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Reports of the Department of Geodetic Science

Report No. 192

REFRACTION EFFECTS OF ATMOSPHERE ON GEODETIC MEASUREMENTS TO CELESTIAL BODIES

by

S. Joshi

CASE FILE COPY

Prepared for

National Aeronautics and Space Administration
Washington, D.C.

Contract No. NGR 36-008-093
OSURF Project No. 2514

The Ohio State University
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Columbus, Ohio 43212

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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and is under the technical direction of James P. Murphy, Special Programs, Code ES, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546.

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I take great pleasure in thanking the United Nations Organization for granting me a fellowship; and the Government of India, in particular the Department of Survey of India, for its permission for my study.

I am thankful to my wife, Sudesh, who has constantly inspired me to complete this work before returning to our home country.

Finally, I thank Miss Martha Borror and Miss Barbara Beer for typing. Last but not the least, I thank Mrs. Evelyn Rist, who was most helpful in providing various references, and who typed part of the formulae/equations in this thesis.

This report was submitted to the Graduate School of the Ohio State University as partial fulfillment of the requirements for the degree Master of Science.

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1. INTRODUCTION

1.1 General

The apparent displacement of an object that results from light rays from a source outside the atmosphere being bent in passing through the atmosphere, is defined as Astronomic Refraction [Mueller and Rockie, 1966, p. 90]. This results in all objects appearing to be higher above the horizon than they actually are. It is also called Celestial Refraction.

The apparent displacement of an object located within the effective atmosphere resulting from light rays being bent in passing through the atmosphere is defined as Atmospheric Refraction.

Also in the reference quoted above, Electronic Refraction is defined as "The refraction due to the effect of the atmosphere and the ionosphere, which introduces appreciable changes in the quantities measured by means of electronic devices, such as in the phase differences measured with interferometers, in the rate of change of phase measured with the Doppler systems and in the change in phase between the times of transmitting and receiving a signal by the ranging instruments."

All of our measurements for the above quantities are, of necessity, to be made through the atmosphere of the earth. This atmosphere is not at all homogeneous but its composition continuously changes with place and time. The non-uniformity of air density (and hence the refractive index) due to the complex composition of the atmosphere introduces

continuous change in the direction and velocity of propagation of light or radio waves passing through the atmosphere. Thus, the observed values of the quantities mentioned in the definitions given above are not what they would have been if there had been vacuum in place of variable atmosphere.

There is perhaps no branch of practical astronomy on which so much has been said and investigated as on this and it is not in a completely satisfactory state. The theoretical difficulties arising from the uncertainty and variability of the density of the atmosphere and the absence of any exact analytical relationship governing it offer main obstructions to the solution of refraction integrals.

Before the advent of artificial satellites and development of electronic techniques the problem was mainly confined to the astronomic refraction. Consequently, various investigators of the Nineteenth Century and early Twentieth Century worked on this very aspect. First, on the basis of an admixture of speculation regarding the constitution of atmosphere, since the meteorological data available by then was scanty. As more and more meteorological data became available, better models were devised on these bases.

Since 1960s, the increasing accuracy of geodetic instruments and development of artificial satellite methods for various geodetic applications, have opened a new era in which refraction of radio waves in the ionosphere also comes into play. The new techniques like laser ranging, very long base line interferometry (VLBI), satellite radar altimetry and direct mapping of gravity field of the earth through range rate measurements, demand a new look at the measurements to spatial

bodies. With accuracies of the order of 5 cm in laser ranging, 0.05 mm/sec in satellite to satellite range rate, 0".001 in VLBI measurements expected to be attained within a decade, it is essential that accurate values of refraction corrections be available. In fact, in most cases it is the refraction correction which is the main obstruction to attaining such accuracies.

It is in this context that an attempt is being made in this report to review the progress which has been made so far and the possibilities which could provide solution to the problem in the near future.

1.2 Arrangement of the report.

The report is being arranged in the following manner.

In the theoretical considerations we shall first state the basic principles of optics, governing the phenomenon of refraction and shall derive differential equations for the refraction corrections under two main subheads: 1. Refraction effects due to change in the direction of propagation, and 2. Refraction effects mainly due to change in the velocity of propagation, of a ray of light or other radiation (e.g., radio waves, etc.) propagating on the principle of wave theory. Then will follow a short description of the atmosphere (including ionosphere, etc.) regarding the factors affecting propagation of light/radio waves.

Next, under the practical considerations, we shall review the various assumptions made by the earlier investigators, and then better empirical relationships being available, the basic principles of improved models designed by the investigators of the Twentieth

century. For this purpose discussion will be divided into two groups as shown in Table 1.1.

The above discussion will be followed by a review of the accuracy problem for various quantities. Next, the future trends and in that the elimination of refraction effect from range, etc., measurements will be discussed.

The report will finally be concluded by summing up conclusions and enlisting recommendations.

Table 1.1

Division of Refraction Effects into Groups

Effect due to variation in	Measurements employing	Type of bodies	Affecting zones	Corrections to basic quantities observed for geodetic accuracies
Direction of propagation	Visible light (optical frequencies)	Natural celestial	Troposphere	Zenith angle (or elevation)
		Artificial celestial	Troposphere	Zenith angle (or elevation)
Velocity of propagation	Radio frequencies	Artificial celestial	Troposphere and ionosphere	Range Doppler Shift Range and Range Rate
		Natural celestial	Troposphere and ionosphere	Phase difference (VLBI)
	Optical frequencies	Artificial celestial	Troposphere	Range
		Natural celestial	Troposphere	Range

2. NOTATION

The following notations are used in this report for various quantities. Exceptions to these will be explained in the report, where applicable. The subscript o is used, in general for the quantities pertaining to the place of observations:

n - refractive index

$\bar{n}$ - refractive index of air under standard conditions (0° C, 760 mm of Mercury, 0.03% Carbon Dioxide).

n_g - group refractive index

N - refractivity = $(n-1)\ 10^6$

u - phase velocity of light or microwave in air

λ - wavelength

f - frequency

z - zenith angle

Δz - refraction correction to zenith angle

S - range

ΔS - refraction correction to range

E - elevation angle = $90^o - z$

T - absolute temperature

P - total pressure of air

e - partial pressure of water vapor

r - radius vector from the center of the earth

H - height above sea level

h - height relative to the place of observations

h_o - height of homogeneous atmosphere above the place of observation

g - acceleration due to gravity

α - coefficient of expansion of air

R - gas constant

$\bar{R}$ - radius of the earth

3. THEORETICAL CONSIDERATIONS

3.1. Laws of Refraction

For any refracting medium the propagation of light or other electromagnetic wave is governed by the basic physical law, formulated by Fermat more than three centuries ago, called FERMAT'S PRINCIPLE, which states that light, for example, will follow that path between two fixed points involving the least travelling time. [Hotine, 1969, p. 209]. Also, if c is the velocity of light in vacuum, v is its velocity in the medium, then the refractive index n of the medium is related by

$$n = \frac{c}{v} \tag{1}$$

From Fermat's principle we deduce the following two laws of refraction:

(i) Considering A and B two points (Fig. 1) in media of refractive indices n_1, n_2, respectively. Let A Q B be a ray of light between them making angles θ_1 and θ_2 as the angles of incidence and of refraction as shown in the figure, then time t for the ray to travel from

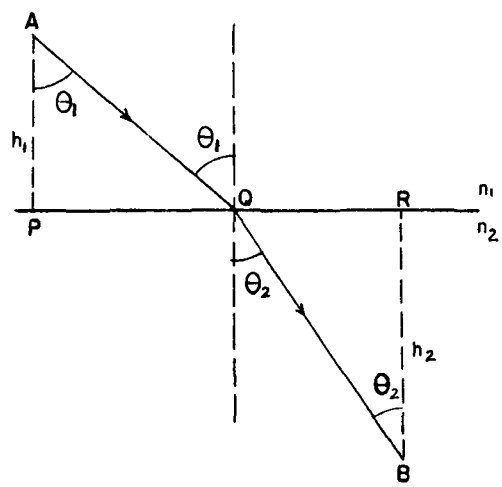

Fig. 1.- Laws of Refraction

A to B is given by

$$t = \frac{AQ}{v_1} + \frac{QB}{v_2}$$

$$= \frac{n_1 AQ}{c} + \frac{n_2 QB}{c} \quad \text{by (1)}$$

$$t = \frac{n_1 h_1}{c \cos \theta_1} + \frac{n_2 h_2}{c \cos \theta_2}$$

According to Fermat's Principle this time is to be minimum. Thus, differentiating and equating to zero, we have (remembering that h_1, h_2, are constant for the above points):

$$c\,dt = n_1 h_1 \sec\theta_1 \tan\theta_1 \, d\theta_1 + n_2 h_2 \sec\theta_2 \tan\theta_2 \, d\theta_2 = 0 \qquad (2)$$

Also, for the above two points we have distance PQ + QR as constant:

But

$$PQ = h_1 \tan\theta_1 \qquad QR = h_2 \tan\theta_2$$

Therefore,

$$h_1 \tan\theta_1 + h_2 \tan\theta_2 = \text{constant}$$

Differentiating we get:

$$h_1 \sec^2\theta_1 \, d\theta_1 + h_2 \sec^2\theta_2 \, d\theta_2 = 0$$

Substituting for $d\theta_2$ from this in equation (2), we get

$$n_1 h_1 \sec\theta_1 \tan\theta_1 \, d\theta_1 - n_2 h_2 \sec\theta_2 \tan\theta_2 \, \frac{h_1 \sec^2\theta_1}{h_2 \sec^2\theta_2} \, d\theta_1 = 0$$

This on simplification gives:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (3)$$

which is Snell's Law.

(ii) Secondly, it is seen that the direct ray, the refrated ray and the perpendicular at the point of refraction lie in the same plane.

3.2 Differential Equation for Change of Direction

Now we substitute for the change in the direction of ray:

$$\theta_1 - \theta_2 = \Delta R$$

also

$$n_2 = n_1 + \Delta n$$

then (3) becomes

$$n_1 \sin\theta_1 = (n_1 + \Delta n) \sin(\theta_1 - \Delta R)$$

Dropping subscript 1 for n_1 and θ_1 we have

$$n \sin\theta = (n + \Delta n) \sin(\theta - \Delta R)$$

$$= (n + \Delta n)(\sin\theta \cos\Delta R - \cos\theta \sin\Delta R)$$

or

$$n \sin\theta = n[\sin\theta \cos\Delta R - \cos\theta \sin\Delta R] + \Delta n[\sin\theta \cos\Delta R - \cos\theta \sin\Delta R] \quad (4)$$

Now if A and B move closer to Q, so that their distance from Q is infinitesimal, then in the limit

$$\Delta n \to dn$$

$$\Delta R \to dR$$

$$\cos\Delta R \to 1$$

$$\sin\Delta R \to dR$$

$$\Delta n \sin\Delta R \to 0$$

Hence (4) becomes

$$n \sin\theta = n \sin\theta - n\, dR \cos\theta + dn \sin\theta$$

or

$$n\, dR \cos\theta = dn \sin\theta$$

i.e.

$$dR = \tan\theta \frac{dn}{n} \quad (5)$$

which is the differential equation for the change of direction of propagation of a ray in a medium in which refractive index is continuously changing from point to point. We may note here that this is a rigorous equation and no assumptions have been made so far.

3.3 Differential Equation for Change of Distance

Again, if A and B are two points in a medium of varying refractive index n, then the departure of the refractive index from unity will cause both a deviation of the ray from its straight line path (S_o) into a curved path (S); and a change in the velocity to v instead of its vacuum velocity c will result in the measured range being different from the geometric range S.

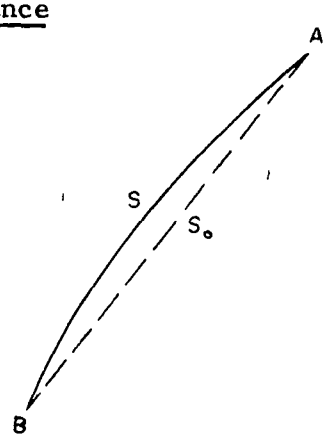

Fig. 2 - Path Difference of a Ray.

If T is the minimum time of travel from A to B, then the measured range S_m will be given by

$$S_m = cT$$

where c is the velocity of light/electromagnetic wave in vacuum. Also we have

$$T = \int_S dt = \int_S \frac{ds}{v}$$

$$= \frac{1}{c} \int_S nds \qquad \text{by (1)}$$

Therefore,

$$S_m = cT = \int_S nds$$

Hence error caused by refraction is

$$\Delta S = S_m - S_o = \int_S nds - S_o$$

Now

$$S_m = \int_S nds = \int_S (1 + n - 1)\, ds$$

$$= \int_S ds + \int_S (n - 1) ds = S + \int_S (n - 1)\, ds$$

consequently

$$\Delta S = S + \int_S (n - 1)\,ds - S_0$$

$$= (S - S_0) + \int_S (n - 1)\,ds$$

Where integration is along the actual path travelled, i.e., S. Here $(S-S_0)$ is the correction due to curvature of the ray from straight line and $\int_S (n - 1)\,ds$ is the retardation due to decrease in the velocity.

Several authors e.g., [Bean and Thayer, 1963] have shown that curvature effect is negligible above about 6° altitude. Since no range distances are measured at elevations less than 5°, the retardation effect only is taken into account. Then

$$\Delta S = \int_S (n - 1)\,ds \qquad (6)$$

which is the basic refraction integral for the correction to measured distance due to the variation in the velocity of propagation. If dS denotes the correction in the measured range due to propagation in an elemental distance ds, then equation (6) can be written as:

$$dS = (n - 1)\,ds \qquad (6a)$$

which is the differential equation for the correction to the measured range.

3.4 Earth's Atmosphere

The practice, initially, had been to regard the complex atmosphere of the earth by various models of constantly decreasing temperatures with height. This was according to the data then available. However, from the beginning of this Century, with more and

more meteorological data pouring in, various layers and the terms Troposphere, Stratosphere have been introduced.

In recent years, on the basis of evidence provided by radar wind-sounding balloons, radio wave investigations, and from rocket and satellite flights, the following division is put forward. Although there is a difference of opinion regarding terminology and extent of layers [Barry and Chorley, 1970, p. 37] give the composition as follows:

(i) Troposphere. It is the lowest layer of the atmosphere. It contains 75% of the total mass of the atmosphere and virtually all the water vapor and aerosols. It is characterized by large scale convective air movements and marked frontal activity involving the movement of fairly well identified air masses. There is a general decrease of temperature with height at an average rate of about 6.5° C/km. The whole zone is capped by a temperature inversion level (i.e., relatively warm air above cold air). This inversion level is called Tropopause, whose height varies with latitude, season and changes in surface pressure. Its height is about 16 km at the equator and about 8 km on the poles.

(ii) Stratosphere. This extends above Tropopause to about 50 km. It contains much of the atmospheric ozone reaching a peak density at about 22 km. It is free from water vapor and clouds. Although it used to be regarded as somewhat isothermal region, recent investigations show some marked seasonal changes in temperature and temperature increase with height, with a warm Stratopause enveloping it.

(iii) Mesosphere. Above Stratopause temperature decreases to about -90° C around 80 km. This layer is called Mesosphere. Above 80 km

temperatures, again, begin rising with height and this inversion is referred to as Mesopause.

The pressure is very low in the Mesosphere, decreasing from 1 mb at 50 km to 0.01 mb at 80 km.

