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EFFECTS OF PROPAGATION PARALLEL TO THE MAGNETIC FIELD
ON THE TYPE I ELECTROJET IRREGULARITY INSTABILITY

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ABSTRACT

A simple analysis indicates that Type I irregularities which have a slight component of propagation along the magnetic field may be more unstable than those which propagate across the field. Since these waves have very large group velocities, detailed ray tracing would be required to establish their true convective amplification. Nevertheless, there remains the possibility that significant irregularity amplitudes may occur at the northern or southern extremities of the equatorial electrojet from those modes with large north-south group velocity, and furthermore, they could significantly change our understanding of nonlinear solutions of the electrojet instability.

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The purpose of this note is to point out several interesting features of the instability commonly thought responsible for Type I irregularities in the equatorial electrojet (Buneman, 1963; Farley, 1963) when propagation parallel to the magnetic field lines is allowed for. The dispersion relation is derived in Appendix A using fluid theory

\[ \omega = \frac{k \cdot V_{ed}}{1 + \alpha} + \frac{\hat{\alpha} k}{1 + \alpha} V_{id} + i \frac{k^2 c_T^2}{\nu_i} \frac{\hat{\alpha}}{1 + \alpha} \left[ \frac{Q^2}{(1 + \alpha)^2} - 1 \right] \]  

(1)

where the notation is defined in Appendix A. \( Q^2 = \left[ \frac{k (V_{ed} - V_{id})}{k c_T} \right]^2 \)

parameterizes the strength of the instability. Equation (1) differs from the \( k_{||} = 0 \) result in a transparent fashion, since the effective electron collision frequency \( \hat{\nu}_e \) is

\[ \hat{\nu}_e = \nu_e \left[ 1 + \frac{2 \Omega_e}{k^2} \right] \]

\[ \alpha = \frac{\nu_e \nu_i}{\Omega_e \Omega_i} \left[ 1 + \frac{2 k^2}{\nu_e} \right] \]  

(2)

Since \( Q^2/\nu_e^2 \gg 1 \) in the E-region even small \( k_{||}/k \) leads to \( \hat{\nu}_e \gg \nu_e \).

Most investigations of the electrojet instability have assumed \( k_{||} = 0 \), because the instability threshold for \( k_{||} \neq 0 \) requires a larger electrojet drift \( Q \).

Of course, eq. (1) indicates that the threshold for instability, given by \( Q^2 = (1 + \alpha)^2 \) increases with \( k_{||} \), and has a minimum, \( Q^2 = 1 + \alpha \), when \( k_{||} = 0 \). However, when \( Q^2 \) exceeds \( (1 + \alpha) \) it is not true that the most rapidly growing modes occur for \( k_{||} = 0 \). Figure 1, which shows a plot of \( \frac{\nu_i \text{ Im} \omega}{k^2 c_T} \) as a function of \( \alpha \), must be interpreted...
as follows. \( \hat{a} \) must exceed \( a \), and \( Q \) must exceed \((1+\alpha)\) for instability. Therefore, if the positive maximum growth rate occurs at \( \hat{a} > a \), the fastest growing mode has \( k_{\parallel} \neq 0 \). A convenient approximate expression for \( \hat{a}_m \) is

\[
\hat{a}_m = \frac{1}{2} \frac{Q^2 - 1}{Q^2 + 1}
\]

so that the fastest growing mode has \( k_{\parallel} \neq 0 \) if \( \frac{1}{2} \frac{Q^2 - 1}{Q^2 + 1} > \alpha \). Since \( \alpha \approx 0.1 \) at 110 km in the equatorial electrojet, it is easy to destabilize \( k_{\parallel} \neq 0 \) modes in the upper electrojet. The parallel wavenumber of the maximally growing wave is given by

\[
k_{\parallel}^2/k^2 = \left( \frac{\nu_i}{\Omega_e} \right)^2 \left[ \frac{1}{2\pi} \left( \frac{Q^2 - 1}{Q^2 + 1} \right) - 1 \right]
\]

which is ordinarily very small. As Kaw (1972) has pointed out, the electrojet must be treated as convectively unstable. To determine convective amplification lengths we must compute the group velocity parallel to the magnetic field

\[
\frac{\Delta \omega}{\Delta k_{\parallel}} = -2 \frac{\nu_i}{\Omega_e} c_T \frac{Q}{(1+\alpha)^2} \frac{k_{\parallel}}{k} \frac{\Omega_e}{\nu_i}
\]

\[
= -2 \left( \frac{\nu_i}{\nu} \right) \frac{\cos \theta}{p} \left( k_{\parallel} \frac{\Omega_e}{\nu} \right)
\]

