Effects of Propagation Parallel to the Magnetic field on the Type I Electrojet Irregularity Instability

K. Lee* and C. F. Kennel
Department of Physics

and

Institute of Geophysics and Planetary Physics

PPG-137 November 1972
Effects of Propagation Parallel to the Magnetic field on the Type I Electrojet Irregularity Instability

K. Lee* and C. F. Kennel

Department of Physics

and

Institute of Geophysics and Planetary Physics

PPG-137 November 1972

This work was supported by NASA grant NGL 05-007-190 S4 and NSF grant GA-34148X.

*Present address: Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544
EFFECTS OF PROPAGATION PARALLEL TO THE MAGNETIC FIELD
ON THE TYPE I ELECTROJET IRREGULARITY INSTABILITY

K. Lee* and C.F. Kennel
Institute of Geophysics and Planetary Physics
University of California
Los Angeles, California 90024

ABSTRACT

A simple analysis indicates that Type I irregularities which have a slight component of propagation along the magnetic field may be more unstable than those which propagate across the field. Since these waves have very large group velocities, detailed ray tracing would be required to establish their true convective amplification. Nevertheless, there remains the possibility that significant irregularity amplitudes may occur at the northern or southern extremities of the equatorial electrojet from those modes with large north-south group velocity, and furthermore, they could significantly change our understanding of nonlinear solutions of the electrojet instability.

Publication No. 1076, Institute of Geophysics and Planetary Physics, University of California, Los Angeles.

*Present address: Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544.
The purpose of this note is to point out several interesting features of the instability commonly thought responsible for Type I irregularities in the equatorial electrojet (Buneman, 1963; Farley, 1963) when propagation parallel to the magnetic field lines is allowed for. The dispersion relation is derived in Appendix A using fluid theory

\[ \omega = \frac{k \cdot V_{ed} + \frac{\alpha}{1 + \alpha} k \cdot V_{id} + i \frac{k^2 c_T^2}{\nu_i} \left( \frac{\nu_e}{1 + \alpha} \right)^2 - 1}{1 + \alpha} \]  

(1)

where the notation is defined in Appendix A. \( Q^2 = \left[ \frac{k (V_{ed} - V_{id})}{k c_T} \right]^2 \) parameterizes the strength of the instability. Equation (1) differs from the \( k \parallel = 0 \) result in a transparent fashion, since the effective electron collision frequency \( \nu_e \) is

\[ \nu_e = \nu_e \left[ 1 + \frac{k \| \Omega_e}{k^2} \right] \quad \hat{\alpha} = \frac{\nu_e \nu_i}{\Omega_e \nu_i} \left[ 1 + \frac{\Omega_e k^2}{\nu_e k^2} \right] \]  

(2)

Since \( Q^2 / \nu_e^2 >> 1 \) in the E-region even small \( k \| / k \) leads to \( \nu_e >> \nu_e \). Most investigations of the electrojet instability have assumed \( k \| = 0 \), because the instability threshold for \( k \| \neq 0 \) requires a larger electrojet drift \( Q \).

Of course, eq. (1) indicates that the threshold for instability, given by \( Q^2 = (1 + \alpha)^2 \) increases with \( k \| \), and has a minimum, \( Q^2 = 1 + \alpha \), when \( k \| = 0 \). However, when \( Q^2 \) exceeds \( (1 + \alpha) \) it is not true that the most rapidly growing modes occur for \( k \| = 0 \). Figure 1, which shows a plot of \( \frac{1}{k^2 c_T} \) as a function of \( \hat{\alpha} \), must be interpreted...
as follows. $\hat{a}$ must exceed $a$, and $Q$ must exceed $(1+a)$ for instability. Therefore, if the positive maximum growth rate occurs at $\hat{a} > a$, the fastest growing mode has $k_{\parallel} \neq 0$. A convenient approximate expression for $\hat{a}_m$ is

