ALUMINUM RUNWAY SURFACE AS POSSIBLE AID TO AIRCRAFT BRAKING

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Page 27: Reference 4 should be replaced with the following reference:


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Several concepts are described for use singly or in combination to improve aircraft braking. All involve a thin layer of aluminum covering all or part of the runway. Advantage would derive from faster heat conduction from the tire-runway interface. Heating of tread surface with consequent softening and loss of friction coefficient should be reduced. Equations are developed indicating that at least 99 percent of friction heat should flow into the aluminum. Preliminary test results indicate a coefficient of sliding friction of 1.4, with predictably slight heating of tread. Elimination of conventional brakes is at least a remote possibility.
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SUMMARY

Possible advantages are discussed that could result from use of aluminum as a full or partial cover for a runway surface. Such advantages are based on the fact that aluminum can conduct frictionally generated heat away from the tire-runway interface at a rate about two orders of magnitude greater than is possible with an asphalt or concrete surface. Very rapid conduction of the heat should substantially reduce heating of the tread rubber with the consequent softening and loss of friction coefficient.

Preliminary results of tests under an NASA grant at the University of Michigan are mentioned, including a coefficient of sliding friction of 1.4 between tread rubber and a smooth aluminum surface with predictably slight heating of the tread.

Equations are developed to predict the ratio of the split of frictionally generated heat between rubber and aluminum. The equations indicate that 99 percent or more of the heat should go into the aluminum.

A possible arrangement is described by which the routine braking of an aircraft with moderate braking forces could be done on the aluminum surface by yawing of wheels, with no need for conventional brakes. A special auxiliary arrangement is suggested for emergency stops.

Rain water, snow, and ice are recognized as possible problems. Reasons are presented to indicate these problems may not be insurmountable. Periodic cleaning of the aluminum surface would be necessary, but preliminary indications are that only moderate cleaning might suffice.

INTRODUCTION

Because aircraft tire-runway friction and wear continue to be a safety and economic concern to aircraft operators, NASA has been investigating ways for providing high runway friction for improved aircraft braking and reduced tire wear. The demonstrated
advantage of grooved runways for achieving high tire-runway friction, explored by the Langley Research Center, are being applied with good results. The relation between rubber chemistry and tire wear is under study by the Ames Research Center. As an alternate approach to this problem, this report suggests that high temperatures that may soften the tire tread surface might be responsible for low tire-runway friction and rapid tire wear. An analysis is presented on the partition of the tire friction heat between the runway and the tire, which indicates that runway surfaces of high thermal conductivity would maintain tire tread temperatures cool enough for good friction coefficients and low wear rates. Such conductive surfaces might be provided by a thin layer of aluminum sheet against which rubber tires develop satisfactory friction coefficients. Preliminary friction coefficient measurements with tire tread free of surface oxide glaze are encouraging.

The concepts developed in this report are presented as a basis for further study to determine how such factors as runway surface dryness or wetness, runway surface texture and cleanliness, and tire tread surface condition relate to the practicality and potential benefits of this approach. The report also offers for consideration methods for relieving the heat load on aircraft brakes by absorbing a meaningful portion of the airplane kinetic energy in frictional heating at the tire-runway interface.

The SI system of units is used as the primary system throughout the text. However, the system of units that is more common in America is used for calculations and the results are then converted to SI units.

The authors collaborated on the main text of this report; the two appendices, however, were solely the work of the first author.

THEORY OF ALUMINUM RUNWAY SURFACE

The idea that aircraft braking might be improved by use of a thin aluminum skin over part or all of the runway surface is based principally on the following two concepts:

1. Greater braking force than presently possible might become available because the increased rate of conduction of heat by use of the aluminum skin should greatly reduce frictional heating of the rubber-runway interface (or footprint), which may cause softening of the rubber surface with consequent reduction of friction coefficient under existing conditions.

2. The aluminum skin might serve as a heat sink in lieu of the thick (and heavy) pads that exist in present-day aircraft brakes. The reduction of interface temperature could possibly yield an incidental advantage of a reduced rate of tire wear. The reduction of interface temperature, combined with use of the aluminum as a heat sink, might enable braking entirely by yawing of wheels with elimination of conventional brakes.
Retreading or replacement of tires is a large item in aircraft maintenance cost. Overhaul or replacement of brakes is also a large item. Moreover, elimination of brakes as such and reduction of tire wear should save much aircraft down time associated with tire and brake servicing.

If a greatly enhanced coefficient of friction should prove to be feasible, of course an optimum compromise might have to be sought between the utility of the higher coefficient and the undesirable aspects of the stronger aircraft undercarriages that would be necessary.

Both the hope of maintaining a relatively cool rubber-aluminum interface and the hope of using the aluminum skin as a heat sink depend on certain tribophysical considerations. Important background material in this field has been published by Blok (ref. 1), by Schaaf (ref. 2), and by Ling and Saibel (ref. 3). However, a somewhat different approach to the subject will now be offered which, though simple, seems to provide good insight regarding the possible advantages of an aluminum surface on a runway.

**Theory of Flow of Frictional Heat**

Basic to a theoretical evaluation of the two concepts we have described is a new boundary equation that approximately defines the split between the flows of frictionally generated heat to one of the rubbing surfaces or the other. A necessary preliminary to a derivation of the new boundary equation is an equation approximately defining the partition of contact conductance across the interface of two materials. For use in the boundary equation, we define the contact conductance $\alpha$ of the interface as the reciprocal of the resistance of the interface to the flow of heat across it. The contact conductance is measured in heat units per unit temperature difference across the interface, per unit time, per unit area of the interface.

The approximate equation for the partition of contact conductance, developed in appendix A, is

$$\frac{\alpha_r}{\alpha_a} = \frac{k_r}{k_a}$$

(A13)

where $\alpha_r$ and $\alpha_a$ are contact conductances ascribed to the rubber and aluminum sides of the interface, respectively, and $k_r$ and $k_a$ are the conductivities of rubber and aluminum. The relation of $\alpha_r$ and $\alpha_a$ to the contact conductance of the interface is

$$\alpha = \left(\frac{1}{\alpha_r} + \frac{1}{\alpha_a}\right)^{-1} = \frac{\alpha_a \alpha_r}{\alpha_r + \alpha_a}$$

(A14)
For general applicability, the derivation of equation (A13) requires an assumption that the flow of heat through voids or contaminants within the interface is negligible in comparison with the flow through the minute areas of firm molecule-to-molecule contact between rubber and aluminum. But for the use of equations (A13) and (A14) that will be made here such assumption is unnecessary.

With use of equations (A13) and (A14) and other well-known relations the new boundary equation, as derived in appendix B, is

$$-k_a \frac{\partial \varphi_a(x, y, z, t)}{\partial x} = \left( \frac{k_a}{k_a + k_r} \right) g(y, z, t) + \alpha \left[ \varphi_r(x, y, z, t) - \varphi_a(x, y, z, t) \right], \quad x = 0$$

(B14)

or

$$-k_r \frac{\partial \varphi_r(x, y, z, t)}{\partial x} = \left( \frac{k_r}{k_r + k_a} \right) g(y, z, t) + \alpha \left[ \varphi_a(x, y, z, t) - \varphi_r(x, y, z, t) \right], \quad x = 0$$

(B15)

where \( x \) is always positive and is the distance from the interface measured in either direction, \( y \) and \( z \) are spatial coordinates within the interface, \( t \) is time, \( \varphi_a(x, y, z, t) \) and \( \varphi_r(x, y, z, t) \) are local temperatures within or on the surface of the aluminum and the rubber, respectively, and \( g(y, z, t) \) is the rate of heat generation by friction per unit area of interface as a function of \( y, z, \) and \( t \). Note that equations (B14) and (B15) are redundant in the sense that either equation implies the other.

