ROCKET ASCENT G-LIMITED MOMENT-BALANCED OPTIMIZATION PROGRAM (RAGMOP)

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August 1972

Final Report

Prepared for

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama 35812
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This document describes the RAGMOP (Rocket Ascent G-limited Moment-balanced Optimization Program) computer program for parametric ascent trajectory optimization. RAGMOP computes optimum polynomial-form attitude control histories, launch azimuth, engine burn-times, and gross liftoff weight for space shuttle type vehicles using a search-eaccelerated, gradient projection parameter optimization technique. The trajectory model available in RAGMOP includes a rotating oblate earth model, the option of input wind tables, discrete and/or continuous throttling for the purposes of limiting the thrust acceleration and/or the maximum dynamic pressure, limitation of the structural load indicators qa and q8 (the product of dynamic pressure with angle-of-attack and sideslip angle), and a wide selection of intermediate and terminal equality constraints. Two step-size control schemes in RAGMOP allow the program to rapidly recover from extremely poor nominal "guess" trajectories and improve the rate of convergence over the classical gradient projection method. RAGMOP is designed to run on the Univac 1108 (Emac 8 version) with less than 30K storage required and typical run times of from two to ten minutes depending upon the quality of the nominal (guess) trajectory and the sophistication of the control program desired.
FOREWORD

This technical report presents work performed by Northrop Services, Inc., Huntsville, Alabama, while under contract to the Aero-Astrodynamics Laboratory of the Marshall Space Flight Center (NAS8-27621). Mr. R. G. Toelle of S&F-AERO-GT was the Contracting Officer's Representative for this task.
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PART I
ENGINEERING MANUAL
Section I
INTRODUCTION

The advent of the space shuttle produced the requirement at NASA, Marshall Space Flight Center, for a computer program that could be used as an analysis tool to evaluate several different vehicle configurations under various launch conditions. A program with the sophistication of the physical model in the ROBOT computer program was desired, with the additional capability of moment-balanced, lifting trajectories in the atmosphere from launch to orbital injection. In addition to the moment-balance, several additional constraints were required to be met such as: maximum dynamic pressure, maximum \( q_a \) and \( q_b \) (product of dynamic pressure with angle-of-attack and sideslip angle), booster flyback fuel requirements, and the calculation of actual payload rather than injected weight. A number of approaches have been taken to the ascent trajectory optimization problem, including: calculus of variations, steepest ascent, \( \text{min-H} \), gradient projection, and conjugate gradient methods. The emergence in recent years of accelerated gradient projection methods as perhaps the most powerful parameter optimization techniques available for highly nonlinear systems led to the selection of gradient projection as the optimization scheme in RAGMOP (Rocket Ascent G-limited, Moment-balanced Optimization Program). The desire was also, of course, for a program that was as compact, fast, and easy to use as possible. The resulting program RAGMOP, has the capability of computing optimal engine burn-times, liftoff weight, launch azimuth, and polynomial-form attitude histories including the effects of atmospheric flight from launch to orbit. A static moment-balance scheme balances moments using thrust vectoring in all stages of the vehicle. A large variety of constraints, equality and inequality, intermediate and terminal, are available as well as a widely variable control program. The program occupies less than 30K storage and converged lifting moment-balanced trajectory runs have been obtained in less than 3 minutes on a Univac 1108 with nominal "guess" trajectories in error as much as 560 meters/sec in velocity, 5 degrees in flight-path angle, and 176 km in radius at injection.

\* Superscript numbers refer to references listed in Section IV.
This document is arranged in three parts plus an appendix. Part I includes the first four sections and may be considered an engineering manual. Part II is comprised of Sections V - VIII and serves the purposes of a programmers manual. Part III is made up of Section IX and is sufficient within itself as a user's manual. The Appendices follow Part III.

**Part I - Engineering Manual**

Section I  Introduction  
Section II  General Description  
Section III  Theory  
Section IV  References

**Part II - Programmer's Manual**

Section V  Program Flow and Operation  
Section VI  Subroutine Descriptions  
Section VII  Program Listing  
Section VIII  Variable Name Cross Reference

**Part III - User's Manual**

Section IX  Input/Output
Section II
GENERAL DESCRIPTION

2.1 GENERAL DESCRIPTION

The RAGMOP (Rocket Ascent G-limited Moment-Balanced Optimization Program) computer program calculates the optimal values of a set of parameters which affect multistage rocket ascent trajectories. The parameter optimization is performed using a search-accelerated gradient projection technique, which includes the capability of satisfying a large number of intermediate and terminal constraints (end conditions). The flexibility and speed of the program, combined with a highly sophisticated physical model, make it a desirable tool for the analysis and design of space shuttle and other ascent rocket vehicles. In addition, output options include tables and plots which are suitable for reports.

2.2 PARAMETER OPTIMIZATION

The parameter optimization scheme used by RAGMOP is a search-accelerated gradient projection method. This method requires complete parameterization of all control variables, and the solution obtained is therefore optimum only to the degree attainable with the chosen parametric form.

The control parameters in RAGMOP describe: (1) the lift-off weight of the vehicle, (2) the duration of a number of engine burn times (thrust events), (3) the launch azimuth, and (4) the vehicle pitch and yaw attitude histories.

The vehicle chi-pitch ($\chi_p$) and chi-yaw ($\chi_y$) attitude histories are given in the form of polynomials in time, i.e.:

$$\chi_p = \chi_{p_0} + a_1(t-t_0) + a_2(t-t_0)^2 + \ldots + a_n(t-t_0)^n$$

and

$$\chi_y = \chi_{y_0} + b_1(t-t_0) + b_2(t-t_0)^2$$

Separate polynomials are used for each stage, with currently up to a fourth-order polynomial available for the first stage $\chi_p$, and second-order polynomials for the second stage $\chi_p$ and the $\chi_y$ of both stages. The $\chi_p$ program may be continuous or discontinuous at staging, while the $\chi_y$ program is always continuous. The validity of the polynomial form for the $\chi_p$ and $\chi_y$ attitudes angles (see Section III for more information concerning the attitude control) is evi-
denced by the close agreement between polynomial forms and solutions obtained using variational methods.

In addition to the optimized parametric form $x_p$ mentioned above, RAGMOP also allows the use of an angle-of-attack profile for the pitch attitude control of a portion of the first stage flight. By the use of an input flag, the user may specify three angle-of-attack control options: (1) zero aerodynamic normal force, (2) zero angle-of-attack, or (3) angle-of-attack as a function of Mach number. The angle of attack control will be used from the end of a tilt-over maneuver to staging. The tilt-over will be performed between the end of the lift-off (vertical rise) phase and the beginning of angle-of-attack control at some time specified by the user. The tilt-over consists of an optimized polynomial $x_p$ control program of the same form as in the complete stage when angle-of-attack control is not used.

Revision 1 adds the capability of enforcing coordinated turns during the first stage. Either positive or negative angle-of-attack is used for this option, the algebraic sign determined separately for each thrust event. This option may be used simultaneously with the angle-of-attack options mentioned above.

2.3 CONSTRAINTS

RAGMOP is extremely flexible in terms of the constraints allowed on the trajectory.

Table 2-1 lists the equality constraints available in the program, which may be enforced both at orbital injection and at staging. In addition to the equality constraint of Table 2-1, the relative velocity at staging may be used as a cutoff criteria for the last thrust event of the first stage. This cutoff criteria will be satisfied regardless of the parameter values (if possible) and does not enter into the parameter update equations.

Also available in RAGMOP are several inequality constraints, namely: (1) the product of dynamic pressure with angle-of-attack ($qu$), (2) the product of
dynamic pressure with sideslip angle \((\alpha)\), and (3) the maximum thrust acceleration of the vehicle \((g\text{-limit})\).

The \(q_2\) and \(q_3\) constr icts are enforced by reducing \(a\) and/or \(\bar{E}\) to produce the maximum acceptable values whenever they are exceeded. This results in temporarily overriding the \(x_3\) and/or \(x_1\) polynomials until such time as the \(q_2\) and \(q_3\) produced by the polynomials is acceptable.

Table 2-1. CONSTRAINT CODES

The codes contained in this table are input into KCDPHI and KCDRES to designate the payoff and the intermediate and terminal constraints desired for the trajectory. The appropriate values desired for these constraints must then be input into PSIRED and PSIRST.

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<td>5</td>
<td>M/SEC²</td>
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<td>6</td>
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<td>7</td>
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</tr>
<tr>
<td>9</td>
<td>DEG.</td>
<td>Colatitude</td>
</tr>
<tr>
<td>10</td>
<td>DEG.</td>
<td>Inclination</td>
</tr>
<tr>
<td>11</td>
<td>DEG.</td>
<td>Line of modes</td>
</tr>
<tr>
<td>12</td>
<td>M</td>
<td>Semi-latus rectum</td>
</tr>
<tr>
<td>13</td>
<td>DEG.</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>14</td>
<td>SEC</td>
<td>Total burn time</td>
</tr>
<tr>
<td>15</td>
<td>LB/FT²</td>
<td>Maximum dynamic pressure</td>
</tr>
<tr>
<td>16</td>
<td>DEG.</td>
<td>True anomaly</td>
</tr>
<tr>
<td>17</td>
<td>DEG.</td>
<td>Argument of perigee</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>Reserved for future use</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>Reserved for future use</td>
</tr>
<tr>
<td>20</td>
<td>NM</td>
<td>Flyback range</td>
</tr>
</tbody>
</table>

The acceleration \((g)\) limiting can be enforced in two ways: (1) continuous throttling may be employed to "ride" the \(g\)-limit, if the actual vehicle engines allow this, or (2) discrete throttling (shutting down one or more engines or reducing the output of all engines in discrete amount:) may be used. Continuous throttling is performed whenever the acceleration reaches the desired limit-
ing value, at which time a gradual continuous reduction of thrust is accomplished until the end of the thrust event. Discrete throttling is accomplished by initiating a new thrust event with a lower fixed (except for the exit plane pressure difference) thrust whenever the desired acceleration limit is reached. The following thrust event will begin with an acceleration less than the limited value, with the acceleration increasing with time until its limit or another thrust event cutoff criteria is reached to begin the next thrust event (see Section IX for a more complete explanation of thrust events).

2.4 PHYSICAL MODEL

The physical model employed in RAGMOP has been designed to be as realistic as possible and yet allow the program to remain within the attempted goals of 32K storage and 3 minute run time. The result is a highly sophisticated trajectory model which provides the user with a number of options in setting up the trajectory run.

The geophysical model presents the options of a spherical or oblate, rotating or nonrotating earth with a tabulated Patrick Reference Atmosphere (1963 version) and the ability to specify input wind directions and speeds as altitude functions. This last option allows the program to bias the trajectory profile to either take advantage of, or to minimize the losses from, winds at various altitudes.

RAGMOP also provides the user with the option of thrust vectoring to produce zero total moment on the vehicle. A two engine equivalent thrust model centered around the actual vehicle thrust centroid is used to reduce the computation time required for this option. Thrust components are computed to balance aerodynamic moments using small angle approximations resulting in negligible error for small gimbal angles. The error, such as it is, will be in the form of slightly unbalanced moments (less than about 1-1/2 percent error for gimbal angles of 10 degrees). The total thrust available remains intact. Refer to Section III and Appendix C for a complete discussion of the moment balance scheme.
2.5 OUTPUT OPTIONS

RAGNOP includes two special output subroutines which provide the user a set of tables and/or plots summarizing the converged trajectory (solution). The tables, which use a fixed format, and the plots, are suitable for publication. The output plots are produced on the CALCOMP plotter, and therefore use of this option is restricted to systems which have the CALCOMP plotter available. Plots are produced for any variable versus any other, in any units the user desires.

Figure 2-1 presents a macro-flow diagram of the RAGNOP computer program.
Figure 2-1. RAGMOP GENERAL FLOW DIAGRAM
Section III
THEORY

3 DISCUSSION

The RAGMOP computer program is designed to solve the rocket ascent optimization problem including the effects of trajectory shaping, engine burn times, liftoff weight, and launch azimuth. The optimization is performed subject to a number of constraints both during and at the end of the trajectory. In order to perform this function, three main requirements have been satisfied by the computer program; (1) a physical model has been programmed into the equations of motion which represents the actual flight of the vehicle as closely as possible, (2) the optimization method attempts to be rapid in terms of computer time without unnecessarily restricting the physical model, and (3) the scheme used to integrate the equations of motion is rapid and flexible with a minimum amount of error in the integration. This section presents a detailed description of the three areas just mentioned.

3.2 PHYSICAL MODEL

The RAGMOP computer program has been designed to include as sophisticated a physical model as possible subject to the computer run time and storage goals of three minutes and 32k, respectively. A three-dimensional trajectory model with a static moment balance is used with tabulated atmospheric, aerodynamic, and center-of-gravity data. The atmosphere model includes a spline-interpolated Patrick Reference Atmosphere (1963) and input wind tables which allow wind direction and speed to be specified at up to 25 altitudes. Aerodynamic data is also spline-interpolated and is input for both stages in the form of force and moment coefficients and their angle-of-attack or sideslip angle derivatives, at up to 25 Mach numbers. The earth model includes a rotating atmosphere and an oblate (Fischer ellipsoid) gravitational model. A complete description of the force and moment equations and the computation of their component parts is presented in the following paragraphs.

*See Appendix B for further information concerning interpolation methods.
3.2.1 Generalized Equations of Motion

The equations of motion in any inertial reference frame are:

\[ \dddot{\mathbf{x}}(t) = \frac{\mathbf{F}(t)}{m(t)} \]
\[ \dddot{\mathbf{x}}(t) = \dot{\mathbf{x}}_{t=0} + \int_{0}^{t} \ddot{\mathbf{x}} \, dt \]
\[ \dddot{\mathbf{x}}(t) = \dot{\mathbf{x}}_{t=0} + \int_{0}^{t} \dot{\mathbf{x}} \, dt + \int_{0}^{t} \int_{0}^{t} \dddot{\mathbf{x}} \, dt \, dt \]

where

\[ \dot{\mathbf{x}} \] is a three-dimensional position vector,
\[ \mathbf{F} \] is the total force acting on the vehicle,
\[ m \] is the instantaneous mass of the vehicle,

and

\[ t \] is time measured from some reference time \( t=0 \).

The moment equation used for the static (3D) moment balance is:

\[ \dot{\mathbf{m}} = \dot{\mathbf{m}}_{A} + \dot{\mathbf{m}}_{T} = 0 \]

where

\[ \dot{\mathbf{m}} \] is the total moment acting on the vehicle,
\[ \dot{\mathbf{m}}_{A} \] is the total aerodynamic moment,

and

\[ \dot{\mathbf{m}}_{T} \] is the total thrust moment.

3.2.2 Coordinate Systems

Several coordinate systems are used in the RAGMOP computer program, and an understanding of these systems (and the transformations which allow
changing from one system to another is essential to a thorough comprehension of the equations of motion. The five coordinate systems used in RAGMOP are:

1. The equatorial inertial system
2. The launch plumbline inertial system,
3. The spherical geocentric system,
4. The body axis system, and
5. The relative velocity system.

3.2.2.1 Equatorial Inertial System. The basic reference coordinate system in RAGMOP is the equatorial inertial geocentric Cartesian coordinate system shown in Fig. 3-1. This coordinate system has the Y axis pointing north, the X and Z-axes in the equatorial plane, and the Z axis contained in the longitudinal plane of the launch site.

Figure 3-1. EQUATORIAL INERTIAL COORDINATE SYSTEM X Y Z.
3.2.2.2 Launch Inertial Plumbline Coordinate System. The launch inertial plumbline coordinate system is the system from which the \( \chi_p \) and \( \chi_y \) attitude angles are defined and in which the equations of motion are written. The \( y \) axis of this system is parallel to the launch site gravity vector (plumbline) but, for an oblate earth, does not pass directly through the launch site. The \( x \) axis is pointed in the direction of the launch azimuth, and the \( z \) axis forms a right-hand system. The origin of the \( xyz \) system is at the center of the earth.

![Figure 3-2. LAUNCH PLUMBLINE INERTIAL COORDINATE SYSTEM X Y Z](image)

3.2.2.3 Spherical Geocentric Polar Coordinate System. The spherical geocentric polar coordinate system moves with the vehicle. The \( \phi \), \( r \) and \( \theta \) axes of this geocentric system point in the directions of increasing \( \phi \), \( r \) and \( \theta \), respectively, where \( \phi \) is measured from the equatorial inertial \( Z \) axis to the
plane containing \( \mathbf{r} \) and the equatorial inertial \( \mathbf{Y} \) axis, and \( \theta \) is measured from the equatorial inertial \( \mathbf{Y} \) axis to the \( \mathbf{r} \) vector. (See Figure 3-3).

![GEOCENTRIC SPHERICAL COORDINATE SYSTEM](image)

**Figure 3-3. GEOCENTRIC SPHERICAL COORDINATE SYSTEM \( \phi, \mathbf{r}, \theta \)**

### 3.2.2.4. Body Axis Coordinate System

The body axis coordinate system, shown in Figure 3-4, is the system in which the aerodynamic and thrust forces and moments are calculated. The forces are then transformed into the launch inertial plumbline system to determine accelerations for integration in the equations of motion. The body axis system is defined with the \( X', Y', \) and \( Z' \) axes such that the vehicle longitudinal axis is parallel to the \( Y' \) axis, the \( X' \) axis is positive "downward" with respect to the vehicle, and the \( Z' \) axis points in the direction of the right wing. This is such that, on the launch pad, the vehicle \( X', Y', \) and \( Z' \) axis are parallel to the launch inertial
plumbline $x$, $y$, and $z$ axes. The origin of this system may be placed anywhere in the $x$-$y$ plane, provided that all input data for the aerodynamics, engine gimbal positions, and center of gravity locations are consistent. (See also Section IX, INPUT/OUTPUT).

Note that for input purposes only, the body axis system is as shown in the insert of Figure 3-4. Input data uses the input body coordinate system. The equations of motion use the $x'y'z'$ system described above. (Data is converted in the input subroutine).

3.2.2.5 Relative Velocity Coordinate System. The relative velocity coordinate system is used whenever coordinated turns are specified for the RAGMOP trajectory (see paragraph 3.2.6.2). This nonorthogonal coordinate system allows the use of simple relationships for the aerodynamic and thrust forces on the vehicle, resulting in a savings of computer time when the coordinated turn option is used. The coordinate system is defined by the plane of the relative velocity vector ($V_R$) and the vehicle longitudinal axis ($y'$). Vectors in the $V_R$ and $y'$ directions, which are not generally orthogonal, are used to specify the forces in the plane, which are the only aerodynamic or thrust forces on the vehicle when using coordinated turns. When this system is used, the unit vectors of the body axis
system are computed in terms of the inertial plumbline unit vectors, from which the inertial attitude angles $\chi_p$, $\chi_y$, and $\chi_R$ are found.

3.2.2.6 Transformations. A vector in any coordinate system may be transformed into any other coordinate system by premultiplication with the proper transformation matrix. This transformation can be performed as:

$$\mathbf{x}' = A\mathbf{x}$$

where

- $\mathbf{x}'$ is a vector in the $x'y'z'$ coordinate system,
- $\mathbf{x}$ is a vector in the $x,y,z$, coordinate system,
- $A$ is the matrix which transforms $\mathbf{x}$ into $\mathbf{x}'$, i.e. the matrix that computes the components of $\mathbf{x}$ in $x'y'z'$ so that $\mathbf{x}$ can be rewritten as $\mathbf{x}'$ ($\mathbf{x}$ and $\mathbf{x}'$ are the same vector since transformation does not change the vector, but only the coordinate system in which it is written).

Equatorial Inertial to Launch Plumbline Inertial. The launch plumbline inertial system is obtained from the equatorial inertial system by first rotating about the equatorial inertial $X$ axis an angle, $\theta = 90^\circ - \text{lat}_{\text{launch site}}$, and then about the $y$ axis (formed from that rotation) an angle, $\phi = -(AZ_{\text{launch}} - 90^\circ)$. The transformation matrix from the equatorial inertial $XYZ$ system to the launch plumbline inertial $xyz$ system is, then:

$$A_{XX} = \begin{bmatrix}
\cos(90-Az) & 0 & -\sin(90-Az) \\
0 & 1 & 0 \\
\sin(90-Az) & 0 & \cos(90-Az)
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos\theta & \sin\theta \\
0 & -\sin\theta & \cos\theta
\end{bmatrix}$$

(noting that $\sin(90-\text{lat}) = \cos(\text{lat})$, $\cos(90-\text{lat}) = \sin(\text{lat})$, $\sin(90-Az) = \cos Az$, and $\cos(90-Az) = \sin Az$).

$$A_{XX} = \begin{bmatrix}
\sin Az & \sin\theta \cos Az & -\cos Az \\
0 & \cos\theta & -\sin\theta \\
-\cos Az & \sin\theta \sin Az & \cos\theta \sin Az
\end{bmatrix}$$

*See Appendix A for further information on coordinate transformations.
where \( \text{lat} \) = geodetic latitude of the launch site and \( \text{Az} \) = launch azimuth.

This transformation is used for computing the earth rotational and gravitational acceleration components in the launch plumbline system from the equatorial system. The reverse transformation uses the transpose of the above matrix, \( A_{xx} = [A_{xx}]^T \).

**Body Axis to Launch Plumbline Inertial System.** The attitude angles chi-pitch \( (\chi_p) \) and chi-yaw \( (\chi_y) \) describe the orientation of the vehicle with respect to the launch plumbline system. The vehicle attitude is obtained by first rotating about the launch plumbline \( z \) axis the angle \( -\chi_p \), and then rotating about the vehicle \( x' \) axis the angle \( \chi_y \). Since, thrust and aerodynamic forces on the vehicle are determined in the body axis system, but their resultant accelerations (added to the gravitational acceleration) are integrated in the launch plumbline system, the transformation matrix from the body axis system to the launch plumbline system is required. This is found by noting that the transformation involves first rotating about the body \( x' \) axis an angle \( -\chi_y \) and then about the launch plumbline \( z \) axis an angle \( \chi_p \) so that, from the body axis \( x'y'z' \) system to the launch plumbline \( xyz \) system we have:

\[
A_{x'x} = \begin{bmatrix}
\cos \chi_p & \sin \chi_p & 0 \\
-sin \chi_p & \cos \chi_p & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(-\chi_y) & \sin(-\chi_y) \\
0 & -\sin(-\chi_y) & \cos(-\chi_y)
\end{bmatrix}
\]

Noting that \( \cos \chi_y = \cos \chi_y \) and \( \sin \chi_y = -\sin \chi_y \):

\[
A_{x'x} = \begin{bmatrix}
\cos \chi_p & \sin \chi_p & \cos \chi_y & -\sin \chi_p & \sin \chi_y \\
-sin \chi_p & \cos \chi_p & \cos \chi_y & -\cos \chi_p & \sin \chi_y \\
0 & \sin \chi_y & \cos \chi_y
\end{bmatrix}
\]

**Equatorial Inertial to Spherical Geocentric Polar Coordinate System.** The use of the spherical geocentric polar coordinate system makes the determination of a number of parameters more convenient, hence the transformation matrices
from the \( x \) stem to the equatorial and vice versa are necessary. The spherical geocentric polar system is obtained by rotating first around the \( Y \) axis an angle \( \phi \) and then about the \( \phi \) axis and angle \( \theta \), so that:

\[
A_X = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi
\end{bmatrix} = \begin{bmatrix}
\cos \phi \cos \theta & -\sin \phi \sin \theta & \cos \phi \cos \theta \\
\sin \phi \sin \theta & \cos \phi \cos \theta & -\sin \phi \cos \theta \\
\sin \phi \cos \theta & \sin \phi \sin \theta & \cos \phi
\end{bmatrix}
\]

Note, however, (see Figure 3-3) that:

\[
\cos \theta = \frac{Y}{r}, \quad \tan \phi = \frac{X}{Z},
\]

\[
\sqrt{X^2 + Z^2} = r \sin \theta, \quad \text{so that} \quad \sin \theta = \frac{\sqrt{X^2 + Z^2}}{r},
\]

\[
\sin \phi = \frac{X}{r \sin \theta} = \frac{X}{\sqrt{X^2 + Z^2}},
\]

and

\[
\cos \phi = \frac{Z}{r \sin \theta} = \frac{Z}{\sqrt{X^2 + Z^2}},
\]

so that

\[
A_{\phi \theta} = \begin{bmatrix}
\frac{Z}{\sqrt{X^2 + Z^2}} & 0 & -\frac{X}{\sqrt{X^2 + Z^2}} \\
\frac{X}{r} & \frac{Y}{r} & \frac{Z}{r} \\
\frac{XY}{r \sqrt{X^2 + Z^2}} & \frac{\sqrt{X^2 + Z^2}}{r} & -\frac{Z}{r \sqrt{X^2 + Z^2}}
\end{bmatrix}
\]

The reverse transformation, from \( \phi \) to \( X \) is accomplished using \( A_{\phi X} = [A_{\phi \theta}]^T \).

**Launch Plumbline to Spherical Geocentric Polar Coordinates.** The transformation from the launch plumbline system to the spherical geocentric polar coordinate system can be obtained by first transforming to the equatorial inertial system and then to the spherical system, so that

\[
\text{Correction page for page 3-9}
\]
The above matrix product yields (letting $A$ denote $A_{xx}$ and $D$ denote $A_{x\Phi}$):

\[
D_{11} = \frac{(A_{22}Z - A_{32}Y)}{\sin \theta} / \text{rsin}\theta
\]
\[
D_{21} = \frac{(A_{32}X - A_{12}Z)}{\sin \theta} / \text{rsin}\theta
\]
\[
D_{31} = \frac{(A_{12}Y - A_{22}X)}{\sin \theta} / \text{rsin}\theta
\]
\[
D_{12} = \frac{X}{r}
\]
\[
D_{22} = \frac{Y}{r}
\]
\[
D_{32} = \frac{Z}{r}
\]
\[
D_{13} = \frac{(D_{12}\cos\theta - A_{12})}{\sin \theta}
\]
\[
D_{23} = \frac{(D_{22}\cos\theta - A_{22})}{\sin \theta}
\]
\[
D_{33} = \frac{(D_{32}\cos\theta - A_{32})}{\sin \theta}
\]

The transformation from $\phi \theta$ to $xyz$ (plumbline) is accomplished using the transpose of the above matrix; $A_{\Phi x} = [A_{x\Phi}]^T$.

2.2.3 Forces and Accelerations

The forces acting on the vehicle in the RAGMOP simulation are the thrust, aerodynamic, and gravitational forces. The aerodynamic and thrust forces are computed in the body axis system and then transformed to the launch plumbline system wherein the accelerations are added to the gravitational acceleration components for integration in the equations of motion.

3.2.3.1 Aerodynamic Forces. The aerodynamic forces acting on the vehicle are the axial, normal, and side forces. These forces are found by determining the proper axial, normal, and side force coefficients for the vehicle at the current Mach number, and then multiplying these coefficients by the product of the dynamic pressure ($Q = 1/2 \rho V^2$) and the aerodynamic reference area ($S$), such

* [$A$] denotes matrix transpose.
\[ F_{AERO_{AXIAL}} = C_{AXIAL} \hat{Q}^S, \]
\[ F_{AERO_{NORMAL}} = C_{NORMAL} \hat{Q}^S, \]
and \[ F_{AERO_{SIDE}} = C_{SIDE} \hat{Q}^S. \]

The axial force is positive aft \((-y')\), the normal force positive upward on the vehicle \((-x')\) and the side force positive toward the right wing \((+z')\). The aerodynamic coefficients are determined from the Mach number and angle-of-attack or sideslip angle by means of a spline interpolation* in the input aerodynamic coefficient tables. The dependence of coefficients on angle-of-attack \((\alpha)\) or sideslip angle \((\beta)\) is assumed linear at each Mach number, thus:

\[
C_{AXIAL} = C_{A0} + (C_{A\alpha}) \alpha
\]
\[
C_{NORMAL} = C_{N0} + (C_{N\alpha}) \alpha
\]
\[
C_{SIDE} = C_{Y\beta} \beta
\]

The axial aerodynamic force is also affected by the base drag term, which is determined using a linear interpolation table lookup from the input tables of base axial force and altitude. Thus

\[
F_{BASE} = F(ALTITUDE)
\]

and then \[ F_{AERO_{AXIAL}} = C_{AXIAL} QS - F_{BASE} \] (The negative sign is used since the axial force (drag) and the base pressure force are opposite in direction. The \( F_{AERO_{AXIAL}} \) will be subtracted from the \( y' \) thrust component since \( F_{AERO_{AXIAL}} \) is positive in the \(-y'\) direction).

Thus, the total aerodynamic forces in the body axis \( x'y'z' \) system are:

\[
F_{Ax'} = -FAN = -\left( C_{N0} + C_{N\alpha} \alpha \right) QS
\]
\[
F_{Ay'} = -FAA = -\left( C_{A0} + C_{A\alpha} \alpha \right) Q \cdot F_{BASE}
\]

* See Appendix B for further information on interpolation methods.
These forces are transformed into the launch plumbline system using the transformation matrix $A_{x'x}$ of Paragraph 3.2.2.6, so that

$$
\begin{bmatrix}
F_x' \\
F_y' \\
F_z'
\end{bmatrix} = \begin{bmatrix}
A_{x'x}
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}.
$$

The Mach number used in the spline interpolation lookup of the aerodynamic coefficients is found by:

$$M = \frac{V_R}{a}$$

where $V_R$ = relative velocity (velocity of vehicle with respect to the atmosphere)

and $a$ = speed of sound at current altitude from the spline-interpolated atmosphere routine PRA63

The relationship between the relative velocity vector and the body axis system determines the angle-of-attack and sideslip angle of the vehicle (See Figure 3-5).
The relative velocity \( VR \) is computed as the difference between the plumbline inertial velocity and the transformed velocity required by an object in order to remain over a given point on the rotating earth (at the same altitude as the vehicle) plus the wind at the current altitude. This results in

\[
\begin{pmatrix}
\omega \\
u \\
v
\end{pmatrix} = \begin{pmatrix}
\omega \\
u \\
v
\end{pmatrix} - D \begin{pmatrix}
r \Omega \sin \theta \\
0 \\
0
\end{pmatrix} \begin{pmatrix}
\omega - (a_{22z} - a_{32y}) \Omega_e \\
u - (a_{32x} - a_{12z}) \Omega_e \\
v - (a_{12y} - a_{22x}) \Omega_e \\
\end{pmatrix} - v_W \left( \cos A_W E + \sin A_W N \right)
\]

where

- \( (\omega, u, v) \) are the relative velocity components in the plumbline system,
- \( (\omega, u, v) \) are the actual inertial velocity components of the vehicle in the plumbline system,
- \( (a_{22z} - a_{32y}) \Omega_e, (a_{32x} - a_{12z}) \Omega_e, \) and \( (a_{12y} - a_{22x}) \Omega_e \) are the inertial velocity components in the plumbline system required to remain stationary over the subpoint on the rotating earth.

\( D = A_W \) (see Paragraph 3.2.2.6),

- \( A_W \) is the wind azimuth (+ CW from north)
- \( V_W \) is the wind speed
- \( E \) is a unit vector in the east direction \( \frac{\hat{r} \times E}{|\hat{r} \times E|} \)
- \( N \) is a unit vector in the north direction \( (\hat{r} \times E) \)

The relative velocity \( VR \) is then equal to:

\[
VR = \sqrt{u^2 + v^2}
\]

The dynamic pressure \( Q \) is obtained from the relation:

\[
Q = \frac{1}{2} \rho (VR)^2
\]

where

- \( \rho \) = atmospheric density from the spline-interpolated Patrick 1963 atmosphere routine.

\( VR \) = relative velocity described above.
3.2.3.2 **Thrust Forces.** Thrust forces in RAGMOP are computed in two ways: (1) the SRM thrust is based on input values of sea level thrust corrected for the atmospheric pressure at the current altitude, (2) the liquid engine thrust is based on vacuum thrust corrected for current atmospheric pressure.

The thrust calculation for the SRM's is as follows:

\[
T = (T_{SL} + A_E (P_{SL} - P_{AM})) N_{ENG}
\]

where

- \(T\) = total thrust available,
- \(T_{SL}\) = sea level thrust per SRM engine from spline-interpolated tables,
- \(A_E\) = exit area per engine,
- \(P_{SL}\) = sea level static pressure,
- \(P_{AM}\) = ambient pressure at current altitude,

and

- \(N_{ENG}\) = number of SRM engines used for the current thrust event.