In a completely polarized vertically stratified electrojet model, appropriate only to very near the magnetic dip equator, the vertical ion and electron drifts are equal and the vertical group velocity \( \Delta \omega/\Delta k_v \) is given by

\[
\frac{\Delta \omega}{\Delta k_v} = \alpha V \frac{\nu_e}{p} \left( \frac{\Omega_e}{\nu_e} \frac{k_{\parallel}}{k} \right) \sin \phi + \frac{\Delta \omega}{\Delta k_v} (k_{\parallel} = 0)
\]

\[
= \frac{\alpha V}{p} \left( \frac{\nu_e}{\nu_e} \frac{k_{\parallel}}{k} \right) \sin \phi + \frac{\nu_i}{\nu} \frac{\Omega_e}{p}
\]
where $\frac{\partial \omega}{\partial k_v}$ $k_\parallel = 0$ denotes the vertical group velocity when $k_\parallel = 0$, which is ordinarily an order of magnitude smaller than $V_p$, the magnitude of the horizontal east-west electron drift velocity. In Eq. (5) above, $\varphi = \tan^{-1}\left(\frac{k_v}{k_n}\right)$, where $k_n$ denotes the horizontal component of the wave vector. Thus, when \( \frac{\Omega_e}{v_e} k_\parallel^2 \approx 1 \)

\[
\frac{\partial \omega}{\partial k_v}(k_\parallel = 0) < \frac{\partial \omega}{\partial k_v}(k_\parallel \neq 0) << \frac{\partial \omega}{\partial k_\parallel}
\]  

(6)

with $\frac{\partial \omega}{\partial k_\parallel}$ larger than $\frac{\partial \omega}{\partial k_v}(k_\parallel \neq 0)$ by about an order of magnitude. Therefore, while the growth rate can peak at $k_\parallel \neq 0$, the group velocities may also increase, which increases the convective amplification length. Since the electrojet scale lengths in the vertical direction and along the magnetic field differ by an order of magnitude, for $\left(\frac{\Omega_e}{v_e}k_\parallel/k > 1\right)$ the increase with increasing $k_\parallel \neq 0$ of the vertical group velocity is as significant as that of the parallel group velocity. Careful ray tracing in good models of the electrojet with latitude structure included (Untiedt, 1967; Sugiura and Poros, 1969) is required to determine the actual convective amplification of these models. However, these simple estimates lead to the speculations that: (1) not all the unstable models in the electrojet have $k_\parallel = 0$; and (2) since observations of the electrojet indicate that $Q^2 > 1 + \alpha$ much of the time, there could be amplitude maxima of irregularities at the north-south extremities of the equatorial electrojet arising from waves with small $k_\parallel/k$ propagating along the field lines. Finally, since the interesting $k_\parallel/k - v_e/\Omega_e << 1$, the waves discussed here will be essentially indistinguishable.
experimentally from those with $k_\parallel = 0$; however, they might significantly change our understanding of the nonlinear solutions of the electrojet instability. For example, unstable waves with $k_\parallel \neq 0$ could carry off considerable wave energy otherwise available for saturation.
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At a given altitude, \( \alpha = \frac{v_{e}v_{i}}{\Omega_{e}\Omega_{i}} \) is given. Then, \[
\hat{\alpha} = \alpha \left[ 1 + \frac{k_{\|}^{2}}{\frac{k^{2}}{2} \frac{\Omega^{2}}{v_{e}^{2}}} \right]
\]
must exceed \( \alpha \). Thus, only the right hand portions of these curves are relevant. If \( Q \geq 1 + \alpha \), instability is possible. The maximum growth rate occurs at \( \hat{\alpha}_{m} = \frac{Q^{2} - 1}{Q^{2} + 1} \). If \( \hat{\alpha}_{m} > \alpha \), the fastest growing mode has \( k_{\|} \neq 0 \); if \( \hat{\alpha}_{m} < \alpha \), the fastest growing mode has \( k_{\|} = 0 \)
Appendix A

FLUID DISPERSION RELATION FOR UNIFORM PLASMA

The dispersion relation will be derived from the following fluid equations

\[ N_j M_j \left( \frac{\partial}{\partial t} + \mathbf{v}_j \right) \cdot \nabla \mathbf{v}_j = -\nabla P_j + q_j N_j \left( E + \frac{\mathbf{v}_j \times \mathbf{B}_0}{c} \right) \]

\[ - \mathbf{v}_j N_j M_j \mathbf{v}_j \]

\[ \frac{\partial N_j}{\partial t} + \nabla \cdot N_j \mathbf{v}_j = 0 \]

\[ P_j = N_j T_j \]

\[ \nabla \cdot \mathbf{E} = 4\pi e (N_i - N_e) \]

\[ \mathbf{E} = \mathbf{E}_s + \mathbf{E}'(r, t) \]