(iv) Ionosphere (Thermosphere). From Mesopause upwards densities are extremely low. The lower portion of this layer consists mainly of nitrogen (N_2) and oxygen in molecular (O_2) and atomic (O) forms. Above 200 km atomic oxygen predominates over nitrogen. Temperatures rise with height owing to absorption of ultraviolet radiation by atomic oxygen, probably approaching 1200° K at 350 km. But these temperatures are essentially theoretical, e.g., artificial satellites do not acquire such temperatures because of the rarefied air.

Ultraviolet radiation from the sun and high energy particles from outer space (cosmic rays) enter the atmosphere above 100 km at high velocity and cause ionization, or electrical charging, by spearating negatively-charged electrons from oxygen atoms and nitrogen molecules.

(v) Exosphere and Magnetosphere. The base of exosphere is between 500 and 750 km. In this atomic oxygen, ionized oxygen, and hydrogen atoms form the tenuous atmosphere and the gas laws cease to be valid. Gas particles, especially helium with low atomic weight, can escape into space since chances of molecular collision to deflect them downwards become less with increasing height.

Neutral particles are predominant, but beyond about 2000 km, in the magnetosphere there are only electrons and protons and the earth's magnetic field becomes more important than gravity in their distribution.

While discussing refraction effects of the atmosphere the term Tropospheric correction is generally meant to include the effect of Troposphere, Stratosphere and Mesosphere; or in other words, the whole of nonionized atmosphere. Similarly, Ionospheric refraction effect is meant to include the effect of all the ionized atmosphere.

As we shall see later that the Troposphere, Stratosphere and Ionosphere are the regions which play the most prominent part in refraction of light/radio waves propagating trhough them.

In addition, we should not forget that the region immediately surrounding the observer is the most turbulent and may cause significant refraction errors.

4. PRACTICAL CONSIDERATION

4.1 Effect Due to Variation in Direction of Propagation

4.1.1 Spherically Symmetric Refractive Index

In order to tackle the complex atmosphere, almost universally-accepted practical assumption made is that the earth is regarded as a sphere and that the index of refraction n is radially symmetric, i.e., it is a function of r the distance of the point from the center of the earth. This also follows from the fact that as we go higher the density of air and, hence, the refractive index (which depends on the density) decreases.

In Figure 3, let SPP'O be a ray of light reaching an observer at O. C is the center of the earth, r, r' the radii vector to points P, P', and r_o the radius vector to the observer.

Then if index of refraction at P is n, after a differential path distance at P' it will be $n+dn$. The angles that ray makes with radius vectors at P and P' be z and $z+dz$, respectively. Also the differential element of refracted ray, i.e., P P' makes angle z' with radius vector at P.

Therefore, by (3)

$$n \sin z = (n + dn) \sin z' \qquad (7)$$

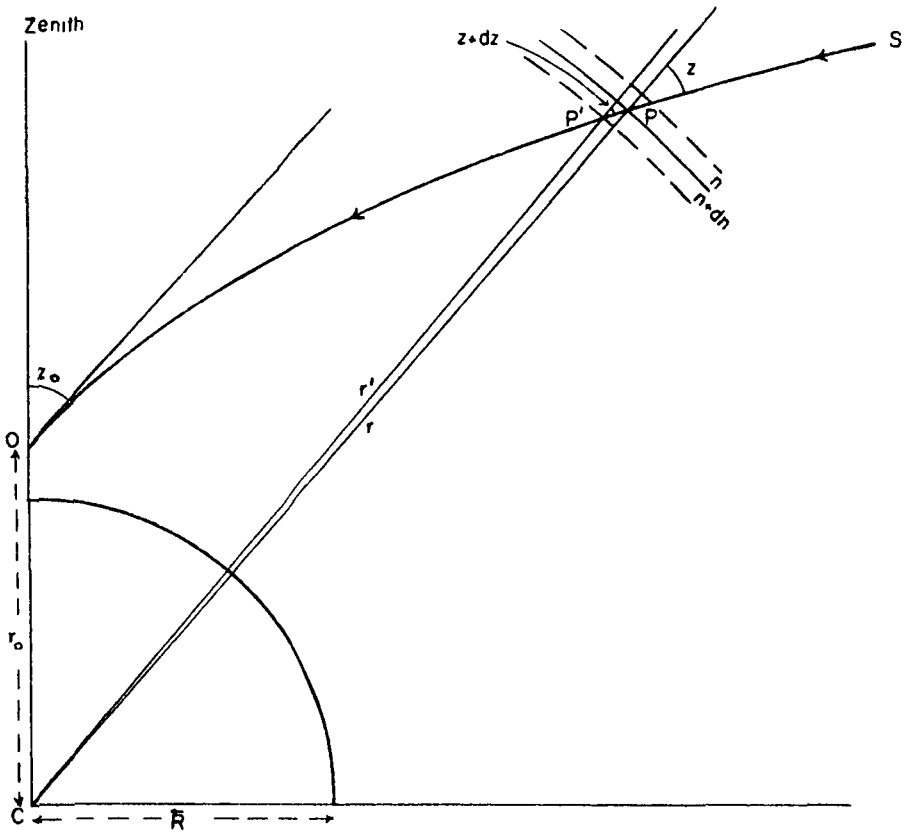

Fig. 3 Deviation in Direction of a Ray

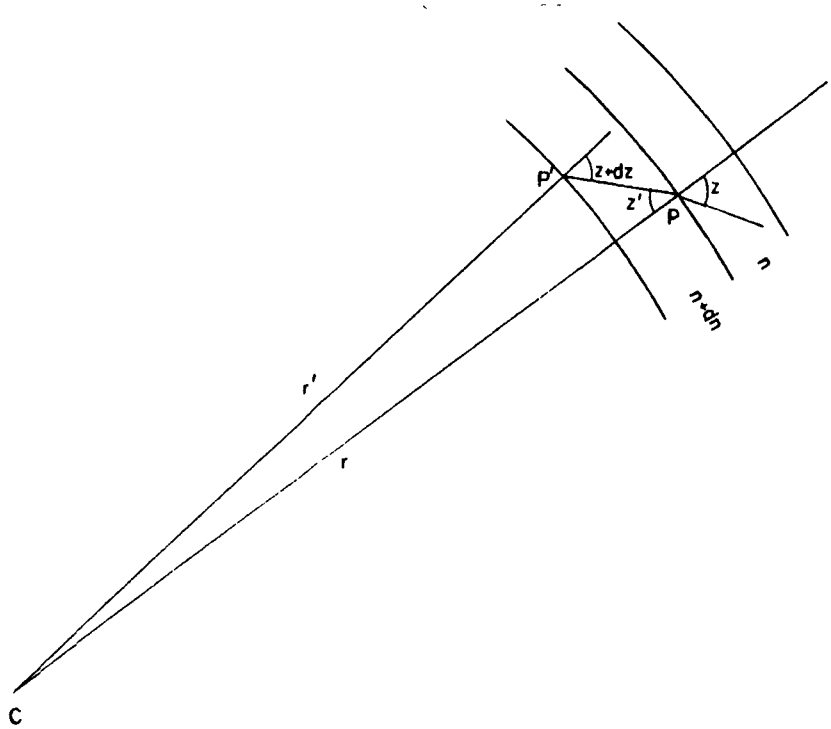

Fig. 3a Deviation in Direction by an Elemental Layer

From triangle CPP'

$$\frac{r}{\sin(z + dz)} = \frac{r'}{\sin z'}$$

eliminating $\sin z'$ with (7) we have

$$\frac{r}{\sin(z + dz)} = r' \frac{(n + dn)}{n \sin z}$$

or $\quad nr \sin z = (n + dn)\, r' \sin(z + dz)$

which gives the invariant

$$nr \sin z = \text{constant} \tag{8}$$

At the observer if $n = n_0$ then we have

$$nr \sin z = n_0 r_0 \sin z_0 \tag{9}$$

i.e.

$$\sin z = \frac{n_0 r_0}{nr} \sin z_0$$

then

$$\cos z = \left(1 - \frac{n_0^2 r_0^2}{n^2 r^2} \sin^2 z_0\right)^{\frac{1}{2}}$$

$$\tan z = \frac{n_0 r_0 \sin z_0}{(n^2 r^2 - n_0^2 r_0^2 \sin^2 z_0)^{\frac{1}{2}}} \tag{10}$$

Now differential equation (5) becomes in the notation of Figure (3) for point P as

$$dR = \tan z \frac{dn}{n} \tag{11}$$

since z the variable angle along the ray will not be known, we eliminate it between (10) and (11) and get

$$dR = \frac{n_0 r_0 \sin z_0}{(n^2 r^2 - n_0^2 r_0^2 \sin^2 z_0)^{\frac{1}{2}}} \frac{dn}{n}$$

Refraction correction for the observed zenith distance $\Delta z = \int dR$

$$\Delta z = n_0 r_0 \sin z_0 \int (n^2 r^2 - n_0^2 r_0^2 \sin^2 z_0)^{-\frac{1}{2}} \frac{dn}{n} \tag{12}$$

where the integral is taken from the limit of the effective atmosphere (for objects outside it) or from the object (if inside effective atmosphere) to the observer.

The equation (12) gives refraction correction in terms of variables n and r and the quantites n_0, r_0, and z_0 which could be known at the observer's position. If we knew the relation between n and r we could integrate (12) and get the correction. Since there is no such exact relationship, the integral is treated on the basis of various models, usually through development into series. The convergence of the series determines the upper limit of z_0, to which the formula is applicable.

Since refractive index depends on the density of the air in the atmosphere, the problem "boils down" to the modelling of density distribution with respect to r or height and a relationship between n and density ρ .

4.1.2 Atmosphere as perfect gas in hydrostatic equilibrium

For the variation of density in the atmosphere, almost universally the air is considered to obey the perfect gas law. The equation of state for a perfect gas being:

$$PV = RT \tag{13}$$

where P - is the pressure

V - volume of the gas

T - absolute temperature

R - the appropriate gas constant.

For unit volume, V can be replaced by $\frac{1}{\rho}$, hence

$$\frac{P}{\rho} = RT$$

where ρ - is the density

or $$\rho = \frac{P}{RT} \tag{14}$$

Differentiating (14) with respect to the height h we have

$$\frac{d\rho}{dh} = \rho\left[\frac{1}{P}\frac{dP}{dh} - \frac{1}{T}\frac{dT}{dh}\right] \tag{15}$$

From the hydrostatic equlibrium condition, if we consider a column of air of unit cross section and of infinitesimal height around a point where density is ρ and value of gravity is g, then pressure dP caused by it is given by

$$dP = -\rho g dh \tag{16}$$

the negative sign indicating that pressure decreases with height. Eliminating dP between (15) and (16) and using (14), we have

$$\frac{d\rho}{dh} = -\frac{\rho}{T}\left(\frac{g}{R} + \frac{dT}{dh}\right) \tag{17}$$

4.1.3 Classical Hypotheses of Atmospheric Density

Among the various hypotheses propounded [Newcomb, 1906, p. 183] we shall give three more important ones here.

(i) Newton's Hypothesis of constant temperature

Newton adopted that the temperature at all altitudes was constant. Thus making T constant, from (17) we get:

$$\frac{d\rho}{\rho} = -\frac{g}{RT}\,dh \tag{18}$$

Disregarding variation of g with altitude i.e. considering it constant and integrating (18):

$$\log \rho = -\frac{g}{RT} h + C$$

when $h = o$, $\rho = \rho_0$, therefore $C = \log \rho_0$

Hence

$$\log \frac{\rho}{\rho_0} = -\frac{g}{RT} h$$

$$\rho = \rho_0 e^{-\frac{g}{RT} h} \tag{19}$$

If we consider a column h_0 of constant density ρ_0 exerting the same pressure at the place of observation as the atmosphere, then pressure at the place is

$$P_0 = \rho_0 g h_0$$

From (14) $P_0 = \rho_0 RT$ T being constant

These give

$$\frac{1}{h_0} = \frac{g}{RT} \tag{20}$$

Hence (19) becomes

$$\rho = \rho_0 e^{-h/h_0} \tag{21}$$

(ii) <u>Bessels' Hypothesis</u>:

This is a modified form of hypothesis of Newton. It is not based on any assumed law of temperature, but expresses density as a function of altitude, in the same exponential form but the exponent being multiplied by a factor k less than unity. Thus

$$\rho = \rho_0 e^{-kh/h_0} \tag{22}$$

Although in most general form it was implied that factor k may vary with height but in practice, the constant value

$$k = 0.9649 \tag{23}$$

was used. On this law were based tables of refraction published in the Tabulae Regiomontanae, which had been widely used [Newcomb, 1906].

According to this, however, pressure of the whole atmospheric column does not integrate to the pressure at the observer but to a quantity $\frac{h_0 \rho_0 g}{k}$.

(iii) Ivory's Hypothesis

Ivory assumed the temperature to decrease at a uniform rate with the height, at all heights. This is in accordance with the Law of adiabetic equilibrium. With this and assuming the rate to be proportional to temperature at the base, we have

$$T = T_0(1 - \beta h) \tag{24}$$

where β is a constant factor. Then constant rate of decrease of temperature is

$$\frac{dT}{dh} = -\beta T_0$$

Then equation (17) becomes

$$\frac{d\rho}{\rho\, dh} = \frac{\beta R T_0 - g}{T_0 R(1 - \beta h)} \tag{25}$$

or

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_0^h \frac{\beta R T_0 - g}{T_0 R(1 - \beta h)}\, dh \tag{26}$$

Again assuming g to be constant and integrating

$$\log \frac{\rho}{\rho_0} = \frac{g - \beta R T_0}{\beta R T_0} \log (1 - \beta h) \tag{27}$$

if we put

$$y = \frac{g - \beta R T_0}{\beta R T_0} = \frac{g}{\beta R T_0} - 1$$

$$= \frac{1}{\beta h_0} - 1 \qquad [\text{by } (20)] \tag{28}$$

Then (27) gives

$$\frac{\rho}{\rho_0} = (1 - \beta h)^{\gamma} \tag{29}$$

From (29) $\rho = o$ when $\beta h = 1$

i.e. atmosphere will terminate when $h = \frac{1}{\beta}$.

4.1.4 Refraction Models

Utilizing some of the above or often better empirical relationships based on further meteorological data available in recent years, various investigators [Chauvenet, 1863; Newcomb, 1906; Willis, 1941; Garfinkel, 1944; Oterma, 1960; Baldini, 1963; Garfinkel, 1967; Saastamoinen, 1971] have derived their models for refraction correction. The basis of some of the recent ones will be discussed here.

It is to be noted that for light rays at optical frequencies, charged particles of ionosphere, etc. have little effect. So in all of these models atmosphere is considered until its density becomes such that its refractive index is not different from unity. Also the water vapor in the air has very small effect on these.

4.1.4.1 Natural Celestial Bodies

(i) Willis 1941 Model

John E. Willis on the consideration that absolute temperature of a particle in high atmosphere will be related throughout the year to the absolute temperature of the effective radiating surface in the lower atmosphere if the particle remains at the same mass height (i.e., if

there is the same fraction of mass of the air below it and above it), assumed the following model for the relative temperature as a power series of relative pressure.

$$(T/T_0) = 0.670 + 0.5925\,(P/P_0) - 0.2625\,(P/P_0)^2 \qquad (30)$$

for (P/P_0) from 1.000 to 0.200 and

$$(T/T_0) = 0.778 \qquad (31)$$

for (P/P_0) from 0.200 to 0.000.