The derivation of equations (B14) and (B15) required an assumption that negligible heat would be generated by slippage between rubber and contaminants or aluminum and contaminants within the interface or by fluid or plastic shearing within contaminant. An assumption was also used that appreciable heat would not be generated by internal friction within the rubber, due, possibly, to local stick-slip conditions within the interface such as might cause cyclic local internal deformations of the rubber. As mentioned before, the assumption of negligible heat flow through contaminant was not needed.

Now that equations (B14) and (B15) are available, some theoretical deductions about the possible use of an aluminum skin on a runway surface to reduce interface temperatures and thereby to maintain a high coefficient of friction will be discussed.

Possible Enhancement of Friction Coefficient with Use of Aluminum Skin on Runway

Slippage between a tire tread and the surface upon which it rolls is known to exist even for a free rolling tire (ref. 4). Figure 1 represents a possible model for longitu-
dinal slide only. The principle to be discussed would be basically unchanged even if the model were considerably wrong in detail. (Lateral slide is also known to exist.) Equal increments of tread circumference \( d\lambda \) are marked in the figure, throughout the parts of the tread that are well removed from the footprint or area of contact with the runway. These increments are assumed to compress steadily to shorter lengths as they approach the footprint and as they progress through the footprint to its transverse centerline. Thereafter they are assumed to expand again. At any position, the length that was \( d\lambda \) becomes \( d\lambda - \Delta\lambda \), where \( \Delta\lambda \) is the amount of longitudinal compression of the increment.

In order that the same number of increments may pass any angular position on the tread per unit time, it is necessary that the tread velocity \( v_{tr} \) at any position shall be proportional to the compressed length of an increment at that angular position, that is, proportional to \( d\lambda - \Delta\lambda \). Hence, the slip rate at any position within the footprint will be

\[
v_{sl} = v_{tr} - v_{wh} = C_{sl}(d\lambda - \Delta\lambda) - v_{wh}
\]

where \( C_{sl} \) is constant for given \( v_{wh} \), which is the linear velocity of the wheel relative to the runway. There should be two positions of no slip where

\[
C_{sl}(d\lambda - \Delta\lambda) = v_{wh}
\]

The slip ahead of the first such position and behind the second should be toward the rear. The slip between those two positions should be forward. The total frictional force exerted by the tread on the runway toward the rear, due to slip in that direction, must equal the total forward frictional force due to forward slip (disregarding rolling resistance).

The rate of heat generation for each differential area within the footprint, approximately constant in time, should be

\[
g(y, z)dy \, dz = pf \left| v_{sl} \right| dy \, dz = pf \left| C_{sl}(d\lambda - \Delta\lambda) - v_{wh} \right| dy \, dz
\]

where \( p \) is the normal pressure and \( f \) is the friction coefficient for the differential area \( dy \, dz \). But, if we assume that \( p, f, \) and \( \Delta\lambda \) are the same for all values of \( v_{wh} \), then \( C_{sl}, v_{sl}, \) and \( g(y, z)dy \, dz \) must all be proportional to \( v_{wh} \).

Therefore, with the high rolling velocity of aircraft tires, and the high values of \( p \) in equation (3), the values of \( g(y, z)dy \, dz \) may be high even without the application of any braking force. Because of the low thermal conductivities of both rubber and asphalt or concrete, we should therefore expect relatively high temperatures within the rubber-runway interface even without braking.
But even more important are the implications of this discussion relative to the heating of the tread surface by braking. If it were theoretically and practically possible that free rolling would not involve slip, then we could reasonably expect that at least moderate braking would be possible without slip. Braking force up to the limit that might be provided by static friction, acting over the whole surface of the footprint, should then be possible without slip. The force would be transmitted from the interface into the carcass of the tire by shearing deformation of the tread in all planes parallel to the interface.

But with only two narrow bands that do not slip even without braking, as shown in figure 1, braking force would be provided only by increasing the area within which forward slip occurs and decreasing the area in which backward slip occurs. The two points of no slip would move farther apart. The result would be that any braking force would increase the heat generation by friction in the interface. With greater braking force greater friction heating would occur, and greater degradation of friction coefficient because of softening of the rubber. Such degradation of friction coefficient, with temperature elevations of the tread surface that could possibly amount to 100 or more degrees Kelvin (several hundred degrees Fahrenheit), would explain the fact that actual braking forces are not nearly as great as should reasonably be predicted from static tests of friction coefficient.

As each part of the tire tread contacts the runway repeatedly, the tread would become hotter than the part of the runway surface within the interface. With heavy braking, under the concept just described, and with poor conductivity of both tread rubber and the runway, the rubber surface could conceivably become very hot. Intuitively it seems obvious that higher conductivity of the runway surface by two orders of magnitude would substantially reduce the temperatures reached by the tread surface at all times during a braking run. A quantitative estimate of the reduction would be too complex to undertake here.

However, the effect of greater conductivity of the runway surface may be perceived qualitatively by inspection of equation (B15). That equation applies only to a differential area of tread surface during a differential time. But the temperature existing within any differential area on the tread surface at any time depends on the application of equation (B15) to that differential area throughout all previous differentials of time, during which the differential area was in contact with the runway, and on the conduction of heat into the tread rubber and into the runway. The term \( k_r/(k_r + k_a) g(y, z, t) \) represents a flow of heat from the tire-runway interface into the surface of the rubber. It should be greatly reduced by a much larger value of \( k_a \). The term \( \alpha [\varphi_a(x, y, z, t) - \varphi_r(x, y, z, t)] \) represents flow of heat from the tread surface into the aluminum surface. This flow should be greatly increased by lower values of \( \varphi_a(x, y, z, t) \) resulting from greater conductivity of the aluminum. The great decrease in the one flow and increase of the other, within all differential areas at all times, should greatly reduce the tread surface tem-
peratures existing at any time within any part of the footprint. Hence, we should expect, with existing brake mechanisms, that with an aluminum-surfaced runway we would suffer much less of the degradation of friction coefficient that is caused by the frictional heating of the tread surface.

This advantage might be gained to a considerable degree if only a fraction of the total runway surface were aluminum. For example, if aluminum particles were sprinkled over the asphalt and pressed in with a roller they might conduct away a large part of the friction heat.

Possible Stopping Distance with Aluminum Skin

Low-speed tests at the University of Michigan under an NASA grant, transmitted in personal communication by Professor S. K. Clark, have consistently shown a coefficient of sliding friction of 1.4 between automotive tire tread rubber and a clean aluminum surface, with a tread loading of about $2 \times 10^5$ newtons per meter squared (30 psi). The rise in temperature of the tread surface was about as should be expected on the basis of equations (B14) and (B15). A 152-meter (500-ft) locked-wheel slide with such loading (with a lower coefficient of friction of about 0.4 because of heavy contamination) showed a temperature rise of less than 0.6 K ($1^\circ$ F) at a depth of less than 0.3 centimeter (1/8 in.) within the tread. These two results approximately support the assumption noted earlier that little heat is generated by internal friction of the rubber under sliding conditions.

