,J) = DUM(J)
19    DO 30 I = 2, 4
20    TZ = T1 + A(I+1)*H
21    DO 35 J = 1, NU
22    UZ(J) = U(J) + A(I+1)*H*FZ(I,J)
23    DUM(J) = FZ(I,J)
24    IF(JOPT .EQ. 2) GO TO 40
25    CALL RKTDIF(TZ, UZ, DUM)
26    GO TO 45
27    CALL RKZDIF(TZ, UZ, DUM)
28    DO 45 J = 1, NU
29    FZ(1,J) = DUM(J)
30    CONTINUE
31    DO 50 J = 1, NU
32    U(J) = U(J) + H*(FZ(1,J) + 2*(FZ(2,J) + FZ(3,J)) + FZ(4,J)) / 6.0
33    GO TO (N0+65)*JOPT
34    CALL RKTDIF(TZ, U, DUM)
35    GO TO 70
36    CALL RKZDIF(TZ, U, DUM)
37    IF(I.EQ.0) GO TO 75
38    PR = K2*U(5) - U(1)*DUM(4) = AF*U(4)
39    PI = N2*U(4) - U(1)*DUM(5) = AF*U(5)
40    PMAS = SORT(NP,PR, PI, PI)
41    PARS = ATAN2(PI/PR)
42    WRITE (6,1001) TZ, PMAS, PARS
43    1001 FORMAT (46X, F8.4, 1X, F10.5, 3X, F10.5)
44    RETURN
45    END
Table A-5. Continued (Page 5 of 10)

1. SUBROUTINE RK2DIF(P,G,GP)
2. COMMON/X1/G4,SVN,ANGLE,RCT,RCC/X2/T/RT*Q,R1,R2,IP,WC,AF
3. COMMON/X3/Z*I,PII
4. DIMENSION G(5), GP(5)
5. U = G(1)
6. Z1 = G(2)
7. ZII = G(3)
8. P+IR = G(4)
9. P+II = G(5)
10. IF(P) 15, 10, 15
11. 10 GP(1) = 4*((GAM+1)*((RCT*RT)**0.5))
12. GP(2) = Z1R
13. GP(3) = ZII
14. GP(4) = Z1R
15. GP(5) = ZII
16. GO TO 20
17. 15 C = 1 - (GAM - 1) * U * 0.5
18. R = 0 + ((C)**(-1.5)/(2*(GAM-1))) * (U**1.25) * 4.9
19. IF(R=1) 22, 22, 50
20. IF(R = R1) 25, 30, 30
21. 25 OR = -(2*RST*(R-RT) - (R-RT)*(R-RT))**0.5/(RT+RCT=R)
22. GO TO 45
23. 30 IF(R=R2) 35, 40, 40
24. 35 OR = -TAN(Y)
25. GO TO 45
26. 40 DN = ((2*RCC*(1-R) - (R-1)*(R-1))**0.5)/(1-R=RCC)
27. 45 DU = -(U**0.75)*(C**1/(2*(GAM-1)/(2*(GAM-1)))/(GAM*(1-(GAM+1)*U**5))
28. 1 )
29. GP(1) = DU+CR
30. SW TO 5
31. 5 C = 1 - (GAM - 1) * U * 0.5
32. IF(A) 3, 3, 3
33. 3 U = P(1)/C + 2*AF*U
34. 3 A = 2*A/C
35. CR = -C**A/C + SVN/5/SVN/(C*(R*R)) - AF*AF
36. 1 = -(GAM-1)*AF*U*G(1)*0.5*(1/C)
37. CI = -((GAM-1)**C+U+SP(1)*0.5*(1/C) = 2*AF*WC
38. G:(P) = (((R**2) - BI*ZI - CI) / A) = ZR+ZI + ZI+ZI
39. G:(S) = (((I+ZI) + BI*ZI - CI) / A) = 2*ZI+ZI
40. G:(4) = 2*PHI - ZI+PHII
41. G:(5) = 2*PHII + ZI+PHIR
42. 20 RTJ=4
43. END
SUBROUTINE RKTDIF(P,R,G)