The derivation of his model is by writing equation (12) in the form

$$\Delta z = \int \frac{\sin z_0}{\left(\frac{n^2r^2}{n_0^2r_0^2} - \sin^2 z_0\right)^{\frac{1}{2}}}\, d\log(n_0/n)$$

$$= \tan z_0 \int \left[\left(\frac{n^2r^2}{n_0^2r_0^2} - 1\right)\sec^2 z_0 + 1\right]^{-\frac{1}{2}} d\log(n_0/n)$$

There being difference of sign with our convention. Then making substitution

$$\frac{n^2r^2}{n_0^2r_0^2} - 1 = M$$

$$\Delta z = \tan z_0 \int (1 + M\sec^2 z_0)^{-\frac{1}{2}}\, d\log(n_0/n) \qquad (32)$$

By binomial expansion

$$\Delta z = \tan z_0 \int \left[1 - (\tfrac{1}{2})M\sec^2 z_0 + (3/8)M^2\sec^4 z_0 - \cdots\right] d\log(n_0/n)$$

Then making use of the model expressed by (30), (31) and hydrostatic equilibrium condition and available data relating n and ρ, numerical integration is performed.

The formulae are claimed to be applicable upto 85° zenith distance.

Final form of the formula given for practical computations is

$$\Delta z = \tan z_o \log_e n_o \; F\left[\left\{\left(2\,\ell_o/r_o + k\,\ell_o^2/r_o^2\right) - \log_e n_o\right\}\sec^2 z_o\right] \qquad (32a)$$

Where apart from known quantities for the place of observations F[. . .] is a function which takes care of departures of lower atmosphere from the standard one, on the basis of observed temperature t_o at the observer. To evaluate this function, first (ℓ_o/r_o) for 0° C is computed from latitude φ_o and height Ho (km) for the place of observations, as below:

Value on meridian = $(0.00125515+0.00000635 \cos 2\varphi_o)(1-0.000157\ Ho)$

Value on prime vertical = $(0.00125093+0.00000211 \cos 2\varphi_o)(1-0.000157 Ho)$

Then value of ℓ_o/r_o at 0° C for direction in azimuth A is

= (meridian value) $\cos^2 A$ + (prime vertical value) $\sin^2 A$

With this, quantity $(2\,\ell_o/r_o + k\,\ell_o^2/r_o^2) = (a + bt)$ for temperature t^o C is computed from a, b given in Table 4.1.

Table 4.1

$(2\,\ell_o/r_o + k\,\ell_o^2/r_o^2) = (a + bt)$ for Willis 1941 model

for 0° C	a	b
0.00124	+513.11	+1.8889
0.00125	+517.26	+1.9042
0.00126	+521.41	+1.9195
0.00127	+525.56	+1.9348

Next, a table for the function F of equation (32a), is computed by Willis and is reproduced in Table 4.2. With these

the correction Δz can be computed using an ordinary calculating machine.

Table 4.2

For Function F of Willis 1941 Model

Argument = $[(2\ell_o/r_o + k\ell_o^3/r_o^3) - \log_e n_o]\ (0''.001 \sec^3 z_o)$

Argument	F	Argument	F	Argument	F	Argument	F
"		"		"		"	
0	1.00000	18	0.96085	36	0.92845	54	0.90070
1	0.99759	19	0.95890	37	0.92680	55	0.89927
2	0.99522	20	0.95697	38	0.92516	56	0.89785
3	0.99287	21	0.95506	39	0.92354	57	0.89644
4	0.99055	22	0.95316	40	0.92193	58	0.89504
5	0.98827	23	0.95129	41	0.92034	59	0.89366
6	0.98601	24	0.94943	42	0.91876	60	0.89228
7	0.98378	25	0.94759	43	0.91718	61	0.89091
8	0.98157	26	0.94577	44	0.91563	62	0.88955
9	0.97939	27	0.94397	45	0.91408	63	0.88820
10	0.97724	28	0.94218	46	0.91255	64	0.88686
11	0.97511	29	0.94041	47	0.91103	65	0.88553
12	0.97301	30	0.93865	48	0.90952	66	0.88422
13	0.97093	31	0.93692	49	0.90802	67	0.88291
14	0.96887	32	0.93519	50	0.90654	68	0.88161
15	0.96683	33	0.93348	51	0.90506	69	0.88033
16	0.96482	34	0.93179	52	0.90360	70	0.87905
17	0.96282	35	0.93011	53	0.90215	71	0.87778
18	0.96085	36	0.92845	54	0.90070	72	0.87652

To compute n_o from Po, t_o, equations were derived by him from the data received from experiments of Barrel and Sears. Perhaps final equations publised in [Barrel and Sears, 1939] had not been available to Willis during his derivations. He used equations in the form:

$$(\bar{n} - 1)10^6 = [0.378167\lambda^2/(\lambda^2 - 0.005761)] \text{ at } \bar{P} = 760 \text{ mm}, \bar{t} = 0°C$$

$$(n_o - 1) = (\bar{n} - 1)\frac{P_o}{1 + 0.003673\, t_o}$$

(ii) Garfinkel 1944 Model

Utilizing the equation (12) in the form

$$\Delta z = \int \left[\left(\frac{nr}{n_0 r_0} \operatorname{cosec} z_0\right)^2 - 1\right]^{-\frac{1}{2}} d \log n \tag{33}$$

A series of substitutions are made adopting new variables connecting the quantities involved.

The essential feature is the polytroptic model of the atmosphere similar to that propounded by Ivory. Here he assumes a piecewise polytropic distribution, composed of spherical shells such that temperature gradient is constant for a particular shell. Two such shells are adopted by him for his model of the atmosphere. A new variable y is introduced that: $y = \dfrac{\text{Dynamical height}}{\text{height of the homogeneous atmosphere for an ideal gas}}$

For a perfect gas in a state of hydrostatic equilibrium in the Earth's gravitational field, the equations (14) and (16) become in terms of relative temperature and pressure as

$$\frac{P}{P_0} = \frac{\rho}{\rho_0} \frac{T}{T_0} \tag{34}$$

$$\frac{d}{dy}\left(\frac{P}{P_0}\right) = -\frac{\rho}{\rho_0} \tag{35}$$

Eliminating P from (34)and (35), the following formula is obtained:

$$\frac{d}{dy}\left(\frac{\rho}{\rho_0}\right) \Big/ \left(\frac{\rho}{\rho_0}\right) + \left[\frac{d}{dy}(T/T_0) + 1\right] \Big/ (T/T_0) = 0 \tag{36}$$

The model adopted for two shells is:

$$\frac{d}{dy}(T/T_0) = c_1 \neq 0, \qquad y \leq y_1 \tag{37}$$

$$\frac{d}{dy}(T/T_0) = c_2 = 0, \qquad y \geq y_1$$

where $y = y_1$ defines the Tropopause.

This causes complications due to discontinuity of $\frac{d}{dy}(T/T_0)$ at the Tropopause. The situation is circumvented by the author by modifying his model as:

$$\frac{d}{dy}(T/T_0) = C, \qquad y \leq y_1 \tag{38}$$

$$T/T_0 = 0, \qquad y = y_1$$

This model thus eliminates Stratosphere and becomes essentially the same as that proposed by Ivory. But the error in adopting model (38) instead of (37) being small is later removed by the author by a differential correction.

Further if $C = -\frac{1}{m+1}$, m being a new constant, the first equation of (38) with (36) becomes:

$$\frac{d}{dy}(\rho/\rho_0)/(\rho/\rho_0) = m\frac{d}{dy}(T/T_0)/(T/T_0) \tag{39}$$

with boundary conditions

$$y = 0 \qquad (\rho/\rho_0) = 1 \qquad (T/T_0) = 1$$

Solution of (39) and first equation of (38) is

$$(\rho/\rho_0) = (T/T_0)^m, \qquad (T/T_0) = 1 - \frac{y}{m+1} \tag{40}$$

The first equation of (40) thus represents a polytropic distribution with its index m given by the second as

$$m = -1 - 1\Big/\frac{d}{dy}(T/T_0) \tag{41}$$

The second equation of (40) shows that the atmosphere extends to a finite height given by

$$y_1 = m + 1$$

or dynamical height $\quad h_1 = (m+1)\,h_0 \qquad$ (41a)

In his derivations, relation between n and ρ used is of the form

$$(n^2 - 1)/(n^2 + 2) = \rho \times \text{constant}$$

The final form of the formula for refraction correction is

$$\Delta z = T_o W (\bar{B}_o + \bar{B}_1 W + \bar{B}_2 W^2 + \bar{B}_3 W^3 + \bar{B}_4 W^4 + \ldots\ldots) \tag{42}$$

where W is called the 'weather factor' and is given by

$$W = P_o/T_o^2 \tag{42a}$$

For coefficients $\bar{B}_i$ (i = 1,2,3,...), some intermediate quantities are defined from a set of standard constants to be adapted. The standard constants are:

$\bar{n}$ - refractive index for a standard temperature $\bar{T}$ and pressure $\bar{P}$

$\bar{R}$ - radius of earth

$\bar{g}$ - standard value of gravity at sea level

R - gas constant for air

m - the polytropic index.

Then intermediate quantities are

$$\bar{A} = (\bar{n}^2 - 1)/2\bar{n}$$

$$\bar{C} = \bar{A}\ (1 + \bar{A})$$

$$\bar{h}o = R\bar{T}/\bar{g}$$

$$\bar{D} = \bar{h}o/\bar{R}$$

Then:

$$\bar{B}_i = \bar{K}\bar{\beta}^i B_i$$

Where

$$\bar{K} = \bar{C}\, m\, [\frac{2}{\bar{D}(m + 1)}]^{\frac{1}{2}}$$

$$\bar{\beta} = \bar{A}/\bar{D}(m+1)$$

The coefficients Bi are functions of an angle θ_o given by:

$$\cot \theta_o = \frac{1}{\sqrt{2\,D(m+1)\,T_o}} \cot z_o$$

Then with a function F_s defined as

$$F_s = \frac{1}{s} \sum_{j=0}^{\infty} \prod_{i=0}^{j} \frac{s-i}{s+i} \tan^{2j+1} \theta_o/2 \qquad (42b)$$

some of the Bi are

$Bo = F_5$

$B_1 = 9\ F_9 - 4\ F_4$

$B_2 = 91\ F_{13} - 72\ F_8 + 6\ F_3$ etc.

Garfinkel adopted the standard values as below:

$\bar{P} = 1.013 \times 10^6$ dynes/cm^2

$\bar{T} = 273^o$ C

$\bar{R} = 6378.4$ km

$\bar{A} = 0.0002942$

$R = 2.87 \times 10^6$ ergs/gram degree

$g = 981$ cm/sec^2

$m = 5$

and computed:

$\bar{h}o = 7.987$ km

$\bar{D} = 0.0012522$

$\bar{C} = 60''.700$

$\bar{\beta} = 0.03916$

$\bar{K} = 4952''$

A table has been constructed in [Garfinkel, 1944] for values of $\bar{B}i$ with argument Θ_o from 0° to 90° (see Table 4.3) to facilitate computations.

Table 4.3

Coefficients for Refraction Correction for $z \leq 90^0$ (Garfinkel 1944 model)

Θ_0	$\bar{B}_0$	$\bar{B}_1$	$\bar{B}_2$	Θ_0	$\bar{B}_0$	$\bar{B}_1$	$\bar{B}_2$	Θ_0	$\bar{B}_0$	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$	$\bar{B}_4$
0°	0.0"	0.0"	0.0"	30°	278.5"	0.8"	0.0"	60°	718.6"	12.4"	0.4"	0.0"	0.0"
1	8.7	0.0	0.0	31	289.2	0.9	0.0	61	740.2	13.4	0.4	0.0	0.0
2	17.3	0.0	0.0	32	300.1	1.0	0.0	62	762.6	14.6	0.5	0.0	0.0
3	25.9	0.0	0.0	33	311.2	1.1	0.0	63	785.7	15.8	0.5	0.0	0.0
4	34.6	0.0	0.0	34	322.4	1.2	0.0	64	809.7	17.1	0.6	0.0	0.0
5	43.3	0.0	0.0	35	333.9	1.4	0.0	65	834.5	18.6	0.7	0.0	0.0
6	52.0	0.0	0.0	36	345.5	1.5	0.0	66	860.3	20.1	0.8	0.0	0.0
7	60.7	0.0	0.0	37	357.3	1.7	0.0	67	887.1	21.8	0.9	0.0	0.0
8	69.5	0.0	0.0	38	369.4	1.9	0.0	68	914.9	23.7	1.1	0.0	0.0
9	78.3	0.0	0.0	39	381.7	2.1	0.0	69	943.9	25.7	1.2	0.0	0.0
10	87.1	0.0	0.0	40	394.2	2.3	0.0	70	974.0	27.9	1.4	0.1	0.0
11	96.0	0.0	0.0	41	407.0	2.5	0.0	71	1005.4	30.3	1.6	0.1	0.0
12	104.9	0.0	0.0	42	420.0	2.7	0.0	72	1038.1	32.9	1.8	0.1	0.0
13	113.8	0.0	0.0	43	433.3	3.0	0.0	73	1072.3	35.9	2.0	0.1	0.0
14	122.9	0.1	0.0	44	446.9	3.3	0.0	74	1108.0	39.0	2.3	0.1	0.0
15	131.9	0.1	0.0	45	460.8	3.6	0.0	75	1145.3	42.4	2.7	0.2	0.0
16	141.0	0.1	0.0	46	475.0	3.9	0.0	76	1184.5	46.2	3.0	0.2	0.0
17	150.2	0.1	0.0	47	489.6	4.3	0.1	77	1225.5	50.4	3.5	0.2	0.0
18	159.6	0.1	0.0	48	504.5	4.7	0.1	78	1268.5	54.9	4.0	0.3	0.0
19	168.9	0.2	0.0	49	519.7	5.1	0.1	79	1313.7	60.0	4.5	0.4	0.0
20	178.3	0.2	0.0	50	535.4	5.5	0.1	80	1361.2	65.5	5.1	0.5	0.0
21	187.8	0.2	0.0	51	551.5	5.9	0.1	81	1411.2	71.6	5.9	0.5	0.0
22	197.4	0.3	0.0	52	567.9	6.5	0.1	82	1463.9	78.4	6.8	0.7	0.0
23	207.1	0.3	0.0	53	584.9	7.0	0.1	83	1519.5	85.9	7.8	0.8	0.1
24	216.9	0.4	0.0	54	602.4	7.6	0.2	84	1578.2	94.2	9.0	1.0	0.1

Table 4.3 continued

+ Θ_0	$\bar{B}_0$	$\bar{B}_1$	$\bar{B}_2$	Θ_0	$\bar{B}_0$	$\bar{B}_1$	$\bar{B}_2$	Θ_0	$\bar{B}_0$	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$	$\bar{B}_4$
25	226.9	0.4	0.0	55	620.3	8.3	0.2	85	1640.4	103.4	10.4	1.2	0.1
26	237.0	0.5	0.0	56	638.8	9.0	0.2	86	1706.1	113.7	12.0	1.5	0.2
27	247.1	0.6	0.0	57	657.8	9.7	0.2	87	1775.8	125.1	13.9	1.8	0.2
28	257.4	0.7	0.0	58	677.4	10.5	0.3	88	1849.8	137.9	16.1	2.1	0.3
29	267.9	0.7	0.0	59	697.7	11.4	0.3	89	1928.5	152.2	18.7	2.6	0.4
30	278.5	0.8	0.0	60	718.6	12.4	0.4	90	2012.2	168.2	21.8	3.2	0.5

For $z > 90^{\circ}$, the expression given is of the form:

$$\Delta z = T_o^{\frac{1}{2}} Y (\bar{C}_o + \bar{C}_1 Y + \bar{C}_2 Y^2 + \bar{C}_3 Y^3 + \ldots\ldots) \qquad (42c)$$

where

$$\bar{C}_i = 2 B_i \cos^2 \theta_o/2 \cot^{2i+1} \theta_o/2$$

$$Y = (P_o/T_o^2) \tan^2 \theta_o/2$$

A table for the coefficients $\bar{C}_i$ has also been given with argument θ_o varying from 90° to 116° (see Table 4.4).

