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Analytical Treatment of Gas Flows Through Multilayer Insulation

Final Report

by

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ABSTRACT

A theoretical investigation of gas flow inside a multilayer insulation system has been made for the case of the broadside pumping process. A set of simultaneous first-order differential equations for the temperature and pressure of the gas mixture was obtained by considering the diffusion mechanism of the gas molecules through the perforations on the insulation layers. A modified Runge-Kutta method was used for numerical experiment. The numerical stability problem was investigated. It has been shown that when the relaxation time is small compared with the time period over which the gas properties change appreciably, the set of differential equations can be replaced by a set of algebraic equations for solution. Numerical examples were given and comparisons with experimental data were made.
Nomenclature

$A$ area of one insulation layer (one side)

$a$ total area of perforation on one insulation layer

$a' = (1 - c)A$

$c$ perforation coefficient $c = a/A$

$k$ Boltzmann constant

$L$ number of interstitial spaces between insulation layers

$M$ number of kinds of absorbed gases plus 1

$m$ mass of a gas molecule or an integer

$N$ number of gas molecules

$n$ number density of gas molecules or an integer

$p$ pressure

$Q, q, R$ outgassing rates, see Equations (4), (7), and (12)

$T$ temperature

$t$ time

$V$ volume

$v$ mean thermal velocity of gas molecules

$\delta$ number of degrees of freedom of a gas molecule

$\tau$ relaxation time

Subscripts: subscript $c$ refers to quantities inside the vacuum chamber, $f$ to condition of the fuel tank, $a$ to atmospheric condition, other subscripts refer to quantities in a certain interstitial space

Superscripts: superscript 1 refers to the purge gas, other superscripts refer to a certain kind of gas molecules
1. Introduction

Recent developments in propulsion technology have stimulated interest in the study of multilayer insulation systems for the fuel tank of a rocket booster. The purpose of multilayer insulations is to eliminate heat conduction between the fuel tank and the environment. It is then desirable that the spaces between the insulation layers be maintained at low pressures. However, it is not practical to require complete vacuum in the spaces between the insulation layers, although it would be ideal in principle, for the reasons that complete vacuum is hardly attainable and that when the pressure gets too low, deformation of the layers would cause solid contacts, which in turn would induce heat conduction. It is therefore customary in practice to require a pressure to be of the order of $10^{-4}$ torr. This low pressure is usually achieved before launch from the ambient pressure by some pumping device. There have been two pumping procedures in practice; namely, the broadside pumping by which the direction of gas flow is perpendicular to the insulation layers, and the edge pumping which cause the gas to flow parallel to the layers. Experiments of both evacuation procedures have been performed and results reported.¹ ² ³ The present work is a theoretical analysis for the broadside pumping process.

In the case of broadside pumping, the insulation layers are perforated so that gas molecules can go through small holes on the layers, resulting in streaming gas motion perpendicular to the insulation layers. It is obvious that, in order to minimize layer deformation and solid contacts, small pressure differentials across the layers should be maintained during evacuation. This can be achieved by small perforation, i.e., the total area of the holes on an insulation layer is small compared to the total area of that layer. If we further require that the diameter of a single hole be small or comparable to the distance between two successive layers, there is the advantage of uniformity of gas motion, which renders convenience during operation.

** A multilayer insulation system consists of a large number (in the order of $10^2$) of extremely thin sheets of low thermal conductivity and lightweight materials, kept parallel to one another with a total thickness of about 1".
2. Formulation

Consider a fuel tank of simple geometry wrapped with multilayer insulation and situated inside a vacuum chamber. Consider the broadside pumping process and assume that the perforation areas are small. Then, a gas molecule in any interstitial space between two successive layers will, on the average, collide many times with the walls or with some other gas molecules in that interstice before getting through a hole to a neighboring interstice. Thus, it is plausible to assume that the gas in any interstitial space is in thermal equilibrium with the temperature and pressure pertinent to that interstice, and the motion of the gas is simply a diffusion process.

