AUTOMATED DESIGN AND OPTIMIZATION
OF FLEXIBLE BOOSTER AUTOPILOTS
VIA LINEAR PROGRAMMING

Authors:
Francis D. Hauser
Glen D. Szollosi
W. Steven Lakins

Prepared for:
George C. Marshall Space Flight Center
Huntsville, Alabama 35812

CASE FILE COPY

Contract NAS8-28482
DCN 1-2-75-20071(IF)

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Approved:

Francis D. Hauser
Program Manager

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This report, Volume II of 2 volumes, was jointly prepared by the Guidance and Controls Section and the Scientific Programming Section of Martin Marietta Corporation, Denver Division, under Contract NAS8-28482. Volume I contains the philosophy and the mathematical basis of the non-linear programming algorithm that led to the development of the COEBRA program. This volume is the User's Manual for the COEBRA program. The purpose of the contract was to convert the COEBRA program from the CDC 6400/6500 digital computer system to the UNIVAC 1108 at the George C. Marshall Space Flight Center, and to provide a manual and instruction on the use of the program. This contract was performed from March 1972 to December 1972, and was administered by the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, Huntsville, Alabama, under the direction of Mr. D. K. Mowery, Dynamics and Control Division, Aeroastrodynamics Laboratory.
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This report, Volume II of 2 volumes, is the User’s Manual for the COEBRA program. Volume I contains the historical background, the philosophy, and the mathematical basis of the nonlinear programming algorithm that led to the development of the COEBRA program. COEBRA is an acronym for the Computerized Optimization of Elastic Booster Autopilots. This volume is written assuming that Volume I has been read.

COEBRA is an automatic autopilot design program. The bulk of the design criteria is in the form of minimum allowed gain/phase stability margins. COEBRA has two optimization phases: (1) a phase to maximize stability margins; and (2) a phase to optimize structural bending moment load relief capability in the presence of minimum requirements on gain/phase stability margins. The following is an outline of this report.

Chapter 1 defines the design criteria (stability margins, closed-loop roots, structural bending moment loads, trajectory drift) that COEBRA is capable of considering in its design procedure. Chapter 2 discusses the constraint equations that COEBRA places on the design criteria. Chapter 3 defines the cost function that is used in the optimization phase that maximizes stability margins. The cost function for the phase that optimizes bending moment load relief capability is presented in Chapter 4. Chapter 5 shows the constraint equations that are placed on the value of each autopilot variable. Chapters 6 and 7 illustrate how the user defines the initial or “first guess” autopilot to the COEBRA program. Chapter 8 illustrates the constraints the user can place on the autopilot parameters to yield the drift minimum condition. Chapter 9 defines additional constraints that
can be used in an attempt to keep COEBRA from designing an autopilot that is sensitive to tolerances on the airframe/autopilot parameters. Chapter 10 summarizes the COEBRA input data, and Chapter 11 contains sample listings of the input data. Appendix A shows a simple example problem that demonstrates the mechanics of the Simplex Algorithm.
CHAPTER 1. THE MARGIN ARRAY

Section 1.1 Introduction

This chapter defines the design criteria that COEBRA is capable of considering. This criteria includes stability margins, rigid-body rotational closed-loop roots, structural bending moment loads, and trajectory drift. This criteria is listed in Table 1.1, the so-called Margin Array Table. This table, even though it contains things other than stability margins, will constantly be referred to throughout this report as the Margin Array. From this table, COEBRA sets up the cost functions (maximize margins or optimize load relief) and the so-called Stability Margin Constraint equations. The Margin Counter and the Figure-of-merit are also calculated from the elements in the Margin Array. Figure 1.1 of Volume I, which illustrates a typical gain/phase frequency response plot, is repeated in this volume to facilitate defining most of the elements of the Margin Array Table. Table 1.2 accompanies Figure 1.1, and contains the definitions of the gain and phase margins.

The SCALF array shown in the Margin Array Table will be defined in Section 1.3 of this chapter. The SPEC array shown in the Margin Array Table, contains the minimum requirements on the gain/phase stability margins. These minimum requirements are used in the so-called Stability Margin Constraint Equations (defined in Chapter 2). The COSPEC Array contains the design objectives for the gain/phase stability margins. These objectives are used in the so-called "Maximize Margins" cost function (defined in Chapter 3). The preset values indicated in the Margin Array Table are to be overridden by the user if he so desires.

Before defining each element of the Margin Array, this section will conclude with a brief overview of how the elements are used. Elements 1 to
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<td></td>
<td>Q</td>
<td>30</td>
<td>1 db/db</td>
<td>-10.0(db)</td>
</tr>
<tr>
<td>4th Mode Peak Phase</td>
<td></td>
<td>Q</td>
<td>31</td>
<td>1 db/9.009 deg.</td>
<td>----</td>
</tr>
<tr>
<td>5th Mode Peak Gain</td>
<td></td>
<td>Q</td>
<td>32</td>
<td>1 db/db</td>
<td>-10.0(db)</td>
</tr>
<tr>
<td>5th Mode Peak Phase</td>
<td></td>
<td>Q</td>
<td>33</td>
<td>1 db/9.009 deg.</td>
<td>----</td>
</tr>
<tr>
<td>6th Mode Peak Gain</td>
<td></td>
<td>Q</td>
<td>34</td>
<td>1 db/db</td>
<td>-10.0(db)</td>
</tr>
<tr>
<td>Margin Array Element</td>
<td>Definition in Figure 1.1</td>
<td>Margin Array Element Number</td>
<td>SCALF Array (Preset Values)</td>
<td>SPEC Array</td>
<td>COSPEC Array</td>
</tr>
<tr>
<td>----------------------</td>
<td>--------------------------</td>
<td>-----------------------------</td>
<td>----------------------------</td>
<td>------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Higher Structural Modes (cont'd.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th Mode Peak Phase</td>
<td>( \Delta Q )</td>
<td>35</td>
<td>1 db/9.009 deg.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7th Mode Peak Gain</td>
<td></td>
<td>36</td>
<td>1 db/db</td>
<td>-10.0(db)</td>
<td>3.162</td>
</tr>
<tr>
<td>8th Mode Peak Phase</td>
<td>( \Delta Q )</td>
<td>37</td>
<td>1 db/9.009 deg.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8th Mode Peak Gain</td>
<td></td>
<td>38</td>
<td>1 db/db</td>
<td>-10.0(db)</td>
<td>3.162</td>
</tr>
</tbody>
</table>

Fuel Slosh Mode Backside Phase Margins:

| | | | | | |
| 1st Slosh Mode | \( \Delta OD \) | 40 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |
| 2nd Slosh Mode | \( \Delta OD \) | 41 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |
| 3rd Slosh Mode | \( \Delta OD \) | 42 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |
| 4th Slosh Mode | \( \Delta OD \) | 43 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |
| 5th Slosh Mode | \( \Delta OD \) | 44 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |
| 6th Slosh Mode | \( \Delta OD \) | 45 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |
| 7th Slosh Mode | \( \Delta OD \) | 46 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |
| 8th Slosh Mode | \( \Delta OD \) | 47 | 1 db/5 deg. | 20.0(deg.) | 1.584 | 30.0(deg.) | 1.995 |

Rigid-body Phase Margin Frequency (rad/sec) | - | 48 | - | 0.0 | 0.0 | 0.0 | 0.0 |
TABLE 1.1 THE MARGIN ARRAY (Continued)

<table>
<thead>
<tr>
<th>MARGIN ARRAY ELEMENT</th>
<th>MARGIN ARRAY ELEMENT NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Value of Angle of Sideslip (rad)</td>
<td>49</td>
</tr>
<tr>
<td>Peak Value of Yaw Control Device (rad)</td>
<td>50</td>
</tr>
<tr>
<td>Peak Value of Roll Control Device (rad)</td>
<td>51</td>
</tr>
<tr>
<td>Amplitude of Case $X$ Autopilot Feedback Loop Evaluated at the Frequency of the $Y$ Element of the MARVEC Array:</td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>$Y$</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>7</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>8</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>9</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>10</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>11</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>12</td>
<td>1, 2, 3</td>
</tr>
</tbody>
</table>
Table 1.2. Definitions of Gain and Phase Margins

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>Reference in Figure 1.1</th>
<th>Typical Design Requirement</th>
<th>Typical Design Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerodynamic Gain Margin</td>
<td></td>
<td>6 dB</td>
<td>6 dB</td>
</tr>
<tr>
<td>Rigid-Body Phase Margin</td>
<td></td>
<td>30°</td>
<td>30°</td>
</tr>
<tr>
<td>Slosh Mode Backside Phase Margin</td>
<td></td>
<td>20°</td>
<td>30°</td>
</tr>
<tr>
<td>Rigid-Body Gain Margin</td>
<td></td>
<td>6 dB</td>
<td>6 dB</td>
</tr>
<tr>
<td>First Mode Frontside Phase Margin</td>
<td></td>
<td>45°</td>
<td>45°</td>
</tr>
<tr>
<td>First Mode Peak Gain</td>
<td></td>
<td>None</td>
<td>20 dB</td>
</tr>
<tr>
<td>First Mode Peak Phase</td>
<td></td>
<td>None</td>
<td>0°</td>
</tr>
<tr>
<td>First Mode Backside Phase Margin</td>
<td></td>
<td>60°</td>
<td>60°</td>
</tr>
<tr>
<td>First Mode Closest Approach Margin (If First Mode Peak &lt; 0 dB)</td>
<td></td>
<td>10 dB</td>
<td>10 dB (letting 4° ≡ 1 dB)</td>
</tr>
<tr>
<td>Second Mode 180° Crossover Gain Margin</td>
<td></td>
<td>10 dB</td>
<td>10 dB</td>
</tr>
<tr>
<td>Second Mode Frontside Phase Margin</td>
<td></td>
<td>60°</td>
<td>60°</td>
</tr>
<tr>
<td>Second Mode Peak Gain</td>
<td></td>
<td>None</td>
<td>20 dB</td>
</tr>
<tr>
<td>Second Mode Peak Phase</td>
<td></td>
<td>None</td>
<td>0°</td>
</tr>
<tr>
<td>Second Mode Backside Phase Margin</td>
<td></td>
<td>60°</td>
<td>60°</td>
</tr>
<tr>
<td>Second Mode Closest Approach Margin (If Second Mode Peak &lt; 0 dB)</td>
<td></td>
<td>10 dB</td>
<td>10 dB (letting 4° ≡ 1 dB)</td>
</tr>
<tr>
<td>Second Mode 180° Crossover Gain Margin</td>
<td></td>
<td>10 dB</td>
<td>10 dB</td>
</tr>
<tr>
<td>Third and Higher Mode Peak Gain</td>
<td></td>
<td>-10 dB</td>
<td>-10 dB</td>
</tr>
<tr>
<td>Third and Higher Mode Peak Phase</td>
<td></td>
<td>None</td>
<td>0°</td>
</tr>
</tbody>
</table>
48 are, at the user's option, candidates for the "Maximize Margins" cost function, where the objective is to maximize all stability margins and attempt to force all structural bending modes to resonate near zero degrees phase. Elements 49, 50, 51 are candidates for the so-called "Optimize Load Relief" cost function, where the objective is to minimize structural bending moment loads, in the presence of minimum requirements on the gain/phase stability margins. Except for the structural bending mode peak phases and elements 49, 50, and 51, all elements of the Margin Array are candidates for the Stability Margin Constraint Equations, where minimum allowed requirements are put on each.

Section 1.2 Definitions of the Margin Array Elements

(1) **Aerodynamic Gain Margin** (denoted |OA| in Figure 1.1)

If the phase angle of the total open-loop frequency response at \( \omega = \text{SLOW} \) is less than \( 180^\circ \) (where SLOW is an input parameter in rad/sec), the aero gain margin will be the first crossing of the \( 180^\circ \) axis as \( \omega \) increases from SLOW. The frequency search interval is:

\[
\text{SLOW} < \omega < \text{W2}
\]

where W2 is the minimum of the following three frequencies.

(a) UPPER (which is an input parameter)

(b) frequency at the peak of the lowest frequency 1st structural mode

(c) frequency at the peak of the highest frequency structural mode.

If the phase angle at SLOW is less than \( 180^\circ \), this margin must exist or COEBRA will terminate (unless the NOTERM option of the COBIN namelist is used).
(2) **Rigid-body Gain Margin** \( (|OE|) \)

This will be the lowest frequency 180° crossover encountered in the following frequency interval.

\[ W_1 < \omega < W_2 \]

where \( W_1 = \max \{ \text{SLOW, aero gain margin frequency} \} \). If fuel slosh modes are included, the rigid-body gain margin will not be considered by COEBRA. Except for fuel slosh and the NOTERM option, COEBRA will terminate if the rigid-body gain margin does not exist.

(3) **Rigid-body Phase Margin** \( (\phi_{OB}) \)

When searching from SLOW, the phase margin will be the first zero db crossing encountered. The following search increment is used:

\[ \text{SLOW} < \omega < \text{UPPER} \]

Unless the user exercises the NOTERM option, COEBRA will terminate if the rigid-body phase margin does not exist.

(4) **Structural Bending and Fuel Slosh Modes**

COEBRA may be run with or without structural bending and/or fuel slosh modes. When modes are included, COEBRA can handle a maximum of eight of them. There are 4 basic types of modes, and each type is constrained differently:

(a) 1st structural bending modes;
(b) 2nd structural bending modes;
(c) 3rd and higher structural bending modes; and
(d) fuel slosh modes.

The user declares the "type" that is to be associated with each mode. He will declare the "type" depending on how he wants to constrain each mode. The modes need not be ordered according to
frequency, e.g., a 3rd mode may be lower in frequency than a 1st mode, etc. Also, bending and slosh modes may be intermixed frequency-wise. There are a few rules that must be obeyed, and these will be explained where appropriate.

(4.1) **First Structural Bending Modes**

Each 1st structural bending mode is defined by the 5 following items:

(a) modal peak gain (|G|)
(b) modal peak phase (\(\Phi G\))
(c) frontside phase margin (\(\Phi OF\))
(d) backside phase margin (\(\Phi OH\))
(e) closest-approach margin (|\(\Delta I\)|)

Stability margin constraints on the 1st mode are handled as follows. When the mode peaks below zero db, the closest approach margin will always be constrained. The mode peak gain will not be constrained and obviously, the front and backside phase margins don't even exist. When the mode peaks above zero db, and the input parameter KAPCH = 0, the closest-approach margin will not be constrained. The mode will be constrained by its peak gain, and its front and backside phase margins. When the mode peaks above zero db and KAPCH = 1, the closest-approach margin, the peak gain, and the front and backside phase margins will all be constrained.

The peak phase of the 1st mode is not constrained. It is a candidate only for the "maximize margins" cost function, where the objective is to force the mode to resonate near zero
degrees phase. The modal peak gain, the front and backside phase margins and the closest approach margin are also candidates for the "maximize margins" cost function.

COEBRA has slots for 2 first structural bending modes. Both 1st modes must resonate after the rigid-body gain margin if it exists and is constrained.

This discussion will conclude with the frequency search intervals that are used for all structural and slosh mode peaks and for all modal front and backside phase margins.

To find each modal peak, COEBRA searches between

\[ 0.9 \omega_{\text{D}} \leq \omega \leq 1.1 \omega_{\text{D}} \]

where \( \omega_{\text{D}} = \omega_n \sqrt{1 - \xi^2} \) where \( \xi \) and \( \omega_n \) are the input values for the damping ratio and undamped natural frequency of each mode.

To find each front and backside phase margin, COEBRA searches between

\[ 0.1 \omega_{\text{p}} \leq \omega \leq 10. \omega_{\text{p}} \]

where \( \omega_{\text{p}} \) is the frequency at each modal peak.

(4.2) **Second Structural Bending Modes**

Each 2nd structural bending mode is defined by the 7 following items:

(a) modal peak gain (\( |L| \))
(b) modal peak phase (\( \xi \) \( L \))
(c) frontside phase margin (\( \xi \) \( OK \))
(d) backside phase margin (\( \xi \) \( OM \))
(e) 180° crossover gain Margin #1 (\( |OJ| \))
(f) 180° crossover gain Margin #2 (|OP|).

(g) closest-approach margin (|ON|).

Except for the 180° crossover gain margins, all elements of the 2nd structural modes are handled exactly like they are for the first structural modes. Because of the 180° crossover margins, 2nd structural modes cannot be included unless the following modes are also included:

(a) something identified as a first mode that is lower than the 2nd modes in frequency; and

(b) something identified as a 3rd or higher mode that is higher than the 2nd modes in frequency.

COEBRA allows for eight 180° crossover margins to exist between the highest frequency 1st mode and the lowest frequency higher mode. Constraints will be placed on the smallest two of them. Note that COEBRA allows for two 2nd structural modes.

(4.3) **Third and Higher Structural Bending Modes**

Each of these modes is defined by its:

(a) modal peak gain (|Q|); and

(b) modal peak phase (φQ).

Each modal peak gain is a candidate for the Stability Margin Constraint Equations, and each modal peak gain and phase is a candidate for the "maximize margins" cost function. COEBRA has slots for 6 of these modes, i.e., for a single 3rd mode, a single 4th mode, ..., a single 8th mode.

An advantage of identifying them in this way is as follows. The 3rd mode at time point #1 can have a different requirement than the 3rd mode at time point #2, by declaring,
for example, that the 3rd mode at time point #2 is a 4th mode. Along this same line, the first mode at time point #2 can be declared one of the two 2nd modes, so that its requirement can be different than the first mode at time point #1.

Finally, the only restriction on 3rd and higher modes, is that they occur after the rigid-body phase margin. They may occur before the rigid-body gain margin.

(4.4) Fuel Slosh Modes

For each fuel slosh mode that resonates above zero db, COEBRA finds its backside phase margin (ΟD). This backside phase margin is a candidate for both the Stability Margin Constraint Equations and the Maximize Margins cost function. COEBRA ignores each slosh mode that resonates below zero db. This backside phase margin is the 1st zero db crossover that is encountered when searching with increasing frequency from the slosh mode peak. It is possible for 2 or more slosh modes to have the same backside phase margin. Slosh modes must peak after the rigid-body phase margin, but slosh modes may be intermixed with the structural modes in any manner according to frequency.

As previously stated when defining the rigid body gain margin, the rigid body gain margin and fuel slosh are mutually exclusive. However, when designing without slosh, the user can bias the rigid body gain margin requirement in order to compensate for the effective rigid body inertia change that
will result when slosh is included. But, there is a better
technique for designing the gain margin with slosh. The
user can input a certain time point twice with identical
airframe transfer functions. The user can tell COEBRA to
ignore the slosh modes at the first time point so as to
design the rigid body gain margin. He can then tell COEBRA
to consider the slosh modes at the 2nd time point.

The following is a closely related item, and is included
here because it is an important design feature and advantage
of the COEBRA algorithm. This technique of entering the same
time point twice can also be used to handle severe tolerance
conditions. Instead of identical transfer functions, the user
can input the nominal airframe transfer functions as the 1st
vehicle state. He can input the "toleranced" airframe transfer
functions as the 2nd vehicle state. By treating the nominal
and the toleranced airframe together, a single autopilot can
be designed that will handle both conditions. Note that the
"toleranced" airframe can even include malfunction conditions
like actuator and sensor failures, etc.

(5) **Rigid-body Phase Margin Frequency**

Chapter 4, Section 4.3, of Volume I includes a discussion of
how COEBRA treats the requirement on the rigid-body rotational roots.
This discussion will not be repeated here, but it can be simply stated
that COEBRA treats these roots by putting a requirement on the frequency
as well as the magnitude of the rigid-body phase margin. The phase
margin frequency is a candidate for both the Stability Margin Constraint Equations and the Maximize Margins cost function.

(6) **Elements 49, 50 and 51**

These elements are only used to form the so-called "optimize load relief" cost function. They are completely defined in Chapter 4, and this definition need not be repeated here.

(7) **Elements 52 thru 69**

These elements are used in the so-called Vector Constraint Equations that are completely defined in Chapter 9. Basically, these constraint equations are used in an attempt to keep COEBRA from designing an autopilot that is sensitive to tolerances on the airframe/autopilot parameters.

As a final note in this section, the subroutine that "finds and identifies" the margins is not completely general. If COEBRA "behaves strangely", the user should check the margin array that is printed out. The first and second structural modes are the major problem areas. The rigid-body margins are more well-behaved. The following is a tip on how the user can circumvent a rigid-body problem that can occur on an aerodynamically stable vehicle. Figure 1.2 shows a frequency response plot that might result from a vehicle that is "bare airframe" stable.

![Figure 1.2 FREQUENCY RESPONSE FOR AERO STABLE VEHICLE](image)
To keep COEBRA from identifying crossover #1 as the rigid-body gain margin, the user can input SLOW at the frequency indicated. In this way, crossover #2 will be called the aero gain margin, and crossover #3 will be called the rigid-body gain margin.

Section 1.3 The SCALF Array

The primary reason for the SCALF array is to scale gain and phase margins and modal peak gains and phases so that they can be equitably treated together in the maximize margins cost function. The SCALF array puts everything on a decibel scale, and from there, COEBRA converts everything to the real axis scale for the Taylor Series Expansion. In other words, all partial derivatives are computed after the conversion to the real axis scale.

For gain and phase margins, the conversion from decibels and degrees to the real axis is rather straightforward since these margins represent distances from the critical point. This includes the rigid-body gain margin where, a 180° crossover at -6 db is really a +6db margin of stability. If SCALF is 1db/db, this would convert to 1.995 on the real axis. Note that a zero db margin of stability converts to unity and a negative stability margin converts to a number less than unity on the real axis scale.

Since all margins are to be maximized, and since all mode peaks are to be "minimized", it was decided to "invert" the scale on the peak gains of all structural bending modes. In this way they can be "maximized" also. For example, if a mode resonates at +6db, and if SCALE is 1db/db, the modal peak gain converts to 0.5011 on the real axis scale.
The scale used for structural mode peak phases is also a special case. If the mode resonates at +90° phase, and if SCALF = 1db/9 deg., the phase angle would convert to +10db and then to +3.162. If the mode resonates at -90° phase and if SCALF = 1db/9 deg., the phase angle would still convert to +10db, but would then convert to -3.162 on the real axis scale. If the mode resonates at 0° phase, this would convert to unity on the real axis. Hence, 0° phase logically serves as the reference. This scaling was necessitated since the modal peak phase angle is not a stability margin and it is generally no more desirable for the mode to resonate at +10° than it is for the mode to resonate at -10°. Also, by scaling this way, the partial derivative of the modal peak phase is always positive in the phase lead direction.

Note that another result of the SCALF array and the corresponding conversion to the real axis, is that all Stability Margin Constraint equations for elements 1 to 48 are "greater than or equal to" type constraints. The SCALF is not applicable to the phase margin crossover frequency (element 48), since it is a real axis quantity already. Finally, the preset values indicated for SCALF in Table 1.1 are to be overridden by the user if he so desires.
Chapter 2. The Margin Constraint Equations

This chapter defines the so-called Margin Constraint Equations. Strictly speaking, the Margin Constraint Equations include constraints on items that are not exactly stability margins. For example, besides the items that can directly be classified as stability margins, these equations include constraints on:

1) The peak gains of structural bending modes;
2) The so-called Drift Minimum Autopilot condition; and
3) The autopilot vector constraints that are defined in Chapter 9.

The phrase "Margin Constraint Equations" is used simply to differentiate this type of constraint from the type that is applied to the magnitudes of the autopilot gains and filter parameters.

There are three types of so-called Margin Constraint Equations:

The type that is used for (1) elements 1 to 48 of the Margin Array (defined in Chapter 1); (2) the "drift minimum" condition (defined in Chapter 8); and (3) the so-called autopilot vector constraints (defined in Chapter 9). This chapter will only define the type that is used for elements 1 to 48 of the Margin Array. The other two types (defined in their respective chapters) are formed using the same philosophy, and hence need not be repeated here.

The following discussion defines the Margin Constraint Equations that are used for elements 1 to 48 of the Margin Array. All elements 1 to 48 are constrained in this way, except those elements that are modal peak phases. Modal peak phases are not "constrained" and they are used only in the "Maximize Margins" cost function (defined in Chapter 3). The equation
for the $i^{th}$ element at the $t^{th}$ time point, $M(i)$, is obtained from a first order Taylor Series expansion about $M(i)_0$, which is the scaled nominal (initial) value for each major iteration. For simplicity, time notation is not used and is merely implied.

Since it is required that $M(i)$ be greater than or equal to some specified requirement, $M(i)_s$, the margin constraint equation can be written:

$$M(i)_0 + \sum_{j=1}^{n} \frac{dM(i)}{dx(j)} _0 \Delta X(j) \geq M(i)_s$$

( for $i = 1, ..., m$)

Where:

1) $\Delta X(j) = X(j) - X(j)_0$ Where: $X(j)$ is the $j^{th}$ autopilot variable whose value COEBRA is to optimize, and $X(j)_0$ is the nominal or initial value of $X(j)$ at the beginning of the major iteration;

2) $\left[ \frac{dM(i)}{dx(j)} \right]_0$ is the partial derivative of $M(i)$ with respect to $X(j)$ evaluated at $X(j)_0$. Note that the method of finite differences is used to calculate this partial derivative. Note also that this derivative is computed after the elements of $M(i)$ have been scaled or converted to the "real number" axis.