With the help of these tables calculation of Δ_z becomes a simple process.

Table 4.4

Coefficients for Refraction Correction $z \geq 90^{\circ}$
(Garfinkel 1944 model)

θ_0	$\bar{C}_0$	$\bar{C}_1$	$\bar{C}_2$	$\bar{C}_3$	$\bar{C}_4$
90°	2012.2	168.1	21.8	3.2	0.5
91	2029.1	173.5	22.8	3.4	0.5
92	2047.4	179.3	24.0	3.6	0.6
93	2067.2	185.5	25.3	3.9	0.6
94	2088.6	192.1	26.7	4.2	0.7
95	2111.7	199.2	28.2	4.5	0.8
96	2136.8	206.9	29.9	4.8	0.8
97	2163.9	215.4	31.8	5.2	0.9
98	2193.2	224.4	33.8	5.7	1.0
99	2224.8	234.2	36.1	6.3	1.2
100	2258.9	244.8	38.6	6.8	1.3
101	2295.8	256.3	41.4	7.5	1.4
102	2335.7	268.9	44.5	8.3	1.6
103	2378.7	282.7	48.0	9.1	1.8
104	2425.3	297.8	51.9	10.1	2.1
105	2475.6	314.3	56.3	11.3	2.4
106	2530.0	332.4	61.3	12.6	2.8
107	2588.9	352.4	66.9	14.2	3.2
108	2652.7	374.5	73.3	16.0	3.7
109	2721.8	398.9	80.7	18.2	4.4
110	2796.7	426.0	89.1	20.7	5.2
111	2878.1	456.2	98.7	23.8	6.1
112	2966.5	489.9	109.9	27.5	7.3
113	3062.6	527.6	122.9	31.8	8.8
114	3167.2	569.9	138.0	37.1	10.6
115	3281.3	617.6	155.6	43.6	13.0
116	3405.7	671.4	176.5	51.6	16.1

(iii) Oterma 1960

Oterma made use of expansion into power series like that of Willis. His model is based on the assumption that temperature decreases uniformly with the distance in passing from the earth's surface to the limit of Stratosphere and then remains constant.

His work was mainly intended for a new method of astronomical triangulation suggested by Y. Vaisala. He derives expressions for objects inside the effective atmosphere as well as for those outside it. Expansion into power series is basically the same as equation (32) written in the form:

$$\Delta z = \tan z_o \; [U_o - \tfrac{1}{2} U_1 \sec^2 z_o + \tfrac{3}{8} U_2 \sec^4 z_o - \tfrac{5}{16} U_3 \sec^6 z_o + \tfrac{25}{128} U_4 \sec^8 z_o - \ldots\ldots] \qquad (43)$$

where

$$U_o = \int_1^{n_o} \frac{dn}{n} \qquad U_1 = \int_1^{n_o} M \frac{dn}{n} \qquad U_2 = \int_1^{n_o} M^2 \frac{dn}{n} \qquad U_3 = \int_1^{n_o} M^3 \frac{dn}{n}, \text{ etc.}$$

and

$$M = \frac{n^2 r^2}{n_o^2 r_o^2} - 1$$

To evaluate the coefficients U_o, U_1, U_2, ... , numerical integration is performed making use of the relation between n and ρ as:

$$(n - 1) = \rho \times \text{constant}$$

and the temperature model stated above. The values for the coefficients for the astronomical refraction correction computed by Oterma are

$$U_o = 60''.17052$$
$$\tfrac{1}{2} U_1 = 6''.6968 \times 10^{-2}$$
$$\tfrac{3}{8} U_2 = 2''.0971 \times 10^{-4}$$

$$\frac{5}{16} U_3 = 1''.0704 \times 10^{-6}$$
$$\frac{35}{128} U_4 = 0''.7655 \times 10^{-8}$$

The formula is valid for zenith distances upto 85°.

(iv) Baldini 1963 Model

The formulae derived by Baldini are for bodies both inside and outside the atmosphere. Utilizing the differential equation (11) and a model for the diminution of density similar to that of Bessel (equation 22), he derived his expressions.

From the available observations and taking into account the fact that the power of reflecting light ceases at about 60 km, he adopted that density ρ decreases exponentially with altitude according to the equation:

$$\rho = \rho_o \, e^{-h/h_o} \tag{44}$$

The constant h_o was computed by him by weighting the observations proportionately to the power of reflected light at the height of observations. The value computed by him was

$$h_o = 9.240 \text{ km} \tag{45}$$

and

$$\frac{1}{h_o} = 0.1082 \text{ km}^{-1}$$

Introducing this value in equation (44) his model for the density becomes

$$\rho = \rho_o \, e^{-0.1082 \, h \, (km)} \tag{46}$$

Then he utilized the relation of Gladstone and Dale as:

$$(n - 1) = \rho \times \text{constant} \tag{47}$$

The value of constant used by Baldini is 0.226.

The expression derived for the refraction correction is:

$$\Delta z = A_o (n_o - 1)\tan z_o + A_1 (n_o - 1)\tan^3 z_o + A_2 (n_o - 1)\tan^5 z_o \qquad (48$$

where $A_o = +0.99827$

$A_1 = -0.00130$

$A_2 = +0.000006$

To compute n_o for the place of observations the author has given the equation of [Barrel and Sears, 1939] as:

$$(\bar{n} - 1)\,10^7 = 2876.04 + \frac{16.288}{\lambda^2} + \frac{0.136}{\lambda^4} \qquad (49)$$

where λ the wavelength of light is in microns.

Then

$$(n_o - 1) = \frac{(\bar{n} - 1)}{1 + \alpha t_o} \frac{P_o}{760} - \frac{0.000000055\, e_o}{1 + \alpha t_o} \qquad (49a)$$

where t_o is the temperature in degrees centigrade at the place of observations. P_o , e_o being in mm of mercury and $\alpha = 0.00367$.

(v) Garfinkel 1967

In March, 1967, the improvement to his model of 1944 was published. The main reason given for the improvement is the advent of electronic computers which did not exist in 1944. Garfinkel's contention is that even at that time the polytropic model of the atmosphere propounded by him in the year 1944, appeared to be the best compromise between accuracy and simplicity.

The author lists the following improvements made by him:

1. The refraction tables were replaced by a Fortran routine.
2. With quick calculation facility provided by electronic computers there was no need to neglect certain higher order terms of small quantities as had been done in the 1944 version.

3. The algorithm based on a double power series, which converged rapidly for all values of the polytropic index $m \geq 1$, was provided.

Proceeding from equation (33), using basically the same model of the atmosphere as discussed in Section 4.1.4.1 (ii), with a set of substitutions and mathematical derivations the expression derived is of the following form:

$$\Delta z = \bar{K} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} b_1^{i} b_2^{j} M_{ij} F_s(\theta) w_1^{2s-1} u_1^{i+1} u_2^{2i-2j+1} \tag{50}$$

Various quantities forming the above expression are defined starting from T_o, P_o, r_o, g_o from the place of observations; also $\bar{P}$, $\bar{T}$, etc., for the standard conditions and polytropic index m. T_o and P_o are measured in terms of $\bar{P}$, $\bar{T}$ as units.

The quantities are:

$$A = n_o - 1$$

$$C = [r_o g_o / 2(m+1) R T_o]^{\frac{1}{2}}$$

$$B = 2 A C^2$$

$$B_1 = B - A$$

$$B_2 = (1 - B_1)/4C^2$$

$$G = [4B_2(1 - B_2)]^{-\frac{1}{2}}$$

$$D = (1 - B_1)(1 - B_2)/(1 - 2B_2)$$

$$K = 2AGmD^m$$

Then

$$b_1 = B_1 D^{m-1}$$

$$b_2 = -B_2(1 - B_2)/(1 - 2B_2)^2$$

and weather factors are given as:

$$u_1 = P_o/T_o^{m+1}$$

$$u_2 = \left[\left(\frac{2G^2\bar{R}}{r_o} - T_o\right)/(2G^2 - 1)\right]^{\frac{1}{2}}$$

$$F = [T_o - BT_o^m u_1 \bar{R}/r_o] / (1 - B)$$

$$w_2 = (1 - 2B_2)/(1 - 2B_2 F)$$

$$w_1 = (w_2)^{\frac{1}{4}} F^{\frac{1}{2}}$$

Also M_{ij} $F_s(\theta)$ is a function of θ given by

$$\cot\theta = G \cot z_o$$

Then
$$F_s(\theta) = \frac{1}{s}\sum_{j=0}^{\infty}\prod_{i=0}^{j}\frac{s-i}{s+i}\tan^{2j+1}\theta/2 \qquad (50a)$$

and

$$M_{ij} = \frac{[m(i+1)-1]!}{i!(m(i+1)-1-i)!}\;\frac{(m(i+1)+2j-1)!}{j!(m(i+1)+2j-i-1)!}$$

A Fortran routine has been designed [Garfinkel, 1967]. In this, the data required to be input are zenith distance z_o, pressure P_o, temperature T_o, height H_o, temperature gradient $\frac{dT}{dr}$; and five geophysical constants for the standard conditions, i.e., standard refractive index $\bar{n}$ at $\bar{P}$ and $\bar{T}$, radius of earth $\bar{R}$, gravity at sea level $\bar{g}$, gas constant for air R and polytropic index m. Consequently, great flexibility is provided to the user to choose his own geophysical constants. This model of Garfinkel is, therefore, one of the strongest ones for astronomical refraction.

(vi) Saastamoinen 1971

During the last few years, J. Saastamoinen of National Research Council of Canada, has published a series of papers [Saastamoinen, 1969, 1970a, 1970b, 1971]. A consolidated report of his work appears in a report "Contributions to the Theory of Atmospheric Refraction" prepared in July, 1971.

His derivations are also based on radially symmetric distribution of n for a spherical earth. He assumes a constant temperature throughout stratosphere and equal to that at the bounding surface the Tropopause. Thus if P', T' etc. denote the conditions of pressure, temperature etc. assumed known, then from condition of hydrostatic equilibrium of air as a perfect gas, eliminating ρ from equations (14) and (16) is derived

$$P = P' \exp\left[-(g/RT')(r - r')\right] \tag{51}$$

disregarding the variation of gravity with altitude.

In the Troposphere the temperature is assumed to be decreasing at a constant rate $\frac{dT}{dr} = \beta$ giving

$$T = T_0 + \beta(r - r_0) \tag{52}$$

which leads to pressure at a point

$$P = P_0 (T/T_0)^{-g/R\beta} \tag{53}$$

or

$$P/T = (P_0/T_0)(T/T_0)^{-(g/R\beta)-1} \tag{54}$$

$$= (P_0/T_0)(T/T_0)^{m'}$$

where m' is a constant.

After a number of mathematical manipulations the author derives a standard formula for the astonomical refraction correction in the following form:

$$\Delta z'' = 16.271 \tan z_0 \left[1+0.0000394 \tan^2 z_0 \left(\frac{P_0-0.156 e_0}{T_0}\right)\right]$$

$$\left(\frac{P_0-0.156\, e_0}{T_0} - 0''.749 \left[\tan^3 z_0 + \tan z_0\right] \frac{P_0}{1000}\right) \tag{55}$$

where P_0, e_0 are in millibars and T_0 in degrees Kelvin.

The above formula is applicable for zenith distances upto 75 degrees.

4.1.4.2 Artificial Celestial Bodies

Our discussion of refraction models so far has been for the natural celestial bodies which are outside the effective atmosphere, so that integration limits for the refraction integral were from n_o to 1. With the advent of artificial satellites and their applications to geodetic purposes, measurements of directions to them had to be made. These being at various finite distances (a few hundred km) cannot be regarded as outside the effective atmosphere.

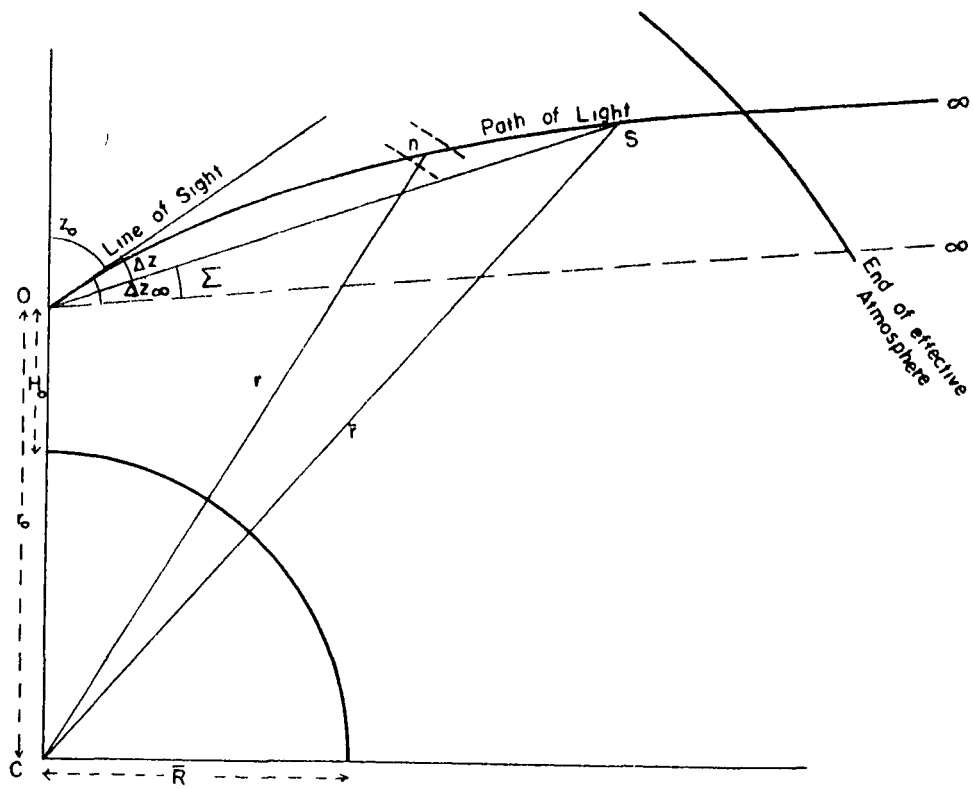

Fig. 4. Atmospheric Refraction

Figure 4 shows a satellite S at a distance $\bar{r}$ from the center of the earth. Thus the refraction angle Δz for the satellite S could be determined by performing integration of (12) up to the point S. Since we

do not know the conditions at S, the refraction angle is usually derived in terms of correction Δz_∞ i.e. the refraction correction if the light through S had come from infinity. If Σ denoted the difference between the two (Fig. 4), then

$$\Delta z = \Delta z_\infty - \Sigma$$

We have already mentioned above that derivations of Oterma 1960, Baldini 1963 include refraction correction for the bodies inside the earth's atmosphere. Several solutions for the differential refraction angle, Σ , have been published in the past, as listed in [Mueller, 1963, p. 304] are[Brown, 1957; Schmid, 1959; Veis, 1960; Holland, 1961; Jones, 1961; and Schmid, 1963].

A formula by Schmid for example is:

$$\Sigma = \frac{r_0 \, s \, \Delta z_\infty}{\bar{r}\left[1 - \frac{\bar{r} \tan z_0}{12,500,000}\right] - H_0} \tag{56}$$

where

H_0 - height of the observer

$\bar{R}$ - mean radius of the earth

$s = R\bar{T}/\bar{R}$

R - appropriate gas constant

$\bar{T} = 273^\circ.16$ K

If $\bar{R}$ = 6,370,000 meters then s = 0.001255

In Part II of the report [Saastamoinen, 1971] also derives formulae for this refraction correction which he has called Photogrammetric Refraction because mostly it comes into play in the photography of the earth's surface taken from an orbiting satellite, or with photography of

an orbiting satellite taken from the surface of the earth against the stellar background.