The liquid engine thrust calculation utilizes input values for vacuum thrust and corrects for the exit plane pressure differential:

\[
T = (T_{VAC} - A_E P_{AM}) N_{ENG}
\]

where

- \(T_{VAC}\) = vacuum thrust per engine

**Acceleration-Limited Thrust.** Two methods exist in RAGMOP for limiting the acceleration of the vehicle (g-limit). One method available is to simply create a new thrust event any time the acceleration reaches the desired limiting value. The new thrust event can have fewer engines, or all engines operating at a reduced thrust level. The second method utilizes continuous throttling to hold the acceleration at the limited value. The approximation
is made that the acceleration of the vehicle due to the thrust force above will
be equal to the total acceleration. Thus, in the \textit{x'y'z'} body axis coordinate
system,

\[ T = \frac{g_{\text{limit}}}{m} \text{ if } g > g_{\text{limit}} \]

where \( g_{\text{limit}} \) = maximum acceptable longitudinal acceleration,
\( m \) = instantaneous mass of the vehicle, and
\( T \) = total vehicle thrust.

In order to enforce the continuous throttling \( g \)-limit constraint, the value of
the thrust acceleration is checked by the integration package until the limit
is reached. The integration package isolates on the exact time that \( g \)-limit
occurs* and then either performs thrust event initiation (discrete or step
throttling) or begins continuous throttling as described above. Note that for
the SRM booster, only the main (liquid) engines are throttled unless all
engines are used in the moment balance (SRM's and main engines).

\[ 3.2.3.3 \text{ Accelerations. The total acceleration components in the launch plumb-} \]
line inertial coordinate system are simply

\[ \begin{align*}
  x &= \frac{F_x}{m} + g_x \\
  y &= \frac{F_y}{m} + g_y \\
  z &= \frac{F_z}{m} + g_z
\end{align*} \]

where \( F_x, F_y, F_z \) are the total forces on the vehicle due to thrust and aerodynamics and \( g_x, g_y, g_z \) are the gravitational accelerations in the launch plumbline coordinate system (see Paragraph 3.2.4). Expanding the above equations we obtain (where \( A_{x'x} \) comes from Paragraph 3.2.2.6)

\[ \begin{align*}
  \begin{bmatrix}
    x \\
    y \\
    z
  \end{bmatrix} &= \begin{bmatrix}
    A_{x'x} \\
  \end{bmatrix} \begin{bmatrix}
    T_x - F_{\text{AERO NORMAL}} \\
    T_y - F_{\text{AERO AXIAL}} \\
    T_z + F_{\text{SIDE}}
  \end{bmatrix} + \begin{bmatrix}
    g_x \\
    g_y \\
    g_z
  \end{bmatrix}
\end{align*} \]

*See Paragraph 3.4 for more information concerning the integration package.
Note that $F_{\text{AERO}_\text{NORMAL}}$ and $F_{\text{AERO}_\text{AXIAL}}$ are positive in the negative $x'$ and $y'$ directions, respectively.

### 3.2.4 Mass Calculation

The instantaneous mass of the vehicle must be known in order to calculate the accelerations in the inertial reference frame. Fuel expended and weight jettisoned must be known in order to perform the mass calculation. Fuel for the liquid engines is integrated whereas SRM fuel overboard is determined using a spline-interpolated table lookup. The jettison weights are discrete weight drops that can occur only at the end of any thrust event. This is necessary since a weight jettison produces discontinuities in the state derivatives and the integration scheme must "restart" when variable step methods are used.

The mass at any instant, $X_M$, is given by

$$X_M = X_{\text{MIAD}} + X_{\text{MAUG}}$$

where

$$X_{\text{MIAD}} = X_{\text{MI}t=0} + \int_{0}^{t} -X_{\text{MDOT}} \, dt$$

$$X_{\text{MI}t=0} = W_{\text{ZERO}} - X_{\text{MAUG}t=0}$$

$$X_{\text{MAUG}t=0} = \frac{W_{\text{JET}(I)}}{\text{NVT}}$$

$W_{\text{JET}(I)}$ = jettison weight per thrust event

$N_{\text{VT}}$ = total number of thrust events

$$X_{\text{MAUG}} = X_{\text{MAUG}t=0} - \sum_{I=1}^{\text{ITHR}-1} W_{\text{JET}(I)}$$

and

$$\text{ITHR} = \text{number of current thrust event}$$

When the SRM booster option is used, SRM mass overboard is found from a spline interpolation of time-dependent values. The effect of this weight loss is included in the mass calculation by using the equation
\[ \text{XMIAD} = \text{XMI} + \text{SRP} \]

where

\[ \text{SRP} = \text{SRMPRP} - \text{SRMDWT} \]
\[ \text{SRMPRP} = \text{total SRM propellant} \]
\[ \text{SRMDWT} = \text{SRM mass overboard} \]

and

\[ \text{XM} = \text{XM}_{t=0} + \int_{0}^{t} - \text{XM}\dot{\text{dt}}. \]

Thus as before,

\[ \text{XM} = \text{XMIAD} + \text{XMAUG}. \]

When the last thrust event using SRM's is completed, the empty cases are jet-
tisoned as one of the WJET(I) and the total fuel expended, SRMPRP, is sub-
tracted from XMAUG, so that the mass equation remains

\[ \text{XM} = \text{XMIAD} + \text{XMAUG} \]

3.2.5 Geophysical Model

RAGMOP uses an oblate earth model (Fischer ellipsoid) with a preset flat-
tening coefficient of \( f = 1/298.3 \) (which may be changed by input). (See
Figure 3-6).

The radius of the earth using this model at any given geocentric co-
latitude \( \theta \) is given by:

\[ R(\theta) = (1-f)R_e / \sqrt{(1-f)^2 \sin^2 \theta + \cos^2 \theta} \]

The derivative of radius with respect to colatitude \( \theta \) (required in order
to calculate the time at which maximum dynamic pressure occurs) is:

\[ \frac{dR(\theta)}{d\theta} = \frac{R^3(\theta)f(2-f)\sin\theta\cos\theta}{R_e^2(1-f)^2}. \]
3.2.5.1 Atmospheric Properties. The atmospheric model in RACMOP is a Patrick Reference Atmosphere, 1963 version (15), using a spline-interpolation technique to obtain the properties of pressure, density, and speed-of-sound, and their altitude derivatives. This atmosphere rotates with the earth at an angular velocity \( \omega = 7.2921158 \times 10^{-5} \) rad/sec, resulting in no wind over the surface of the earth. Winds may be added as functions of altitude so that the trajectory shaping performed by the program is biased to include the wind effects.

The initial inertial velocity of the vehicle (at liftoff) will be the velocity produced by the earth's rotation. The relative velocity at time = 0 will be zero unless input wind tables specify some ground wind direction and speed.

3.2.5.2 Gravitational Model. (1) The Fischer ellipsoid gravitational model (14) is used unless the user specifies a spherical earth model in the input. The gravitational potential function used for the oblate model is given by:

* Superscript numbers refer to references in Section IV.
\[ U(r, \theta) = \frac{\mu_e}{r} \left( 1 + \frac{CJ}{3} \left( \frac{R_e}{r} \right)^2 (1-3 \cos^2 \theta) + \frac{H}{5} \left( \frac{R_e}{r} \right)^3 (3-5 \cos^2 \theta) \cos \theta \right. \]
\[ \left. + \frac{DJ}{35} \left( \frac{R_e}{r} \right)^4 (3-30 \cos^2 \theta + 35 \cos^4 \theta) \right] \]
where
\[ CJ = 1.62345 \times 10^{-3} \]
\[ H = -0.575 \times 10^{-5} \]
\[ DJ = 0.7875 \times 10^{-5} \]
\[ R_e = \text{earth equatorial radius} = 6378165 \text{ m} \]
\[ \mu_e = \text{Product of universal gravity constant and earth mass} = 3.986032 \times 10^{14} \text{ m}^3/\text{sec}^2. \]

Each of the above may be changed by input if desired by the user.

The components of the gravitational acceleration vector in the launch plumbline inertial system are calculated as the first partial derivatives of the potential function with respect to the plumbline coordinate axes. Thus,

\[
\begin{align*}
\{ g_x \} &= \left\{ \frac{\partial U}{\partial x} \right\} = \left\{ \frac{\partial r}{\partial x} \right\} + \frac{\partial U}{\partial \theta} \left\{ \frac{\partial \theta}{\partial x} \right\} \\
\{ g_y \} &= \left\{ \frac{\partial U}{\partial y} \right\} = \left\{ \frac{\partial r}{\partial y} \right\} + \frac{\partial U}{\partial \theta} \left\{ \frac{\partial \theta}{\partial y} \right\} \\
\{ g_z \} &= \left\{ \frac{\partial U}{\partial z} \right\} = \left\{ \frac{\partial r}{\partial z} \right\} + \frac{\partial U}{\partial \theta} \left\{ \frac{\partial \theta}{\partial z} \right\}
\end{align*}
\]

These equations may be rearranged into the form:

\[
\begin{align*}
\{ g_x \} &= G_{11} \{ x \} - G_{12} \{ y \} + G_{12} \{ a_{12} \} \\
\{ g_y \} &= G_{21} \{ y \} - G_{22} \{ z \} + G_{22} \{ a_{22} \} \\
\{ g_z \} &= G_{31} \{ z \} + G_{32} \{ a_{32} \}
\end{align*}
\]

where
\[
G_{11} = -\frac{\mu_e}{r^3} \left[ 1 + CJ \left( \frac{R_e}{r} \right)^2 (1-5 \cos^2 \theta) + H \left( \frac{R_e}{r} \right)^3 (3-7 \cos^2 \theta) \cos \theta \right. \\
\left. + DJ \left( \frac{R_e}{r} \right)^4 \left[ \frac{3}{7} - (6-9 \cos^2 \theta) \cos^2 \theta \right] \right]
\]

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\[ G_{T0} = \frac{u_e}{r^2} \left( 2CJ \left( \frac{R_e}{r} \right)^2 \cos^3 - H \left( \frac{R_e}{r} \right)^3 \left( \frac{3}{5} - 3 \cos^2 \right) \right) \]

\[ + DJ \left( \frac{R_e}{r} \right)^4 \left( \frac{12}{7} - 4 \cos^2 \theta \right) \cos \theta \]

and

\[ \begin{bmatrix} a_{12} \\ a_{22} \\ a_{23} \end{bmatrix} \]

is the second column of the transformation matrix from the launch plumbline system to the equatorial inertial system \( A_{xx} \) (see Paragraph 3.2.2.6).

This general form is the same as that used in the Saturn V flight computer.\(^{(16)}\)

In the event a spherical earth is specified the above relations reduce to:

\[ G_{11} = -\frac{u_e}{r^3} \]

\[ G_{T0} = 0. \]

### 3.2.6 Moments

RAGMOP employs a static (3-D) moment balance scheme using thrust vectoring to balance aerodynamic moments in order to more accurately model the actual vehicle performance. Solutions obtained without moment-balancing assume the entire thrust to be directed through the vehicle center-of-gravity, and are suitable for symmetric vehicles such as the Saturn V. However, for a non-symmetric vehicle with offset center-of-gravity and thrust centroid (as in space shuttle configurations) the results obtained without moment balancing can differ greatly from those obtained with the moment balance enforced.

Moment balancing requires redirecting the vehicle thrust vector so as to produce no net moments acting on the vehicle. Thus, the thrust component acting through the center-of-gravity (and hence the total vehicle acceleration) will be less than that obtained for the nonmoment-balanced case. The moment balance scheme employed in RAGMOP makes use of two approximations: (1) the controllable engines are lumped into a two-engine equivalent thrust model, and (2) a small angle approximation is used in the solution for the required gimbal
angles. These approximations, which yield a slight residual unbalanced moment, serve to reduce the run time of the program considerably.

3.2.6.1 Aerodynamic Moments. The aerodynamic moments $M_{AERO}$ are found in a manner similar to the aerodynamic forces. Moment coefficients are determined using spline interpolation for the tabulated values of the moment coefficients and their angle-of-attack/sideslip angle derivatives as functions of Mach number. The coefficients used are:

- $CMO$ - pitching moment coefficient at zero angle-of-attack, positive about $z'$ according to the right-hand rule
- $CMALP$ - partial derivative of pitching moment coefficient with respect to angle-of-attack.
- $CLBETA$ - partial derivative of rolling moment coefficient with respect to sideslip angle. Rolling moment is positive about $y'$ according to the right-hand rule.
- $CNBETA$ - partial derivative of yawing moment coefficient with respect to sideslip angle. Yawing moment is positive about $x'$ according to the right-hand rule.

The total aerodynamic moment about the center-of-gravity is obtained by determining the total moment coefficient about the aerodynamic reference point for which the coefficients are specified (and which is not, in general, coincident with the center-of-gravity) and then translating these moments to the center-of-gravity. The aerodynamic moments about the body axes at the center-of-gravity are then determined by multiplying the coefficients by the product of dynamic pressure, aerodynamic reference area, and aerodynamic reference length. Thus,

$$C_{Mz'} = C_{Mo} + C_{Ma} \alpha$$
$$C_{My'} = C_{L\beta} \beta$$
$$C_{Mx'} = C_{n\beta} \beta$$

about the aerodynamic reference point.

and

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and $C_{Ax}$, $C_{Ay}$, and $C_S$ are the normal, axial, and side force coefficients of paragraph 3.2.3.1, and $l_{REF}$ is the aerodynamic reference length of the vehicle.

The actual aerodynamic moments about the cg are then found by:

$$M_{Az} = C_{Az} \cdot l_{REF} = C_{Az} \cdot Q_S \cdot l_{REF}$$

$$M_{Ay} = C_{Ay} \cdot Q_S \cdot l_{REF}$$

3.2.6.2 Thrust Moments. The moment balance scheme employed in RAGMOP computes the thrust components required to balance moments using a two engine equivalent thrust model for the controllable engines. The moment balance computations are carried out in one of several different ways, depending upon the option desired by the user. Three options for SRM engines are available: (1) SRM engines thrusting through a fixed point in the vehicle $x' - y'$ plane, (2) SRM's tracking the c.g., and (3) SRM's balancing moments along with the main (liquid) engines. In addition, the coordinated turn option may or may not be used. With coordinated turns, only pitch plane moment balancing is required.

When SRM's are not used for moment balancing, positions of the two equivalent engines are determined by summing the total liquid engine gimbal point $X'$ and $Y'$ locations and dividing by the total number of engines to obtain the average $X'$ and $Y'$ gimbal points, and by summing all of the positive $Z'$ positions and dividing by half the number of engines. The two equivalent engines are then placed at:
These locations are calculated assuming equal thrust for all engines of a given stage, and are calculated separately for each stage. (See Figure 3-7).

When the SRM engines are also used for moment balancing, the equivalent engine locations are determined by averaging the thrust-weighted gimbal point positions for the liquid and solid engines. The thrust weighting is accomplished using the first thrust value of the SRM thrust table and the liquid engine thrust for the first thrust event. The equivalent engine locations are then given by:

\[
X_{GP_{avg}} = \frac{X_{GP_{liq}} \ast FLBS}{N_{eng_{liq}}} + \frac{X_{GP_{SRM}} \ast SRMTHR}{N_{SRM_{eng}}}
\]

and similarly for \(Y_{GP_{avg}}\) and \(Z_{GP_{avg}}\).

The components of thrust at each equivalent engine location required to balance moments are determined using the following equations:

\[
\vec{N}_T = -\vec{N}_A
\]

\[
T_1 = T_2 = \frac{T_T}{2}
\]

where

\(\vec{N}_T\) is the total thrust moment vector
\(\vec{N}_A\) is the total aerodynamic moment vector
\(T_1\) and \(T_2\) are the equivalent engine total thrust values

and
Figure 3-7. TWO-ENGINE EQUIVALENT THRUST COMPONENTS
$T_T$ is the total vehicle thrust at the current altitude.

The solution of the above equations is obtained as follows:

Define the coordinate system $xyz$ where the origin is located at the vehicle center-of-gravity and the two equivalent engines are contained in the $yz$ plane (See Figure 3-8).

![Diagram of Center of Gravity/Gimbal Plane Coordinate System]

Define the pitch and yaw gimbal angles for each engine as follows:

\[
\delta_{p1} = \tan^{-1} \frac{T_{x1}}{T_{y1}}
\]

\[
\delta_{v1} = \tan^{-1} \frac{T_{z1}}{T_{y1}}
\]

\[
\delta_{p2} = \tan^{-1} \frac{T_{x2}}{T_{y2}}
\]
\[
\delta_{Y_2} = \tan^{-1} \left( \frac{T_{Z_2}}{T_{Y_2}} \right)
\]

The moment equations about each axis can be written:

\[
(T_Y \tan \delta_{Y_1} + T_{Y_2} \tan \delta_{Y_2})dy + (T_Y - T_{Y_2})dz = -M_{AX}
\]

\[
(T_Y \tan \delta_{p_1} - T_{Y_2} \tan \delta_{p_2})dz - (T_Y \tan \delta_{Y_1} + T_{Y_2} \tan \delta_{Y_2})dx = -M_{AY}
\]

\[
(T_Y + T_{Y_2})dx + (T_Y \tan \delta_{p_1} + T_{Y_2} \tan \delta_{p_2})dy = -M_{AZ}
\]

where:

\[T_X, T_Y, T_Z, T_{X_1}, T_{Y_2}, T_{Z_2}\] are the thrust components of each engine in the \(xyz\) coordinate system,

\[M_{AX}, M_{AY}, M_{AZ}\] are the components of the total aerodynamic moment in the \(xyz\) system,

\[
dx \equiv -X_{GP_1} - X_{CG} = X_{GP_2} - X_{CG} = 0,
\]

\[
dy \equiv Y_{GP_1} - Y_{CG} = Y_{GP_2} - Y_{CG},
\]

and

\[
dz \equiv Z_{GP_2} - Z_{CG} = -(Z_{GP_1} - Z_{CG}).
\]

Since there are only three equations to determine the four gimbal angles, the assumption is made that

\[
\delta_{Y_1} = \delta_{Y_2}.
\]
We now have four equations and four unknowns. The solution of these equations is presented in Appendix C and yields:

\[
\begin{align*}
    \text{TX} & = -\frac{T_T}{2} \left[ \frac{\tan \delta_p'}{\sqrt{1 + \tan^2 \delta_p' + \tan^2 \delta_y'}} + \frac{\tan \delta_p'}{\sqrt{1 + \tan^2 \delta_p' + \tan^2 \delta_y'}} \right] \\
    \text{TY} & = \frac{T_T}{2} \left[ \frac{1}{\sqrt{1 + \tan^2 \delta_p' + \tan^2 \delta_y'}} + \frac{1}{\sqrt{1 + \tan^2 \delta_p' + \tan^2 \delta_y'}} \right] \\
    \text{TZZ} & = \text{TY} \tan \delta_y' \\
\end{align*}
\]

**SRM Booster Moment Balance.** When the SRM booster shuttle is used, the moment balance equations are the same, but the values of the moments to be balanced and of the thrust used for moment balancing may change. With the fixed SRM option (MOMBAL = 1), the moment produced by the SRM’s is added to \(M_{AZ}\) and the \(x'\) and \(y'\) thrust components produced by the SRM’s are added to the main engine \(x'\) and \(y'\) thrust components. With SRM’s tracking the c.g. (MOMBAL=2), no moment is produced by the SRM’s. The \(x'\) and \(y'\) thrust components are added to the main engine \(x'\) and \(y'\) components (which along with the \(z'\) component, balance the aerodynamic moments) as before. When the SRM’s are used in conjunction with the main engines for moment balance, the two engine equivalent thrust model represents the total vehicle thrust available and moments are balanced using this total thrust.

**Coordinated Turn Moment Balance.** The thrust components \(Txx\) and \(Tyy\) are found from a simplified moment balance when coordinated turns are used. Since only pitch plane moments are encountered with this option, the moment balance problem reduces simply to finding the angle \(\delta\) (see Figure 3-9) required to produce no net moment on the vehicle.

From Figure 3-9 we see that the magnitude of the moment produced by the thrust vector is simply

\[
M_T = R_T T
\]

and
Figure 3-9. COORDINATED TURN MOMENT BALANCE

\[ \sin(\delta - \theta) = \frac{R_T}{\sqrt{dx^2 + dy^2}} \]

where

\[ dx \equiv x_{G_p} - x_c \]

and

\[ dy \equiv y_{G_p} - y_c \]

also

\[ \cos \theta = \frac{dy}{\sqrt{dx^2 + dy^2}} \]

The thrust components are

\[ T_{xx} = T \sin \delta \]
\[ T_{yy} = T \cos \delta \]

and \( \delta \) is given by

\[ \delta = \theta + (\delta - \theta) \]
\[
\delta = \cos^{-1} \frac{dy}{\sqrt{dx^2 + dy^2}} + \sin^{-1} \frac{M_I/T}{\sqrt{dx^2 + dy^2}}
\]

But we desire \( M_T = -M_{AZ} \) so that

\[
\delta = \cos^{-1} \frac{dy}{\sqrt{dx^2 + dy^2}} + \sin^{-1} \frac{-M_{AZ}}{T \sqrt{dx^2 + dy^2}}
\]

### 3.2.7 Control Law

The control law used by RAGMOP is a set of time-dependent polynomials for the vehicle attitude angles chi-pitch, \( \chi_p \), and chi-yaw, \( \chi_y \). The attitude polynomials are separated for each stage, so that altogether four attitude polynomials exist in the program - one for \( \chi_p \) in each stage and one for \( \chi_y \) in each stage. The current version of the program allows a quartic polynomial for the first stage \( \chi_p \), and a quadratic polynomial for the remaining three control angles. The \( \chi_p \) profile may be discontinuous at staging, at the user's option, however, the \( \chi_y \) profile may not. The \( \chi_p \) and \( \chi_y \) angles, defined in Paragraph 3.2.2.5, are determined at any given time \( T \) from the polynomial equations:

\[
\chi_p^{\text{BOOSTER}}(T) = \chi_p^{\text{T=TLIFT}} + \text{AP}(1,1)(T-\text{TLIFT}) + \text{AP}(2,1)(T-\text{TLIFT})^2 + \text{AP}(3,1)(T-\text{TLIFT})^3 + \text{AP}(4,1)(T-\text{TLIFT})^4
\]

where \( \chi_p^{\text{T=TLIFT}} \) is the value of \( \chi_p \) at the end of the vertical rise from the launch \( \text{rad} \), and \( \text{AP}(1,1) \) through \( \text{AP}(4,1) \) are the coefficients of the polynomial, some of which may be zero if a lower order polynomial is desired.

\[
\chi_p^{\text{ORBITE}}(T) = \chi_p^{\text{T=TSTG}} + \text{AP}(1,2)(T-\text{TSTG}) + \text{AP}(2,2)(T-\text{TSTG})^2
\]

where \( \chi_p^{\text{T=TSTG}} \) is the value of \( \chi_p \) at the beginning of the second stage. This may or may not be the value of \( \chi_p \) at the end of the first stage, at the user's option.
\[ x_y^{\text{BOOSTER}} = x_y^{T=\text{TLIFT}} + AY(1,1)(T-\text{TLIFT}) + AY(2,1)(T-\text{TLIFT})^2 \]

\[ x_y^{\text{ORBITER}} = x_y^{T=\text{TSTG}} + AY(1,2)(T-\text{TSTG}) + AY(2,2)(T-\text{TSTG})^2 \]

The polynomial coefficients used in the above equations are determined by curve-fitting the appropriate number of points (three for a quadratic, four for a cubic, etc) of \( x_y \) and time. The time points are not all required to lie within the time spans of the particular stages; however, an even distribution over each stage's time span would probably be most efficient in the optimization scheme. The actual attitude control parameters optimized by the program are the \( x_p \) and \( x_y \) values at the various time points which are used to form the polynomials.

3.2.7.1 Optional Angle-of-Attack Control for First Stage. In addition to the optimized \( x_p \) polynomial, the vehicle pitch attitude may be controlled with any of three angle-of-attack options: (1) zero angle-of-attack, (2) zero normal force angle-of-attack, or (3) Mach number-dependent angle-of-attack.

The zero normal force angle-of-attack is found by noting that

\[ F_{\text{AERO, NORMAL}} = \frac{1}{2} \rho V^2 S (C_N + C_{N\alpha} \alpha) \]

and that \( F_{\text{AERO, NORMAL}} = 0 \) when \( C_N + C_{N\alpha} \alpha = 0 \)

or, solving for \( \alpha \):

\[ \alpha = - \frac{C_N}{C_{N\alpha}}. \]

The Mach number dependent angle-of-attack is found using a spline interpolation scheme in an input Mach number/angle-of-attack table.

Information concerning the use of these options may be found in Section IX, INPUT/OUTPUT.
3.2.7.2 Coordinated Turn Option. The coordinated turn option incorporated into RAGMOP takes advantage of the fact that simple relationships exist for describing the total aerodynamic and thrust forces on the vehicle during coordinated turns. The simplification occurs by noting that the relative velocity vector, $\mathbf{\dot{V}}_R$, lies in the vehicle $x' - y'$ plane as shown in Figure 3-10. The forces on the vehicle will also lie entirely in the $x' - y'$ plane since aerodynamic side forces, yawing moments, and rolling moments are all zero for zero sideslip angle in RAGMOP. Thus, no net aerodynamic or thrust forces on the vehicle are produced or required normal to the $x' - y'$ plane.

![Figure 3-10. VEHICLE PITCH PLANE](image)

The $x_p$ and $x_y$ polynomials orient the vehicle longitudinal ($y'$) axis in the inertial reference frame. The current state determines the orientation of the relative velocity vector, $\mathbf{\dot{V}}_R$. For coordinated turns, the plane containing $y'$ and $\mathbf{\dot{V}}_R$ must also contain the vertical axis of the vehicle, $x'$. Two possibilities exist for doing this: one with positive angle-of-attack and one with negative angle-of-attack. Note that the position of the $y'$ axis does not change for the two. Rather, the vehicle is rolled about $y'$ and $\mathbf{\dot{V}}_R$ is either above (negative $\alpha$) or below (positive $\alpha$) the cockpit. We define the unit vectors $\mathbf{\hat{y}}'$ and $\mathbf{\hat{V}}_R$ in the directions of the body $y'$ axis and the relative velocity vector, respectively. These are:

$$\mathbf{\hat{y}}' = \sin x_p \cos x_y \mathbf{i} + \cos x_p \cos x_y \mathbf{j} + \sin x_y \mathbf{k}$$

$$\mathbf{\hat{V}}_R = \frac{\mathbf{\hat{v}}_y}{\mathbf{\hat{V}}_R} \mathbf{i} + \frac{\mathbf{\hat{v}}_y}{\mathbf{\hat{V}}_R} \mathbf{j} + \frac{\mathbf{\hat{v}}_z}{\mathbf{\hat{V}}_R} \mathbf{k},$$

where $\mathbf{\hat{v}}_y$ and $\mathbf{\hat{v}}_z$ are the projections of the vehicle velocity components onto $y'$ and $z'$, respectively.
Consider the nonorthogonal coordinate system formed by the unit vectors $y'$ and $V_R$ in the $y'$ and $V_R$ directions. The aerodynamic and thrust forces in this system are described as follows, taking note of Figure 3-11.

The $x'$ force $(T_{xx} + T_{xs} - F_{AN})$ may be formed by adding a force $f_1$ in the $V_R$ direction and a force $f_2$ in the $y'$ direction as seen by the parallelogram of Figure 3-11 where

$$f_1 = [(T_{xx} + T_{xs} - F_{AN})/\sin\alpha] V_R$$

$$f_2 = -f_1 \cos\alpha = [(F_{AN} - T_{xx} - T_{xs})\cos\alpha/\sin\alpha] y$$

and $\alpha = \cos^{-1}(V_R \cdot \hat{y})$ where the sign is specified by input.

The total force acting on the vehicle becomes then:

$$\ddot{F} = [T_{yy} + T_{ys} - F_{AA} + (F_{AN} - T_{xx} - T_{xs})/\tan\alpha] \dot{y} + [(T_{xx} + T_{xs} - F_{AN})/\sin\alpha] V_R$$

Since $\dot{y}$ and $\dot{V}_R$ are described in the launch plumbline $XYZ$ system as

$$\dot{y} = X \sin \psi \cos \chi + Y \cos \psi \cos \chi + Z \sin \chi$$

and $\dot{V}_R = X \frac{\dot{u}}{V_R} + Y \frac{\dot{u}}{V_R} + Z \frac{\dot{v}}{V_R}$,

the total force becomes

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\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = F_y \begin{bmatrix}
\sin \chi_p \cos \chi_y \\
\cos \chi_p \cos \chi_y \\
\sin \chi_y
\end{bmatrix} + F_v \begin{bmatrix}
\frac{u}{v} \\
\frac{v}{v} \\
\frac{v}{v}
\end{bmatrix}
\]

The event may arise that \( \alpha \) is too large and must be reduced in order to meet structural load (qa) requirements. If this is required, a rotation of the above \( y \) axis through the angle \((\alpha_{\text{new}} - \alpha)\) is necessary, resulting in a new \( \chi_p \) and \( \chi_y \). This rotation, contained in the \( x - y - V_R \) plane, is also shown in Figure 3-12.

By inspection of Figure 3-13, we see that

\[
y_{\text{new}} = \cos(\alpha - \alpha_{\text{new}}) - \frac{\sin(\alpha - \alpha_{\text{new}})}{\sin \alpha} \cos \alpha y + \left[ \frac{\sin(\alpha - \alpha_{\text{new}})}{\sin \alpha} \right] V_R
\]

\[
y_{\text{new}} = V_R \left[ \frac{\sin(\alpha - \alpha_{\text{new}})}{\sin \alpha} \right] + y \left[ \frac{\sin \alpha_{\text{new}}}{\sin \alpha} \right]
\]

![Figure 3-12. ANGLE-OF-ATTACK LIMITING WITH COORDINATED TURNS](image-url)
The total force acting on the vehicle in the inertial plumb line XYZ system is then

\[ \dot{\mathbf{F}} = \mathbf{F}_{\text{new}} + \mathbf{F}_{\mathbf{V}} \mathbf{V} \]

where

\[ \mathbf{F}_{\text{new}} = \mathbf{Y} + \mathbf{F}_{\text{new}} \]

and

\[ \mathbf{F}_{\mathbf{V}} = (\mathbf{Txx} + \mathbf{Txs} - \mathbf{F}_{\text{AN}}) / \sin \alpha \]

The thrust and aerodynamic force components used in these equations are recalculated based on the new angle-of-attack.

The algebraic sign of the angle-of-attack will be taken as a constant for each thrust event. A new input variable, NCORD(15) allows the use of coordinated turns with positive or negative angle-of-attack for NCORD (ITHR) = a or -a, respectively, where ITHR denotes the number of the thrust event. If NCORD (ITHR) = 0, or if the vertical rise is not yet completed, coordinated turns are not used except by coincidence.

When the coordinated turn option is used, the inertial roll attitude angle, \( \psi_R \), must be determined for print output in subroutine APRTN. Also, when the angle-of-attack must be changed, the \( \chi_p \) and \( \chi_y \) angles are changed. Although the new values of \( \chi_p \) and \( \chi_y \) are not required in the equations of motion, the values are required for the trajectory block output of subroutine APRTN. Subroutine ADER calculates the components of the body axis y system and stores them in YHAT(1-3) so that (see Figure 3-13):

\[ (\mathbf{Y}) = \text{YHAT}(1) \hat{\mathbf{i}} + \text{YHAT}(2) \hat{\mathbf{j}} + \text{YHAT}(3) \hat{\mathbf{k}}. \]

The \( \chi_p \) and \( \chi_y \) angles are then given by

\[ \chi_p = \tan^{-1} \frac{\text{YHAT}(1)}{\text{YHAT}(2)} \]
and

\[ x_y = \left( \cos^{-1} \sqrt{\text{YHAT}(1)^2 + \text{YHAT}(2)^2} \right) \frac{\text{YHAT}(3)}{\text{ABS}(\text{YHAT}(3))} \]

The \( x_R \) angle has two possible solutions depending upon whether positive or negative angle-of-attack was specified. We have, for positive angle-of-attack:

\[ x_R = \tan^{-1} \frac{-v \cos \alpha \sin \gamma + w \sin \alpha \sin \gamma + u \cos \alpha \sin \gamma}{-u \sin \alpha + w \cos \alpha} \]

For negative angle of attack \( x_R \) is the above value plus \( \pi \).

\[ \text{Figure 3-13. } x_p, x_y, x_R \]
3.3 OPTIMIZATION/RESTORATION

RAGMOP uses a search-accelerated gradient projection technique to compute the optimum values of a chosen set of parameters used in controlling a rocket ascent trajectory. As with any other steepest-descent method, an initial guess parameter set is used to start the program. Changes to this parameter set are calculated to meet the required end conditions and optimize (extremize) the payoff quantity. Since the control is limited to a parametric form, the solution obtained is limited to the best control of that chosen form. RAGMOP requires parameterization of the chi-pitch and chi-yaw attitude histories (see Paragraph 3.2.6) with polynomials in time, which have generally been seen to be in close agreement with variational solutions for the attitude histories.

The initial guess set of parameters used to start the iteration process is arbitrary; however, the better the initial guess, the faster the solution will be found. Updating of the parameter set is accomplished in RAGMOP in two ways: (1) a constraint-error-reduction, or restoration, step; and (2) a payoff-extremizing, or optimization, step.