There are 2 cases for $M(i)_s$:

Case 1: If the requirement on $M(i)$ is already met, then $M(i)_s = SPEC(i)$ where $SPEC(i)$ is the scaled requirement for $M(i)$. For this case, the constraint is "loose";
Case 2: If the requirement on \( M(i) \) is not yet met, then 
\[
M(i)_s = M(i)_o + \text{STEP} \cdot [\text{SPEC(i)} - M(i)_o].
\]
For this case, the constraint is "tight".

As discussed in Chapter 3, Section 3.5, of Volume I, \text{STEP} is maximized in the Inner Loop. At the beginning of each "inner" iteration, \text{STEP} begins with its input value. If no feasible solution can be obtained (i.e., if the feasible region defined by the Margin Constraints does not overlap the feasible region defined by the Autopilot Variable Constraints), \text{STEP} is reduced via

\[
\text{STEP} \leftarrow \text{STEP} - \text{DSTEP}
\]

until a feasible solution is obtained. \text{DSTEP} is an input parameter. The minimum value for \text{STEP} is zero. Note that when \text{STEP} = 0, a feasible solution is virtually guaranteed since the nominal autopilot should always satisfy this condition which requires no improvement. Note also that \text{STEP} has the same value for all the "not-met" Margin Constraint Equations.

This chapter will conclude with 3 notes.

**Note #1** The Margin Constraint Equations are comprised of constraints from all the time points that are being designed together.

**Note #2** Any structural bending mode whose peak value is less than \text{AMPOUT}, is not included in the constraint equations for elements 1 to 48. (\text{AMPOUT} is a COBIN namelist parameter).

**Note #3** For each 1st and 2nd structural mode, the constraints on the front and backside phase margins, on the modal peak gain, and on the closest-approach margin, are handled as follows:

1) If the mode peaks below zero db, the closest-approach margin will always be constrained. The modal peak will not be constrained, and
obviously, the phase margins don't even exist;

2) When KAPCH = 1, and the modal peak gain is greater than zero db, constraints will be put on the front and backside phase margins, on the modal peak gain, and on the closest-approach margin;

3) When KAPCH = 0, and the modal peak gain is greater than zero db, constraints will be put on the front and backside phase margins and on the modal peak gain, but not on the closest-approach margin.
CHAPTER 3. THE STABILITY MARGIN COST FUNCTION

This chapter contains the definition of the so-called Stability Margin cost function. The objective of this cost function is to attempt to: (1) maximize all stability margins; (2) optimize the location of the rigid-body closed-loop rotational roots; and (3) force all structural bending modes to resonate near zero degrees phase. This cost function \( Y_1 \) is a weighted linear combination of the variable portion of the first order terms in the Taylor series expansion of each element of the Margin Array that is to be "maximized". \( Y_1 \), which is given by the following expression, is to be maximized.

\[
Y_1 = \sum_{j=1}^{n} \sum_{t=1}^{m} \sum_{i=1}^{48} \text{WTFA}(i,t) \times W(i,t) \times U(i,t) \times \left[ \frac{\partial M(i,t)}{\partial X(j)} \right] \times X(j)
\]

where:

1. \( j \) is the index of the autopilot variables (total of \( n \)).
2. \( t \) is the index of the time points or vehicle states (total of \( m \)).
3. \( i \) is the index of the elements of the Margin Array (\( i = 1 \) to 48).
4. \( X(j) \) is the \( j \)th autopilot variable whose value is to be determined by COEBRA.
5. \( \frac{\partial M(i,t)}{\partial X(j)} \) is the partial derivative of the \( i \)th element of the scaled Margin Array at time \( t \), with respect to \( X(j) \), evaluated at \( X(j) \).
6. \( U(i,t) \): The purpose of this variable is to account for the following definitions on the signs of the partial derivatives. For all structural mode peak phases, the derivative is positive in the phase lead direction. For all the stability margins...
including the root requirement, the derivative is positive when the scaled margin increases. Therefore: \( U(i,t) \) is unity for all elements of the Margin Array except for the structural mode peak phases. For the structural mode peak phases:

(a) \( U(i,t) = +1.0 \) if the mode peaks between \( 0^\circ \) and \( -180^\circ \).

(b) \( U(i,t) = -1.0 \) if the mode peaks between \( 0^\circ \) and \( +180^\circ \).

This is in keeping with the philosophy to seek to have all structural modes resonate near zero degrees phase. **NOTE:** For all structural modes, COEBRA will attempt to push each towards its nearest zero degree point.

(7) \( W(i,t) \): This is a weighting factor that is computed by COEBRA. Generally, this weighting factor is the ratio of the desired "margins" over the actual "margins". When \( WTYPE \) is unity, all the \( W(i,t) \)'s are set to unity. When \( WTYPE \) is zero, then

(a) for all elements of the Margin Array except for the structural mode peak phases:

\[
W(i,t) = \frac{COSPEC(i)}{M(i,t)_0}
\]

where:

(1) \( COSPEC(i) \) is the scaled **objective** of the \( i^{th} \) margin.

(2) \( M(i,t)_0 \) is the scaled nominal value of the \( i^{th} \) margin at the \( t^{th} \) time point.

(b) for the structural mode peak phases:

\[
W(i,t) = \frac{\text{antilog} \left\{ -\frac{1}{20} \times \text{SCALF}(i) \times \left| \text{UM}(i,t)_0 \right| \right\} -1.0}{\text{antilog} \left\{ -\frac{1}{20} \times \text{SCALF}(i) \times 90 \text{ degrees} \right\} -1.0}
\]
where: (1) \( |\text{UM}(i,t)| \) is the absolute value in degrees, of the nominal unscaled structural mode peak phase at time \( t \), and has a value between \( 0^\circ \) and \( +180^\circ \).

(2) \( \text{SCALF}(i) \) is the scale factor or multiplying factor that converts \( |\text{UM}(i,t)| \) to decibels. The main purpose of \( \text{SCALF} \) is to scale the elements of the Margin Array so that gain margins and phase margins can be treated together.

Note that \( \text{W}(i,t) \) for the structural mode peak phases is derived from a real number scale that is referenced to \( 90 \) degrees. That is, if the mode peaks between \( 0^\circ \) and \( +90^\circ \) or between \( 0^\circ \) and \( -90^\circ \), then

\[
0.0 \leq \text{W}(i,t) \leq 1.0
\]

If the mode peaks at \( +90^\circ \) or \( -90^\circ \),

\[
\text{W}(i,t) = 1.0
\]

If the mode peaks between \( +90^\circ \) and \( +180^\circ \) or between \( -90^\circ \) and \( -180^\circ \),

\[
1.0 < \text{W}(i,t) \leq \text{(a value that depends on SCALF}(i))\]

(8) \( \text{WTFAC}(i,t) \) is a weighting factor that is input by the user, that will allow him to emphasize or de-emphasize certain elements of \( Y_1 \) relative to others. This array is an additional multiplicative weighting factor to that which is provided by the WTYPE option. This array also allows the user to "build-his-own" cost function if the "built in" cost function will not solve his problem (See Note 1 below).

The following are a series of important notes dealing with the formation of the Stability Margin cost function.
The "built in" cost function is the result of the preset values for WTFAC(i,t). These values, given next, are to be overridden by the user if he so desires.

(a) 1.0 for the first three rigid body margins.
(b) 1.0 for all structural mode peak gains.
(c) 1.0 for all fuel slosh backside phase margins.
(d) 0.0 for all the other elements of the Margin Array from $i = 1$ to 48.

In other words, if the user does not exercise the WTFAC option, only the rigid body margins, the fuel slosh margins, and the structural mode peak gains will get into the optimize-margins cost function, with weighting factors given by $W(i,t)$.

WTFAC(i,t) not only allows the user to build his own cost function, and emphasize or de-emphasize certain elements, but it also allows him to correct the following situation. When a time point is entered into the problem two times, (e.g. once for fuel slosh, and once for the rigid body gain margin), WTFAC(i,t) can be used to keep identical margins from being entered into the cost function twice, which has the effect of doubling their weights.

The user must be warned not to include conflicting elements in the cost function when he exercises the WTFAC option. An example of this is a dual second mode, where the first peak resonates at greater than 180° phase, and the second peak at less than 180° phase, and it is not possible to "push" them apart. NOTE: This note is premature, but will be very important on the second reading.
The IDMODE array (defined in COBDE namelist) might be used to solve the conflict in the above example, by telling COEBRA to simply ignore one of the dual second modes.

NOTE 4

The following is another example of using the WTFAC(i,t) array to keep identical margins from being entered into the cost function two or more times. It is possible for two or more fuel slosh modes to have the same backside phase margin. Hence, the WTFAC(i,t) elements for all of these slosh modes except one, can be set to zero.
Chapter 4. The Load Relief Cost Function

Section 4.1 of this chapter presents the philosophy and the formula that COEBRA uses for the cost function when optimizing the load relief autopilot. Section 4.2 shows the airframe/autopilot equations that COEBRA uses when setting up this cost function. Section 4.3 discusses the so-called wind forcing function that COEBRA uses as the input to the airframe/autopilot equations.

Section 4.1 Formula for the Load Relief Cost Function

As stated earlier, COEBRA has two phases: (1) the so-called "Maximize Margins" phase; and (2) the so-called "Load Relief Optimization" phase. The only difference between these two phases is the cost function. The matrix of constraint equations on the stability margins, on the closed-loop roots and on the autopilot variables, remains the same for either of the two phases.

Philosophy of the load relief cost function is as follows:
When the objective is maximize structural bending moment load relief capability, the cost function is comprised of the response of the angle of attack ($\beta$) and the control deflections ($\delta$) due to a wind forcing function ($\beta_\omega$). When the cost function is maximized, the peak values of $\beta$ and $\delta$ are minimized, thereby minimizing total structural bending moment loads.

A separate linear transient response routine is used to calculate the peak values of angle of attack ($\beta_\rho$) and control deflections ($\delta_\rho$) due to $\beta_\omega$. In order to save computer time, a linear transient response approach was used, rather than a nonlinear time-varying trajectory simulation. For illustration, if the user wants to optimize the load relief autopilot during the
portion of flight from "load-relief switch-in" to "max-q", he might input airframe/autopilot data at the following two time points: (1) load relief switch-in; and (2) max-q. COEBRA will then optimize the autopilot based on a transient response at each of these two time points. That is, COEBRA will calculate the rate of change of $\beta_\rho$ and $\delta_\rho$ with respect to each autopilot variable at the time point of load relief switch-in and at the time point of max-q. COEBRA will then form the cost function from these sensitivities at these two time points. Via this method, hopefully load relief optimization will also occur at all the intermediate time points. Since COEBRA uses a linear transient response routine, the actual values of $\beta_\rho$ and $\delta_\rho$ do not represent the values that would result from a nonlinear time-varying simulation. However, the actual values are not really germane. Only the trend or sensitivity with respect to the autopilot variables is important.

As will be shown later, the load relief cost function will be formed from a "weighted" linear combination of the sensitivities of $\beta_\rho$ and $\delta_\rho$. This is a reasonable definition, since it makes the cost function an equation very similar to that used to compute total bending moment loads [Harris, 4]:

\[
\text{Total Bending Moment Load} = K_1 \beta(t) + K_2 \delta(t) + f (\text{gusts, buffets, etc.})
\]

By a judicious choice of the wind profile that is used to force the linearized equations at each flight time, a load relief autopilot can be designed that possesses a good response characteristic to both low frequency and high frequency winds.

When the user desires to optimize load relief capability, he sets the flag LOADOP to unity and he supplies the necessary airframe/autopilot data needed to form the load relief function. As with the stability margin cost function, the load relief cost function ($\gamma_2$) is a weighted linear combination of the variable portion of the first order terms in the Taylor Series.
\( Y_{2} \) is given as follows:

\[
Y_{2} = \sum_{j} \sum_{t} \sum_{i=49}^{51} \left\{ \text{WTFAC}(i,t) \times \left[ \frac{\partial M(i,t)}{\partial X(j)} \right]_{o} \right\} \times X(j)
\]

In the above expression:

1. \( j \) refers to the summation over all the autopilot variables denoted \( X(j) \). (Note that COEBRA will optimize the value of \( X(j) \).)
2. \( t \) refers to the summation over all the time points or vehicle states.
3. \( i \) refers to the element of the so-called Margin Array table that was defined in Chapter 1. When \( i = 49 \), \( M(i,t) \) is the peak value of the angle of attack \( (\beta_{p}) \) at the \( t^{th} \) time point. Similarly, when \( i = 50 \), \( M(50,t) \) is the peak value of the yaw (or pitch) control device deflection \( (\delta_{\psi_{p}}) \). Finally, \( M(51,t) \) is the peak value of the roll control device deflection \( (\delta_{\phi_{p}}) \) if the yaw/roll coupled airframe/autopilot equations are used when setting up the cost function.
4. \( \text{WTFAC}(i,t) \) refers to arbitrary weighting factors that are input by the user. (Note: \( \text{WTFAC}(i,t) \) for \( i = 49, 50 \) and 51 has a preset value of unity to be overridden by the user).
5. \( \left[ \frac{\partial M(i,t)}{\partial X(j)} \right]_{o} \) is the partial derivative of \( M(i,t) \) with respect to the \( j^{th} \) autopilot variable, evaluated at \( X(j)_{o} \), the present nominal value of the \( j^{th} \) autopilot variable. As with stability margins, the method of finite differences is used to compute the partial derivatives of \( \beta_{p}, \delta_{\psi_{p}} \) and \( \delta_{\phi_{p}} \) with respect to each autopilot variable.
In conclusion, when maximizing load relief capability, the COEBRA design algorithm will maximize the negative of $Y_2$ in the presence of the constraint equations on the minimum allowed gain/phase stability margins and closed-loop root locations, and on the allowed ranges of the individual autopilot variables. Note that multiple time point design is handled just as it is when maximizing stability margins.
Section 4.2 Load Relief Airframe/Autopilot Equations

Figure 4.1 defines the linearized airframe equations that are used to calculate $\beta_p$, $\delta \psi_p$, and $\delta \phi_p$. The five equations of Figure 4.1 are:

1. The normal force equation, where $\beta(s)$ is the Laplace Transform of $\beta(t)$ and $\beta_w(s)$ is the Laplace Transform of the wind that will be defined in Section 4.3.

2. The yaw (or pitch) moment equation where $\psi_p$ is the perturbated yaw attitude.

3. The accelerometer equation which defines the output ($V_{acc}$) of a lateral body-mounted accelerometer located a distance $L_{AY}$ from the center of gravity.

4. The yaw (or pitch) control law which is really the load relief autopilot control law. The parameters denoted $A_1(s)$ and $A_2(s)$ contain the gains and filters that are used in the load relief autopilot. COEBRA optimizes load relief by varying the gains and filters in this control law. Chapter 7 will discuss how the user inputs the initial values for these gains and filters. It is not necessary to use all of the parameters in this control law. Chapter 7 will also explain how additional filtering may be used in this control law. This additional filtering (e.g. filters in the attitude loop ($K_D$) and in the rate loops ($K_{R1}, K_{R2}$)) can be used to calculate stability margins, but it is assumed that this additional filtering will not affect load relief capability. Hence, this additional filtering will not be used to calculate $\beta_p$, $\delta \psi_p$, and $\delta \phi_p$. This additional filtering will only be used to "meet" stability margin requirements.
1. NORMAL FORCE
\[
\begin{bmatrix}
(BS + Y_\beta) & (US - G_{R1}) & Y_{\delta_\psi} & 0 & (-WS - G_{R2}) & 0
\end{bmatrix}
\begin{bmatrix}
\beta(s)
\end{bmatrix}
\begin{bmatrix}
BS
\end{bmatrix}
\]

2. YAW MOMENT
\[
\begin{bmatrix}
N'_\beta & S^2 & -N'_{\delta_\psi} & 0 & 0 & N'_{\delta_\phi}
\end{bmatrix}
\begin{bmatrix}
\psi_b(s)
\end{bmatrix}
= 0
\]

3. ACCELEROMETER
\[
\begin{bmatrix}
Y_\beta & -L_\delta S^2 & Y_{\delta_\psi} & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_\mu(s)
\end{bmatrix}
\begin{bmatrix}
\dot{V}_{acc}(s)
\end{bmatrix}
= 0
\]

4. YAW CONTROL LAW
\[
\begin{bmatrix}
0 & A_1(s) & 1 & A_2(s) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\beta(s)
\end{bmatrix}
\begin{bmatrix}
\dot{V}_{acc}(s)
\end{bmatrix}
= 0
\]

5. ROLL MOMENT
\[
\begin{bmatrix}
L'_\beta & 0 & L'_{\delta_\psi} & 0 & S^2 & -L'_{\delta_\phi}
\end{bmatrix}
\begin{bmatrix}
\phi_b(s)
\end{bmatrix}
= 0
\]

6. ROLL CONTROL LAW
\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & A_3(s) & 1
\end{bmatrix}
\begin{bmatrix}
\delta_\phi(s)
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]

**NOTE:** All parameters are assumed to be in a body-axis reference system. Aerodynamic moment derivatives are referenced to the c.g. and are non-dimensionalized.

**FIGURE 4-1.**

**LINEARIZED EQUATIONS FOR LOAD RELIEF OPTIMIZATION**
YAW CONTROL LAW

\[
A_1(s) = \left[ K_D + K_R s - \frac{K_{DA} s^2}{(1 + \tau_1 s)(1 + \tau_2 s)(1 + \tau_3 s)} \right] K_{TVC}
\]

\[
A_2(s) = \left[ \frac{K_{A1}}{(1 + A_1 s)(1 + A_2 s)(1 + A_3 s)} + \frac{K_{A2} (1 + A_4 s)}{(1 + A_5 s)(1 + A_6 s)(1 + A_7 s)(1 + A_8 s)} \right] K_{TVC}
\]

ROLL CONTROL LAW

\[
A_3(s) = \left[ K_{DR} + K_{RR} s \right] K_{TVC}
\]

where \( K_{TVC} \) is the actuator gain

Figure 4-1 (Continued)
(5) The roll moment equation, where $\phi_b$ is the linearized roll attitude. This equation can be used when yaw/roll coupling is not negligible and is needed when designing the yaw load relief autopilot.

(6) The roll control law which is also used when yaw/roll coupling is important. $A_3(s)$ contains the roll attitude gain ($K_{DR}$) and the roll rate gain ($K_{RR}$) that are used in the assumed roll control law. These gains are treated as constants since COEBRA only uses this equation to define yaw/roll coupling. In other words, $K_{DR}$ and $K_{RR}$ are not considered a part of the load relief autopilot.

The equations of Figure 4.1 ignore structural bending and fuel slosh modes, since the rigid-body angle of attack and control deflections are the principal factors in determining structural bending moment loads. Another reason for using only rigid-body equations and, in fact, for using only these specific airframe/autopilot equations, was to minimize computer time. The COEBRA program does not invert the matrix of Figure 4.1. The matrix was inverted by hand, and this "manual inversion" was programmed in such a manner that all COEBRA does is compute S-plane polynomial coefficients from the input data, factor these numerator and denominator polynomials, compute residues, and insert values of time in order to calculate $\beta_r$, $\delta_{\psi_r}$, and $\delta_{\phi_r}$. If the equations of Figure 4.1 were generalized so that COEBRA would have to invert the matrix, this would result in a marked increase in computer time, and is left as a possible future item in the development of COEBRA. In fact, the reason for optimizing the load relief autopilot via a linear transient response approach that uses rather restricted and fixed airframe/autopilot equations, is to minimize computer time. The examples contained in Volume I of this report, demonstrate that this rather restricted approach is effective for at least a certain class of vehicles like Martin Marietta Corporation's Titan booster.
The equations of Figure 4.1 also assume a single control torque source for yaw (\(\delta_\psi\)) and a single control torque source for roll (\(\delta_\phi\)). By appropriate input data, these sources can be either: (1) thrust vector control via gimbaled engines, secondary injection, or jet vanes; or (2) control via aerodynamic surfaces. Also, the equations of Figure 4.1 include yaw/roll coupling from inertial and aerodynamic effects, as well as from control effects.

The equations of Figure 4.1 can be used for pitch only, by simply using the indicated yaw plane notation and sign convention, and by not inputting roll data. Obviously, these equations can also be used for yaw only, by simply not inputting the roll data.

As stated earlier, Chapter 7 explains how the user inputs the initial values for the yaw (or pitch) control law gains and filters. The airframe and the roll autopilot parameters that are needed in Figure 4.1, are input via the COBDE namelist. Since the equations of Figure 4.1 are to be used for the pitch plane as well as for the yaw/roll plane, the following sign convention is used in order to avoid confusion between pitch and yaw.

The usual sign convention for forces and moments is:

1. a positive X-axis force points "out the nose";
2. a positive sideforce is "out the right wing";
3. a positive normal force is "down";
4. a positive pitching moment is "nose up";
5. a positive yawing moment is "nose right"; and
6. a positive rolling moment is clockwise looking forward.

With this in mind, all COEBRA data is input as positive values for an aerodynamically unstable vehicle (negative \(C_{n\beta}\) or positive \(C_{m\alpha}\) with a positive dihedral effect (negative \(C_{l\beta}\)), and with "normal" control torque sources. By "normal" control torque sources is meant:
4.10

(1) a positive yaw deflection produces a negative sideforce, a positive yawing moment, and a negative rolling moment; (2) a positive pitch deflection produces a positive normal force, a positive pitching moment, and zero rolling moment; and (3) a positive roll deflection produces a negligible sideforce, a negative yawing moment, and a positive rolling moment. If any of these conditions are not satisfied, the user must compensate by making the appropriate changes in the signs of the input data. For example, if the vehicle is aerodynamically stable (positive $C_n\beta$ or negative $C_m\alpha$), the corresponding COBDE namelist parameter, i.e., CNB, is input with a negative sign. Table 4.1 defines the coefficients in Figure 4.1, explains the sign convention that is used, and explains how these coefficients are related to the data that is input via the COBDE namelist. A complete summary (with definitions) of these parameters is also contained in Chapter 10.

From the equations of Figure 4.1 and the wind forcing function that will be defined in Section 4.3, COEBRA computes the 3 peak values: $\beta_\rho$, $\delta_\psi_\rho$, and $\delta_\phi_\rho$. The autopilot sensitivities of $\beta_\rho$, $\delta_\psi_\rho$ and $\delta_\phi_\rho$ from each time point are then used to form the load relief cost function. Via the so-called "LRCASE" option, the user does have control over which combination of the 3 peak values COEBRA is to compute. LRCASE is a COBDE namelist parameter, and if LOADOP = 1, and:

(1) LRCASE (1) ≠ 0, COEBRA will compute $\beta_\rho$;
(2) LRCASE (2) ≠ 0, COEBRA will compute $\delta_\psi_\rho$; and
(3) LRCASE (3) ≠ 0, COEBRA will compute $\delta_\phi_\rho$.

In addition, the user may direct COEBRA to compute the peak values, but need not allow all of them to get into the cost function, simply by exercising the WTFAC (i,t) array.
<table>
<thead>
<tr>
<th>Airframe Parameter</th>
<th>Definition</th>
<th>COEBRA Notation</th>
</tr>
</thead>
</table>
| B                  | $B = \frac{V_{rw}}{U_r}$ for yaw/roll and $\phi \leq 15^\circ$  
|                    | $B = \frac{V^2_{rw}}{U_r}$ for pitch and $\alpha \leq 90^\circ$ | B               |
| U                  | Linear velocity along the X body axis | U               |
| W                  | Linear velocity along the Z body axis | W               |
| L<sub>AY</sub>     | Accelerometer moment arm $L_{AY} = X_{cg} - X_{acc}$  
|                    | Sign convention: LAY is positive if the accelerometer is forward of the c.g. | LAY             |
| G<sub>R1</sub>     | Gravity component $G_{R1} = g \sin (\theta_o)$  
|                    | Where $g$ is gravitational acceleration ($9.807 \, \text{m/sec}^2$)  
|                    | Define $\theta_o = \text{vehicle attitude relative to local horizontal}$ | $G_{R1} = g\sin(\theta)$ |
| G<sub>R2</sub>     | Gravity component $G_{R2} = g \cos (\theta_o) \cos (\phi_o)$  
|                    | Define $\phi_o = \text{euler roll angle}$ | $G_{R2} = g\cos(\theta_o)\cos(\phi)$ |
| Y<sub>\theta</sub> | Yaw aerodynamic sideforce coefficient  
|                    | $Y_{\theta} = \frac{q S_F C_{\gamma\theta}}{M}$ | YB = QBAR*SF*CYB |

\*YB is input as positive if positive $\theta$ produces negative sideforce or positive $\alpha$ produces negative normal force.