The coefficient of 1.4 would allow an airplane at 82.5 meters per second (160 knots) to stop in 246 meters (807 ft). According to the theory that has now been explained, actual realization of such a stopping distance could not be ruled out without actual test results. It is true that so high a coefficient has not been approached in any known earlier tests with an aircraft tire and an aluminum surface. But, in view of the achievement of such a coefficient in a low-speed test, and in view of the theory that has been presented concerning possible degradation of friction coefficient at higher speeds, intense research effort would appear to be justified to make certain whether such a coefficient could be obtained at high speeds in practice. All the options that will be discussed are predicated to some extent on an assumption of success in such research.

Options Involving Possible Use of Aluminum Skin on Runway as Heat Sink

Many possible variations in manner of implementation of the concept of braking with use of an aluminum skin on the runway surface as a heat sink exist at least in theory. For example, the aluminum surface could exist over the entire runway for routine use,
or it could be placed only near the end of the runway, for emergency use, as illustrated in figure 2. In any application, however, use of the aluminum skin as a heat sink to absorb part of the kinetic energy of an aircraft would require a large component of slip between rubber and aluminum. Use of the aluminum skin to absorb all the kinetic energy in the form of heat would require a slip rate between rubber and aluminum equal to the ground velocity of the aircraft. Some of the options available at least theoretically for use singly or in combination follow.

**Total slide to a stop.** - One of the possible options with the arrangement illustrated in figure 2 would be a total slide to a stop. Under this option, a pilot's permanent instructions might be most simple. If his aircraft while not airborne ever reached the aluminum-surfaced extension of the runway he should lock all wheels at that instant and apply to the aluminum surface any auxiliary rubber surfaces that might exist for braking purposes. This instruction could well substitute for the present-day decision speed \( V_1 \), and would more frequently be complied with by the pilot because it would agree rather than conflict with his survival instinct in cases where frightening malfunction occurs after the \( V_1 \) speed is reached and before the aircraft is airborne.

The effect of a strong crosswind component should not be a serious deterrent to the use of this option. Figure 3 shows a trajectory determined by numerical integration for a total slide from 82.5 meters per second (160 knots) with deceleration of 10.4 meters per second squared (1.4 times gravity) applied at all times exactly in the reverse direction to the existing velocity. A crosswind component was assumed sufficient to produce a lateral force equal to one-tenth the aircraft weight. The total forward movement is 247 meters (809 ft). The lateral movement is only 8.8 meters (29 ft).

Figure 4 illustrates the condition that would exist on the aluminum-surfaced extension of the runway illustrated in figure 2. The distribution of the heat as roughly 99 percent into the aluminum is based on equation (B14), but with due regard for the fact the aluminum skin will have an oxide film whose thickness would almost certainly be much greater than the dimensions of the microscopic asperities in the aluminum surface. In such a case, with an assumption that the thermal resistance of the oxide film will be substantially less than \( \alpha^{-1} \), it can easily be shown that equations (B14) and (B15) will apply, but with use of the thermal conductivity of aluminum oxide instead of \( k_a \) in the terms \( \left[ k_a/(k_a + k_r) \right] g(y, z, t) \) and \( \left[ k_r/(k_r + k_a) \right] g(y, z, t) \). The term \( k_a \left[ \partial \varphi_a(x, y, z, t)/\partial x \right] \) in equation (B4) then would still use the conductivity of aluminum \( k_a \) and the partial derivative \( \partial \varphi_a(x, y, z, t)/\partial x \) as it would exist on the aluminum side of the interface between aluminum and aluminum oxide.

We will assume the aluminum oxide to be polycrystalline, dense, and at a temperature between 311 and 478 K (100° and 400° F), with conductivity of about 27.7 W/(m)(K) or 16 Btu/(hr)(ft)(°F), (ref. 5). We will take the conductivity of tread rubber as 0.28 W/(m)(K) or 0.16 Btu/(hr)(ft)(°F), (ref. 6). Thus, from equation (B14) with the modified use as just described,
\[
\kappa_a \left( \frac{\partial \phi}{\partial x} \right) = 0.99 g(y, z, t) + \alpha \left[ \phi_r(x, y, z, t) - \phi_a(x, y, z, t) \right], \quad x = 0 \quad (4)
\]

Because the same rubber surface is continuously exposed to the frictional heat in the contact patch, while new aluminum surface is continually entering the patch, the temperature \( \phi_r(x, y, z, t) \bigg|_{x=0} \) must inevitably rise above \( \phi_a(x, y, z, t) \bigg|_{x=0} \) so that the term \( \alpha \left[ \phi_r(x, y, z, t) - \phi_a(x, y, z, t) \right] \bigg|_{x=0} \) in equation (4) must be positive. The magnitude of \( \alpha \) is unknown. Partly completed investigation leads us to believe that it is high and that an assumption \( \alpha \left[ \phi_r(x, y, z, t) - \phi_a(x, y, z, t) \right] \bigg|_{x=0} \equiv 0 \) would be strongly conservative.

The partition of the friction heat with 99 percent into the aluminum is based on equation (4) with that conservative assumption.

We now wish to estimate the amount of tread rubber that would be destroyed if an aircraft weighing 90 000 kilograms (200 000 lb) were stopped from 82.5 meters per second (160 knots) in a total slide, with 1 percent of the total friction heat flowing into the tread rubber. The calculation will be independent of stopping distance because the aircraft's kinetic energy must be dissipated as heat regardless of that distance. We will assume an initial tread temperature of 311 K (100° F) and assume that tread rubber will smear off onto the runway at 478 K (400° F). A close estimate of the temperature at which tread rubber would smear off cannot be made without actual test. The value of 478 K (400° F) is near the curing temperature in some fast-curing processes (ref. 7). Nybakken, Staples, and Clark (ref. 8) found that rubber reversion occurred at temperatures ranging from 422 to 587 K (300° to 600° F) for various types of elastomers. In a personal communication those authors observed that the rubber reversion as indicated by loss of friction coefficient seemed to correlate with smearing. As the decomposition associated with smearing is a time-consuming process, the value of 478 K (400° F) seems conservative for the very short time intervals under consideration here. The specific heat of the rubber will be taken as 1750 J/(kg)(K) or 0.42 Btu/(lb)(°F), calculated from values given in reference 9 for 55 parts carbon black and 100 parts natural rubber. The weight of rubber lost should be

\[
w_r = \frac{1}{2} \left( \frac{160 \times 1.152 \times 5280}{3600} \right)^2 \times \frac{200,000}{32.2} \times \frac{1}{778} \times \frac{1}{0.42 \times (400 - 100)} \times 0.01
\]

\[= 23.1 \text{ lb} (6.55 \text{ kg}) \quad (5)\]