3.3.1 Restoration Step

The first task to be accomplished by the parameter update scheme is meeting the desired end conditions (constraints). To this end, the initial guess parameter set is updated using the relation

\[ \{dS\} = -H \lambda^\psi (\lambda^\psi)^T H^{-1} \lambda^\psi \]  \hspace{1cm} (3-1)

where

\[ \{dS\} \text{ is the vector of parameter update values,} \]
\[ H \text{ is a diagonal weighting matrix which attempts to normalize the effects of the various parameters (see Paragraph 3.3.5),} \]
\[ \lambda^\psi \text{ is the } m \times n \text{ matrix of partial derivatives of the constraints } \psi_i (i=1,m) \text{ with respect to the parameters, } \beta_j (j=1,n), \]
\( \lambda^T_B \) is the transpose of \( \lambda_B \),

and

\( \psi \) is the vector of constraint errors.

Since more parameters than constraints \((n > m)\) are required if optimization is to be performed, more than one solution exists to the problem of updating the parameters to satisfy the constraints. The solution obtained from equation (3-1) is the one which minimizes the weighted sum of the squares of the parameter changes, i.e.,

\[(d\beta^T H d\beta)\] is minimized.

The minimum weighted parameter change is desirable since the update is made using linear prediction, and the actual problem is in general very nonlinear. (Small changes will be more in the region of linear prediction).

The restoration step of equation (3-1) is scaled whenever any of the constraint errors is greater in absolute value than fifty times the acceptable error for a converged run: i.e.

\[
d\beta = E_2 [ -H^\psi_B (\lambda_B^T H_B \lambda_B)^{-1} \psi ]
\]

(3-2)

where \( E_2 \) is the restoration step scale factor. The value of \( E_2 \) is found assuming a quadratic relationship between a performance index \( P_\psi \) and the scale factor \( E_2 \):

\[
P_\psi = P_{\psi E_2=0} + aE_2 + bE_2^2
\]

where

\( P_\psi \) is a constraint performance index for the total constraint errors,
and

\[ a \text{ and } b \text{ are the coefficients in the quadratic relationship between } P_\psi \text{ and } E_2. \]

The constraint performance index used in RAGMOP is

\[ P_\psi = \sum_{i=1}^{m} k_i \psi_i^2 \]

where

\[ k_i = \frac{|\text{GNU}_i|}{\text{END}_i}, \]

\[ \{\text{GNU}_i\} = - (\lambda_B^T H_\psi \lambda_B) (\lambda_B^T H_\psi \lambda_B)^{-1}, \]

\[ \text{END}_i \text{ is the maximum acceptable absolute error in the } i^{th} \text{ constraint for a converged run,} \]

\[ \lambda_B^\phi \text{ is the vector of partial derivatives of the payoff with respect to the parameters,} \]

and

\[ \phi \text{ is the payoff.} \]

The assumed quadratic relationship between \( P_\psi \) and \( E_2 \) is established using three bits of information: (1) the value of \( P_\psi \) when \( E_2 = 0 \), (2) the slope of the \( P_\psi(E_2) \) function at \( E_2 = 0 \), and (3) the value of \( P_\psi \) at some value of \( E_2 \neq 0 \). The first two bits of information are readily available since

\[ P_\psi_{E_2=0} = P_{\psi \text{ current}} \approx P_{\psi \text{ o}} \]

and
Since
\[
\left( \frac{3P_{\psi}}{3E_2} \right)_{E_2=0} = 2 \left( \sum_{\text{CNL}} \frac{1}{E_2} \right)_{E_2=0}
\]
and
\[
\left( \frac{3P_{\psi}}{3E_2} \right)_{E_2=0} = -\psi_1
\]

The third bit of information is the value of $P_{\psi}$ at some nonzero value of $E_2$. This is obtained by choosing $E_2 = .1$, using the parameter update with that value of $E_2$, running the trajectory, and evaluating $P_{\psi_{E_2}}$. The quadratic relation between $P_{\psi}$ and $E_2$ is, then:

\[
P_{\psi_{E_2}} = P_{\psi} + \left( \frac{3P_{\psi}}{3E_2} \right)_{E_2} E_2 + b E_2^2
\]

which yields

\[
b = \frac{P_{\psi_{E_2}} - P_{\psi_{E_2}}} {E_2^2} = \frac{P_{\psi_{E_2}} - P_{\psi} (1 + 2E_2)} {E_2^2}
\]

Since we desire all of the constraint errors to be nulled we have $P_{\psi_{\text{desired}}} = 0$ so that:

\[
0 = P_{\psi_{C}} = 2P_{\psi} E_2 + b E_2^2
\]

or solving for $E_2$:

\[
3-39
\]
The value of $E_2$ used in RAGMOP is based on the value of $b$. For $b < 0$, 

$$E_2 = \frac{2P_{\psi_o} + \sqrt{4P_{\psi_o}^2 - 4bP_{\psi_o}}}{2b}$$

The value of $E_2$ is limited by RAGMOP to $0.05 < E_2 < 1.0$.

3.3.2 Optimization Step

The second task to be accomplished by the parameter update scheme (after the constraints are satisfied) is the extremization of some payoff quantity. The step which accomplishes this (6) is given by

$$d\delta = QY \left\{ -H\left( \lambda_{\psi_o}^{\psi} - \lambda_{\psi_o}^{\psi T} H_{\psi_o}^{\psi -1} \lambda_{\psi_o}^{\psi T} H_{\psi_o}^{\psi} \right) \right\}$$

(3-3)

where

$QY$ is a step scale factor to be determined by a search.

This step assumes linearity of the payoff and constraints with respect to the parameters and also assumes that the optimum on the intersection of the constraint hyperplanes (which describes the allowable surface of parameter values over which the optimum payoff is sought) can be approached using a linear step. Since the linearity assumption is false, the step is scaled to increase the payoff as much as possible without excessive increase in the constraint errors. In other words, the attempt is made to remain within some defined region of the constraint boundary during the optimization step by limiting the step size taken.
The value of the optimization step scale factor $Q_Y$ is found using a one-dimensional search on the change in value of a composite payoff index, $\Delta \phi$:

$$\Delta \phi = \Delta \phi + v \Delta \psi$$

where

$$\Delta \phi = \phi - \phi_{Q_Y=0}$$

$$\Delta \psi = \psi - \psi_{Q_Y=0}$$

and

$$v = - (\lambda_B^T H_B^T \lambda_B)^{-1} \phi_{Q_Y=0}$$

The search begins with a value of $Q_Y$ calculated to give some minimum parameter change, below which the significance of the search is lost due to machine noise and/or roundoff error. (A change in liftoff weight of .1 lb for a 5 million-pound vehicle is not significant, for example.) The parameter update is temporarily performed with this $Q_Y$, a trajectory run is made, and the composite payoff index evaluated. The value of $Q_Y$ is then increased, the temporary parameter update performed, the trajectory run made, and the composite payoff index reevaluated. This procedure continues until the value of the payoff index increases, denoting that the minimum has been passed. If more than four values of $Q_Y$ are tried before the minimum is passed, only the most recent four points are used in a cubic curve-fit to locate the minimum. If the minimum is passed before four points are obtained, three points are run and a quadratic curve-fit performed for the minimum. The result ($Q_Y$) from this curve-fit is then used to form a fourth data point for a cubic curve fit. Once the first cubic curve-fit is performed, the resultant $Q_Y$ is used for another data point, and a second cubic curve-fit is performed for the value of $Q_Y$ that minimizes the payoff index $\phi$ (Note that for maximized payoff a negative sign is introduced so that the minimization logic is still valid).
3.3.2.1 Cubic Curve Fit

The cubic form is written

\[ Y = aX^3 + bX^2 + cX + d, \text{ where } Y = \Psi \text{ and } X = QY. \]  

(3-4)

The value d is just the value of Y at some base point, (usually at \( X = 0 \), but not necessarily so) so choosing the fourth data point as the base point we have:

\[ Y - Y_4 = a(X - X_4)^3 + b(X - X_4)^2 + c(X - X_4) \]  

(3-5)

At the data points \((X_1, Y_1), (X_2, Y_2), (X_3, Y_3), (X_4, Y_4)\) we can write:

\[
\begin{align*}
Y_1 - Y_4 &= a(X_1 - X_4)^3 + b(X_1 - X_4)^2 + c(X_1 - X_4) \\
Y_2 - Y_4 &= a(X_2 - X_4)^3 + b(X_2 - X_4)^2 + c(X_2 - X_4) \\
Y_3 - Y_4 &= a(X_3 - X_4)^3 + b(X_3 - X_4)^2 + c(X_3 - X_4).
\end{align*}
\]  

(3-6)

We now have three equations in the three unknown coefficients \( a, b, \) and \( c \). The solution for these coefficients follows:

Let \[ dX_1 = X_1 - X_4 \]
\[ dX_2 = X_2 - X_4 \]
\[ dX_3 = X_3 - X_4 \]
\[ dY_1 = Y_1 - Y_4 \]
\[ dY_2 = Y_2 - Y_4 \]
\[ dY_3 = Y_3 - Y_4 \]

then Equation (3-6) can be written:

\[ \frac{dY_1}{dX_1} = adX_1^2 + bdX_1 + c \]  

(3-7)

\[ \frac{dY_2}{dX_2} = adX_2^2 + bdX_2 + c \]

3-42
\[
\frac{dy_3}{dx_3} = adx_3^2 + bdx_3 + c
\]  

(3-9)

Subtracting Equation (3-8) from Equation (3-7) gives

\[
\frac{dy_1}{dx_1} - \frac{dy_2}{dx_2} = a(dx_1^2 - dx_2^2) + b(dx_1 - dx_2)
\]  

(3-10)

and similarly (3-9) from (3-7) gives

\[
\frac{dy_1}{dx_1} - \frac{dy_3}{dx_3} = a(dx_1^2 - dx_3^2) + b(dx_1 - dx_3)
\]  

(3-11)

which reduce to

\[
\left( \frac{dy_1}{dx_1} - \frac{dy_2}{dx_2} \right) / (dx_1 - dx_2) = a(dx_1 + dx_2) + b
\]  

(3-12)

\[
\left( \frac{dy_1}{dx_1} - \frac{dy_3}{dx_3} \right) / (dx_1 - dx_3) = a(dx_1 + dx_3) + b
\]  

(3-13)

Defining \( dx_{12} = dx_1 - dx_2 \)

and \( dx_{13} = dx_1 - dx_3 \)

and subtracting (3-13) from (3-12) we have

\[
a(dx_2 - dx_3) = \frac{\left( \frac{dy_1}{dx_1} - \frac{dy_2}{dx_2} \right)}{dx_{12}} - \frac{\left( \frac{dy_1}{dx_1} - \frac{dy_3}{dx_3} \right)}{dx_{13}}
\]  

(3-14)

and now defining \( dx_{23} = dx_2 - dx_3 \)
we have

\[ a = \frac{\left( \frac{dy_1}{dx_1} - \frac{dy_2}{dx_2} \right) / dx_{12} - \left( \frac{dy_1}{dx_1} - \frac{dy_3}{dx_3} \right) / dx_{13}}{dx_{23}} \]  

(3-15)

and then from (3-13)

\[ b = \left( \frac{dy_1}{dx_1} - \frac{dy_3}{dx_3} \right) / dx_{13} - a(dx_1 + dx_3) \]  

(3-16)

and from (3-9)

\[ c = \frac{dy_3}{dx_3} + adx_3^2 + bdx_3 \]  

(3-17)

Using (3-15), (3-16) and (3-17) in (3-5) we have

\[ x_{\text{MIN}} = \frac{-2b \pm \sqrt{b^2 - 4ac}}{2a} \]  

or

\[ x_{\text{MIN}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]  

(3-18)

Thus (16) yields the extrema of the function. In order to choose between the + or - sign we look at the second derivative at the extrema:

\[ \frac{d^2y}{dx^2} = 6ax + 2b = \frac{-6ab \pm 6a\sqrt{b^2 - 3ac}}{3a} + 2b > 0 \]

which becomes
\[ \pm \sqrt{b^2 - 3ac} > 0 \]

from which the choice of the + sign is obvious and

\[
QY_{\text{min}} = -b + \sqrt{b^2 - 3ac} \\
3a
\]

This value of \( QY \) is then used to perform a permanent parameter update (the optimization step). The trajectory is run with the permanent update, a new set of partial derivatives is obtained at the new control state, and the convergence test is made. If the solution has not yet converged, another \( QY \) search will be performed as soon as the constraint errors from the optimization step are restored to acceptable levels (if they were not already acceptable).

The search is terminated prior to passing the minimum if either of two criteria are not satisfied: (1) parameter changes must be below some maximum level, and (2) the constraint errors must not be excessive. The parameter change is monitored by determining a maximum acceptable change for a single parameter which directly affects the payoff. The constraint errors are monitored by determining the maximum acceptable error of a single constraint. If the parameter change or the constraint error exceeds the maximum value, the search is terminated. The current value of \( QY \) is then used for the optimization step calculation rather than a curve-fitted value.

3.3.4 Partial Derivatives

The partial derivatives used in the parameter update equations

\[
\begin{bmatrix}
\frac{\partial \psi_1}{\partial \theta_1} & \frac{\partial \psi_2}{\partial \theta_1} & \cdots & \frac{\partial \psi_m}{\partial \theta_1} \\
\frac{\partial \psi_1}{\partial \theta_2} & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots \\
\frac{\partial \psi_1}{\partial \theta_n} & \cdots & \cdots & \frac{\partial \psi_m}{\partial \theta_n}
\end{bmatrix}
\]
are determined using numerical difference techniques. Each parameter is perturbed by some slight amount, a trajectory is run, and the effect of the perturbation on the equality constraints and on the payoff is calculated. This process must be performed separately for each parameter. The partial derivatives are then found as the ratio of the change in the constraint or payoff to the change in the parameter:

\[
\frac{\Delta \psi_i}{\Delta \beta_j} = \frac{\psi_i(\beta_j + \Delta \beta_j) - \psi_i(\beta_j)}{\Delta \beta_j}
\]

or

\[
\frac{\Delta \phi}{\Delta \beta_j} = \frac{\phi(\beta_j + \Delta \beta_j) - \phi(\beta_j)}{\Delta \beta_j}
\]

Derivatives formed in this manner (forward differences) are used whenever a restoration step is made. If the optimization step (QY) search is to be performed (signalled by all constraint errors being within the acceptable limits for a converged run), the parameters are also perturbed in the negative direction and the average slope (partial derivative) is found:

\[
\frac{\Delta \psi_i}{2 \Delta \beta_j} = \frac{\psi_i(\beta_j + \Delta \beta_j) - \psi_i(\beta_j - \Delta \beta_j)}{2 \Delta \beta_j}
\]
This method is used since the optimization step (QY) search is more sensitive to errors in the partial derivatives than the restoration step. (A considerable savings in computer time results from using only the forward differences whenever practical.)

The perturbation values used in RAGMOP are:

- Δ burn times = 1 second
- Δ liftoff weight = .01%
- Δ launch azimuth = .25°
- Δ first stage χₚ = .1°
- Δ first stage χᵧ = .5°
- Δ second stage χₚ = .1°
- Δ second stage χᵧ = .5°

Preset to these values. May also be changed by input.

The one exception to the above method of determining the partial derivatives occurs when the final thrust event burn time is optimized without the payload option. In this case, the partial derivatives are obtained analytically rather than by a trajectory integration. The approach used is that

\[
\frac{\partial \phi}{\partial \beta} = \frac{\phi_{\beta + \Delta \beta} - \phi_{\beta - \Delta \beta}}{2\Delta \beta}
\]

where

- \( \tilde{x} \) denotes the final state (position, velocity mass).

The derivatives

\[
\frac{\partial (\psi, \phi)}{\partial \tilde{x}}
\]
are presented in Appendix G. The use of this formulation both saves computer
time and increases the accuracy of the partial derivatives by eliminating the
numerical difference technique.

3.3.5 The Weighting Matrix

The purpose of the weighting matrix is to normalize the partial derivatives
of the constraints with respect to the various parameters. The numerical
value of each partial derivative

$$\frac{\partial \psi_i}{\partial \beta_j}$$

is dependent upon the units chosen for the parameter \(\beta_j\). Consider for example
the case of the partial derivative of the radius vector at orbital injection
with respect to one of the \(x_p\) values used to form the first stage \(x_p\) polynomial.
At some time suppose the value of this partial derivative is

$$\frac{\partial R}{\partial x_p} = 1000 \text{ meters/degree}$$

If the units of \(x_p\) were radians rather than degrees, the value would be 17.45
instead of 1000, and the square of the partial (used in the parameter update
scheme) would be 304.57 instead of \(10^6\). Now suppose that the partial derivative
of the radius vector with respect to liftoff weight is

$$\frac{\partial R}{\partial w_{l}} = 1 \text{ meter/kilogram}$$

The square of this is 1. With \(x_p\) in radians, the ratio of the square of the
liftoff weight partial to the \(x_p\) partial is \(1/304.57 = .00328\), but with \(x_p\) in
degrees this ratio is \(1/10^6 = .000001\). Thus, merely by changing the units of
\(x_p\), the effect of the \(x_p\) partial derivative on the sum of the squares

$$\frac{\partial R}{\partial x_p}^2 + \frac{\partial R}{\partial w_{l}}^2$$

is changed from the third significant digit to the sixth. It is easily seen
that for some parameters and constraints, the significance of one or more
parameters on a particular constraint could be completely lost on an eight digit computer when summing the squares as shown above. The following paragraph explains the significance of this sum.

The matrix

\[ I_{\psi\psi} = \lambda_{\psi} \psi^T H \lambda_{\psi} \]

must be inverted in the parameter update calculation for both the restoration step and the optimization step. The diagonal elements of this matrix contain the weighted sum of the squares of the partial derivatives of the parameters with respect to each constraint, i.e.,

\[ I_{\psi j} = H_{11} \frac{\psi_1^2}{\psi_1} + H_{22} \frac{\psi_2^2}{\psi_2} + \cdots + H_{nn} \frac{\psi_n^2}{\psi_n} \]

Without a weighting matrix the numerical value of the terms comprising each \( I_{\psi j} \) are dependent upon two factors: (1) the actual effect of the various parameters on the \( j \)th constraint, and (2) the units chosen for the parameters. The weighting matrix attempts to eliminate the second factor by normalizing all of the squared terms to the same order of magnitude.

The weighting matrix is calculated as follows:

Define the vector

\[ v = \begin{pmatrix} \frac{\psi_1^2}{\beta_1} \\ \frac{\psi_2^2}{\beta_1} \\ \vdots \\ \frac{\psi_n^2}{\beta_1} \end{pmatrix} \]
where it is assumed that all the \( V_i > 0 \). Using \( V \), a second vector \( S \) is chosen such that

\[
S_j = \frac{\sum_{i=1}^{m} \log_{10} \left( \frac{\partial V_i}{\partial \phi_j} \right)^2}{m} V_i
\]

In other words, \( S_j \) is the average base ten logarithm of the ratio of the squares of the constraint partial derivatives of each parameter with respect to the first parameter. Thus, \( S_1 = 1 \).

The first parameter is arbitrarily chosen as the basis for comparison, however, any parameter with all the \( V_i > 0 \) could be used. (The parameter chosen affects the scaling of \( QY \) in the optimization step.)

The weighting matrix \( H \) is then given by

\[
H_{jj} = \frac{W_j}{10^S_j}
\]

where \( W_j \) is an additional weighting factor found by experience to improve the performance of certain parameters. This definition of the weighting matrix has been seen to contribute to essentially uniform convergence of the parameters.

The weighting matrix calculation is performed at each restoration step after the initial guess (nominal) trajectory until an optimization step scale factor (QY) search is completed. At this time the weighting matrix is updated using the Fletcher and Powell form of Davidon's update formula such that:

\[
H_{i+1} = H_i + \Delta \phi \Delta \phi^T \frac{H_i \Delta \phi \Delta \phi^T H_i}{\Delta \phi^T \Delta \phi} - \frac{\Delta \phi^T H_i \Delta \phi}{\Delta \phi^T \Delta \phi} H_i
\]
where

$$\Delta \phi_\beta = \frac{D \phi}{D B_1} - \frac{D \phi}{D B_{1-1}}$$

and

$$\Delta \beta = \beta_1 - \beta_{1-1}$$

Using the updated weighting matrix, the restoration step is again performed until the constraints are satisfied. At this time, the convergence tests as described in subsection 3.3.6 are tested, if the tests are met convergence is denoted and a final trajectory printed. If the tests are not met another optimization step search is begun and the weighting matrix updated upon successful completion of the search. This process of updating the weighting matrix is continued until either the convergence tests are met or the updating process is performed more times than the difference between the number of parameters and the number constraints. In the latter case the weighting matrix is reinitialized based upon the aforementioned procedure. Experience has shown that the reinitialization process speeds the convergence since this process tends to eliminate the inherent numerical errors.

3.3.6 Convergence Test

Three criteria must be met at the end of a QY search (See Paragraph 3.3.2) in order for a given trajectory solution to be considered converged: (1) the constraint errors must all be within acceptable limits, (2) the scaled total derivative of the payoff with respect to each parameter ($P_{con_i}$) must be acceptably small, (3) the parameter changes computed using the curve-fit value of QY must be trivial.

The acceptable limits on the constraint errors are input values.
The total derivatives of the payoff with respect to each of the parameter are given by:

\[
\frac{D\phi}{D\theta_j} = \frac{\partial \phi}{\partial \theta_j} + \sum_{j=1}^{m} \frac{\partial \phi}{\partial \psi_j} \frac{\partial \psi_j}{\partial \theta_j}
\]

The convergence parameters mentioned in (2) above are given by

\[
P_{\text{con}_i} = \frac{D\phi}{P_{\text{scale}_i}}
\]

where \( P_{\text{scale}_i} \) is the absolute value of largest of the terms added together in calculating \( \frac{D\phi}{D\theta_j} \). This scaling assures that each \( \frac{D\phi}{D\theta_j} \) is zero to the same number of significant digits. The test on \( P_{\text{con}_i} \) in RAGMOP is set at \( P_{\text{con}_i} < .025 \). (On the optimum \( \frac{D\phi}{D\theta_j} = 0 \) and \( P_{\text{con}_i} = 0 \).)

The absolute values of the parameter changes at the end of a QY search (i.e. using the value of QY produced by the search) must all be trivial, where trivial for this purpose is defined as:

\[
\Delta \text{burn times} < 1\%
\]

\[
\Delta \text{liftoff weight} < .001\%
\]

\[
\Delta \text{launch azimuth} < .00085^\circ
\]

\[
\Delta \chi_p \text{ values} < 1\%
\]

\[
\Delta \chi_y \text{ values} < 0.1\%
\]

3.4 INTEGRATION PACKAGE

Numerical integration is performed in RAGMOP using the DESOLV integration package (17). The DESOLV package provides executive control of the integration of the equations of motion from initialization to termination. Three integration schemes are available in the DESOLV package: 1) a fixed step fourth
order Runge-Kutta, 2) a variable step-size, variable order. Adams-Moulton (maximum 8th order) with fourth order Runge-Kutta starter, and 3) a fixed step size, variable order Adams predictor (maximum 8th order) with fourth order Runge-Kutta starter.

Control of the trajectory integration by DESOLV is performed using a series of integration interrupt triggers. These triggers serve two purposes: (1) to allow a print of the state at various times during the trajectory, and (2) to allow changes in the equations of motion to be made during the trajectory integration (such as termination of the thrust events based on time, fuel, relative velocity, or acceleration). The integration package recognizes the difference between these two types of triggers, and will restart the integration for the second type.
Section IV
REFERENCES


PART II
PROGRAMMER'S MANUAL
Section V

MAIN PROGRAM FLOW AND OPERATION

This section describes the flow and operation of the RAGHOP main program. Individual subroutines, treated as "black box" units in this section, are described in detail in Section VI. The main program is best described in six segments:

- Initialization,
- Trajectory integration and evaluation,
- Convergence test,
- Partial derivative calculation,
- Parameter update,
- Plot and table output.

These six segments are described in detail in the following paragraphs. A flow diagram of the main program is presented in Figure 5-1.

5.1 INITIALIZATION

The main program begins by presetting a number of variables and constants used in the program. Section IX presents the preset values of all input variables. In addition to the preset input variables, several program control variables are initialized and the weighting matrix is preset to an $n \times n$ identity matrix where $n$ is the number of parameters to be optimized. The input subroutine AINIT is then called to read the input data for the case to be run. This input data includes a description of the vehicle, the type of trajectory to be flown, the parameters to be optimized, and the output desired (see Section IX). The maximum allowable parameter change during the optimization step is calculated based on which parameters are used.

5.2 TRAJECTORY INTEGRATION AND EVALUATION

The trajectory integration and evaluation is performed by calling AFORUN and AFOFEND. Upon completion of these two subroutines, the values of the constraint errors and the payoff are known for a particular parameter set. The constraint errors are tested to determine whether any exceed the acceptable level in END and fifty times that level.
Figure 5-1. RAGMOP MAIN PROGRAM FLOW DIAGRAM
PARAMETER UPDATE (Continue)

Figure 5-1. RAGMOP MAIN PROGRAM FLOW DIAGRAM (Concluded)
5.3 CONVERGENCE TEST

A test for convergence is the next step. A run is considered converged when, using the parameter update at the completion of an optimization step scale-factor search, the constraint errors are small and the convergence tests of subroutine ANEYCH have passed (see Sections III and VI). If the convergence test is passed, NMAX is set to zero and the trajectory integration is performed again to obtain a detailed print of the final trajectory and to create a trajectory tape for the plot and table routine if they are to be used. If the convergence test is not passed, the payoff and the constraint error values are stored in PHITES for later use as baseline values in the restoration and optimization step scaling.

5.4 PARTIAL DERIVATIVE CALCULATION

The partial derivatives of the constraint errors and the payoff with respect to the parameters are then determined by calling subroutine BADLX. Upon the return to MAIN, the partial derivatives will be stored in the array XLAMB. These derivatives are determined using forward numerical differences when the constraint errors are large and central differences when the constraints are within acceptable convergence limits (e.g., at the beginning of an optimization step scale-factor search).

5.5 PARAMETER UPDATE

The parameter update is performed in one of two ways: (1) as a constraint-restoring step, or (2) as an optimization step after the constraints are met.

Once the partials have been obtained, the values of the parameters used for the last trajectory (before the partial derivative trajectories in BADLX) are stored in the vector PARSAV.

The variable NTNST is checked to determine whether any of the constraints were greater in absolute value than fifty times the allowable limits in END (indicated by NTNST=1). If NTNST=1, the restoration step is scaled to
minimize the total constraint error based on a quadratic assumption for the relationship between a constraint performance index and the scale factor DP2. If NTNSTV≥1 (indicating that all constraint errors are less than 50 times the acceptable limits) the restoration step is not scaled. For either case subroutine ANEWCH is called to calculate the matrix products and inverses required in the parameter update. When the restoration step is not scaled and the constraint errors are not yet sufficiently small to perform an optimization step search, this call to ANEWCH (with DP2=1.) is also used to perform a permanent parameter update and the flow returns to the trajectory integration of Paragraph 5.2.

When scaling of the restoration step is required, subroutine ANEWCH is called with DP2=1, a temporary parameter update is performed, and the trajectory is run in order to obtain the value of a constraint performance index. This value serves as a data point in calculating the value of the scale factor (DP2) that minimizes the performance index for use in the permanent parameter update. A quadratic relationship is assumed between the performance index and the scale factor for the purpose of this scaling. After performing the parameter update with the calculated scale factor, the flow returns to the trajectory integration of Paragraph 5.2.

If all constraint errors are less than the acceptable levels for a converged run, the optimization step scale-factor (QY) search is begun. The value of QY which will produce a minimum acceptable parameter change is found and used as the starting value for the search. This is done to eliminate searching in the machine "noise" region with very small parameter changes which might result from an arbitrarily small starting value for QY. The parameter update is performed and a composite payoff index is checked for improvement over the baseline value at the beginning of the search. When the minimum of the payoff index is passed, and at least three values of QY have been tried, a curve fit scheme predicts the value of QY which yields the minimum. If the minimum has been passed with the third data point, a quadratic fit for QY is made. If after three values of QY have been tried, the payoff index is still decreasing,
the value of QY is repeatedly increased until an unacceptably large parameter change or constraint error is obtained or until the minimum is passed, with the most recent four data points (QY and payoff index) retained each step. When the payoff index increases, (indicating that the minimum has been passed) or after the quadratic fit mentioned above when the minimum three data points are used to predict QY (which is then used to produce a fourth point), a cubic polynomial is used to estimate the peak value of QY. This value is then used to produce another data point, the first point is dropped, and the cubic fit repeated one time. The parameter update scheme is then used to permanently change the parameter set with the final value of QY and flow returns to the trajectory integration of Paragraph 5.2. The value of QY in subsequent restoration steps will be zero. The search is terminated early (before the minimum is found) if the test parameter change (W0l or the last optimized TAUT) becomes too large or if the error in the radius vector at insertion is greater than 1000 meters.

5.6 PLOT AND TABLE OUTPUT

Once a converged run is obtained, or when the maximum number of iterations has been reached, a test is made of the input variable NTABLE to determine whether special output tables (NTABLE=1) or plots (NTABLE=2) or both (NTABLE=3) are desired. If tables are desired, subroutine BOPTBL is called. If plots are desired, subroutine BOPPLT is called. The input variable LAST is then decremented (LAST=LAST-1) and checked for multiple cases (LAST>0). If another case is to be run, the flow returns to the initialization of Paragraph 5.1; otherwise the program stops.
Section VI

SUBROUTINE DESCRIPTIONS

This section describes in detail the subroutines (with exception of the integration package) used in the RAGMOP program. For each subroutine, the functional flow is given as a word description and a macro-flow diagram. The integration package subroutines DPIR, DESOLV, and RTMRS are not presented in a detailed fashion since documentation for these routines is available in reference 17.

6.1 ALPHABETICALLY ORDERED LIST OF SUBROUTINE NAMES AND FUNCTIONS

<table>
<thead>
<tr>
<th>Subroutine Name</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACSTQP</td>
<td>Evaluates the payoff and determines the value of terminal and intermediate equality constraints.</td>
</tr>
<tr>
<td>ADER</td>
<td>Evaluates the state derivatives.</td>
</tr>
<tr>
<td>ADER1</td>
<td>Evaluates time dependent variables used in ADER.</td>
</tr>
<tr>
<td>APFRND</td>
<td>Controls the call to ACSTQP to determine terminal conditions, computes errors from desired values, and prints terminal summary.</td>
</tr>
<tr>
<td>APFRUN</td>
<td>Controls the setup logic for the trajectory integration and calls the integration package.</td>
</tr>
<tr>
<td>AGEQ</td>
<td>Evaluates oblate gravitational properties.</td>
</tr>
<tr>
<td>AINIT</td>
<td>Reads input data, performs initialization.</td>
</tr>
<tr>
<td>AKALT</td>
<td>Trigger routine for 10 km and 14 km altitude printout.</td>
</tr>
<tr>
<td>AMACH</td>
<td>Trigger routine for MACH = 1 printout.</td>
</tr>
<tr>
<td>AMULG</td>
<td>Linear interpolation scheme.</td>
</tr>
<tr>
<td>ANEWCH</td>
<td>Evaluates the optimization and restoration step equations to find the required changes in the controlling parameters. NNEWCH is an entry point in this subroutine.</td>
</tr>
<tr>
<td>APRTN</td>
<td>Performs output editing, prints data, and writes output tape for output tables and plots.</td>
</tr>
<tr>
<td>AQMAX</td>
<td>Trigger routine for maximum dynamic pressure printout.</td>
</tr>
<tr>
<td>ASIMP</td>
<td>Determines the flyback range using empirical data interpolation.</td>
</tr>
<tr>
<td>ATHREV</td>
<td>Thrust event control routine; sets up vehicle geometry, aerodynamics, number of engines, thrust limits, flow rates, and trigger flags for thrust event cutoff.</td>
</tr>
</tbody>
</table>
### Subroutine Name | Function
--- | ---
**ATILT** | Trigger routine for end of vertical rise (begin tilt over or attitude control). The subroutines AKALT, AMACH, AQMAX, AXPRT, AND GLIMT, are contained within this subroutine as entry points.
**AXPRT** | Trigger routine for normal printout.
**BADLX** | Determines the influence coefficients (partial derivatives) for computation of the changes in the controlling parameters.
**BØPPLT** | Generates output plots on CALCOMP plotter.
**BØPTBL** | Generates output table summary.
**CHIPØL** | Evaluates the pitch and yaw attitude polynomials.
**DESØLV** | *Integration package*  
**DPIR** | 
**FIND** | Determines attitude polynomials based on attitude time history information.
**GLIMT** | Trigger routine for acceleration limit printout.
**MATINV** | Performs matrix inversion.
**NNEWCH** | Determines parameter update step during one dimensional search.
**PRA63** | Evaluates the atmospheric parameters as well as their partial derivatives w.r.t. altitude using the spline interpolation method of subroutine SPLINE.
**RTMRK** | Part of integration package which flags termination of the trajectory integration.
**SPLINE** | Interpolation routine using SPLINE method for aerodynamic coefficients.**
**SPLINE2** | Interpolation routine using SPLINE method for SRM thrust and weight drop tables**.
**TEST** | Linear scheme used in ASIMP for determining slope of the dependent variable w.r.t. the independent variable.
**TRIVLP** | Trivariant lookup (of flyback range) for ASIMP.