4.11
<table>
<thead>
<tr>
<th>Airframe Parameter</th>
<th>Definition</th>
<th>COEBRA Notation</th>
</tr>
</thead>
</table>
| \( N'_\beta \)     | Yaw aerodynamic torque coefficient (primed coefficient) \[
\begin{align*}
N'_\beta &= \frac{\bar{q} S_F b C_n \sigma}{I_{zz}} \\
\end{align*}
\] | \( NB = \text{QBAR}\text{SF}\text{D}\text{CNB} \) IZZ |
|                   | Note 1: The "primed coefficients" are defined by \[
\begin{align*}
N'_i &= \frac{N_i + \left( \frac{I_{xz}}{I_{zz}} \right) L_i}{(1 - \frac{I_{xz}}{I_{xx} I_{zz}})} \\
L'_i &= \frac{L_i + \left( \frac{I_{xz}}{I_{xx}} \right) N_i}{(1 - \frac{I_{xz}}{I_{xx} I_{zz}})} \\
\end{align*}
\] \( \text{YAW COEFFICIENT} \) \( \text{ROLL COEFFICIENT} \) | (If IXZ is input as non-zero, \( NB \) is corrected to the primed coefficient form.) |
|                   | Obviously for the uncoupled case \((I_{xz} = 0)\) \[
\begin{align*}
N'_i &= N_i \text{ and } L'_i = L_i \\
\end{align*}
\] | \( \text{CNB} \) is input as positive if vehicle is aerodynamically unstable. \( \text{CNB} \) is defined in the body axis system and is referenced about the c.g. of the vehicle. |
| \( L'_\beta \)     | Roll aerodynamic torque coefficient (primed coefficient) \[
\begin{align*}
L'_\beta &= \frac{\bar{q} S_F b C_{\ell \beta}}{I_{xx}} \\
\end{align*}
\] and if yaw/roll inertial coupling is input, \( L'_\beta \) is computed as per Note 1. \( C_{\ell \beta} \) is assumed to be in the body axes about the c.g. | \( LB = \text{QBAR}\text{SF}\text{D}\text{CLB} \) IXX |
<p>|                   | (If IXZ is input as non-zero, ( LB ) is corrected to the primed derivative form) ( \text{CLB} ) is input as positive if dihedral effect is positive. |</p>
<table>
<thead>
<tr>
<th>Airframe Parameter</th>
<th>Definition</th>
<th>COEBRA Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_{\delta \psi} )</td>
<td>Yaw sideforce due to yaw control deflection. For TVC: ( Y_{\delta \psi} = \frac{T_Y}{M} ) For Aerodynamic Surface: ( Y_{\delta \psi} = \frac{q S F C_Y \delta}{M} )</td>
<td>NOTE: ( Y_{\delta \psi} ) cannot be a combination of TVC and aero surface. The user must choose one or the other. TVC: ( YD_{\psi} = \frac{TY}{M\text{ASS}} ) Aero Surface: ( YD_{\psi} = \frac{Q\text{BAR}\times SF\times CYDR}{M\text{ASS}} ) CYDR is input as positive if positive yaw deflection produces negative sideforce, or positive pitch deflection produces positive normal force.</td>
</tr>
</tbody>
</table>

NOTE 2: If CYDR is input as non-zero, COEBRA assumes \( YD_{\psi} \) to be from an aero surface. This rule applies to all such yaw and roll matrix coefficients: \( Y_{\delta \psi} \), \( N_{\delta \psi} \), \( N_{\delta \phi} \), \( L_{\delta \psi} \), and \( L_{\delta \phi} \).
### TABLE 4-1 (CONTINUED)

**LOAD RELIEF MATRIX COEFFICIENTS**

<table>
<thead>
<tr>
<th>Airframe Parameter</th>
<th>Definition</th>
<th>COEGBRA Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N'_{\psi} )</td>
<td>Yaw torque due to yaw control deflection. For TVC: For Aerodynamic Surface</td>
<td>TVC: ( N_{\psi} = \frac{T_Y L_{GY}}{I_{zz}} ) ( N_{\psi} = \frac{-\dot{q} S_F b C_n s}{I_{zz}} ) Aero Surface: ( N_{\psi} = \frac{QBAR<em>SF</em>D*CNDR}{I_{zz}} ) CNDR (or LGY) is input as positive for pitch and yaw. See Note 2.</td>
</tr>
<tr>
<td>( L'_{\psi} )</td>
<td>Roll torque due to yaw control deflection. For TVC For Aerodynamic Surface</td>
<td>TVC: ( L_{\psi} = \frac{T_Y Z_{OFF}}{I_{xx}} ) ( L_{\psi} = \frac{-\dot{q} S_F b C_{l\delta r}}{I_{xx}} ) Aero Surface: ( L_{\psi} = \frac{QBAR<em>SF</em>D*CLDR}{I_{xx}} ) CLDR (or ZOFF) is input as positive if positive yaw deflection produces negative rolling moment. See Note 2.</td>
</tr>
</tbody>
</table>

Note 1: The "primed coefficient" is then computed as in Note 1.

Note 2: See Note 2.
### TABLE 4-1 (CONTINUED)
LOAD RELIEF MATRIX COEFFICIENTS

<table>
<thead>
<tr>
<th>Airframe Parameter</th>
<th>Definition</th>
<th>COEBRA Notation</th>
</tr>
</thead>
</table>
| \( N' \delta \phi \) | Yaw torque due to roll control deflection.  
For TVC \( N^0 \phi = 0 \)  
For Aerodynamic Surface \( N^0 \phi = \frac{q S_p b C_{n\delta}}{I_{zz}} \)  
The "primed coefficient" is then computed as in Note 1. | TVC:  
ND \( \phi \) = 0  
Aero Surface:  
ND \( \phi \) = \( \bar{Q} \text{BAR} \times S \text{F} \times D \times C \text{NDA} \)  
IZZ  
CNDA is input as positive if positive aileron deflection produces negative yawing moment. See Note 2. |
| \( L' \delta \phi \) | Roll torque due to roll control deflection.  
For TVC \( L^0 \phi = \frac{T_R L_{GR}}{I_{xx}} \)  
For Aerodynamic Surface \( L^0 \phi = \frac{q S_F b C_{l\delta}}{I_{xx}} \)  
The "primed coefficient" is then computed as in Note 1. | TVC:  
LD \( \phi \) = \( T_R \times L_{GR} \)  
IXX  
Aero Surface:  
LD \( \phi \) = \( \bar{Q} \text{BAR} \times S \text{F} \times D \times C \text{LDA} \)  
IXX  
CLDA (or LGR) is input as positive. See Note 2. |
This section will conclude with the following note. The 3 peak values \((\beta_\rho, \delta_\phi_\rho, \delta_\phi)\) are computed via the S-plane equations of Figure 4.1. When designing an analog autopilot, stability margins, etc., are also computed via the S-plane. But with the digital autopilot, margins, etc., are computed via the W-plane. Therefore, with the digital system, autopilot filters must be transformed between the S and W-planes for compatibility between the subroutine that calculates the W-plane frequency response and the subroutine that calculates the time response via S-plane equations.

Because of the simplified model, only the filters in the accelerometer and in the derived attitude acceleration feedback loops require transformation via the following equation:

\[
\text{(Time constant in S-plane)} = T_s \frac{2}{\pi} \times \text{(Time constant in W-plane)}
\]

where \(T_s\) is the sampling period. This simplified formula is valid since the filters in the load relief loop and in the derived attitude acceleration loop always have extremely low break frequencies. The gains in the autopilot require no transformation.
Section 4.3 The Wind Forcing Function

The wind forcing function is a four segment piecewise-linear function of time. A typical shape for \( \beta_\omega (t) \) is shown in Figure 4.2, where the user inputs the slopes denoted as \( U_1, U_2, U_3, \) and \( U_4, \) and the break-times denoted as \( TW_1, TW_2 \) and \( TW_3. \)

The general shape of \( \beta_\omega (t) \) is fixed, but the user has control over such things as the "size" or "weighting" of the wind shear relative to the total wind profile. As can be seen, \( \beta_\omega (t) \) is a series of steps and/or ramps that can be made to approximate the shape of the commonly used synthetic wind profile [Harris, 4]. As stated earlier, by a judicious choice of this wind profile, a load relief autopilot can be designed that possesses a good response characteristic to both low frequency and high frequency winds.

By careful selection of the slopes and break times, the user may choose any relative effective frequency content that he wishes. The following values represent the typical wind profile that is preset in COEBRA, to be overridden by the user.

(a) \( U_1 = .0021 \) rad/sec.
(b) \( U_2 = .01 \) rad/sec.
(c) \( U_3 = .0228 \) rad/sec.
(d) \( U_4 = -.0038 \) rad/sec.
(e) \( TW_1 = 40. \text{ sec.} \)

(f) \( TW_2 = 52.5 \text{ sec.} \)

(g) \( TW_3 = 55. \text{ sec.} \)

To minimize machine time used in computing the time responses, a full response is computed only for the nominal case. This "full response" is computed from \( T_{\text{START}} \) to \( T_{\text{STOP}} \). When computing the peaks for the partial derivatives, COEBRA searches only a very small time region: The time at which the nominal case peak occurred is stored and a search is done in a small region about this time for the disturbed case. The two input parameters of interest here are \( \Delta T \) which is the computational time increment, and \( D_{\text{INCRE}} \) which is the number of increments on either side of the time of the nominal peaks over which COEBRA searches for the disturbed peak. The preset values (to be overridden by the user) are:

\[
\begin{align*}
T_{\text{START}} &= 30 \text{ sec} \\
T_{\text{STOP}} &= 70 \text{ sec} \\
\Delta T &= .05 \text{ sec} \\
D_{\text{INCRE}} &= 10
\end{align*}
\]

All nominal case time responses are plotted for each time point. Also, the peaks of \( \beta, \delta_\psi \) and \( \delta_\phi \) need not occur at the same time.
Chapter 5. Autopilot Variable Constraint Equations

Section 5.1 The Constraint Equation

For each autopilot variable, \( X(j) \), the following is the detailed expression of the autopilot variable constraint equation.

\[
\text{MAX} \left\{ (1+P)^{-1} x(j)_0, \text{XMIN}(j) \right\} \leq X(j) \leq \text{MIN} \left\{ (1+P)x(j)_0, \text{XMAX}(j) \right\}
\]

for \( j = 1, \ldots, n \)

Where:

1) \( X(j) \) is the \( j \)th autopilot variable, whose value is to be optimized by COEBRA. \( x(j)_0 \) refers to the value of \( X(j) \) at the beginning of the so-called Major Iteration. Note that the point defined by \( x(j)_0 \) for all \( j \), is the point about which the partial derivatives are computed, and the Taylor Series is expanded.

2) \( P \) refers to the autopilot variable step-size, and has the same value for all the autopilot variables.

3) The \( \text{XMIN}(j) \) and \( \text{XMAX}(j) \) arrays refer to the minimum and maximum values ever allowed for \( X(j) \).

In words, if \( \text{XMIN} \) and \( \text{XMAX} \) are not encountered on a particular iteration, the above constraint equation says that \( X(j) \) is allowed to vary no more than about \( \pm P \%) \) from \( x(j)_0 \) on any iteration. Since it is desirable to maximize the step-size on each iteration, thereby getting the maximum "mileage" out of each set of partial derivatives, it is desirable to have a Minor Loop that increases the size of \( P \) until improvement in that "search direction" is no longer possible. In other words, the Minor Loop serves to maximize the autopilot variable step-size. In maximizing \( P \), the Minor Loop uses two
"indicators": (1) a counter that keeps track of the number of stability margins that are already met, and (2) a figure-of-merit that is a linear combination of the actual margins. The so-called Margin Counter and Figure-of-Merit are used as follows:

1) If the value of the Margin Counter increases, the value of $P$ can be increased regardless of the value of the Figure-of-Merit.

2) If the value of the Margin Counter does not change, the Figure-of-Merit is used to decide whether $P$ should be increased or decreased.

3) If the value of the Margin Counter decreases, $P$ must be reduced.

The procedure for changing the value of $P$, is either to double it or to average the present $P$ with the "best-so-far" $P$.

In the optimization of the value of $P$, the input parameter $PSMALL$ serves as a convergence criteria on two items:

1) $PSMALL$ is the smallest value allowed for $P$. COEBRA will terminate when $P$ becomes less than $PSMALL$, since this means that the present nominal autopilot (the autopilot at the beginning of this major iteration) is the best autopilot that can be obtained.

2) $PSMALL$ is the smallest value allowed for $\Delta P$ which is defined as $|P(\text{present}) - P(\text{last})|$. When $\Delta P$ becomes less than $PSMALL$, COEBRA will begin the next major iteration.

By inputting $PSMALL$ with the same value as the initial value of $P$, the minor loop will be bypassed. The initial value of $P$ will then be used unchanged on each major iteration.

As discussed in Chapter 3, Section 3.5, of Volume I, a major benefit of the minor loop is that it allows the algorithm to converge steadily to an
"interior" optimum. This is explained as follows. Since the solution to the linear programming problem always lies at a vertex of the feasible region defined by the constraint equations, it is the Minor Loop that allows the algorithm to converge to a local optimum that is interior to the stability margin constraint equations.

Section 5.2 Definitions of the Margin Counter and Figure-of-Merit

Referring to the Margin Array of Chapter 1, the only elements of that array that can be included in the Margin Counter and the Figure-of-Merit are:

1) The 3 rigid-body margins and the phase margin crossover frequency;
2) For the 1st and 2nd structural modes, the front and backside phase margins, and the closest-approach;
3) For the 3rd to the 8th structural modes, the modal peak gain; and
4) For the fuel slosh modes, the backside phase margins.

Section 5.2.1 The Margin Counter

The i\textsuperscript{th} candidate element of the Margin Array, \(M(i)\), is counted by the Margin Counter if: 1) \(M(i)\) exceeds its "toleranced" SPEC value; and 2) the i\textsuperscript{th} element of the WTMARG array is not equal to zero. The so-called "toleranced" SPEC value is defined as

\[
(1 - \text{TOLMAC}) \times \text{SPEC} (i)
\]

The tolerance is applied to SPEC in order to avoid oscillations in the Margin Counter.

NOTE: WTMARG and TOLMAC are input parameters.

Table 5.1 shows the so-called weighting values given to each element in the Margin Counter. For example, if the 3 rigid-body margins and the frontside phase margin of a stable 1st structural mode were the only elements that exceeded their "toleranced" SPEC values, the Margin Counter would equal 7. This assumes that the corresponding values of the WTMARG array were not equal to zero.
Table 5.1 Weighting Values in the Margin Counter

<table>
<thead>
<tr>
<th>Weighting Value</th>
<th>Candidate Element of Margin Array</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>for each of the 3 rigid-body margins</td>
</tr>
<tr>
<td>1</td>
<td>for each front and backside phase margin of the 1st and 2nd structural modes if the modes are stable.*</td>
</tr>
<tr>
<td>2</td>
<td>for each closest-approach margin of the 1st and 2nd structural modes if the modes are stable.*</td>
</tr>
<tr>
<td>2</td>
<td>for each peak gain of structural modes 3 to 8.</td>
</tr>
<tr>
<td>2</td>
<td>for each backside phase margin of the fuel slosh modes.</td>
</tr>
<tr>
<td>2</td>
<td>for the rigid-body phase margin crossover frequency.</td>
</tr>
</tbody>
</table>

*If a first or second structural mode is unstable, nothing is counted for that mode. Also, the front and backside phase margins are mutually exclusive with the closest-approach margin. If the mode resonates above zero db, only the front and backside phase margins are eligible to be counted. If the mode resonates below zero db, obviously only the closest-approach margin is eligible.
Section 5.2.2 The Figure-of-Merit

Whereas there is only one Margin Counter, there are two Figures-of-Merit: (1) a figure-of-merit (FM₁) for the phase of COEBRA where stability margins are to be optimized; and (2) a different figure-of-merit (FM₂) for the phase of COEBRA where the objective is to maximize structural bending moment load relief capability. For the "optimize margins" phase:

\[ FM₁ = \sum WTMARG(i) \times \min \{ \text{COSPEC}(i), M(i) \} \]

where:

1) The summation is carried out over all time points, and over all the "candidate" elements of the Margin Array for i = 1 to 48. These "candidate" elements are summarized in Table 5.2;
2) WTMARG is an array of weighting factors that is input by the user; and
3) COSPEC is the array of desired objectives, that is also included in the Cost Function (Chapter 3).

In other words, as discussed in Volume I, the figure-of-merit (FM₁) is "rewarded" for each value of i, only up to a certain value (given by COSPEC). Note also that COSPEC and M(i), are scaled when they are included in the figure-of-merit.

For the "load relief optimization" phase:

\[ FM₂ = \sum WTMARG(i) \times M(i) \]

Where:

1) The summation is carried out over all time points, and for i = 49, 50, and 51 (depending on the LRCOST array).
2) M(49) is of course the peak value of the angle of sideslip.
M(50) and M(51) are the peak values of the control deflections in yaw and roll respectively.
Table 5.2 Candidate Elements for FM1

| (1) | Each of the 3 rigid-body margins. |
| (2) | For **stable** 1st or 2nd structural modes, the closest approach margin. |
| (3) | For **unstable** 1st or 2nd structural modes, the unstable front or backside phase margin. |
| (4) | The peak gain of structural modes 3 to 8. |
| (5) | The backside phase margin of each fuel slosh mode. |
| (6) | The rigid-body phase margin crossover frequency. |
Section 5.3 Termination

This section discusses the 4 ways in which the optimization process can be terminated.

1) The first way might be referred to as self-termination. This occurs when $P$ becomes less than $PSMALL$ due to the following two situations:
   a) when the Margin Counter always decreases for any value of $P$ that is greater than $PSMALL$; and
   b) when the figure-of-merit (FM) is used to "break ties" and the following relationship is not satisfied:
      \[ FM \text{ (present)} \geq (1+TOLFGM) \times FM\text{(last)} \]
      where TOLFGM is an input parameter.
   This relationship will not be satisfied when all $M(i)$ exceed their COSPEC values (which means that no further improvement is desired), or when all $M(i)$ do not increase sufficiently (which means that no further improvement is possible). For this latter case, if all margins do not yet meet their requirements, the user must then either (1) try another initial condition for the autopilot variables, (2) add more complexity to the autopilot, and/or (3) relax some of the design requirements and/or alter some of the design objectives.

2) The second method of termination is when $NITER$ major iterations have been completed. ($NITER$ is an input parameter.)

3) The third way is when the user's estimated computer run time is exceeded.
4) The fourth way is due to various error messages like:

a) shutdown because the user's first guess autopilot does not lie within the range of XMIN to XMAX; and

b) shutdown due to the nearly impossible situation where STEP (defined in Chapter 2) goes to zero and no feasible solution can be achieved.
Chapter 6. The QD030 Phase

The user inputs the initial autopilot for COEBRA via the INDATA namelist of the QD030 phase of the program. However, the QD030 phase may be run by itself. This chapter describes the QD030 phase.

Section 6.1 The QD030 Block Diagram

Figure 6.1 is the block diagram that is used by the QD030 phase of the program. COEBRA assumes that the autopilot gains and filters are located in Blocks A thru N. When running COEBRA, the airframe transfer functions (including the actuator/engine) may appear anywhere in Blocks A thru U. But normally, they will be input in Blocks 0 thru U, with the denominator input into Block 0, and the numerators input into Blocks P thru U. When running the QD030 by itself, airframe and autopilot transfer functions may, of course, appear anywhere.

Obviously, all transfer functions may be defined in either the S-plane or the W-plane. COEBRA assumes everything in Blocks A thru U is input in Bode form, i.e., time constants, effective damping ratios, and undamped natural frequencies. However, when running QD030 by itself, only Block 0 thru U need be in Bode form. For the QD030 phase by itself, Blocks A thru N may contain data in polynomial form.

COEBRA optimizes the total open-loop frequency response. The QD030 by itself can compute 13 different open and closed-loop transfer functions, and run frequency and transient responses on any of them. The closed-loop transfer functions assume that all feedback loops are closed, and that the system is forced by $\Theta_1$. The open-loop transfer functions assume that $\Theta_1 = 0$, and that the open-loop forcing function is $\delta_i$. It is emphasized again that
COEBRA deals only with the total open-loop transfer function defined by $\delta_F/\delta_i$.

Table 6.1 summarizes the 13 open and closed-loop transfer functions that the QD030 phase can compute.

As mentioned earlier, COEBRA assumes everything is input in Bode form. Table 6.2 lists the INDATA namelist names that are associated with Bode form input to Blocks A thru U. Following the INDATA namelist name, the data in each block is input in the following order:

1) No. of real roots in the numerator
2) No. of complex pairs in the numerator
3) No. of free S's (or free W's) in the numerator
4) No. of real roots in the denominator
5) No. of complex pairs in the denominator
6) No. of free S's (or free W's) in the denominator
7) Bode gain
8) Thru . . . . . . . . . .
    numerator real roots ($\gamma$)
    numerator complex pairs ($\gamma$, $\omega$)
    denominator real roots ($\gamma$)
    denominator complex pairs. ($\gamma$, $\omega$)

For example, assume the following Bode form transfer function is put into Block C:

$$C = (3,0) \frac{(1+0.4S)(1+0.3S)(S)(S)}{(1+0.2S)(1+\frac{2(0.1S)}{20} + (S)(S))}$$

Associated input:

$$RC = 2., 0., 2., 1., 1., 0., 3., .4, .3, .2, .1, 20.,$$
Table 6.1 QD030 Transfer Functions

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Closed-Loop Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( p/\theta_i )</td>
</tr>
<tr>
<td>2</td>
<td>( q/\theta_i )</td>
</tr>
<tr>
<td>3</td>
<td>( r/\theta_i )</td>
</tr>
<tr>
<td>4</td>
<td>( s/\theta_i )</td>
</tr>
<tr>
<td>5</td>
<td>( t/\theta_i )</td>
</tr>
<tr>
<td>6</td>
<td>( u/\theta_i )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Open-Loop Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>( i = \text{GOPI} )</td>
</tr>
<tr>
<td>8</td>
<td>( j = \text{FGOQJ} )</td>
</tr>
<tr>
<td>9</td>
<td>( k = \text{EFGORK} )</td>
</tr>
<tr>
<td>10</td>
<td>( l = \text{DEFGOSL} )</td>
</tr>
<tr>
<td>11</td>
<td>( m = \text{CDEFGOHTM} )</td>
</tr>
<tr>
<td>12</td>
<td>( n = \text{BCDEFGOHUN} )</td>
</tr>
<tr>
<td>13</td>
<td>( \delta_f/\delta_1 = i+j+k+l+m+n )</td>
</tr>
</tbody>
</table>
Table 6.2  INDATA Namelist Names for the Bode Form

<table>
<thead>
<tr>
<th>INDATA Namelist Name</th>
<th>Block</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>A</td>
</tr>
<tr>
<td>RB</td>
<td>B</td>
</tr>
<tr>
<td>RC</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>RO*</td>
<td>O</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>RT</td>
<td>T</td>
</tr>
<tr>
<td>RU</td>
<td>U</td>
</tr>
</tbody>
</table>

* Block O is restricted to denominator roots only.
Table 6.3 lists the INDATA namelist names that are associated with so-called polynomial form input that can be used in Blocks A thru N when running the QD030 by itself. Following the INDATA namelist name, the data in each block is input in the following order:

1) The degree of the numerator
2) The degree of the denominator
3) The gain factor
4) Thru . . . . . . . . . . . . . . .

All coefficients of the numerator in ascending orders of powers
All coefficients of the denominator in ascending orders of powers

For example, assume the following polynomial form transfer function is put into Block C:

$$ C = (6.0) \frac{(5.0s^3 + 3.0s^2 -1.0)}{(4.0s^2 -2.0s)} $$

associated input:

$$ PC = 3., 2., 6., -1., 0., 3., 5., 0., -2., 4., $$

Section 6.2 Additional INDATA Namelist Variables

This section will define the additional INDATA namelist variables besides those contained in Tables 6.2 and 6.3. It is noted that in the optimization phase, COEBRA uses only the variables in Table 6.2 and the variables defined below as SLOW and UPPER. Of course, all of the INDATA namelist variables are applicable when running the QD030 phase by itself. If preset values are indicated, they can be overridden by the user if he so desires.
Table 6.3 INDATA Namelist Names for Polynomial Form

<table>
<thead>
<tr>
<th>INDATA Namelist Name</th>
<th>Block</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA</td>
<td>A</td>
</tr>
<tr>
<td>PB</td>
<td>B</td>
</tr>
<tr>
<td>PC</td>
<td>C</td>
</tr>
<tr>
<td>PM</td>
<td>M</td>
</tr>
<tr>
<td>PN</td>
<td>N</td>
</tr>
</tbody>
</table>
6.2.1 **SLOW**, Floating point, preset value = .01

All frequency responses will be calculated using SLOW (radians per second) as the lower limit on the frequency.

6.2.2 **UPPER**, Floating point, preset value = 1000.

All frequency responses will be calculated between the limits SLOW and UPPER (radians per second).

6.2.3 **NCASE**, Fixed point

The transfer functions (or cases) to be computed. Refer to Table 6.1.

6.2.4 **NFREQ**, Fixed point

Having computed the transfer functions (via NCASE), NFREQ specifies those transfer functions on which a frequency response is to be calculated. If plots are desired, the negative of the transfer function number must be input.

6.2.5 **NTRAN**, Fixed point

Having computed the transfer functions (via NCASE), NTRAN specifies those transfer functions on which a transient response is to be calculated. If plots are desired, the negative of the transfer function number must be input.

6.2.6 **BT**, Floating point

The starting time for the transient response.

6.2.7 **DT**, Floating point

The time increment for the transient response.

6.2.8 **FT**, Floating point

The final time for the transient response.

6.2.9 **NUMZ**, Fixed point

Number of free S's (or W's) to be added to the numerator before computing the transient response.
6.2.10 KDENZ, Fixed point

Number of free S's (or W's) to be added to the denominator before computing the transient response.