Now if we assume a load on the contact patch approximately equal to the air pressure in the tires, say \( 1.38 \times 10^6 \) newtons per meter squared (200 psi), the total area of contact would be about 0.6452 meter squared (1000 in. \(^2\)). With a tread rubber density
of about 1200 kilograms per meter cubed (0.043 lb/in.\(^3\)), (ref. 6), the loss of 6.55 kilograms (23.1 lb) of rubber would reduce the tread thickness in the areas of contact by about 1.4 centimeters (0.54 in.). If the air pressure within the tires were less, the thickness of rubber lost would be less. Loss of 1.4 centimeters (0.54 in.) of tread thickness would blow out tires of some types. However, after blowout the tire on a locked wheel could still absorb much kinetic energy before it would allow contact between the wheel flanges and the aluminum runway surface. Also, if a wheel were allowed to rotate by even as much as one complete revolution during the slide the loss of 6.55 kilograms (23.1 lb) of rubber should not blow any tire out. The loss of all tires on an airplane would be a small price to pay for an emergency stop that avoided the wreckage of the aircraft with possible extensive loss of life. Note that the loss of 6.55 kilograms (23.1 lb) of rubber is a conservative estimate because of the conservatism of the estimate of 1 percent of the total heat into the rubber, because it assumed no aerodynamic loss of kinetic energy, and because it neglected heat into parts of the tread that did not reach 478 K (400\(^\circ\) F).

The next question we wish to consider relative to use of an aluminum skin as a heat sink is the ability of such a skin to absorb the heat. Presumably, the temperature reached by the aluminum surface could not exceed that at which the tread rubber would smear off, about 478 K (400\(^\circ\) F). To what extent the depths of the aluminum skin would approach that temperature would depend on the length of time interval during which an element of surface area of the aluminum would be exposed to heat flux (that is, contact with tread rubber). If an optimum time interval were approached, the surface area of aluminum needed to absorb the kinetic energy, under the same conditions specified by equation (5) would be

\[
A_{al} = \frac{1}{2} \left( \frac{160 \times 1.152 \times 5280}{3600} \right)^2 \times \frac{200 \, 000 \times 1}{32.2} \times \frac{1}{778} \times \frac{1}{400 - 100} \times \frac{16 \times 12}{0.207 \times 62.4 \times 2.7}
\]

\[
= 5353 \text{ ft}^2 (497 \text{ m}^2) \quad (6)
\]

with an assumed aluminum thickness of 1.6 millimeters (1/16 in.), specific heat of 865 J/(kg)(K) or 0.207 Btu/(lb)(\(^\circ\)F), and specific gravity of 2.7.

For a stopping distance of 305 meters (1000 ft), as an example, equation (6) would call for a bearing 1.63 meters (5.35 ft) wide (measured at right angles to the direction of aircraft motion) between rubber and aluminum. The length of uniformly loaded bearing (measured in the direction of motion) would need to be great enough that the aluminum skin would be heated to an approximately uniform temperature of 478 K (400\(^\circ\) F).

It is known (ref. 10), that substantially uniform temperature will be reached with a constant heat flux through the aluminum surface if
\[ \frac{\kappa t}{l^2} > 1 \]  

where \( \kappa \) is diffusivity, \( t \) is the time interval throughout which the flux exists, and \( l \) is thickness. With diffusivity of \( 8.58 \times 10^{-5} \) square meter per second (3.326 ft\(^2\)/hr) for aluminum and with \( l = 1.6 \) millimeters (1/16 in.), the criterion of equation (7) calls for

\[ t > 0.0294 \text{ sec} \]  

At 82.5 meters per second (160 knots) the time \( t = 0.0294 \) second would call for a bearing length of

\[ l_b = \frac{160 \times 1.152 \times 5280}{3600} \times 0.0294 = 7.93 \text{ ft (2.42 m)} \]  

The loading on the bearing would be

\[ p_b = \frac{200 \times 10^5}{5.35 \times 7.93 \times 144} = 32.8 \text{ psi (2.26} \times 10^5 \text{ N/m}^2) \]

Obviously the locked wheels of a conventional aircraft as illustrated in figure 4 could not provide the total bearing width of 1.63 meters (5.35 ft) nor the bearing length of 2.42 meters (7.93 ft). But auxiliary braking surfaces would not be out of the question for emergency stopping. For example, several flat rubber pads totalling 1.63 meters (5.35 ft) in width and 2.42 meters (7.93 ft) in length might be provided. The underside of a conventional bogie could be a flat rubber surface. Under emergency conditions, the wheels could be partly unloaded and the under surfaces of the bogies allowed to slide on the aluminum.

The bearing length \( l_b = 2.42 \) meters (7.93 ft) would be required only at 82.5 meters per second (160 knots). At all lower speeds during the braking process, the length of the bearing could be less. Hence, a compromise might be made with a longer braking run and a smaller length \( l_b \). The arrangement of figure 4 would be such a compromise, with coefficient of friction automatically limited to that which would smear rubber from the tires until the speed became low enough that smearing would stop. Of course, in the example cited earlier, the stopping distance would then be greater than 246 meters (807 ft).

\textbf{Modified antiskid device.} - With a deceleration of 1.4 times gravity, the aluminum surface in the locked wheel stop as illustrated in figure 4 would of course get too hot. However, a modified antiskid device could conceivably control the skid so as to allow the
1.4 coefficient for the entire 246 meters (807 ft), letting the aluminum take as much of
the heat as it could and letting conventional brakes take the remainder. The only changes
necessary would be in the antiskid mechanisms. It might be assumed that too much slip
would cause degradation of friction coefficients through production of excessive temper-
atures in the tire-runway interfaces. A decelerometer could be incorporated, which
would be integrated with other parts of the servomechanism in such a manner as to de-
crease the braking torque uniformly on all wheels whenever the decelerating force de-
creased, or to increase the braking torque under the reverse condition. Decreasing
braking torques would allow the angular velocities of the wheels to increase, thus reduc-
ing the slip and reducing the interfacial heating caused by the slip, allowing the coeffi-
cients of friction to recover their higher values. Simultaneously, the antiskid mecha-
nism for any individual wheel could continuously monitor the angular speed of that wheel
relative to the average angular speed of all the other wheels and make minor adjustments
of braking torque on the individual wheel to keep its angular velocity about the same as
for all other wheels.

Possible routine braking without conventional brakes. - A runway could be entirely
covered with a thin layer of aluminum, as in figure 5, and routine braking could be ob-
tained without conventional brakes by yawing of wheels. Auxiliary braking surfaces
would need to be provided as earlier discussed. Except near the ends of the runways,
the aluminum skin could be much thinner than for the emergency stops that might be
necessary near the ends.

In the arrangement shown, optimum cooling by the aluminum would be obtained by
permanent yawing of the bogie, so that trailing wheels would never slide over aluminum
already heated by leading wheels. Moderate yawing of the wheels would produce both
sliding components and rolling components of the wheels. The sliding components would
be in the direction of the axis of each wheel, which would be at right angles to the longi-
tudinal grooves that are standard in the tire treads of jet aircraft, so that the grooves
would provide a maximum scraping effect on a wet runway. The yawing arrangement
shown would balance both lateral components of friction forces and torques about the
vertical axis.