COMMON/X1/GAM,SVN,ANGLE,RCT,RCC/X2/T,R,G,R1,R2,IP,W,C,AF

DIMENSION G(5), GP(5)

U = G(1)

TR = G(2)

TI = G(3)

PHIR = G(4)

PHII = G(5)

C = 1 - (GAM-1)*U*0.5

R = Q * ((C)**(-1/(2*(GAM-1)))) * (U**-0.25) *4.0

IF(R-1) 22,23,00

22 IF(R=R1) 25, 30, 30

23 25 DR = -((2*RT*(R-RT) - (R-RT)*(R-RT))*0.5)/(RT+RCT-R)

GO TO 45

30 IF(R=R2) 35, 40, 40

35 DR = -TAN(T)

GO TO 45

40 DR = ((2*RCT*(1-R) - (R-1)*(R-1))*0.5)/(1-R-RCC)

45 DU = -(U**0.75)*((C**((2*GAM-1)/(2*(GAM-1)))) / (Q*(1-(GAM+1)*U**1

1)

20

GP(1) = DU*DR

22

GO TO 55

23

55 GP(1) = 0

24

A = U*(C-U)

25

BR = U*5P(1)/C + 2*AF*U

26

BI = 2*W*C*U

27

CR = W*C*WC - SVN**2/SV**2/(R*R) - AF*AF

28

CI = -((GAM-1)*AF*U*GP(1)**0.5*(1/C)

29

GP(2) = 1 - ((3R*TR-BI*TI) - (CR*(TR*TR-TI*TII)-2*CI*TR*TI))/ A

30

GP(3) = -TR*TI - BI*TR + CI*(TR*TR-TI*TII) + 2*CR*TR*TI) / A

32

T2 = TR*TR + TI*TII

33

GP(4) = (TR+P+IR - TI*PHIR)/T2

34

GP(5) = (TR+P+II + TI*PHII)/T2

35

RC1 = U

36

END
Table A-5. Continued (Page 7 of 10)

1* SUBROUTINE ZADAMS(N,H,Y,Y0,Y1)
2* COMMON/XI,GAM,SVN,ANGLR,RECT,RC/C/RT0,RT1,R2,IP,NC,AF
3* COMMON/X6/CV
4* DIMENSION COR(5),DP(5),DY(5,4),PRED(5),Y(5),Q(5),GP(5)
5* 10 CONTINUE
6* DO 15 I = 1,N
7* PRED(I) = Y(I)+H*(55.*DY(I,4)-59.*DY(I,3)+37.*DY(I,2)-9.*DY(I,1))/24.0
8* 15 CONTINUE
9* X = X+H
10* U = PRED(1)
11* ZR = PRED(2)
12* ZI = PRED(3)
13* PRI = PRED(4)
14* PII = PRED(5)
15* C = 1 - (GAM-1)*U*0.5
16* R = C + (C)**(-(1/(2*(GAM**-1))))*(U**-0.25)***4.0
17* IF(R<1) 17,17,100
18* 17 IF(R<1) 19,20,25
19* SO T = 40
20* 20 DR = -((2*RC*(R-RT)-(R-RT)*(R-RT))*0.5)/(RT+RCT-R)
21* 25 GP T = 40
22* 35 DR = ((2*RC*(R-RT)-(R-RT)*(R-RT))*0.5)/(1-R-RC)
23* 40 DY = -((U**0.75)*((C***(2*GAM**-1)/(2*(GAM**-1)))))/(Q*(1-(GAM+1)*U*0.5
24* 1) )
25* DP(1) = DR+U
26* A = U*(C-I)
27* BR = U**P(1)/C+2*AF+U
28* BI = 2*AF+I
29* C = (GAM**-1)*AF+U+DP(1)*0.5/C
30* CI = (GAM**-1)*AF+U+DP(1)+0.5/C = 2*AF+WC
31* DP(2) = ((1+2R-BI+ZI-CA)/A) = ZR*2R+ZI*ZI
32* DP(3) = ((1+2R+B+ZI-CA)/A) = ZR*2R+ZI*ZI
33* DP(4) = ZR*P+1R = ZR*1R
34* DP(5) = ZR*P+II = ZR*1R
35* 40 U = 1 + Y
36* C = 1 - (GAM-1)*U*0.5
37* 45 Y(I) = (251.*COR(I) + 19.*PRED(I))/270.
38* 42 U = Y(1)
39* ZR = Y(2)
40* ZI = Y(3)
41* PIR = Y(4)
42* PII = Y(5)
43* C = 1 - (GAM-1)*U*0.5
44* 52 DZ 5 : I = 1+N
45* DY(I,1) = DY(I,2)
46* DY(I,2) = DY(I,3)
47* DY(I,3) = DY(I,4)
48* 55 ZMAS = (ZMA*RT+ZI*ZI)**0.5
49* IF(R>10) 55,50,50
50* 55 ZMAS = (ZMA*RT+ZI**0.5)
51* 56 R = 0*(C**(-(1/(2*GAM**-1))))*(U**-0.25)***4.0
52* IF(R<1) 56,62,62
53* 56 R = 0*(C**(-(1/(2*GAM**-1))))*(U**-0.25)***4.0
54* 55 IF(R<1) 56,62,62
55* 62 IF(R<1) 63,70,70
56* 63 DR = -((2*RC*(R-RT)-(R-RT)*(R-RT))*0.5)/(RT+RCT-R)
57* 65 GP T = 85
58* 70 IF(R<1) 75,80,80
59* 75 DR = -TAU(R)
60* 80 GP T = 85