Consider the gas inside the insulation system as a mixture of a certain kind of purge gas and a number of different kinds of gases originally absorbed in the insulation materials. Let $A_1$ be the total area of the outermost insulation layer (one side), and $c_1$ the perforation coefficient of the same layer (the ratio of the total perforation area on the first layer to $A_1$), and so on. Let $V_1$ be the volume of the interstice between the first and the second insulation layers, and $N_1$ the number of molecules in $V_1$, $V_2$ the volume of the interstice between the second and the third insulation layers, and $N_2$ the number of molecules in $V_2$, and so on. Since the gases in any interstice are in thermal equilibrium, each component gas has a Maxwellian distribution with its local density and the common local temperature. From the kinetic theory of gases, we have, for the gases in the $i$th interstice,

\[
\frac{d N_i^{(j)}}{dt} = \frac{a_i}{4} \left( n_i^{(j)} v_i^{(j)} - n_{i-1}^{(j)} v_i^{(j)} - n_i^{(j)} v_{i-1}^{(j)} \right) \\
+ \frac{a_i + 1}{4} \left( n_{i+1}^{(j)} v_{i+1}^{(j)} - n_i^{(j)} v_{i+1}^{(j)} - n_{i+1}^{(j)} v_i^{(j)} \right) + \left( a_i' + a_{i+1}' \right) q_i^{(j)}
\]

\[i = 1, 2, \ldots, L; \quad j = 1, 2, \ldots, M. \quad (1)\]
\[
\frac{d}{dt} \left( k \sum_{j=1}^{M} \delta(j) N_i(j) \right) = \frac{a_i}{4} \sum_{j=1}^{M} k \delta(j) \left( n_{i-1}(j) v_i(j) T_i - n_i(j) v_i(j) T_i \right) \\
+ \frac{a_i+1}{4} \sum_{j=1}^{M} k \delta(j) \left( n_{i+1}(j) v_i(j) T_i - n_i(j) v_i(j) T_i \right) \\
+ \sum_{j=1}^{M} k \delta(j) T_i \left( a_i + a_i+1 \right) q_i(j) \\
i = 1, 2, \ldots, L-1.
\]

where \( a_i = c_i A_i \), \( a_i' = (1-c_i) A_i \), \( k \) is the Boltzmann constant, \( \delta \) the number of degrees of freedom of motion of a gas molecule, \( n_i, v_i \) and \( T_i \) are the number density, the mean thermal velocity and the temperature of the gas molecules in the \( i \) th interstice, respectively. The superscript \( j \) refers to the purge gas if \( j = 1 \), or the absorbed gas of kind \( j \) otherwise. The number of interstitial spaces is \( L \), and the number of kinds of the absorbed gases is \( M-1 \) so that

\[
n_i = \sum_{j=1}^{M} n_i(j) \quad \text{and} \quad N_i = \sum_{j=1}^{M} N_i(j) \quad \text{where} \quad N_i(j) = \nu_i n_i(j)
\]

Finally,

\[
q_i(j) = \begin{cases} 
0 & (j = 1) \\
\text{number of outgassing molecules, of kind } j \text{ from the} & \\
\text{walls binding the } i \text { th interstice, per unit area and} & \\
\text{time } (j \neq 1). & 
\end{cases}
\]

Equations (1) state the conservation of mass and Eqs. (2) the conservation of energy.
Since the $L$ th interstice is next to the fuel tank, we may assume that $a_{L+1} = 0$ and $T_L = T_f$, where $T_f$ is the temperature of the fuel tank. Since interstice 1 is the outermost, denoting the number densities, the mean thermal velocities and the temperature of the molecules in the vacuum chamber by $n_c^{(i)}$, $v_c^{(i)}$ and $T_c$, respectively, we have $n_o^{(1)} = n_c^{(1)}$, $v_o^{(1)} = v_c^{(1)}$ and $T_o = T_c$. Notice that as the volume of the vacuum chamber is very large, $n_c^{(j)} (j \neq 1)$ are vanishingly small compared to $n_c^{(1)}$ at all times and we may assume that $n_c^{(j)} = 0 (j \neq 1)$. By making this assumption, $v_o^{(j)} (j \neq 1)$ become meaningless and they drop out from Eqs. (1) and (2) automatically. Therefore, in Eqs. (1) and (2), we use

$$
\begin{align*}
\left\{ a_{L+1} &= 0 , \ T_L = T_f, \ T_o = T_c, \ v_o^{(1)} = v_c^{(1)} \\
n_o^{(1)} &= n_c^{(1)} \ \text{and} \ n_o^{(j)} = 0 (j \neq 1) \right. 
\end{align*}
$$

(5)

Following $p_i = n_i k T_i$, where $p_i$ is the pressure in the $i$th interstice, we may assume that

$$
p_i^{(j)} = n_i^{(j)} k T_i \ \text{so that} \ p_i = \sum_{j=1}^{M} p_i^{(j)}
$$

(6)

which is plausible when the pressure is not too high.