#### 6.2 ALPHABETICALLY ORDERED LIST OF SYSTEM SUBROUTINE NAMES AND FUNCTIONS

| Subroutine Name | Function |
--- | --- |
**ACØS** | Determines arccosine (rad). |
**AINT** | Truncates real variables to integer form. |
**ALØG** | Determine the logarithm of a variable (base e). |

*Documentation of the integration package can be found in reference 17.*

*See Appendix C for further information concerning interpolation techniques.*
<table>
<thead>
<tr>
<th>Subroutine Name</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALOG10</td>
<td>Determine the logarithm of a variable (base 10).</td>
</tr>
<tr>
<td>AMAX1</td>
<td>Selects maximum value from argument list.</td>
</tr>
<tr>
<td>ASIN</td>
<td>Determines arcsine (rad).</td>
</tr>
<tr>
<td>ATAN</td>
<td>Determine arctangent based on one argument $-\frac{\pi}{2} \leq 0 \leq \frac{\pi}{2}$ (rad).</td>
</tr>
<tr>
<td>ATAN2</td>
<td>Determine arctangent based on two arguments $-\pi \leq 0 \leq \pi$ (rad).</td>
</tr>
<tr>
<td>AXIS</td>
<td>CALCOMP system routine, draws axis with &quot;tic&quot; marks and scales at every inch.</td>
</tr>
<tr>
<td>CALEND</td>
<td>CALCOMP plotter routine: plot an ending identification frame and end the current plot file.</td>
</tr>
<tr>
<td>CALID</td>
<td>CALCOMP plotter routine: initializes the CALCOMP plotter uses interface package.</td>
</tr>
<tr>
<td>COS</td>
<td>Determines the cosine of the argument.</td>
</tr>
<tr>
<td>DCSOS</td>
<td>Double precision COS.</td>
</tr>
<tr>
<td>DSIN</td>
<td>Double precision SIN.</td>
</tr>
<tr>
<td>DSQRT</td>
<td>Double precision SQRT.</td>
</tr>
<tr>
<td>EXP</td>
<td>Exponential function.</td>
</tr>
<tr>
<td>LINF</td>
<td>CALCOMP plotter routine: plots a line or curve from a set of points.</td>
</tr>
<tr>
<td>NAMELIST</td>
<td>Variable format input routine that reads data cards and places the values in the proper common blocks.</td>
</tr>
<tr>
<td>PLOT</td>
<td>CALCOMP plotter routine: moves pen to a new coordinate position, or establishes a new reference position.</td>
</tr>
<tr>
<td>SCALE</td>
<td>CALCOMP plotter routine: obtains maximum and minimum scale values to adjust plot values.</td>
</tr>
<tr>
<td>SIN</td>
<td>Determines the sine of the argument.</td>
</tr>
<tr>
<td>SQRT</td>
<td>Determines the square root of the argument.</td>
</tr>
<tr>
<td>SYMBOL</td>
<td>CALCOMP plotter routine: draws a symbolic label at a specified position.</td>
</tr>
</tbody>
</table>

6.3 **Detailed Description of Subroutines**

The following paragraphs describe each subroutine in detail.
6.3.1 ACSTQP

Subroutine Identification
- Title
  ACSTQP
- Calling sequence
  CALL ACSTQP (M,VALE,I), where M - option flag see Table 6-1
  VALE - value of option used
  I - not used.

Function
Evaluates the payoff and determines the values of the terminal and intermediate equality constraints.

ACSTQP also provides the analytical partial derivatives of the constraints, with respect to the state variables when final burn time is a parameter and payload (as opposed to final mass) is not instant or payoff.

Functional Flow
Called by the subroutine AFFORD at the end of trajectory integration to evaluate the current payoff and to determine values of the terminal and intermediate constraints. The calling argument, M, of the subroutine denotes the calculation required of the routine, and VALE denotes the value produced by that calculation. The options for M are defined in Table 6-1.

Table 6-1. FUNCTION LIBRARY FOR CONSTRAINTS

<table>
<thead>
<tr>
<th>Code No. (M)</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Payload (or final mass)</td>
</tr>
<tr>
<td>2</td>
<td>Inertial velocity</td>
</tr>
<tr>
<td>3</td>
<td>Inertial flight path angle</td>
</tr>
<tr>
<td>4</td>
<td>Radius</td>
</tr>
<tr>
<td>5</td>
<td>Energy</td>
</tr>
<tr>
<td>6</td>
<td>Angular Momentum</td>
</tr>
<tr>
<td>7</td>
<td>Inertial longitude</td>
</tr>
<tr>
<td>8</td>
<td>Inertial heading angle</td>
</tr>
<tr>
<td>9</td>
<td>Colatitude</td>
</tr>
<tr>
<td>10</td>
<td>Inclination</td>
</tr>
<tr>
<td>11</td>
<td>Line of nodes</td>
</tr>
<tr>
<td>12</td>
<td>Semilatus rectum</td>
</tr>
<tr>
<td>13</td>
<td>Eccentricity</td>
</tr>
</tbody>
</table>
Table 6-1. (Concluded)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>Burn Time</td>
</tr>
<tr>
<td>15</td>
<td>Maximum dynamic pressure</td>
</tr>
<tr>
<td>16</td>
<td>True anomaly</td>
</tr>
<tr>
<td>17</td>
<td>Argument of perigee</td>
</tr>
<tr>
<td>18</td>
<td>Not used</td>
</tr>
<tr>
<td>19</td>
<td>Not used</td>
</tr>
<tr>
<td>20</td>
<td>Flyback range</td>
</tr>
</tbody>
</table>

The value of $M$ is determined by input of the integer variables KCDPHI and KCDRES.

ACSTOP also provides the analytical partial derivatives of the constraints with respect to the state variables when final burn time is a parametric and payload (as opposed to final mass) is not a constraint or payoff.

Functional Flow Diagram of ACSTOP
4.3.2 ADER

Subroutine Identification

- Title
  ADER

- Calling Sequence
  CALL ADER

Function

Evaluates the state derivatives

Functional Flow

Called by the subroutines AP0RUN, ATHREV, and the integration routine (DPTR) to calculate the values and derivatives of variables which are not totally dependent on time. These include the current velocity and acceleration of the vehicle, the mass of the vehicle, the thrust, propellant flowrate, the aerodynamic forces and moments, the thrust vectoring required to balance moments, angle-of-attack, sideslip angle, dynamic pressure and its derivatives, and the attitude requirements when an angle-of-attack history is specified or when intermediate constraints of qa or q8 are exceeded.

The first part of the subroutine finds the magnitudes of the vehicle position and velocity vectors. The gravitational acceleration is found by calling subroutine AG30. The altitude is found using either an oblate or a spherical earth model, as desired. The inertial velocity of the atmosphere at the current vehicle altitude due to the rotation of the earth is calculated, and wind bias values (if any) are added. The difference between the vehicle's inertial velocity and that of the atmosphere is then used to determine the relative velocity. The atmospheric routine, FRA6J, is called to determine the pressure, density, speed of sound, and partial derivative of density w.r.t. altitude at the current altitude. Mach number is calculated and used for the interpolation of the aerodynamic force and moment coefficients. Axial force due to base pressure is then determined by interpolation based on current altitude. Dynamic pressure, q, is determined based on the density and the relative velocity. Angle-of-attack and sideslip angles are determined by either coordinated turn requirements or transforming the components of the relative
velocity into the body axis system. The value of the input variable FACT is examined to determine whether an angle-of-attack profile is required. The values of the structural load indicators, \( \alpha \) and \( \beta \), are checked to determine violation of input constraints. If the variables violate the constraints, \( \alpha \) and/or \( \beta \) are set equal to the constrained values and the values of pitch and/or yaw attitude are calculated to satisfy the constraint relationships, or the desired profile, overriding predetermined values.

The next portion of the subroutine calculates the aerodynamic and thrust forces and moments acting on the vehicle. Aerodynamic force and moment coefficients are found using the zero angle-of-attack coefficients, the slope of coefficients w.r.t. angle-of-attack or sideslip, and the aerodynamic moment arms. The aerodynamic forces and moments are calculated with respect to the body axes.

The value of the thrust at the current altitude is found and tested for order of magnitude. If the thrust is too low (arbitrarily chosen 10,000 newtons) or no moment balance is required, the moment balance scheme is bypassed. If sufficient thrust exists, and moment balance is required, the thrust components needed to balance the moments are found depending on the type of moment balance as given by the input variable MBA. A two-engine equivalent is used in the moment balance scheme for the controllable engines regardless of the actual number of engines. If acceleration (g) limiting is required, special calculations are made to determine the thrust requirements and propellant flowrate of the main (orbiter) engines. Only liquid fuel engines are throttled, and these are throttled only to zero thrust. If the SRM thrust alone causes the g-limit to be exceeded no further action is taken.

The total forces acting on the vehicle are calculated and transformed to the inertial plumbline system. Total acceleration in the plumbline system is found by summation of the aerodynamic, thrust, and gravitational accelerations. The final portion of the subroutine calculates the derivatives of the velocity losses and of the dynamic pressure. (See Appendix F for an explanation of the velocity loss equations).
Functional Flow Diagram of ADER

START

CALCULATE MACH & VELOCITY

CALCULATE ATTITUDE & RELATIVE VELOCITY

CALL THRUST DETERMINE ROLL & YAW PROPERTIES

CALCULATE FORCES & MOMENTS & SPACE INITIAL FORCE

COORDINATES TIME

NO

CALCULATE MACH OF ATTITUDE AND SPACE TIME BASED ON DYNAMICS

CALCULATE MACH OF ATTITUDE & SPACE TIME BASED ON DYNAMICS

FACT 0

NO

FACT 1.0

NO

FACT 2.0

NO

ATTITUDE BASED ON CONSTRAINTS

CALCULATE FORCES & MOMENTS BASED ON

END

ESTABLISH FORCES & MOMENT FORCES & MOMENTS FOR NORMAL OPTIONS

CALCULATE MOMENT BALANCE Transit needs

CONTINUE THURST BALANCE Transit needs

RECULATE MOMENT BALANCE Transit needs

CALCULATE TOTAL FORCES, MOMENTS, & ACCELERATIONS

CALCULATE  VELOCITY LOSS EFFECTS OF DYNAMIC PRESSURE

CONTINUE

6-9
6.3.3 ADER1

Subroutine Description

- Title
  ADER1
- Calling Sequence
  CALL ADER1

Function

Evaluates time dependent variables used in ADER.

Functional Flow

Calculates all time dependent variables, i.e. thrust, weight, and attitude control values. The subroutine is called initially by AFORUN and subsequently by the integration package. This subroutine must be called at the beginning of each integration time step and called before ADER, since calculations in ADER require values computed in ADER1.

The first step of this subroutine is to test to see if SRM engines are being used. If so, the value of SRM thrust and weight loss are determined from SPLINE interpolation of time dependent tables. The value of the propellant flowrate for the liquid engines is stored into the variable DVAR(7) for integration of mass loss. A check is made to determine whether the liftoff phase has been terminated (KSl>2) and if so, the appropriate attitude values are found. Finally the location of the center-of-gravity at the present vehicle weight is found, s are the distances of the engine gimbal points from the present center-of-gravity position and several relations among these distances used in the moment balance equations.
Functional Flow Diagram of ADERI

START

JTHR = 0

YES

NO

DETERMINE SRM THRUST AND WEIGHT LOSS

STORE PROPELLANT FLOWRATE

KS1 >= 2

YES

NO

x_p * x_y = a_0 + a_1 t + a_2 t^2 + ... + a_n t^n

x_p * x_y = LIFTOFF

MOMENT ARM CALCULATIONS

RETURN

6-11
6.3.4 AF\textsuperscript{ORD}

Subroutine Identification

- Title
  AF\textsuperscript{ORD}

- Calling Sequence
  CALL AF\textsuperscript{ORD}

Function

Controls the call to ACST\textsuperscript{OP} to determine terminal conditions, computes errors from desired values, and prints terminal summary.

Functional Flow

AF\textsuperscript{ORD} is called at the end of each trajectory run. The subroutine checks the constraints by calling the subroutine ACST\textsuperscript{OP} for the payoff and terminal constraints requested by input and evaluates the differences between the desired terminal values and the actual values. If AF\textsuperscript{ORD} has been called by BADLX (indicated by LSB=1) the constraint errors are evaluated and returned to BADLX to calculate influence coefficients. If AF\textsuperscript{ORD} has been called by MAIN (LSB=0) an additional trajectory summary printout is printed and returned to MAIN.
Functional Flow Diagram of AFORD

START
ACSTOP
CALL FOR PAYOFF
AND EACH EQUALITY
CONSTRAINT
CALCULATE
ERRORS FROM
DESIRED
LSB?
= 1
RETURN
= 0
TRAJECTORY
SUMMARY PRINTOUT
6.3.5 AF0RUN

Subroutine Identification

- Title
  AF0RUN

- Calling Sequence
  CALL AF0RUN

Function

AF0RUN controls the setup logic for the trajectory integration.

Functional Flow

Calling the AF0RUN subroutine is synonymous to calling the integration of the trajectory from the initial conditions to termination. The subroutine is called either from MAIN or from BADLX.

The AF0RUN subroutine first calculates a transformation matrix for rotation from an inertial equatorial coordinate system to the inertial plumbline system. If the trajectory is initiated at liftoff (JUMP=1), the components of the radius and velocity are determined. For JUMP>1, the radius and velocity vectors are initialized by plumbline-state components stored in the input array VIV. After initialization of several variables, trigger values are set for the integration package for print, thrust events, tilt-over time, discrete altitudes, maximum dynamic pressure, Mach number equal to 1, and acceleration limits. Initialization of the trajectory is performed by first calling the subroutine ATHREV to determine the vehicle geometry, thrust levels, propellant flowrate, aerodynamics, integration steps, thrust event triggers, etc. ADER1 and ADER respectively, are called to determine the initial values of derivatives. The subroutine APRTN is then called with the header title "liftoff".

The integration package is called and the trajectory is integrated to termination. A check for integration error (KERR#0) is made. If no error has been made, the program flow is returned to the calling routine, otherwise, an
error message "INTEGRATION ERROR DUMPED" is printed.

Functional Flow Diagram of AP/RUN

START
CALCULATE A MATRIX

JUMP #1
YES \[
\frac{\dot{x}}{x} = \text{VIV}
\]

CALCULATE \[
\frac{\dot{x}}{x}
\]

INITIALIZE VARIABLES

SET TRIGGERS
ATHREV
ADER1
ADER
APRTRN

INTEGRATION PACKAGE
KERR=0
NO STOP
YES RETURN

6-15
6.3.6 AGEØ

Subroutine Identification
- Title
  AGEØ
- Calling sequence
  CALL AGEØ

Function
Evaluates oblate gravitational properties

Functional Flow
This subroutine is used for calculation of terms necessary for determining the gravitational accelerations when the oblate model is required.

Functional Flow diagram of AGEØ
6.3.7 AINIT

Subroutine Identification
- Title
  AINIT
- Calling Sequence
  CALL AINIT

Function
AINIT reads input data, and performs initialization.

Functional Flow
Subroutine AINIT initializes variables and calls for NAMELIST input. After calling the input, specific values are converted to MKS system and most angles converted to radians. The number of optimized values and the number of constraints are checked and counted to insure being within the current program limits (ten parameters and ten constraints). Initial (launch or jump-start) values of the state variables and environmental terms are determined. The NAMELIST input is then printed for reference.

Functional Flow Diagram of AINIT

START
READ NAMELIST "INPUT"
DATA CONVERSION

TOTAL NUMBER OF PAYOFF + CONSTRAINTS
YES STOP
NO

VARIABLE INITIALIZATION
WRITE NAMELIST
RETURN

6-17
6.3.8 AKALT

Subroutine Identification

- Title
  AKALT (Entry point in ATILT)
- Calling sequence
  CALL AKALT

Function

Trigger routine used for 10 km and 14 km altitude printout.

Functional Flow

This subroutine is called by the integration package when the vehicle altitude reaches 10 and 14 kilometers. The subroutine APRTN is called to print the state variables with special printouts of "10 KMS." or "14 KMS."

Functional Flow Diagram of AKALT

[Diagram showing the flow of operations with decision points and actions labeled accordingly.]
6.3.9 AMACH

Subroutine Identification

- Title
  AMACH (Entry point in ATILT)
- Calling sequence
  CALL AMACH

Function

Trigger routine for MACH=1 printout.

Functional Flow

This subroutine is called when the Mach number is equal to 1. The subroutine APRTN is called to print the state variables with special printout of "'ACH ONE".

Functional Flow Diagram of AMACH

START

TURN OFF TRIGGER

APRTN

RETURN
6.3.10 AMULG

Subroutine Identification
- Title
  AMULG
- Calling sequence
  C: L AMULG (L,M,N,X,XT,Y1, Y1T,Y2,Y2T), where
  L - number of dependent variables (1 or 2)
  M - previous value used in table location
  N - number of points in table
  X - independent variable
  XT - independent variable table
  Y1 - first dependent variable
  Y1T - first dependent variable table
  Y2 - second dependent variable
  Y2T - second dependent variable table

Function
  Linear interpolation scheme

Functional Flow
  (see appendix B)

Functional Flow Diagram of AMULG
  (not required)
6.3.11 ANEWCH AND NNEWCH

Subroutine Identification

- Title
  ANEWCH; note that NNEWCH is an entry point in ANEWCH.
- Calling sequence
  CALL ANEWCH or CALL NNEWCH

Function

Evaluates the parameter update equations to find the required changes in the controlling parameters.

Functional Flow

This subroutine uses the influence coefficients, constraint errors and the payoff values to determine the amount of change required in each controlling parameter to meet the constraints and optimize the payoff.

Initially, the matrix of influence coefficients is adjusted such that those required in the optimization are relocated to the upper left portion of that total matrix. If two QY searches (see Section III) have not been initiated (indicated by NOSRCH < 2) a weighting matrix is determined which allows all variables to have similar weights; otherwise, the previous weighting matrix is used. The $I_{\psi}$ matrix is calculated as the product of the transpose of the constraint influence coefficient matrix, the weighting matrix, and the constraint influence coefficient matrix. The $I_{\psi}$ vector is calculated as the product of the first two matrices used to form $I_{\psi}$ and the payoff influence coefficient vector.

The inverse of the $I_{\psi}$ matrix is calculated by calling MATINV and the result is made symmetric by averaging across the leading diagonal. The product $I_{\psi}^{-1}$ is found and the program flow proceeds to find the amount of controlling parameter correction desired to optimize the payoff and/or the amount desired to null constraint errors. The controlling parameter changes, $d\theta$, are then computed (see Section III) and added to the appropriate parameters. A test for convergence is made. If the desired convergence indicator tolerances
are satisfied, the required change in each parameter is small, and LSB is not equal to 1, the logical variable "BETCON" is set TRUE and the integer NOMI is set equal to 1. If these conditions are not met, "BETCON" is set FALSE and control is returned to the calling routine.

The entry point NNEWCH is called when a one-dimensional search is required, which does not require recalculation of the matrix products.

Functional Flow Diagram of ANEWCH and NNEWCH
6.3.12 APRTN

Subroutine Identification

- Title
  APRTN
- Calling Sequence
  CALL APRTN(NN), where
  NN - specifies the extra identification printout.

Function

Performs output editing, prints data, and creates tape file used for output tables and plots.

Functional Flow

Subroutine APRTN is a print subroutine called by subroutines ATHREV, ATILT, AMACH, AQMAX, AXPRT, GLIMT, AFQRUN, and AKALT. If the subroutine is called during an influence coefficient determination run (LSB=1), an immediate return is made; otherwise, the program checks the value of LPRINT (see Table 6 for an explanation of the LPRINT options) to determine the amount and frequency of printed information desired. The subroutine calculates the variables desired in the printout which have not been defined through common block storage and converts those which have to the desired units of output. The print is made with a label determined by the variable NN. A check is made of the value of NTABLE to determine if tape output is desired. If not, the program returns to the calling subroutine. If tape output is desired, the pertinent data is stored on tape unit 9.
Functional Flow Diagram of APRTN

START

LSB≠0

YES → RETURN

NO → LPRINT>1

NO → NMAX≠O

NMAX≠INMAX

YES → RETURN

NO → YES → VARIABLE

CONVERSION

AND CALCULATION

PRINT BLOCK

FORMAT

NTABLE=0

YES → RETURN

NO → WRITE OUTPUT

TAPE
6.3.13 AQMAX

Subroutine Identification

- Title
  AQMAX (Entry point in ATILT)
- Calling sequence
  CALL AQMAX

Function

Trigger routine for maximum dynamic pressure printout.

Functional Flow

Subroutine AQMAX is a print trigger routine called by the integration package when the time derivative of the dynamic pressure is zero, i.e., at the point where maximum dynamic pressure occurs. The routine turns the trigger off, and prints the state variable with a special printout of "Q MAXIMUM".

Functional Flow Diagram of AQMAX

```
START
    TURN TRIGGER OFF
        APRTN
            RETURN
```
6.3.14 ASIMP

Subroutine Identification
- Title
  ASIMP
- Calling sequence
  CALL ASIMP

Function
Determines the flyback range using empirical data interpolation.

Functional Flow
The subroutine ASIMP is used to determine the flyback range based on the altitude, relative velocity, and relative flight-path angle at booster staging. A trivariant interpolation* is used to determine the flyback range based on empirical data.

Functional Flow Diagram of ASIMP

*See Appendix B for further information concerning interpolation routines.
6.3.15 ATHREV

Subroutine Identification

- Title
  ATHREV
- Calling Sequence
  CALL ATHREV

Function

Thrust event control routine, sets up vehicle geometry, aerodynamics, number of engines, thrust levels, flowrates, and trigger flags for thrust event cutoff.

Functional Flow

Subroutine ATHREV is called at the beginning of each thrust event either by AFDRUN before the integration starts or by the integration package during the trajectory run. This subroutine sets up the vehicle geometry, aerodynamics, number of engines, thrust levels, flowrates, etc., for the ensuing thrust event. ATHREV also stops the integration at the end of the last thrust event by calling the subroutine RTMRK.

ATHREV first determines which thrust event is being initiated from the value of ITHR. If the first event is being initiated (liftoff) ITHR will be equal to one and the first stage center-of-gravity, engine gimbal position, and aerodynamic data are entered into the tables used in the calculations, and the first stage pitch and yaw attitude polynomials are determined. If ITHR is greater than one, the value of LSTGE (ITHR) is checked to see if it differs from the previous thrust event (ITHR-1), indicating that staging has occurred at the end of the last event. If staging has occurred, the second stage center-of-gravity, engine gimbal, position, and aerodynamic data are entered into the working tables, the flyback fuel required is calculated if desired,* and the second stage pitch and yaw attitude polynomials are determined. Also, the state at staging is stored if the current trajectory integration is not part of the influence coefficient calculation or the optimization step search.

*See Appendix E for further information concerning flyback calculations.
Printout is required if the value of ITHR is not equal to (JUMP-1). A test is then made to determine if the previous thrust event was the last event required (by the variable NVNT). If the final thrust event has been completed, a check is made to determine if the routine is being executed during an influence coefficient trajectory (LSB=1). If not, the subroutine calls APRTN for a final printout using the header "INJECTION" and stores the state derivatives. Whether or not the LSB variable is equal to 1, the final cutoff weight is calculated and the subroutines ADER1, ADER, and APRTN are called for a final pass through the equations of motion and a final printout. A call to the routine RTMRK terminates the integration package to return to the subroutine APFORUN.

If the final thrust event has not been completed, selection of cutoff triggers is made by the input variable MSWCH (see input section). Integration step size, print step size, exit area, propellant flow rate and thrust are calculated for the thrust event and a test is made to see if ITHR is equal to 1. If ITHR=1, the program flow is returned to the calling routine, APFORUN. If ITHR is greater than 1, the value of the previous thrust event, JT, is checked to see if an intermediate equality constraint is required at the end of the last event, noted by NVRST. If JT=NVRST, the state variables are stored at that time. The time derivative of dynamic pressure is then checked by calling ADER1 and ADER to see if the next thrust event will result in a change in sign of the derivative. If there is a sign change, the subroutine AQMAX is called to mark maximum dynamic pressure. If the variable LSB=1 (indicating influence coefficient trajectory), the program flow is returned to the calling routine. If LSB=0, the calculations for flight performance reserves are made if required and if the thrust event (ITHR) is equal to the input variable IPR.

If the number of the previous thrust event is equal to NVRST, the state and derivatives of the state are stored and APRTN is called with a header "INJECTION". Whether the number of the previous thrust event was equal to NVRST or not, the subroutines ADER1 and ADER are called to update the equations of motion. The APRTN subroutine is called with a header "THRUST EVENT".
Functional Flow Diagram of ATHREV

START

JTH-1

YES

JTH+1

NO

CALCULATE FUEL

YES

STORE STATE

NO

FIND

YES

KBACK+1

NO

ATMP

YES

LSB+1

NO

STORE STATE

SETUP AERODYNAMIC AND BODY REFERENCES

APRTN

YES

JTH#1

NO

APRTN

YES

LTMP

YES

JTH+1

NO

ADVR

YES

ADVR

NO

CALCULATE PERFORMANCE RESERVES

YES

STORE STATE

NO

APRTN

RETURN
6.3.16 ATILT

Subroutine Identification

- Title
  ATILT
- Calling Sequence
  CALL ATILT

Function

ATILT is a trigger routine for end of vertical rise (begin tilt-over or attitude control) and end of tilt over (if used). The subroutines AKALT, AMACH, AQMAX, AXPRT, and GLIMT are contained within this subroutine as entry points.

Functional Flow

Subroutine ATILT is a print and control trigger routine that is called by the integration package at the end of the liftoff phase of the trajectory (specified by the input variable TLIFT) and again at the end of the tilt-over phase if angle-of-attack attitude control is used.

The subroutine first sets KSI=2 denoting end of vertical rise and turns off the ATILT trigger. A test is then made to determine if a programmed tilt-over is to be used (in conjunction with angle-of-attack attitude control). If it is, the ATILT trigger is turned back on and the trigger time is the time at the end of the tilt-over, TILT. If the call to ATILT was the end of tilt-over, the ATILT trigger is turned off. APRTN is called with the header "END TILT" and flow returns to the integration package. If a programmed tilt is not used, or if it is and the call to ATILT occurred at the end of the vertical rise, APRTN is called with the header "BEGIN TILT", the maximum dynamic pressure trigger is turned on, the $\chi_p$ and $\chi_y$ attitude polynomials are determined by calling subroutine FIND, and flow returns to the integration package. If the call to ATILT has occurred at the end of a programmed tilt-over, the ATILT trigger is turned off, subroutine APRTN is called with the header "END TILT", and flow returns to the integration package.
Functional Flow Diagram for ATILT

START

TURN OFF ATILT TRIGGER
SET END VERTICAL RISE FLAG

IS PROGRAMMED TILT-OVER TO BE USED WITH ANGLE-OF-ATTACK ATTITUDE CONTROL?

YES

TURN ATILT TRIGGER ON
SET TRIGGER TIME

NO

CALL APRTN
PRINT STATE WITH HEADER "BEGIN TILT"

WAS THE CURRENT CALL MADE AT END OF VERTICAL RISE?

YES

RETURN

NO

TURN ATILT TRIGGER OFF
CALL APRTN
PRINT STATE WITH HEADER "END TILT"

RETURN

STORt CURRENT $x_p$ AND $x_y$ IN FIRST TABLE LOCATIONS

DETERMINE $x_p$ AND $x_y$ POLYNOMIALS FROM TABULATED VALUES

RETURN
6.3.17 AXPRT

Subroutine Identification
- Title
  AXPRT (entry routine in ATILT)
- Calling Sequence
  CALL AXPRT

Function
  Trigger routine for normal printout.

Functional Flow
  Subroutine AXPRT provides a call to the print routine APRTN during the trajectory integration indicated by the input variable LPRINT. The subroutine first calls APRTN with a blank header and updates the value of the next print time required. Using the function AINT the decimal portion of time is truncated such that print time will occur at whole number times.

Functional Flow Diagram of AXPRT
6.3.18 BADLX

Subroutine Identification

- Title
  BADLX
- Calling sequence
  CALL BADLX

Function

BADLX determines the influence coefficients (partial derivatives) used in the computation of the changes in the controlling parameters.

Functional Flow

Subroutine BADLX is used to determine the influence coefficients (the partial derivatives of the payoff and constraints with respect to the controlling parameters) which are used in the calculation of the changes in the controlling parameters. The subroutine first sets the flag LSEb=1, which is used in other routines to identify the type of integration run being made. The increment amounts for the parameters are determined and used according to the parameters required by the input KDB. A call to the subroutines AF0RUN and AF0RND is executed for each independent change of the controlling parameters. The influence coefficients are then stored in the array XLAMB for use in ANEWCH. If the variable NPSTST is equal to 1, the printout "FORWARD DIFFERENCES" is printed, indicating that only one change will be made in the parameters. If the variable NPSTST is not equal to 1, the printout "CENTRAL DIFFERENCES" is printed which indicates that a plus and minus increment will be made in each parameter and the associated influence coefficients taken as the average of the plus and minus values.

An exception to the above procedure for obtaining the partial derivatives occurs when final burn time is an optimized parameter and payload is not a payoff or constraint. In this case, the subroutine ACSTOP provides BADLX a set of analytical partial derivatives based on the partial of the constraint with respect to the state variables. BADLX then determines the influence coefficients of the constraints with respect to burn time without integration of a trajectory simulation.
Functional Flow Diagram of BADLX

START

MPSTST+1

YES
WRITE FORWARD DIFFERENCES

NO

WRITE CENTRAL DIFFERENCES

DETERMINE INCREMENTS

KD8(1-10)=0

YES

KD8(11)=0

NO
LIFTOFF HEIGHT

YES

KD8(12)=0

NO
LAUNCH AZIMUTH

YES

KD8(13-16)=0

NO
BOOSTER PITCH ATTITUDE

YES

KD8(17-18)=0

NO
BOOSTER YAW ATTITUDE

YES

KD8(19-21)=0

NO
ORBITER PITCH ATTITUDE

RESTART AT ORBITER IGNITION

YES

KD8(22-24)=0

NO
ORBITER YAW ATTITUDE

RESTART AT ORBITER IGNITION

YES

FINAL BURN TIME W/O PAYLOAD OPTION

CALCULATE ANALYTIC PARTIALS

YES

CHANGE PARAMETERS(+)

AFTERM ABOUND

NO
RESET PARAMETERS

MPSTST+1

NO
CALCULATE ILAMB

YES

CHANGE PARAMETERS(-)

AFTERM ABOUND

NO
RESET PARAMETERS

CALCULATE ILAMB

RETURN

RETURN

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6.3.19 BQPPLT

Subroutine Identification

- Title
  BQPPLT
- Calling sequence
  CALL BQPPLT

Function

BQPPLT generates output plots on CALCOMP plotter

Function of Flow

The subroutine BQPPLT will be called from MAIN after completion of a converged trajectory by setting the input variable NTABLE equal to 2 or 3. This subroutine uses special CALCOMP library functions during execution. The programmer should verify that these routines are on the system library before execution of BQPPLT.

The subroutine starts by reading a three-card, 27-word, field identification message used for communication with the plot operator. The identification message is printed and a call is made to the CALID routine which plots an identification print and begins the plotting process. The PLOT subroutine is called which sets up the location of the origin in terms of location on the plot. Plotting information is supplied by an input card which determines the variables required for plotting and the desired units for output. If the word ENDPLT is input, the CALEN subroutine is called which draws a final identification block and terminates the plotting routine.

For each input card the variables required and the units desired are identified and the correct variables are taken from the trajectory tape provided by APRTN. These variables are read from the tape and placed into storage arrays. The SCALE subroutine is called which determines the scaling required for the plotting process.
The AXIS subroutine is called which labels the plot and draws the axes. The subroutine LINE is then called to draw the generated curve.

If labeling is required, a label is read from cards and the subroutine SYMBOL is called. Each plot is concluded by calling the PLOT subroutine to establish a new origin. Program flow returns to read another plot variable input card.

Functional Flow Diagram of NPPLT
6.3.20 BOPTBL

Subroutine Identification
- Title
  BOPTBL
- Calling Sequence
  CALL BOPTBL

Function
  BOPTBL generates output table summary.

Functional Flow
  The subroutine BOPTBL will be called from MAIN after completion of a
  converged trajectory by setting the input variable NTABLE equal to 1 or 3.
  The subroutine uses a special NAMELIST call, therefore additional input is
  required when using this subroutine.