Table A-5. Continued (Page 8 of 10)

62* 83  DY = (1.2+P-C* (1-R) - (1-R)* (1-R)* 0.5) / (1-R-RCC)
63* 84  DJ = -(U*QR,75) * (C+c+{2+G/4-1}/(2*(GAM-1))) / (G* (1- (GAM+1) * U/2))
64* 85  DY(1,4)= D3*U
65* 86  A = U*(C-U)
66* 87  BR = U*Y(1,4)/C + 2*AF*U
67* 88  BI = 2*WC*U
68* 89  CR = WC*MC - (SVH*SVH+C) / (R*R) = AF*AF
70* 90  1 = (GAM-1) * AF*U*DY(1,4)*0.5/C
71* 91  CI = -(GAM-1) * WC*U*Y(1,4)*0.5/C - 2*AF*WC
72* 92  DY(2,4) = (3R*ZR - R*ZI - CR) / A = ZR*ZR + ZI*ZI
73* 93  DY(3,4) = (3I*ZR + R*ZI - CI) / A = 2*ZR*ZI
74  94  DY(4,4) = ZR*PHIR - ZI*PHII
75  95  IF (ZI*PHII + ZI*PHIR)
76  96  IF (1) * EQ (0) GOTO 87
77  97  PR = WC*PHII = U*DY(4,4) = AF*PHIR
78  98  PI = WC*PHII = U*DY(5,4) = AF*PHII
79  99  PMAG = (PR*PR + PI*PI) ** 0.5
80 100  PARG = ATAM (PI/PR)
81 101  WRITE (6,1000) X, PMAG, PARG
82 102  GOTO 10
83 103  90  IQZ = 1
84 104  Z2 = ZMG**ZMG
85 105  Y(2) = ZR/Z2
86 106  Y(3) = ZI/Z2
87 107  ZPR = DY(2,4)
88 108  ZPI = DY(3,4)
89 109  DY(2,4) = (2*R*ZI - ZI*ZI) + 2*ZR*ZI**ZP1) / (Z2**Z2)
90 110  DY(3,4) = (2*R*ZI - ZI*ZI) + 2*ZR*ZI**ZPI) / (Z2**Z2)
91 111  G(1) = U
92 112  G(2) = Y(2)
93 113  G(3) = Y(3)
94 114  G(4) = PHIR
95 115  G(5) = PHII
96 116  DY(1,1) = DY(1,4)
97 117  DY(2,1) = DY(2,4)
98 118  DY(3,1) = DY(3,4)
99 119  DY(4,1) = PHIR*ZR = PHII*ZI
100 120  DY(5,1) = PHII*ZR = PHIR*ZI
101 121  DO 95 I = 2,4
102 122  CALL RKTIZ(5*,H, X, G, GP, IQZ)
103 123  X = X+H
104 124  U = G(1)
105 125  TR = G(2)
106 126  TI = G(3)
107 127  P*IR = G(4)
108 128  PHII = G(5)
109 129  DY(1,1) = GP(1)
110 130  DY(2,1) = GP(2)
111 131  DY(3,1) = GP(3)
112 132  95  DY(5,1) = GP(5)
113 133  Y(1) = U
114 134  Y(2) = TR
115 135  Y(3) = TI
116 136  Y(4) = PHIR
117 137  Y(5) = PHII
118 138  CALL TADAMS(NH, X, Y, DY, IQZ, IQ)
119 139  GOTO (10, 100, 10)
120 140  100 FORMAT (46X, F6.4, IX, F10.5, 3X, F10.5)
121 141  100 RETURN
122 142  END
Table A-5. Continued (Page 9 of 10)