In all of these the refraction angle Σ is obtained in terms of Δz^{∞}, so is directly dependent on the theoretical and practical considerations according to which Δz^{∞} is calculated. The distance of the satellite generally involved in the expression is calculated from geometry of Figure (4) with some approximations or the other, or can be calculated from orbital elements as it is not required very accurately for this purpose.

Due to the limitations of this method for desirable geodetic accuracies we are not going into it further.

4.2. Effect Due to Variation in Velocity of Propagation

The development of electronic and electromagnetic techniques have played a very significant role in the geodetic measurements, during the last two decades. The advent of artificial satellites being a boon to the advancement of geodesy in various respects, increasing interest has centered around distance measurements to them by radio and optical (laser) methods.

The promise of accuracy in the development of these techniques is again limited by the earth's atmosphere, refractivity of which causes a change in the velocity of propagation of these signals and hence the significant errors in measured quantities.

Since the atmospheric effect on measured range and other quantities (range rate, differential range, velocity, etc.) connected with it are dependent on the velocity of propagation of light or microwaves as the case may be, we shall first discuss their relationship to the refractive index. The refractive index depends on wavelength in a rather irregular way. In the neighborhood of strong absorption lines the variation (or dispersion) is considerable, but elsewhere it is small and could be neglected. As we shall see in the following discussion that for light waves the dispersion is required to be allowed for, but for microwave instruments the wavelength can be so chosen that the dispersion is negligible.

4.2.1. Refractive Properties of Light and Radio Waves

4.2.1.1. Refraction of Light Waves

For convenience the quantity

$$(n - 1)10^6 \text{ is denoted by } N$$

where N is called refractivity of the medium.

The refractivity of light waves for dry air under standard conditions (i.e. 0° C, 760 mm of Hg with 0.03 percent of carbon dioxide) is given by Barrel and Sears equation (compare (49)).

$$(\bar{n} - 1)10^{6} = \bar{N} = 287.604 + \frac{1.6288}{\lambda^{2}} + \frac{0.0136}{\lambda^{4}} = A + \frac{B}{\lambda^{2}} + \frac{C}{\lambda^{4}} \tag{57}$$

where λ is the wavelength of light wave.

Refractive index in other atmospheric conditions being given by (same as (50))

$$(n - 1) = \frac{(\bar{n} - 1)}{\alpha T} \cdot \frac{P}{760} - \frac{0.000\,000\,055\ e}{\alpha T} \tag{58}$$

where

T - absolute temperature in ^{o}K

P - total air pressure in millimeters of mercury

e - partial pressure of water vapor in millimeters of mercury

α - coefficient of expansion of air, 0.003661 or 1/273.16

If P and e are expressed in millibars, (58) becomes:

$$(n - 1) = \frac{(\bar{n} - 1)}{\alpha T} \cdot \frac{P}{1013.25} - \frac{0.000\,000\,042\ e}{\alpha T} \tag{59}$$

This shows there is very small effect of moisture on the refractive index. The formula (1) gives the velocity of a single pure wave. If two or more waves of slightly different wavelengths are involved, the resulting modulated wave form will travel with a different velocity, called the GROUP VELOCITY, v_g given by

$$v_g = v - \lambda \frac{dv}{d\lambda} \tag{60}$$

Then comparing with (1) is defined

$$v_g = \frac{c}{n_g} \tag{61}$$

where n_g is the GROUP REFRACTIVE INDEX. It is, however, to be noted that n_g has no particular connection with refraction of light. It is just defined as above. Thus (60) and (61) with (1) give:

$$n_g = n - \lambda \frac{dn}{d\lambda} \tag{62}$$

The charged particles of ionosphere, etc. cause no refraction for them. They are affected in Troposphere and Stratosphere only.

4.2.1.2. Refraction of Radiowaves

The behavior of radiowaves (frequencies up to 15000 MHz) in a few tens of kilometers (Troposphere and Stratosphere) is about the same as that of light waves, except that their refractivity is given by [Smith and Weintraub, 1953, p. 1035] formula:

$$(n - 1)10^6 = N = 77.6 \frac{P}{T} + 3.73 \times 10^5 \frac{e}{T^2} \tag{63}$$

Where P and e are in millibar units. Comparison of (63) with (59) shows much greater effect of water vapor on the refractivity in this case.

The velocity of microwaves is almost independent of wavelength, and consequently there is generally no question of a group velocity differing from the phase velocity.

In the higher atmosphere i.e, in Ionosphere, the radio waves are affected by the electrons detached from some of the atoms as a result of solar radiation. The refraction of radio waves in this region is dependent on various factors, like the electron density, electron charge, electron gyro frequency, earth's magnetic field,

etc., most of which change with place and time. Thus the refractive index for the ionosphere at a point at r and time t is given by the Appleton-Hartree formula [Weiffenbach, 1965, p. 347]:

$$n(r,t) = \left[1 - \frac{f_N^2(r,t)}{f^2} \cdot \frac{1}{\alpha_{+,-}}\right]^{\frac{1}{2}} \tag{64}$$

where

$f_N(r,t)$ - electron plasma resonance frequency at point r, t

$$= \left[\frac{N(r,t)\, e^2}{\pi\, m}\right]^{\frac{1}{2}} \tag{65}$$

$N(r,t)$ - electron density at position r and time t

e - electron charge

m - electron mass

$$\alpha_{+,-} = 1 - \frac{(f_T/f)^2}{2(1 - f_N^2/f^2)} \pm \left[\frac{f_T^4/f^4}{4(1 - f_N^2/f^2)} + \frac{f_L^2}{f^2}\right]^{\frac{1}{2}}$$

$f_L = f_B(r) \cos\theta$

$f_T = f_B(r) \sin\theta$

$f_B(r)$ - electron gyro frequency = Be/cm

$B(r)$ - Earth's magnetic field at r

θ - angle between propagation vector and magnetic field direction at r

As a first approximation (64) can be written as, not writing (r,t) with the functions

$$n = 1 - \frac{Ne^2}{2\pi mf^2} \tag{66}$$

If $m = 9 \times 10^{-28}$ gram

$e = 4.8 \times 10^{-10}$ e.s.u.

Then (66) gives

$$n = 1 - 41 \left(\frac{N}{f^2}\right) \tag{67}$$

where N is the number of electrons per cubic meter.

Then phase velocity, i.e., the velocity of a single pure wave:

$$v = \frac{c}{n} \tag{68}$$

$$= c\left[1 + 41\left(\frac{N}{f^2}\right)\right]$$

From (67), since (n-1) is negative, the ray curves towards the areas of high electron density. Also phase velocity v exceeds the velocity of light in vacuum, a remarkable situation, but see group velocity below. The electron density varies greatly with time and is most unpredictable. It is dependent on solar activity, being maximum during the day and minimum at night.

Because refractive index varies with the frequency, the group velocity of microwaves in the ionosphere is not the same as phase velocity. The group velocity is given by

$$v_g = v - \lambda \frac{dv}{d\lambda}$$

since $\lambda \simeq \frac{c}{f}$, substituting this in (68), differentiating it with respect to λ and simplifying, we get:

$$\lambda \frac{dv}{d\lambda} = c\,\frac{82N}{f^2}$$

Thus:

$$v_g = c\left(1 - \frac{41N}{f^2}\right) \tag{69}$$

So the group velocity is less than c , as is proper.

4.2.2 Refraction Corrections

With the developments of the techniques of electromagnetic measurements a host of refraction formulae by various authors have appeared and continue to appear. Some of these especially the ones being used will be reviewed as below.

4.2.2.1 Measurements by Radio Waves

As we have already discussed, the radio waves are refracted both in the Troposphere and Ionosphere, the correction for each quantity measured with their help will be in two parts, each one pertaining to each of these regions.

(A) Artificial Celestial Bodies

Measurements in this category are to artificial earth satellites for geodetic purposes. We shall discuss them according to the basic quantity measured:

(i) Range

Range from a ground station to an artificial satellite is measured, e.g., by transmitting a phase modulated electromagnetic wave (carrier). This is received by a satellite borne transponder which retransmits the signal as a phase modulation on an offset carrier frequency to avoid conflict with the incoming signal. This is received back at the ground station and the phase shift of the modulation is measured by an electronic servo phase meter. The phase shift is proportional to the total distance traveled.

For a given frequency f, the range S to a satellite can be represented as:

$$S = \tfrac{1}{2}(\Delta\lambda + \bar{N}\lambda) \tag{70}$$

where $\Delta\lambda$ - is the phase displacement of the wave

$\bar{N}$ - number of full periods of the wave

in its total distance travelled

Apart from determination of other factors, if the wave had travelled in vacuum, then simply

$$\lambda = \frac{c}{f}$$

c - being the adopted speed of light.

But due to atmospheric refraction it is not so. Hence corrections are required to the measured range calculated, using c instead of the actual velocity of propagation.

The American tracking system which utilizes this principle is SECOR (Sequential Collation of Range).

Basic differential equation for the refraction correction is equation (6). From the situation of the troposphere and ionosphere already discussed actual integration of this equation is rather difficult. Often empirical formulae are used. For SECOR reductions for example the following formulae are used [Culley and Sherman, 1967].

Tropospheric Correction

Two formulae are used. The first is the one suggested by Cubic corporation as:

$$\Delta S = \frac{k_1(1-e^{-H/k_3})}{k_2 \cos E_0 + \sin E_0} \qquad (71)$$

where ΔS - is the correction to the observed range

k_1 - is the zenith refractive correction (=2.7 meters)

k_2 - is the horizontal scaling correction (=0.0236)

k_3 - a constant (scale height = 7000 meters)

E_0 - is the elevation angle

H - is the height of the satellite in meters

The other, more sophisticated one which takes into account the changes in temperature, humidity, pressure and geographic location is [Culley and Sherman, 1967]:

$$\Delta S = \frac{10^{-6} N_s}{C \sin E_0} \Psi \qquad (72)$$

where N_o - is the surface refractivity = $\frac{77.6}{T_o}\left(P_o + 4,810 \frac{e_o}{T_o}\right)$

C - is a parameter varying with location and seasonal factors

Ψ - is a correction factor used when Eo is less than 10° but is taken as unity when Eo greater than 10°

P_o - is the total pressure in millibars

e_o - is the partial pressure of water vapor in millibars

T_o - is the absolute temperature

Other Models

There are other models, investigations for the refraction corrections to measured range, for example:

[Saastamoinen, 1971, p. 46-63] with a similar treatment as discussed earlier (4.1.4.1 (vi)) from equation (6) derives expression for the Tropospheric correction for the measured range of the form:

$$\Delta S = 0.002277 \sec z_o \left[P_o + \left(\frac{1255}{T_o} + 0.05\right) e_o - B \tan^2 z_o\right] + \delta R \qquad (73)$$

where ΔS - is range correction in meters

z_o - apparent (radio) zenith distance of satellite

P_o - is total pressure in millibars

e_o - is partial pressure of water vapor in millibars

T_o - is absolute temperature in ^oK

B and δR are correction quantities for which tables are given. These depend on station height and apparent zenith distance, respectively. These values as tabulated by Saastamoinen are given in Tables 4.5 and 4.6.

H.S. Hopfield has brought out a series of papers on tropospheric range correction [Hopfield, 1970, 1971, 1972]. Her investigations include fitting of theoretically derived expressions to observed data and thus giving expressions for tropospheric correction with improved parameters. Theoretical consideration basically is the integration of equation (6), and

Table 4.5

Standard Values of Correction B for Tropospheric Range Correction

Station Height Above Sea Level	B, mb	Station Height Above Sea Level	B, mb
0 km	1.156	2 km	0.874
0.5 km	1.079	2.5 km	0.813
1 km	1.006	3 km	0.757
1.5 km	0.938	4 km	0.654
2 km	0.874	5 km	0.563

Table 4.6

Correction Term δL in Meters, for Tropospheric Range Correction ($\delta R \simeq \delta L$)

Apparent Zenith Distance	Station Height Above Sea Level							
	0 km	0.5 km	1 km	1.5 km	2 km	3 km	4 km	5 km
60°00'	+0.003	+0.003	+0.002	+0.002	+0.002	+0.002	+0.001	+0.001
66 00	0.006	0.006	0.005	0.005	0.004	0.003	0.003	0.002
70 00	0.012	0.011	0.010	0.009	0.008	0.006	0.005	0.004
73 00	0.020	0.018	0.017	0.015	0.013	0.011	0.009	0.007
75 00	0.031	0.028	0.025	0.023	0.021	0.017	0.014	0.011
76 00	0.039	0.035	0.032	0.029	0.026	0.021	0.017	0.014
77 00	0.050	0.045	0.041	0.037	0.033	0.027	0.022	0.018
78 00	0.065	0.059	0.054	0.049	0.044	0.036	0.030	0.024
78 30	0.075	0.068	0.062	0.056	0.051	0.042	0.034	0.028
79 00	0.087	0.079	0.072	0.065	0.059	0.049	0.040	0.033
79 30	0.102	0.093	0.085	0.077	0.070	0.058	0.047	0.039
79 45	0.111	0.101	0.092	0.083	0.076	0.063	0.052	0.043
80 00	0.121	0.110	0.100	0.091	0.083	0.068	0.056	0.047

making use of equation (63) for the radio refractivity. The refractivity N is expressed as

$$N = Nd + Nw \tag{74}$$

where dry component $Nd = \frac{77.6\ P}{T}$ pertains to dry air

wet component $Nw = \frac{3.73 \times 10^5\ e}{T^2}$ pertains to atmospheric water vapor

In her derivation the atmospheric mathematical model is by assuming air as a perfect gas in hydrostatic equilibrium and a constant lapse rate $\alpha = -\frac{dT}{dh}$. With these neglecting variation of gravity for Tropospheric heights, is derived by her [Hopfield, 1969] that the N profile is a polynomial function of height (not exponential) of the form

$$N = N_o\left(\frac{h_d - h}{hd}\right)^{\mu} \qquad h \le h_d \tag{75}$$

where

$h_d = Ts/\alpha$

$\mu = g/R\alpha - 1$

R - being gas constant per gram of dry air and subscript 'o' used for surface values.

The smaller is α, higher is the degree of the polynomial, approaching an exponential as $\alpha \to 0$.

The tropospheric contribution to a vertical range measurement is shown to be

$$\Delta h_{trod} = 10^{-6} \int N_d dh = kP_o \tag{76}$$

Where k is a constant for a given location. Its values for various places, investigated from one year set of data, are tabulated. The rms

value of predicting the height integral from this value of k is estimated between 1 and 2 mm for each one year data set. These values are reproduced in Table 4.7.

There is no such expression of comparable accuracy for the wet part yet, and for that, investigations are still being done by her.