6.2.11 TOL, Floating point

Tolerance by which roots in the numerator and denominator are cancelled before computing the transient response.

6.2.12 DAMP, Floating point

Any underdamped quadratic roots in the denominator of the total open loop transfer function (CASE 13) that have an effective damping ratio with an absolute value less than DAMP, are considered to be bending or fuel slosh modes. The namelist variable called the IDMODE array is then used to identify the modal type (e.g., 1st mode, etc.) to be associated with each quadratic pair that is selected by DAMP. DAMP serves to eliminate from consideration as modes, things such as sensor dynamics, autopilot filters and prefilters, and even certain modes including perhaps the engine.

6.2.13 AMPOUT, Floating point

Modes that are selected by DAMP whose peaks resonate below AMPOUT, are not considered when calculating stability margins via the INDATA variables called KASK, KVECT, and KVECPL. Units on AMPOUT are in decibels, i.e., if AMPOUT = -20., modes that resonate below -20. decibels will not be considered.

6.2.14 IDMODE (k), Integer Array (8 dimensional)

Referring to the underdamped quadratic roots in the denominator of the total open loop transfer function (CASE 13), any root-pair whose damping ratio is less than DAMP is considered to be from a structural bending or fuel slosh mode. The program lists these bending and fuel slosh modes according to
frequency, beginning with the lowest frequency. The IDMODE (k) array is then used to identify these modes according to this ordered list, beginning with the lowest frequency.

IDMODE (k) = 1, if the $k^{th}$ mode is a first structural mode.

IDMODE (k) = 2, if the $k^{th}$ mode is a second structural mode.

e tc., until

IDMODE (k) = 8, if the $k^{th}$ mode is an eighth structural mode.

IDMODE (k) = 10 or higher, if the $k^{th}$ mode is a fuel slosh mode.

IDMODE (k) = 0, if the $k^{th}$ mode is to be ignored.

NOTE: When identifying these modes, they need not be ordered according to frequency, e.g., the 1st mode may be higher in frequency than the 3rd mode, or the 4th mode may be lower in frequency than the slosh modes, etc. The exception to this is the 2nd mode, which must be run with something identified as a 1st mode that is lower in frequency, and something identified as a higher mode that is higher in frequency.

6.2.15 KASK, Fixed point, preset value = 1

If KASK = 1, this enables printout at the end of each CASE 13 frequency response of the following stability margins:

1) Aerodynamic gain margin

2) Rigid-body gain margin

3) Rigid-body phase margin

4) The following modal margins, where IDMODE and DAMP identify whether the modes are structural or fuel slosh,
   a) Amplitude of all structural mode peaks
b) Front and backside phase margins of the 1st and 2nd structural modes.

c) Backside phase margins of the slosh modes.

If KASK = 0, this flag is disabled.

6.2.16 KVECT, Fixed point, preset value (disable value) = 0

If KVECT = 1:

1) Polar coordinate plots of the open loop cases asked for by NFREQ are generated at the frequency of each of the stability margins generated by KASK. Since this option works in conjunction with KASK, KASK must be set to unity.

2) Polar vector plots will be generated at up to 10 user specified frequencies supplied by FQVECT in radians per second.

3) In addition to the above, a summary of the plotted vectors is printed out.

6.2.17 FQVECT, Floating point array (10 dimensional)

FQVECT is an array of frequencies (in radians per second) that is input by the user. At each of these frequencies, the program will compute and plot all the vectors that are called for by NFREQ.

6.2.18 KVECPL, Fixed point, preset value (disable value) = 1

If KVECPL = 0, the plots generated by the KVECT option will be suppressed. However, the vector plot summary will still be printed out.

6.2.19 NFLAGG, Fixed point

If NFLAGG = 0, this implies that immediately following this QD030 case will be another QD030 case that will use the same data that is input into Blocks 0 thru U, but different data for Blocks A thru N. Hence, Blocks A
6.12
thru H will be reset to unity, and Blocks I thru N will be reset to zero.

If NF1AGG = 1, all QD030 Blocks will be reset to their initial values,
i.e., Blocks A thru H will be reset to unity, Blocks I thru N will be reset
to zero, Block 0 will be reset to unity, and Blocks P thru U will be reset
to zero.

6.2.20 In conclusion, the options NCASE, NFREQ, NTRAN, KASK, etc., are not
executed during COEBRA's optimization phase, but can be executed when the
optimization process terminates. In other words, when optimization is termi-
nated, the program will return to the INDATA namelist and execute all the
options that are called for.

Section 6.3 QD030 Input Instructions
1) If QD030 is run without the COEBRA optimization phase, the first card
must be:

Col. 1 2 3 4 5 6
Q D 0 3 0

2) The second card is a title card and will be printed at the top of
each page of output. There must not be a $ as the first character
of the title card in column two.

3) The title card is followed by the input data as defined in Sections
6.1 and 6.2.

The input data must have a $ as the first character in the first card.
The dollar sign is placed in column two and is immediately followed by the
word INDATA. $INDATA must be followed by at least one blank. The data is
then placed in columns 10 thru 80. The data items must be separated by
commas: however, the use of a comma after the last item is optional.
The end of the data group is indicated by $END beginning in column 2 of a new card.

If more than one card is needed for input data, the last item of each card must be constant followed by a comma. Blanks are permissible between the comma and the next data value. Blanks are prohibited between digits of a data value. The succeeding card must not have a $ in column two and may start in any column except column one. Each successive card will continue with the data elements of input. The data on a continuation card may end in any column.

The form data may take is either as its input variable name or its equivalent array element name. The form is variable name = constant. Subscripts must be integer constants. When the input is in an array in sequentially ascending steps, the form is variable name = constant, constant, . . . .

4) Transfer function input to Blocks 0 thru U must be in Bode form.

Block 0 is used for the common denominator of Blocks P thru U as is suggested by their relative positions in the block diagram. Therefore, only denominator dynamics are allowed in Block 0. If only a portion of the common denominator of Blocks P thru U is input to Block 0, the program will extract the remaining common roots and store them in Block 0.

5) When running QDO30 by itself, transfer function input to Blocks A thru N may be in polynomial or Bode form.

NOTE: When running COEBRA's optimization phase, only Bode form may be used.

6) The program initializes all forward loop blocks (A thru H and 0) to a unity transfer function, and all feedback blocks (I thru N and P thru U) to a zero transfer function.
Section 6.4 QD030 Output Data

The transfer functions that are computed by the QD030 are output in three (3) forms:

1) Form A, Polynomial Form:

\[ G = \frac{\text{(Gain)} S^k (1 + a_1 S + a_2 S^2 + \ldots + a_l S^l)}{S^m (1 + b_1 S + \ldots + b_n S^n)} \]

2) Form B, Bode form:

   First lines: time constants and free S's (or W's) (denoted by zeros)
   Last lines: pairs of damping ratios and natural frequencies.

3) Form C, Root Locus Form:

   First lines: frequencies of the real roots
   Last lines: pairs of real parts of the roots and imaginary parts of the roots.

The following is a list of the output in the order in which it appears:

1) Title and page number
2) Input data for Blocks A thru U
3) Transient responses
   a) Case number
   b) Transfer function in Forms A, B and C
   c) Partial Fraction Expansion
   d) Transient Response
4) Frequency Responses
   a) Case number
   b) Transfer function in Forms A, B and C
   c) Frequency Response
5) Stability Margin Summary and Printout of Vector Plots

Section 6.5 Error Comments

This section briefly defines some error comments that may appear.

xxxxxUSER INPUT ERRORxxxxx This is the heading that will be output if the program finds an error in the input. Following this heading the user will find one of the following messages. These messages are explained below:

1) Element xx is equal to zero.
2) In the xx element, the absolute value of the effective damping ratio of the complex pair is LE 0 or GE 1.
3) Gain is equal to zero.
4) Highest degree of polynomial is equal to zero.
5) Inconsistency between NTYPE array and root input.
6) Inconsistency in IDMODE array input.

The messages 1) and 2) are generated by subroutine CKROOT. This routine checks the user's root input for proper form. These messages will be printed directly below the input if they are applicable to that case.

The messages 3) and 4) are generated by subroutine CKPOLY. This routine checks the user's polynomial input for proper form. These follow the output as above.

Message 5) is generated by subroutine CKLPRV. This routine checks for consistency between NTYPE array input and the root input.
Message 6) is generated by subroutine CKIDEN. This routine checks the IDMODE array for proper form.

All of the user's input data is checked, and if any of these messages appear, the program is terminated and user should correct all input errors before resubmittal.
CHAPTER 7. ALLOWED AUTOPILOT CONFIGURATION

This chapter is written assuming that the reader has read Chapter 6, and understands how to use the INDATA namelist. The INDATA namelist is used to input the initial autopilot. Section 7.1 discusses how the user defines this initial autopilot to the COEBRA optimization routines. Section 7.2 contains additional items that are required in order to define the configuration of the load relief and/or drift minimum autopilots. Section 7.3 contains a discussion of the so-called ROOTCH subroutine that allows COEBRA to change a quadratic filter into 2 single break filters, or vice versa.

Section 7.1 Defining the Initial Autopilot

This section discusses how the user defines the initial autopilot to COEBRA. The items in this section are applicable to both phases of COEBRA, namely, stability margin optimization and so-called load relief optimization. Section 7.2 will discuss the additional items that are needed to define the load relief autopilot to COEBRA.

The user inputs the autopilot parameters via the INDATA namelist, into any of the QD030 blocks A through N (see Figure 6.1 in Chapter 6). COEBRA deals with the autopilot parameters in Bode form, i.e., time constants for single break filters, and effective damping ratios and undamped natural frequencies for quadratic break filters.

In order to allow for multiple time point design, three types of designations are assigned to the autopilot parameters (assigned by the NTYPE(j) array defined later). Each autopilot parameter (Bode gain, time constant, damping ratio, or natural frequency) is one of the following:

(1) **Constant**, whose value is not to be altered by COEBRA.
(2) Stand-alone variable, whose value, when altered by COEBRA, need not be the same as the parameter that appears in the same block and element location at the immediately preceding time point.

(3) Common variable, whose value, when altered by COEBRA, must be the same as the variable that appears in the same QD030 block and element location at the preceding time point.

The user may input parameters (variables and constants) and free S's into any of the QD030 blocks A through N (see Figure 6.1). The following discussion shows the general configuration allowed for the autopilot variables. In each block, a maximum of 41 variables is allowed, and these may be configured as follows:

\[
K \begin{pmatrix} (10 \ - \ 1st \ order \ functions) & (5-2nd \ order \ functions) \\ (10 \ - \ 1st \ order \ functions) & (5-2nd \ order \ functions) \end{pmatrix}
\]

Note that this is the maximum configuration allowed for the variables (stand-alones and commons). Any parameter that appears outside the allowed field must be designated as a constant. Also, free S's may appear, but are not assigned a parameter type. The first order functions are specified by a single parameter, the time constant \( \gamma \), and the second order functions by two parameters, effective damping ratio \( \gamma \) and undamped natural frequency \( \omega \).

For example, if, in the numerator of a block, it is desired to have 10 variable single break filters, 1 constant single break filter, 1 variable quadratic pair of roots, 1 constant quadratic pair of roots, and a free S, the numerator will be configured as follows:

\[
(\gamma_1) \ (\gamma_2) \ (\gamma_3) \ (\gamma_4) \ (\gamma_5) \ (\gamma_6) \ (\gamma_7) \ (\gamma_8) \ (\gamma_9) \ (\gamma_{10}) \ (\gamma_{11}) \ (\gamma_1, \omega_1) \ (\gamma_2, \omega_2) S
\]

Where: (1) \( \gamma_1 \) through \( \gamma_{10} \) are the variable single break filters.

(2) \( \gamma_{11} \) must be the constant single break filter.
(3) \((\gamma_1 \omega_1)\) may be designated as either the variable or the constant quadratic pair.

(4) \((\gamma_2 \omega_2)\) may also be designated as either the variable or the constant quadratic pair.

**NOTE #1:** \(\gamma\) and \(\omega\) of a quadratic pair need not have the same type designation. For example, \(\gamma\) may be a constant and \(\omega\) may be a variable or vice versa. Obviously, \(\gamma\) and \(\omega\) may both be constants or may both be variables.

**NOTE #2:** COEBRA is dimensioned for a maximum of 60 parameters (not including free S's) per time point, including a maximum of 50 variables (stand-alone and commons).

**NOTE #3:** All of the gains and filters that are denoted as variables must have positive values. Constants may have negative values. If a variable gain must have a negative value, the minus sign must be accounted for in another QD030 block where the minus gain can be designated as a constant. For example, if the variable gain in Block K must be negative, the user must put minus-one gains (designated as constants) in blocks D and E.

The following is a discussion of how the parameter "type designations" are assigned.

In the input namelist called COBDE, which is input for each time point:

(1) \(\text{NTOTAL}\) is the total number of parameters (variables and constants) for this time point;

(2) \(\text{NTYPE}(j)\) is a compressed array that identifies the type of each parameter at this time point. This array corresponds
to the parameters in the order in which they are input in the QD030 (INDATA namelist). This array must identify even those constants that lie outside the allowed field.  

\[ \text{NTYPE}(j) = \begin{cases} 
0 & \text{if the } j^{\text{th}} \text{ parameter is a constant.} \\
1 & \text{if the } j^{\text{th}} \text{ parameter is a common variable.} \\
-1 & \text{if the } j^{\text{th}} \text{ parameter is a stand-alone variable.} 
\end{cases} \]

Note that for the first time point entered, only 0's and -1's can be used in the NTYPE(j) array.

NTOTAL and NTYPE(j) are used to identify gains, time constants, damping ratios and natural frequencies, but are not used to identify free S's. Finally, at the beginning of each run, QD030 blocks A through H are each initialized with unity gain and no roots. QD030 blocks I through N are each initialized with zero gain and no roots. Hence:

(a) if in blocks A through H, any block contains roots (including free S's) and/or a gain not equal to unity, then NTOTAL and NTYPE(j) must identify the parameters in that block. If the user desires a block (A through H) to contain simply a variable gain whose initial value is unity, he must enter the gain with a value of one-plus-epsilon.

(b) if in blocks I through N, any block contains a nonzero gain, the parameters in that block must be identified by NTOTAL and NTYPE(j). NTOTAL and NTYPE(j) must not identify blocks that do not fit into categories (a) and (b) above. Obviously, NTOTAL and NTYPE(j) assign the "types" whether the user is optimizing margins or optimizing load relief.
Section 7.2 Defining the Load Relief (and Drift Minimum) Autopilot Configuration

The section discusses the additional items that are needed to define to COEBRA, the configuration of the load relief autopilot and/or the drift minimum autopilot. These items are in addition to those presented in Section 7.1.

As discussed in Chapter 4, COEBRA uses the control law of Equation 7.1 to optimize load relief capability. (Chapter 8 will show that the same control law is assumed when designing a drift minimum autopilot.)

\[
\Delta \psi = (K_D + K_{R1}S + K_{R2}S) (-\Delta \psi_b) + \left[ \frac{K_{DA}S^2}{(1+T_1S)(1+T_2S)(1+T_3S)} \right] (\Delta \psi_b) \\
+ \left[ \frac{K_{A1}}{(1+A_1S)(1+A_2S)(1+A_3S)} \right] + \left[ \frac{K_{A2}}{(1+A_5S)(1+A_6S)(1+A_7S)(1+A_8S)} \right] (-\Delta \dot{V}_{ACC})
\]

(Equation 7.1)

Equation 7.1 uses yaw plane notation, but it is also applicable to pitch, provided that yaw plane notation and sign convention are used. All gains in Equation 7.1 have positive signs.

The COEBRA program must be told how to equate the autopilot parameters in the above control law with the autopilot parameters in the INDATA namelist. This will be done via the following example.

Suppose that the autopilot is input into the QD030 block diagram as shown in Figure 7.1. The filter networks that are denoted \( F_i \) do not get into the load relief cost function, but they do contribute to the open-loop frequency response. Hence the \( F_i \) that are declared to be variables (by the NTYPE(j) array) will get into the constraint matrix. Note that the \( F_i \) might be used for other things besides bending and fuel slosh filter networks. They might also be used for such things as sensor dynamics, noise prefilters, etc.
FIGURE 7.1 LOAD RELIEF AUTOPILOT EXAMPLE #1

ATTITUDE LOOP

\[
\begin{align*}
\text{QD030 Block I} & \\
\left[ K_D \right] & \frac{F_1}{F_2} \\
\rightarrow & \Delta \psi_b
\end{align*}
\]

RATE 1 LOOP

\[
\begin{align*}
\text{QD030 Block J} & \\
\left[ K_{R1} \right] & \frac{F_3}{F_4} \\
\rightarrow & \Delta \psi_b (1)
\end{align*}
\]

RATE 2 LOOP

\[
\begin{align*}
\text{QD030 Block K} & \\
\left[ K_{R2} \right] & \frac{F_5}{F_6} \\
\rightarrow & \Delta \psi_b (2)
\end{align*}
\]

ATTITUDE ACCELERATION LOOP

\[
\begin{align*}
\text{QD030 Block L} & \\
\left[ K_{D_A} \right] & \frac{1}{(1+\tau_1 s)(1+\tau_2 s)(1+\tau_3 s)} \\
\rightarrow & \Delta \dot{\psi}_b
\end{align*}
\]

KA1 LOAD RELIEF LOOP

\[
\begin{align*}
\text{QD03Q Block M} & \\
\left[ K_{A1} \right] & \frac{1}{(1+A_1 s)(1+A_2 s)(1+A_3 s)} \\
\rightarrow & \Delta \dot{V}_{ACC}
\end{align*}
\]

KA2 LOAD RELIEF LOOP

\[
\begin{align*}
\text{QD030 Block N} & \\
\left[ K_{A2} \right] & \frac{1}{(1+A_5 s)(1+A_6 s)(1+A_7 s)(1+A_8 s)} \\
\rightarrow & \Delta \dot{V}_{ACC}
\end{align*}
\]
The COBDE namelist name called LKD specifies in Hollerith form, the QD030 location of the parameter $K_D$. For Figure 7.1, the user would input

$$LKD = \text{SHRI(7)}$$

Before continuing with the definition of the parameters in Figure 7.1, the COBDE namelist names associated with the remaining load relief control law parameters are given in Table 7.1.
### TABLE 7.1
COBDE NAMELIST NAMES FOR THE LOAD RELIEF PARAMETERS

<table>
<thead>
<tr>
<th>LOAD RELIEF CONTROL LAW PARAMETER (EQUATION 7.1)</th>
<th>COBDE NAMELIST NAME ASSOCIATED WITH ITS QD030 LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_D$</td>
<td>LKD</td>
</tr>
<tr>
<td>$K_{R1}$</td>
<td>LKR1</td>
</tr>
<tr>
<td>$K_{R2}$</td>
<td>LKR2</td>
</tr>
<tr>
<td>$K_{DA}$</td>
<td>LKDA</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>LT1</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>LT2</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>LT3</td>
</tr>
<tr>
<td>$K_{A1}$</td>
<td>LKA1</td>
</tr>
<tr>
<td>$A_1$</td>
<td>LA1</td>
</tr>
<tr>
<td>$A_2$</td>
<td>LA2</td>
</tr>
<tr>
<td>$A_3$</td>
<td>LA3</td>
</tr>
<tr>
<td>$K_{A2}$</td>
<td>LKA2</td>
</tr>
<tr>
<td>$A_4$</td>
<td>LA4</td>
</tr>
<tr>
<td>$A_5$</td>
<td>LA5</td>
</tr>
<tr>
<td>$A_6$</td>
<td>LA6</td>
</tr>
<tr>
<td>$A_7$</td>
<td>LA7</td>
</tr>
<tr>
<td>$A_8$</td>
<td>LA8</td>
</tr>
</tbody>
</table>
Table 7.2 will now summarize how the user will equate the autopilot parameters in Figure 7.1 with the parameters in the load relief (and drift minimum) control law of Equation 7.1. The following assumptions are made in forming Table 7.2: (1) The filter networks denoted as \( F_7, F_9 \) and \( F_{11} \) are assumed to be first order filters and hence need only one parameter (namely, the time constant) to define them; and (2) the filter networks denoted as \( F_8, F_{10}, \) and \( F_{12} \) are assumed to appear as the last elements in the denominators of their respective blocks.

For this example, the time constant \( A_4 \) was not used and hence need not be declared. \( A_4 \) is needed when the KA1 and the KA2 feedback loops share the same filters. To illustrate this condition, suppose the QD030 blocks M and N of Figure 7.1 are replaced with QD030 blocks H, M and N of Figure 7.2. Table 7.3 summarizes how the user equates the parameters of Figure 7.2 with the parameters of Equation 7.1. In Table 7.3, it is assumed that \( F_9 \) is a first order filter, and \( F_{10} \) appears as the last element in the denominator of Block H.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKD</td>
<td>5HRJ(7)</td>
</tr>
<tr>
<td>LKR1</td>
<td>5HRK(7)</td>
</tr>
<tr>
<td>LKR2</td>
<td>5HRK(7)</td>
</tr>
<tr>
<td>LKDA</td>
<td>5HRL(7)</td>
</tr>
<tr>
<td>LT1</td>
<td>5HRL(9)</td>
</tr>
<tr>
<td>LT2</td>
<td>6HRL(10)</td>
</tr>
<tr>
<td>LT3</td>
<td>6HRL(11)</td>
</tr>
<tr>
<td>LKA1</td>
<td>5HRM(7)</td>
</tr>
<tr>
<td>LA1</td>
<td>5HRM(9)</td>
</tr>
<tr>
<td>LA2</td>
<td>6HRM(10)</td>
</tr>
<tr>
<td>LA3</td>
<td>6HRM(11)</td>
</tr>
<tr>
<td>LKA2</td>
<td>5HRN(7)</td>
</tr>
<tr>
<td>LA4</td>
<td>is not used and hence is undeclared</td>
</tr>
<tr>
<td>LA5</td>
<td>5HRN(9)</td>
</tr>
<tr>
<td>LA6</td>
<td>6HRN(10)</td>
</tr>
<tr>
<td>LA7</td>
<td>6HRN(11)</td>
</tr>
<tr>
<td>LA8</td>
<td>6HRN(12)</td>
</tr>
</tbody>
</table>
where, using the notation of Figure 7.1:

\[
\begin{align*}
K_L &= K_{A1} + K_{A2} \\
A_4 &= (K_{A1}A_5) / K_L
\end{align*}
\]
### TABLE 7.3

**LOAD RELIEF AUTOPILOT EXAMPLE #2**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKA1</td>
<td>not used and hence not declared</td>
</tr>
<tr>
<td>IA1</td>
<td></td>
</tr>
<tr>
<td>IA2</td>
<td></td>
</tr>
<tr>
<td>IA3</td>
<td></td>
</tr>
<tr>
<td>LKA2</td>
<td>5HRM(7)</td>
</tr>
<tr>
<td>LA4</td>
<td>5HRM(8)</td>
</tr>
<tr>
<td>LA5</td>
<td>5HRM(9)</td>
</tr>
<tr>
<td>LA6</td>
<td>5HRH(9)</td>
</tr>
<tr>
<td>LA7</td>
<td>6HRH(10)</td>
</tr>
<tr>
<td>LA8</td>
<td>6HRH(11)</td>
</tr>
</tbody>
</table>
The user must exercise care when specifying the autopilot configuration of Figure 7.2, since the combining of the $K_{A1}$ and $K_{A2}$ loops into a single channel may result in undesirable load relief optimization by COEBRA. This is explained as follows. The $K_{A2}$ feedback loop is a pseudo-integration loop on the accelerometer signal. That is,

$$K_{A2} \frac{1}{(1 + A_5 S)} = (K_v \gamma) \frac{1}{(1 + \gamma S)}$$

where $\gamma$ is typically a 7.5 sec. time constant, and $K_v$ is the loop gain. Since $K_L = K_{A1} + K_{A2} \equiv K_a + K_v \gamma$

and $\Delta 4 \equiv \gamma_L = (K_{A1}) (A_5) / K_L \equiv K_a \gamma / K_L$

then if the autopilot parameters $K_L$ and $\gamma_L$ are disturbed by a factor of $P$, the perturbed equations are (assuming that $\gamma$ of the pseudo integration circuit is a constant):

$$(K_L + P K_L) = (K_a + \Delta K_a) + (K_v + \Delta K_v) \gamma$$

so that

$$\Delta K_a = P K_L - (\Delta K_v) \gamma$$

and

$$(\gamma_L + P \gamma_L) = \frac{(K_a + \Delta K_a) \gamma}{K_L + \Delta K_a + (\Delta K_v) \gamma}$$

which gives

$$\Delta K_v = \frac{P}{\gamma} \left[ K_v \gamma - K_a (1 + P) \right]$$

$$\Delta K_a = P (2 + P) K_a$$

We see that $K_a$ increases by more than $P$ and $K_v$ can effectively increase or decrease depending on the nominal or initial values of $K_a$ and $K_v \gamma$. 
As a final note, each of the parameters in Equation 7.1 (or Table 7.1) can be designated as a common or stand-alone variable, or as a constant. This "type designation" is done by the NTYPE(j) array which declares the parameters in the order in which they appear in QD030 blocks A through N. Obviously, not all of the parameters in Equation 7.1 need be used. Also, the time constants in Equation 7.1 need not be the first elements in the numerators and denominators of the respective QD030 blocks.