The net braking force should depend primarily on the yaw angle and very little on
the speed, unless the sliding component were great enough to overheat the tire-runway
interface. As reduced speed made possible a more uniformly elevated temperature
throughout the depth of the aluminum skin, the yaw angle could be somewhat increased
without overheating the interface.

In an all-out emergency, use could be made of the arrangement described earlier,
in which the wheels would be unloaded and a thin rubber layer covering the entire under-
side of the bogie would be allowed to bear on the aluminum surface of the runway.
EFFECTS OF SURFACE CONTAMINANTS

The effects of various surface contaminants such as oil, grease, various kinds of dirt, water, snow, and ice must be considered. No definitive tests have been made. Yet, some pertinent observations and reasonable speculations might well be mentioned.

Effects of Oil, Grease, and Dirt

Good wet coefficient of friction, and to a lesser extent good dry coefficient, require moderate cleanliness of the aluminum surface. For that reason periodic cleaning would be required. Tests have not yet been made to determine whether common types of contaminant other than oil and grease have appreciable deleterious effect. However, in tests reported in personal communication by Professor S. K. Clark of the University of Michigan, any minute film of tread rubber that may have been left on the aluminum surface in the sliding tests did not degrade the coefficient of friction. In simple tests with small sheets of aluminum, small pieces of rubber, and low normal pressure, we have found that after the friction coefficient had been destroyed by a drop of oil it could be restored by wetting the surface with water, sprinkling an ordinary household powder-type detergent over the surface, light rubbing with a wet cloth for 2 or 3 seconds, and rinsing with clean water.

Simple tests such as we made for approximate determination of friction coefficient between a small rubber specimen and a smooth sheet of aluminum can be improvised in a matter of minutes by any engineer. Such a test is illustrated in sketch (a). To minimize the effect of gravity, the slender rod may be oriented vertically. It may be tapered to a point at the top so that a force $F$ may be applied with a finger tip without applying appreciable torque. The aluminum sheet may be tilted slowly in a manner to decrease
the angle $\beta$. The coefficient of sliding friction then, for very light loading and low rate of slide, will be the cotangent of the angle $\beta$ at which the rubber tip slowly moves on the aluminum sheet.

**Effect of Water**

In tests such as just described, with moderately clean aluminum and rubber, the value of $\beta$ at which slow slippage begins is usually about 45° or less, whether the rubber tip is a pencil eraser, or a piece of tread rubber from either an automobile or aircraft tire. Such values of $\beta$ indicate a friction coefficient of 1.0 or greater. The value of $\beta$ is little changed if the test is performed in a heavy rainfall, under a flood of water from a faucet, or even with the entire arrangement immersed in a tank of water.

Wet tests with an automobile-type tire, but with a bald tread, reported in personal communication by Professor S. K. Clark of the University of Michigan, have shown a very low coefficient of friction quite inconsistent with the negligible effect of water just mentioned. Because of the good wet test results with small pieces of rubber, it is anticipated that tests with a grooved tire tread may show good wet coefficients. With enough grooves in the tread it should, so far as the rubber-aluminum interface is concerned, be the equivalent of many small pieces of rubber sliding on the aluminum surface in different relative positions.

Of course tests with higher speeds and greater loads need to be made.

A smooth aluminum surface could not have the sponge-like effect of concrete or asphalt. It might therefore be hoped that an air jet ahead of a braking tire would be more effective in improving wet performance with aluminum than with asphalt or concrete. The absence of the sponge-like surface could conceivably be a great advantage for aluminum in a manner that can be verified or disproved only by test. A tread land (contact surface between grooves or outside of the outermost groove on each side of the tread) might wipe the macroscopic surface of asphalt or concrete free of water, but cannot wipe out water that has gone deeply into pores. At high rates of slip, friction heating would vaporize water within the pores, causing a large fraction of the bearing surface of the tread to float on a cushion of steam. This effect could not exist with a smooth aluminum surface.

The phenomenon of hydroplaning might possibly be a most important problem. For dynamic hydroplaning, we would expect the aluminum surface to be neither better nor worse than conventional surfaces, given the existence of a layer of water. Viscous hydroplaning might tend to be worse with aluminum than with concrete or asphalt because the porous surface of concrete or asphalt might tend to break up patterns of viscous flow. On the other hand, the fact that water does not adhere to moderately clean aluminum
should make continuous drainage of the surface easier and should allow the use of air jets to combat viscous hydroplaning.

**Effect of Snow or Ice**

Given an accumulation of snow or ice on top of which an aircraft tire rolls or slides, the aluminum-surfaced runway would be no worse nor better than existing runways except possibly if we are considering the specific implementation in which routine use would be made of yawed wheels, with conventional brakes eliminated.

Under present conditions runways are often not thoroughly cleaned of snow, so that aircraft are forced to land on packed snow, or even with patches of ice. Low adhesion of snow or ice to the aluminum because of the absence of pores should be a distinct advantage in removal of snow. Blowing, or removal with large rotating brushes, should be more effective with aluminum than with asphalt or concrete because of the lower adhesion. Lower adhesion to the aluminum would also make it easier for a sliding tire to scrape the surface free of snow and to achieve the coefficient of friction that exists between rubber and aluminum. The degree to which this effect would be possible with high-pressure aircraft tires could be determined only by test.

Removal of thin residual layers of snow by heating might be economically feasible if a satisfactory way could be found to distribute the heat uniformly to the surface. For example, with a 13-millimeter (1/2-in.) layer of snow, a specific gravity of 0.125, and electric power at 1.5 cents per kilowatt-hour, the snow could be melted from a 3050-meter (10 000-ft) runway 51 meters (200 ft) wide at a cost of

\[
\frac{1}{2} \times \frac{1}{12} \times 200 \times 10 000 \times 0.125 \times 62.4 \times 144 \times \frac{1}{3413} \times 0.015 = $411
\]

If natural gas were used at 0.025 cents per cubic meter (70¢ per 1000 ft\(^3\)), with a heating value of 3.72x10\(^4\) joules per meter cubed (1000 Btu/ft\(^3\)), the cost would be

\[
\frac{1}{2} \times \frac{1}{12} \times 200 \times 10 000 \times 0.125 \times 62.4 \times 144 \times \frac{1}{1000} \times \frac{1}{1000} \times 0.7 = $65.52
\]

The problem of uniform distribution of applied heat would be somewhat ameliorated by the fact that, if unevenness of heating existed, every landing and takeoff of an aircraft would redistribute some of the snow from the relatively unheated areas into the more strongly heated areas.
CONCLUDING REMARKS

The theoretical treatment and preliminary test results that have been presented here appear to indicate a sufficient probability of success in some of the objectives outlined, with at least a possibility of success in others, to justify definitive tests. Those objectives, in approximately the order of their probabilities of success are as follows:

1. Attainment of a substantially greater braking force than is now possible, perhaps as great as 1.4 g, with use of braking equipment of the type now in use, and with a textured runway surface that exposes tread rubber to small areas of aluminum and small areas of abrasive material. (Reduced tire wear a possible incidental advantage.)

2. The same as objective 1, but with a runway surface completely covered with a patterned aluminum sheet.

3. The same as objective 2, but with smooth aluminum.

4. Use of an aluminum skin as a heat sink, relieving conventional brakes of a substantial part of the heat load, with result of better braking, less brake wear, and possibly less tire wear.