$$\hat{a}_m = \frac{1}{2} \frac{Q^2 - 1}{Q^2 + 1}$$

so that the fastest growing mode has $k_{\parallel} \neq 0$ if $\frac{1}{2} \frac{Q^2 - 1}{Q^2 + 1} > a$. Since $a \approx 0.1$ at 110 km in the equatorial electrojet, it is easy to destabilize $k_{\parallel} \neq 0$ modes in the upper electrojet. The parallel wavenumber of the maximally growing wave is given by

$$k_{\parallel}^2 / k^2 = \left( \frac{v_e}{\Omega_e} \right)^2 \left[ \frac{1}{2} \frac{Q^2 - 1}{Q^2 + 1} - 1 \right]$$

which is ordinarily very small. As Kaw (1972) has pointed out, the electrojet must be treated as convectively unstable. To determine convective amplification lengths we must compute the group velocity parallel to the magnetic field

$$\frac{\partial \omega}{\partial k_{\parallel}} = -2 \frac{v_i}{\Omega} c_T \frac{Q}{(1+a)^2} \frac{k_{\parallel} \Omega_e}{v_e} k$$

$$= -2 \frac{v_i}{\Omega} c_T \frac{\cos \phi}{p (1+a)^2} \frac{k_{\parallel} \Omega_e}{v_e}$$

In a completely polarized vertically stratified electrojet model, appropriate only to very near the magnetic dip equator, the vertical ion and electron drifts are equal and the vertical group velocity $\partial \omega / \partial k_v$ is given by

$$\frac{\partial \omega}{\partial k_v} = \alpha V p \frac{(\Omega_e/k)^2}{(1+a)^2} \sin 2\phi + \frac{\partial \omega}{\partial k_v} (k_{\parallel} = 0)$$

$$= \frac{\alpha V p (\Omega_e/k)^2 \sin 2\phi}{(1+a)^2} + \frac{\Omega_i v_i}{v_e}$$

$$= \frac{\alpha V p (\Omega_e/k)^2 \sin 2\phi}{(1+a)^2} + \frac{\Omega_i v_i}{v_e}$$
where $\frac{\partial \omega}{\partial k_v}(k_\parallel = 0)$ denotes the vertical group velocity when $k_\parallel = 0$, which is ordinarily an order of magnitude smaller than $V_p$, the magnitude of the horizontal east-west electron drift velocity. In Eq. (5) above, $\varphi = \tan^{-1}\left(\frac{k_v}{k_n}\right)$, where $k_n$ denotes the horizontal component of the wave vector. Thus, when $(n_e k_\parallel)^2 > 1$

$$\frac{\partial \omega}{\partial k_v}(k_\parallel = 0) < \frac{\partial \omega}{\partial k_v}(k_\parallel \neq 0) << \frac{\partial \omega}{\partial k_\parallel}$$

(6)

with $\frac{\partial \omega}{\partial k_\parallel}$ larger than $\frac{\partial \omega}{\partial k_v}(k_\parallel \neq 0)$ by about an order of magnitude. Therefore, while the growth rate can peak at $k_\parallel \neq 0$, the group velocities may also increase, which increases the convective amplification length. Since the electrojet scale lengths in the vertical direction and along the magnetic field differ by an order of magnitude, for $(n_e k_\parallel)^2/k > 1$ the increase with increasing $k_\parallel \neq 0$ of the vertical group velocity is as significant as that of the parallel group velocity. Careful ray tracing in good models of the electrojet with latitude structure included (Untiedt, 1967; Sugiura and Poros, 1969) is required to determine the actual convective amplification of these models. However, these simple estimates lead to the speculations that: (1) not all the unstable models in the electrojet have $k_\parallel = 0$; and (2) since observations of the electrojet indicate that $Q^2 > 1 + \alpha$ much of the time, there could be amplitude maxima of irregularities at the north-south extremities of the equatorial electrojet arising from waves with small $k_\parallel/k$ propagating along the field lines. Finally, since the interesting $k_\parallel/k \sim n_e/\Omega_e \ll 1$, the waves discussed here will be essentially undistinguishable
experimentally from those with $k_\parallel = 0$; however, they might significantly change our understanding of the nonlinear solutions of the electrojet instability. For example, unstable waves with $k_\parallel \neq 0$ could carry off considerable wave energy otherwise available for saturation.
REFERENCES