Using Eqs. (3) and (6), and if the outgassing rates are expressed in the customary units of pressure times volume per unit area and time denoted by

$$
Q_i^{(j)} = k T_i q_i^{(j)}
$$

(7)

then, Eqs. (1), (2), and (5) can be written as
\[ V_i \left( \frac{1}{T_i} \frac{dp_i(j)}{dt} - \frac{p_i(j)}{T_i^2} \frac{dT_i}{dt} \right) = \frac{a_i}{4} \left( \frac{p_{i-1}(j)}{T_i} v_i(j) - \frac{p_i(j)}{T_i} v_i(j) \right) \]

\[ + \frac{a_i + 1}{4} \left( \frac{p_{i+1}(j)}{T_i+1} v_i(j) - \frac{p_i(j)}{T_i} v_i(j) \right) \]

\[ + \left( a_i' + a_i' + 1 \right) \frac{Q_i(j)}{T_i} \]

\[ i = 1, 2, \ldots, L; \quad j = 1, 2, \ldots, M. \]  \hspace{2cm} (8)

\[ V_i \sum_{j=1}^{M} \delta(j) \frac{dp_i(j)}{dt} = \frac{a_i}{4} \sum_{j=1}^{M} \delta(j) \left( \frac{p_{i-1}(j)}{T_i} v_i(j) - \frac{p_i(j)}{T_i} v_i(j) \right) \]

\[ + \frac{a_i + 1}{4} \sum_{j=1}^{M} \delta(j) \left( \frac{p_{i+1}(j)}{T_i+1} v_i(j) - \frac{p_i(j)}{T_i} v_i(j) \right) \]

\[ + \sum_{j=1}^{M} \delta(j) \left( a_i' + a_i' + 1 \right) Q_i(j) \]

\[ i = 1, 2, \ldots, L - 1. \]  \hspace{2cm} (9)

\[ \begin{cases} a_L + 1 = 0, \ T_L = T_f', T_o = T_c', v_o^{(1)} = v_c^{(1)} \\ p_o^{(1)} = p_c^{(1)} \text{ and } p_o^{(j)} = 0 \ (j \neq 1). \end{cases} \]  \hspace{2cm} (10)
where \( p_c^{(1)} \) is the chamber pressure due to the purge gas.

The mean thermal velocities are given by

\[
v_{ij} = \left( \frac{8 k_i T_i}{\pi m_j} \right)^{1/2}, \quad i = 0, 1, \ldots; j = 1, 2, \ldots, M.
\]

(11)

where \( m_j \) is the mass of a molecule of the \( j \)th kind.

To obtain a solution to the problem, the outgassing rates \( Q_i^{(j)} \) must be known a priori, usually determined experimentally. Outgassing is a process in which gas molecules originally adsorbed or absorbed in a solid material leave that material under reduced pressure or elevated temperature. The outgassing rate of the insulation layers depends on the material and preconditioning of the layers, the temperature, pressure, time and the kinds of absorbed gases. For a specific material and preconditioning and a specific absorbed gas, \( Q_i^{(j)} = Q_i^{(j)}(T, p, t) \).

Around the room temperature range, there is not outgassing at atmospheric or higher pressures. Outgassing occurs when the background pressure is reduced considerably below the atmospheric pressure. The outgassing rate increases as the pressure decreases, and attains appreciable values only at very low pressures. However, in very low pressure ranges, the outgassing rate is a weak function of pressure. Therefore, its dependence on the pressure may be approximated by

\[
Q_i^{(j)} = \left( 1 - \frac{p_i}{p_a} \right) R_i^{(j)}(T, t)
\]

(12)

where \( p_i = \sum_{j=1}^{M} p_i^{(j)} \), \( p_a \) is the atmospheric pressure and \( R_i^{(j)} \) are the outgassing rates at extremely low pressures.