Table 4.7

Prediction of $\int N_d dh$ from Surface Pressure
($\int N_d dh = k\ P_o$)

Station	Year	Latitude	Longitude	Height (meters)	k (meters/mb)	Prediction Error in $\int N_d\ dh$, σ (meters)
Weather Ship E	1963	35°N	48°W	10	0.002281504	0.0017736
Weather Ship E	1965	35 N	48 W	10	0.002281285	0.0016183
Weather Ship E	1967	35 N	48 W	10	0.002281130	0.0016839
Ascension Island	1967	7 55'S	14 24'W	79	0.002290524	0.0011532
Caribou, Maine	1967	46 52 N	68 01 W	191	0.002277725	0.0019329
Washington,D.C. (Dulles Airport)	1967	38 59 N	77 28 W	85	0.002280275	0.0020481
St.Cloud, Minn.	1967	45 35 N	94 11 W	318	0.002278233	0.0015620
Columbia, Mo.	1967	38 58 N	92 22 W	239	0.002280504	0.0019135
Albuquerque, New Mexico	1967	35 03 N	106 37 W	1620-	0.002280765	0.0014815
El Paso, Texas	1967	31 48 N	106 24 W	1193	0.002282555	0.0016598
Vandenberg AFB, California	1967	34 44 N	120 34 W	100	0.002280797	0.0015237
Pago Pago, Samoa	1967	14 20 S	170 43 W	5	0.002287643	0.0017371
Wake Island	1963	19 17 N	166 39 E	5	0.002286083	0.0015023
Wake Island	1965	19 17 N	166 39 E	5	0.002286238	0.0015791
Wake Island	1967	19 17 N	166 39 E	5	0.002286287	0.0017215
Majuro Island	1967	7 05 N	171 23 E	3	0.002289389	0.0015692
Point Barrow, Alaska	1967	71 18 N	156 47 W	8	0.002273335	0.0014273
Byrd Station, Antarctica	1967	80 01 S	119 32 W	1543	0.002272051	0.0011065

Ionospheric Correction

SECOR employs $f_1 = 420.9$ MHz radio waves emitted by ground station, and transponded back by satellite on both of $f_2 = 449$ MHz and $f_3 = 224.5$ MHz frequencies.

From the assumption that the retardation, to a first approximation varies inversely as the square of frequency, the range correction due to ionosphere is:

$$\Delta S = k(D_1 - IC) \tag{77}$$

where

$$k = \frac{(1/f_1^2) + (1/f_2^2)}{(1/f_2^2) - (1/f_3^2)} = -0.7125 \tag{78}$$

D_1 - is the range component on the highest modulating frequency on the 449 MHz carrier.

IC - is the range component of the highest modulating frequency on the 224.5 MHz carrier.

Sometimes interference on the low-frequency carrier makes it impossible to get a range measurement on that frequency. Using samples of ionospheric refraction data measured during operations in the Pacific Ocean area, following analytical model was developed [Culley and Sherman, 1967]:

$$\Delta S = \frac{40.3}{f^2} S(\varphi)\, F(X, R') \frac{\bar{H}\left[\tan^{-1}\left(\frac{H_s - H_m}{\bar{H}}\right) + \tan^{-1}\left(\frac{H_m}{\bar{H}}\right)\right]}{\left[1 - \frac{\cos^2 E}{(1 + H_m/\bar{R})^2}\right]} \tag{79}$$

where $\bar{H}$ - is a parameter from ionospheric model and is called scale height

H_s - height of satellite

H_m - height of maximum electron density

$\bar{R}$ - mean radius of the earth

E - elevation angle of satellite

$S(\Phi)$ - function of the earth's magnetic field

Φ - effective magnetic latitude

f - frequency in megacycles per second

$f(x,R')$ - sun zenith angle function

x - the effective sun zenith angle

R' - function of satellite height

All factors except $S(\Phi)$ can be found and put into program. $S(\Phi)$ is a function of 3 unknown coefficients.

$S(\Phi) = C_1 \Phi^0 + C_2 \Phi^1 + C_3 \Phi^2$ these parameters are found by fitting this linear function to the measured data and to the rest of the model.

Ionospheric correction in case of resonance

Since in the use of dual frequency, e.g., in SECOR, imperfection of equipment performance and radio interference sometimes cause poor or useless ionospheric correction data, study of the behavior and modeling of complicated ionosphere has always been felt essential for the recovery of such data. [Rhode , 1969] did a feasibility study of the ionospheric model particularly for SECOR range measurements. As a result he concluded:

> Ionospheric correction curves are usually smooth curves superimposed by noise of 3 to 5 meters. The curves can be frequently approximated by straight lines. If the curves show an erratic behavior, equipment malfunction or radio interference may be suspected.

From the conclusions drawn by Rhode, it is evident that to detect the erratic ionospheric refraction correction given by equation (77), a curve of the computed correction versus range for various determinations at a tracking station should be plotted. This should give a smooth curve approximating to a straight line. Any particular values abruptly departing from this curve are found to be erratic ones caused by equipment malfunction and should be rejected.

(ii) Doppler Shift

It was an austrian physicist, Christian Doppler (1803-1853), who first explained successfully the relationship between the change in received frequency and the relative motion between the frequency source and a receptor. This variation in received frequency is now called Doppler shift. Shortly after Sputnik I was launched (October, 1957), the staff at John Hopkins University APL noted a pronounced Doppler Shift in the received frequency of Sputnik I transmissions. Research was then conducted by various people and the results published. [Guier and Weiffenbach, 1958] showed that from observations at one station, satellite period, time and distance of its closest approach and its relative velocity could be determined. From 3 stations, orbital parameters also could be determined.

The principle of the method is that the satellite sends unmodulated wave at a fixed frequency fo (say) which is received at tracking station as a varying frequency f and is a function of transmitted frequency fo, phase velocity of propagation v and rate of change of slant range $\frac{ds}{dt}$. Then

$$f = fo\,[1 - (1/v)\,(ds/dt)] \tag{80}$$

Although elementary considerations give $f = fo\,[1 - ds/dt \div (v - ds/dt)]$ as for sound waves in air; but for electromagnetic waves, relativity principles change this expression into equation (80) above. [Bomford, 1971, p. 405.]

Therefore Doppler Shift, Δf is given by

$$\Delta f = f - fo = -(fo/v)(ds/dt) \tag{81}$$

so if v is known $\dot{s} = ds/dt$ is immediately available. But due to the presence of the atmosphere, the phase velocity v varies during propagation through the atmosphere. It is, therefore, necessary to rewrite the equation (81) in the form

$$\Delta f = -fo \frac{d}{dt} \int_s \frac{ds}{v} \tag{82}$$

with (1), equation (82) becomes

$$\Delta f = -\frac{fo}{c} \frac{d}{dt} \int_s n\,ds \tag{83}$$

Thus corrections are needed to correct observed Doppler shift to that of vacuum value. For this correction also, dual frequence can be used to remove bulk of the ionospheric correction, but at the radio frequencies refractivity of air being independent of frequency, tropospheric effects are to be considered carefully.

The U.S. Navy Doppler Tracking Network (TRANET) utilizes this method. Here the signals from the satellite are transmitted on two frequencies which are coherent and related by simple ratio.

Tropospheric Correction

This is as given in [Hopfield, 1963]:

$$\Delta f_{tro} = -\frac{f_o}{c}\frac{d}{dt}(\Delta S_{tro}) \tag{84}$$

Δf_{tro} - tropospheric refraction correction, which is applied to the observed Doppler shift for each data point.

fo - satellite transmitter frequency

c - speed of light in vacuum

ΔS_{tro} - range error in received signal due to tropospheric refraction.

Then, assuming the atmosphere to be horizontally stratified and not changing during the time of a pass and using a two-parameter quadratic expression as an approximation to the refractivity profile, she derives an expression for the contribution of the Tropospheric refraction to the Doppler Shift of a satellite signal. The quadratic expression used is of the form:

$$N = 10^6 (n - 1) = a[r - (\overline{R} + \overline{H})]^2 \tag{85}$$

where r - is the radial distance from center of earth

$\overline{R}$ - is the radius of the earth

$\overline{H}$ - is the height at which tropospheric refraction becomes negligible.

When a value of $\bar{H}$ is postulated, the coefficient 'a' is evaluated from the boundary condition, $N = N_o$ when $r = \bar{R} + Ho$, the subscript o referring to the observing station.

In [Hopfield, 1965] the author on the basis of observed values of N at a variety of geographic locations, altitudes and seasons, concludes that the ratio between the tropospheric error due to her earlier quadratic model and that due to observed refractivity profile, is not, in general unity but is a linear function of the surface refractivity. A few typical values of the ratio of $\int N\, dh$ computed from her quadratic theory of equation (87) and its value from observed N values computed by her are shown in Table 4.8.

The above led the author to investigate further. As a result of further investigation, superseding her earlier two models, is given in [Hopfield, 1969] a new model. A fourth degree function of height, separately for dry and wet components of refractivity, in the form:

$$Nd = \frac{Nd_o}{(\bar{H}d - Ho)^4} (\bar{H}d - H)^4 \qquad H \leq \bar{H}d$$

$$Nw = \frac{Nw_o}{(\bar{H}w - Ho)^4} (\bar{H}w - H)^4 \qquad H \leq \bar{H}w \tag{85a}$$

Table 4.8

Ratio of Computed N Profile and Observed One
(Signal Arriving Vertically)
[Hopfield, 1965]

Place	Latitude °	Latitude '		Longitude °	Longitude '		Height (meters)	Date	No	Ratio of $\int N \cdot dh$ (computed/observed)
Anchorage, Alaska	61	10	N	144	59	W	40	Jan.1964	304.5	1.038
								May 1964	308.7	1.017
Weather Ship D	44		N	41		W	0	Jan.1964	317.3	1.049
								Mar.1964	315.8	1.046
Brownsville, Texas	25	54	N	97	26	W	6	Jan.1964	329.2	1.049
								Mar.1964	342.7	1.091
Canton Island	2	46	S	171	43	W	3	Jan.1964	386.6	1.140
								Mar.1964	378.2	1.161
Petoria, South Africa	25	45	S	28	14	E	1368	Feb.1964	289.2	1.008
Christ Church, New Zealand	43	29	S	172	32	E	34	Jan.1964	313.7	1.019
								May 1964	321.0	1.042
Cape Hallett, Antarctica	72	18	S	170	18	E	5	Feb.1964	301.5	1.021

The parameters $\bar{H}_d$, $\bar{H}_w$ are empirically selected by fitting the observed zenith integrals of refractivity to the computed values. The value of $\bar{H}_d$ and $\bar{H}_w$ estimated by the author are of the order of 40 km and 12 km, respectively.

This model is stated to be capable of closely matching any local average N profile observed in a worldwide sample of locations throughout the height range of meteorological balloon data (upto 24 km)

The corrections based on it are stated to be usable at all angles of elevation. Tropospheric refraction correction computed from this is recommended to be used in equation [84).

For her further work on $\int N_d \, dh$ also see Section 4.2.2.1(A) (i).

Ionospheric Correction

Since to a first approximation, the ionospheric refraction varies inversely as the square of the frequency, by measuring apparent Doppler Shift at each of the two frequencies, the effect of first order refraction is eliminated as below [Gross, 1968].

The refraction error Δf_1 is given by:

$$\Delta f_1 \left(\frac{f_2^2 - f_1^2}{f_2^2} \right) = \Delta f_1 - \left(\frac{f_1}{f_2} \right) \Delta f_2 \tag{86}$$

where f_1 - lower frequency of the coherent pair

f_2 - higher frequency of the coherent pair

$\Delta f_1, \Delta f_2$ - measured Doppler Shifts in the respective frequencies.

The Doppler Shift corrected for first order ionospheric refraction correction Δf_D is:

$$\Delta f_D \left(\frac{f_2^2 - f_1^2}{f_2 f_1} \right) = \Delta f_2 - \left(\frac{f_1}{f_2} \right) \Delta f_1 \tag{87}$$

For frequency pair of 162 MHz/324 MHz:

$$\frac{3}{4} \Delta f_1 = \Delta f_1 - \frac{\Delta f_2}{2}$$

$$\frac{3}{2} \Delta f_D = \Delta f_2 - \frac{\Delta f_1}{2}$$

Residual Ionospheric Correction

Even after removal of first order ionospheric correction, there is residual ionospheric refraction correction which needs further consideration. The full effect of ionosphere is derived by [Guider, 1963] by

substituting equation (64) in (83) from a rigorous treatment of Maxwell's equation as:

$$\Delta f = \frac{1}{c}\frac{ds}{dt}f + \frac{a_1}{f} + \frac{a_2^{(\pm)}}{f^2} + \frac{a_3}{f^3} + \cdots \tag{88}$$

where

Δf - is the observed Doppler shift

a_1 - is the first order term, proportional to the time derivative of $N_{(r,t)}$ integrated along geometric slant range.

$a_2^{(\pm)}$ - second order (Faraday rotation) term which depends on polarization, and on time derivative of $N_{(r,t)}$ and θ.

a_3 - third order term which depends on various powers of $N_{(r,t)}$ and its spatial gradients.

First term in (88) on the right is vacuum Doppler shift. It contains geometrical range rate $\frac{ds}{dt}$ needed for geodetic analysis and is directly proportional to the transmitter frequency f while all other terms are inversely proportional to f.

In principle, therefore, one must choose high enough frequency to reduce the undesirable terms to a negligible value. But practical considerations dictate that one must use lower frequencies to get optimum efficiency from the equipment.

A lot of data are available for a_1, a_2 from ionospheric studies, however, knowledge of a_3 is poor. Data indicate that only terms of equation (88) that cause significant geodetic errors are a_1/f and a_3/f^3, the former being the dominant one.

(iii) Range and Range Rate

Sometimes both range and range rate are simultaneously observed. Considerations for the refraction correction are similar as before.

A system operated on this is by NASA in the Goddard Range and Range Rate System (GRARR). In this system, range is measured from ground transmitted sidetones and satellite transmitter returned sidetones. Phase shift being proportional to distance traveled. Range rate is determined from two-way Doppler shift of uplink carrier frequency (station to satellite). The Doppler shift of signal being due to satellite motion relative to tracking station.

The refraction model used for the above is given below. It was formulated by J.J. Freeman Associates, Inc. [Gross, 1968].

$$\Delta S = \frac{1}{\sin E}\left[\int_0^\infty N dh - \frac{\cot^2 E}{\bar{R}}\int_0^\infty N dh\right] \tag{89}$$

where:

ΔS - refraction correction to be subtracted from the observed range.

E - elevation angle of satellite

$\bar{R}$ - radius of earth

N - refractivity

Thus corrections applied to range data are:

a. Tropospheric Refraction Correction

$$\Delta S = \frac{1}{\sin E}\left[\frac{N_o}{k} - \frac{\cot^2 E}{\bar{R}}\left(\frac{N_o}{k^2}\right)\right] \tag{90}$$

where:

N_o - refractivity at the observing station

k - a tabulated function of N_o

b. Ionospheric Refraction Correction

$$\Delta S = \frac{1}{\sin E}\left[\bar{H}N_m e - \frac{\cot^2 E}{\bar{R}}\left\{\bar{H}^2 N_m e\left(1 + \frac{h_m}{\bar{H}}\right)\right\}\right] \tag{91}$$

where:

$\bar{H} = 1.66\ [30+0.2\ (h_m - 200)]$ km

$h_m = 1393.1 \exp(-0.5014M)$

$M = \frac{MUF(3000)}{F_o}$ (Both values are obtained for a given month and position from the CRPL Ionospheric Prediction Map).

MUF - maximum usable frequency at 3000 km

e - 2.71828

F_o - plasma frequency at the maximum in MHz

N_m - $0.502\left(\frac{F_o}{Feq}\right)^2$ (This is maximum index of refraction at frequency Feq)

F_{eq} - 1928 Mc (This equivalent frequency is used due to the difference in the uplink and downlink carrier frequencies.)

For range rate data, total refraction correction as formulated by Freeman, is:

$$C_R = \frac{\cos E}{\sin^2 E}\dot{E}\left[\left(\int_0^\infty N dh + \frac{N_o}{k}\right) + \frac{3}{\frac{1 - \sin^2 E}{\bar{R}}}\left(\int_0^\infty N h dh + \frac{N_o}{k^2}\right)\right] \tag{92}$$

where

C_R - is the correction to be added to measured range rate.