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<th>Column 3</th>
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<td>DO 15 I = 1,N</td>
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<td>PRED(I) = Y(I)+H*(S6<em>D(I+1)-S9</em>D(I+2)+376<em>D(I,2)-9</em>D(I-1))/24.0</td>
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<td>X = X+H</td>
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<td>U = PRED(1)</td>
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<td>TR = PRED(2)</td>
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<td>TI = PRED(3)</td>
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<td>PH = PRED(4)</td>
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<td>PH1 = PRED(5)</td>
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<td>C = 1 - (5AM-1)*U**5</td>
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<td>R = 0 + (C)<strong>(-1/(2*(GAM-1))) * (U</strong>5 = 0.25) * 4.0</td>
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<td>20 IF((R-1)**1.7+1.7 &gt; 100</td>
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<td>J = -TA(T)</td>
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<td>5U.TC = 4U</td>
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<td>40 J = -((5AM-1)*U**5)/(1-R-RTC)</td>
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</table>
Table A-5. Continued (Page 10 of 10)

63* 86  \( D_1 = \frac{(2 R C (1-R) - (1-R)(1-R)) \ast 5}{(1-R) \ast CC} \)
64* 85  \( D_2 = \frac{-u \ast (75 \ast C \ast ((2 \ast GM - 1) / (2 \ast GM - 1))) / (1 \ast (GM + 1) \ast U) \ast 5}{(1 \ast (GM + 1) \ast U) \ast 5} \)
65* 81  \( A = u \ast (C - U) \)
66* 82  \( B = u \ast Y(1,4) / C + 2 \ast AF \ast U \)
67* 83  \( B_1 = 2 \ast \ast C \ast U \)
68* 84  CR = WC \ast X \ast C - (SVN \ast SVN \ast C) / (R \ast R) - AF \ast AF
69* 85  1 = (GM - 1) \ast AF \ast U \ast D \ast Y(1,4) / (1,4) \ast 0,5 / C
70* 86  CI = (GM - 1) \ast WC \ast U \ast \ast D \ast Y(1,4) / (1,4) \ast 0,5 / C - 2 \ast AF \ast WC
71* 87  DY(2,4) = 1 + (-BR \ast TR + BI \ast TI + CR \ast (TR \ast TR - TI \ast TI) - 2 \ast CR \ast TR \ast TI) / A
72* 88  DY(3,4) = (-TR \ast TR + BI \ast TR + CI \ast (TR \ast TR - TI \ast TI) + 2 \ast CR \ast TR \ast TI) / A
73* 89  DY(4,4) = (TR \ast PHIR - PHII \ast TI) / T2
74* 90  DY(5,4) = (TR \ast PHII + PHIR \ast TI) / T2
75* 91  IF(IP = EQ, 0, GO TO 87
76* 92  PR = WC \ast PHII - U \ast DY(4,4) = AF \ast PHIR
77* 93  PI = WC \ast PHIR + U \ast DY(5,4) = AF \ast PHII
78* 94  PMAG = (PR \ast PR + PI \ast PI) \ast 5