At a given altitude, \( \alpha = \frac{\nu_e \nu_i}{\Omega_e \Omega_i} \) is given. Then, \( \hat{\alpha} = \alpha \left[ 1 + \frac{k \Omega_e^2}{\nu_e^2} \right] \) must exceed \( \alpha \). Thus, only the right hand portions of these curves are relevant. If \( Q > 1 + \alpha \), instability is possible. The maximum growth rate occurs at \( \hat{\alpha}_m = \frac{Q^2 - 1}{Q^2 + 1} \). If \( \hat{\alpha}_m > \alpha \), the fastest growing mode has \( k \neq 0 \); if \( \hat{\alpha}_m < \alpha \), the fastest growing mode has \( k = 0 \).
Appendix A

FLUID DISPERSION RELATION FOR UNIFORM PLASMA

The dispersion relation will be derived from the following fluid equations

$$N_j M_j \left( \frac{\partial}{\partial t} + \mathbf{v}_j \right) \cdot \nabla \mathbf{v}_j = - \mathbf{v}_p + q_j N_j (E + \frac{\mathbf{v}_j \times B_0}{c})$$

$$- \mathbf{v}_j N_j M_j \mathbf{v}_j$$

$$\frac{\partial N_j}{\partial t} + \nabla \cdot N_j \mathbf{v}_j = 0$$

$$P_j = N_j T_j$$

$$\nabla \cdot E = 4\pi q_e (N_i - N_e)$$

$$E = E_S + E'(r, t)$$

The subscript j (i, e) refers to ions or electrons; $N_j$ = the number density, $M_j$ = the mass; $\mathbf{v}_j$ = the fluid velocity, $P_j$ = the pressure, $T_j$ = the temperature, $q_j$ = the charge, $E$ = the electric field, $B_0$ = the magnetic field, and $\nu_j$ = the neutral collision frequency. The neutrals have been assumed immobile. The steady state solution of the above equations are
\( N_0 = \text{constant} \)

\[
\nu_j = \frac{\varepsilon_j \Omega_j \tau_j}{1 + \Omega_j^2 \frac{\tau_j^2}{B_0}} + \frac{\Omega_j^2 \Gamma_j^2}{1 + \Omega_j^2 \frac{\tau_j^2}{B_0}} \text{cE}_s \times B_0 \\
\quad (A.1)
\]

\( j = i, e \)

\( \varepsilon_i = 1; \quad \varepsilon_e = -1 \)

and where \( E_s \) is the static electric field linearizing the above fluid equations and Fourier transforming in space and time, with 
\( k = \text{wave vector and } \omega = \text{frequency} \)

\[
(-i \omega + ik \cdot \nu_j + \nu_j + \varepsilon_j \Omega_j \varepsilon B_0 x) \nu_j ' (k, \omega) \\
= \frac{\varepsilon_j E_j}{M_j} E'(k, \omega) - \frac{ik c_j^2 N_j (k, \omega)}{N_0} \\
\quad (A.2)
\]

\[
N_j/N_0 = \frac{k \cdot \nu_j ' (k, \omega)}{\omega - k \cdot \nu_j} \\
\quad (A.3)
\]

\[
i k E'(k, \omega) = 4 \pi e (N_i ' - N_e ') \\
\quad (A.4)
\]

where \( e_B = \frac{B_0}{B_0}; \quad c_j = \frac{T_j}{M_j} = \frac{T}{M_j}. \)
Define $e_{||} = \frac{k_{||}}{k_{||}}; \quad e_{\perp} = \frac{k_{\perp}}{k_{\perp}}; \quad \hat{e}_{\perp} = e_{\perp} \times e_{B0}$

$$v_{j}' = (e_{||} \cdot v_{j}')e_{||} + (e_{\perp} \cdot v_{j}')e_{\perp} + (\hat{e}_{\perp} \cdot v_{j}')\hat{e}_{\perp}$$

where $k_{||}$ and $k_{\perp}$ are components of the propagation vector parallel and perpendicular to the magnetic field respectively.