$\dot{E}$ - elevation rate of satellite.

The integrals are as defined above, except that:

$F_{eq} = 3648$ Mc.

(B) Natural Celestial Bodies

The observations in this category are those connected with Very Long Baseline Interferometry (VLBI).

The principle of interferometry, well known in optics, had been employed at radio wavelengths since 1946, when McCready, et al., used an interferometer for solar observations [Cohen, 1969]. Radio interferometry rapidly developed thereafter but the maximum length of baseline was restricted.

In a simple form, a conventional radio interferometer consists of a pair of antenna arrays separated by a distance d and connected by a transmission line. The antennae collect radiation (radio frequency signals) from a radio source, which are amplified and fed to a common point for direct multiplication and recording the amplitude and phase of the fringes. If λ is the wavelength of the radiation, the resolution of the interferometer is λ/d radians. Due to the restriction on d sufficient accuracies for geodetic purposes were not available from conventional interferometers.

The development of atomic frequency and time standards eliminated inter-connection between two antenna stations of the pair as independent atomic oscillators could be used. Thus, Very Long Baseline Interferometry (VLBI) differs from the conventional one, in the sense that no direct link exists between the two observing stations. The signals received from a celestial source at each antenna simultaneously are mixed with a local oscillator and translated to the video band. The video signal is subjected to clipping and sampling and is then recorded in a digital form on a magnetic tape. Simultaneously, timing

information is also recorded on the tape. The two tapes are later correlated in a computer to determine the delay τ (Fig. 5). Extreme accuracy is required in the timing information. Also extreme stability and accuracy are required from the local oscillator. Usually rubidium vapor or hydrogen maser clocks are used to generate both the local oscillator signals and the timing information.

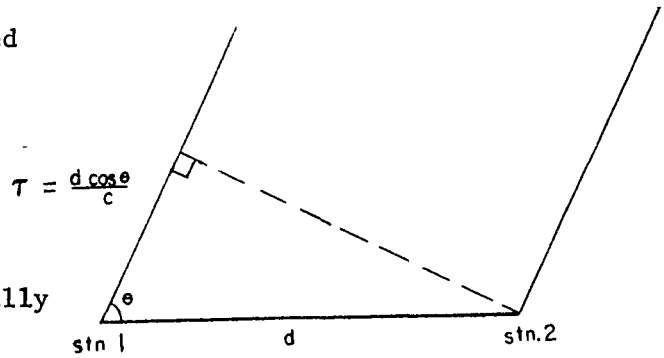

Fig. 5. Geometry of VLBI

Thus the elimination of interconnection between two antenna stations, makes it possible to have base line at intercontinental distances. With VLBI, resolution of distant radio sources with angular diameter of 0".0006 has been achieved. In the inverse problem, which is more important to the geodesists is if the position of point radio source is available, then baseline could be measured. The method, theoretically is capable of measuring intercontinental distances to centimeter accuracy and has other useful geodetic applications [Cohen et al., 1968].

The delay τ between arrival of signal wavefront at the two antenna stations depends primarily on the length and direction of the baseline and the direction of the source. Measurement of τ can, therefore, be used to determine these quantities.

But, the geometric delay, as indicated in Figure 5, differs from the observed delay because of the effects of the atmosphere on the propagation of signal through it. Here, too, the ultimate accuracy

attainable is being limited by the effects of the atmosphere. Corrections are to be applied for the atmospheric effects from the best available knowledge of the atmospheric parameters.

Ray tracing analysis for this problem has been done by [Mathur, 1969]. The basic principle of the method is depicted in Figure 6. Each of the two rays suffers a path difference as per equation (6) of 3.3 where the integration in each case is to be taken along the actual ray path. Here since the two stations may be at the intercontinental distances and, consequently, the atmospheric conditions at two places may be entirely different. Thus each of the two rays are to be treated rigorously through the Ionosphere and Troposphere to obtain the effect of the atmosphere on path difference of the two rays, and hence, the effect on the delay.

In [Mathur, 1969] ray tracing by the author has been done, solving a set of six differential equations developed by [Haselgrove, 1954] for the ray path, in three dimensional space. Even though the effect of Ionosphere (with $n < 1$) and Troposphere (with $n > 1$) is opposite to each other, but their magnitudes differ. As a result of his analysis, the above-mentioned author concluded:

> When interferometric baseline determinations are to be made to accuracies better than about 50 m, the ionospheric and tropospheric effects cannot be ignored. These media introduce a differential phase path that is positive for the ionosphere and negative for the troposphere. The differential phase path increases with increasing zenigh angle z, and for zenith angles less than 70° , the dependence is essentially sec z. The ionospheric phase path is strongly frequency dependent; and, for frequencies greater than 5 GHz, it is very small compared with the tropospheric contribution. For frequencies of 1 GHz and less, the ionospheric contribution predominates. The effects of the magnetic field are less than 10 cm at 1 GHz and can usually be ignored.

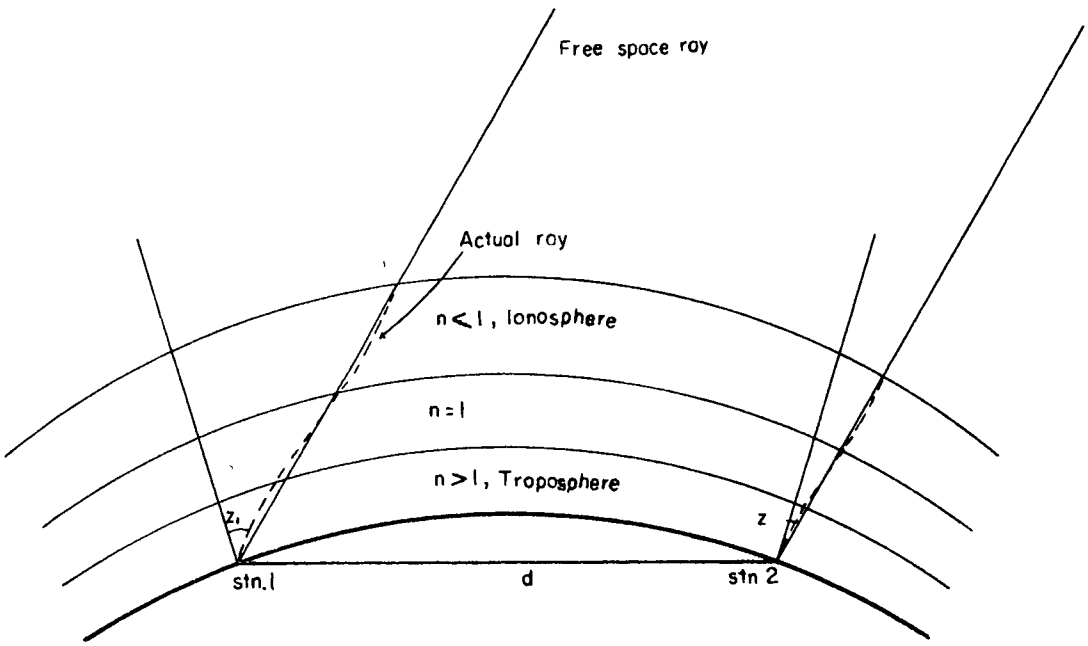

Fig. 6. Refraction Effect on VLBI

It is suggested by the author that best information for applying ionospheric corrections be obtained from incoherent back scatter on topside sounding experiments.

A similar investigation is presented in [Mathur, et al., 1970].

[Dickenson, et al., 1970] state that they have calculated from models using a ray tracing program differential phase delays (geometric length minus electro magnetic length) for paths through the atmosphere and ionosphere. Typical values given by them are tabulated in Table 4.9.

Table 4.9
Typical Values of Refractive Bias

Zenith Angle		X Band 10 GHz m	C Band 5 GHz m	L Band* 1.5 GHz m
	Atmospheric	-2.3	-2.3	-2.3
0°	Ionospheric	0.1	0.3	3.3
	Total	-2.2	-2.0	1.0
	Atmospheric	-3.2	-3.2	-3.2
45°	Ionospheric	0.1	0.4	4.4
	Total	-3.1	-2.8	1.2
	Atmospheric	-86.6	-86.6	-86.6
90°	Ionospheric	0.2	1.0	11.0
(Horizon)	Total	-86.4	-85.6	-75.6

*Derived from the C-Band column, by taking into account $1/f^2$ dependence of plasma effects.

The above authors state, too, that the only reasonable approach for gathering the needed correction factors appears to be a direct probing performed from each terminal at the same time as the VLBI observations and along the same radio path. For the ionosphere, the probing could be performed by using an incoherent backscattering radar to gather the electron density distribution along the direction of interferometric observations.

4.2.2.2 Measurement by Light Waves (Optical Frequencies)

Measurements in this category are, the range measurements by laser. The extension of radio frequency techniques to the optical region of the spectrum, i.e., the invention of laser (Light Amplification by Stimulated Emission of Radiation) has made possible many types of measurements which were not feasible before. One such application is the measurement of range to celestial bodies, and the effect of refraction on this, is the subject of discussion in two sub-sections of this section.

Lasers are electromagnetic radiation having frequency of the visible spectrum region and have coherence properties so far available in radio region only. They are emitted by excited atoms (excited by optical pumping by light of high frequency) e.g., chromium atoms within ruby. Pulses are highly monochromatic, unidirectional and intense.

Since lasers transmit at wavelengths of visible spectrum region, their refractive properties are the same as discussed in section 4.2.1.1, for light waves. Consequently, from the refraction point of view they have great advantage over radiowaves because ionospheric

refraction effects are not observed and the atmospheric water vapor has very little effect on them, as seen from equations (58)or (59) compared to equation (63) for radiowaves. Due to these reasons total atmospheric refraction correction for a laser range measurement is between 2 to 6 meters and its value can be determined within about one percent [Lehr, 1969].

(A) Artificial Celestial Bodies

The application of lasers to satellite tracking has become practicable during the last decade since launching of satellites (e.g. BE-B 1964, BE-C 1965, GEOS 1 1965, D1-C 1967, D1-D 1967, GEOS 2 1968) equipped with retroreflectors.

The quantity measured is the range from ground station to the satellite from the time of travel of the pulse. The principle of measurement is very simple. The laser ranging system measures the time interval required for a laser pulse to travel from the transmitter of the system to the satellite and back to the receiver after being reflected by the satellite retro-reflector. Range to the satellite is then obtained by multiplying this time interval by one-half the velocity of light in vacuum.

Apart from other corrections to be applied is the refraction correction since due to the refraction effect of troposphere the pulse does not travel with vacuum velocity 'c' but travels with a varying velocity v . But the refraction effects in this case are small and could be derived in the same manner as for radio waves ignoring the effect of ionosphere and of atmospheric water vapor.

The U.S. laser ranging system operated by the Goddard Space Flight Center (GSFC) for example, had been using a simple refraction model as:

$$\Delta S = \frac{2.10}{\sin E_0} \text{ meters} \tag{93}$$

where:

ΔS - atmospheric correction to be subtracted from computed range

E_0 - elevation angle of the observation

However, after January 1, 1968, a new formula has been recommended [Lehr, 1967] which is from the work of Thayer.

$$\Delta S = \frac{2.238 + 0.0414(P_0/T_0) \quad -0.238\, H_0}{\sin E_0 + 10^{-3} \cot E_0} \tag{93a}$$

where:

P_0 - atmospheric pressure, in millibars at the observing station

T_0 - temperature in ^{o}K, at the observing station.

H_0 - laser's elevation above mean sea level, in km

E_0 - elevation of satellite

These corrections are from the integration of equation (6) throughout the Troposphere regarding dry component [Lehr, et al., 1967].

[Saastamoinen, 1971] from his treatment discussed before in this report gives an expression of the following form for correction to laser ranging:

$$\Delta S = 0.002357 \sec z \,(P_0 + 0.06\, e_0 - B \tan^2 z) + \delta L \tag{94}$$

where:

ΔS - is range correction in meters

z - apparent zenith distance of satellite

P_o - is total pressure in hillibars

e_o - is partial pressure of water vapor in millibars

B and δL are correction quantities for which tables are given by the author. These depend on station height and apparent zenith distance, respectively. Their values are tabulated in Tables 4.5 and 4.6, respectively.

(B) Natural Celestial Bodies

Of the natural celestial bodies the moon is the one to which laser ranging of geodetic accuracy has become possible. After successful emplacement of Laser Ranging Retroreflector on the surface of the moon by the crew of Apollo 11 and the capability of laser transmitters for generating extremely short (5 nsec) and high power (1 GW) pulses at optical frequencies which can be concentrated into beam widths of as narrow as a few seconds of arc, such measurements have been put into practice.

The basic operating principles are the same as for satellites. With current laser and timing techniques, an uncertainty of $\pm$ 1 nsec in the absolute measurements of the round-trip travel time which is equivalent to $\pm$ 15 cm in one-way distance, is possible.

The treatment of refraction correction for this is the same as for artificial satellites.

5. ACCURACY PROBLEM

Refraction anomalies exist in all regions of the atmosphere. To start with the turbulence of air in the telescope or camera may give rise to complicated pattern of convection inside the telescope and may affect the image. The simplest way to overcome these is to let the instrument acclimatize before starting observations. More sophisticated ways are to create vacuum in the telescope tube or fill in a gas of low refractive index, e.g. helium, but this has obviously disadvantages for cameras.

Next is the turbulence in the air layers immediately outside the objective lens. The camera housing, the instrument itself, or the surrounding ground and vegetation may act as heat sources or sinks and may cause errors. Partial remedies being removal of the camera housing from the instrument during observations and the positioning of camera away from likely sources of temperature anomalies, as well as some distance above the ground. Results quoted by [Meyer-Arendt and Emmanuel, 1965, p. 140] indicate that an elevation of the instrument to about 7 to 10 m above the ground level is desirable.

For room refraction and for refraction in the telescope tube [Willis, 1941] suggested the following equation:

$$d\,(\text{refraction}) = (ds)(d \log_e n) / (dp) \tag{95}$$

where

s - is distance along the ray

p - is distance perpendicular to the ray

n - is the index of refraction

Then comes the atmosphere from about 100 m above the ground and is beyond our control. For this we have seen the various models etc. designed. In order to appreciate the accuracy problem, we shall now review the validity of applying various laws/formulae from physical and meteorological sciences and the practical assumptions made in computing the refraction correction. A recent exhaustive analysis of the problem is by [Teleki, 1972].

5.1 Validity of Various Physical Laws/formulae

First of all, the laws of refraction are examined, which are deduced from the Fermat's Principle. G. Teleki cites [Born and Wolf, 1964] judgment that the strong formulation of Fermat's Principle is only valid if two points are sufficiently close together, i.e., if there is no image of either of them in between these points on the ray connecting them. In the earth's atmospheric layers, this condition is not expected to hold and consequently only the weak formulation of Fermat's Principle is applicable.

Next comes the important parameter 'n', the refractive index. For its relationship to the density we have seen the use of Gladstone and Dale equation:

$$(n - 1) = C\rho$$

or sometimes used is the Newton-Laplace formula:

$$(n^2 - 1) = c_1\rho$$

These formulae are not rigorously obtained but are based on wholly empirical data. They are now replaced by more exact formulae. For example, the best, theoretically-grounded expression is regarded as that of Lorenz and Lorentz, as below [Teleki, 1972]:

$$\frac{n^2 - 1}{n^2 + 2} = R\rho \qquad (96)$$

where R is the dispersion factor.