**Section 7.3 The ROOTCH Subroutine**

There is a subroutine in COEBRA called ROOTCH that allows the filters to change planes. For example, a quadratic filter can become 2 real break filters, if COEBRA seeks to drive its damping ratio greater than unity. Also, 2 real break filters can become a quadratic if COEBRA drives them toward one another. When using ROOTCH (by setting the flag KROOT = 1), the allowed autopilot configuration is not as general as it is when not using ROOTCH. When using ROOTCH, filters should only be designated as commons or constants (except for the first time point when commons are entered as stand-alones). Also, the commons must be entered before the constants.

Obviously, when a filter changes planes, COEBRA adjusts the XMAX(j) and XMIN(j) arrays accordingly. As discussed in Chapter 5, the user inputs to COEBRA the minimum (XMIN(j)) and maximum (XMAX(j)) values that are allowed for each stand-alone autopilot variable. When a quadratic filter becomes 2 real break filters, COEBRA will use for the min-max values of the real-filter time constants, the inverted min-max values of the quadratic filter break frequency. When 2 real filters become complex, the inverted min-max values of the time constants are applied to the frequency of the quadratic filter. The min-max values for the damping ratio (\( \zeta \)) of the quadratic filter will be set to 1.0E-4 and 0.9999 respectively.
CHAPTER 8. THE DRIFT MINIMUM CONSTRAINT EQUATIONS

The purpose of this chapter is to (1) illustrate the derivation of the so-called "Drift Minimum" principle and (2) show in detail how COEBRA constrains the autopilot gains and filters to meet this Drift Minimum criteria.

Section 8.1

Hoelker [5] and Greensite [3] derive the Drift Minimum Principle using pitch plane notation. This section will outline this derivation using yaw plane notation. The five linearized or perturbated yaw plane equations are as follows:

1. The equation for the linear acceleration of the center of gravity of the vehicle perpendicular to the trajectory or standard path:
   \[ \ddot{Y} = (g\sin\Theta_L) \psi_b + (Y_b)\dot{\theta} + (Y_s)\delta \]

2. The moment equation about the vehicle's center of gravity:
   \[ \dot{\psi}_b = (N_\theta)\dot{\theta} + (N_\delta)\delta \]

3. The kinematic equation relating \(\dot{\psi}\) and \(\psi_b\) to the inertial angle(\(\psi/V_I\)) and the angle of sideslip of the wind (\(\beta_W\)):
   \[ \beta = -\psi_b + (\dot{Y}/V_I) + \beta_W \]

4. The equation for the output of a lateral body-mounted accelerometer located a distance \(L_A\) forward of the vehicle's center of gravity:
   \[ \dot{V}(X) = (Y)\beta + (L_A)\dot{\psi}_b + (Y_s)\delta \]

5. The equation defining the autopilot control law:
   \[ \delta = K_{TVC} \left[ K_D(\psi_c - \psi_b) - K_R\dot{\psi}_b + K_{\psi}\ddot{\psi}_b - (K_\alpha + \frac{K_v\gamma}{1 + \zeta_s}) \dot{V}(X) \right] \]

Note that in Section 7.2 of Chapter 7, the following notation was used:

\[ K_{DA} \equiv K_{\dot{\psi}} \]
\[ K_{A1} \equiv K_{\alpha} \]
\[ K_{A2} \equiv K_v \gamma \]
\[ \zeta_5 \equiv \gamma \]
In this equation, all of the gains have positive values.

As shown by Hoelker [5], these five equations can be manipulated into the following form:

\[ (K_4)\dddot{\gamma} + \dddot{\gamma} = (K_3)\dddot{\psi}_b + (K_2)\ddot{\psi}_b + (K_1)\dot{\psi}_b + (K_0)\psi_b \]

(Equation 8.1)

where \( K_0, K_1, K_2, K_3, \) and \( K_4 \) are constants and functions of the airframe and autopilot parameters.

Equation 8.1 can be written:

\[ \dddot{\gamma} = \frac{K_3}{1+K_4s} \dddot{\psi}_b + \frac{K_2}{1+K_4s} \ddot{\psi}_b + \frac{K_1}{1+K_4s} \dot{\psi}_b + \frac{K_0}{1+K_4s} \psi_b \]

(Equation 8.2)

Now the response of the vehicle to any disturbance can be thought of as being made up of two kinds of motion, rotation and translation. The frequency of the "rotational mode" is almost always five to ten times higher than the frequency region of the "translational modes". Hence, assuming that changes in the wind velocity and in the lateral linear velocity (\( Y \)) are negligible during a cycle of the rotational mode, and assuming that the rotational mode is relatively well damped, a so-called quasi-steady state condition will be reached soon after any disturbance. During this quasi-steady state period, \( \dddot{\psi}_b, \ddot{\psi}_b, \) and \( \dot{\psi}_b \) will be negligible, and Equation 8.2 then becomes:

\[ \dddot{\gamma} = \frac{K_0}{1+K_4s} \psi_b \]

If \( K_0 = 0 \), then \( \dddot{\gamma} \) will also equal zero during this quasi-steady state period. The condition, \( K_0 = 0 \), is referred to as the Drift Minimum condition since the vehicle theoretically will not be accelerating away from the trajectory following a wind disturbance. Writing \( K_0 \) in terms of airframe and autopilot parameters, the Drift Minimum condition is:
In equation 8.3, $A = Y_N N_s - N_q Y_s$

Equation 8.3 is a linear relationship between $K_D$, $K_a$, and the gain referred to as $(K_v \gamma)$. Note that this linear relationship is independent of the magnitude of the wind velocity. The Drift Minimum phenomenon can be looked on as a condition in which $\Psi_b \theta$, and $\delta$ are blended or combined via the autopilot feedback loops in such a way that the forces normal to the trajectory sum to zero. In other words, the forces due to gravity, aerodynamics and control, cancel one another when summed normal to the trajectory.

In general, the characteristic equation ($\Delta$) of this system factors as follows:

$$\Delta = (s^2 + 2 \Psi \omega s + \omega^2) (s + \omega_1) (s + \omega_2)$$

The quadratic pair corresponds to the so-called dominant rigid-body closed-loop roots (the dutch-roll mode), and $\omega_1$ and $\omega_2$ are usually real roots that lie near the origin. Even though all four roots contribute to both rotational and translational motion, usually the quadratic roots of the dutch roll mode represent most of the rotary motion, and as shown by Hoelker [5], the roots given by $\omega_1$ and $\omega_2$ represent most of the path motion. This being the case, $\Delta$ can be approximated by the following polynomial:

$$\Delta \approx s^4 + (2 \Psi \omega) s^3 + (\omega^2) s^2 + (\omega^2 (\omega_1 \omega_2)) s + \omega^2 \omega_1 \omega_2$$

As might be expected, $K_0$ is related to the constant term $(\omega^2 \omega_1 \omega_2)$.

In fact,

$$\frac{K_0}{\gamma} \approx \frac{\omega^2}{\omega_1 \omega_2}$$

(Equation 8.4)
Hence, setting $K_0$ to zero is the same as putting one of the roots that represents path motion right at the origin. This discussion leads to another insight into the drift minimum principle. When $K_0 = 0$, let's refer to the root that goes to the origin as $\omega_2$. Hoelker [5] shows that if $\omega_2$ is in the right half plane (unstable), the vehicle will accelerate into the wind. If $\omega_2$ is in the left half plane (stable), the vehicle will accelerate with the wind.

Section 8.2

This section will show in detail how COEBRA constrains the autopilot gains and filters to meet the Drift Minimum criteria.

Section 8.2.1

This subsection contains the derivation of COEBRA's Drift Minimum Constraint Equations. As shown by Equation 8.3, the Drift Minimum criterion is:

$$K_0 = a(K_D) + b(K_a) + b(K_v \gamma) + c = 0$$

where $a$, $b$ and $c$ are constants and functions of the airframe parameters, and the gains ($K_D$, $K_a$, and $K_v \gamma$) all have positive values. Now, at the beginning of each iteration of COEBRA, $K_D = K_{D0}$

$$K_a = K_{a0}$$

and, $(K_v \gamma) = (K_v \gamma)_0$

Hence, $(K_0) = a K_{D0} + b K_{a0} + b (K_v \gamma)_0 + c$

Now, expanding $K_0$ in a first order Taylor Series, it is desired that

$$K_0 + \Delta K_0 = 0 + \epsilon$$

where

$$\Delta K_0 = \frac{\partial K_0}{\partial K_D} \Delta K_D + \frac{\partial K_0}{\partial K_a} \Delta K_a + \frac{\partial K_0}{\partial (K_v \gamma)} \Delta (K_v \gamma)$$

$$= a(K_D - K_{D0}) + b (K_a - K_{a0}) + b ((K_v \gamma) - (K_v \gamma)_0)$$
and $\varepsilon$ is some tolerance allowed on the Drift Minimum Condition ($\varepsilon$ is input by the user).

At the beginning of any COEABRA iteration, there are 3 possible conditions on $K_0$.

**Condition 1:**

The drift minimum criterion is already met and

$$-\varepsilon \leq K_0 \leq +\varepsilon$$

**Condition 2:**

The drift minimum criterion is not met and

$$-\varepsilon \leq K_0 > +\varepsilon$$

**Condition 3:**

The drift minimum criterion is not met and

$$-\varepsilon > K_0 \leq +\varepsilon$$

For each condition on $K_0$, COEABRA will put 2 constraint equations on Drift Minimum.

**Condition 1:** If

$$-\varepsilon \leq K_0 \leq +\varepsilon$$

the 2 drift minimum constraint equations will be:

**Eq. 1**

$$K_0 + \Delta K_0 \leq +\varepsilon$$

or

$$aK_D + bK_d + b(K_v \zeta) \leq +\varepsilon - \varepsilon$$

**Eq. 2**

$$K_0 + \Delta K_0 \geq -\varepsilon$$

or

$$aK_D + bK_d + b(K_v \zeta) \geq -\varepsilon - \varepsilon$$
Condition 2: If \(-\varepsilon \leq K_o > +\varepsilon\) 

the 2 drift minimum constraint equations will be:

Eq. 1 \[ K_o + \Delta K_o \leq K_o + \text{STEP} \cdot (\varepsilon - K_o) \]

or \[ aK_D + bK_a + b(K_V Z) \leq K_o + \text{STEP} \cdot (\varepsilon - K_o) - C \]

Eq. 2 \[ K_o + \Delta K_o \geq -\varepsilon \]

or \[ aK_D + bK_a + b(K_V Z) \geq -\varepsilon - C \]

NOTE: STEP is a COBIN namelist variable that is defined in Chapter 2 and again in Chapter 10. STEP specifies the percent "improvement" required in the "not-met" constraint equations on each COEBRA iteration. Recall that STEP is adjusted in the so-called Inner Loop.

Condition 3: If \(-\varepsilon > K_o \leq \varepsilon\) 

the 2 drift minimum constraint equations will be:

Eq. 1 \[ K_o + \Delta K_o \leq + \varepsilon \]

or \[ aK_D + bK_a + b(K_V Z) \leq + \varepsilon - C \]

Eq. 2 \[ K_o + \Delta K_o \geq K_o + \text{STEP} \cdot (-\varepsilon - K_o) \]

or \[ aK_D + bK_a + b(K_V Z) \geq K_o + \text{STEP} \cdot (-\varepsilon - K_o) - C \]
8.7

Section 8.2.2

This subsection defines how COEBRA calculates the airframe/actuator coefficients in the Drift Minimum Constraint equations. These coefficients, denoted in Section 8.2.1 as a, b, and c, are calculated from the same COBDE namelist data that COEBRA uses to calculate $\theta_p$, $\psi_p$, and $\phi_p$ for the Load Relief Cost Function. In keeping with the philosophy used for the Load Relief Cost Function, all of the airframe and actuator parameters in the COBDE namelist that are used to calculate a, b, and c, are input with positive values for an aerodynamically unstable vehicle. The reason for defining the sign convention in this way is to avoid confusion between pitch and yaw. When running pitch, the user must use yaw plane notation and the indicated sign convention (i.e., all data positive for unstable vehicle). If the vehicle is stable (i.e., center of pressure aft of the center of gravity), the only parameter that should change is $N_C$ (yawing aero moment due to $\beta$), and it should then be input as a negative number.

Using yaw plane notation and the sign convention that all data is positive for an unstable vehicle:

$$a = \left(\frac{K_{TVC}}{B}\right) \left(N_C Y_{\delta_r} + Y_C N_{\delta_r}\right)$$
$$b = -\left(\frac{K_{TVC}}{B}\right) \left(N_C Y_{\delta_r} + Y_C N_{\delta_r}\right) (g \sin \theta_L)$$
$$c = \left(\frac{1}{B}\right) \left(N_C\right) (g \sin \theta_L)$$

Table 8.1 lists the COBDE namelist variables that are required to calculate a, b, and c. Chapter 10 which summarizes all of the variables that are used in the COBDE namelist, contains the detailed definitions of each
TABLE 8.1: COBDE NAMELIST VARIABLES THAT ARE REQUIRED FOR THE DRIFT MINIMUM CONSTRAINTS

<table>
<thead>
<tr>
<th>COBDE NAMELIST</th>
<th>YAW PLANE SYMBOL</th>
<th>PITCH PLANE EQUIVALENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNB</td>
<td>$C_n \phi$</td>
<td>$C_m \alpha$</td>
</tr>
<tr>
<td>CYB</td>
<td>$C_y \phi$</td>
<td>$C_z \alpha$</td>
</tr>
<tr>
<td>CNDR$^1$</td>
<td>$C_{n\delta r}$</td>
<td>$C_m \delta e$</td>
</tr>
<tr>
<td>CYDR$^1$</td>
<td>$C_{y\delta r}$</td>
<td>$C_z \delta e$</td>
</tr>
<tr>
<td>QBAR$^2,1$</td>
<td>$\bar{b}$</td>
<td>$\bar{b}$</td>
</tr>
<tr>
<td>RHO$^2,1$</td>
<td>$\rho$</td>
<td>$\rho$</td>
</tr>
<tr>
<td>SF</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>D</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>TY$^1,1$</td>
<td>$T_y$</td>
<td>$T_p$</td>
</tr>
<tr>
<td>LGY$^1$</td>
<td>$L_{gy}$</td>
<td>$L_{gy}$</td>
</tr>
<tr>
<td>IZZ</td>
<td>$I_{zz}$</td>
<td>$L_y y$</td>
</tr>
<tr>
<td>MASS</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>G</td>
<td>g</td>
<td>g</td>
</tr>
<tr>
<td>THETA</td>
<td>$\Theta_L$</td>
<td>$\Theta_L$</td>
</tr>
<tr>
<td>B</td>
<td>$V_{rw}$</td>
<td>$V_{rw}/U_{r}$</td>
</tr>
<tr>
<td>KTVC</td>
<td>KTVC</td>
<td>KTVC</td>
</tr>
</tbody>
</table>

1. COEBRA assumes a single control torque source, either TVC or an aero surface. Hence, CNDR and CYDR must be zero if TVC is used, and TY and LGY must be zero if aero surfaces are used.

2. If QBAR is input as zero, COEBRA calculates QBAR from $(0.5 \times \text{RHO} \times \text{B} \times \text{B})$. 
of these parameters. Table 8.1 merely lists these parameters and shows the yaw plane symbol usually used for each as well as the pitch plane equivalent. Table 8.2 illustrates how COEBRA combines these variables to calculate $a$, $b$, and $c$. 
### TABLE 8.2: CALCULATING THE DRIFT MINIMUM COEFFICIENTS

<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>COEBRA NOTATION IN COBDE NAMELIST</th>
<th>PITCH PLANE EQUIVALENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{a}^{2}$</td>
<td>$\frac{q_{b} c_{n_{a}}}{I_{ZZ}}$</td>
<td>$\frac{QBAR<em>SF</em>D*CNB}{IZZ}$</td>
</tr>
<tr>
<td>$Y_{a}$</td>
<td>$\frac{q_{b} S_{C_{y}a}}{M}$</td>
<td>$\frac{QBAR<em>SF</em>CYB}{MASS}$</td>
</tr>
<tr>
<td>$N_{s_{r}}$ from an aero surface</td>
<td>$\frac{q_{b} S_{bC_{n_{s_{r}}}}}{I_{ZZ}}$</td>
<td>$\frac{QBAR<em>SF</em>D*CNDR}{IZZ}$</td>
</tr>
<tr>
<td>$N_{s_{r}}$ from TVC</td>
<td>$T_{Y}*L_{GY}$</td>
<td>$T_{Y}*L_{GY}$</td>
</tr>
<tr>
<td>$Y_{s_{r}}$ from an aero surface</td>
<td>$\frac{q_{b} S_{C_{y}a}}{M}$</td>
<td>$\frac{QBAR<em>SF</em>CYDR}{MASS}$</td>
</tr>
<tr>
<td>$Y_{s_{r}}$ from TVC</td>
<td>$T_{Y}$</td>
<td>$T_{Y}$</td>
</tr>
</tbody>
</table>

1. By definition, all of these parameters are positive for an aerodynamically unstable vehicle.

2. Note that $N_{a}$ and $N_{s_{r}}$ are unprimed aero stability coefficients.
Section 8.2.3

The purpose of this subsection is to discuss the parameter called $\varepsilon$. This parameter is input by the user, and is a tolerance allowed on the drift minimum condition. In order to define the units on $\varepsilon$, Equation 8.4 is repeated:

$$\frac{K_0}{\ell} = \omega^2 \omega_1 \omega_2$$  \hspace{1cm} \text{(Equation 8.4)}

Now, $\omega^2$ is usually around unity in magnitude, and $\omega_1 \omega_2 = (1/\tau)$ where $\tau$ is the time constant of the pseudo-integrator in the $(K_0\ell)$ feedback loop. Hence, Equation 8.4 becomes

$$\frac{K_0}{\ell} = \frac{\omega^2}{\ell}$$

and $K_0$ is seen to be approximately equal to the frequency of the real root that is right at the origin for "perfect" drift minimum. Since it is desired that

$$-\varepsilon \leq K_0 \leq \varepsilon + \varepsilon$$

then $\varepsilon$ is a tolerance on the frequency of the "drift minimum" root. For example, if $\varepsilon = .1$, this requires that the root be within $\pm .1$ rad/sec of the origin. This allowed tolerance ($\varepsilon$) is input by the variable called "DRFTOL" in the COBIN namelist. "DRFTOL" is applicable to all the time points.
Section 8.2.4

The purpose of this subsection is simply to state that COEBRA identifies the autopilot parameters \((K_D, K_a, \text{and the gain } K_v \mathcal{Z})\) in the drift minimum constraint equations using the same method that is required for "load relief optimization". This method is explained in detail in Chapter 7, and briefly repeated here.

1. The COBDE namelist name corresponding to \(K_D\) is LKD.
   
   LKD = (location in Hollerith of \(K_D\) in the QD030 block diagram). For example, if \(K_D\) is the gain in Block J, then LKD = 5HRJ(7).

2. \(K_a\) is identified by LKA1 which is set equal to the location of \(K_a\) in the QD030 block diagram.

3. \((K_v \mathcal{Z})\) is similarly identified via LKA2.

Further, \(K_D\), \(K_a\) and \((K_v \mathcal{Z})\) must individually be declared either:

1. A constant;
2. A "stand-alone" variable;
3. A "common" variable; or

4. They may individually have a value of zero, in which case, they will not be declared, and will not even appear in the QD030 Block Diagram. As an example, \(K_a\) may be a stand-alone variable, \(K_D\) may be a constant, and \((K_v \mathcal{Z})\) may not even be used.

Finally, note that \(K_D\), \(K_a\) and \((K_v \mathcal{Z})\) all have positive values when used in the Drift Minimum Constraint Equations.
Section 8.2.5

Additional Notes:

**Note 1.** The user will select those time points at which the Drift Minimum Constraints are to be applied. He will do this via the COBDE namelist variable called "MINDRF".

If MINDRF = 0, the drift minimum constraints are not to be used at this time point.

If MINDRF = 1, the drift minimum constraints are to be used at this time point.

**Note 2.** "Drift Minimum" and "Load Relief Optimization" are not linked together by COEBRA. In other words, the user may ask for the drift minimum constraints at any or all of the time points, completely independent of whether LOADOP = 0 (no load relief optimization) or LOADOP = 1 (load relief optimization).

**Note 3.** The Drift Minimum Constraints can be used whether or not the airframe includes structural bending and/or fuel slosh modes.

**Note 4.** The Drift Minimum option allows COEBRA to design a "drift minimum" autopilot that meets the stability margin requirements, and if desired, at the same time has the maximum amount of load relief capability. Note that a "high frequency" drift minimum autopilot yields the "least" residual drift rate and hence least trajectory drift. A "high load relief" drift minimum autopilot results in larger drift rates and hence more drift.
9.1 Introduction

The purpose of the so-called Vector Constraints is to keep the individual autopilot vectors (e.g., the attitude error vector, the rate vector, the accelerometer loop vector, etc.) from getting very much larger than the total resultant vector at all frequencies. When the individual vectors do get much larger than the resultant, this means that vector cancellation exists, and this can lead to problems when tolerances on the airframe/autopilot parameters are considered.

Figure 9.1 illustrates the situation that might exist at the resonant frequency or frequency at the peak of the 1st structural bending mode. The figure shows that the resultant (peak of the 1st structural bending mode) is comprised of the following vectors or feedback loops: (1) attitude error; (2) rate; and (3) accelerometer. Referring to Figure 6.1 and Table 6.1 on the QD030 block diagram, suppose that the attitude error feedback loop was defined by the Case 7 open-loop transfer function, the rate feedback loop by the Case 8 transfer function, and the accelerometer feedback loop by the Case 9 transfer function. Then, Case 7 + Case 8 + Case 9 = Case 13 (Resultant). Now, it is desired to constrain the magnitude of each vector at the frequency of the 1st structural bending mode peak. For example, it is desired that:

| Case 7 | ≤ C* | Case 13 |
| Case 8 | ≤ C* | Case 13 |
| Case 9 | ≤ C* | Case 13 |

where C is an arbitrary constant.
Figure 9.1  ILLUSTRATING THE VECTOR CONSTRAINTS
9.2 The Vector Constraint Equations

At each frequency, the resultant is comprised of \( K \) vectors, \( V(k) \). In turn, each vector is comprised of \( L \) autopilot variables, \( X(\ell) \). Expanding each \( V(k) \) in a first order Taylor Series about \( V(k)_{\text{nom}} \) (nominal value of \( V(k) \) evaluated at the \( i \)th frequency), it is desired that,

\[
\left| V(k)_{\text{nom}} \right| + \sum_{\ell=1}^{L} \left[ \frac{\partial |V(k)|}{\partial X(\ell)} \right]_{0} \Delta X(\ell) \leq V(k)_{s} \quad \text{(for} \; k = 1, \ldots, K) \]

If the requirement on \( V(k) \) is already satisfied, then

\[
V(k)_{s} = \text{SPEC}(k)
\]

If not, then

\[
V(k)_{s} = \left| V(k)_{\text{nom}} \right| + \text{STEP}^{*} \left[ \text{SPEC}(k) - \left| V(k)_{\text{nom}} \right| \right]
\]

Where, at the user's option:

(1) \( \text{SPEC}(k) = \text{PERVEC}(k)^{*} \) \quad \text{Case 13}

or

(2) \( \text{SPEC}(k) = \text{AMPVEC}(k) \)

where, \( \text{PERVEC} \) and \( \text{AMPVEC} \) are input parameters.

9.3 Vector Constraint Input Parameters

This section defines the four input parameters that deal with the so-called vector constraints. \( \text{COEBRA} \) allows for a total of 18 vector constraints per time point. This is because the user can request that vector constraints be applied to a maximum of 3 elements of the Margin Array, and because each element of the Margin Array can be comprised of a maximum of 6 vectors or feedback loops.
The following 4 input parameters are input via the COBDE namelist.

1) **MARVEC(k)**, Integer array (3 dimensional), preset value = 3*0

   This array specifies which elements of the Margin Array are to have the so-called vector constraints. A maximum of three margins per time point can be constrained in this way, and any of the first 47 elements of the Margin Array is a valid choice (except, of course, the structural mode peak phases). For example, if MARVEC = 4, 14, 28, then vector constraints are to be applied to the peaks of the 1st, 2nd and 3rd structural bending modes.

2) **KVEC(k)**, Integer array (6 dimensional), preset value = 6*0

   This array specifies which of the six QD030 feedback loops are to be constrained for the margins flagged by MARVEC. These six QD030 feedback loops are identified by the NCASE option (of the INDATA namelist) for NCASE = 7 through 12. For example, if all the QD030 feedback loops are to be constrained, then KVEC = 7, 8, 9, 10, 11, 12.

3) **AMPVEC(k)**, Floating point array (3 dimensional), preset value = 3*0

   This is "vector constraint" option #1. At the frequency of each margin specified by MARVEC, AMPVEC(k) is the maximum absolute real value that any QD030 feedback vector (specified by KVEC) may have. There is one value of AMPVEC for each margin called out in MARVEC. For example, if AMPVEC(2) = .5, then each of the QD030 feedback vectors that are called out by KVEC, will be constrained to an amplitude of less than or equal to 0.5, when evaluated at the frequency of the 2nd element of the MARVEC array.