Taking the scalar product of (A.2) with $e_{||}$ gives

$$-i\omega_j e_{||} \cdot v_{j}' + ik_{||}c_{j} \frac{2(k \cdot v_{j}')}{\omega_j} = \frac{ee_{||}}{M_j} e_{||} \cdot E' \tag{A.5}$$

where $\omega_j = \omega - k \cdot v_{jd}; \quad \omega_j' = \bar{\omega}_j + iv_j$

Taking the scalar product of (A.2) with $e_{\perp}$ gives

$$-i\omega_j (e_{\perp} \cdot v_{j}') + c_{j}\hat{e}_{j} (\hat{e}_{\perp} \cdot v_{j}')e_{\perp} \cdot (e_{B0} \times e_{\perp})$$

$$+ ik_{\perp}c_{j} \frac{2(k \cdot v_{j}')}{\omega_j}$$

$$= \frac{ee_{\perp}}{M_j} e_{\perp} \cdot E' \tag{A.6}$$

Taking the scalar product of (A.2) with $\hat{e}_{\perp}$ gives

$$-i\omega_j (\hat{e}_{\perp} \cdot v_{j}') + c_{j}(e_{\perp} \cdot v_{j}')e_{\perp} \cdot (e_{B0} \times e_{\perp}) = 0 \tag{A.7}$$
Note that $\mathbf{e}_1 \cdot [\mathbf{e}_B \times (\mathbf{e}_1 \times \mathbf{e}_B)] = 1$. (A.7) then becomes

$$
(\hat{\mathbf{e}}_1 \cdot v_j') = -\frac{\epsilon_j \Omega_j}{i\omega_j} (\mathbf{e}_1 \cdot v_j') \tag{A.8}
$$

Substituting (A.8) into (A.6)

$$
\left[-i\omega_j' - Q_j^2 - i\frac{k_2^2 c_j^2}{\omega_j'} + \frac{i k_2^2 c_j^2}{\omega_j} \right] (\mathbf{e}_1 \cdot v_j') + ik_2 k_2 (\mathbf{e}_1 \cdot v_j') \frac{c_j^2}{\omega_j'} = \frac{e\epsilon_j}{M_j} (\mathbf{e}_1 \cdot \mathbf{E}') \tag{A.9}
$$

Using (A.5) and (A.9) to eliminate $(\mathbf{e}_1 \cdot v_j')$ gives

$$
\left\{ \frac{k_2^2 k_{11} c_j^4}{\omega_j} + \left[ -i\omega_j' - Q_j^2 - i\frac{k_2^2 c_j^2}{\omega_j} \right] \left[ -i\omega_j' + \frac{i k_2^2 c_j^2}{\omega_j} \right] \right\} (\mathbf{e}_1 \cdot v_j') = -\frac{e\epsilon_j}{M_j} \left[ ik_2 k_2 (\mathbf{e}_1 \cdot \mathbf{E}') + \left[ i\omega_j' - Q_j^2 + \frac{i k_2^2 c_j^2}{\omega_j} \right] (\mathbf{e}_1 \cdot \mathbf{E}') \right] \tag{A.10}
$$