79* 95  PARG = ATAN(P1/PR)
80* 96  WRITE(6,1000) X, PMAG, PARG
81* 97  GO TO 10
82* 98  WRITE(6,1000) X, PMAG, PARG
83* 99  10 \# = 2
84* 100  Y(2) = TR / T2
85* 101  Y(3) = TI / T2
86* 102  TPR = DY(2,4)
87* 103  TPI = DY(4,4)
88* 104  DY(2,4) = (TR \ast TR - TI \ast TI) \ast 2 \ast TR \ast TI \ast TPI / (T2 \ast T2)
89* 105  DY(3,4) = (2 \ast TR \ast TR \ast TI - TP \ast TI \ast TI) / (T2 \ast T2)
90* 106  G(1) = U
91* 107  G(2) = Y(2)
92* 108  G(3) = Y(3)
93* 109  G(4) = PHIR
94* 110  G(5) = PHII
95* 111  DY(1,1) = DY(1,4)
96* 112  DY(2,1) = DY(2,4)
97* 113  DY(3,1) = DY(3,4)
98* 114  DY(4,1) = (PHIR \ast TR - PHII \ast TI) / T2
99* 115  DY(5,1) = (PHIR \ast TR - PHII \ast TI) / T2
100* 116  DD 95 I = 2
101* 117  CALL RK7PZ(5, X, G, GP, I0Z)
102* 118  X = X + H
103* 119  U = G(1)
104* 120  ZR = G(2)
105* 121  ZI = G(3)
106* 122  PHIR = G(4)
107* 123  PHII = G(5)
108* 124  DY(1,1) = GP(1)
109* 125  DY(2,1) = GP(2)
110* 126  DY(3,1) = GP(3)
111* 127  DY(4,1) = GP(4)
112* 128  DY(5,1) = GP(5)
113* 129  Y(1) = U
114* 130  Y(2) = ZR
115* 131  Y(3) = ZI
116* 132  Y(4) = PHIR
117* 133  Y(5) = PHII
118* 134  IQ = 1
119* 135  GO TO 105
120* 136  IQ = 2
121* 137  1000 FORMAT(46X, F6.4, 1X, F10.5, 3X, F10.5)
122* 138  105 RETURN
123* 139  END

43
### Table A-6. Sample Output

**Theoretical Nozzle Admittances**

- Mach Number = 0.25
- SVN = 1.4913
- Gamma = 1.2
- Decay Coefficient = -0.6500
- Nozzle Angle = 20.3
- Radii of Curvature: Throat = 0.9234
- Entrance = 1.0000

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<td>-0.08772</td>
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<td>1.06453</td>
</tr>
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<td>-0.00632</td>
<td>-0.09690</td>
<td>2.55527</td>
<td>-0.05945</td>
<td>1.13557</td>
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Figure A-1. Flow Chart for the Nozzle Admittance Computer Program (Page 1 of 10)
SUBROUTINE NOZADM

STEPPIZE
DP = -0.001

WRITE CM, SVN, RCT, RCC, ANGLE

DO THROUGH \( \alpha \)
FOR N = 1,NPT

WC = WC + DWC
COMPUTE F, RT, RL, R2

COMPUTE INITIAL VALUES OF PHIR, PHI2, U, A,B,C,CLR,CII,
B1R,B1I,ZR,ZI

ZO = ZOR
ZI = ZO1
G(1) = U
G(2) = ZR
G(3) = ZI
G(4) = PHIR
G(5) = PHI2
DY(1,1) to
DY(4,1), JOPT

DO THROUGH \( \beta \)
FOR I = 2,4

Y(1) = U
Y(2) = ZR
Y(3) = ZI
Y(4) = PHIR
Y(5) = PHI2

CALL ZADAMS

CALL RKTZ

P = P + DP
U = G(1)
ZR = G(2)
ZI = G(3)
PHIR = G(4)
PHI2 = G(5)
COMPUTE DERIVATIVES

COMPUTE NEW VALUES OF C, U, CM

DY(1, I) = GP(1)
DY(2, I) = GP(2)
DY(3, I) = GP(3)
DY(4, I) = GP(4)
DY(5, I) = GP(5)