  The general flow of the subroutine is to read data from a trajectory
  tape provided by the APRTN subroutine and place this data in the fourteen
  output tables. These tables (see subsection 9.3) are comprised of two sets of
  seven tables each which provide output in the MKS system and the English
  system.
Functional Flow Diagram of BØPTEL

START

READ & WRITE NAMELIST INPUT

READ TRAJECTORY TAPE

END OF TAPE

YES

NO

NUMBER OF TABLES REQ'D ?

PRINT CORRECT TABLES

NUMBER OF TABLES SETS REQ'D ?

YES

RETURN

NO
6.3.21 CHIPQL

Subroutine Identification

- Title
  CHIPQL

- Calling sequence
  CALL CHIPQL (N1,N2,T,Y1,A1,Y2,A2) where
  N1 - order of A1 polynomial
  N2 - order of A2 polynomial
  T - independent variable
  Y1 - value of polynomial A1 evaluated at T
  A1 - polynomial coefficients
  Y2 - value of polynomial A2 evaluated at T
  A2 - polynomial coefficients.

Function

Evaluate the pitch and yaw attitude polynomials.

Functional Flow

(not required)

Functional Flow Diagram of CHIPQL

(not required)
6.3.22 FIND

Subroutine Identification

- Title
  FIND
- Calling Sequence
  CALL FIND (CHI,A,DT,NORDER) where
  CHI - attitude table
  A - output polynomials
  DT - time table based on beginning of stage time
  NORDER - order of the polynomial desired.

Function

FIND determines attitude polynomials based on tabulated attitude time
history information.

Functional Flow

(not required)

Functional Flow Diagram of FIND

(not required)
6.3.23 GLIMT

Title
GLIMT (Entry point in ATILT)

Calling Sequence
CALL GLIMT

Function
Trigger routine for acceleration limit printout.

Functional Flow
The subroutine GLIMT is called from the integration package when the axial acceleration limit has been reached.

The acceleration trigger is turned off and the APRTN subroutine is called with the header "BEGIN GLIMIT". If discrete throttling is required (MSWCH<0) the subroutine ATHREV is called; otherwise, the flow is returned to the calling routine.

Functional Flow Diagram of GLIMT
6.3.24 MATINV

Subroutine Identification

- Title
  MATINV
- Calling Sequence
  CALL MATINV (B,N) where
  B - matrix to be inverted (the inverted matrix is stored in B also)
  N - number of rows of matrix B.

Function

MATINV performs matrix inversion, calculating the inverse in double precision arithmetic, although the entry and exit matrices are truncated to single precision.

Functional Flow
(not required)

Functional Flow Diagram of MATINV
(not required)
6.3.25 PRA63

Subroutine Identification

- Title
  PRA63

- Calling Sequence
  CALL PRA63 (PR, ERR@R) where
  PR - storage array for atmospheric parameters
  ERR@R - error flag (not used).

Function

PRA63 evaluates the atmospheric parameters as well as their partial derivatives w.r.t. altitude using the spline interpolation method of the subroutine SPLINE.

Functional Flow

Interpolation method used in this routine is defined in appendix B.

Functional Flow Diagram of PRA63

(not required)
6.3 SPLINE, SPLIN2

Subroutine Identification

- Title
  SPLINE, SPLIN2

- Calling sequence
  CALL SPLINE (L,M,N,X,XT,Y,TY,YD,K,LAST)
  CALL SPLIN2 (L,M,N,X,XT,Y,TY,YD,K,LAST) where
  L - number of dependent variables (stored in common storage)
  M - location index
  N - number of points in table
  X - independent variable
  XT - independent variable table
  Y - dependent variables
  YT - dependent variables tables
  YD - first derivative of the dependent variable w.r.t. the independent variable
  K - flag to denote calculation of partial derivatives
    K=1, do not calculate partials; K=2 calculate partials
  LAST - stored value of M.

Function

SPLINE determines the aerodynamic coefficients using a SPLINE interpolation method.

SPLIN2 is identical to SPLINE in logic, however, SPLIN2 is used for interpolation of thrust and weight loss for the SRM engines.

Functional Flow

(see appendix B)

Functional Flow diagram of SPLINE

(not required)
6.3.27 TEST

Subroutine Identification

- Title
  TEST

- Calling Sequence
  CALL TEST (X,XT,NX,I,R) where
  X - independent variable
  XT - independent variable table
  NX - number of points in XT table
  I - location index
  R - dependent variable slope

Function

TEST embodies the linear scheme used in ASIMP for determining slope of
the dependent variable w.r.t. the independent variable.

Functional Flow

(not required)

Functional Flow Diagram of TEST

```
START
  \arrow{down}
  TEST
  \arrow{down}
  TRIVLP
  \arrow{down}
  CALCULATE FLYBACK RANGE
  \arrow{down}
  RETURN
```
6.3.2 TRIVLP

Subroutine Identification

- Title
  TRIVLP

- Calling Sequence
  CALL TRIVLP (G,GT,NG,V,VT,NV,A,AT,NA,ϕT,ϕ) where
  G - value of relative flight-path angle
  GT - relative flight-path angle table
  NG - number of points in GT table
  V - value of relative velocity
  VT - relative velocity table
  NV - number of points in VT table
  A - value of altitude
  AT - altitude table
  NA - number of points in AT table
  ϕT - flyback range table
  ϕ - flyback range (output).

Function

Trivariant lookup of flyback range (for use in ASIMP).

Functional Flow

The subroutine TRIVLP is called by the subroutine ASIMP to determine the flyback range as a function of relative flight-path angle, relative velocity, and altitude.

ASIMP calls the subroutine TEST to determine the linear slopes of the altitude, relative velocity, and relative flight-path angle w.r.t. the flyback range. The flyback range is then calculated using these slopes.

Functional Flow Diagram of TRIVLP

(See TEST)
PART III
USER'S MANUAL
Section VII

PROGRAM LISTING

Consult MSFC, S&E, AERO-GT for Program Listing
Section VIII

VARIABLE NAME CROSS REFERENCE OF MAJOR SUBROUTINES

Consult MSFC, S&E, AERO-GT for variable name cross reference of major subroutines.
Section IX
INPUT/OUTPUT

The usefulness of any computer program as an analysis tool is dependent largely upon how well the user understands the operation of the program and the input and output involved in its use. This section has therefore been designed to provide the user a thorough understanding of the basic mechanics of the program, the input required, and the output available. The following paragraphs describe in detail: 1) how to formulate the trajectory problem for presentation to RAGMOP, 2) how to select the various options available, 3) the physical input data required, 4) the various types of output produced by the program, and 5) some suggestions concerning technique which are helpful in making the most efficient use of the program.

RAGMOP users familiar with the ROBOT computer program will note the similarity in the input of the two programs. The input philosophy of RAGMOP is intended to coincide with that of ROBOT whenever possible. This was done (due to the wide use of ROBOT) as a convenience to potential users of RAGMOP. Note, however, that beyond this similarity the two programs are vastly different.

9.1 PROBLEM FORMULATION

This section describes how to formulate the ascent trajectory for presentation to RAGMOP. The terminology used in describing various events is defined, and information pertinent to the selection of various program options is presented.

9.1.1 Thrust Profile

The RAGMOP trajectory is described as a succession of a number of thrust events, one or more of which may comprise a complete stage. A thrust event is a period of time characterized by a continuous thrust profile (constant, zero, or varying in a continuous fashion) (Figure 9-1). Each thrust event is initiated at the termination of the previous event except, of course, for the first event which initiates the trajectory. A thrust event is terminated
on one of four criteria: (1) time (the duration of the thrust event, not the absolute final time), (2) liquid fuel depletion (fuel is input in the first thrust event of a liquid burn and any fuel remaining at the end of a given event is carried forward to the next event of the same stage, if any), (3) g-limit, or (4) relative velocity (which, of course, should be used only for one thrust event). The data used to determine the vehicle geometry, propulsion, and aerodynamics is described either by thrust event or by stage. Data which is input by stage consists of the aerodynamic coefficient, base drag, and center-of-gravity tables, the aerodynamic reference length, the order of the $x_p$ and $x_y$ attitude polynomials, number of thrust events comprising the stage, engine gimbal point locations, and the location of the aerodynamic data moment ref-
ference point. Data which is input by thrust event consists of the number of engines, vacuum thrust per liquid engine, exit area per SRM or liquid engine, mass flow rate per liquid engine, specific impulse for liquid engine when continuous throttling is used, liquid fuel per thrust event (actually input in the first thrust event of the liquid stage which begins the liquid burn), acceleration (g) limit, liquid thrust event cut-off flag, print and integration step sizes, aerodynamic reference area, thrust event duration, and jettison weight (which is dropped at the end of the thrust event).

Data for the SRM engines consisting of sea level thrust and weight loss are input as tabular functions of time from liftoff.

9.1.2 Control Program

In formulating the trajectory problem, the user must decide what kind of attitude control is required. If the optimized polynomial form is used for the \( x_p \) program, the degree of the polynomial desired for each stage must be specified. Also, the decision must be made as to whether or not the yaw program is to be optimized, and if so, the degree of the yaw polynomial in each (or either) stage must be selected. If angle-of-attack control is to be used for the first stage, the optimized tilt-over maneuver must be used. This requires selection of the polynomial degree and the length of time desired for the pitchover.

Note that a polynomial for \( x_p \) and \( x_y \) can be used without being optimized or without each data point used in the curvefit being optimized. Suppose, for example, that the value of \( x_p \) at a certain time during the first stage was required to be a fixed value, and that a cubic polynomial was desired for the \( x_p \) of the stage. The cubic polynomial could be used with only two optimized data points, with the first point and any one of the other three points (four points required to perform cubic curvefit) remaining fixed throughout the run. As another example, a \( x_y \) polynomial could be used without optimization of any of the data points.

9.1.3 Efficient Use of RAGMOP

While a great many combinations of parameters and constraints exist within RAGMOP, careful selection of the optimized parameters and the constraints en-
forced can streamline the program greatly with little or no effect on the solution obtained. Several comments and suggestions are presented below which provide information pertinent to using RAGMOP most efficiently.

9.1.3.1. Body axis Coordinate System. In order to agree with the present Apollo Standard coordinate system, all vehicle input data uses the coordinate system shown in Figure 9-2, with the longitudinal axis as the x-axis, the lateral axis as the y-axis (out the right wing), and the vertical axis as the z-axis (down). At liftoff, the vehicle will be oriented with the z-axis pointed in the direction of the launch azimuth (downrange).

9.1.3.2 Constrained qa and/or q8. Although RAGMOP has the capability of enforcing the inequality qa and q8 constraints, experience has shown that often these constraints will not be violated when a q_{\text{max}} constraint is simultaneously enforced. Since the use of these constraints presents discontinuities in the trajectory angle-of-attack and/or sideslip angle derivatives, the solution using the constraints is generally more difficult to obtain and hence more computer time will be required. If some doubt exists as to whether or not the qa and/or q8 limits will be exceeded, a solution can first be obtained without enforcing the constraints. If the maximum values are exceeded, the solution may be used as the initial "guess" (nominal) trajectory for a run with the constraints enforced.

9.1.3.3 Constrained q_{\text{max}}. The q_{\text{max}} constraint in RAGMOP is enforced as an equality constraint. Therefore, as with the qa and q8 constraints mentioned above, the constraint should not be enforced unless the user is sure that the maximum allowable value will be exceeded. Since the q_{\text{max}} constraint is an equality constraint, the optimized parameters will be adjusted until the constraint is met. Thus, all parameters (launch weight, azimuth, burn times, and attitude control) are affected. Experience has shown that creating a thrust event prior to q_{\text{max}} where thrust is reduced by a discrete amount (at an optimized time) will aid in satisfying the constraint with a minimum of reshaping of the trajectory. This combination of throttling and reshaping has been seen to be much more efficient than reshaping alone.

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9.1.3.4. **Payload Option.** The payload calculation (see Appendix D) has been designed especially for space shuttle type vehicles and is therefore preferable to the FPR option for these vehicles.

9.1.3.5. **Attitude Control Program.** The vehicle attitude control polynomials are stage-dependent and therefore are carried over thrust events of a given stage. The order of the polynomial used to describe the attitude histories should be chosen with the following points in mind:

- The higher the order of the polynomials, the more optimum the trajectory can be.
- The higher the order of the polynomials, the more parameters are required and therefore the longer the run will take to converge. Therefore, the lowest order polynomial which yields an acceptable solution should be used when rapid solutions are required.
- An optimized launch azimuth or yaw control program is generally not required unless a specific orbital inclination is a constraint, or when the coordinated turn option is used.
- The second stage $\chi_P$ program is normally initiated at the last value of $\chi_P$ from the first stage (continuous $\chi_P$). The use of a discontinuous $\chi_P$ is optional (KDB(19)=1) and selection of discontinuous $\chi_P$ can be based largely on two points: (1) the discontinuous $\chi_P$ is in general more optimum and (2) the use of $\chi_P$ discontinuity at staging may not be realistic if the dynamic pressure is above a certain level (i.e., difficulties in executing the $\chi_P$ discontinuity may arise in actual flight). If the second stage is flown essentially in a vacuum, (i.e. no aero-dynamic coefficients input) there is no need for more than a linear $\chi_P$ profile since, even with a quadratic form available, the $\chi_F$ program produced the RACMOP will be linear (due to the nature of the vacuum flight problem).

In addition to the above points concerning selection of the order of the attitude polynomials, one should also note that proper spacing of the points in the first and the second stage time tables can help increase the efficiency of the program. The input tables TTBL (first stage) and TOBL (second stage) contains the time points at which the angles in the input tables CPTBL, CYTBL (first stage), CPÔTBL, and CYÔTBL (second stage) apply. The $\chi_P$ and $\chi_Y$ values in the latter set of tables will be varied at each parameter update step, and curvefits for the appropriate polynomials performed. In light of this, it is easily seen that an even distribution of time points over the duration of each stage will provide the most stable operating conditions for the program. (Consider, for example, the difficulty of fitting a curve to points grouped very closely together, and
the sensitivity of the curve to very slight variations in the points when they are so grouped).

9.1.3.6 Run Time. It should be noted that RAGMOP, while appearing similar in some ways to ROBOT, may at times require considerably more computer time to obtain a converged trajectory solution. This is due to the more complex physical model (atmosphere in both stages, optimized from launch to orbit, lifting trajectory, etc.) and to the necessarily different optimization scheme. If a particular run is returned and has not obtained a converged solution, the user may take advantage of the work performed during the first run by merely updating the values of the optimized parameters and resubmitting the run. Generally, the parameter set obtained during a run will meet the required constraints, but if the run is terminated due to maximum time, will not be optimum. Thus, a run which is not yet converged may still provide the user with meaningful information, unlike other techniques wherein intermediate results have no significance. Trajectory solutions with RAGMOP have been obtained in from two to ten minutes of computer time starting from very poor nominal trajectories. Good initial guess (nominal) trajectories will increase speed of convergence.

Figure 9-2. INPUT BODY AXIS SYSTEM
### Itemized Description of Namelist Input

<table>
<thead>
<tr>
<th>NAME</th>
<th>DIMENSIONS</th>
<th>UNITS</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>AE</td>
<td>(15)</td>
<td>M²</td>
<td>Exit area per engine for each thrust event. Preset to zero.</td>
</tr>
<tr>
<td>ALAT</td>
<td></td>
<td>DEG</td>
<td>Launch site latitude. Preset to 28.531885.</td>
</tr>
<tr>
<td>ALONG</td>
<td></td>
<td>DEG</td>
<td>Launch site longitude. Preset to 80.5649528.</td>
</tr>
<tr>
<td>ALTBas</td>
<td>(25,2)</td>
<td>M</td>
<td>Altitudes at which the base pressure differentials in BAXIAL apply. Up to 25 altitudes may be used. The second index denotes the combined booster-orbiter (1) or the orbiter alone (2). Preset to zero.</td>
</tr>
<tr>
<td>ALTLS</td>
<td></td>
<td>M</td>
<td>Altitude of the launch site above the spheroid. Preset to zero.</td>
</tr>
<tr>
<td>ALTtBL</td>
<td>(25)</td>
<td>M</td>
<td>Altitude values at which the wind speeds and directions in WTBL and AZWTBL apply.</td>
</tr>
<tr>
<td>AYL</td>
<td></td>
<td></td>
<td>Used for error check in integration. The preset value should be used. (No input required). Preset to 0.002.</td>
</tr>
<tr>
<td>AZ</td>
<td></td>
<td>DEG</td>
<td>Launch azimuth. Preset to 90.</td>
</tr>
<tr>
<td>AZWTBL</td>
<td>(25)</td>
<td>DEG</td>
<td>Azimuth angles denoting the directions that apply to the wind speeds and altitudes in WTBL and ALTtBL. Denotes the direction toward which the wind is blowing.</td>
</tr>
<tr>
<td>BAXIAL</td>
<td>(25,2)</td>
<td>LBF</td>
<td>Base axial force.</td>
</tr>
<tr>
<td>BLØD</td>
<td></td>
<td></td>
<td>$C_L/C_D$ of booster. Used in booster flyback fuel computation.</td>
</tr>
<tr>
<td>CAALP</td>
<td>(25,2)</td>
<td>/DEG</td>
<td>Partial of axial force coefficient with respect to angle-of-attack. Indices as in CAØ. Preset to zero.</td>
</tr>
<tr>
<td>CAØ</td>
<td>(25,2)</td>
<td></td>
<td>Zero lift axial force coefficient corresponding to Mach numbers in PNM. Second index denotes stage. Preset to zero.</td>
</tr>
</tbody>
</table>

9-7
<table>
<thead>
<tr>
<th>NAME</th>
<th>DIMENSIONS</th>
<th>UNITS</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE</td>
<td></td>
<td></td>
<td>The number of the case being run. For output use only. Preset to 1.</td>
</tr>
<tr>
<td>CISP</td>
<td>(15)</td>
<td>SEC</td>
<td>Vacuum I of liquid engines used during continuous throttling.</td>
</tr>
<tr>
<td>CI</td>
<td></td>
<td></td>
<td>First coefficient in the Fischer ellipsoid gravitational expansion. The preset value should be used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(No input required)</td>
</tr>
<tr>
<td>CLBETA</td>
<td>(25,2)</td>
<td>/DEG</td>
<td>Partial of rolling moment coefficient with respect to sideslip angle. Indices as in CA0. Preset to zero.</td>
</tr>
<tr>
<td>CMALP</td>
<td>(25,2)</td>
<td>/DEG</td>
<td>Partial of pitching moment coefficient with respect to angle-of-attack. Indices as in CA0. Preset to zero.</td>
</tr>
<tr>
<td>CM0</td>
<td>(25,2)</td>
<td></td>
<td>Zero lift pitching moment coefficient. Indices as in CA0. Preset to zero.</td>
</tr>
<tr>
<td>CMUE</td>
<td></td>
<td>M^3/SEC^2</td>
<td>Product of the universal gravitational constant and the mass of the earth. The preset value should be used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(No input required)</td>
</tr>
<tr>
<td>CNALP</td>
<td>(25,2)</td>
<td>/DEG</td>
<td>Partial of normal force coefficient with respect to angle-of-attack. Indices as in CA0. Preset to zero.</td>
</tr>
<tr>
<td>CNBETA</td>
<td>(25,2)</td>
<td>/DEG</td>
<td>Partial of yawing moment coefficient with respect to sideslip angle. Indices as in CA0. Preset to zero.</td>
</tr>
<tr>
<td>CN0</td>
<td>(25,2)</td>
<td></td>
<td>Zero angle of attack normal force coefficient. Indices as for CNALP. Preset to zero.</td>
</tr>
<tr>
<td>CORBWT</td>
<td></td>
<td>LBM</td>
<td>Constant orbiter weight used in payload calculation.</td>
</tr>
<tr>
<td>CPQTBL</td>
<td>(20)</td>
<td>DEG</td>
<td>Values of chi-pitch (xₚ) to be used in forming the nominal trajectory for the second stage. Values correspond to time in T0BL(1-10)</td>
</tr>
<tr>
<td>CPTBL</td>
<td>(20)</td>
<td>DEG</td>
<td>Same as CPQTBL but for first stage, and values correspond to times in TTBBL(1-10)</td>
</tr>
<tr>
<td>CRSDWT</td>
<td></td>
<td>LBM</td>
<td>Constant residual weight used if SRESID = 0. Preset to zero.</td>
</tr>
<tr>
<td>NAME</td>
<td>DIMENSIONS</td>
<td>UNITS</td>
<td>EXPLANATION</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>CTNKWT</td>
<td>LBM</td>
<td>Constant tank weight used if SCALE = 0. Preset to zero.</td>
<td></td>
</tr>
<tr>
<td>CWDQT</td>
<td>(15) LBM/SEC</td>
<td>Critical flow rate per engine for each thrust event.</td>
<td></td>
</tr>
<tr>
<td>CYBETA</td>
<td>(25,2) /DEG</td>
<td>Partial of side force coefficient with respect to sideslip angle. Indices as in CAØ. Preset to zero.</td>
<td></td>
</tr>
<tr>
<td>CYQTBL</td>
<td>(20) DEG</td>
<td>Same as CPQTBL but for yaw(correspond to TØBL(11-20))</td>
<td></td>
</tr>
<tr>
<td>CYTBL</td>
<td>(20) DEG</td>
<td>Same as CPTBL but for yaw(correspond to TTBL(11-20))</td>
<td></td>
</tr>
<tr>
<td>DELVG</td>
<td>M/SEC</td>
<td>Delta velocity required for geometry reserves. Preset to zero.</td>
<td></td>
</tr>
<tr>
<td>DELVP</td>
<td>M/SEC</td>
<td>Delta velocity required for performance reserves. Preset to zero.</td>
<td></td>
</tr>
<tr>
<td>DJ</td>
<td></td>
<td>Third coefficient in the Fischer ellipsoid gravitational expansion. The preset value should be used. (No input required).</td>
<td></td>
</tr>
<tr>
<td>DIZ</td>
<td>SEC</td>
<td>Time from ground reference release (GRR) to liftoff. GRR is the point in the countdown at which the launch inertial coordinate system is established. Must be input for jump starts as well as for ground-launch trajectories. Preset to zero.</td>
<td></td>
</tr>
<tr>
<td>DVØNS</td>
<td>FT/SEC</td>
<td>Delta velocity required of orbital maneuvering system, used in payload calculation.</td>
<td></td>
</tr>
<tr>
<td>END</td>
<td>(10)</td>
<td>Maximum allowable absolute values of the constraint errors. END(I) corresponds to KCDPHI(I+1), and then to KCDRES(I+N) where N is the number of constraints in KCDPHI. This set of tolerances must be met before a run is considered to be converged.</td>
<td></td>
</tr>
<tr>
<td>EØPCT</td>
<td></td>
<td>Additional fuel percentage needed for booster flyback with one engine out. Percentage of landed weight of booster.</td>
<td></td>
</tr>
<tr>
<td>ZU</td>
<td></td>
<td>Upper error bound in integration. The preset value should be used. (No input required).</td>
<td></td>
</tr>
<tr>
<td>NAME</td>
<td>DIMENSIONS</td>
<td>UNITS</td>
<td>EXPLANATION</td>
</tr>
<tr>
<td>----------</td>
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<td>---------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>FACT</td>
<td></td>
<td></td>
<td>Denotes whether optimized lifting (FACT = -1), zero normal force (FACT = 1), zero angle-of-attack (FACT = 2) or Mach number dependent (FACT = 3) is to be flown. If FACT is 1, 2, or 3, a programmed tilt over time should be used (See TILT). Preset to -1.</td>
</tr>
<tr>
<td>FLBS</td>
<td>(15)</td>
<td>LBF</td>
<td>Liquid engine vacuum thrust per engine for each thrust event. Preset to zero.</td>
</tr>
<tr>
<td>FPRFAC</td>
<td></td>
<td></td>
<td>Factor used in calculation of delta velocity for performance reserves (FPR)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>FPR = FPRFAC*[\frac{\sqrt{\text{COW}}}{g_o} \ln \frac{(\text{LOW} + g_o \text{VISPO})}{\sqrt{\text{COW}}} + \frac{g_o \text{VISPO}}{\sqrt{\text{COW}}}]</td>
</tr>
<tr>
<td>FUELIQ</td>
<td>(15)</td>
<td>LBM</td>
<td>Liquid engine fuel. For parallel burn, all liquid engine fuel should be put into FUELIQ (1) and the indicator NPARN set equal to 1. For series burn, the fuel is input in the first thrust event of each stage. Must be input whenever fuel consumed is used as a cutoff criteria for any thrust event in a stage.</td>
</tr>
<tr>
<td>GAPFCT</td>
<td></td>
<td></td>
<td>Additional fuel percentage required by booster for a go-around at the end of flyback. Percentage of landed weight of booster.</td>
</tr>
<tr>
<td>GLIM</td>
<td>(15)</td>
<td>G's</td>
<td>Maximum longitudinal acceleration, in G's for each thrust event. Preset to 0.1E20.</td>
</tr>
<tr>
<td>GZERØ</td>
<td>m/SEC²</td>
<td></td>
<td>Gravitational acceleration of earth at the equator. The preset value should be used. (No input required).</td>
</tr>
<tr>
<td>H</td>
<td></td>
<td></td>
<td>Second coefficient in the Fischer ellipsoid gravitational expansion. The preset value should be used. (No input required).</td>
</tr>
<tr>
<td>HEAD</td>
<td>90 Spaces</td>
<td></td>
<td>90 Space Hollerith Field used for identification of each page of trajectory print.</td>
</tr>
<tr>
<td>HEDWND</td>
<td>ft/sec</td>
<td></td>
<td>Headwind used in flyback fuel calculation. Preset to zero.</td>
</tr>
<tr>
<td>HSN</td>
<td></td>
<td></td>
<td>Minimum step-size for integration. The preset value should be used. (No input required).</td>
</tr>
<tr>
<td>NAME</td>
<td>DIMENSIONS</td>
<td>UNITS</td>
<td>EXPLANATION</td>
</tr>
<tr>
<td>------</td>
<td>------------</td>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>IPR</td>
<td></td>
<td></td>
<td>Denotes the thrust event from which the performance reserves are taken. Must be in the last stage and no thrust event may be optimized after this one. If IPR is greater than zero, WPM and CWDG must be input. Preset to zero.</td>
</tr>
<tr>
<td>JB0RB</td>
<td></td>
<td></td>
<td>Denotes whether spherical (JB0RB = 1) or oblate (JB0RB=0) earth model is desired. Preset to zero.</td>
</tr>
<tr>
<td>JTHR</td>
<td>(15)</td>
<td></td>
<td>Denotes use of SRM thrust and delta weight tables for each thrust event (JTHR = 1) or no SRM thrusts or delta weight (JTHR = 0). Preset to zero.</td>
</tr>
<tr>
<td>JUMP</td>
<td></td>
<td></td>
<td>Denotes the number of the thrust event at which the start is to occur. A normal ground launch would start at the first thrust event (JUMP=1). A jump start can start at any later thrust event. For a jump start the array VIV must be input. Preset to 1.</td>
</tr>
<tr>
<td>KBACK</td>
<td></td>
<td></td>
<td>Indicator to denote computation of booster flyback fuel desired (KBACK = 1) or not desired (KBACK = 0). Preset to zero.</td>
</tr>
<tr>
<td>KCDPHI</td>
<td>(10)</td>
<td></td>
<td>Terminal constraint and payoff codes. KCDPHI(1) denotes the payoff desired. KCDPHI(2-10) are the code numbers of the terminal constraints desired. See Table 9-1. Preset to KCDPHI = 1,2,3,4.</td>
</tr>
<tr>
<td>KCDRES</td>
<td>(6)</td>
<td></td>
<td>Intermediate constraint codes. See Table 9-1. Preset to zero.</td>
</tr>
<tr>
<td>KDB</td>
<td>(30)</td>
<td></td>
<td>Parameter optimization indicator. Each location corresponds to a particular control parameter. (See Table 9-2 of this section. KDB(1)=1 denotes that the corresponding parameter is to be optimized. KDB(1)=0 denotes no optimization of that parameter. Preset to zero.</td>
</tr>
<tr>
<td>KDT</td>
<td>(10)</td>
<td></td>
<td>A companion matrix to KDB(1-10). Denotes the number of the thrust event from that in KDB(I) which is to be altered to hold tank limits. If the first four thrust events are to be optimized (i.e., their burn times) with</td>
</tr>
</tbody>
</table>
the tank limits held in thrust event
3 for the first two events and thrust event
8 for the second two, input KDB=1, 1,1,1,
and KDT=4,3,5,4. Preset to zero.

KIND
Denotes the type of integration desired.
KIND=1,2,3 denotes: variable step-size
Adams-Moulton, Runge-Kutta, or fixed step-
size Adams, respectively. KIND=1 is usually
preferable. Preset to one.

KP (2)
Denotes the order of the chi-pitch polynomi-
al desired for each stage. Preset to one.

KRDER
Order of differences in the integration
package. The preset value should be used.
(no input required). Preset to 3.

KY (2)
Same as KP but for yaw. Preset to zero.

LAST
Denotes only one data pack (case) to be
evaluated (LAST=1), or more than one (LAST=
number of cases). "reset to one.

LPRINT
Print option indicator. See Table 9-4.
Preset to zero.

MOMBAL
Moment balance option indicator. Preset
to zero. (MOMBAL = 0, no moment balance;
MOMBAL = 1, SRM's thrust through specified
point (SRXCGF,SRZCGF), liquid engines bal-
ance moments; MOMBAL = 2, SRM's track c.g.,
liquid engines balance moment; MOMBAL = 3,
all engines balance moments is collective
two engine equivalent model.)

MSWCH (15)
Thrust event cutoff indicator. Tells the
program which cutoff triggers are to be
turned on for each thrust event. See Table
9-3. Preset to 4 for each thrust event.

NCORD (15)
Coordinated turn option indicator per thrust
event. NCORD = 0, no coordinated turn;
NCORD = +1, coordinated turn with positive
angle of attack; NCORD = -1, coordinated
turn with negative angle of attack.