5. Use of an aluminum skin as a heat sink for all braking heat, with elimination of conventional brakes and their maintenance expense, and possibly with reduced tire wear.

The effects of rain water, snow, and ice are yet to be determined. However, reasons are presented to indicate these effects may not be an insurmountable problem.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 27, 1972,
501-38.
APPENDIX A

THEORY OF PARTITION OF CONTACT CONDUCTANCE

Contact conductance $\alpha$, as the term will be used here, signifies the reciprocal of the resistance of an interface to the flow of heat across it. According to widely accepted theory (ref. 11), the reason for existence of finite rather than infinite thermal contact conductance is that conduction of heat through an interface between two solids is reduced below the rate that might otherwise exist because firm contact within the interface occurs only at a relatively few isolated points and the thermal streamlines are consequently crowded together throughout appreciable distances along which streamlines approach and leave those few isolated points. Consequently, in the neighborhood of the interface, the capacity of the materials to conduct heat is not efficiently used. The conductivity of the contaminant filling the voids between the areas that are in firm contact may often be neglected, as will be done here.

This effect is illustrated for two metal blocks in contact in figure 6, in which only three points of firm contact are shown. Firm contact is prevented elsewhere by crevices, assumed of depth $\delta_1$ and $\delta_2$ in blocks 1 and 2, respectively.

If the upper block in figure 6 were replaced by rubber under substantial pressure, while the lower block remained as a metal, or were replaced by concrete or blacktop, presumably the rubber would displace into the crevices of the lower block. As a result, the crowding of streamlines should be eliminated or substantially reduced. However, even if the surface of the lower block were absolutely flat and smooth, the same effect might occur due to inclusion of minute bubbles of air or other contaminant.

In figure 7, which will now be used as a basis for derivation of equations, only one area of firm contact is assumed. It is further assumed that all the thermal streamlines passing through the total area of the blocks $A_t$, and only those streamlines, will pass through the area of firm contact. The area $A_t$ at each end of the assembly will be completely filled with a large number $n$ of streamtubes of which one only, streamtube $i$, is shown. The distances $L$ are shown equal and are assumed great enough that the thermal streamlines will be substantially parallel at the upper and lower ends of the assembly.

The crevice depths $\delta_1$ and $\delta_2$ will be assumed equal and, in consequence of this assumption, flow streamlines will be parallel at the plane of the interface. Actually the crevice depth $\delta_1$ should be greater than $\delta_2$ because the rubber will yield to accommodate the contaminant. Assumption of $\delta_1$ equal to $\delta_2$ is believed to be conservative for the purpose of this treatment.

Now, under the conditions specified, after steady state is established, the shapes and lengths of the $n$ streamtubes in the rubber will be a mirror image of the shapes
and lengths in the rigid solid. The potential-flow solution for the streamlines under the steady-state condition is independent of the thermal properties of the material.

The total thermal conductance for all the streamtubes in one of the blocks will be the sum of the conductances for all of the \( n \) streamtubes within that block. The thermal conductance for streamtube \( i \) in the rubber will be

\[
K_{i(r)} = k_r^{-1} \int_0^{\xi_i} A_i^{-1} \, dS_i^{-1}
\]

and in aluminum, for example,

\[
K_{i(a)} = k_a^{-1} \int_0^{\xi_i} A_i^{-1} \, dS_i^{-1}
\]

where \( k_r \) and \( k_a \) are the thermal conductivities of rubber and aluminum, \( A_i(s) \) is the cross-sectional area of streamtube \( i \) at position \( S_i \) along its length, and \( \xi_i \) is the total length of streamtube \( i \) within the rubber or the aluminum. For the present we ignore the fact that the streamtube on the aluminum side may actually consist of aluminum oxide.

Accordingly, the total conductance in the rubber for the area \( A_t \) will be

\[
K_r = k_r \sum_{i=0}^{n} \left[ \int_0^{\xi_i} A_i^{-1} \, dS_i \right]^{-1}
\]

and in the aluminum,

\[
K_a = k_a \sum_{i=0}^{n} \left[ \int_0^{\xi_i} A_i^{-1} \, dS_i \right]^{-1}
\]

But, because the streamline pattern in the aluminum is a mirror image of that in the rubber, the summations in equations (A3) and (A4) are identical, and
To a close approximation, but not with theoretical exactness, we could write

\[ \alpha \equiv \left( K_r^{-1} + K_a^{-1} \right)^{-1} \]  

(A6)

where \( \alpha \) is the conventional contact conductance (film coefficient) of the interface. We will now seek an equation in the same form as equation (A6), which will be theoretically exact, with use of parameters \( \alpha_r \) and \( \alpha_a \) that will be nearly identical with \( K_r \) and \( K_a \). The theoretical inexactness of equation (A6) is due to the fact that the conventional contact conductance is the reciprocal of the excess resistance, within the distance \( 2L \) of figure 7, above the resistance that would exist if there were no voids or contaminants within the interface. Thus, in practice, the contact conductance for both materials together is calculated from the result of an experimental determination of the actual total resistance of the assembly. The theoretical total resistance of the assembly that would be expected for zero contact resistance is easily calculated. The excess of the experimentally determined total resistance over this theoretical total resistance is taken as the thermal contact resistance. For rubber and aluminum, the contact conductance as measured is

\[ \alpha = \left[ K_r^{-1} + K_a^{-1} - L A_t^{-1} \left( k_r^{-1} + k_a^{-1} \right) \right]^{-1} \]  

(A7)

or, from equations (A3), (A4), and (A7),

\[ \alpha = \left[ k_r^{-1} \left( X - L A_t^{-1} \right) + k_a^{-1} \left( X - L A_t^{-1} \right) \right]^{-1} \]  

(A8)

where \( X \) represents the reciprocal of the summation in equation (A3) or (A4). If we let

\[ X' = \left( X - L A_t^{-1} \right) \]  

(A9)

equation (A8) becomes

\[ \alpha^{-1} = k_r^{-1} X' + k_a^{-1} X' \]  

(A10)
From equation (A10) it is seen that the total contact resistance $\alpha^{-1}$ can be divided into two parts, one ascribable to the rubber and the other to the aluminum. Thus, for the rubber

$$\alpha^{-1}_r = k^{-1}_r X'$$  \hfill (A11)

and for the aluminum

$$\alpha^{-1}_a = k^{-1}_a X'$$  \hfill (A12)

So, from equations (A11) and (A12),

$$\frac{\alpha_r}{\alpha_a} = \frac{k_r}{k_a}$$  \hfill (A13)

Also

$$\alpha = \left(\alpha^{-1}_r + \alpha^{-1}_a\right)^{-1} = \frac{\alpha_a \alpha_r}{\alpha_r + \alpha_a}$$  \hfill (A14)

Thus, if $\alpha$ is measured, $\alpha_r$ and $\alpha_a$ may be obtained with the simultaneous equations (A13) and (A14), a fact that will be of only academic interest here, though not so in anticipated future work. Equations (A13) and (A14) will be useful for present purposes in derivation of a boundary condition governing partition of frictional heat between a rubber surface and an aluminum surface on which it slides.