WRITE WC, YR, YI,
W, SIR, SYI

RETURN

END

Figure A-1. Continued (Page 2 of 10)
SUBROUTINE RKTZ

COMPUTE
A(1) → A(5)
TZ = TL
U(1) → U(5)
FZ(1,1) → FZ(1,5)

DO THROUGH α
J = 1, NU

UZ(J) = U(J)
DUM(J) = FZ(1,J)

α

JOPT = 2
CALL SUBROUTINE RKZDIF

CALL SUBROUTINE RKTDIF

DO THROUGH β
J = 1, NU

CALL SUBROUTINE RKZDIF

Figure A-1. Continued (Page 3 of 10)
FOR SECTION I

SUBROUTINE RKZDIF

\[ U = G(1) \]
\[ ZF = G(2) \]
\[ ZI = G(3) \]
\[ PHIR = G(4) \]
\[ PHI1 = G(5) \]

COMPUTE R

\[ P = 0 \]

COMPUTE DR FOR SECTION I

\[ GP(1) = \frac{4}{(SQR\{RT\times(GAM + 1)\)} \]
\[ GP(2) = Z1R \]
\[ GP(3) = Z1I \]
\[ GP(4) = Z1R \]
\[ GP(5) = 0, 0 \]

\[ GP(2) \to GP(5) \]

RETURN

END

Figure A-1. Continued (Page 5 of 10)
Figure A-1. Continued (Page 6 of 10)
SUBROUTINE ZADAMS

A

DO THROUGH α
I = 1,N

α

X = X + H
U = PRED(1)
ZR = PRED(2)
ZI = PRED(3)
PHIR = PRED(4)
PHII = PRED(5)
COMPUTE C,R

R-R1
≥ 0

R-R2
< 0

COMPUTE DR FOR SECTION I

R - l
≥ 0

≥ 0

< 0

< 0

R - R2
≥ 0

DO THROUGH β
I = 1,N

β

COMPUTE COR(1),Y(I)

R - 1
≥ 0

< 0

< 0

DO THROUGH Γ
I = 1,N

γ

DY(I,1) = DY(I,2)
DY(I,2) = DY(I,3)
DY(I,3) = DY(I,4)

Figure A-1. Continued (Page 7 of 10)
Figure A-1. Continued (Page 8 of 10)
SUBROUTINE TADAMS

DO THROUGH $\alpha$
$I = 1, N$

COMPUTE PRED(I)

$X = X + H$
$U = PRED(1)$
$TR = PRED(2)$
$TI = PRED(3)$
$PHIR = PRED(4)$
$PHII = PRED(5)$
COMPUTE $C_R$

$R - R1 \geq 0$
$R - R2 < 0$

COMPUTE DR FOR SECTION I

$R - R1 < 0$

COMPUTE DR FOR SECTION II

$R - R2 \geq 0$

COMPUTE DR FOR SECTION III

$DU, A, BR, BI, CR, CI,$
$DF(1) \to DF(5)$

DO THROUGH $\beta$
$I = 1, N$

COMPUTE COR(I)

$\beta^T$

COMPUTE $C_T$

$R - 1 \geq 0$

$R - 1 < 0$

DO THROUGH $\Gamma$
$I = 1, N$

$\text{DY}(I,1) = \text{DY}(I,2)$
$\text{DY}(I,2) = \text{DY}(I,3)$
$\text{DY}(I,3) = \text{DY}(I,4)$

COMPUTE $\text{DY}(1,4) - \text{DY}(1,5)$

2

$\Gamma$
Figure A-1. Concluded (Page 10 of 10)
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