4) **PERVEC(k)**, Floating point array (3 dimensional), preset value = 3*0

   This is "vector constraint" option #2. At the frequency of each margin specified by MARVEC, none of the magnitudes of the QD030 feedback vectors (specified by KVEC) may exceed the product:

   \[ \text{PERVEC}(k) \times (\text{magnitude of the } k^{\text{th}} \text{ element of MARVEC}) \]
This option allows the user to constrain each feedback vector to have a magnitude less than or equal to some percentage of the magnitude of the total or resultant vector. For example, suppose $\text{MARVEC}(3) = 14$, and suppose that element #14 (peak of the 2nd structural bending mode) has a value of 0.4. If $\text{PERVEC}(3) = 1.5$, then, at the frequency of Margin array element #14, the magnitude of each QD030 feedback vector that is called out by KVEC, will be constrained to be less than or equal to $(1.5) \times (0.4)$ or 0.6.
Chapter 10. COEBRA Input Data Summary

This chapter summarizes and defines the data that is input to COEBRA via the COBDE and COBIN namelists. The COBDE namelist is required for each time point or vehicle state, and the COBIN namelist is input once and is applicable to all time points.

Note that the data in these namelists may appear anywhere in columns 2 through 80. Also, the preset values indicated for these variables are to be overridden by the user if he so desires.

Part A of this chapter defines the COBDE namelist variables. For convenience, the following is a list of these variables, along with the aspect of COEBRA each is concerned with.

(1) Section 1, The Margin Array
   1.1 IOMEGA
   1.2 IDMODE
   1.3 KAPCH
   1.4 MARVEC
   1.5 KVEC
   1.6 AMPVEC
   1.7 PERVEC

(2) Section 2, The Cost Functions
   2.1 WTFAC

(3) Section 3, The Autopilot Parameters
   3.1 NTOTAL
   3.2 NTYPE

(4) Section 4, The Figures-of-merit
   WTMARG
Section 5, Load Relief Optimization and Drift Minimum Constraints

5.1 The airframe/actuator data that is needed for uncoupled yaw.

- CNB
- CYB
- CNDR
- CYDR
- QBAR
- RHO
- SF
- D
- TY

The additional airframe/autopilot data needed for yaw/roll coupling.

- CLB
- CLDR
- CLDA
- CNDA
- TR
- LGR
- ZOFF

5.2 Autopilot Parameters

5.2.1 LKD LKDA LKA1 LKA2
    LKR1 LT1 LA1 LA4
    LKR2 LT2 LA2 LA5
    LT3 LA3 LA6 LA7
    LA8

5.2.2 TS
(5) Section 5 - (Continued)

5.3 Wind and Time Search Parameters

- U1
- U2
- U3
- U4
- TW1
- TW2
- TW3
- TSTART
- TSTOP
- DELT
- DINCRE

5.4 Miscellaneous Parameters

- MINDRF
- LRCOST
- ROOTOL
Part A, Section 1 - The seven COBDE namelist variables that are used when forming the so-called Margin Array (Chapter 1) are discussed in this section.

1.1 IOMEGA, Integer, preset value = 0

- IOMEGA = 0, if the rigid-body phase margin frequency requirement is not to be used.
- IOMEGA = 1, if the requirement is to be used.

1.2 IDMODE (k), Integer array (8 dimensional), preset value = 1, 2, 3, 4, 5, 6, 7, 8, corresponding to i = 1 to 8

Referring to the underdamped quadratic roots in the denominator of the total open loop transfer function (CASE 13), any root-pair whose damping ratio is less than DAMP (COBIN namelist variable) is considered to be from a structural bending or fuel slosh mode. COEBRA lists these bending and fuel slosh modes according to frequency, beginning with the lowest frequency. The IDMODE (k) array is then used to identify these modes according to this ordered list, beginning with the lowest frequency.

- IDMODE (k) = 1, if the k\(^{th}\) mode is a first structural mode.
- IDMODE (k) = 2, if the k\(^{th}\) mode is a second structural mode.
- etc., until
- IDMODE (k) = 8, if the k\(^{th}\) mode is an eighth structural mode.
- IDMODE (k) = 10 or higher, if the k\(^{th}\) mode is a fuel slosh mode.
- IDMODE (k) = 0, if the k\(^{th}\) mode is to be ignored.

NOTE: COEBRA may be run with or without slosh and/or bending modes. When running with modes, the modes need not be ordered according to frequency; e.g., the 1st mode may be higher in frequency than the 3rd mode, or the 4th mode may be lower in frequency than the slosh modes, etc. The exception to this is the
NOTE: (Continued)

2nd mode, which must be run with something identified as a 1st mode that is lower in frequency and something identified as a higher mode that is higher in frequency.

1.3 KAPCH, Integer, preset value = 0

This option is applicable to all first and second structural modes. When KAPCH = 0 and the modal peak gain is greater than zero db, the closest-approach margin of the mode will not be constrained. The mode will be constrained by its peak gain, peak phase, and front and backside phase margins.

When KAPCH = 1 and the modal peak gain is greater than zero db, the closest-approach margin of the mode will be constrained, along with its peak gain, peak phase, and front and backside phase margins. When the mode peaks below zero db, the closest-approach margin will always be constrained. For this case, the mode peak will not be constrained, but the peak phase can still enter the cost function at the user's option.

1.4 MARVEC (k), Integer array (3 dimensional), preset value = 3*0

This array specifies which elements of the Margin Array are to have the so-called vector constraints (Chapter 9). A maximum of three margins can be constrained in this way, and any of the first 47 elements of the Margin Array is a valid choice (except, of course, the structural mode peak phases). For example, if MARVEC = 4, 14, 28, then vector constraints are to be applied to the peaks of the 1st, 2nd and 3rd structural bending modes.
1.5 KVEC (k), Integer array (6 dimensional), preset value = 6*0

This array is associated with the vector constraints (Chapter 9). It specifies which of the six QD030 feedback loops are to be constrained for the margins flagged by MARVEC (k). These six QD030 feedback loops are identified by the NCASE option (of the INDATA namelist) for NCASE = 7 through 12. If all the QD030 feedback loops contain autopilot variables, and are to be constrained, the KVEC = 7, 8, 9, 10, 11, 12.

1.6 AMPVEC (k), Floating point array (3 dimensional), preset value = 3*0

This is "vector constraint" option #1. At the frequency of each margin specified by MARVEC, AMPVEC (k) is the maximum absolute real value that any QD030 feedback vector (specified by KVEC) may have. There is one value of AMPVEC for each margin called out in MARVEC. For example, if AMPVEC (2) = .5, then each of the QD030 feedback vectors that are called out by KVEC, will be constrained to an amplitude of less than or equal to 0.5, when evaluated at the frequency of the 2nd element of the MARVEC array.

1.7 PERVEC (k), Floating point array (3 dimensional), preset value = 3*0

This is "vector constraint" option #2. At the frequency of each margin specified by MARVEC, none of the magnitudes of the QD030 feedback vectors (specified by KVEC) may exceed the product:

PERVEC (k) * (magnitude of the k-th element of MARVEC)

This option allows the user to constrain each feedback vector to have a magnitude less than or equal to some percentage of the magnitude of the total or resultant vector. For example, suppose MARVEC (3) = 14, and suppose that Margin Array element #14 (peak of the 2nd structural bending mode) has a
value of 0.4. If PERVEC (3) = 1.5, then, at the frequency of Margin
Array element #14, the magnitude of each QD030 feedback vector that is
called out by KVEC, will be constrained to be less than or equal to (1.5) *
(0.4) or 0.6.
Part A, Section 2 - This section is used to define the COBDE namelist variable called WTFAC which is an arbitrary weighting factor in both cost functions: (1) "Maximize margins" in Chapter 3; and (2) "Maximize Load Relief" in Chapter 4.

2.1 WTFAC (i), Floating point array (51 dimensional). Since the index i corresponds to the elements of the Margin Array Table (Chapter 1), the following table lists the preset values for WTFAC (i).

<table>
<thead>
<tr>
<th>Preset values for i=1 to 51</th>
<th>Corresponding Margin Array Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 * 1, 1, 0, 3 * 0, 1, 0, 3 * 0, 1, 0, 5 * 0, 1, 0, 5 * 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1</td>
<td>rigid body margins both first modes both second modes 3rd to 8th mode eighth fuel slosh modes phase margin crossover frequency $\beta_p, \delta_p, \delta_{fp}$</td>
</tr>
</tbody>
</table>

This is an array of arbitrary cost function weighting factors that is input by the user. This array is used for both types of cost functions.

For the "maximize margins" cost function, WTFAC (i) serves as an additional multiplicative weighting factor to the one that is provided by the WTYPE option. The WTYPE option either provides a "unity" or "desired over nominal" type weighting factor. WTFAC (i) allows the user to emphasize or de-emphasize certain elements of the cost function relative to others. This array also serves another function. Since there is a standard preset cost function, WTFAC (i) allows the user to add or delete elements from this preset cost function. In other words, WTFAC (i) also allows the user to "build his own" cost function, if the "built-in" cost function will not solve his problem.
Part A, Section 3 - This section defines the two COBDE namelist variables that are used to define the "parameter-type" that is to be associated with each autopilot parameter.

3.1 NTOTAL, Integer, Preset value - none, problem dependent.

NTOTAL is the total number of parameters (Bode gains, time constants, damping ratios and natural frequencies) that appear in the QD030 blocks A through N for this time point. NOTE: Free S's are not considered as parameters, though they may be used in any of the QD030 blocks.

3.2 NTYPE (j), Integer array (50 dimensional), preset values - none, problem dependent.

This is a compressed array that corresponds to the parameters in the order in which they are input in the QD030 blocks A through N, and identifies their "type." By order is meant, by ordered QD030 block and by ordered element number within each block.

(a) NTYPE (j) = 0 if the $j^{th}$ parameter is a constant whose value is not to be altered by COEBRA.

(b) NTYPE (j) = -1 if the $j^{th}$ parameter is a stand-alone variable whose value, when altered by COEBRA, need not be the same as the parameter that appears in the same QD030 block and element location at the immediately preceeding time point.

(c) NTYPE (j) = 1 if the $j^{th}$ parameter is a common variable whose value, when altered by COEBRA, must be the same as the variable that appears in the same QD030 block and element location at the preceding timepoint.

The following three notes are very important.

(1) For each time point, before reading in any data, COEBRA initializes the QD030 block diagram as follows:
(a) QD030 blocks A through H are initialized with a gain of unity and no roots including free S's;

(b) QD030 blocks I through N are completely zeroed out. After reading the autopilot parameters into the QD030 block diagram (via the INDATA namelist), COEBRA checks to see how blocks A through N have been changed from their initial values. NTOTAL and NTYPE (j) must only include those parameters in those QD030 blocks A through N that have been changed from their initial values. If a variable gain whose initial value is unity, must be used in any of the blocks A through H, this variable gain must be initialized with a value of one-plus-epsilon.

(2) NTOTAL and NTYPE (j) must identify even those constants that lie outside the allowed field as defined in Chapter 7 on the Allowed Autopilot Configuration.

(3) For the first time point entered, only 0's and -1's are to be used in the NTYPE (j) array.
Part A. Section 4

This section is used to define the COBDE namelist parameter WTMARG (i). This parameter is a weighting factor in both of the figures-of-merit (Chapter 5). It is a 51 dimensional integer array. The index i corresponds to the elements of the Margin Array (Chapter 1), and if WTMARG (i) ≠ 0, the i\textsuperscript{th} element of the Margin Array will be included in the figure-of-merit.

For the "maximize margins" figure of merit, only the following elements of the Margin Array can ever be included. All of the other elements are not applicable.

1. The three rigid body margins. (i = 1, 2, and 3).
2. If the 1\textsuperscript{st} and/or 2\textsuperscript{nd} modes are stable, only their "closest approach" margins can be included. (i = 8, 13, 20, and 27).
3. If the 1\textsuperscript{st} and/or 2\textsuperscript{nd} modes are unstable, only their "unstable" front or backside phase margins can be included. (i = 6 or 7, 11 or 12, 16 or 17, and 23 or 24).
4. For the 3\textsuperscript{rd} to the 8\textsuperscript{th} mode, only their peak gains can be included. (i = 28, 30, 32, 34, 36 and 38).
5. For all 8 slosh modes, their backside phase margins can be included. (i = 40 through 47).
6. The rigid body phase margin frequency (i=48). WTMARG (48) is preset to zero. The rest of the above elements are preset to unity to be overridden by the user.

For the "optimize load relief" figure-of-merit, $\beta\rho$, $\delta\psi\rho$ and $\delta\phi\rho$ can all be included. (i = 49, 50 and 51). WTMARG (i) for i = 49, 50 and 51 is preset to unity to be overridden by the user.
Part A, Section 5 - This section defines those COBDE namelist parameters that are input only when load relief optimization and/or the drift minimum constraint equations are used. When optimizing load relief and/or constraining drift minimum on a multiple time point run, the user must not enter any of the data of this section, at those time points at which he does not want to consider load relief optimization and/or the drift minimum constraints.

5.1 Airframe, actuator and roll autopilot data

Table 10.1 defines the airframe and actuator data needed for uncoupled load-relief optimization and for the drift minimum constraint equations. Table 10.2 defines the additional airframe and roll autopilot data that is needed for yaw/roll coupled load relief optimization. The following notes are applicable to the data in Tables 10.1 and 10.2.

Note #1: All the data is input in floating point, and all the data is preset to zero.

Note #2: Yaw and roll plane notation is used. A pitch plane design is performed using the indicated yaw plane notation and sign convention, and by not inputting roll data. Obviously, uncoupled yaw is designed by simply not inputting the roll data. Since this same data is to be used for the pitch plane as well as for the yaw/roll plane, the following sign convention is used in order to avoid confusion between pitch and yaw.

The usual sign convention for forces and moments is:

(1) a positive X-axis force points "out the nose";
(2) a positive sideforce is "out the right wing";
(3) a positive normal force is "down";
(4) a positive pitching moment is "nose up";
(5) a positive yawing moment is "nose right"; and
(6) a positive rolling moment is clockwise looking forward.
<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
<th>Pitch Plane Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNB</td>
<td>$C_{n\beta}$ - aero moment due to angle of sideslip</td>
<td>$C_{m\alpha}$</td>
</tr>
<tr>
<td>CYB</td>
<td>$C_{y\beta}$ - aero sideforce due to angle of sideslip</td>
<td>$C_{r\alpha}$</td>
</tr>
<tr>
<td>CNDR</td>
<td>$C_{n\delta r}$ - aero moment due to rudder deflection</td>
<td>$C_{m\delta e}$</td>
</tr>
<tr>
<td>CYDR</td>
<td>$C_{y\delta r}$ - aero sideforce due to rudder deflection</td>
<td>$C_{r\delta e}$</td>
</tr>
<tr>
<td>QBAR</td>
<td>$\dot{q}$ - dynamic pressure</td>
<td>--</td>
</tr>
<tr>
<td>RHO</td>
<td>$\rho$ - air density. Note: if QBAR is zero, COEBRA calculates QBAR from $(0.5 \ast \rho \ast B \ast B)$</td>
<td>--</td>
</tr>
<tr>
<td>SF</td>
<td>$S_F$ - aero reference area</td>
<td>--</td>
</tr>
<tr>
<td>D</td>
<td>$b$ - aero reference length</td>
<td>$C$</td>
</tr>
<tr>
<td>TY</td>
<td>Controlled thrust</td>
<td>--</td>
</tr>
<tr>
<td>LGY</td>
<td>Controlled thrust moment arm (positive number)</td>
<td>--</td>
</tr>
<tr>
<td>IZZ</td>
<td>$I_{zz}$ - body axis moment of inertia about the center of gravity</td>
<td>$I_{yy}$</td>
</tr>
<tr>
<td>MASS</td>
<td>$m$ - total vehicle mass</td>
<td>--</td>
</tr>
<tr>
<td>G</td>
<td>$g$ - gravitational acceleration</td>
<td>--</td>
</tr>
<tr>
<td>THETA</td>
<td>$\theta_L$ - vehicle pitch attitude relative to local horizontal (degrees)</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>For yaw, $\text{\bf B}=\text{\bf V}_{r\text{w}}$ = total velocity relative to the wind</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>For pitch, $\text{\bf B}=\text{\bf V}<em>{r\text{w}} \ast \text{\bf V}</em>{r\text{w}} / U$ where $U_r$ is body X-axis relative velocity.</td>
<td>--</td>
</tr>
<tr>
<td>KTVC</td>
<td>Effective actuator/control-device gain</td>
<td>--</td>
</tr>
<tr>
<td>LAY</td>
<td>Lateral body mounted accelerometer moment arm (positive if forward of center of gravity). LAY is not used for Drift Minimum.</td>
<td>--</td>
</tr>
<tr>
<td>U</td>
<td>Velocity along the body X-axis. U is not used for Drift Minimum</td>
<td>--</td>
</tr>
</tbody>
</table>
Table 10.2 DATA FOR YAW/ROLL COUPLED LOAD RELIEF

<table>
<thead>
<tr>
<th>NAME</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLB</td>
<td>$C_l \beta$ - Aero roll moment: due to angle of sideslip</td>
</tr>
<tr>
<td>CLDR</td>
<td>$C_l \delta r$ - Aero roll moment: due to rudder deflection</td>
</tr>
<tr>
<td>CLDA</td>
<td>$C_l \delta a$ - Aero roll moment: due to aileron deflection</td>
</tr>
<tr>
<td>CNDA</td>
<td>$C_n \delta a$ - Aero yaw moment: due to aileron deflection</td>
</tr>
<tr>
<td>TR</td>
<td>Controlled thrust in roll</td>
</tr>
<tr>
<td>LGR</td>
<td>Controlled thrust moment arm in roll (positive number)</td>
</tr>
<tr>
<td>ZOFF</td>
<td>Moment arm through which yaw controlled thrust produces a rolling moment. Positive, if positive yaw deflection produces counterclockwise roll.</td>
</tr>
<tr>
<td>IXX</td>
<td>Body axis roll moment of inertia about the center of gravity</td>
</tr>
<tr>
<td>IXZ</td>
<td>Body axis yaw-roll crossproduct of inertia about the center of gravity.</td>
</tr>
<tr>
<td>PHI</td>
<td>$\phi_o$ - Euler roll angle used to orient the gravity vector with respect to the vehicle's yaw plane (degrees).</td>
</tr>
<tr>
<td>W</td>
<td>Velocity in the body Z-axis direction</td>
</tr>
<tr>
<td>KDR</td>
<td>Attitude error gain used in the roll autopilot equation.</td>
</tr>
<tr>
<td>KRR</td>
<td>Attitude rate gain used in the roll autopilot equation.</td>
</tr>
</tbody>
</table>
With this in mind, all COEBRA data is input as positive values for an aerodynamically unstable vehicle (negative $C_{n\beta}$ or positive $C_{m\alpha}$) with a positive dihedral effect (negative $C_{\ell\beta}$), and with "normal" control torque sources. By "normal" control torque sources is meant: (1) A positive yaw deflection produces a negative sideforce, a positive yawing moment and a negative rolling moment; (2) A positive pitch deflection produces a positive normal force, a positive pitching moment, and zero rolling moment; and (3) A positive roll deflection produces a negligible sideforce, a negative yawing moment, and a positive rolling moment. If any of these conditions are not satisfied, the user must compensate by making the appropriate changes in the signs of the input data. For example, if the vehicle is aerodynamically stable (positive $C_{n\beta}$ or negative $C_{m\alpha}$), the corresponding COBDE namelist parameter in Table 10.1, i.e., CNB, is input with a negative sign.

**Note #3:** In optimizing load relief, a single control torque source is assumed for yaw (or pitch), and a single control torque source is assumed for roll. By appropriate input data, these sources can be either:

1. Thrust vector control via gimballed engines, secondary injection, or jet vanes; or
2. Control via aerodynamic surfaces.

If aero control surface data is not input, thrust vector control is assumed.

**Note #4:** All aerodynamic stability derivatives are input in the body axis with units of per radian. All aero moment coefficients are assumed about the center of gravity.

**Note #5:** The angles THETA and PHI must be input in degrees. All other units are arbitrary, but must be consistent.
5.2 Autopilot Parameters

This section defines the COBDE namelist variables that are associated with the autopilot parameters when optimizing load relief and/or constraining drift minimum.

5.2.1 Identifying Each Parameter

As discussed in Chapter 7, the user must tell COEBRA how to equate the autopilot parameters in the QD030 Block Diagram with the parameters that are used in the load relief control law (Equation 7.1 in Chapter 7). Table 10.3 lists the Namelist names that are used to identify the locations in the QD030 Block Diagram of the load relief control law parameters of Equation 7.1. Each namelist name is equated via Hollerith form, to the location of the respective parameter. Obviously, the parameters of Table 10.3 are preset with blank locations.
Table 10.3  LOCATIONS OF THE PARAMETERS OF EQUATION 7.1

<table>
<thead>
<tr>
<th>Namelist Name</th>
<th>Corresponding QD030 location, in Hollerith, of...</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKD</td>
<td>$K_D$</td>
</tr>
<tr>
<td>LKR1</td>
<td>$K_{R1}$</td>
</tr>
<tr>
<td>LKR2</td>
<td>$K_{R2}$</td>
</tr>
<tr>
<td>LKDA</td>
<td>$K_{DA}$</td>
</tr>
<tr>
<td>LT1</td>
<td>$\gamma_1$</td>
</tr>
<tr>
<td>LT2</td>
<td>$\gamma_2$</td>
</tr>
<tr>
<td>LT3</td>
<td>$\gamma_3$</td>
</tr>
<tr>
<td>LKA1</td>
<td>$K_{A1}$</td>
</tr>
<tr>
<td>LA1</td>
<td>$A_1$</td>
</tr>
<tr>
<td>LA2</td>
<td>$A_2$</td>
</tr>
<tr>
<td>LA3</td>
<td>$A_3$</td>
</tr>
<tr>
<td>LKA2</td>
<td>$K_{A2}$</td>
</tr>
<tr>
<td>LA4</td>
<td>$A_4$</td>
</tr>
<tr>
<td>LA5</td>
<td>$A_5$</td>
</tr>
<tr>
<td>LA6</td>
<td>$A_6$</td>
</tr>
<tr>
<td>LA7</td>
<td>$A_7$</td>
</tr>
<tr>
<td>LA8</td>
<td>$A_8$</td>
</tr>
</tbody>
</table>
5.2.2 TS, Floating point, preset value = 0

TS is used when designing a digital load relief autopilot, and is the sampling period in seconds. Since the user inputs the initial values of the digital autopilot into the QD030 block diagram in W-plane form (because the frequency response is calculated in the W-plane), and since $\beta_p$, $\delta_{\phi p}$ and $\delta_{\phi p}$ are calculated via S-plane equations, TS is required to transform the filters in Equation 7.1 back and forth between the S and W-planes. TS is only required for the following filters: LT1, LT2, LT3, LA1, LA2, LA3, LA4, LA5, LA6, LA7, and LA8. Since these are always very low break frequency filters, the following approximate formula is sufficient.

\[(S\text{-plane time constant}) = 0.5 \times TS \times (W\text{-plane time constant})\]

If TS $\neq 0$, COEBRA assumes a digital autopilot, and will use the above equation.

5.3 Wind Parameters

This section defines the COBDE namelist variables that are used to define the wind profile when optimizing load relief capability. The wind profile is completely defined in Chapter 4, but Table 10.4 summarizes the parameters that determine the shape of the wind profile. Table 10.5 summarizes the parameters that are used when searching the time response for $\beta_p$, $\delta_{\phi p}$ and $\delta_{\phi p}$.