Eliminating $(\mathbf{e}_1 \cdot v_j')$ from (A.5) and (A.9) gives

$$
\left\{ \frac{k_2^2 k_{11} c_j^4}{\omega_j} + \left[ -i\omega_j' + \frac{Q_j^2}{i\omega_j'} - i\frac{k_2^2 c_j^2}{\omega_j} \right] \left[ -i\omega_j' + \frac{i k_2^2 c_j^2}{\omega_j} \right] \right\} (\mathbf{e}_1 \cdot v_j') = -\frac{e\epsilon_j}{M_j} \left[ ik_2 k_2 (\mathbf{e}_1 \cdot \mathbf{E}') + \left[ i\omega_j' - i\frac{k_2^2 c_j^2}{\omega_j} \right] (\mathbf{e}_1 \cdot \mathbf{E}') \right] \tag{A.11}
$$
Since only electrostatic perturbations are being considered, 

\[ E'(k) = \frac{e}{k}, \text{(A.8), (A.10), and (A.11)} \]

can be combined to give

\[
\nu'_j = \frac{-e\epsilon_j}{M_j} E' \left\{ \left[ \frac{\Omega_j^2 - \omega'_j}{\omega_j} \right] - \frac{k_{\parallel}^2 c_j^2 \Omega_j^2}{\omega_j} + \frac{\omega'_j - k_{\parallel}^2 c_j^2}{\omega_j} \right\}^{-1}
\]

\[
\cdot \left\{ - \frac{\Omega_j^2}{k_{\parallel}} \frac{k_{\parallel}^2}{k} \epsilon_j \Omega_j \frac{k_{\parallel} \times \epsilon B_0}{k} \right\} \text{(A.12)}
\]

Combining (A.3), (A.4), and (A.12)

\[
1 = \sum_{j=i,e} \omega_{pj}^2 \left\{ \frac{\Omega_j^2 - \omega'_j}{\omega_j} \right\} \left\{ \frac{\Omega_j^2 - \omega'_j}{\omega_j} \right\} \nu'_j - k_{\parallel}^2 c_j^2
\]

\[
\cdot \left\{ \frac{k_{\parallel}^2}{k^2 \Omega_j - \omega'_j} \right\}^{-1}
\text{(A.13)}
\]

where \( \omega_{pj}^2 = \frac{4\pi N_0 e^2}{M_j} \)

In the ionospheric E-layer \( \nu_i >> \Omega_i \), so that the ion Hall drift can be neglected to lowest orders. For long wavelengths and low frequencies, unity may be neglected in (A.13). Furthermore, we assume \( \bar{\omega}_i, \bar{\omega}_e << \nu_e, \nu_i \). Since \( \nu_e >> \nu_i \), first order terms in \( \bar{\omega}_i/\nu_i \) will be kept; those in \( \bar{\omega}_e/\nu_e \) will be dropped. With these approximations (A.13) reduces to
\[ \frac{\omega^2_{pi}}{\omega_i(\omega_i + i\nu_i)} - k^2 c_i^2 \quad = \quad \frac{1\omega^2_{pc} \hat{\nu}_e}{\Omega_e^2/\nu_e + ik^2 c_e^2 \nu_e} \quad (A.14) \]

where \( \hat{\nu}_e = \nu_e \left[ 1 + k^2 / k^2 \Omega_e^2 / \nu_e^2 \right] \). The quadratic (A.14) has one damped solution, and the growing solution is given by

\[ \omega = \frac{k \cdot \nu_{ed}}{1 + \hat{\alpha}} + \frac{1 \cdot \nu_e k^2 c_T^2}{\Omega_e \Omega_i^{(1+\hat{\alpha})} \left[ \left( \frac{1}{1+\hat{\alpha}} \right)^2 - 1 \right]} \quad (A.15) \]