9-12
<table>
<thead>
<tr>
<th>NAME</th>
<th>DIMENSIONS</th>
<th>UNITS</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMAX</td>
<td></td>
<td></td>
<td>Maximum number of trajectory integrations allowed in constraint-zeroing portion of program. Preset to zero.</td>
</tr>
<tr>
<td>NÆEVNT</td>
<td>(5)</td>
<td></td>
<td>Number of thrust events in each stage, e.g., if there are two stages, with five thrust events in the first stage and two in the second, input NÆEVNT=5,2. Preset to zero.</td>
</tr>
<tr>
<td>NPARBN</td>
<td></td>
<td></td>
<td>Parallel burn indicator. Set NPARB=1 if parallel burn is desired, and place all fuel for orbiter engines into FUELQ(1) (if fuel cutoff is required). If parallel burn is not desired, fuel for liquid engines must be input in the first thrust event of each stage for which fuel cutoff is required.</td>
</tr>
<tr>
<td>NTABLE</td>
<td></td>
<td></td>
<td>Denotes output of tables and/or plots at end of converged run. Preset to zero. (NTABLE = 0, no post-processing is required; NTABLE = 1, output tables only; NTABLE = 2, output plots only, NTABLE = 3, output tables and plots.)</td>
</tr>
<tr>
<td>NVRST</td>
<td></td>
<td></td>
<td>Denotes the number of the thrust event at the end of which the intermediate constraints are to be imposed. Must be zero if no intermediate constraints are desired. Preset to zero.</td>
</tr>
<tr>
<td>NWIND</td>
<td></td>
<td></td>
<td>The number of points used in the wind tables WTBTL, AXWTBL, ALTTBL. If NWIND=0 no tables are used. Preset to zero.</td>
</tr>
<tr>
<td>ÔMEGA</td>
<td></td>
<td>RAD/SEC</td>
<td>Angular rotational velocity of the earth. The preset value should be used. (No input required).</td>
</tr>
<tr>
<td>ÔMSISP</td>
<td></td>
<td>SEC</td>
<td>Specific impulse of the orbital maneuvering system.</td>
</tr>
<tr>
<td>PNM</td>
<td>(25,2)</td>
<td></td>
<td>Mach numbers at which the values of the various aerodynamic coefficients and the alpha history in TALFP apply. Indices as in CA0. (Any number of points may be used for either stage.) Preset to zero.</td>
</tr>
<tr>
<td>PRINT</td>
<td>(15)</td>
<td>SEC</td>
<td>Print time increment for each thrust event during a long trajectory print. Preset to 10 seconds for each event.</td>
</tr>
<tr>
<td>NAME</td>
<td>DIMENSIONS</td>
<td>UNITS</td>
<td>EXPLANATION</td>
</tr>
<tr>
<td>-----------</td>
<td>------------</td>
<td>--------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>PSIREQ</td>
<td>(10)</td>
<td></td>
<td>Desired values of the end constraints. PSIREQ (1) corresponds to KCDPML(1+1). See Table 9-1. Preset to 7876.4195, .01, 6470762. (V, \gamma, and R for 50 &amp; 100 n mi orbit).</td>
</tr>
<tr>
<td>PSIRST</td>
<td>(6)</td>
<td></td>
<td>Desired values of the intermediate constraints. PSIRST(1) corresponds to KCDRES(1). Preset to zero.</td>
</tr>
<tr>
<td>QALPM</td>
<td></td>
<td>LB DEG/FT$^2$</td>
<td>Maximum allowed value of the product of dynamic pressure and angle-of-attack. If the $x_p$ history causes this value to be exceeded, alpha is reduced to hold the limit, overriding the $x_p$ polynomial. Preset to 1.E20.</td>
</tr>
<tr>
<td>NAME</td>
<td>DIMENSIONS</td>
<td>UNITS</td>
<td>EXPLANATION</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
<td>-------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>SRMTRP</td>
<td></td>
<td>LBM</td>
<td>Total SRM fuel available. Should be the same as the last data point in the SRDWTB array.</td>
</tr>
<tr>
<td>SRMFTB</td>
<td>(15)</td>
<td>SEC</td>
<td>Time points at which the thrust levels and delta weights in SRMFTB and SRDWTB apply (note: a thrust event should be created whenever the slope of the thrust-time curve is discontinuous.)</td>
</tr>
<tr>
<td>SRXCGF</td>
<td></td>
<td>M</td>
<td>Longitudinal coordinate of SRM aiming point for fixed SRM's (MONBAL=1)</td>
</tr>
<tr>
<td>SRXGP</td>
<td>(5)</td>
<td>M</td>
<td>Longitudinal coordinate of the SRM gimbal points.</td>
</tr>
<tr>
<td>SRYGP</td>
<td>(5)</td>
<td>M</td>
<td>Lateral coordinate of the SRM gimbal points.</td>
</tr>
<tr>
<td>SRZCGF</td>
<td></td>
<td>M</td>
<td>Vertical coordinate of SRM aiming point for fixed SRM's.</td>
</tr>
<tr>
<td>SRZGP</td>
<td>(5)</td>
<td>M</td>
<td>Vertical coordinate of the SRM gimbal points.</td>
</tr>
<tr>
<td>STEP</td>
<td>(15)</td>
<td>SEC</td>
<td>Integration step-size increment for each thrust event when Runge-Kutta or fixed step-size Adams-Moulton integration is used. Preset to 1 second for the first thrust event and 8 seconds thereafter.</td>
</tr>
<tr>
<td>SYRADB</td>
<td></td>
<td></td>
<td>Same as SPRADB but for yaw.</td>
</tr>
<tr>
<td>SYRADØ</td>
<td></td>
<td></td>
<td>Same as SPRADØ but for yaw.</td>
</tr>
<tr>
<td>TALFP</td>
<td>(25,2)</td>
<td>DEG</td>
<td>Angle-of-attack values corresponding to Mach numbers of PNMM, Preset to zero.</td>
</tr>
<tr>
<td>TAUT</td>
<td>(15)</td>
<td>SEC</td>
<td>Duration of each thrust event. If a thrust event time is being optimized, this value is used as the initial estimate. Preset to zero.</td>
</tr>
<tr>
<td>TBDWT</td>
<td>(15,2)</td>
<td>LBM</td>
<td>Delta weight values corresponding to the c.g. locations in TXCG and TZCG. Up to 15 points may be used for each stage. The second index denotes the stage.</td>
</tr>
<tr>
<td>TLIFT</td>
<td></td>
<td>SEC</td>
<td>Time at end of liftoff phase and the beginning of the $x_p$ and/or $x_y$ control program. Referenced to TZERØ. Preset to 8 seconds.</td>
</tr>
<tr>
<td>NAME</td>
<td>DIMENSIONS</td>
<td>UNITS</td>
<td>EXPLANATION</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>-------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>TNE</td>
<td>(4,15)</td>
<td></td>
<td>Number of liquid engines per thrust event (TNE(1,ITHR)), number of SRM engines per thrust event (TNE(2,ITHR)), and SRM pointing angles for fixed SRM's (TNE(364,1)). The latter two values are calculated internally and are not input requirements.</td>
</tr>
<tr>
<td>TØBL</td>
<td>(20)</td>
<td>SEC</td>
<td>Times, from staging, at which the $\chi_p$ and $\chi_y$ values in CPØTBL and CYØTBL apply. (TØBL (1-10), refer to $\chi_p$, TOBL (11-20) refer to $\chi_y$)</td>
</tr>
<tr>
<td>TTBL</td>
<td>(20)</td>
<td>SEC</td>
<td>Same as TØBL but for first stage and referenced from TLIFT.</td>
</tr>
<tr>
<td>TTILT</td>
<td></td>
<td>SEC</td>
<td>Time at end of programmed tilt-over from which zero-lift, zero angle-of-attack, or Mach number dependent alpha profile is to be flown. The programmed tilt should be accomplished using a linear $\chi_p$ (KP=-1) for the first stage. One value of CPITBL and TTBL should be input, and the tilt may or may not be optimized, according to the coding in KDB.</td>
</tr>
<tr>
<td>TXCG</td>
<td>(15,2)</td>
<td>M</td>
<td>Longitudinal coordinate of the center of gravity corresponding to delta weights in TBDWT. See Figure 9-2.</td>
</tr>
<tr>
<td>TZCG</td>
<td>(15,2)</td>
<td>M</td>
<td>Vertical coordinate of the center of gravity corresponding to delta weights in TBDWT. See Figure 9-2.</td>
</tr>
<tr>
<td>TZEROØ</td>
<td></td>
<td>SEC</td>
<td>Time at the start of the trajectory. For a normal launch TZEROØ is the time at lift-off. For a jump start TZEROØ is the time at the initiation of the trajectory.</td>
</tr>
<tr>
<td>VCRUS</td>
<td></td>
<td>FT/SEC</td>
<td>Booster flyback cruise velocity desired.</td>
</tr>
<tr>
<td>VISPB</td>
<td></td>
<td>SEC</td>
<td>Vacuum I$^\text{SP}$ of booster. Used in payload calculation.</td>
</tr>
<tr>
<td>VISPØ</td>
<td></td>
<td>SEC</td>
<td>Vacuum I$^\text{SP}$ of orbiter. Used in payload calculation.</td>
</tr>
<tr>
<td>VIV</td>
<td>(15)</td>
<td></td>
<td>Initial state for a jump start. If VIV(7)=0, the plumbine state vector, $W_u,U,V,X,Y,Z$ ($Z,X,Y,Z,X,Y$ in the Apollo 13) must be input</td>
</tr>
</tbody>
</table>
into VIV(1-6). If VIV(7)=2., the state variables \( V_i, Y, r, AZ \), geodetic latitude \( \phi \), and node \( \omega_i \) must be input into VIV(1-6).

If payload is required as a constraint VIV(8-10) should contain initial values of ideal velocity, back pressure loss, and gimbal loss, respectively.

<table>
<thead>
<tr>
<th>NAME</th>
<th>DIMENSIONS</th>
<th>UNITS</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRCUT</td>
<td>FT/SEC</td>
<td></td>
<td>Relative velocity at which the last thrust event of the first stage is to be cut off, if desired. Preset to zero.</td>
</tr>
<tr>
<td>WD(\dot{T})</td>
<td>(15)</td>
<td>LBM/SEC</td>
<td>Flowrate per engine for each thrust event.</td>
</tr>
<tr>
<td>W(\phi)</td>
<td>LBM</td>
<td></td>
<td>Liftoff weight at TZERO.</td>
</tr>
<tr>
<td>WPM</td>
<td>LBM</td>
<td></td>
<td>Maximum critical propellant in a stage from which performance reserves are taken. This value is assumed to include the performance reserves and must be input if IPR is greater than zero. Preset to zero.</td>
</tr>
<tr>
<td>WTBL</td>
<td>(25)</td>
<td>M/SEC</td>
<td>Wind speeds corresponding to altitude in ALTTBL and azimuth angles in AZWTBL.</td>
</tr>
<tr>
<td>WTJET</td>
<td>(15)</td>
<td>LBM</td>
<td>Jettison weights per thrust event. The weight jettison occurs at the end of the thrust event, such as releasing the empty first stage. Preset to zero.</td>
</tr>
<tr>
<td>W(L)(1)%</td>
<td>M</td>
<td></td>
<td>Landed weight of booster for flyback fuel calculation.</td>
</tr>
<tr>
<td>XGP</td>
<td>(15,2)</td>
<td>M</td>
<td>Longitudinal coordinate of engine thrust point location for each engine of each stage. See Figure 9-2.</td>
</tr>
<tr>
<td>XLEN</td>
<td>(2)</td>
<td>M</td>
<td>Aerodynamic reference length for each stage.</td>
</tr>
<tr>
<td>XREF</td>
<td>M</td>
<td></td>
<td>Longitudinal coordinates of the aerodynamic reference point. See Figure 9-2.</td>
</tr>
<tr>
<td>YGP</td>
<td>(15,2)</td>
<td>M</td>
<td>Lateral coordinate of engine thrust point location for each engine of each stage. See Figure 9-2.</td>
</tr>
<tr>
<td>ZGP</td>
<td>(15,2)</td>
<td>M</td>
<td>Vertical coordinate of engine thrust point locations for each engine of each stage. See Figure 9-2.</td>
</tr>
</tbody>
</table>

9-17
<table>
<thead>
<tr>
<th>NAME</th>
<th>DIMENSION</th>
<th>UNITS</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZREF</td>
<td>M</td>
<td></td>
<td>Vertical coordinate of the aerodynamic moment reference point. See Figure 9-2.</td>
</tr>
</tbody>
</table>
Table 9-1. CONSTRAINT CODES

The codes contained in this table are input into KCPHI and KCDRES to designate the payoff and the intermediate and terminal constraints desired for the trajectory. The appropriate values desired for these constraints must then be input into PSTREL and PSTSER.

<table>
<thead>
<tr>
<th>CODE NUMBER</th>
<th>UNITS</th>
<th>CONSTRAINT OR PAYOFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KG</td>
<td>Payload</td>
</tr>
<tr>
<td>2</td>
<td>M/SEC</td>
<td>Inertial velocity</td>
</tr>
<tr>
<td>3</td>
<td>DEG.</td>
<td>Inertial flight path angle</td>
</tr>
<tr>
<td>4</td>
<td>M</td>
<td>Radius</td>
</tr>
<tr>
<td>5</td>
<td>M^2/SEC^2</td>
<td>Energy</td>
</tr>
<tr>
<td>6</td>
<td>DEG.</td>
<td>Angular momentum</td>
</tr>
<tr>
<td>7</td>
<td>DEG.</td>
<td>Inertial longitude</td>
</tr>
<tr>
<td>8</td>
<td>DEG.</td>
<td>Inertial heading angle (+ East from South)</td>
</tr>
<tr>
<td>9</td>
<td>DEG.</td>
<td>Colatitude</td>
</tr>
<tr>
<td>10</td>
<td>DEG.</td>
<td>Inclination</td>
</tr>
<tr>
<td>11</td>
<td>DEG.</td>
<td>Line of nodes</td>
</tr>
<tr>
<td>12</td>
<td>M</td>
<td>Semi-latus rectum</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>Eccentricity</td>
</tr>
<tr>
<td>14</td>
<td>SEC</td>
<td>Burn Time</td>
</tr>
<tr>
<td>15</td>
<td>LB/FT^2</td>
<td>Maximum dynamic pressure</td>
</tr>
<tr>
<td>16</td>
<td>DEG.</td>
<td>True anomaly</td>
</tr>
<tr>
<td>17</td>
<td>DEG.</td>
<td>Argument of perigee</td>
</tr>
<tr>
<td>18</td>
<td>DEG.</td>
<td>Not used</td>
</tr>
<tr>
<td>19</td>
<td>DEG.</td>
<td>Not used</td>
</tr>
<tr>
<td>20</td>
<td>NM</td>
<td>Flyback range</td>
</tr>
</tbody>
</table>

Table 9-2. OPTIMIZATION CODING

The KDB locations listed below will provide optimization of the corresponding parameter when the location contains a 1. For no optimization of given parameter, the parameter is either held at a constant value or is determined as a consequence of other parameters. All KBD locations contain zero unless input otherwise.

<table>
<thead>
<tr>
<th>KDB LOCATION</th>
<th>UNITS</th>
<th>CONTROL PARAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>SEC</td>
<td>TAUT(1-10) (burn time)</td>
</tr>
<tr>
<td>11</td>
<td>KG</td>
<td>W01 (liftoff weight)</td>
</tr>
<tr>
<td>12</td>
<td>DEG.</td>
<td>AZ (launch azimuth)</td>
</tr>
<tr>
<td>13</td>
<td>DEG.</td>
<td>CPTBL(2) Booster pitch attitude at time TTBL(2)</td>
</tr>
<tr>
<td>14</td>
<td>DEG.</td>
<td>CPTBL(3) Booster pitch attitude at time TTBL(3)</td>
</tr>
<tr>
<td>15</td>
<td>DEG.</td>
<td>CPTBL(4) Booster pitch attitude at time TTBL(4)</td>
</tr>
<tr>
<td>16</td>
<td>DEG.</td>
<td>CPTBL(5) Booster pitch attitude at time TTBL(5)</td>
</tr>
</tbody>
</table>
### Table 9-3. THRUST EVENT CUTOFF OPTIONS

The table below lists the triggers that are turned on for a particular thrust event according to the value of MSWCH(ITHR), where ITHR is the number of the thrust event. The thrust event will be terminated when the trajectory reaches the first trigger value selected for that event, except for the g-limit trigger when used with continuous throttling, in which case the trigger (if it is the first trigger reached during the thrust event) will cause only a print of the state at the time when the g-limit was reached. The thrust event will continue then until the next trigger is reached, with the thrust adjusted so that the g-limit is held for the remainder of the thrust event.

The absolute value of MSWCH is used to denote the triggers to be turned on, while the algebraic sign tells the program whether to use continuous (+) or discrete (-) throttling at the g-limit.

<table>
<thead>
<tr>
<th>ABS(MSWCH(ITHR))</th>
<th>TRIGGER USED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>TIME</td>
</tr>
<tr>
<td>1</td>
<td>TIME, G-LIMIT</td>
</tr>
<tr>
<td>2</td>
<td>TIME, VSUBR</td>
</tr>
<tr>
<td>3</td>
<td>TIME, G-LIMIT, VSUBR</td>
</tr>
<tr>
<td>4</td>
<td>FUEL</td>
</tr>
<tr>
<td>5</td>
<td>FUEL, G-LIMIT</td>
</tr>
<tr>
<td>6</td>
<td>FUEL, VSUBR</td>
</tr>
<tr>
<td>7</td>
<td>FUEL, G-LIMIT, VSUBR</td>
</tr>
</tbody>
</table>

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Table 9-4. USE OF LPRINT OPTION

<table>
<thead>
<tr>
<th>BASIS FOR PARAMETER UPDATE</th>
<th>LPRINT = 2</th>
<th>LPRINT = 1</th>
<th>LPRINT = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint Restoration Step</td>
<td>Trajectory Block Output</td>
<td>Trajectory Block Output</td>
<td>Trajectory Block Output</td>
</tr>
<tr>
<td></td>
<td>The trajectory block is printed for every iteration*.</td>
<td>Only first and final trajectory blocks are printed. The weight summary table is provided at the end of the restoration step printout.</td>
<td>Same as LPRINT = 1</td>
</tr>
<tr>
<td>Optimization Procedure Output</td>
<td>Optimization Procedure Output</td>
<td>Optimization Procedure Output</td>
<td>Optimization Procedure Output</td>
</tr>
<tr>
<td></td>
<td>Output formats 1 thru 6</td>
<td>Same as LPRINT = 2</td>
<td>Output formats 1, 5, and 6</td>
</tr>
<tr>
<td>PAYOFF OPTIMIZATION STEP</td>
<td>Trajectory Block Output</td>
<td>Trajectory Block Output</td>
<td>Trajectory Block Output</td>
</tr>
<tr>
<td></td>
<td>Same as above</td>
<td>Only first and final trajectory blocks are printed. The weight summary table is printed at the end of the optimization step.</td>
<td>Same as LPRINT = 1</td>
</tr>
<tr>
<td>Optimization Procedure Output</td>
<td>Optimization Procedure Output</td>
<td>Optimization Procedure Output</td>
<td>Optimization Procedure Output</td>
</tr>
<tr>
<td></td>
<td>Output formats 1 thru 8</td>
<td>Same as LPRINT = 2</td>
<td>Output formats 1, 5, 6, 7, &amp; 8</td>
</tr>
</tbody>
</table>

* An iteration is defined as a permanent parameter update.
9.3 DETAILED OUTPUT DESCRIPTION

The RAGMOP output is divided into two groups; 1) the trajectory output, which displays the running account of the restoration and optimization procedure, and 2) postprocessor output in the form of publishable output tables and plots. This subsection describes the output of the trajectory and the input requirements and the output of the postprocessor modules.

9.3.1 Trajectory Output

The RAGMOP trajectory output can be classified into the following three general areas:

- **Namelist Input.** All Namelist data is printed describing the current status of each variable in the Namelist input array. Included with the Namelist, immediately preceding the first Namelist variable, the locations (in meters) of the equivalent engines used in the moment balance scheme for each stage are printed for reference.

- **Trajectory Block Output.** Variation of the equations of motion as well as other pertinent data is printed by a block printout based on print interval, discrete events, or discontinuities in the equations of motion. A page of output consists of a header row printed at the top of the page (containing the case number, the HEAD input card hollerith field, and the page number) and four block printouts (see Figure 9-3). If the block format is describing an event, the block will have displayed to its left one of the following captions:
  
  LIFT\text{OFF} - initiation of the trajectory from the launch site.
  
  IGNITION - jump start.
  
  THRUST\_EVENT - specifies the beginning of a thrust event.
  
  END\_TILT - specifies beginning of angle-of-attack control history used only if FACT>0).
  
  10\_KMS - 10 kilometers altitude has been reached.
  
  14\_KMS - 14 kilometers altitude has been reached.
  
  Q\_MAXIMUM - maximum value of dynamic pressure.
  
  MACH\_ONE - the value MACH = 1 has been reached.
  
  BEGIN\_GLIMIT - specifies the beginning of either a discrete or continuous throttling event.
  
  INJECTION - intermediate and terminal trajectory constraints are imposed at these time points.

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The block printout consists of eleven lines of output having the following labeled mnemonics:

**Line 1**
- **TIME** - time from liftoff (sec)
- **RANGE** - relative range from the launch site to vehicle subpoint (n mi)
- **RNGAN** - inertial range angle between the launch site and the vehicle radius vector (deg)
- **THRST** - total vehicle thrust (lbs)
- **XMLB** - total weight (lbs)
- **LACC** - longitudinal acceleration (g's)

**Line 2**
- **Z13-X** - Apollo 13 Z position coordinate, RAGMOP X coordinate (m).
- **X13-Y** - Apollo 13 X position coordinate, RAGMOP Y coordinate (m).
- **Y13-Z** - Apollo 13 Y position coordinate, RAGMOP Z coordinate (m).
- **Z013X** - Apollo 13 Z velocity coordinate, RAGMOP X coordinate (m/sec).
- **XD13Y** - Apollo 13 X velocity coordinate, RAGMOP Y coordinate (m/sec).
- **YD13Z** - Apollo 13 Y velocity coordinate, RAGMOP Z coordinate (m/sec).

**Line 3**
- **R** - radius from center of earth (m)
- **VSUBI** - inertial velocity (m/sec)
- **GAMI** - inertial flight path angle (deg)
- **AZI** - inertial azimuth (deg)
- **LAT** - geocentric latitude (deg)
- **LONG** - relative longitude (deg)

**Line 4**
- **ALT** - altitude above ellipsoid (m).
- **VSUBR** - relative velocity (m/sec).
- **GAMR** - relative flight path angle (deg)
- **AZR** - relative azimuth (deg)
- **NØDS** - inertial descending node (deg)
- **INCL** - inclination (deg)
Line 5
GDLAT - geodetic latitude (deg)
LTIMP - latitude of the instantaneous impact point (deg)
LNGMP - longitude of the instantaneous impact point (deg)
CHIP - pitch attitude (deg)
CHIY - yaw attitude (deg)
CHIR - roll attitude (deg)

Line 6
MACH - Mach number (unitless)
ALPHA - angle of attack (deg)
BETA - sideslip angle (deg)
DELPC - pitch gimbal command (deg)
DELYC - yaw gimbal command (deg)
DELRC - roll gimbal command (set=0)(deg)

Line 7
Q - dynamic pressure (lbs/ft^2)
QALPH - product of dynamic pressure and angle of attack (lbs deg/ft^2)
QBETA - product of dynamic pressure and sideslip angle (lbs deg/ft^2)
PIT M - total pitch moment (newton-m)
YAW M - total yaw moment (newton-m)
ROLL M - total roll moment (newton-m)

Line 8
FAA - aerodynamic axial force (lbs)
FAS - aerodynamic side force (lbs)
FAN - aerodynamic normal force (lbs)
WDOT - propellant flowrate of liquid engines (lbs/sec)
TORB - thrust of the orbiter engines (lbs)
DWORB - weight loss of the orbiter engines (lbs)
Line 9
QCBl - product of dynamic pressure and the aerodynamic normal force coefficient (lbs/ft²)
VWIND - velocity of the input wind (m/sec)
AZW - direction to which the wind is blowing (deg)
ISP - liquid engine specific impulse (sec)
TSRM - thrust of the SRM engines (lbs)
DWSRM - weight loss of the SRM engines (lbs)

Line 10
CH.VL - characteristic velocity (m/sec)
TN.L - turning loss (m/sec)
GV,L - gravitational loss (m/sec)
DR.L - drag loss (m/sec) (See Appendix F)
BKP.L - back pressure loss (m/sec)
GIM.L - gimbal loss (m/sec)

Line 11
ID.VL - ideal velocity (see Appendix F)
GIMAN - pitch gimbal angle of two engine equivalent for zero aerodynamic or SRM moments
Figure 9-3. TRAJECTORY BLOCK PRINTOUT
After completion of the trajectory simulation, a second block of data is output which presents a weight summary of the subsystems of the vehicle. These weights are used in the definition of the payload weight. This block is shown in Figure 9-4. The variables are defined as follows:

- **WPMB** - orbiter propellant burned (lb)
- **CHVEL** - characteristic velocity (ft/sec)
- **DELVP** - delta velocity for performance reserves (ft/sec)
- **FPR** - flight performance reserves (lb)
- **WRESID** - computed residual weight (lb)
- **CRSDWT** - constant residual weight (lb)
- **WTANK** - computed tank weight (lb)
- **CTNKWT** - constant tank weight (lb)
- **WDROP** - drop weight (lb)
- **WPOMS** - OMS propellant weigh. (lb)
- **PAYLOD** - payload (lb) (the value will be negative when payload is the payoff since maximization is performed by minimizing a negative number.)

![Table](image)

**Figure 9-4. PAYLOAD WEIGHT BLOCK**

A trajectory constraint summary table is then printed which presents the payoff, the error in the desired equality constraints in the present and previous trajectories, the amount of change requested from the previous run, the amount of change obtained, and the percentage of predictability (see Figure 9-5).

![Table](image)

**Figure 9-5. TRAJECTORY CONSTRAINT SUMMARY TABLE**
Optimization Procedure Output. Following the trajectory output are several variables, vectors, and matrices used in the optimization procedure. The input variable LPRINT however will limit the amount of printout obtained from this area. Table 9-4 lists the output obtained using the various LPRINT options. To separate the various outputs a number will be used which will agree with the nomenclature of Table 9-4.

1) Influence Coefficients - This block of output provides the user with the various partial derivatives of the constraints with respect to the optimization parameters. The size of this output block is a function of the number of equality constraints (left to right on the page) and of the number of optimization parameters (top to bottom). The header "FORWARD DIFFERENCES" or "CENTRAL DIFFERENCES" relates to the method used for determining these numerical partial derivations (see Figure 9-6).

<table>
<thead>
<tr>
<th>CENTRAL DIFFERENCES</th>
<th>FORWARD DIFFERENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.12947457*00</td>
<td>+.10880500*01</td>
</tr>
<tr>
<td>+.00000000</td>
<td>-.00000000</td>
</tr>
<tr>
<td>-.15247460*01</td>
<td>+.13147567*01</td>
</tr>
<tr>
<td>-.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>-.12437460*01</td>
<td>+.16537567*01</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>-.11437460*01</td>
<td>+.12437567*01</td>
</tr>
<tr>
<td>-.00000000</td>
<td>-.00000000</td>
</tr>
<tr>
<td>-.10437460*01</td>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>-.11637460*01</td>
<td>+.19137567*01</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>-.11537460*01</td>
<td>+.12437567*01</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>-.10637460*01</td>
<td>+.12437567*01</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>-.11737460*01</td>
<td>+.12437567*01</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
</tbody>
</table>

Figure 9-6. INFLUENCE COEFFICIENTS TABLE

2) Weighting Matrix (HWIBT) - Identifies the weighting matrix used in the optimization procedure to normalize the various parameters (consult Section III) (See Figure 9-7).

<table>
<thead>
<tr>
<th>WEIGHTING MATRIX</th>
<th>HWIBT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00000000</td>
<td>.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
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<tr>
<td>+.00000000</td>
<td>+.00000000</td>
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<td>+.00000000</td>
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<td>+.00000000</td>
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<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
<td>+.00000000</td>
</tr>
</tbody>
</table>

Figure 9-7. WEIGHTING MATRIX OUTPUT

3) Total I-SY-SY Matrix - Matrix products used in the optimization procedure (consult Section III) (see Figure 9-8).

<table>
<thead>
<tr>
<th>TOTAL I-SY-SY MATRIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2486734*03</td>
</tr>
<tr>
<td>+.0827926*01</td>
</tr>
<tr>
<td>+.22840255*01</td>
</tr>
<tr>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
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<tr>
<td>+.00000000</td>
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</tr>
<tr>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
</tr>
<tr>
<td>+.00000000</td>
</tr>
</tbody>
</table>

Figure 9-8. TOTAL I-SY-SY MATRIX OUTPUT
4) **DRHΦ, WDS, GNU** - The variables are defined in the following manner (see Figure 9-9):

DRHΦ - equality constraint errors used in parameter update equation.

WDS - product of total I-SY-SY matrix and DRHΦ vector.

GNU - matrix product used in optimization procedure.

<table>
<thead>
<tr>
<th>DRHΦ, WDS, GNU</th>
</tr>
</thead>
<tbody>
<tr>
<td>935.00515001</td>
</tr>
<tr>
<td>172.93319004</td>
</tr>
<tr>
<td>354.99429004</td>
</tr>
<tr>
<td>296.93568001</td>
</tr>
</tbody>
</table>

Figure 9-9. DRHO, WDS, GNU OPTIMIZATION PARAMETERS

5) **E1, E2** - Values of the restoration and optimization control step variables E2 and E1 (see Figure 9-10)

| E1 = 0.00000000 | E2 = 1.00000000 |

Figure 9-10. CONTROL STEP VARIABLES PRINTOUT

6) **Parameter update and convergence test block** - The following variables are printed in this block (see Figure 9-11).

DPAR - optimization parameter update requirement (one per parameter).

OLD.PC0N - post convergence factor for parameters (one per parameter).

NEW.PC0N - present convergence factor for parameter (one per parameter).

P.SCALE - convergence parameter normalization factor (one per parameter).

TAUT(I) - if time is being optimized, prints out updated time.

W01 - if liftoff weight is being optimized, prints out updated weight

AZ - if launch azimuth is being optimized, prints out updated launch azimuth.

CPTBL(I) - if the booster pitch attitude is being optimized, prints out updated values.

CYTBL(I) - if the booster yaw attitude is being optimized, prints out updated values.

CP0TBL(I) - if the orbiter pitch attitude is being optimized, prints out updated values.

CY0TBL(I) - if the orbiter yaw attitude is being optimized, prints out updated values.

BETO0N - convergence indicator (BETO0N=T - all convergence criteria have met in subroutine ANEWCH, BETO0N=F - nonconvergence)
7) The QY (or El) search output - after the program has met the terminal conditions, the optimization procedure starts by varying the value of the control step El (Section III refers to this parameter as QY). When performing this search several additional parameters are printed (see Figure 9-12).

TQSLP - the slope of the performance index with respect to the control step variable El.

NSRCH - indicates the number of points in the QY search.

COMPOSITE.PAYOFF INDEX - total derivative of the constrained payoff.

QY - value of the control step used.

PREDICTED PAYOFF INDEX - the estimated value of the payoff index using the coefficients of the quadratic or cubic fits.

QUADRATIC.FIT.PAYOFF.QY - value of QY predicted by the quadratic approximation.

CUBIC.FIT.PAYOFF.QY - value of QY predicted by the cubic approximation.

8) After the converged trajectory has been printed, the following output will occur:

Orbital element summary table - the following injection variables are printed (See Figure 9-13).

TIME - time (sec).

RANGE - relative range (m).

RANGE ANGLE - relative range angle (deg).

INERTIAL VELOCITY - inertial velocity (m/sec).

RADIUS - radius (m).

FLIGHT_PATH ANGLE - inertial flight-path angle (deg).

INCL - inclination (deg).
Figure 9-12. QY SEARCH OUTPUT
<table>
<thead>
<tr>
<th>Time</th>
<th>Range</th>
<th>Range Angle</th>
<th>Flight Path Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.242321846</td>
<td>1.5539488078</td>
<td>1.6818860773</td>
<td>-2.7903449887</td>
</tr>
<tr>
<td>Inertial Velocity</td>
<td>Radius</td>
<td>DSE</td>
<td>Flight DSE</td>
</tr>
<tr>
<td>Inclination</td>
<td>DSE</td>
<td>Flight DSE</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
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</tr>
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<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
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</tr>
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<td></td>
</tr>
<tr>
<td>999.999999</td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 9-13. ORBITAL ELEMENT SUMMARY TABLE
DES NODE - descending node (deg).
FLIGHT AZIMUTH - launch azimuth (deg).
GDLAT - geodetic latitude (deg).
GCLA - geocentric latitude (deg).
LØNG - longitude (deg).
INERTIAL AZIMUTH - inertial azimuth (deg).
C1 - angular momentum magnitude (m²/sec).
WEIGHT - mass (lb).
C3 - vis viva integral (twice the energy) (m²/sec²).
ECNTRICY - orbital eccentricity (unitless).
APØGEE.RADIUS - radius of apoee (m).
PERIGEE.RADIUS - radius of perigee (m).
APØGEE. ALTITUDE - apoee altitude above ellipsoid (n mi).
PERIGEE. ALTITUDE - perigee altitude above ellipsoid (n mi).

After the orbital element summary table the parameter set that yields the converged trajectory is printed (see Figure 9-14).

Parameter summary table -- The values of all parameters, whether optimized or fixed, are printed for reference.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>401</td>
<td>.6000000000</td>
</tr>
<tr>
<td>402</td>
<td>.6000000000</td>
</tr>
<tr>
<td>403</td>
<td>.6000000000</td>
</tr>
<tr>
<td>404</td>
<td>.6000000000</td>
</tr>
<tr>
<td>405</td>
<td>.6000000000</td>
</tr>
<tr>
<td>406</td>
<td>.6000000000</td>
</tr>
<tr>
<td>407</td>
<td>.6000000000</td>
</tr>
<tr>
<td>408</td>
<td>.6000000000</td>
</tr>
<tr>
<td>409</td>
<td>.6000000000</td>
</tr>
<tr>
<td>410</td>
<td>.6000000000</td>
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<td>411</td>
<td>.6000000000</td>
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<tr>
<td>412</td>
<td>.6000000000</td>
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<td>413</td>
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<td>414</td>
<td>.6000000000</td>
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<td>419</td>
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</tr>
<tr>
<td>420</td>
<td>.6000000000</td>
</tr>
</tbody>
</table>

Figure 9-14. CHI PARAMETER VALUES

The last printout of the converged trajectory is the comment,

"TELL EM RAGMOP DID IT".

9-33
9.3.2 Postprocessor Option

The user has the option using the variable NTABLE to require the execution of the postprocessor modules BOPTIL and BOPTBL. The following paragraphs describe both the input requirements and the output for use of these modules.

9.3.2.1 BOPTBL Input/Output. The selection of the first or third option under the input integer variable NTABLE indicates the use of the output report table and subroutine, BOPTBL. A second Namelist input package, $INPUT2, is required after the trajectory input, $INPUT. The definitions for this Namelist package are described in Table 9-5.