From this is obtained the following formula for a mixture of gases [Owens, 1967]:

$$\frac{n^2 - 1}{n^2 + 2} = \sum_i R_i \rho_i \qquad (97)$$

where R_i is called the specific refraction and ρ_i the partial density of the ith component of the mixture. R_i being given by:

$$R_i = \frac{4}{3} \pi \frac{N_A}{M_i} \alpha_i \qquad (98)$$

where N_A - is Avogadro's number

M_i - the molecular weight of the ith component

α_i - the polarizability of the ith component,

This formula is recommended to be used.

For calculation of refractive index, the formula used has been that of Barell and Sears (1939), given in (49). This is partly based on laboratory experiments at temperatures of 12°-31° C, and pressure of 100-800 mm of mercury. But according to theoretical research of the authors, it can also be used in larger ranges of temperature and pressure. [Teleki, 1972] has mentioned various other formulae, based on better and the latest considerations, but observes that even with these, absolute accuracy will not be better than 5×10^{-8} but the change in composition can reduce this accuracy sensibly. The main problem for this case is not the formula but its application. To retain accuracy of 10^{-8} meteorological factors should be precisely known, i.e., pressure to 0.025 mm Hg, temperature 0.01^o, relative humidity to 0.01% (at 20° C) and carbon dioxide to 0.00006 parts by volume. Such accuracies are unreachable in practice. The author states that in

practice we cannot expect a higher accuracy of determination than $\pm 0.02 \tan z$.

Moreover, the atmosphere being a dynamical medium, formulae derived in the laboratory are not applicable to it. One of the possible solutions suggested is permanent determination of refractive index when observations are going on.

5.2 Atmospheric Factors

In examining the various factors assumed in modelling the atmosphere, the first one almost universally assumed is the spherically symmetric layers. Although, according to modern meteorological observations, atmosphere can be regarded as consisting of layers with rather smudgy boundaries. [Saastamoinen, 1971] discusses that there is a tilt of layers since the refractivity of the air at sea level increases from the thermal equator towards the colder climate at the poles. The systematic error as a result of general inclination of surfaces of constant refractivity, is investigated by the author. For average conditions at mid-latitude, change in refraction is approximately given by the author as:

$$-0''.20\ h\ (dT/ds)\ (\tan^2 z_0 + \tfrac{1}{2}) \qquad (99)$$

where h - is the depth of locally affected layer in km

dT/ds - is the lapse rate in $^{\circ}$ C/km

z_0 - observed zenith distance

According to the author, usually in the case of sea breeze, h is of the order of 500 meters.

Next, there are formed different standard atmospheres which do not materially differ from each other. Since it is impossible to represent

atmosphere with one standard atmosphere, supplemental corrections are designed for the variation in latitudes and particularly with seasons. The question is how much the deviations from the mean state of atmosphere influence the value of normal (pure) refraction [Teleki, 1969], on the basis of data over Belgrade, points out that the change in value of normal (pure) refraction; at $z=45^{o}$ is approximately $0''.01 - 0''.02$ and at $z = 60^{o}$becomes about $0''.1$.

Next is the question of hydrostatic equilibrium of the atmosphere, which is usually taken for granted. But the atmosphere certainly is not a quiet medium. There are circulations and it is a complex phenomenon, the structure of which is not known with any sufficient accuracy. Investigations of atmospheric turbulence started since about 1950 and are still going on.

So far as ionospheric factors are concerned, we have seen in Section 4.2.1.2 that ionospheric refractivity is a complicated function of factors like electron density and earth's magnetic field, studies which although fairly advanced are still short of giving exact information for calculation of exact ionospheric refraction correction. The alternative usually considered is to eliminate this correction for electromagnetic measurements rather than to actually calculate and apply it.

5.3 Accuracy Figures

In view of various factors discussed above, it is very difficult to quote any figures for the absolute accuracy of measurements. [Teleki, 1972] is of the opinion that hundredth parts of a second of arc are under grave suspicion, meaning thereby that the best models at

present are expected to give refraction correction for direction within $0''.1$.

[Saastamoinen, 1971] estimates that using his model, the maximum error in laser range correction (tropospheric) at observed zenith distance of 80° as 3.4 cm and concludes that standard error of range correction would be 1 to 2 cm.

The standard error of radio ranging due to troposphere is estimated by him to be about 10 times larger than laser ranging.

For accuracies of measurements in which variation in velocity of propagation is affected in the troposphere [Kaula, 1970], sums up that ultimate accuracy of single wavelength optical ranging to satellites or the moon, using surface measurements of refractive index to estimate these corrections is limited to about 6 cm, or 2.5% of the total correction at zenith. The range error increases approximately by secant of zenith angle as zenith angle increases. Radio systems are worse by a factor of about two. For a VLBI with a baseline of about the radius of earth, say 6×10^{6} m, when observing a radio star, this error is about 14 cm for each antenna. This causes range difference uncertainty in two paths about $14\sqrt{2} \simeq 20$ cm resulting in angular uncertainty about $0''.007$. Applying the same, i.e., 5% topospheric uncertainty factor to radio Doppler refraction error a limit of 0.5 mm/sec at 45° zenith angle is estimated.

For radio ranging to keep the ionospheric error below 3 cm, it is observed [Kaula, 1970] that the lower frequency of a pair should be one GHz or greater.

6. ELIMINATION OF REFRACTION EFFECTS - FUTURE TRENDS

From the discussion in Chapter 5, it becomes clear that the complexity and variability of atmospheric (including ionospheric) composition and the limitations of various physical laws/formulae to model it, coupled with the practical difficulty of measuring various meteorological and ionospheric factors beyond a certain accuracy, do not allow us to achieve better accuracies than those summed up in Section 5.3. These accuracies, however, fall far short of the overall accuracies expected in the near future for various measurements as mentioned in the penultimate paragraph of Section 1.1. Moreover, working on the approach discussed so far, there appears to be no prospect of a radical improvement in the accuracies of determination of refraction effects in the coming years. This state of affairs, naturally suggests to the ever-exploring human mind to investigate some other methods of altogether eliminating this refraction effect. Two such approaches, which are in the offing are discussed in the following two sections.

6.1 Dispersion Methods

In Section 4.2.2.1 (A) it was seen how the measurements employing two frequencies could be used to eliminate first order ionospheric correction. Since the refractivity of radio waves (upto 15 GHz) is independent of the frequency or the wavelength in the uncharged atmosphere, no part of the tropospheric correction can be eliminated by them in a similar manner. However, the advent of lasers whose refractivity in the Troposphere is a function of wavelength, showed promise of eliminating tropospheric refraction effects using

their dispersive property. After [Bender and Owens, 1965] described the use of two optical wavelengths for eliminating tropospheric refraction effects (which is the only one at optical frequencies), it has been increasingly realized that this method holds a real prospect of eliminating refraction effects or in the process determining average refractive index just where it is wanted most.

Assuming that electromagnetic devices can simultaneously measure at several frequencies, the range between the same two points [Thayer, 1967] discusses at length the theory and the sources of error. The basic principle of the theory is very simple. If several frequencies operating simultaneously to measure range between two points were assumed to follow the same straight line between end points (although there will be little deviation), then the only source of error in each of the measurements is the retardation due to refraction of the atmosphere.

Now, refractivity (n-1) of air for lasers can be expressed as

$$n - 1 = D(\lambda)\, F(P,T) \tag{100}$$

where

$D(\lambda)$ - is a dispersive constant which depends on the frequencey or wavelength only and does not depend on atmospheric factors.

$F(P,T)$ - is a function of pressure (P) and temperature (T).

If measurements were made at two wavelengths λ_1, λ_2 of the optical region, then range corrections would be given by average value of (n-1) multiplied by the path in this case assumed straight, i.e., S_o.

If $\bar{F}(P,T)$ denotes the average value of $F(P,T)$ along the path followed by rays:

$$\Delta S_1 = (\bar{n}_1 - 1)\, S_0 = D(\lambda_1)\, \bar{F}(P,T) S_0$$

$$\Delta S_2 = (\bar{n}_2 - 1)\, S_0 = D(\lambda_2)\, \bar{F}(P,T) S_0 \qquad (101)$$

Subtracting, we get differential correction:

$$\delta S_{2,1} = \bar{n}_2 S_0 - \bar{n}_1 S_0 = \bar{F}(P,T) S_0 \left[D(\lambda_2) - D(\lambda_1) \right] \qquad (102)$$

This can be solved for $\bar{F}(P,T)$ and result substituted in one of the two equations for true path length gives S_0, as

$$S_0 = S_1 - \frac{D(\lambda_1)}{D(\lambda_2) - D(\lambda_1)}\, \delta S_{2,1} \qquad (103$$

where

S_1 - is measured value of range with wavelength λ_1

The fraction involving $D(\lambda_1)$, $D(\lambda_2)$ does not depend on any atmospheric terms and may be calculated for any (say, standard) conditions. For example, for

$\lambda_1 = 6328\mathring{A}$ of the helium-neon laser and

$\lambda_2 = 3660\mathring{A}$ of the mean of three principal mercury-arc lines

At standard conditions ($P = 1013.25$ mb, $T = 288^{\circ}.16$ K, i.e., 15° C for dry air with 0.03% CO_2 by volume):

$$\frac{D(\lambda_1)}{D(\lambda_2) - D(\lambda_1)} = 10.6634$$

Since for the accuracies of 1 in 10^7 or better the effect of atmospheric water vapor cannot be ignored, the above theory is extended by writing group refractivity as

$$(n - 1) = D(\lambda)\ F(P,T) + W(\lambda)\ G(e,T)$$

where

$W(\lambda)$ - is the dispersive constant for the water vapor term at wavelength λ.

$G(e,T)$ - is the atmospheric dependence of water vapor term and is a function of partial pressure of water vapour e and temperature T.

Thus by using a third wavelength say λ_3 and writing similar equations as above, it is possible to get a pair of simultaneous equations in two unknowns. This is the principle of triple frequency technique.

Here, since water vapor dispersion term $W(\lambda)$ is very small for optical wavelengths, it will be very difficult to get good results to the required precision. It is, therefore, recommended that two optical and one radio frequency be used. [Thayer, 1967] analyzes the problem and discusses the likely sources of error.

The author points out that multifrequency range measurements are potentially capable of advancing the state-of-the-art in distance measurements by one to two orders of magnitude; from the standpoint of atmospheric effects.

The instruments for this are in the process of development. Since relatively small effect is to be measured by a still smaller effect of dispersion, we need instruments of very high sensitivity. Some such

instruments for small terrestrial distances have been tested [Owens, 1967], [Owens and Earnshaw, 1968].

From the above approach it is apparent that such instruments can measure the average value of the functions F(P,T) and G(e,T) along the path of its operation and such developments may lead to the determination of average refractivity, which could be used to correct direction observations to celestial bodies provided instrument could be made to operate in the desired direction employing radio controlled balloons mentioned in the next paragraph. Further research in this direction is needed.

For tropospheric refractivity, in connection with VLBI observations [Dickinson et al., 1970] suggest that "a balloon-borne array of retro-reflectors could conceivably be located at high altitude (20 km) for each one of the interferometer terminals and could provide effective echoing cross-section to a radar system located at the terminal and operating at two optical wavelengths and one microwave frequency." The authors observing that though radio-controlled, powered-balloon systems suitable for such work had been partially developed, the practicability of such a system did not appear encouraging. If, however, it could be overcome, then from each interferometer terminal, balloons could be made to follow the apparent position of the chosen radio sources for the duration of the VLBI observations. From the difference between phase delays measured at the three probing frequencies, the columnar refractivity of the tropsphere could be determined - the two optical frequencies giving dry term and the radio-optical interval giving wet term.

For the ionospheric effect on VLBI, the above authors suggest dual-frequency observations of the radio source to provide directly the difference in columnar ionospheric refractivity for the two terminals.

The authors state that for a pair of frequencies like 0.5 and 1.0 GHz they have verified that residual error due to the ionosphere is well below the overall error permissible in the observational program.

[Kaula, 1970] states that use of dispersion effects by two optical wavelengths can reduce the uncertainty in range measurements by a factor of 20 to 30. A further reduction by a factor of 40 or more could probably be gained by adding one or two radio wavelengths, depending on the wavelengths chosen, to reduce the effects of tropospheric water vapor and of the ionosphere.

6.2 Satellite to Satellite (Range Rate) Tracking

Another approach suggested is to make measurements beyond the effective atmosphere especially the Troposphere and the Stratosphere. The possibility of such a system is by making one or more very high geostationery satellites to track a low satellite. The tracking satellites would be required to be equipped with instruments for tracking including the facilities for data storage and later transmitting it to ground station. [Kaula, 1970] states that this is the only way out for direct mapping of earth's gravitation field accurately. Of the various tracking systems used from ground stations, the method of measuring Doppler shift to determine range rate between the satellites is preferred. Optical systems require precise orientation and, therefore, will not be suitable.

In such a system apart from other factors improving accuracies, the complete elimination of tropospheric refraction effects, will be a major step forward. Further improvement is suggested for eliminating

ionospheric refraction effects by using two tracking frequencies at one to two GHz or by using one single frequency above 20 GHz. Such high frequency, which would be absorbed by the Troposphere for ground tracking, can be usefully employed for satellite to satellite tracking above the Troposphere. [Kaula, 1970] observed that such a system should produce a range rate accuracy of 0.1 mm/sec and in the forseeable future it is expected to improve to a level of 0.03 mm/sec.

7. CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The conclusions from this report are: In spite of the best possible modelling of the atmosphere our insufficient knowledge of characteristics and variabilities of physical, meteorological and ionospheric factors does not allow us to make exact determinations of the refraction correction. Moreover, lack of complete knowledge of these factors can often substantially affect the calculation of refraction correction and many times it is very difficult to estimate the total amount of influence of these uncertainties.

At present, however, using latest relationships available, for the above factors, it should be possible to determine astronomical refraction correction within one-tenth of a second of arc and laser range corrections within a few centimeters provided observations are not made under turbulent atmospheric conditions.

For very high accuracies compatible with future needs the development of instrumentation and techniques for eliminating effects of refraction by dispersion methods of Section 6.1 and employing satellite to satellite tracking discussed in Section 6.2, are the answers to the problem. Just as the development of electronics and electromagnetic techniques threw a challenge to the accurate determination of refraction effects, the need for developing instrumentation for the systems discussed in Sections 6.1 and 6.2 seems to throw a challenge back to the development of electronics and electromagnetic techniques to perfect instruments of high sensitivity to serve the purpose. Further research and development in this direction are needed.

7.2 Recommendations

The following recommendations are, therefore, made:

(i) For direction observations by visible light to natural celestial bodies a model like that of Garfinkel, 1967, improved by using the latest formulae for relative refractivity as function of the air composition be used for accuracies better than 0".1. This, incidentally, is within the accuracies of present star catalogues with accuracies ranging from 0".3 to 0".5.

(ii) A model like the above would suffice for camera tracking of artificial satellites, i.e., photographing them against the star background, for which accuracy by other factors like limitation of cameras, star positions, etc., is limited to 0".5 to 1".0 [Kaula, 1970].

(iii) For laser ranging, models in use be improved by measuring and using tropospheric refractivity profile. However, models like (94) give correction better than the limitation of present-day receivers. This is about one nsec corresponding to about 15 cm in range difference.

(iv) For observations employing radio waves, too, measurement of tropospheric refractivity profile in the direction of observations should be employed in addition to using two frequencies with the lower one of the order of one GHz or greater.

(v) For extreme accuracies to cater to the future needs, instruments and techniques be developed and perfected based on the approach discussed in Chapter 6.

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