where \( \hat{\alpha} = \frac{\nu_e \nu_i}{\Omega_e \Omega_i} \), \( c_T^2 = \frac{T_e + T_l}{M_i} \) and \( Q = \frac{k \cdot (\nu_{ed} - \nu_{id})}{k c_T} \).
<table>
<thead>
<tr>
<th>Report Number</th>
<th>Title</th>
<th>Authors/Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-1</td>
<td>&quot;Propagation of Ion Acoustic Waves Along Cylindrical Plasma Columns&quot;</td>
<td>A.Y. Wong (July 1965)</td>
</tr>
<tr>
<td>R-2</td>
<td>&quot;Stability Limits for Longitudinal Waves in Ion Beam-Plasma Interaction&quot;</td>
<td>B.D. Fried and A.Y. Wong</td>
</tr>
<tr>
<td>R-3</td>
<td>&quot;The Kinetic Equation for an Unstable Plasma in Parallel Electric and Magnetic Fields&quot;</td>
<td>B.D. Fried and S.L. Osakow</td>
</tr>
<tr>
<td>R-5</td>
<td>&quot;Effects of Collisions on Electrostatic Ion Cyclotron Waves&quot;</td>
<td>A.Y. Wong, D. Judd and F. Hai</td>
</tr>
<tr>
<td>R-6</td>
<td>&quot;Interaction Between Ion Beams and Plasmas&quot;</td>
<td>R. Rowberg, A.Y. Wong and J.M. Sellen</td>
</tr>
<tr>
<td>R-7</td>
<td>&quot;Observation of Cyclotron Echoes from a Highly Ionized Plasma&quot;</td>
<td>D.E. Kaplan and R.M. Hill</td>
</tr>
<tr>
<td>R-8</td>
<td>&quot;Excitation and Damping of Drift Waves&quot;</td>
<td>A.Y. Wong and R. Rowberg</td>
</tr>
<tr>
<td>R-9</td>
<td>&quot;The Guiding Center Approximation in Lowest Order&quot;</td>
<td>Alfredo Baños, Jr.</td>
</tr>
<tr>
<td>R-10</td>
<td>&quot;Plasma Streaming into a Magnetic Field&quot;</td>
<td>S.L. Ossakow (November 1966)</td>
</tr>
<tr>
<td>R-11</td>
<td>&quot;Cooperative Effects in Plasma Echo Phenomena&quot;</td>
<td>A.Y. Wong (March 1966)</td>
</tr>
<tr>
<td>R-12</td>
<td>&quot;A Quantum Mechanical Study of the Electron Gas Via the Test Particle Method&quot;</td>
<td>M.E. Rensink (March 1967)</td>
</tr>
<tr>
<td>R-14</td>
<td>&quot;The Expansion and Diffusion of an Isolated Plasma Column&quot;</td>
<td>J. Hyman (May 1967)</td>
</tr>
<tr>
<td>R-15</td>
<td>&quot;Two-pole Approximation for the Plasma Dispersion Function&quot;</td>
<td>B.D. Fried, C.L. Hedrick and J. McCune</td>
</tr>
<tr>
<td>R-17</td>
<td>&quot;Parametric Coupling Between Drift Waves&quot;</td>
<td>F. Hai, R. Rowberg and A.Y. Wong</td>
</tr>
<tr>
<td>R-18</td>
<td>&quot;Cyclotron Echoes from Doppler Effects&quot;</td>
<td>A.Y. Wong (March 1968)</td>
</tr>
<tr>
<td>R-19</td>
<td>&quot;Ion Wave Echoes&quot;</td>
<td>D.R. Baker, N.R. Ahern and A.Y. Wong</td>
</tr>
<tr>
<td>R-20</td>
<td>&quot;Cyclotron Echoes in Plasmas&quot;</td>
<td>D. Judd, Thesis (March 1968)</td>
</tr>
<tr>
<td>R-21</td>
<td>&quot;Test Particle Theory for Quantum Plasmas&quot;</td>
<td>M.E. Rensink (October 1967)</td>
</tr>
<tr>
<td>R-23</td>
<td>&quot;Landau Damping of Ion Acoustic Waves in a Cesium Plasma with Variable Electron-Ion Temperature Ratio&quot;</td>
<td>K.B. Rajangam (October 1967)</td>
</tr>
<tr>
<td>R-24</td>
<td>&quot;The Inhomogeneous Two-Stream Instability&quot;</td>
<td>G. Knorr (September 1967)</td>
</tr>
<tr>
<td>R-25</td>
<td>&quot;Magnetic Turbulence in Shocks&quot;</td>