In partial support of the foregoing treatment, we may examine equation (24) in Chapter 13 of reference 11. (Note that a typographical error exists in that equation, namely, $\eta$ should have been $n$. ) That equation, applied to aluminum and rubber and with neglect of the conductivity of the contaminant, reads as follows

$$\alpha = \frac{\epsilon^2}{1 - \epsilon^2} \left(\frac{\delta_r}{k_r} + \frac{\delta_a}{k_a} + \frac{0.46 \left(\frac{\epsilon}{n}\right)^{1/2}}{k_s}\right)$$  \hfill (A15)

where $\epsilon^2$ is the ratio of the area of hard molecular contact to the area $A$, $n$ is the number of areas of hard contact per unit total area, and
\[
\frac{1}{k_s} = \frac{1}{2} \left( \frac{1}{k_r} + \frac{1}{k_a} \right) \tag{A16}
\]

For all values of \( \epsilon, \delta_r, \) and \( n, \) if \( \delta_r = \delta_a, \) equation (A15) can be converted to the same form as equation (A10) with \( X' \) replaced by a constant \( (1 - \epsilon^2)(\delta_r + 0.23 \epsilon/\sqrt{n})/\epsilon^2. \)

Even if \( \delta_r \neq \delta_a, \) we see by inspection of equation (A15) that it will reduce approximately to the form of equation (A10) if

\[
\frac{\epsilon^2}{n} \gg 18.9 \delta_{\text{max}}^2 \tag{A17}
\]

where \( \delta_{\text{max}} \) is \( \delta_r \) or \( \delta_a, \) whichever is larger. In the physical picture (fig. 6), it is true that a complete potential flow solution would show many a streamline that would flow in turn through the stagnation area \( St_1, \) one of the minute areas of firm contact, and then through the stagnation area \( St_2. \) But for the pattern as it appears in the figure, such streamlines would represent only a minute fraction of the total heat flow. Thus, if either \( \delta_1 \) or \( \delta_2 \) were greatly increased so that a void would cut deeply into the area \( St_1 \) or the area \( St_2, \) the flow pattern would be but little changed. For two metals in contact, failure of the criterion (A17) to be satisfied might reasonably be expected in many cases. But, in the case of rubber in contact with any rigid solid, because of the readiness of rubber to conform, it seems more plausible to expect \( \epsilon^2/n \) to be relatively great.
APPENDIX B

BOUNDARY CONDITION GOVERNING PARTITION OF FRICTION HEAT

The treatment of the question of the partition of frictionally generated heat here will be based on an assumption that only negligible heat will be generated by slippage between rubber and the contaminant or between aluminum and the contaminant that fills the microscopic voids shown in figure 7. It is assumed that negligible heat will be generated by slippage between contaminant and contaminant, or by viscous friction within fluid or plastic contaminant. It is also assumed that negligible heating will be caused by internal friction of the rubber due, for example, to local stick-slip conditions within the interface such as might cause cyclical local deformations within the body of the rubber. Although the discussion will deal specifically with rubber and aluminum, it would have general applicability to other materials, so long as the conductivities of those materials were great relative to the conductivities of contaminants and the frictional heat generated within the contaminants were negligible compared with the frictional heat generated within the minute interfaces of firm contact between the two basic materials. The fact we may really be dealing with aluminum oxide will be disregarded for the present.

For simplicity, this treatment will deal with a one-dimensional situation. That is, we will seek a result that applies, per unit area, to a specific differential area within the interface $S_i$ shown in sketch (b). Also, our result will apply only within a specific differential time $dt$. We will designate $g$ as the rate of heat generation within the interface between the rubber and the aluminum. We define $q_a$ and $q_r$ as rate of heat flow into the interface from aluminum and rubber, respectively. We denote the value of $x$ at the absolute interface as $x_i$. By absolute interface, we mean a plane in which microscopic or submicroscopic areas of aluminum and rubber surfaces come into hard atom-to-atom contact. We denote by $x_{as}$ the value of $x$ at a plane $S_{a'}$, the closest plane to $S_i$ and parallel to $S_i$, within the body of the aluminum, at which the heat flow
streamlines have become substantially uniformly distributed throughout the entire os-
tensible area of the interface. We denote by $x_{rs}$ the value of $x$ for an analogous plane $S_r$ within the body of rubber. Then,

$$x_{as} \approx x_1 \approx x_{rs} \quad (B1)$$

We may write two independent expressions for $q_a$, and another pair of independent equations for $q_r$, as follows,

$$q_a = -k_a \left( \frac{\partial \varphi}{\partial x} \right)_{as} \quad (B2)$$

where $k_a$ is the thermal conductivity of aluminum, $\varphi = \varphi(x)$ is temperature within a body of material, whether aluminum or rubber, and the subscript $as$ denotes that the derivative applies at $x_{as}$ approached from lower values of $x$.

$$q_a = \alpha_a (\varphi_{as} - \varphi_1) \quad (B3)$$

where $\alpha_a$ as earlier defined applies to conduction of heat from the plane $S_a$ into the plane $S_i$, $\varphi_{as}$ is the temperature within plane $S_a$, and $\varphi_1$ is the temperature within plane $S_i$. (The temperatures of the monomolecular layers of aluminum and rubber adjacent to $S_i$ will each be treated as equal to $\varphi_1$.)

$$q_r = k_r \left( \frac{\partial \varphi}{\partial x} \right)_{rs} \quad (B4)$$

$$q_r = \alpha_r (\varphi_{rs} - \varphi_1) \quad (B5)$$

Notations in equations (B4) and (B5) for rubber are all analogous to those in equations (B2) and (B3) for aluminum. The signs are opposite in equations (B2) and (B4) because both $q_a$ and $q_r$ are defined as flows into plane $S_i$ and the derivative in equation (B4) applies to the condition at $x_{as}$ in the direction of greater values of $x$.

Now, because there is zero heat capacity within plane $S_i$, the net heat flow into that plane, including the heat generated within it, must equal zero at all times. Hence,

$$g + q_a + q_r = 0 \quad (B6)$$
We now have seven equations (A13), (A14), and (B2) to (B6) in nine unknowns, namely, $q_a$, $q_r$, $(\partial \phi / \partial x)_{as}$, $(\partial \phi / \partial x)_{rs}$, $\alpha_a$, $\alpha_r$, $\varphi_{as}$, $\varphi_{rs}$, and $\varphi_i$. We may reduce these seven equations to one equation in three unknowns. We now wish to do so in two ways: (1) to obtain an equation in $(\partial \phi / \partial x)_{as}$, $\varphi_{as}$, and $\varphi_{rs}$, and (2) to obtain an equation in $(\partial \phi / \partial x)_{rs}$, $\varphi_{rs}$, and $\varphi_{as}$.