<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
<th>Preset</th>
<th>Floating Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>Slope of $\beta_\omega$ from 0 to TW1 sec. (rad/sec)</td>
<td></td>
<td>.0021</td>
</tr>
<tr>
<td>U2</td>
<td>Slope of $\beta_\omega$ from TW1 to TW2 sec. (rad/sec)</td>
<td></td>
<td>.0100</td>
</tr>
<tr>
<td>U3</td>
<td>Slope of $\beta_\omega$ from TW2 to TW3 sec. (rad/sec)</td>
<td></td>
<td>.0228</td>
</tr>
<tr>
<td>U4</td>
<td>Slope of $\beta_\omega$ from TW3 to TSTOP sec. (rad/sec) - .0038</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TW1</td>
<td>Wind slope break time #1 (sec.)</td>
<td></td>
<td>40.0</td>
</tr>
<tr>
<td>TW2</td>
<td>Wind slope break time #2 (sec.)</td>
<td></td>
<td>52.5</td>
</tr>
<tr>
<td>TW3</td>
<td>Wind slope break time #3 (sec.)</td>
<td></td>
<td>55.0</td>
</tr>
</tbody>
</table>

TABLE 10.4 SHAPE OF THE WIND PROFILE
TABLE 10.5 DEFINING THE TIME RESPONSE SEARCH

<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
<th>Preset Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSTART</td>
<td>Starting time for calculating and plotting the nominal time response (sec.)</td>
<td>30. (floating point)</td>
</tr>
<tr>
<td>TSTOP</td>
<td>Stop time for calculating and plotting the nominal time response (sec.)</td>
<td>70. (floating point)</td>
</tr>
<tr>
<td>DELT</td>
<td>Time increment used in calculating the time response (sec.). Because of storage limitations, the following relationship must be satisfied: (TSTART - TSTOP) / (DELT) ≤ 2000.</td>
<td>.05 (floating point)</td>
</tr>
<tr>
<td>DINCRE</td>
<td>When determining the &quot;disturbed&quot; peak values for the partial derivatives, COEBRA computes the response for only ± DINCRE time increments on either side of the time at which each (βₚ, δᵥₚ, or δₚₚ) nominal peak occurred. Note that βₚ, δᵥₚ, and δₚₚ need not occur at the same time.</td>
<td>10 (Integer)</td>
</tr>
</tbody>
</table>

5.4 Miscellaneous

Table 10.6 contains the definitions of 3 miscellaneous COBDE namelist variables that are used when optimizing load relief and/or constraining drift minimum.
### TABLE 10.6 MISCELLANEOUS PARAMETERS

<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
<th>Preset Value</th>
</tr>
</thead>
</table>
| MINDRF | = 1, the drift minimum constraints will be used.  
= 0, they will not be used                                                                                                                                   | 0 (Integer)           |
| LRCOST (k) | (a) if LRCOST (1) ≠ 0, $\beta_\rho$ will be computed  
(b) if LRCOST (2) ≠ 0, $\delta_{\Psi_\rho}$ will be computed  
(c) if LRCOST (3) ≠ 0, $\delta_{\Phi_\rho}$ will be computed  
Via the WTFAC option, any combination of these peaks can be included in the load relief cost function. For example, even though LRCOST = 3 * 1, if WTFAC (49) and WTFAC (50) are unity and WTFAC (51) is zero, then only $\beta_\rho$ and $\delta_{\Psi_\rho}$ will enter the load relief cost function. | 3 * 0 (3 dimensional integer array) |
| ROOTOL | Tolerance used for root cancellation in the three "load relief" transfer functions. If a real root in the numerator and denominator have a frequency such that $\left| \frac{W(\text{num}) - W(\text{den})}{W(\text{den})} \right| < \text{ROOTOL}$, the roots will automatically be cancelled. Also, for complex roots, the magnitude of the roots replaces the frequency in the above formula. | .0001 (Floating point) |
Part B of this chapter defines the COBIN namelist variables. The following is a list of these variables.

(1) Section 1, the Margin Array:
   1.1 DAMP
   1.2 AMPOUT
   1.3 SCALF
   1.4 MAROPT
   1.5 NOTERM

(2) Section 2, the Cost Functions:
   2.1 COSPEC
   2.2 WTYPE
   2.3 LOADOP

(3) Section 3, the Autopilot Variable Constraints:
   3.1 XMIN
   3.2 XMAX
   3.3 P
   3.4 PSMALL

(4) Section 4, the Figures-of-Merit and the Margin Counter:
   4.1 TOLMAC
   4.2 TOLFGM
Section 5, the Stability Margin and Drift Minimum Constraints:

5.1 SPEC
5.2 STEP
5.3 DSTEP
5.4 DRFTOL

Section 6, Miscellaneous:

6.1 NITER
6.2 MFRESP
6.3 DELPAR
6.4 PRINT
6.5 NPRINT
6.6 KROOT
Part B, Section 1

This section defines the COBIN namelist variables that are used to form the Margin Array.

1.1 DAMP, Floating point, preset value = 0.05

Any underdamped quadratic roots in the denominator of the total open loop transfer function (CASE 13) that have an effective damping ratio with an absolute value less than DAMP, are considered to be bending or fuel slosh modes. The IDMODE array is then used to identify the modal type (e.g., 1st mode, etc.) to be associated with each quadratic pair that is selected by DAMP. DAMP serves to eliminate from consideration as modes, things such as sensor dynamics, autopilot filters and prefilters, and even certain modes including perhaps the engine.

1.2 AMPOUT, Floating point, preset value = -20.0

Modes that are selected by DAMP whose peaks resonate below AMPOUT, are not considered when forming the constraint equations or cost function. Units on AMPOUT are in decibels. However, all modes that are selected by DAMP do enter the "maximize margins" figure-of-merit and the Margin Counter.

1.3 SCALF (i), Floating point array (51 dimensional), preset values are given in Table 10.7, where the index i corresponds to the first 51 elements of the Margin Array.
### TABLE 10.7 SCALF

| 1., 1., 2, | rigid body margins |
| 1., 111, .2, .2, 1., 1., .111, .2, .2, 1., | 1st structural modes |
| 1., .111, 2 * .2, 3 * 1., 1., .111, 2 * .2, 3 * 1., | 2nd structural modes |
| 1., .111, 1., .111, 1., .111, 1., .111, | 3rd to 8th structural modes |
| 1., | all 8 fuel slosh modes |
| 3 * 1., | crossover frequency |
| | load relief variables |

SCALF is an array of multiplying factors that are used to scale the elements of the Margin Array, and has units of (1) db/db, (2) db/deg, or (3) nondimensional.

1.4 MAROPT, Integer, preset value = 0

MAROPT is used to distinguish between the 1st mode of Figure 10.1 and the 1st mode of Figure 10.2. In both cases, the "frontside" phase margin is between 180° and 360°, and the "backside" phase margin is between 0° and 180°. If MAROPT = 1, COEBRA assumes Figure 10.1, and (a) sets the frontside phase margin to a negative margin, (b) sets the backside margin to a positive margin, and (c) attempts to "lag" the mode in the cost function.
If \( \text{MAROPT} = 0 \), \text{COEBRA} assumes Figure 10.2, and (a) sets the frontside to a positive margin, (b) sets the backside to a negative margin, and (c) "leads" the mode.

**Figure 10.1** \( \text{MAROPT} = 1 \)

**Figure 10.2** \( \text{MAROPT} = 0 \)

1.5 \( \text{NOTERM} \), Integer, Preset value = 0

If \( \text{NOTERM} = 0 \), the first 3 rigid body margins must exist, or \text{COEBRA} will terminate. Exceptions: (1) the aerodynamic gain margin if it does not exist at all, and (2) the rigid-body gain margin if fuel slosh modes are included.

If \( \text{NOTERM} = 1 \), the first 3 rigid body margins do not have to exist.
This section defines the COBIN namelist variables that are used to form the Cost Functions.

2.1 COSPEC (i), Floating point array (48 dimensional), preset values are given in Table 10.8, where the index i corresponds to the first 48 elements of the Margin Array.

<table>
<thead>
<tr>
<th></th>
<th>rigid body margins</th>
<th>1st structural modes</th>
<th>2nd structural modes</th>
<th>3rd to 8th structural modes</th>
<th>all 8 fuel slosh modes</th>
<th>crossover frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>6., 6., 30.,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20., 0., 45.,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45., 60., 10.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20., 0., 60.,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60., 60., 3 *</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60., -10., 0.,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10., 0., -10.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 * 30.,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The COSPEC array contains the stability margin objectives that are used to form the "maximize margins" cost function. The COSPEC array is input in decibels, degrees, and rad/sec.
2.2 WTYPE Floating point, preset value = 0.0

The "maximize margins" cost function has two weighting factors:
(1) W that is computed by the program; and (2) WTFAC that is input by
the user.

If WTYPE = 1, all the elements of W will be unity. If WTYPE = 0,
all the elements of W will be the "desired over nominal" type as defined
in Chapter 3.

2.3 LOADOP, Integer, preset value = 0

If LOADOP = 0, COEBRA will optimize stability margins.

If LOADOP = 1, COEBRA will optimize load relief.
This section defines the COBIN namelist variables that are used to form the autopilot variable constraint equations.

3.1 $X_{MIN}(i)$, Floating point array (65 dimensional), preset values - none, problem dependent.

This array specifies the minimum values allowed for the autopilot variables. The index $j$ corresponds to the stand-alone autopilot variables in the order in which they are input in the QD030 namelists for all time points. In other words, each "-1" value in all the NTYPE arrays, must have its corresponding $X_{MIN}$ value.

3.2 $X_{MAX}(i)$, Floating point array (65 dimensional), preset values - none, problem dependent

This array specifies the maximum values allowed for the autopilot variables. Each value in the $X_{MIN}$ array must have a corresponding value in the $X_{MAX}$ array.

3.3 $P$, Floating point, preset value = 0.1

This variable is the initial value to be used for $P$ at the beginning of each major iteration. $P$ is the autopilot variable step-size, and on any iteration, has the same value for all the autopilot variables. $P$ is optimized on what are referred to as minor iterations.
3.4 PSMALL, Floating point, preset value - 0.1

In the optimization of the value of P, PSMALL serves as a convergence criteria on two items:

(1) PSMALL is the smallest value allowed for P. COEBRA will terminate when P becomes less than PSMALL, since this means that the present nominal autopilot (the autopilot at the beginning of this major iteration) is the best autopilot that can be obtained.

(2) PSMALL is the smallest value allowed for ΔP which is defined as $|P_{\text{present}} - P_{\text{last}}|$. When ΔP becomes less than PSMALL, COEBRA will begin the next major iteration.

By inputting PSMALL with the same value as P, the minor loop will be bypassed. The initial value of P will then be used unchanged on each major iteration.
Part B, Section 4

This section defines the COBIN namelist variables that are associated with the 2 Figures-of-merit and the Margin Counter.

4.1 **TOLMAC**, Floating point, preset value = 0.03

TOLMAC is a one-sided tolerance on the decisions made in forming the Margin Counter. A margin is counted as "met" if its scaled value exceeds the quantity:

\[(1.0 - \text{TOLMAC}) \times \text{(the corresponding scaled SPEC value)}\].

This tolerance is needed to avoid the situation where a margin is met on one iteration, and then, due to nonlinearities, is only slightly not met on the next. Table 10.9 shows the effect of TOLMAC = 0.03.

4.2 **TOLFGM**, Floating point, preset value = 0.01

TOLFGM is a tolerance on the decisions in the minor loop that are determined by the Figure-of-merit. If the Figure-of-merit from the present iteration, exceeds the quantity.

\[(1.0 + \text{TOLFGM}) \times \text{(the Figure-of-merit from the last iteration)}\],

the present iteration is considered to be an improvement over the last. Otherwise, the last is assumed better than the present.
<table>
<thead>
<tr>
<th>Margin</th>
<th>SPEC value</th>
<th>Scaled SPEC value*</th>
<th>Scaled Value</th>
<th>Unscaled value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) aero gain</td>
<td>6. db</td>
<td>1.995</td>
<td>1.935</td>
<td>5.75 db</td>
</tr>
<tr>
<td>(2) rigid-body gain</td>
<td>6. db</td>
<td>1.995</td>
<td>1.935</td>
<td>5.75 db</td>
</tr>
<tr>
<td>(3) rigid-body phase</td>
<td>30. deg.</td>
<td>1.995</td>
<td>1.935</td>
<td>28.75 deg.</td>
</tr>
<tr>
<td>(4) 1st mode front-side phase</td>
<td>45. deg.</td>
<td>2.818</td>
<td>2.733</td>
<td>43.75 deg.</td>
</tr>
<tr>
<td>(5) 1st mode back-side phase</td>
<td>60. deg.</td>
<td>3.981</td>
<td>3.862</td>
<td>58.75 deg.</td>
</tr>
<tr>
<td>(6) 1st mode closest approach</td>
<td>10. db</td>
<td>3.162</td>
<td>3.067</td>
<td>9.75 db</td>
</tr>
<tr>
<td>(7) 2nd mode front-side phase</td>
<td>60. deg.</td>
<td>3.981</td>
<td>3.862</td>
<td>58.75 deg.</td>
</tr>
<tr>
<td>(8) 2nd mode back-side phase</td>
<td>60. deg.</td>
<td>3.981</td>
<td>3.862</td>
<td>58.75 deg.</td>
</tr>
<tr>
<td>(9) 2nd mode closest approach</td>
<td>10. db</td>
<td>3.162</td>
<td>3.067</td>
<td>9.75 db</td>
</tr>
<tr>
<td>(10) 3rd to 8th mode peak gain</td>
<td>10. db</td>
<td>3.162</td>
<td>3.067</td>
<td>9.75 db</td>
</tr>
<tr>
<td>(11) Fuel slosh back-side phase</td>
<td>20. deg.</td>
<td>1.584</td>
<td>1.536</td>
<td>18.75 deg.</td>
</tr>
<tr>
<td>(12) Crossover frequency</td>
<td>2. rps</td>
<td>2.</td>
<td>1.94</td>
<td>1.94 rps</td>
</tr>
</tbody>
</table>

*This is computed using the preset values of the SCALF array.
Part B, Section 5

This section defines the COBIN namelist variables that are used when setting up the Stability Margin and Drift Minimum Constraint Equations.

5.1 SPEC (i), Floating point array (48 dimensional), preset values are given in Table 10.10, when the index i corresponds to the elements in the Margin Array. The units on the SPEC array are in decibels, degrees, and rad/sec.

<table>
<thead>
<tr>
<th>SPEC</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6., 6., 30.,</td>
<td>the rigid-body margins</td>
</tr>
<tr>
<td>20., 0., 45., 60., 10., 20., 0., 45., 60., 10.,</td>
<td>1st structural modes</td>
</tr>
<tr>
<td>20., 0., 60., 60., 3 * 10., 20., 0., 60., 60., 3 * 10.,</td>
<td>2nd structural modes</td>
</tr>
<tr>
<td>-10., 0., -10., 0., -10., 0., -10., 0., -10., 0.,</td>
<td>3rd to 8th structural modes</td>
</tr>
<tr>
<td>8 * 20.,</td>
<td>fuel slosh modes</td>
</tr>
<tr>
<td>0.,</td>
<td>crossover frequency</td>
</tr>
</tbody>
</table>

The SPEC array specifies the minimum requirements that are put on the stability margins and closed-loop root locations. The SPEC array is used when forming the Stability Margin constraint equations.
5.2 \texttt{STEP}, Floating point, preset value = 1.0

This is the initial value to be used for \texttt{STEP} at the beginning of each minor iteration. \texttt{STEP} is the "percent improvement" required in each "margin" relative to its SPEC value or required value. On each minor iteration (each value of \( P \)), COEBSA maximizes the value of \texttt{STEP}. This is done in the inner loop. \texttt{STEP} applies to all of the so-called "not-met Margin Constraint Equations" on stability margins, root locations, drift minimum, and the autopilot vector constraints.

5.3 \texttt{DSTEP}, Floating point, preset value = 0.2

In the inner loop, \texttt{STEP} is maximized as follows. \texttt{STEP} begins with its initial value (input as \texttt{STEP}). If a feasible solution does not exist, \texttt{STEP} is reduced by \texttt{DSTEP} using the following equation

\[
\texttt{STEP} \leftarrow \texttt{STEP} - \texttt{DSTEP}
\]

until a feasible solution is obtained. The minimum value for \texttt{STEP} is zero, for which a feasible solution is virtually guaranteed.

5.4 \texttt{DRFTOL}, Floating point, preset value = 0.0

\texttt{DRFTOL} is the tolerance used in the Drift Minimum Constraint Equations. This tolerance is referred to as \( \zeta \) in Chapter 8. As shown in Chapter 8, this is approximately the tolerance that is allowed on the frequency of the drift root that would be exactly at the origin for "perfect" drift minimum. For example, if \texttt{DRFTOL} = \( .1 \), this requires that the drift root be within \( \pm 0.1 \) rad/sec of the origin.
Part B, Section 6

This section defines miscellaneous COBIN namelist variables.

6.1 NITER, Integer, preset value - none, problem dependent

COEBRA will terminate after NITER major iterations.

6.2 MFRESP, Integer, preset value = 0

If MFRESP ≠ 0, the frequency response resulting from each minor iteration (each value of P), will be printed out and plotted.

6.3 DELPAR, Floating point, preset value = 0.02

DELPAR is the percent disturbance that is made on each autopilot variable when computing all the partial derivatives. The partial derivative of the \( i \)th element of the Margin Array with respect to the \( j \)th autopilot variable is defined as:

\[
\frac{\text{Disturbed } M(i) - \text{Nominal } M(i)}{\text{DELPAR} \times \text{Nominal } X(j)}
\]

Sometimes when disturbing an autopilot variable in order to compute partial derivatives, the "disturbed" Margin Array will differ from the "nominal" array because one or more elements have been added or taken away due to the disturbance. When this occurs, the sign on DELPAR is reversed. If an element again has been added or taken away, the derivative of that element with respect to that variable is set to zero. The solution of reversing the sign on DELPAR is also used when a disturbance causes the damping ratio of an autopilot quadratic filter to become greater than or equal to unity.
When computing the partial derivatives, the elements of the Margin Array are always refound. In other words, the derivative is not calculated at a certain frequency.

Obviously, the partial derivatives are calculated after the elements of the Margin Array have been scaled, and are calculated only once per major iteration.

**6.4 PRINT, Integer, preset value = 0**

PRINT is a flag that can be used to obtain additional printout for checkout and debugging purposes.

If PRINT = 1, (a) Case 13 numerator roots are printed out for all calls to the polynomial factoring routine, and (b) various computer dumps are printed out.

If PRINT = 0, this flag is disabled.

**6.5 NPRINT, Integer, preset value = 0**

NPRINT is a flag that can be used to obtain additional printout for checkout and debugging purposes.

If NPRINT = 1, the initial tableau for each entry to the Simplex Algorithm is printed out.

If NPRINT = 2, all tableaus of the Simplex are printed out.

If NPRINT = 0, this flag is disabled.
If \( KROOT \neq 0 \), the ROOTCH subroutine (Chapter 7) is to be used. The ROOTCH subroutine allows autopilot quadratic roots to become real roots if the effective damping ratio becomes unity, or vice-versa for real roots to become complex. The ROOTCH subroutine will not be used when optimizing load relief since the filters in the load relief control law must be real roots.
### 11.1 COEBRA Deck Setup

Table 11.1 summarizes the deck setup for a COEBRA run.

**TABLE 11.1 COEBRA DECK SETUP**

<table>
<thead>
<tr>
<th>Explanation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>COEBRA</td>
<td>COEBRA caller card (columns 2 to 7)</td>
</tr>
<tr>
<td>Title Card</td>
<td>Title card for time point #1* (columns 2 to 43)</td>
</tr>
<tr>
<td>$INDATA</td>
<td>INDATA namelist data for time point #1 ($ in column 2)</td>
</tr>
<tr>
<td>$END</td>
<td></td>
</tr>
<tr>
<td>$COBDE</td>
<td>COBDE namelist data for time point #1 ($ in column 2)</td>
</tr>
<tr>
<td>$END</td>
<td></td>
</tr>
<tr>
<td>Title Card</td>
<td>Title card for time point #2*</td>
</tr>
<tr>
<td>$INDATA</td>
<td>INDATA namelist data for time point #2</td>
</tr>
<tr>
<td>$END</td>
<td></td>
</tr>
<tr>
<td>$COBDE</td>
<td>COBDE namelist data for time point #2</td>
</tr>
<tr>
<td>$END</td>
<td></td>
</tr>
<tr>
<td>:</td>
<td></td>
</tr>
<tr>
<td>:</td>
<td>Etc. for remaining time points or vehicle states.</td>
</tr>
<tr>
<td>ENDBLK</td>
<td>ENDBLK card (columns 2 to 7)</td>
</tr>
<tr>
<td>$COBIN</td>
<td>COBIN namelist data</td>
</tr>
<tr>
<td>$END</td>
<td></td>
</tr>
<tr>
<td>ENDRUN</td>
<td>ENDRUN card (columns 2 to 7)</td>
</tr>
</tbody>
</table>

* Title card must not have a $ in column 2.
11.2 QD030 Deck Setup

Table 11.2 summarizes the deck setup for a QD030 run.

**TABLE 11.2 QD030 DECK SETUP**

<table>
<thead>
<tr>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>QD030</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
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<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
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<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Title Card</td>
</tr>
<tr>
<td>$INDATA</td>
</tr>
<tr>
<td>•</td>
</tr>
<tr>
<td>$END</td>
</tr>
<tr>
<td>Etc. for remaining cases.</td>
</tr>
<tr>
<td>ETCRUN card (columns 2 to 7)</td>
</tr>
</tbody>
</table>

* Title card must not have a $ in column 2.

**Note the function of the variable called NFLAGG for multiple case runs.
11.3 Sample Listings

This section contains the listings of the input data used to generate Examples 1 to 6 of Volume I of this report. This section also includes a sample listing of a QD030 test case. The following is a summary of these listings.

(1) Example #1 (Run 1) of Volume I
(2) Example #2 (Run 2) of Volume I
(3) Example #3 of Volume I
(4) Example #4 of Volume I
(5) Example #5 of Volume I
(6) Example #6 of Volume I
(7) QD030 Sample Listing
EXAMPLE #1 (RUN 1) OF VOLUME I

COBRA

EXAMPlE 1 RUN 1 VOLUME I

$INDATA
RF=1.0.0.1.0.0.1.20.20.  
RG=3.0.0.2.0.75.-1.9.-134.1.2.17.  
RN(7)=1.  
RO(7)=1.  RU(7)=1.  
NFLAGC=1  
$END

$COBDE
INTOTAL=10.  NTYPE=0.-1.-1.7.0.  
INHGA=1.  W1FAC(48)=0.  
$END

ENDBLK
$COBIN NITER=20.  P=.2.  PSMALL=.05.  
STEP=1.  DSTEP=.2.  
SPEC=6.12.50.  SPEC(48)=.2.  COSPEC=6.12.50.  COSPEC(48)=.2.  
TOLFH=.01.  
MFRESP=1.  TOLMAG=.03  
$END

ENDRUN
EXAMPLE #2 (RUN 2) OF VOLUME I

COFRA
EXAMPLE 2 RUN 2 VOLUME 1
$INDATA
NCASE=13., NFREQ=-13.,
RF=2.,0.,0.,2.,0.,0.,1.,10.,2.,10.,2.,
RC=0.,0.,0.,2.,0.,1.,100.,125.,0.5,
RN(7)=1.,
AO(7)=1.,RU(7)=1.,
NFLAG=1
$END
$COBDE
NTOAL=9., NTYPE=0.,4.=1.,4.,
IOMEGA=1., WFACT(48)=1.,
$END
ENDBLK
$COBIN NITER=10., P=2., PSKALL=.05,
XMAX=4.*1.E6, XMIN=4.*1.E-6,
STEP=1., DSTEP=2.
SPEC=6.,10.,45., SPEC(48)=10.,
CSPEC=6.,10.,45., CSPEC(48)=10.,
TOLCM=.01,
MFRESP=1., TOLMAC=.03
$END
ENDRUN
<table>
<thead>
<tr>
<th>COBRA</th>
<th>EXAMPLE 3</th>
<th>VOLUME 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{INDA}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathbf{RO}$</td>
<td>0.0.0, 3.8, 1.0</td>
<td>1.00000000E+00, -1.86076583E+01</td>
</tr>
<tr>
<td></td>
<td>-1.36262989E+00, 1.23270333E+00, -1.86076583E+01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.49987917E+02, 8.66379519E+01, 1.49089949E-02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.10546336E+01, 1.49810447E-02, 4.36472340E+01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.49923601E+02, 4.10520076E+01, 1.49752393E-02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.365625962E+01, 9.99467577E-03, 3.31305402E+01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.92141654E-03, 6.21674075E-00, 1.49827592E-02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.12915627E+02</td>
<td></td>
</tr>
<tr>
<td>$\mathbf{RS}$</td>
<td>3.0, 7.1, 0.0, 0.0</td>
<td>4.66325646E+00, 2.63380666E+02</td>
</tr>
<tr>
<td></td>
<td>-2.46849628E+01, 8.55145836E+01, 2.63380666E+02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.55150711E+01, 6.79866314E-02, 2.76766317E-01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.75577872E+02, 4.25261830E+01, -1.46242991E-01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.17005789E+01, 1.17155895E+01, 3.12444746E+01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.34779036E-02, 1.01403810E+01</td>
<td></td>
</tr>
<tr>
<td>$\mathbf{RT}$</td>
<td>1.0, 8.1, 0.0, 0.0</td>
<td>4.66325646E+00, 2.63380666E+02</td>
</tr>
<tr>
<td></td>
<td>2.46102938E+01, 1.48004139E-02, 8.95946461E+01</td>
<td></td>
</tr>
<tr>
<td></td>
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11.6
EXAMPLE #3 OF VOLUME I (Cont'd.)

NCASE=4*10*11*12*13
NFREQ=10*11*12*13
KVECT=1
RO(4)=2.0*0.0*0.0*1.0*0.01*0.01
RG(7)=9.40
RL(4)=2.0*0.0*1.0*0.0333*0.0333
RL(7)=4.105
RM(3)=1.2*0.0*1.0*2.0*2
RM(7)=2.392
RN(3)=1.2*0.0*1.0*2.0*2
RN(7)=0.01
RL(7)=4.0
RM(7)=1.6
RN(7)=0.8
NLAG=1
$END
$COBDE NTOTAL=12 NTYPE=3*0.9*1 IDMODE=1*2*3*4*5*6*7*8
WFA(4)=0.1
WFA(14)=0.1
WTHA(34)=0.0
WTHA(36)=0.0
WTHA(38)=0.0
IO=4
WFA(48)=0
$END
ENDRUN
$COBON XMAX=9.100 XMIN=9.01 MFRSP=1 NOTERM=1 TOLMAC=0.03
P=.2
PSMALL=.05
STEP=1
DSTEP=4
COSPEC(6)=100
COSPEC(7)=100
COSPEC(16)=100
COSPEC(17)=100
COSPEC(18)=20
COSPEC(20)=20
SPEC(48)=2
TOLF=0.005
NITER=5
$END
ENDRUN
### COBRA

**EXAMPLE 4  VOLUME 1**

**SINDATA**

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118
EXAMPLE #4 OF VOLUME I (Cont'd.)