<td>C.F. Kennel and H.E. Petschek</td>
</tr>
<tr>
<td>R-26</td>
<td>&quot;Small Amplitude Waves in High Beta Plasmas&quot;</td>
<td>V. Formisano and C. Kennel (February 1968)</td>
</tr>
</tbody>
</table>
R-27 "Low Beta Plasma Penetration Across a Magnetic Field", B.D. Fried and S. Ossakow (March 1968)†
R-29 "The Theorist's Magnetosphere", C. Kennel (April 1968)
R-31 "Electromagnetic Echoes in Collisionless Plasmas", A.Y. Wong (April 1968)*
R-32 "Parametric Excitation of Drift Waves in a Resistive Plasma", G. Weyl and M. Goldman (June 1968)†
R-33 "Parametric Excitation from Thermal Fluctuations at Plasma Drift Wave Frequencies", A.Y. Wong, M.V. Goldman, F. Hai, R. Rowberg (May 1968)*
R-34 "Current Decay in a Streaming Plasma Due to Weak Turbulence", S.L. Ossakow and B.D. Fried (June 1968)†
R-35 "Temperature Gradient Instabilities in Axisymmetric Systems", C.S. Liu (August 1968)†
R-37 "Transverse Plasma Wave Echoes", B.D. Fried and Craig Olson (October 1968)†
R-38 "Low Frequency Interchange Instabilities of the Ring Current Belt", C.S. Liu (January 1969)†
R-39 "Drift Waves in the Linear Regime", R.E. Rowberg and A.Y. Wong (February 1969)*
R-41 "Nonlinear Oscillatory Phenomena with Drift Waves in Plasmas", F. Hai and A.Y. Wong (September 1970)
R-42 "Ion-Burst Excited by a Grid in a Plasma", H. Ikezi and R.J. Taylor (February 1969)
R-43 "Measurements of Diffusion in Velocity Space from Ion-Ion Collisions", A. Wong and D. Baker (March 1969)*
R-44 "Nonlinear Excitation in the Ionosphere", A.Y. Wong (March 1969)
R-46 "A New Representative for the Conductivity Tensor of a Collisionless Plasma in a Magnetic Field", B.D. Fried and C. Hedrick (March 1969)†
R-47 "Direct Measurements of Linear Growth Rates and Nonlinear Saturation Coefficients", A.Y. Wong and F. Hai (April 1969)*
R-49 "Auroral Micropulsation Instability", F. Coroniti and C.F. Kennel (May 1969)†
R-50 "Effect of Fokker-Planck Collisions on Plasma Wave Echoes", G. Johnston (June 1969)†
R-52 "Theory of Stability of Large Amplitude Periodic (BGK) Waves in Collisionless Plasmas", M.V. Goldman (June 1969)†
R-55 "Optical Mixing in a Magnetoactive Plasma", G. Weyl (August 1969)†
R-56 "Trapped Particles and Echoes", A.Y. Wong and R. Taylor (October 1969)†
R-60 "Efficient Modulation Coupling Between Electron and Ion Resonances in Magnetoactive Plasmas", A. Wong, D.R. Baker, N. Booth (December 1969)*
R-63 "Perturbed Ion Distributions in Ion Waves and Echoes", H. Ikezi and R. Taylor (January 1970)*
R-67 "Dispersion Discontinuities of Strong Collisionless Shocks", F.V. Coroniti (March 1970)†
R-68 "An Ion Cyclotron Instability", E.S. Weibel (April 1970)†
R-72 "A Note on the Differential Equation $g'' + x^2 g = 0$", E.S. Weibel (April 1970)
R-74 "The UC Mathematical On-Line Systems as a Tool for Teaching Physics", B.D. Fried and R. White (August 1970)†
R-75 "High Frequency Hall Current Instability", K. Lee, C.F. Kennel, J.M. Kindel (August 1970)†
R-76 "Laminar Wave Train Structure of Collisionless Magnetic Slow Shocks", F.V. Coroniti (September 1970)†