*** The absorbed gases are mainly water vapor, with small amounts of CO\(_2\) and N\(_2\) (Ref. 7).
The initial conditions associated with the system of Equations (8) and (9) are prescribed according to a specific situation encountered. Usually the evacuation process starts with atmospheric pressure inside the insulation system. In this case, the initial conditions can be written as

\[
p_i^{(1)}(0) = p_a, \quad p_i^{(j)}(0) = 0 \quad (j \neq 1), \quad i = 1, 2, \ldots, L
\]

\[
T_i(0) = T_i, \quad i = 1, 2, \ldots, L - 1.
\]

Now, with the \((M + 1) L - 1\) initial conditions given by Eqs. (13) or prescribed otherwise and the outgassing rates \(Q_i\) known, and using Eqs. (10) and (11), Eqs. (8) and (9) constitute an appropriate set of \((M + 1) L - 1\) simultaneous first order differential equations for the same number of unknown functions of time, namely, \(T_i(t), \quad i = 1, 2, \ldots, L - 1\) and \(p_i^{(j)}(t), \quad i = 1, \ldots, L; \quad j = 1, \ldots, M\).

In some engineering applications, e.g., the Apollo Telescope Mount insulation, the temperature is quite uniform across the entire insulation system, and the gas flow inside the system is nearly isothermal. In this case, the problem reduces to the solution of Eqs. (8) with Eqs. (10) and (11), and the initial conditions (13), when all temperatures are set equal to a constant.
3. Numerical Analysis

In a multilayer insulation system, the number \( L \) usually is of the order of \( 10^2 \). Thus, the governing Eqs. (8) and (9) consist of a large number of first order differential equations which are non-linear and coupled. The analytical solution of this system of equations can hardly be obtained, one therefore uses a direct numerical method for its solution.

For simplicity in numerical experiments, we took an isothermal case, and assumed water vapor was the only absorbed gas present \( (M = 2) \). Furthermore, a hypothetical \( R(2)(T_o, t) \) and chamber conditions were used. When the temperature is constant, the energy equations (Eqs. (9)) are not needed, and the problem reduces to the solution of Eqs. (8) with initial conditions (13). In the process of numerical computation, however, the problem of instability occurred. A numerical program using a modified Runge-Kutta method \(^8\) has been set up for this initial value problem. Computer experiments on this scheme showed that the system was stable when the values of \( \tau_i(1) = (4V_i/a_i)(\pi m(1)/8kT_o)1/2 \), which have a dimension of the time, were large. However, in order to establish numerical stability for small values of \( \tau_i(1) \), the step sizes of integration had to be so small that a solution could not be obtained with a reasonable length of computer time. Since in ordinary applications the values of \( \tau_i(1) \) are very small, we need to seek some alternative or approximation to the system of Eqs. (8), (9) and (13).

For the case \( j = 1 \), Eqs. (8) can be written as

\[
\frac{dP_i}{dt} - \frac{P_i}{T_i} \frac{dT_i}{dt} = r_{i-1} \frac{P_{i-1}}{\tau_i-1} - \frac{P_{i+1}}{\tau_i+1} + \frac{P_i}{\tau_{ip}}
\]

where \( \tau_i = \tau(a_i, T_i) = (4V_i/a_i)(\pi m(1)/8kT_o)1/2 \), \( \tau_{i-1} = \tau(a_i, T_{i-1}) \), \( \tau_{i+1} = \tau(a_{i+1}, T_{i+1}) \) and \( \tau_{ip} = \tau(a_{i+1}, T_i) \) have a dimension of the time, and where \( r_{i-1} = T_i / T_{i-1} \) and \( r_{i+1} = T_i / T_i + 1 \) are dimensionless quantities or order 1.
Let the largest of all the four \( \tau \)'s defined above be \( \tau_m \). Define the non-dimensional temperatures and pressures as
\[
T'_i = \frac{T_i}{T_r}, \quad p'_i = \frac{p_i}{p_a}.
\]
where \( T_r \) is a reference temperature (say the room temperature), and \( T, p \) be the time period over which \( T'_i \) and \( p'_i \) change appreciably, and define the non-dimensional time as
\[
t' = t/T_e
\]
so that \( dT'_i/dt' \) and \( dp'_i/dt' \) have their magnitudes of order 1.