The subscript \( j \) (i, e) refers to ions or electrons; \( N_j \) = the number density, \( M_j \) = the mass; \( \mathbf{v}_j \) = the fluid velocity, \( P_j \) = the pressure, \( T_j \) = the temperature, \( q_j \) = the charge, \( \mathbf{E} \) = the electric field, \( \mathbf{B}_0 \) = the magnetic field, and \( \nu_j \) = the neutral collision frequency. The neutrals have been assumed immobile. The steady state solution of the above equations are
\[ \mathcal{N}_0 = \text{constant} \]

\[ \nu_j = \frac{\varepsilon_j \Omega_j \tau_j c E_s}{1 + \Omega_j \tau_j} \frac{c E_s}{B_0} + \frac{\Omega_j^2 \tau_j^2 c E_s \times B_0}{1 + \Omega_j \tau_j} \frac{c E_s}{B_0^2} \quad (A.1) \]

\[ j = i, e \]

\[ \varepsilon_i = 1; \quad \varepsilon_e = -1 \]

and where \( E_s \) is the static electric field linearizing the above fluid equations and Fourier transforming in space and time, with \( k = \) wave vector and \( \omega = \) frequency

\[ (-i \omega + i k \cdot \nu_j d + \nu_j + \varepsilon_j \Omega_j e_B x^j) \nu_j'(k, \omega) \]

\[ = \frac{e \varepsilon_j E'(k, \omega)}{M_j} - \frac{i k c_j^2 N_j'(k, \omega)}{N_0} \quad (A.2) \]

\[ \frac{N_j}{N_0} = \frac{k \cdot \nu_j'(k, \omega)}{\omega - k \cdot \nu_j d} \quad (A.3) \]

\[ i k E'(k, \omega) = 4 \pi e (N_i' - N_e') \quad (A.4) \]

where \( e_B = \frac{B_0}{B_0} \); \( c_j = \frac{T_j}{M_j} = \frac{T}{M_j} \).
Define $e_\parallel = \frac{k_\parallel}{k}; \quad e_\perp = \frac{k_\perp}{k}; \quad \hat{e}_\perp = e_\perp \times e_{B0}$

$$v_j' = (e_\parallel \cdot v_j')e_\parallel + (e_\perp \cdot v_j')e_\perp + (\hat{e}_\perp \cdot v_j')\hat{e}_\perp$$

where $k_\parallel$ and $k_\perp$ are components of the propagation vector parallel and perpendicular to the magnetic field respectively.

Taking the scalar product of (A.2) with $e_\parallel$ gives

$$-i\omega_j \cdot e_\parallel \cdot v_j' + ik_\parallel c_j \frac{2(k \cdot v_j')}{\omega_j} = \frac{e\epsilon_j}{M_j} e_\parallel \cdot E'$$  \hspace{1cm} (A.5)

where $\omega_j = \omega - k \cdot v_j d; \quad \omega_j' = \bar{\omega}_j + i\nu j$

Taking the scalar product of (A.2) with $e_\perp$ gives

$$-i\omega_j \cdot (e_\perp \cdot v_j') + \epsilon_j n_j (\hat{e}_\perp \cdot v_j')e_\perp \cdot (e_{B0} \times \hat{e}_\perp)$$

$$+ ik_\perp c_j \frac{2(k \cdot v_j')}{\omega_j}$$

$$= \frac{e\epsilon_j}{M_j} e_\perp \cdot E'$$  \hspace{1cm} (A.6)

Taking the scalar product of (A.2) with $\hat{e}_\perp$ gives

$$-i\omega_j \cdot (\hat{e}_\perp \cdot v_j') + \epsilon_j (e_\perp \cdot v_j')e_\perp \cdot (e_{B0} \times e_\perp) = 0$$  \hspace{1cm} (A.7)
Note that $e_\perp \cdot \left[ e_0 \times (e_\perp \times e_0) \right] = 1$. (A.7) then becomes

$$\left( e_\perp \cdot v_j \right) = \frac{-e_j \Omega_j}{i\omega_j} (e_\perp \cdot v_j) \tag{A.8}$$