NCASE=4*10*11*12*13,
NFREQ=10*11*12*13*
PO(4)=2.0*0.0*0.0*1.0*0.01*0.01,
RE(7)=.490,
PL=0.*0.*0.*2.*0.*0.*6.*.0333*-.0333,
RM=0.*0.*1.*2.*0.*0.*3.*.0333*-.0333,
RN=0.*0.*1.*2.*0.*0.*5.*.0333*-.0333,
NFLAGG=1
$END
$COBDE  NTOTAL=12  NTYPE=3*0*9*1  IDHODE=1,2,3,4,5,6,7,8,
WTFAC(4)=0.*1.*  WTFAC(14)=0.*1.*
WTMARG(16)=0.*  WTMARG(17)=0.*
WTMARG(36)=0.*  WTMARG(38)=0.*
TCMECA=1.  WTFAC(48)=1.
$END
ENDBLK
$COUTIN  XMAX=9*100.*  XMID=9.*01.*  MFRESP=1.  NOTERM=1.  TOUMAC=.03,
F=.2.  PSKALL=.05.  STEP=1.*  DSTEP=.4, 
COSPEC(16)=100.*  COSPEC(17)=100.*  COSPEC(16)=100.*  COSPEC(17)=100.*
SPEC(48)=2.*  COSPEC(48)=2.*
COSPEC(8)=20.*  COSPEC(20)=20.*
TOLFGM=.01,
TOLFGM=.005,
NITER=5,
$END
ENDRUN
EXAMPLE #5 OF VOLUME I (Cont'd.)

RU=9.93*1.0*2.0*3.8813388E+2*
2.98099843E+03* 2.87633795E-00* 9.99972044E-01*
-2.81125906E-00* 9.24695562E-01* -9.69932688E-01*
-0.2087*0.57034379*1.01011324*
-0.3134832*0.01170666*0.512918*0.9599376*
1.012925*0.999896*1.017521*0.8908525*1.36203*
SLOW=.001*UPPER=5*
NFLAG=1*
$END
$CODE  U=8.46E3* D=8.46E3* QBAR=2.96786* CYB=7.26* TV=2.06716*
CNB=21.3* IZZ=2.44618* LGY=478.9* LAY=463.1*
KFWC=1.075* MASS=2931.1* THETA=73.31* G=386.9*
SF=1.13E4* D=120.* U1=-0.021* U2=-0.028* U3=-.038*
TW1=52.5* TW2=55.* TW3=80.* TS=-.04* LROCST=1.0*0*
LKD=5HRL(7)*
LKR1=5HRM(7)*
LKR2=5HRN(7)*
LKA2=5HRK(7)*
LA1=SHRK(8)*
LA5=SHRK(9)*
LA6=SHRK(10)*
LA7=SHRK(11)*
NTOAL=16*
NTYPE=-1-1*0*2*-1-1*2*-1-1*3*-1*
IDMODE=1*2*3*
WTFAC(49)=1.*0.*0.*
WMARG(49)=1.*0.*0.*
$END
EXAMPLE 5 VOLUME I  T=60
$ENDATA
NCASC=4.13* NFREQ=13*
RK=1.0*0.0*3.0*0.*1.*80.*374.*16.7*20.*
RK(7)=.005407*
AL=0.*1.*0.*1.*6.*135*
RL(7)=.9211*
EXAMPLE #5 OF VOLUME I (Cont'd.)

$COBDC  U=1.90015E+0,  B=1.90015E+4,  QBAR=5.91634,  CYB=8.745,  TY=1.845804E6,
CMO=28.354,  IZZ=2.10317E8,  LGO=490.06E6,  LAY=451.938E6,
KTC=1.142,  MASA=2322.132,  THETA=52.,  G=386.,
SF=1.13E4,  D=120.,  U1=0.021,  U2=0.0228,  U3=0.0038,
TW1=52.5,  TW2=55.,  TW3=80.,  TS=.04,  LRCOST=1.0*.0

LKD=SHRL(7),
LK1=SHRM(7),
LK2=SHRN(7),
LK4=SHRK(7),
LA4=SHRK(8),
LA5=SHRK(9),
LA6=SHRK(10),
LA7=SHRK(11),
NTP=16,
NTYPE=1*,  1*0*2* 1*-1*2* 1*-1*3* 1*-1*3* 1*
IPR=1*1*3*,
WFACT(49)=1.*0.0*0.0*
WMARG(49)=1.*0.0*0.0*
SEND
EXAMPLE 5  VOLUME I  T=80BGC
$TMDATA
NCASE=4*.13*,  NFR=13,
RL=1.0*0.0.*,  0.0*1.0*0.0*0.1*80.*,  374.*16.7*20.,
RL(7)=0.0053,
RN=0.*0.0.1*0.*,  1*6*.135*,
RN(7)=.564*,
RM=0.*0.0.*1.1.*0.1.*1.19*.4*35*,
RM(7)=178*,
RN=0.*0.*1.*0.*1.*1.19*.5*3*,
RN(7)=.0445,
EXAMPLE #5 OF VOLUME I (Cont'd.)

```
$CODE
NTOTAL=16,
NTYPE=1 1 1 0 1 2 1 1 1 1 1 3 1 1 1 3 1,
IMODE=1 2 3,
LRCOST=0 0 0 0
$END
ENDBLK
$COBIZIN DAMP=.02, LOADP=1, P=.05, PSMAH=.05, STEP=0, DSTEP=0
NPRINT=1,
XMIN=23*1.E-5, XMAX=23*1.E5, NITER=1
$END
ENDRUN
```
EXAMPLE #6 OF VOLUME I

COBDA4

EXAMPLE 6 VOLUME I T=27 AgC

!TINDATA
NCASE=413
NFREQ=-13
RR=0.00.00.20.0.0.0.0.0.0.0.0.0.0
RO(7)=.702
RK=1.0.0.04.00.00.0005043.04525.2575
RL=0.0.0.0.20.0.0.0.0.0.1.1
PL(7)=.55
RM=0.0.1.3.10.0.0.0.0714.0714.001471.761475
RM(7)=1.0
RN=0.0.0.0.12.00.0.0.0.0.0.0833.0714
RN(7)=.135
PO=0.00.030.01001.00000000E+00
-1.63958849E+00
9.90006447E-03
9.62832509E-01
RR=1.04.01.00.00
-2.11387064E+01
-6.51269248E-02
1.49309948E+01
4.53690771E-02
4.53690771E-02
1.00019467E-02
1.52586588E+01
RT=1.03.010.00
5.37675663E+01
9.92025612E-03
8.76949880E-00
RU=3.02.01.00
4.53690771E-02
1.00019467E-02
1.52586588E+01
NFLAGC=1

$END
EXAMPLE #6 OF VOLUME I (Cont'd.)

**CODE**  U=8009,  D=8009,  QBAR=2.8447,  CYB=6.88,  TY=2.151098E6,
CND=26.13,  IZZ=2.46E6,  LGY=470.6,  LAY=620.4,
KTV=702,  MASS=2921.65G,  THETA=82.4716G=386.77,
SF=1.1304E4,  D=120.,  U1=.0021,  U2=.0228,  U3=-.038,
TW=52.5,  TW2=55.,  TW3=80.,  LRCOST=1.000,
LKD=5HRK(7),
LKR=5HRK(7),
LPK=5HRK(7),
LA2=5HRK(7),
LA4=5HRK(3),
LA5=5HRK(9),
LAR=5HRK(10),
LA7=5HRK(11),
LAR=5HRK(12),
NTOTA=21,
NTYPE=3*0+12*-1;3*0.3*1,
NTYPE(9)=0,
TMODE=1,2,3,
WTPAQ(49)=1.0*0.0,
WTPARG(49)=1.0*0.0,
$END

EXAMPLE G VOLUME I T=53

$INDATA
NCASE=4*13,  NFREQ=-13,
RO=0.0, 0.0, 2.0, 0.0, 1.0, 0.01, 0.01,
RG(7)=.678,
RM=1.0, 0.0, 4.0, 0.0, .00504, 3.045, 25.0, 25.75,
RL=0.0, 0.0, 1.0, 0.0, 1.0, 1.0,
RL(7)=.655,
RM=0.0, 0.0, 1.0, 0.0, 1.0, 1.0, 0.0714, 0.0714, 0.001471, 0.761475,
RM(7)=1.0,
RN=0.0, 0.0, 1.0, 0.0, 1.0, 0.0833, 0.0714,
RN(7)=1.355,
RCO=0, 0, 0, 3.0, 4.0, 1.0, 1.0, 0.00000000E+00,
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9.99410198E-03, 1.73218744E+01, 7.95729722E-03,
9.98952627E-03, 9.99986501E-03, 2.43799650E+01,
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EXAMPLE #6 OF VOLUME I (Cont'd.)

RR=1.0*1.0*1.0*0  *0  *0  -1.56726113E+03
-4.95539236E-01*  1.53577303E-02*  3.09891265E+01
1.37103831E-02*  2.44469562E+01*  5.34197545E-03
1.56826376E+01*  1.55346855E-02*  1.47759095E+01
2.81936308E-02*  1.03447811E+00
RS=3.0*3.0*1.0*0  *0  *0  4.6115383E+00
-4.85364255E-02*  5.01959466E-02*  3.70900584E+01
1.03651111E-02*  2.44430515E+01*  1.09437450E-02
2.32951635E+01*  9.96676533E+00*  1.65723933E+00
RT=1.0*4.0*1.0*0  *0  *0  4.52557052E+00
3.58576923E+01*  1.34445600E-02*  2.44418084E+01
1.19020492E+02*  2.51796085E+01*  9.97590356E+00
1.72633056E+01*  6.88790708E-03*  8.72567612E+00
RU=3.0*3.0*1.0*0  *0  *0  4.46115368E+00
-4.85364255E-02*  5.01959466E-02*  3.70900584E+01
1.03651111E-02*  2.44430515E+01*  1.09437450E-02
2.32951635E+01*  9.96676533E-03*  1.65723933E+00
NFLAG=1
$END
%C089E  U=1.7612E4,  B=1.7612E4,  QBAR=6.033,  CYB=8.69,  TY=1.8869376E6,
CNB=33.4,  IZL=2.095E8,  LGY=477.4,  LAY=613.6,
KTV=678,  MASS=2374.546,  THETA=67.4028,  CO=386.27,
SF=1.1304E4,  D=120,  U1=.0021,  U2=.0220,  U3=-.0038,
TW1=52.5,  TW2=55.5,  TW3=80,  LRCOST=1.0*0
LKD=5HRL(7),
LRK1=5HRK(7),
LKP2=5HRN(7),
LKA2=5HRK(7),
LAK=5HRK(3),
LAK5=5HRK(9),
LGA6=6HRK(10),
LA7=6HRK(11),
LA8=6HRK(12),
NTRAL=21
### Example #6 of Volume I (Cont’d)

```plaintext
NTYPE=3*0.12* 1.3*0.3* 1
NTYPE(9)=0
TMDT=1*2*3*4
WTAC(49)=1.*0.*0.*
WITMPG(49)=1.*0.*0.*
$END
EXAMPLE 6 VOLUME I T=758GC
$INDATA
NCASE=4*13, NFREQ=13*
RO=0.*0.*0.*2.*0.*0.*1.*0.*0.*1.*
RS(7)=.641*
RK=1.*0.*0.*4.*0.*0.*0.*0.*0504*3.*4.*5.*25.*25.*7.*5.
RL=0.*0.*0.*2.*0.*0.*0.*1.*1.*1.*
PL(7)=.655*
RM=0.*0.*1.*3.*1.*0.*1.*0.*0714*0.*0714*0.*01471*76.*147.5*
RM(7)=1.0*
RN=0.*0.*1.*2.*0.*0.*0.*0833.*0.*0714*
RN(7)=.135*
RO=0.*0.*0.*3.*0.*4.*0.*1.*0.* 1.00000000E+00*
-1.07899963E+00 1.05769414E+00 -9.07283062E+01*
7.9447336E-03 1.01128072E+01 9.9903462E-03*
1.7965522E+01 9.99295811E-03 2.60680976E+01*
1.49973389E-02 3.08596988E+01 9.9903462E-03*
RR=3.*0.*4.*1.*0.*0.*0.*-2.01811288E+03*
2.51520279E-02 -2.41762230E-02 -9.07834082E+01*
1.25867380E-02 2.85753184E+01 9.0395773E-03*
1.76061838E+01 9.87021535E-03 1.35457027E+01*
1.59868632E-02 9.36713126E-01 9.9903462E-03*
RS=3.*0.*3.*0.*1.*0.*0.* 6.44680604E+00*
6.633720594E-02 -6.49663916E+02 7.61543319E+01*
1.22125139E-02 2.79729717E+01 1.07473337E-02*
2.03368555E+01 9.99465555E-03 1.75132772E+01*
```

EXAMPLE 6 OF VOLUME I (Cont'd.)

RY=1.0*4.7*1.0*0.0*0.0  6.52397993E+00
  7.41800380E+01  7.49959656E-02  2.68733070E+01
-5.16490396E-02  2.69080234E+01  9.98079690E-03
  1.79372224E+01  6.73775530E-02  8.68368353E-00
RU=3.0*3.0*1.0*0.0*0.0  6.44680604E+00
  6.37205942E-02  -6.49663916E-02  7.61543319E+01
  1.22125139E-02  2.78729717E+01  1.07473337E-02
  2.03368553E+01  9.99465555E-03  1.75132772E+01

NFLAGG=1
$END
$COBDE
NTYPE=3*0.12  1*3*0.3*1
NTYPE(9)=0
IMODE=1*2*3*4
LRCOST=0*0*0
$END
ENDBLK
$COBIN DMAP=0.02  LOADOP=1  P=0.05  PSMALL=.05  STEP=1.0  DSTEP=.2
PRINT=0  NPRINT=1
XMIN=16*1.0-5* XMAX=16*1.05  NITER=1
$END
ENDRUN
### QD030 SAMPLE LISTING

**QD030**

**TEST CASE QD030**

```
*INDATA
NTRAN=-4*KDENZ=2*B1=0.*DT=.1*C=10.*
KVECT=1.*
FOVCT=6.7844* 16.67686*
NCASE=4*8*9*10*11*12*13*
NFREQ=8*9*10*11*12*13*
RD(7)=-1.* PF(7)=-1.*
RJ=0.*0.*2.*3.*0.*0.*3.*2768E-3*1.24416*1.24416*1.24416*
RK=1.*0.*0.*4.*0.*0.*1.1271E-4*6.63981*7.5*24576.*24576.*24576.*
RL=0.*0.*0.*2.*0.*0.*1.15947*0.081906*1.16122*
PM=0.*0.*1.*2.*0.*0.*0.*74*0.0606948*0.0426265*
PN=0.*0.*1.*2.*0.*0.*0.*74*0.0606948*0.0426265*
RJ(7)=0.15*3*0.3333*
RK(7)=2.84E-3*2.96*1.3*3*0.3333*
RL(7)=1.4*0.110592*0.110592*
PM(7)=0.13108*2.*0.08333*
PN(7)=0.833973*2.*0.083333*
RJ(7)=.095*
RK(7)=1.562E-3*
PL(7)=.882*
RM(7)=.26*
RH(7)=.525*
RJ(7)=.0191232*4.24404*1.2595*1.8983*
RK(7)=13.4E-4*6.15*1.75*7.5*0.751834*1.728*1.728*
PL(7)=1.59526*10.0786*2.0736*
RH(7)=1.65888*6.29319*0.04608*
RN(7)=.397672*0.302956*0.0262533*
```
QD030 SAMPLE LISTING (Cont'd)

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APPENDIX A. THE SIMPLEX ALGORITHM

The purpose of this appendix is to illustrate the mechanics of the Simplex Algorithm. This will be done in Section 1 via an example problem. Dantzig [1] is the author of the Simplex Algorithm, which is a method for solving Linear Programming problems in a finite number of steps. Ficken [2] and Wilde and Beightler [6] also contain very good illustrations and definitions of the mechanics of the Simplex Algorithm. Section 2 of this appendix shows the setup of the initial tableau of the Simplex Algorithm as it is mechanized in COE BRA.

Section 1. Simplex Algorithm Example Problem

The problem is to maximize the cost function

\[ Y = 3X_1 + 2X_2 \]

subject to the following constraints:

- Constraint #1: \( 2X_1 + X_2 \leq 8 \)
- Constraint #2: \( X_1 + 3X_2 \leq 15 \)
- and: \( X_1, X_2 \geq 0 \)

The feasible region defined by these constraints is illustrated in Figure A.1.

Step #1

The first step begins with denoting \( X_1 \) and \( X_2 \) as so-called decision variables. Next, so-called slack variables (\( X_3 \) and \( X_4 \)) are introduced to turn the inequality constraints #1 and #2 into equality constraints.

- Constraint #1: \( 2X_1 + X_2 + X_3 = 8 \) (Eq. A.1)
- Constraint #2: \( X_1 + 3X_2 + X_4 = 15 \) (Eq. A.2)

In addition, it is now required that \( X_1, X_2, X_3 \) and \( X_4 \) all be greater than zero. Next, instead of
Step #2 (Optimal Solution)

\[ 2X_1 + X_2 = 8 \]

Step #1

\[ X_1 + 3X_2 = 15 \]

Feasible Region

Increasing \( Y \)

FIGURE A.1 THE EXAMPLE PROBLEM
maximizing \( Y \), change the cost function to one of minimizing \( Z \) which will be defined as the negative of \( Y \). Hence,

\[
Z = -3X_1 - 2X_2
\]

At this point, let us call \( X_3 \) and \( X_4 \) so-called state variables. Note that \( Z \) is not a function of these state variables.

As a first feasible solution, choose \( X_1 = X_2 = 0 \).

From equations A.1 and A.2 (constraints #1 and #2), it is seen that the first value for \( X_3 \) is 8, and the first value for \( X_4 \) is 15. Also, initially \( Y = 0 \). Now,

\[
\frac{\partial Z}{\partial X_1} = -3
\]

which says that \( Z \) decreases as \( X_1 \) increases. This is desirable. Also,

\[
\frac{\partial Z}{\partial X_2} = -2
\]

which says that \( Z \) decreases as \( X_2 \) increases. At each step, the rule is to choose the decision variable that yields the largest payoff. Since \( X_1 \) yields the fastest rate of decrease, it will be increased from zero. With \( X_2 \) remaining at zero, constraint #1 says that \( X_1 \) can be increased to a value of 4 before \( X_3 \) is driven to zero. Also, with \( X_2 = 0 \), constraint #2 says that \( X_1 \) can be increased to 15 before \( X_4 \) is driven to zero. Since all the variables must remain greater than or equal to zero, we can only increase \( X_1 \) to 4. With \( X_2 = 0 \), and \( X_1 = 4 \), \( Y = 12 \), and Step 1 is complete. Step 1 is illustrated in Figure A.1.

Step #2

Since \( X_3 \) is now zero, we must "control" it to keep it greater than or equal to zero. Hence, we interchange the role of \( X_1 \), and \( X_3 \), and now call \( X_3 \) a decision variable and \( X_1 \), a state variable. Another of the basic rules of the Simplex Algorithm
is to keep the cost function ($Z$), a function of only the decision variables. Solving equation A.1 (constraint #1) for $X_1$ yields:

Constraint #1: $X_1 = 4 - 0.5X_2 - 0.5X_3$  (Eq. A.3)

Substituting $X_1$ into $Z$ yields:

$$Z = -12 - 0.5X_2 + 1.5X_3$$

Substituting $X_1$ into equation A.2 (constraint #2) yields:

Constraint #2: $2.5X_2 - 0.5X_3 + X_4 = 11$ (Eq. A.4)

Summarizing, the decision variables are now:

$$X_2 = 0$$
$$X_3 = 0$$

The state variables are now:

$$X_1 = 4$$
$$X_4 = 11$$

Now:

$$\frac{\partial Z}{\partial X_2} = -0.5$$
$$\frac{\partial Z}{\partial X_3} = 1.5$$

At this point, in order to further reduce $Z$, we must hold $X_3$ to zero, and increase $X_2$.

From equation A.3 (constraint #1) we can increase $X_2$ to 8 before driving $X_1$ to zero. From equation A.4 (constraint #2), we can increase $X_2$ to 4.4 before driving $X_4$ to zero. Hence, we can increase $X_2$ only to 4.4. Step 2 is now complete, with $X_2 = 4.4$, $X_4 = 0$, $X_3 = 0$, and $Z = -14.2$. Step 2 is illustrated in Figure A.1.
Step #3

With $X_4 = 0$, we must now interchange the roles of $X_4$ and $X_2$ by now calling $X_4$ a decision variable, and $X_2$ a state variable. In order to keep $Z$ a function of the decision variables, we must solve equation A.4 (constraint #2) for $X_2$:

Constraint #2: $X_2 = 4.4 + 0.2X_3 - 0.4X_4$ (Eq. A.5)

Substituting $X_2$ into $Z$ yields:

$Z = -14.2 + 1.4X_3 + 0.2X_4$

Substituting $X_2$ into equation A.3 (constraint #1) yields:

Constraint #1: $X_1 = 1.8 - 0.6X_3 + 0.2X_4$ (Eq. A.6)

Now:

$$\frac{\partial Z}{\partial X_3} = 1.4$$

$$\frac{\partial Z}{\partial X_4} = 0.2$$

At this point, since $X_3$ and $X_4$ are both zero, $Z$ cannot be further reduced, and the optimal solution has been obtained. From equations A.5 and A.6 (constraints #1 and #2), the optimal solution is:

$$X_1 = 1.8$$

$$X_2 = 4.4$$

and $$Z = -14.2$$

or $$Y = 14.2$$

Figure A.1 verifies that this is indeed the optimal solution.
Section 2. Initial Tableau of the Simplex Algorithm

The Simplex Algorithm solves the following problem. Find the maximum value of the linear cost function:

\[ Y = \sum_{j=1}^{n} a_j X_j \]

subject to a matrix of \( m \) linear constraint equations

\[ \sum_{j=1}^{n} b_{ij} X_j \begin{cases} \leq \sum_{j=1}^{n} C_i \\
\geq \sum_{j=1}^{n} C_i \end{cases} \quad (i = 1, \ldots, m) \]

and all \( X_j \geq 0 \).

In COEBRA, \( X_j \) is the \( j^{th} \) stand-alone autopilot variable. Also in COEBRA, the constraint equations are a mixture of " \( \leq \) " and " \( \geq \) " type constraints. As shown in Wilde and Beightler [6], the Simplex Algorithm uses slack and artificial variables in addition to the original variables \( (X_j) \). Slack variables are used for the " \( \leq \) " constraints. Slack and artificial variables are used for the " \( \geq \) " constraints. In the final solution, the artificial variables must have zero value in order for the solution to be feasible.

Table A.1 shows the setup used in COEBRA for the initial tableau of the Simplex Algorithm. The first row is for the cost function coefficients. The next \( n \) rows are for the constants \( C_i \) and the coefficients \( (b_{ij}) \) in the " \( \leq \) " autopilot variable constraint equations. The next \( K \) rows are for the " \( \leq \) " so-called margin constraint equations. Following these, are \( M \) rows for the " \( \geq \) " so-called margin constraint equations. The last \( n \) rows are for the " \( \geq \) " autopilot variable constraint equations. If NPRINT = 1, this tableau will be printed out for each entry into the routine that solves the Simplex Algorithm.
### TABLE A.1 INITIAL TABLEAU OF THE SIMPLEX ALGORITHM

<table>
<thead>
<tr>
<th>1st Column</th>
<th>n Columns</th>
<th>(n + K)Columns</th>
<th>(n + M) Columns</th>
<th>(n + M)Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint Equation Constants</td>
<td>Autopilot Variables ($X_i$) for all time points in the order in which they are input</td>
<td>Slack variables for the &quot;( \leq )&quot; constraint equations.</td>
<td>Artificial variables for the &quot;( \geq )&quot; constraint equations</td>
<td>Slack variables for the &quot;( \geq )&quot; constraint equations</td>
</tr>
<tr>
<td>1st Row</td>
<td>---</td>
<td>Cost function coefficients ($a_j$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n Rows</td>
<td>($C_i$)</td>
<td>Coefficients for the &quot;( \leq )&quot; autopilot variable constraint equations ($b_{ij}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K Rows</td>
<td>($C_i$)</td>
<td>Coefficients for the &quot;( \leq )&quot; so-called margin constraint equations ($b_{ij}$)</td>
<td></td>
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<tr>
<td>M Rows</td>
<td>($C_i$)</td>
<td>Coefficients for the &quot;( \geq )&quot; so-called margin constraint equations ($b_{ij}$)</td>
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<tr>
<td>n Rows</td>
<td>($C_i$)</td>
<td>Coefficients for the &quot;( \geq )&quot; autopilot variable constraint equations ($b_{ij}$)</td>
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</table>

In the notation used when the tableau is printed out:
- \( L = \) total number of constraint equations
  - \( = 2n + K + M \)
  - \( K1 = n + K \)
  - \( K2 = 0 \)
  - \( K3 = n + M \)
APPENDIX B.  BIBLIOGRAPHY