In terms of \( T'_i, p'_i \) and \( t' \), Eqs. (14) can be written as

\[
- \left[ T'_i \left( \frac{d^2 p'_i}{dt'^2} - p'_i \frac{dT'_i}{dt'} - \frac{d T'_i}{dt'} \right) \right] = T'_i \left( r_{i-1} s_{i-1} p'_{i-1} - (s_i + s_{ip}) p'_i + r_{i+1} s_{i+1} p'_{i+1} \right)
\]

where \( s_i = \frac{\tau_i}{\tau_e} \), etc., are dimensionless quantities of order 1.

We see that \( \tau_m \) is a measure of the relaxation time. If \( \tau_m << \tau_e \), the two terms on the left hand side of Eqs. (15) are vanishingly small compared to any term on the right hand side, since the magnitudes of the two terms inside the brackets on the left hand side are of order 1. In this case, component 1 of the gas mixture in the \( i \) th interstice is "quasi-steady", which means that at any instant component 1 in interstice \( i \) can be considered instantaneously steady and the derivative terms in its governing equation can be dropped. This argument applies to all other components of the mixture in every interstice, i.e., applies equally well to each equation of Eqs. (8) for cases \( j \neq 1 \) and Eqs. (9) individually.

As a final step towards completion of this argument, let \( 1 \leq m \leq L \) and \( 1 \leq n \leq M \) be two integers such that \( \tau_m(i) \) is the largest of all \( \tau_i(j) \) defined by

\[
\tau_i(j) = \left( \frac{4 V_i}{a_i} \right) \left( m(j)/8 k T_i \right)^{1/2}
\]

(16)
Then, the relaxation time of the gas mixture in the entire insulation system \( \tau_s \) is bounded by \( L \tau_m^{(n)} \), i.e.,

\[
\tau_s \leq L \tau_m^{(n)} \tag{17}
\]

Denoting the time period over which the properties of the gas mixture change appreciably by \( \tau_t \), we can drop all the derivative terms in Eqs. (8) and (9) simultaneously if

\[
\tau_s \text{ or } L \tau_m^{(n)} \ll \tau_t
\]

Thus, this system of differential equations reduces to a set of algebraic equations.

Now, for ordinary engineering applications, \( \tau_s \) is of the order \( 10^{-2} \sim 10^{-1} \) sec., while \( \tau_t \) is measured by hours. Therefore, condition (18) is usually fulfilled, and Eqs. (8) and (9) reduce to

\[
4 \left( a_i + a_{i+1} \right) \frac{Q_i^{(j)}}{T_i} + a_i \left( \frac{P_{i-1}^{(j)}}{T_{i-1}} v_{i-1} - \frac{P_i^{(j)}}{T_i} v_i \right) + a_{i+1} \left( \frac{P_{i+1}^{(j)}}{T_{i+1}} v_{i+1} - \frac{P_i^{(j)}}{T_i} v_i \right) = 0
\]

\[i = 1, 2, \ldots, L; \quad j = 1, 2, \ldots, M.\]

\[
\sum_{j=1}^{M} 4 \delta^{(j)} \left( a_i + a_{i+1} \right) Q_i^{(j)} + a_i \sum_{j=1}^{M} \delta^{(j)} \left( P_{i-1}^{(j)} v_{i-1} - P_i^{(j)} v_i \right) + a_{i+1} \sum_{j=1}^{M} \delta^{(j)} \left( P_{i+1}^{(j)} v_{i+1} - P_i^{(j)} v_i \right) = 0
\]

\[i = 1, 2, \ldots, L - 1.\]
As Eqs. (19) and (20) are algebraic equations, there is no need for the prescription of initial conditions.