R-77 "Field Aligned Current Instabilities in the Topside Ionosphere", J.M. Kindel and C.F. Kennel (August 1970)†
R-78 "Spatial Cyclotron Damping", Craig Olson (September 1970)
R-79 "Electromagnetic Plasma Wave Propagation Along a Magnetic Field", C.L. Olson (September 1970)†
R-83 "Nonlinear Collisionless Interaction between Electron and Ion Modes in Inhomogeneous Magnetoactive Plasmas", N. Booth (December 1970)*
R-84 "Observations of Parametrically Excited Ion Acoustic Waves", R. Stenzel (March 1971)
R-85 "Remote Double Resonance Coupling of Radar Energy to Ionospheric Irregularities", C.F. Kennel (January 1971)
R-86 "Ion Acoustic Waves in a Multi-Ion Plasma", B.D. Fried, R. White, T. Samec (January 1971)
R-87 "Current-Driven Electrostatic and Electromagnetic Ion Cyclotron Instabilities", D.W. Forslund, C.F. Kennel, J. Kindel (February 1971)
R-88 "Locating the Magnetospheric Ring Current", C.F. Kennel and Richard Thorne (March 1971)
R-89 "Ion Acoustic Instabilities Due to Ions Streaming Across Magnetic Field", P.J. Barrett, R.J. Taylor (March 1971)
R-90 "Evolution of Turbulent Electronic Shocks", A.Y. Wong and R. Means (July 1971)*
R-91 "Density Step Production of Large Amplitude Collisionless Electrostatic Shocks and Solitons", David B. Cohen (June 1971)
R-95 "3-D Velocity Space Diffusion in Beam-Plasma Interaction without Magnetic Field", P.J. Barrett, D. Gresillon and A.Y. Wong (September 1971)
PPG-96 "Dayside Auroral Oval Plasma Density and Conductivity Enhancements due to Magnetosheath Electron Precipitation", C.F. Kennel and M.H. Rees (September 1971)
PPG-97 "Collisionless Wave-Particle Interactions Perpendicular to the Magnetic Field", A.Y. Wong, D.L. Jassby (September 1971)
PPG-98 "Magnetospheric Substorms", F.V. Coroniti and C.F. Kennel (September 1971)
PPG-100 "Structure of Ion Acoustic Solitons and Shock Waves in a Two-Component Plasma", R.B. White, B.D. Fried and F.V. Coroniti (September 1971)
PPG-101 "Solar Wind Interaction with Lunar Magnetic Field", G. Siscoe (Meteorology Dept.) and Bruce Goldstein (JPL) (November 1971)
PPG-102 "Changes in Magnetospheric Configuration During Substorm Growth Phase", F.V. Coroniti and C.F. Kennel (November 1971)


PPG-108 "Threshold and Saturation of the Parametric Decay Instability," R. Stenzel and A. Y. Wong, November 1971


PPG-112 "Polarization of the Auroral Electrojet," F. V. Coroniti and C. F. Kennel, February

PPG-113 "Mode Coupling and Wave Particle Interactions for Unstable Ion Acoustic Waves," Pablo Martin and Burton D. Fried, February 1972


PPG-116 "Large Diameter, Quiescent Plasma in a Magnetospheric Field," Earl Ault, Thesis, April 1972


PPG-124 "Calculation of Reflection and Transmission Coefficients for a Class of One-Dimensional Wave Propagation Problems in Inhomogeneous Media," A. Baños, Jr., September 1972

PPG-125 "Electromagnetic Wave Functions for Parabolic Plasma Density Profiles," A. Baños, Jr. and D. L. Kelly, September 1972


PPG-128 "Can the Ionosphere Regulate Magnetospheric Convection?" F. V. Coroniti and C. F. Kennel, October, 1972