Substituting (A.8) into (A.6)

$$\left[-i\omega_j - \frac{Q_j^2}{i\omega_j} + \frac{ik_\perp c_j^2}{\omega_j}\right] (e_\perp \cdot v_j) + ik_\parallel k (e_\parallel \cdot v_j) \frac{c_j^2}{\omega_j} =$$

$$\frac{ee_j}{M_j} (e_\perp \cdot E') \tag{A.9}$$

Using (A.5) and (A.9) to eliminate $(e_\perp \cdot v_j')$ gives

$$\left\{ \frac{k_\parallel k_\perp c_j^4}{\omega_j^2} + \left[-i\omega_j - \frac{Q_j^2}{i\omega_j} + \frac{ik_\perp c_j^2}{\omega_j}\right] \left[-i\omega_j + \frac{ik_\parallel c_j^2}{\omega_j}\right] \right\} (e_\parallel \cdot v_j')$$

$$= -\frac{ee_j}{M_j} \left( i k_\parallel k_\perp c_j^2 \frac{e_\perp \cdot E'}{\omega_j} + \left[i\omega_j + \frac{Q_j^2}{i\omega_j} + \frac{ik_\parallel c_j^2}{\omega_j}\right] (e_\perp \cdot E') \right) \tag{A.10}$$

Eliminating $(e_\parallel \cdot v_j')$ from (A.5) and (A.9) gives

$$\left\{ \frac{k_\parallel k_\perp c_j^4}{\omega_j^2} + \left[i\omega_j + \frac{Q_j^2}{i\omega_j} + \frac{ik_\parallel c_j^2}{\omega_j}\right] \left[i\omega_j - \frac{ik_\parallel c_j^2}{\omega_j}\right] \right\} (e_\perp \cdot v_j')$$

$$= \frac{-ee_j}{M_j} \left( i k_\parallel k_\perp c_j^2 \frac{e_\perp \cdot E'}{\omega_j} + \left[i\omega_j - \frac{ik_\parallel c_j^2}{\omega_j}\right] (e_\perp \cdot E') \right) \tag{A.11}$$
Since only electrostatic perturbations are being considered, 
$E'(k) = E\frac{k}{k}$, (A.8), (A.10), and (A.11) can be combined to give

$$v_j' = \frac{-e\epsilon_j}{M_j} E' \left\{ \left[ \frac{\Omega_j^2 - \omega_j^2}{\omega_j} \right] \frac{k_j^2 c_j^2}{\omega_j \omega_j} + \frac{\omega_j'}{\omega_j k_j^2 c_j^2} \right\}^{-1}$$

$$+ \left\{ - \left[ \Omega_j^2 - \omega_j^2 \right] \frac{i k_{||}^2}{k \omega_j^2} + i \omega_j \frac{k_j}{k} - \epsilon_j \Omega_j \frac{k_j \times \epsilon_{B_0}}{k} \right\}$$

(A.12)

Combining (A.3), (A.4), and (A.12)

$$1 = \sum_{j=i, e} w^2_{pj} \left\{ \left[ \Omega_j^2 - \omega_j^2 \right] \left[ \Omega_j^2 - \omega_j^2 \right] - \omega_j^2 \right\} \left\{ \left[ \Omega_j^2 - \omega_j^2 \right] \omega_j v_j - k_j^2 c_j^2 \right\}$$

$$\cdot \left\{ \frac{k_j^2}{k_j^2 \Omega_j^2 - \omega_j^2} \right\}^{-1}$$

(A.13)

where $w^2_{pj} = \frac{4\pi N_0 e^2}{M_j}$

In the ionospheric E-layer $v_i \gg \Omega_i$, so that the ion Hall drift can be neglected to lowest orders. For long wavelengths and low frequencies, unity may be neglected in (A.13). Furthermore, we assume $w_i, \bar{w}_e << v_e, v_i$. Since $v_e \gg v_i$, first order terms in $\bar{w}_i/v_i$ will be kept; those in $\bar{w}_e/v_e$ will be dropped. With these approximations (A.13) reduces to
where \( \hat{v}_e = v_e \left[ 1 + \frac{k_i^2}{k^2} \frac{\Omega_e^2}{\nu_e^2} \right] \). The quadratic (A.14) has one damped solution, and the growing solution is given by

\[
\omega = \frac{k \cdot \nu_{ed}}{1 + \alpha} + \frac{\alpha}{1 + \alpha} \nu_{ld} + \frac{1}{\Omega_e \Omega_i (1 + \alpha)} \left[ \frac{\Omega_e^2}{\nu_e^2} \right] ^2 \left[ \left( \frac{Q}{1 + \alpha} \right)^2 - 1 \right] \tag{A.15}
\]

where \( \hat{\alpha} = \frac{\nu_e \nu_i}{\Omega_e \Omega_i} \), \( c_T^2 = \frac{T_e + T_l}{M_l} \) and \( Q = \frac{k \cdot (\nu_{ed} - \nu_{ld})}{kc_T} \).
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