To obtain the solution for a particular problem, we first choose a sequence of the time \((t_1, t_2, \ldots, t_n)\). The solution at any chosen time, say \(t_1\), is obtained by solving Eqs. (19) and (20) with the chamber conditions and fuel tank temperature at \(t_1\) (see Eqs. (10)) and \(R(i)\) \((T, t_1)\) (see Eq. (12)) which are pertinent to that problem. After we finish with the time sequence, we have the pressures and temperatures in all the interstices of the insulation system as functions of the time. If the process is isothermal, one only has to solve Eqs. (19) with the relevant chamber conditions and outgassing rates.
4. Numerical Results and Comparison with Experimental Data

For the purpose of comparison with experiment, we choose examples, the experimental data of which are available.

A circular disk of 1" thick and 6" in diameter is composed of a variety of numbers of insulation sheets of crinkled single-aluminized mylar. The edge of the disk is sealed with a solid insulation sheet (i.e., without perforation). The last layer (the back sheet) is also solid, but the rest of the insulation layers are perforated with a perforation coefficient equal to 0.0138. The perforation holes are 0.09375" in diameter. The distance between two adjacent holes is 0.707" center to center.

The disk is placed in a large vacuum chamber. The chamber and hence the multilayer system (i.e., the disk) are maintained at room temperature all the time, so it is an isothermal process. At the beginning of the experiment, the vacuum chamber and the multilayer system are filled with nitrogen (purge gas), and the pressure inside each interstitial space between two successive insulation layers and the pressure in the chamber are atmospheric. The chamber is then evacuated. The pressure in the chamber and the pressure in the last interstice of the insulation system are recorded as time proceeds.

The outgassing rates, $R(t)$ as functions of the time, of the insulation sheets were obtained beforehand by separate experiments. The outgas components are mainly water vapor, with small amounts of nitrogen and carbon dioxide. For simplicity, we assume only one kind of outgas (water vapor) present, i.e., only $R(2)$ has non-zero values. The outgassing rate $R(2)(t)$ of crinkled single-aluminized mylar is shown in Fig. 1.

In our analysis, we use a set of $R(2)(t)$ values at different times successively. To obtain the solution at any chosen time, say $t_1$, we use the values of $R(2)(t_1)$ and the chamber pressure $p_c(1)(t_1)$ in Eqs. (19). Thus an appropriately chosen set of $R(2)(t)$ and $p_c(1)(t)$ will result in a solution of the $P_i(t)$, the pressure history of the gas mixture inside the multilayer insulation system. However, the $R(2)(t)$ values are available only up to $t = 48$ hours, thus our computations have to stop there.
Theoretical computation includes the pressures $P_i(t)$ inside each interstice, while only the pressure in the last interstice $P_L(t)$, i.e., the backside pressure, is experimentally measured. Therefore, only the comparison for $P_L(t)$ can be made. Comparisons of the theoretical $P_L(t)$ and the experimental data are presented in Figures 2 to 6.
5. Conclusions

First of all, it should be noted that the comparison between the computed and measured values of the $p_L$'s presented in Figs. 2 to 6 cannot be taken seriously, since it is known that the experiment for the determination of the outgassing rate was not appropriately performed, thus the values of $R^{(2)}(t)$ presented in Fig. 1, on which our calculation was based, is not reliable. Nevertheless, Figs. 2 to 6 serve to show a qualitative comparison and to indicate some possible experimental errors.

It can be seen from the figures that at early times the computed $p_L$ is much higher than the experimental values. However, the computed and the measured values are coming closer and closer toward each other when time proceeds, and good agreement is established after reasonably large time. Since the effect of preconditioning on the outgassing rate is likely to be a weak function of time when $t$ is large, one probable and important course of experimental discrepancy would arise from different preconditionings of samples, i.e., the samples for the measurement of $p_L(t)$ and the sample for the determination of $R^{(2)}(t)$ were differently pre-treated. Another source of error would be the contamination of equipments inside the vacuum chamber.

It is believed that agreement between the theory and experiment can be achieved if, with special attention to preconditioning, a set of consistent experiments is performed.

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*At the time of the writing of this report, the Marshall Space Flight Center is planning to reconduct the outgassing rate experiment.*
References


FIG. 1. Outgassing Rate for Crinkled Single-Aluminized Mylar.
FIG. 2. Comparison of Computed and Measured Backside Pressures, 10-Layer Sample.
FIG. 5. Comparison of Computed and Measured Backside Pressures, 70-Layer Sample.
FIG. 6. Comparison of Computed and Measured Backside Pressures, 105-Layer Sample.