Table 9-5. BOPTBL INPUT VARIABLES

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DIMENSION</th>
<th>DEFINITION</th>
<th>PRESET VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DATE</td>
<td>2*</td>
<td>Hollerith Field for calendar data</td>
<td>BLANK</td>
</tr>
<tr>
<td>NCASE</td>
<td>1</td>
<td>Case number desired on output</td>
<td>0</td>
</tr>
<tr>
<td>OFFICE</td>
<td>2*</td>
<td>Hollerith Field for Office Identification</td>
<td>BLANK</td>
</tr>
<tr>
<td>SRID</td>
<td>60*</td>
<td>Hollerith Field for optional thrust event notation. One notation requires 2 fields. (Replaces the header &quot;THRUST EVENT&quot; with data found in SRID)</td>
<td>BLANK</td>
</tr>
<tr>
<td>TITLE</td>
<td>8*</td>
<td>Hollerith Field for desired table</td>
<td>BLANK</td>
</tr>
</tbody>
</table>

*A six letter Hollerith field makes up one dimension length.

Output for subroutine BOPTBL consists of fifteen tables suitable for publication. The first seven tables give the output variables in the MKS system, the next seven tables give the same variables in the English system, and the last table is a table of contents and a list of definitions for the variables in the various tables. Presented in Figure 9-15 are examples of the output tables one through fourteen. Table fifteen is shown first to describe the location and definition of the variables. Also shown is a data setup for the sample problem.
Table 9-15. BOPTBL OUTPUT

### Definitions and Symbols for Trajectory Tables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Table</th>
<th>Units</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tinel</td>
<td>ALL</td>
<td>SECONDS</td>
<td>Instantaneous Time from Lift-off</td>
</tr>
<tr>
<td>R</td>
<td>1</td>
<td>METERS</td>
<td>Instantaneous Radius from Center of Earth</td>
</tr>
<tr>
<td>VI</td>
<td>1</td>
<td>M/SEC</td>
<td>Inertial Velocity</td>
</tr>
<tr>
<td>#</td>
<td>8</td>
<td>S/SEC</td>
<td>Inertial Acceleration</td>
</tr>
<tr>
<td>GAMMA</td>
<td>1.8</td>
<td>DEGREES</td>
<td>Inertial Flight Path Angle</td>
</tr>
<tr>
<td>LAT-DO</td>
<td>1.8</td>
<td>DEGREES</td>
<td>Geodetic Latitude</td>
</tr>
<tr>
<td>LONG</td>
<td>1.8</td>
<td>DEGREES</td>
<td>Relative Longitude</td>
</tr>
<tr>
<td>AZI</td>
<td>1.8</td>
<td>DEGREES</td>
<td>Inertial Azimuthal Measurement of VI in Local Horizontal Plane</td>
</tr>
<tr>
<td>LAT</td>
<td>2</td>
<td>METERS</td>
<td>Instantaneous Altitude Above Reference Ellipsoid</td>
</tr>
<tr>
<td>INC</td>
<td>2.9</td>
<td>DEGREES</td>
<td>Instantaneous Inclination</td>
</tr>
<tr>
<td>NODE</td>
<td>2.9</td>
<td>DEGREES</td>
<td>Angular Measurement of the Descending Node from the Launch Meridian</td>
</tr>
<tr>
<td>VR</td>
<td>2</td>
<td>M/SEC</td>
<td>Relative Velocity</td>
</tr>
<tr>
<td>#</td>
<td>9</td>
<td>S/SEC</td>
<td>Relative Acceleration</td>
</tr>
<tr>
<td>GAMMA</td>
<td>2.9</td>
<td>DEGREES</td>
<td>Relative Flight Path Angle</td>
</tr>
<tr>
<td>AZR</td>
<td>2.9</td>
<td>DEGREES</td>
<td>Relative Azimuth Angle</td>
</tr>
<tr>
<td>MACH</td>
<td>3.10</td>
<td>------</td>
<td>MACH Number</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>M/SEC</td>
<td>Dynamic Pressure</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>LBS/FT²</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>DEGREES</td>
<td>Angle of Attack Measured in Vehicle Flight Plane</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>LBS/FT²</td>
<td></td>
</tr>
<tr>
<td>CALPHA</td>
<td>3</td>
<td>DEGREES</td>
<td>Sideslip Angle (Lateral Angle of Attack)</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>LBS/FT²</td>
<td></td>
</tr>
<tr>
<td>QELTA</td>
<td>3</td>
<td>DEGREES</td>
<td>Product of 0 and ALPHA</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>LBS/FT²</td>
<td></td>
</tr>
<tr>
<td>THRST</td>
<td>4</td>
<td>NETOMS</td>
<td>Instantaneous Thrust</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>LBS</td>
<td></td>
</tr>
<tr>
<td>M11AS</td>
<td>4</td>
<td>KILOGRAMS</td>
<td>Instantaneous Mass</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>LBS</td>
<td></td>
</tr>
<tr>
<td>W11GT</td>
<td>4</td>
<td>NETOMS</td>
<td>Aerodynamic Axial Force</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>LBS</td>
<td></td>
</tr>
<tr>
<td>W11GT</td>
<td>4</td>
<td>NETOMS</td>
<td>Aerodynamic Normal Force</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>LBS</td>
<td></td>
</tr>
<tr>
<td>SIDE11GT</td>
<td>4</td>
<td>NETOMS</td>
<td>Aerodynamic Side Force</td>
</tr>
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<td>11</td>
<td></td>
<td>LBS</td>
<td></td>
</tr>
<tr>
<td>L11ONG</td>
<td>4.11</td>
<td>G/SEC</td>
<td>Longitudinal Acceleration</td>
</tr>
<tr>
<td>CH11R</td>
<td>5.12</td>
<td>DEGREES</td>
<td>Inertial Roll Attitude Angle</td>
</tr>
<tr>
<td>CHIP</td>
<td>5.12</td>
<td>DEGREES</td>
<td>Inertial Pitch Attitude Angle</td>
</tr>
</tbody>
</table>

Figure 9-15. BOPTBL OUTPUT
### Figure 9-15. BOPTBL OUTPUT (Continued)

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>UNITS</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMIP</td>
<td>5,12</td>
<td>DEGREES INERTIAL YAW ATTITUDE ANGLE</td>
</tr>
<tr>
<td>DFLRC</td>
<td>5,12</td>
<td>DEGREES ROLL THRUST GIMBAL COMMAND</td>
</tr>
<tr>
<td>DELPC</td>
<td>5,12</td>
<td>DEGREES PITCH THRUST GIMBAL COMMAND</td>
</tr>
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<td>DELYC</td>
<td>5,12</td>
<td>DEGREES YAW THRUST GIMBAL COMMAND</td>
</tr>
<tr>
<td>RANGE</td>
<td>6</td>
<td>KILOMETERS RELATIVE SURFACE RANGE FROM INSTANTANEOUS LAUNCH POINT TO THE LAUNCH POINT</td>
</tr>
<tr>
<td>RANGE ′WOLE</td>
<td>6,13</td>
<td>DEGREES RELATIVE RANGE ANGLE</td>
</tr>
<tr>
<td>TIP LAT</td>
<td>6,13</td>
<td>DEGREES INSTANTANEOUS IMPACT POINT LATITUDE</td>
</tr>
<tr>
<td>TIP LONG</td>
<td>6,13</td>
<td>DEGREES INSTANTANEOUS IMPACT POINT LONGITUDE</td>
</tr>
<tr>
<td>VCH</td>
<td>m/sec</td>
<td>CHARACTERISTIC VELOCITY</td>
</tr>
<tr>
<td>VIDEA</td>
<td>m/sec</td>
<td>IDEAL VELOCITY</td>
</tr>
<tr>
<td>X</td>
<td>m</td>
<td>APOLLO 13 POSITION X COORDINATE</td>
</tr>
<tr>
<td>Y</td>
<td>m</td>
<td>APOLLO 13 POSITION Y COORDINATE</td>
</tr>
<tr>
<td>Z</td>
<td>m</td>
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</tr>
<tr>
<td>ZDOT</td>
<td>m/sec</td>
<td>APOLLO 13 VELOCITY Z COORDINATE</td>
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</tbody>
</table>
### Table 1

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<tr>
<th>TIME</th>
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<th>VT</th>
<th>GAMMA</th>
<th>L</th>
<th>D</th>
<th>E</th>
<th>D</th>
<th>E</th>
<th>LIFT-OFF</th>
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<tbody>
<tr>
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<tr>
<td>2.00</td>
<td>6373.746</td>
<td>408.3</td>
<td>-1.240</td>
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<td>-1.556</td>
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<tr>
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<td>408.3</td>
<td>2.235</td>
<td>28.661</td>
<td>8.556</td>
<td>89.999</td>
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<td></td>
<td></td>
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<tr>
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<td>408.3</td>
<td>3.703</td>
<td>28.661</td>
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<td>28.661</td>
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<td>89.999</td>
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### Table 2

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**Figure 9-15.** BOPTBL OUTPUT (Continued)
### TABLE 1

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<tr>
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<tr>
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### TABLE 2

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### TABLE 3

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</tr>
<tr>
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<td>318.0</td>
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Figure 9-15. BOPTBL OUTPUT (Continued)
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<tr>
<th>TIME</th>
<th>RAMP</th>
<th>DORP</th>
<th>BETA</th>
<th>GOLPHA</th>
<th>OBEIA</th>
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<tr>
<td>SEC</td>
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<td>DEG</td>
<td>DEG</td>
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<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.127</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.127</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>0.00</td>
<td>0.127</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Figure 9-15.** B0PTBL OUTPUT (Continued)
9.3.2.2. **BOPPLT Input/Output.** The selection of the second or third option under integer variable NTABLEL indicates the use of the output CALCOMP plotting subroutine, BOPPLT. If the second option is used, the BOPPLT subroutine is used and required input for this subroutine directly follows the trajectory input; however, if the third option is used, the BOPPLT input follows the BOPPTBL input. The input for the BOPPLT subroutine consists of a series of formatted input cards. These formatted input cards will be described as a series of card sets.

**Card Set 1.** (13A6/13A6/A6) The first card set consists of three cards making up 27 Hollerith fields. (The first two cards consist of 13 hollerith fields each and the third card has one hollerith field, all start in column 1). This card set is used for one-way communication between the user and the operator of the CALCOMP plotter. (An example of this communication would be to tell the operator where to send the plot, what pen to use for the plotter, and what color ink to use.)

**Card Set 2.** (4A6,2E10.5,4A6,E10.5,I2) This card set will be referred to as the plot card since the information on this card is used to determine what will be plotted.

The input card is made up of a combination of alphanumeric and numeric words which are used in the description of the required plot. Alphanumeric words on the plot card are used in the label of the axis on the CALCOMP plotter. Mnemonics corresponding to this card set are INAME(1-4), XFAC, XMAX, INAME(5-8), YFAC, N and are defined as follows:

- **INAME(1)** - the alphanumeric name of the X variable required for plotting (see Table 9-6 for options)
- **INAME(2)** - the alphanumeric name of the units of the X variable required for plotting (see Table 9-7 for options)
- **INAME(3-4)** - additional alphanumeric words required by user on X axis label (optional)
- **XFAC** - input multiplication factor (optional, if less than or equal to zero, XFAC is set to one)
- **XMAX** - The maximum value of the X component required for plotting (optional, set to 1.E + 20 if input less than or equal to zero)
- **INAME(5)** - the alphanumeric name of the Y variable required for plotting (see Table 9-6 for options)
INAME(6) - the alphanumeric name of the units of the Y variable required for plotting (see Table 9-7 for options)

INAME(7-8) - additional alphanumeric words required by user on Y axis label. (optional)

N - the number of label cards to follow the plot used.

<table>
<thead>
<tr>
<th>Table 9-6. ALPHANUMERIC PARAMETER WORDS FOR BOPPLT INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHA</td>
</tr>
<tr>
<td>ALT</td>
</tr>
<tr>
<td>AZI</td>
</tr>
<tr>
<td>AZR</td>
</tr>
<tr>
<td>AZW</td>
</tr>
<tr>
<td>BETA</td>
</tr>
<tr>
<td>BKP.L</td>
</tr>
<tr>
<td>CHIP</td>
</tr>
</tbody>
</table>

See General Program Output, Section 9.3 for definitions of parameters.

<table>
<thead>
<tr>
<th>Table 9-7. ALPHANUMERIC UNIT WORDS FOR BOPPLT INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPUT WORD</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>_____</td>
</tr>
<tr>
<td>BTU/FT</td>
</tr>
<tr>
<td>CAL/MM*</td>
</tr>
<tr>
<td>DEGREE</td>
</tr>
<tr>
<td>FEET</td>
</tr>
<tr>
<td>FT/SEC</td>
</tr>
<tr>
<td>G'S</td>
</tr>
<tr>
<td>KILOGR</td>
</tr>
<tr>
<td>KILOME</td>
</tr>
<tr>
<td>LB/FT*</td>
</tr>
<tr>
<td>LB-FT/</td>
</tr>
</tbody>
</table>

9-45
It is important that the user be careful in his selection of the alphanumeric words since the subroutine will only recognize the words of Tables 9-6 and 9-7. If the user does misspell a word, the subroutine writes out the comment "______ IS NOT A RECOGNIZABLE PARAMETER WORD, CONSULT USER'S GUIDE/ THIS PLOT HAS BEEN ABORTED" if the word is a parameter, and "______ IS NOT A RECOGNIZABLE UNIT WORD, CONSULT USER'S GUIDE/ THIS PLOT HAS BEEN ABORTED" if the word is a unit. If either word is in error, the present plot is aborted and the next plot card is read.

The plot card can also be used for two additional purposes; 1) a comment can be used for identification by placing a "C" in column one with five spaces (see subsection 9.4.3). The rest of the card (other than the numeric portion) can be used for alphanumeric words. A comment card, when read is printed and ignored. 2) A plot card can also be used to identify the end of the plot cards and the end of the plotting requirements. Placing the word "ENDPLT" starting in column one calls the final CALCOMP subroutine and terminates the plotting sequence (see subsection 9.4.3).

Card Set 3. (E'.5,2E1C.5,5A6) This card set, which will be called the label card, is an optional set based on the value of N in the plot card. If N=0, no label cards are required, and if N>0, the number of label cards will be a function of the value of N. Mnemonic corresponding to the label card are the following:

- X, Y, YHT, LØCN where,
- X - the lateral location of the label referenced to the origin of the axes (input in inches)
- Y - the horizontal location of the label referenced to the origin of the axes (input in inches)
- YHT - the height of the letters in the label (Input in units of 1/10 of an inch. For most labels a value of 1 is adequate)
- LØCN - Alphanumeric words used for the label (not to exceed 30 spaces).

Output for the BØPPLT subroutine is shown in Figures 9-16 and 9-17. Figure 9-16 shows the paper output of the plot cards, whereas Figure 9-17 shows the plotted information.
Figure 9-16. PAPER OUTPUT FOR BOPPLT

WOLTOS

BIN215

432711

RAGMOP C50217

072172 075203

Figure 9-17. BOPPLT PLOT OUTPUT
Figure 9-17. (Continued)
Figure 9-17. (Continued)
Figure 9-17. (Concluded)
9.4 CONSTRUCTION OF THE DATA DECK

9.4.1 Control Cards

The card input sequence for a given case on the RAGMOP program is initiated by the computer system dependent control cards. The MSFC Univac 1108 Exec "III system control cards are set up and arranged corresponding to the sample control data input of Figure 9-18. A brief description of the control cards is presented below (for a better understanding of them or any other control cards the user should consult the appropriate Programmer's Procedures Manual.)

Figure 9-18. SAMPLE CONTROL SETUP FOR MSFC UNIVAC 1108 EXEC VIII SYSTEM
Card 1. 
"@RUN,/T.RACMOP,Seq#,NAMEBIN,TIME,PAGE/CARDS"
This card is the run card which executes the control stream logic and provides the computation laboratory with pertinent information concerning the name of the job, the job charge number, the name of the user, the user's bin number (for return), the maximum C.P.U. time of the run, the maximum number of pages, and the maximum number of output cards.

Card 2. 
"@ASC,T.RACMOP,T,TAPE" 
If the program is stored on magnetic tape, this card is used to identify the tape number and assign a name for the tape for execution purposes.

Card 3. 
"@REWIND.RACMOP"
Assures a rewind of the program tapes to the first record.

Card 4. 
"@COPIN.RACMOP,TPFS"
Copies the program tape onto a temporary storage drum file.

Card 5. 
"@FREE.RACMOP"
Frees the program tape from the control stream, rewinds the tape and turns off the tape unit.

Card 6. 
"@MAP,IS,MO,MOP"
Processor call statement for the collector, the name "MOP" is given to the program file containing the binary elements of the program subroutines and the required external (library) subroutines.

Card 7. 
"LIB SYS&MSFC" 
Control card to obtain MSFC library subroutine package.

Card 8-13. Program overlay cards (required to get within 32k limits at MSFC); optional if the 32k case limit is not imposed.)

Card 14. 
"@XQT,MOP"
Demands execution of the program file MOP.

Trajectory input. Namelist input for trajectory simulation.

"@FIN". Specifies finalization of the control flow.

7.4.2 Namelist Input
Program input (except for the BOPPLT input) uses the Namelist format. Namelist is a FOR..AN input routine which groups and stores the input into various sets of input data depending upon the alphanumeric name of the variables.
and the dimension of the variables in the calling subroutine. Before using the Namelist type of input, the user should be aware of the following rules.

1. No Namelist input (names and data) may start in column 1.

2. The first control card of a Namelist group of data must contain a dollar sign ($) followed by an identification name. This name is followed by at least one blank character.

3. Namelist data must take one of the following forms
   a. variable name = constant, where variable name may be an array element name or a simple variable name. Subscripts must be integer constants.
   
   b. array name = set of constants (separated by commas), where $K$ constant, may be included to represent $K$ constants ($K$ must be an unsigned integer). The following are two examples of how TIM may be expressed.

   $$TIM = 52., 52., 52., 52.,$$
   $$TIM = 4 * 52.,$$

   c. subscript variable = set of constants (separated by commas). This causes elements to be loaded, starting with the element designated by the subscript, in consecutive order.

4. All data elements must be separated by commas.

5. Table 9-8 lists the types of constants which may be utilized with Namelist, the definition of the constants and their magnitude limits on the Univac 1108 digital computer.

### Table 9-8. NAMELIST CONSTANTS

<table>
<thead>
<tr>
<th>CONSTANT</th>
<th>DEFINITION</th>
<th>MAGNITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integer</td>
<td>1 to 11 digits written without a decimal point. (fixed point number)</td>
<td>$\pm 10^{10}$</td>
</tr>
<tr>
<td>Real</td>
<td>1 to 9 significant digits written with a decimal point (floating point number)</td>
<td>$10^{-38} - 10^{38}$</td>
</tr>
<tr>
<td>Double Precision</td>
<td>1 to 18 significant digits written with a decimal point</td>
<td>$10^{-308} - 10^{+308}$</td>
</tr>
</tbody>
</table>
6. Fortran V assumes that all variable names beginning with I, J, K, L, M or N are integers and thus allows no decimal. (Unless the variable is REAL in the input subroutine). Variable names beginning in other than these six letters must contain a decimal.

7. Each card in the Namelist which contains data must end with a comma.

8. If an array is being input and the amount of data exceeds one card the data may be continued on the next card by either continuing the array or by redefining the variable.

9. Real numbers may be written with or without the decimal exponent specified. The following example shows three ways in which a real number may be written:

   2230.5
   22305.E-1
   .22305E4

A double precision constant must be followed by the letter D with a signed (+ is optional) one, two or three digit exponent. The following are acceptable double precision constants:

   1.00D0 (means 1.0)
   75.3D1
   7.53D2
   75300.D-2

10. The end of a Namelist requires a $END card either on the same card with the last data item or as the last card of the deck.

11. The number of elements read into an array must not exceed the dimensions.

12. Namelist input can include columns 2 through 80.

   The trajectory input for each case is initiated by a $INPUT card and terminated by a $END card. No particular order of input variables has to be maintained when setting up data cards, but it is suggested that alphabetizing the input or defining some order or sequence should be used to minimize both checkout procedures and input errors.

   Multiple cases can be input by continued use of the $INPUT and $END cards. However if multiple cases are required, the variable LAST must be set equal to the number of cases in the initial data set to signify that subsequent cases will follow. Multiple case data is made up of the variables desired to be changed from the initial or preceding case.
9.4.3 Input Categories

Input to the RAGMOP program can be grouped into four general categories. These are 1) trajectory physical model, 2) program control, 3) trajectory optimization and control, and 4) postprocessor control. Tables 9-9 through 9-11 define the variables used under each category. Definitions of these four categories are presented below.

**Trajectory Physical Model.** This category relates to all variables concerned with either the earth model or the vehicle physical model. Subdivision of this category is used to classify various topics which make up the physical model. These topics are the following.

- **Earth Model Constants** - constants associated with the earth model shape and gravity model (not required input, preset values are normally used).
- **Earth Model Variables** - variables associated with the earth model, such as launch condition (preset values can be used).
- **Vehicle Model Variables** - variables associated with the definition of the vehicle model. These variables are broken into the areas of aero dynamic, vehicle geometry, flight performance reserves, flyback, payload, and propulsion system.

**Program Control.** This category describes the input which is critical for the setup and operation of the trajectory simulation. Included in this category are the integration inputs, logic flags, and thrust event configuration requirements.

**Trajectory Optimization and Control.** This category consists of all parameters associated with either the optimization or attitude control input requirements.

**Postprocessor Control.** The variable NTABLE in the $INPUT Namelist is used to specify to the program that a postprocessing module will be initiated after completion of the trajectory simulation.

*See subparagraphs 9.3.2.1 and 9.3.2.2*
Table 9-9. RAGMC? NAMELIST INPUT SYMBOLS (ORDERED ACCORDING TO USE) - TRAJECTORY PHYSICAL MODEL

<table>
<thead>
<tr>
<th>INPUT SYMBOL</th>
<th>DIMENSION</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CJ</td>
<td>1</td>
<td>Second harmonic in gravity potential equation</td>
<td>(0.162345E+03)</td>
<td>unitless</td>
</tr>
<tr>
<td>CMUE</td>
<td>1</td>
<td>Product of the universal gravitational constant and the Earth's mass</td>
<td>(1.986032E+14)</td>
<td>m(^3)/sec(^2)</td>
</tr>
<tr>
<td>DJ</td>
<td>1</td>
<td>Fourth harmonic in gravity potential equation</td>
<td>(0.7875E-05)</td>
<td>unitless</td>
</tr>
<tr>
<td>FLAT</td>
<td>1</td>
<td>Flattening coefficients of the Fischer ellipsoid</td>
<td>(1.298.3)</td>
<td>unitless</td>
</tr>
<tr>
<td>GZERO</td>
<td>1</td>
<td>Earth's gravitational acceleration constant</td>
<td>(9.80665)</td>
<td>m/sec(^2)</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>Third harmonic in gravity potential equation</td>
<td>(-5.75E-06)</td>
<td>unitless</td>
</tr>
<tr>
<td>OMEGA</td>
<td>1</td>
<td>Earth's angular rotation rate</td>
<td>(7.2921158E-04)</td>
<td>rad/sec</td>
</tr>
<tr>
<td>RE</td>
<td>1</td>
<td>Earth's equatorial radius</td>
<td>(6378165.0)</td>
<td>meters</td>
</tr>
</tbody>
</table>

| EARTH MODEL VARIABLES |
|------------------------|-----------------|-----------------|
| ALAT                   | 1               | Geodetic latitude of the launch site (real variable) | \(28.531855\) | deg |
| ALONG                  | 1               | Longitude of the launch site (measured positive west) | \(80.5649528\) | deg |
| ALTLS                  | 1               | Altitude of the launch site above model ellipsoid | \(0.\) | meters |
| ALTBL                  | 25              | Altitude table for wind and wind azimuth tables | \(0.\) | meters |
| A2                     | 1               | Launch azimuth | \(90.\) | deg |
| AZNTBL                 | 25              | Wind azimuth table | \(0.\) | deg |
| DTZ                    | 1               | Time from GRR to liftoff | \(0.\) | sec |
| NIND                   | 1               | Number of points in wind table | \(0.\) | unitless |
| WBL                    | 25              | Wind table | \(0.\) | m/sec |

<table>
<thead>
<tr>
<th>TRAJECTORY MODEL VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLIM</td>
</tr>
<tr>
<td>JTHR</td>
</tr>
<tr>
<td>MSWCH</td>
</tr>
<tr>
<td>NBEVL</td>
</tr>
<tr>
<td>MNAMEAL</td>
</tr>
</tbody>
</table>

*Full description is given in Paragraph 9.3.
Table 9-9. (Continued)

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DIMENSION</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TLIFT</td>
<td>1</td>
<td>Time indicating end of vertical rise</td>
<td>8.0</td>
<td>sec</td>
</tr>
<tr>
<td>TILZ</td>
<td>1</td>
<td>Time indicating end of tilt-over maneuver (optional)</td>
<td>0.0</td>
<td>sec</td>
</tr>
<tr>
<td>TZERO</td>
<td>1</td>
<td>Liftoff time</td>
<td>0.0</td>
<td>sec</td>
</tr>
<tr>
<td>VIV</td>
<td>8</td>
<td>Initial state vector for jump start</td>
<td>0.0</td>
<td>variable depending on value of VIV(I)</td>
</tr>
</tbody>
</table>

**VEHICLE MODEL VARIABLES**

**AERODYNAMICS**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DIMENSION</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAALP</td>
<td>(25,2)</td>
<td>Coefficient of slope of CA w.r.t. angle-of-attack per stage</td>
<td>0.0</td>
<td>deg</td>
</tr>
<tr>
<td>CA</td>
<td>(25,2)</td>
<td>Coefficient of CA at zero angle-of-attack per stage</td>
<td>0.0</td>
<td>unitless</td>
</tr>
<tr>
<td>CLBETA</td>
<td>(25,2)</td>
<td>Coefficient of slope of CL w.r.t. sideslip angle per stage</td>
<td>0.0</td>
<td>deg</td>
</tr>
<tr>
<td>CMALP</td>
<td>(25,2)</td>
<td>Coefficient of slope of CM w.r.t. angle of attack per stage</td>
<td>0.0</td>
<td>deg</td>
</tr>
<tr>
<td>CM</td>
<td>(25,2)</td>
<td>Coefficient of CM at zero angle of attack per stage</td>
<td>0.0</td>
<td>unitless</td>
</tr>
<tr>
<td>CNMALP</td>
<td>(25,2)</td>
<td>Coefficient of slope of &quot;N (normal force) w.r.t. angle of attack per stage</td>
<td>0.0</td>
<td>deg</td>
</tr>
<tr>
<td>CNBETA</td>
<td>(25,2)</td>
<td>Coefficient of slope of CN (yaw moment) w.r.t. sideslip angle per stage</td>
<td>0.0</td>
<td>deg</td>
</tr>
<tr>
<td>CN</td>
<td>(25,2)</td>
<td>Coefficient of CN (normal force) at zero angle of attack per stage</td>
<td>0.0</td>
<td>unitless</td>
</tr>
<tr>
<td>CYBETA</td>
<td>(25,2)</td>
<td>Coefficient of slope of CY w.r.t. sideslip angle per stage</td>
<td>0.0</td>
<td>deg</td>
</tr>
<tr>
<td>PNM</td>
<td>(25,2)</td>
<td>Mach number table for all aerodynamic tables &amp; TALFP table per stage</td>
<td>0.0</td>
<td>unitless</td>
</tr>
<tr>
<td>S</td>
<td>15</td>
<td>Aerodynamic reference area per thrust event</td>
<td>0.0</td>
<td>meters</td>
</tr>
<tr>
<td>XLEN</td>
<td>2</td>
<td>Aerodynamic reference length per stage</td>
<td>0.0</td>
<td>meters</td>
</tr>
<tr>
<td>XREF</td>
<td>2</td>
<td>Longitudinal location of aerodynamic moment reference point per stage</td>
<td>0.0</td>
<td>meters</td>
</tr>
<tr>
<td>ZREF</td>
<td>2</td>
<td>Vertical location of aerodynamic moment reference point per stage</td>
<td>0.0</td>
<td>meters</td>
</tr>
</tbody>
</table>

**BASE AXIAL FORCE**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DIMENSION</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALT BAS</td>
<td>(25,2)</td>
<td>Altitude table for base axial force table per stage</td>
<td>0.0</td>
<td>meters</td>
</tr>
<tr>
<td>BAXIAL</td>
<td>(25,2)</td>
<td>Base axial force table per stage</td>
<td>0.0</td>
<td>lbs</td>
</tr>
</tbody>
</table>

*Full description is given in Paragraph 9.3.
### Table 9-9. (Continued)

<table>
<thead>
<tr>
<th>INPUT SYMBOL</th>
<th>DIMENSION</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOMETRY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P(nc)</td>
<td>1</td>
<td>Recip al of the nose cone radius</td>
<td>0.</td>
<td>1/ft</td>
</tr>
<tr>
<td>TCG</td>
<td>(15,2)</td>
<td>Delta weight history for center of gravity</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>LCG</td>
<td>(15,2)</td>
<td>Longitudinal component of the center of gravity</td>
<td>0.</td>
<td>meters</td>
</tr>
<tr>
<td>VCG</td>
<td>(15,2)</td>
<td>Vertical component of the center of gravity</td>
<td>0.</td>
<td>meters</td>
</tr>
<tr>
<td>WB1</td>
<td>1</td>
<td>Liftoff weight</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>WTJET</td>
<td>15</td>
<td>Jettison weights</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>FLIGHT PERFORMANCE RESERVES (OPTIONAL METHOD)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCGS</td>
<td>15</td>
<td>Critical flowrate</td>
<td>0.</td>
<td>lbs/sec</td>
</tr>
<tr>
<td>DELVG</td>
<td>1</td>
<td>.V for geometry reserves</td>
<td>0.</td>
<td>m/sec</td>
</tr>
<tr>
<td>DELVP</td>
<td>1</td>
<td>.V for performance reserves</td>
<td>0.</td>
<td>m/sec</td>
</tr>
<tr>
<td>KPM</td>
<td>1</td>
<td>Maximum critical propellant</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>IPR</td>
<td>1</td>
<td>Specifies thrust event from which flight reserves are taken</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>FLYBACK</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SLD0</td>
<td>1</td>
<td>Booster lift to drag ratio</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>CAFRT</td>
<td>1</td>
<td>Percentage of go around fuel</td>
<td>0.</td>
<td>decimal fraction</td>
</tr>
<tr>
<td>EBPCT</td>
<td>1</td>
<td>Additional flyback fuel percentage for engine out (of loaded weight)</td>
<td>0.</td>
<td>decimal fraction</td>
</tr>
<tr>
<td>MGWV</td>
<td>1</td>
<td>Flyback headwind</td>
<td>0.</td>
<td>ft/sec</td>
</tr>
<tr>
<td>KBACK</td>
<td>1</td>
<td>=1, flyback computation required, =0, no flyback</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>SFC</td>
<td>1</td>
<td>Thrust specific fuel consumption of the flyback engines</td>
<td>0.</td>
<td>ft/lb-hr.</td>
</tr>
<tr>
<td>VCRUS</td>
<td>1</td>
<td>Flyback cruise velocity</td>
<td>0.</td>
<td>ft/sec</td>
</tr>
<tr>
<td>WTLAN</td>
<td>1</td>
<td>Landed weight of booster</td>
<td>0.</td>
<td>lbs</td>
</tr>
</tbody>
</table>

*Full description is given in Paragraph 9.3.
### Table 9-9. (Concluded)