We observe that only the five equations (B2) to (B6) contain $q_a$ and $q_r$. We reduce these five equations to the following three, in which both $q_a$ and $q_r$ are absent:

\[ \varphi_{as} - \varphi_i = -\frac{k_a}{\alpha_a} \left( \frac{\partial \phi}{\partial x} \right)_{as} \quad (B7) \]

\[ \varphi_{rs} - \varphi_i = \frac{k_r}{\alpha_r} \left( \frac{\partial \phi}{\partial x} \right)_{rs} \quad (B8) \]

\[ g - k_a \left( \frac{\partial \phi}{\partial x} \right)_{as} + k_r \left( \frac{\partial \phi}{\partial x} \right)_{rs} = 0 \quad (B9) \]

We now have five simultaneous equations (A13), (A14), and (B7) to (B9) in seven unknowns, namely, $(\partial \phi / \partial x)_{as}$, $(\partial \phi / \partial x)_{rs}$, $\alpha_a$, $\alpha_r$, $\varphi_{as}$, $\varphi_{rs}$, and $\varphi_i$. We observe that among these five equations only equations (B7) and (B8) contain $\varphi_i$. Eliminating $\varphi_i$ between these two equations, we get

\[ \varphi_{as} - \varphi_{rs} = -\frac{k_a}{\alpha_a} \left( \frac{\partial \phi}{\partial x} \right)_{as} - \frac{k_a}{\alpha_r} \left( \frac{\partial \phi}{\partial x} \right)_{rs} \quad (B10) \]

We now have only four simultaneous equations, namely, equations (A13), (A14), (B9), and (B10). The unknowns $\alpha_a$ and $\alpha_r$ appear only in equations (A13), (A14), and (B10) among the remaining four equations. We now eliminate $\alpha_a$ and $\alpha_r$ from these three equations, getting

\[ \varphi_{as} - \varphi_{rs} = -\frac{k_a k_r}{(k_r + k_a) \alpha} \left[ \left( \frac{\partial \phi}{\partial x} \right)_{as} + \left( \frac{\partial \phi}{\partial x} \right)_{rs} \right] \quad (B11) \]
Only two simultaneous equations now remain, namely, equations (B9) and (B11). We now redefine \( x \) to be measured positively in either direction from the plane \( S_1 \). Then, eliminating \( \frac{\partial \varphi}{\partial x} \) from equations (B9) and (B11) we get

\[
-k_a \left( \frac{\partial \varphi}{\partial x} \right)_{as} = \left( \frac{k_a}{k_a + k_r} \right) g + \alpha (\varphi_{rs} - \varphi_{as}) \tag{B12}
\]

Eliminating \( \frac{\partial \varphi}{\partial x} \) from equations (B9) and (B11) we get

\[
-k_r \left( \frac{\partial \varphi}{\partial x} \right)_{rs} = \left( \frac{k_r}{k_r + k_a} \right) g + \alpha (\varphi_{as} - \varphi_{rs}) \tag{B13}
\]

Passing now to three physical dimensions, and including time as a variable, we may rewrite equations (B12) and (B13) as

\[
-k_a \frac{\partial \varphi_a(x, y, z, t)}{\partial x} = \left( \frac{k_a}{k_a + k_r} \right) g(y, z, t) + \alpha \left[ \varphi_r(x, y, z, t) - \varphi_a(x, y, z, t) \right], \quad x = 0 \tag{B14}
\]

and

\[
-k_r \frac{\partial \varphi_r(x, y, z, t)}{\partial x} = \left( \frac{k_r}{k_r + k_a} \right) g(y, z, t) + \alpha \left[ \varphi_a(x, y, z, t) - \varphi_r(x, y, z, t) \right], \quad x = 0 \tag{B15}
\]

where \( y \) and \( z \) are spatial coordinates within the plane \( S_1 \), \( \varphi_a(x, y, z, t) \) and \( \varphi_r(x, y, z, t) \) are temperatures within or on the surface of aluminum and rubber, respectively, as functions of \( x, y, z, \) and \( t \), and \( g(y, z, t) \) is the rate of heat generation per unit area within plane \( S_1 \) as a function of \( y, z, \) and \( t \).

Equations (B12) and (B13), or (B14) and (B15), are redundant boundary conditions that must be satisfied at any time \( t \) and in any part of the planes \( S_a, S_r, \) and \( S_1 \), which may now be treated as all the same plane.

This treatment has used equation (A13), whose derivation neglected conduction of heat by contaminants that fill the voids between the aluminum and the rubber. However, it may be noted that equations (B12) and (B13), and hence also equations (B14) and (B15), are valid even if conduction of heat by the contaminants is not negligible, so long as heat generation within the contaminants or on their surfaces is negligible.

Figure 8 illustrates the condition with thermal streamlines passing through the contaminant. Only one area of hard contact is shown, with all streamlines within regions...
designated A passing through it. Streamlines within the regions designated B and C pass through the contaminant on either side. A type of symmetry in relation to other areas of hard contact is assumed such that streamlines at the extreme right and left sides of the sketch are vertical straight lines.

Now if we consider only the surface area $A^t$ as designated in figure 8, and the regions A, remembering that all frictional heat is generated within the area of hard contact, we have exactly the same condition as illustrated in figure 7, but with substitution of $A^t$ for $A^o$. So, if we designate $\alpha_t$ as the part of the total contact conductance provided by the area $A^t$, we may apply equation (B12) in the form

$$- \frac{A^A}{A^t} k \frac{\partial \varphi}{\partial x} = \left( \frac{k_a}{k_a + k_r} \right) \alpha + \alpha_t (\varphi_{rs} - \varphi_{as})$$  \hspace{1cm} (B16)

Now regions B and C are paths for heat flow parallel to region A. They must have a total conductance

$$\alpha_{BC} = \alpha - \alpha_t$$ \hspace{1cm} (B17)

As no heat is generated at the interface between the two regions B or between the two regions C, and as steady-state conditions are assumed at all times, a simple steady-state equation for heat conduction may be written for these regions as

$$- \frac{A_B + A_C}{A^t} k \frac{\partial \varphi}{\partial x} = \alpha_{BC} (\varphi_{rs} - \varphi_{as})$$  \hspace{1cm} (B18)

Finally, as the expressions on the left sides of equations (B16) and (B18) are parallel flows, they may be added. Hence, from equations (B16), (B17), and (B18), we get equation (B12), the same as for the condition when no heat flow through the contaminant was assumed. Equation (B13) may also be reproduced in a similar manner.
REFERENCES


Figure 1. - Nature of sliding of free-rolling tire.

Figure 2. - Aluminum-surfaced extension of runway for locked-wheel stop of aircraft with conventional brakes.
Figure 3. - Computed path of aircraft with locked wheels in crosswind. Deceleration, 10.4 meters per second squared (1.4 x gravity); initial speed, 82.5 meters per second (160 knots); crosswind force, 0.1 x aircraft weight.

Figure 4. - Distribution of flow of heat of friction between aluminum and rubber.
(a) Bogie in unbraked condition, always yawed to avoid tracking of leading wheels by trailing wheels.

(b) Bogie moderately braked by yawed wheels, partly rolling, partly sliding. Directional control possible. All torques and forces balanced except brake force.

(c) 90° Yaw probably always undesirable. No rolling. No directional control. Single spot on tread receives all heat that flows into rubber.

Figure 5. - Braking of aircraft bogie by yawing of wheels on aluminum-surfaced runway.

Figure 6. - Illustration of crowding of thermal streamlines near interface of two metals.
Figure 7. Illustration of heat flow through rubber and rigid solid in contact.

Figure 8. Thermal streamlines through hard contact area and through contaminant.
"The aeronautical and space activities of the United States shall be conducted so as to contribute ... to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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