<table>
<thead>
<tr>
<th>INPUT SYMBOL</th>
<th>UNITS</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAYLOAD COMPUTATION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRRWMT</td>
<td>1</td>
<td>Constant orbiter weight</td>
<td>.</td>
<td>lbs</td>
</tr>
<tr>
<td>CRRWRT</td>
<td>1</td>
<td>Constant residual weight</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>CTRWMT</td>
<td>1</td>
<td>Constant tank weight</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>VMHIS</td>
<td>1</td>
<td>Velocity potential of OMS engine</td>
<td>0.</td>
<td>ft/sec</td>
</tr>
<tr>
<td>FFPFAC</td>
<td>1</td>
<td>Scale factor for flight performance reserves</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>OMISP</td>
<td>1</td>
<td>Specific impulse of the OMS engine</td>
<td>0.</td>
<td>sec</td>
</tr>
<tr>
<td>SCALC</td>
<td>1</td>
<td>Specific impulse for booster</td>
<td>0.</td>
<td>sec</td>
</tr>
<tr>
<td>SCALE</td>
<td>1</td>
<td>Scale factor for orbiter tank</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>SCRRD</td>
<td>1</td>
<td>Scale factor for orbiter tank</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>VISPB</td>
<td>1</td>
<td>Specific impulse for orbiter tank</td>
<td>0.</td>
<td>sec</td>
</tr>
<tr>
<td>VISPO</td>
<td>1</td>
<td>Specific impulse for orbiter tank</td>
<td>0.</td>
<td>sec</td>
</tr>
<tr>
<td>PROPELLION SYSTEM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AE</td>
<td>15</td>
<td>Exit area per liquid engine per thrust event</td>
<td>0.</td>
<td>m²</td>
</tr>
<tr>
<td>CISP</td>
<td>15</td>
<td>Specific impulse of liquid engines used for continuous throttling per thrust event</td>
<td>0.</td>
<td>sec</td>
</tr>
<tr>
<td>FGBS</td>
<td>15</td>
<td>Vacuum thrust per liquid engine per thrust event</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>FUELIQ</td>
<td>15</td>
<td>Liquid engines fuel per thrust event</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>NPARBN</td>
<td>1</td>
<td>Parallel burn indicator; +1, parallel burn liquid</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>SMENB</td>
<td>1</td>
<td>SRM exit area per engine</td>
<td>0.</td>
<td>m²</td>
</tr>
<tr>
<td>SRMDT</td>
<td>15</td>
<td>Total SRM weight overboard table</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>SRMMTB</td>
<td>15</td>
<td>SRM sea level thrust per engine table</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>SRMTPP</td>
<td>1</td>
<td>Total SRM fuel available</td>
<td>0.</td>
<td>lbs</td>
</tr>
<tr>
<td>SRMTT</td>
<td>15</td>
<td>Independent time table for SRMDT &amp; SRMMTB</td>
<td>0.</td>
<td>sec</td>
</tr>
<tr>
<td>SRGCGF</td>
<td>1</td>
<td>Longitudinal SRM aiming point</td>
<td>0.</td>
<td>m</td>
</tr>
<tr>
<td>SRLGP</td>
<td>5</td>
<td>Longitudinal SRM gimbal points</td>
<td>0.</td>
<td>m</td>
</tr>
<tr>
<td>SRXGP</td>
<td>5</td>
<td>Lateral SRM gimbal points</td>
<td>0.</td>
<td>m</td>
</tr>
<tr>
<td>SRSLGF</td>
<td>1</td>
<td>Vertical SRM aiming point</td>
<td>0.</td>
<td>m</td>
</tr>
<tr>
<td>SRLGP</td>
<td>5</td>
<td>Vertical SRM gimbal points</td>
<td>0.</td>
<td>m</td>
</tr>
<tr>
<td>THE</td>
<td>(4.1)</td>
<td>Number of liquid engines per thrust event</td>
<td>0.</td>
<td>unitless</td>
</tr>
<tr>
<td>MTET</td>
<td>15</td>
<td>Propellant flowrate per liquid engine per thrust event</td>
<td>0.</td>
<td>lbs/sec</td>
</tr>
<tr>
<td>AGP</td>
<td>(15,2)</td>
<td>Longitudinal gimbal plan location per liquid engine per stage</td>
<td>0.</td>
<td>m</td>
</tr>
<tr>
<td>YGP</td>
<td>(15,2)</td>
<td>Lateral gimbal plan location per liquid engine per stage</td>
<td>0.</td>
<td>m</td>
</tr>
<tr>
<td>ZGP</td>
<td>(15,2)</td>
<td>Lateral gimbal plan location per liquid engine per stage</td>
<td>0.</td>
<td>m</td>
</tr>
</tbody>
</table>

*Full description is given in Paragraph 9.3.
Table 9-10. **RAGMOP NAMELIST INPUT SYMBOLS (ORDERED ACCORDING TO USE)**  
**- PROGRAM CONTROL**

<table>
<thead>
<tr>
<th>INPUT SYMBOL</th>
<th>DIMENSION</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATL</td>
<td>1</td>
<td>Integration error check variable</td>
<td>2.23</td>
<td>unitless</td>
</tr>
<tr>
<td>CASE</td>
<td>1</td>
<td>Case number</td>
<td>1</td>
<td>unitless</td>
</tr>
<tr>
<td>E3</td>
<td>1</td>
<td>Upper bound on integration error</td>
<td>1 E-6</td>
<td>unitless</td>
</tr>
<tr>
<td>HEAD</td>
<td>15</td>
<td>Aperture term for header output on trajectory</td>
<td>BLANK</td>
<td></td>
</tr>
<tr>
<td>HMIN</td>
<td>1</td>
<td>Minimum step size for integration</td>
<td>25</td>
<td>sec</td>
</tr>
<tr>
<td>HNEW</td>
<td>1</td>
<td>Earth model flag. 0= Oblate. -1 spherical</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>JUMP</td>
<td>1</td>
<td>Jump start flag</td>
<td>1</td>
<td>unitless</td>
</tr>
<tr>
<td>KIND</td>
<td>1</td>
<td>Integration type flag. +1, variable step-size</td>
<td>1</td>
<td>unitless</td>
</tr>
<tr>
<td>ORDER</td>
<td>1</td>
<td>Order of differences in integration</td>
<td>3</td>
<td>unitless</td>
</tr>
<tr>
<td>LAST</td>
<td>1</td>
<td>0 if only one case is run, +1 if more than one case is run</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>LEFINT</td>
<td>1</td>
<td>Print indicator (see table 9-4)</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>MAX</td>
<td>1</td>
<td>Maximum number of iterations</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>NVALUE</td>
<td>3</td>
<td>(Plot table or plot required. +0, no tables or plots. +1, tables only. +2, plots only. +3, tables &amp; plots)</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>PRINT</td>
<td>15</td>
<td>Desired print interval per thrust event</td>
<td>10</td>
<td>sec</td>
</tr>
<tr>
<td>STEP</td>
<td>15</td>
<td>Integration step size per thrust event</td>
<td>8</td>
<td>sec</td>
</tr>
</tbody>
</table>

Table 9-11. **RAGMOP NAMELIST INPUT SYMBOLS (ORDERED ACCORDING TO USE)**  
**- TRAJECTORY AND CONTROL**

<table>
<thead>
<tr>
<th>INPUT SYMBOL</th>
<th>DIMENSION</th>
<th>BRIEF DEFINITION*</th>
<th>PRESET VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHA</td>
<td>3</td>
<td>Attitude control flag. +1, optimize controller, -1, control based on zero aerodynamic force; +2, control based on zero angle of attack, -3, control based on angle of attack history as a function of Mach number</td>
<td>A</td>
<td>unitless</td>
</tr>
<tr>
<td>CONTROL</td>
<td>20</td>
<td>Pitch attitude history for orbiter</td>
<td>n</td>
<td>deg</td>
</tr>
<tr>
<td>CPITCH</td>
<td>20</td>
<td>Pitch attitude history for booster</td>
<td>n</td>
<td>deg</td>
</tr>
<tr>
<td>CYTHET</td>
<td>20</td>
<td>Yaw attitude history for orbiter</td>
<td>n</td>
<td>deg</td>
</tr>
<tr>
<td>CYTHET</td>
<td>20</td>
<td>Yaw attitude history for booster</td>
<td>n</td>
<td>deg</td>
</tr>
<tr>
<td>NO</td>
<td>10</td>
<td>Tolerance on terminal end conditions</td>
<td>0</td>
<td>variable</td>
</tr>
<tr>
<td>KCOMP</td>
<td>10</td>
<td>Terminal function codes (See帏KCOMP (1) is say &quot;0&quot; (see table 9-3)</td>
<td>KCOMP(1)=1</td>
<td>unitless</td>
</tr>
<tr>
<td>KCODES</td>
<td>6</td>
<td>Intermediate constraint codes (See帏KCODES (1) is say &quot;0&quot; (see table 9-3)</td>
<td>KCODES(1)=6</td>
<td>unitless</td>
</tr>
<tr>
<td>KOB</td>
<td>30</td>
<td>Codes to identify optimization parameters (See帏KOB (1) is say &quot;0&quot; (see table 9-3)</td>
<td>KOB(1)=0</td>
<td>unitless</td>
</tr>
<tr>
<td>KST</td>
<td>30</td>
<td>Companion matrix to KOB for calculation of flight reference reserves</td>
<td>KST(1)=0</td>
<td></td>
</tr>
<tr>
<td>KP</td>
<td>2</td>
<td>Order of the desired pitch polynomial per stage</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>KFY</td>
<td>2</td>
<td>Order of the desired yaw polynomial per stage</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>KWST</td>
<td>1</td>
<td>Intermediate constraint imposed at termination of this thrust event number</td>
<td>0</td>
<td>unitless</td>
</tr>
<tr>
<td>PSREO</td>
<td>10</td>
<td>Required values of the terminal constraints at termination</td>
<td>PSREO(1)=1</td>
<td>variable</td>
</tr>
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*Full description is given in Paragraph 9.3.*

9-62
If NTABLE is equal to zero, no postprocessor is used and the data deck is set up corresponding to Figure 9-19; however, if NTABLE is greater than zero, three options are available which relate to NTABLE equal to one, two, or three.

NTABLE equal to one signifies the use of the subroutine BOPTBL. This subroutine outputs the trajectory parameters into fifteen output tables suitable for publishing. The subroutine BOPTBL requires a separate group of NAMELIST input under the NAMELIST name $INPUT2. Therefore the input data set requires not only the $INPUT input but the $INPUT2 input (see Figure 9-20).

NTABLE equal to two signifies the use of subroutine BOPPLT. This subroutine plots required variables using the CALCOMP plotter. Similar to BOPTBL, subroutine BOPPLT also requires a separate input package. However, formatted input is used rather than the NAMELIST form. When using the BOPPLT postprocessor the data package consists of the trajectory input plus the BOPPLT input requirements (see Figure 9-21).

NTABLE equal to three indicates that the subroutines BOPTBL and BOPPLT, respectively, are used for output tables and plots. In this option, the data is set up such that the BOPTBL input and BOPPLT input, respectively, follow the trajectory input (see Figure 9-22).

Figure 9-19. SAMPLE DECK SETUP WITH NTABLE=0

9-63
Figure 9-20. **BØPTBL SAMPLE DECK SETUP**

Figure 9-21. **BØPPLT SAMPLE DECK SETUP**

9-64
9.5 SAMPLE PROBLEM

The following pages contain excerpts from the computer printout of a typical sample problem.

The vehicle used for this problem was the SRM shuttle with 049 HO orbiter. The mission was a 90-degree launch from Cape Kennedy to a 50 x 100-nautical mile orbit. Maximum payload for fixed liftoff weight was desired. Intermediate constraints are:

\[ q_1 = 3000 \text{ lb deg/ft}^2 \]

\[ g_{\text{limit}} = 3 \text{ g's (continuous throttling in the orbiter).} \]

Parameters selected were:
- two \( \chi_p \) values for a quadratic first stage \( \chi_p \) polynomial,
- two \( \chi_p \) values for a linear second stage \( \chi_p \) polynomial with discontinuous \( \chi_p \) at staging,

and liftoff weight.
**AXOR RAGMP**

**FIRST STAGE EQUIVALENT ENGINE LOCATION ARE**

\[
\begin{align*}
& ( -0.47955200 \times 0.0 - 0.89466700 \times -1.7799200 \times 0.02 ) \text{ IN INPUT BODY AXIS SYSTEM}
\end{align*}
\]

**SECOND STAGE EQUIVALENT ENGINE LOCATION ARE**

\[
\begin{align*}
& ( 0.0000000 \times 0.0 - 0.0000000 \times 0.0 - 0.0000000 \times 0.0 ) \text{ IN INPUT BODY AXIS SYSTEM}
\end{align*}
\]

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**Figure 9-23. NAMELIST INPUT.**

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Figure 9-23. (Continued)
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Figure 9-23. (Continued)  
9-69
### Figure 9-23. (Continued)

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9-70
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Figure 9-23. (Continued)
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**Figure 9-25. NOMINAL TRAJECTORY SUMMARY. BEGIN QY SEARCH**

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RETURN F

CURRIC FIT FOR GTY = 1.3269420 + 04

PAVHOF INDEX: -0.25193591 + 02

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E2 = 0.0000000

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CPTMT 21 = 0.7054694 + 01

CPTMT 31 = -1.0677739 + 02

CPTMT 41 = 0.5367744 + 02

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RETURN F

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Figure 9-26. RESULTS OF QY SEARCH STEP, CONVERGENCE TEST PASSED
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**Figure 9-26. (Concluded)**
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| 0.149 | 0.002589864+0 | 0.02395675+0 | 0.36319737+0 | 0.6321974+0 | 3.390769+0 | -1.9656752+0 |
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| 0.214 | 0.002589864+0 | 0.02395675+0 | 0.36319737+0 | 0.6321974+0 | 3.390769+0 | -1.9656752+0 |

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<td>CHEV</td>
<td>.9321160+04 TL</td>
<td>.46766078+01 GVL</td>
<td>.7194982+03 DRL</td>
<td>.7649618+02 BKP</td>
</tr>
</tbody>
</table>

**Figure 9-27.** (Continued)
Figure 9-27. (Concluded)
APPENDICES
Appendix A
COORDINATE TRANSFORMATIONS

A vector or tensor in any orthogonal coordinate system can be transformed into any other orthogonal coordinate system by premultiplication of the vector or tensor with the proper transformation matrix. Any coordinate system can be obtained from any other by at most three successive rotations about the coordinate axes. For any rotation about an x (or l) axis, the transformation matrix is given by

$$A_{xx'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} = A_1$$

where x'y'z' are the new axes. A vector in xyz is transformed into x'y'z' by premultiplication with A\(_{xx'}\), e.g.

$$\vec{x}' = A_{xx'} \vec{x}$$

or

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where \(\vec{x}' = x'\hat{e}_1 + y'\hat{e}_2 + z'\hat{e}_3\) and \(\vec{x} = x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3\).

Similarly, for any rotation through the angle \(\phi\) about a y (or 2) axis the transformation matrix is:

$$A_{xy} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} = A_2$$

where x'y'z' is rotated about y' to form x*y*z*. Also, for a rotation of \(\gamma\) about the z* axis we would have:
where the XYZ system is obtained by rotating \( x'y'z' \) about the \( z' \) the angle \( \gamma \).

Since any coordinate system can be obtained from any other by three successive rotations, the above matrices \( A_1, A_2, \) and \( A_3 \) may be used together to form the transformation matrix from any one system to another. Note that for each rotation, premultiplication of the proper matrix is required to transform the old coordinates into the new ones. Thus, if a coordinate system XYZ is obtained by rotating the \( x'y'z' \) system first about \( y \) an angle \( \phi \) to form \( x*y*z' \), then about \( x' \) an angle \( \theta \) to form \( x'y*z \), and then about \( z \) an angle \( \gamma \) to form XYZ, the transformation matrices are:

\[
A_{x'*x} = \begin{bmatrix} 
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi 
\end{bmatrix} = A_2
\]

\[
A_{x*x} = \begin{bmatrix} 
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta 
\end{bmatrix} = A_1
\]

\[
A_{x*x} = \begin{bmatrix} 
\cos \gamma & \sin \gamma & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1 
\end{bmatrix} = A_3
\]

and so the total transformation matrix is

\[
A_{x'*X} = A_{x*X} A_{x'*x} A_{x'x'} = \begin{bmatrix} 
\cos \gamma & \sin \gamma & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1 
\end{bmatrix} \begin{bmatrix} 
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta 
\end{bmatrix} \begin{bmatrix} 
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi 
\end{bmatrix} = \begin{bmatrix} 
\cos \gamma \cos \phi & \sin \gamma \cos \phi - \sin \phi \sin \gamma & -\sin \gamma \sin \phi + \cos \phi \sin \gamma \\
-\sin \gamma \cos \phi & \cos \gamma \cos \phi + \sin \phi \sin \gamma & \sin \gamma \sin \phi + \cos \phi \sin \gamma \\
\cos \phi & -\sin \phi & 0 
\end{bmatrix}
\]
\[
\begin{bmatrix}
\cos \gamma & \sin \gamma \cos \phi & \sin \gamma \sin \phi \\
-sin \gamma & \cos \gamma \cos \phi & \cos \gamma \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi
\end{bmatrix}
\]

\[
A_{x',x} = \begin{bmatrix}
(\cos \gamma \cos \phi + \sin \gamma \sin \phi \sin \phi) & \sin \gamma \cos \phi & (-\cos \gamma \sin \phi + \sin \gamma \sin \phi \cos \phi) \\
(-\sin \gamma \cos \phi + \cos \gamma \sin \phi \sin \phi) & \cos \gamma \cos \phi & (\sin \gamma \sin \phi + \cos \gamma \sin \phi \cos \phi) \\
\cos \gamma \sin \phi & -\sin \phi & \cos \phi \cos \phi
\end{bmatrix}
\]

Note that the order of rotations is important, but that, if the proper order is maintained, the total transformation matrix may be obtained from any orthogonal system to another in the manner demonstrated above. The transformation of any vector or tensor in the x' y' z' system above to the XYZ system is accomplished then by simple premultiplication with \( A_{x',x} \):

\[
\vec{R}_x = A_{x',x} \vec{R}_{x'}
\]
Appendix B

INTERPOLATION METHODS

Rapid, consistent, and accurate interpolation of tabulated data is essential to the operation of RAGMOP. To this end, two very efficient interpolation schemes are used in the program. The linear interpolation subroutine, AMULG, is used for the determination of base pressure force as a function of altitude, wind direction and speed as a function of altitude, and center-of-gravity location as a function of total vehicle delta weight. A cubic spline interpolation routine, SPLINE, which ensures continuous first derivatives of the data across the data points, is used to find the aerodynamic coefficients as functions of Mach number and to find the atmospheric properties as functions of altitude in subroutine PRA63. Both interpolation routines keep track of the lower data point from the interval used in the previous call to the routine for each particular table. In this manner the routine is not required to repeatedly start from the beginning of the table to find the current interval containing the independent variable. Also, both routines will extrapolate beyond the range of the independent variable data set.

The linear interpolation routine, AMULG, searches for the interval containing the current value of the independent variable. If, of course, \( x = x_1 \) (a data point), then \( y = y_1 \). If the independent data point \( x_{i+1} \) is less than \( x_i \), it is assumed that the last valid data point has been passed and that extrapolation beyond that point will now be required. (Note that this allows the number of data points to be less than the maximum number of points available in each table). If \( x < x_1 \), then \( m \) is chosen as \( m = 2 \), (where \( m \) denotes the number of the base point of the interval) and backward extrapolation is performed. Once the base point, \( x_m \), is located, the general linear interpolation is:

\[
y = y_{m-1} + \frac{x - x_{m-1}}{x_m - x_{m-1}} (y_m - y_{m-1})
\]

The cubic spline interpolation routine, SPLINE, first locates the independent variable within the central span of a four point group, i.e.,
$x_m \leq x \leq x_{m+1}$. If $x < x_1$, $m = 1$. If all of the available points in the independent variable table are filled, extrapolation will be performed when $x > x_m$ (where $m$ is the number of locations in the table). If $x_{i+2} < x_{i+1}$, implying (as in AMULG) that the last valid data point has been passed, the routine sets $x_{i+2} = x_{i+1} + 10$ and sets the $y_{i+2}$ values in all the dependent variable tables equal to the $y_{i+1}$ values. Thus, if fewer than the total number of available points are used in the tables, the scheme assumes that the dependent variables remain unchanged after the last valid input point. Having established the interval containing $x$ ($x_m < x < x_{m+1}$), the dependent variables are found as:

$$y_i = a_i x^3 + b_i x^2 + c_i x + y_m.$$  \hspace{1cm} (B-1)

The coefficients $a_i$, $b_i$, and $c_i$ are derived below.

$$s = \frac{y_{m+1} - y_m}{x_{m+1} - x_m}$$

$$s_m = \frac{y_m - y_{m-1}}{x_m - x_{m-1}}$$

$$s_p = \frac{y_{m+2} - y_{m+1}}{x_{m+2} - x_{m+1}}$$

If $m = 1$, $s_m$ is chosen such that

$$s = \frac{s + s_m}{2} \text{ or, in other words,}$$

$$s_m = 2s - s_p$$

If $m = n-1$, $s_p$ is chosen such that

$$s = \frac{s + s_p}{2} \text{ or in other words}$$

$$s_p = 2s - s_m.$$
Equation (B-1) can be rewritten as

$$\Delta y_1 = y_{m+1} - y_m = a\Delta x_1^3 + b\Delta x_1^2 + c\Delta x_1$$

where

$$\Delta x_1 = x_{m+1} - x_m$$

This yields, dividing by $\Delta x_1$:

$$s = a\Delta x_1^2 + b\Delta x_1 + c.$$

The spline assumption is

$$\frac{dy}{dx} = c = \frac{s + s_m}{2}$$

and

$$\frac{dy}{dx} = 3a\Delta x_1^2 + 2b\Delta x_1 + c = \frac{s + s_p}{2}.$$  

Note that this leaves the first derivatives continuous as $m$ changes.

Solving for $a$, $b$, and $c$ we find:

$$a = \frac{1}{2} \left( \frac{s_p - s_m}{2} - s + s_m \right)$$

$$b = \frac{1}{\Delta x_1} \left( s - s_p + s - s_m \right)$$

$$c = \frac{s + s_m}{2}.$$  

With $R = \Delta x / \Delta x_1 = \frac{x - x_m}{x_{m+1} - x_m}$

we have

$$y = y_m + ((aR + b) R + c) \Delta x$$

B-3
and \[
\frac{dy}{dx} = (3aR + 2b)R + c.
\]

Since \(a\), \(b\), and \(c\) are functions of the data points only (and are therefore constant for a given interval of the independent variable data set), their values are stored and reused without recalculation if interpolation within the same interval is required more than once.
Appendix C

SOLUTION OF MOMENT BALANCE EQUATIONS

From Paragraph 3.2.5.2 we have the moment equations in the center-of-
gravity/gimbal point coordinate system:

\[
\begin{align*}
(T_1 \tan \delta + T_2 \tan \delta) dy + (T_2 - T_1) dz &= -M_{xy} \\
(T_1 \tan \delta - T_2 \tan \delta) dz &= -M_{xz} \\
(T_1 \tan \delta + T_2 \tan \delta) dy &= -M_{xz} \\
\delta_1 &= \delta \\
\delta_2 &= \delta
\end{align*}
\]

Since the total thrust of each engine is fixed \((T_1 = T_2 = T/2)\) we can write:

\[
\begin{align*}
T_x^2 + T_y^2 + T_z^2 &= \left(\frac{T}{2}\right)^2 \\
T_x &= \frac{T_1 \tan \delta}{y_1 y_2} \\
T_y &= \frac{T_2 \tan \delta}{y_1 y_2} \\
T_z &= \frac{T_1 \tan \delta}{y_1 y_2}
\end{align*}
\]

But, from the definitions of \(\delta_p\) and \(\delta_y\):

\[
\begin{align*}
T_x &= -T_1 \tan \delta \\
T_y &= -T_2 \tan \delta \\
T_z &= T_1 \tan \delta = T_2 \tan \delta
\end{align*}
\]

so that

\[
\begin{align*}
T_y &= \frac{T_{T/2}}{\sqrt{1 + \tan^2 \delta_1 + \tan^2 \delta_2}} \\
T_y &= \frac{T_{T/2}}{\sqrt{1 + \tan^2 \delta_1 + \tan^2 \delta_2}}
\end{align*}
\]
We now assume that \( \delta_{p_1}, \delta_{p_2}, \) and \( \delta_y \) are small enough so that

\[
\tan^2 \delta_{p_1} \ll 1 \\
\tag{C-9}
\]

\[
\tan^2 \delta_{p_2} \ll 1 \\
\tag{C-10}
\]

and

\[
\tan^2 \delta_y \ll 1 \\
\tag{C-11}
\]

This approximation yields, from equation (C-7) and (C-8),

\[
T_{y_1} = \frac{T_T}{2} \\
\tag{C-12}
\]

and

\[
T_{y_2} = \frac{T_T}{2}. \\
\tag{C-13}
\]

Using (C-4), (C-12), and (C-13) in (C-1), (C-2), and (C-3) we have

\[
(T_T \tan \delta_y) dy = -\text{Max} \\
\tag{C-14}
\]

\[
\frac{T_T}{2} (\tan \delta_{p_1} - \tan \delta_{p_2}) dz = -\text{Max} \\
\tag{C-15}
\]

\[
\frac{T_T}{2} (\tan \delta_{p_1} + \tan \delta_{p_2}) dy = -\text{Max} \\
\tag{C-16}
\]

which can be rewritten as:

\[
\tan \delta_y = -\frac{\text{Max}}{T_T dy} \\
\tag{C-17}
\]

\[
\tan \delta_{p_1} - \tan \delta_{p_2} = -\frac{2\text{Max}}{T_T dz} \\
\tag{C-18}
\]

and

\[
\tan \delta_{p_1} + \tan \delta_{p_2} = -\frac{2\text{Max}}{T_T dy}. \\
\tag{C-19}
\]

Adding (C-18) and (C-19) we obtain

\[
\tan \delta_{p_1} = -\frac{1}{T_T} \left( \frac{\text{Max}}{dz} + \frac{\text{Max}}{dy} \right) \\
\tag{C-20}
\]
Subtracting (C-18) from (C-19) yields

\[ \tan \delta = -\frac{1}{T_p} \left( \frac{M_{ax}}{dy} - \frac{M_{ay}}{dz} \right). \]  

(C-21)

Since the total thrust is inherently enforced in the solution, the error amounts to an error in the moments produced by the engine to balance the aerodynamic moments. (It is possible to obtain an approximate solution that produces no net unbalanced moment but changes the total thrust slightly. The solution used in RAGMOP does not do this.) The philosophy in RAGMOF is that the purpose of the moment balance scheme — to determine the performance of a vehicle based on realistic thrust vectoring — is satisfied acceptably when the moment on the vehicle has been reduced to a very small level without changing the overall thrust of the vehicle.

As expected, for no aerodynamic moment \( \delta_p = \delta_y = 0 \). The thrust vector of each engine is parallel to the center of gravity/gimbal point \( y \) axis when aerodynamic moments are zero.

Note that the moment and distance components in the above solution are in the XYZ (center-of-gravity/gimbal point) coordinate system. The moments must be transformed into this system from the body axis \( X'Y'Z' \) system in which they are computed, and the resulting thrust components must be transformed from the XYZ system back to the \( X'Y'Z' \) system in order to determine the total forces acting on the vehicle. The XYZ system is obtained from the \( X'Y'Z' \) system by rotating about \( Z' \) the angle \(-\rho\), where

\[ \rho = -\tan^{-1} \left( \frac{X'_{GP} - X'_{CG}}{Y'_{GP} - Y'_{CG}} \right) \]  

(C-22)

Thus, the transformation matrix (see Appendix A) from \( X'Y'Z' \) to XYZ is
The aerodynamic moments in the XYZ system are, then:

\[
M_{AX} = M'_{AX} \cos \theta + M''_{AX} \sin \theta
\]

\[
M_{AY} = M'_{AX} \sin \theta + M''_{AX} \cos \theta
\]

\[
M_{AZ} = M''_{AX}
\]

We define the distances

\[
DX = X'_{GP_1} - X'_{CG} = X'_{GP_2} - X'_{CG}
\]

\[
DY = Y'_{GP_1} - Y'_{CG} = Y'_{GP_2} - Y'_{CG}
\]

\[
DZ = Z'_{GP_2} - Z'_{CG} = -(Z'_{GP_1} - Z'_{CG})
\]

so that

\[
\rho = -\frac{DX}{DY}
\]

\[
\cos \rho = \frac{\sqrt{DX^2 + DY^2}}{DX}
\]

\[
\sin \rho = \frac{\sqrt{DX^2 + DY^2}}{DY}
\]
\[
dx = DX \cos \rho - DY \sin \rho = 0 \quad \text{(C-25)}
\]
\[
dy = DA \sin \rho + DY \cos \rho = \sqrt{DX^2 + DY^2}
\]
\[
dz = DZ
\]

Using (C-21), (C-22), and (C-23) in (C-14), (C-17) and (C-18) we obtain

\[
y = - \tan^{-1} \left[ \frac{M_{AY}'}{DY + \frac{M_{AX}'}{DY} \sqrt{1 + \frac{DX^2}{DY^2}}} \right]
\]

\[
\delta p_1 = - \tan^{-1} \left[ \frac{M_{AZ}'}{DY} \frac{M_{AY}'}{DZ} + \frac{M_{AX}'}{DYDZ} \frac{DX}{DY} \right]
\]

\[
\delta p_2 = - \tan^{-1} \left[ \frac{M_{AZ}'}{DY} \frac{M_{AY}'}{DZ} + \frac{M_{AX}'}{DYDZ} \frac{DX}{DY} \right]
\]

Note that the gimbal angles in the X'Y'Z' body axis coordinate system are given by

\[
\delta p_1 = \delta - \tan^{-1} \frac{DX}{DY} = \delta - \rho
\]

\[
\delta p_2 = \delta - \tan^{-1} \frac{DX}{DY} = \delta - \rho
\]

and

\[
\delta y = \delta_y
\]

The total thrust components in the X'Y'Z' body axis coordinate system are then found by noting that

\[
T_{x_1}' = T_y ' \tan \delta _1 p_1
\]

\[
T_{x_2}' = T_y ' \tan \delta _2 p_2
\]

\[
T_z' = T_y ' \tan \delta
\]

C-5
\[ T_{Z_2}' = T_{y_2} \tan \delta \]
\[ T_{y_1}' = \frac{T_T}{\sqrt{2\frac{1 + \tan^2 \delta_{p_1} + \tan^2 \delta_{y}}{}}}, \]

and

\[ T_{y_2}' = \frac{T_T}{\sqrt{2\frac{1 + \tan^2 \delta_{y} p_2}}}, \]

so that

\[ T_{XX} = -\frac{T_T}{2} \left[ \frac{\tan \delta_{p_1}}{\sqrt{\tan^2 \delta_{p_1} + \tan^2 \delta_{y}}} + \frac{\tan \delta_{p_2}}{\sqrt{\tan^2 \delta_{p_2} + \tan^2 \delta_{y}}} \right] \]
\[ T_{YY} = \frac{T_T}{2} \left[ \frac{1}{\sqrt{\tan^2 \delta_{p_1} + \tan^2 \delta_{y}}} + \frac{1}{\sqrt{\tan^2 \delta_{p_2} + \tan^2 \delta_{y}}} \right] \]
\[ T_{ZZ} = T_{YY} \tan \delta_{y} \]

where \( T_{XX}, T_{YY}, \) and \( T_{ZZ} \) are the total thrust components in the body axis \( X'Y'Z' \) coordinate system.
Appendix D

PAYLOAD CALCULATION

RAPMOP includes a payload calculation so that, with proper input, the payload as well as the orbiter (last stage) cutoff weight can be optimized. This calculation is based on a two stage space shuttle type vehicle. Payload is determined from the orbiter cutoff weight by including the requirements for:

1. flight performance reserves (FPR) for both stages based on the ratio of initial to final weight and the vacuum I\textsubscript{sp} for each stage,
2. orbiter tank weight based on total second stage fuel (including reserves) using an input scale factor, and
3. orbital maneuvering system (OMS) propellant requirements based on the delta velocity required and the I\textsubscript{sp} of the OMS.

The calculations in this option are used at all times to determine the final mass printed with each trajectory summary. These calculations are performed as follows:

\[
\begin{align*}
W_p &= W_{OORB} - W_{COORB} - W_o \\
\Delta V_p &= \delta FPR \text{ characteristic velocity}^* \\
FPR &= W_{COORB} \left(1 - e^{-\Delta V_p/g_o I_{sp}}\right) \\
W_{TANK} &= \eta(W_p + \text{FPR}) \\
W_{ON ORBIT} &= W_{COORB} - \text{FPR} - W_{TANK} \\
W_{OMS} &= W_{ON ORBIT} \left(1 - e^{-\Delta V_{OMS}/g_o I_{sp}}\right) \\
W_{PAYLOAD} &= W_{ON ORBIT} - W_{OMS} - W_{\text{CONSTORB}} \\
\end{align*}
\]

*see Appendix F
where

\[ \text{FPR} = \text{flight performance reserves for both stages} \]

\[ g_o = \text{sea level gravitational acceleration (9.80665 m/sec}^2) \]

\[ I_{SPB} = \text{booster (first stage) vacuum specific impulse} \]

\[ I_{SPO} = \text{orbiter (second stage) vacuum specific impulse} \]

\[ I_{SP_{OMS}} = \text{orbital maneuvering system vacuum specific impulse} \]

\[ \Delta V_{OMS} = \text{delta velocity required of orbital maneuvering system} \]

\[ \Delta V_p = \text{delta velocity required for flight performance reserves} \]

\[ W_{COB} = \text{booster cutoff weight} \]

\[ W_{COORB} = \text{orbiter cutoff weight} \]

\[ W_{CONSTORB} = \text{constant orbiter weight (dry orbiter shell, no fuel or OMS propellant, no fuel tank weight)} \]

\[ W_o = \text{booster liftoff weight (total vehicle)} \]

\[ W_{OB} = \text{orbiter initial weight (at staging)} \]

\[ W_{ORB} = \text{on orbit weight (orbiter cutoff weight minus fuel, FPR, and tank weight)} \]

\[ \text{ON ORBIT includes payload, OMS propellant, and constant orbiter weight.} \]

\[ W_{Pcons} = \text{fuel consumed by orbiter} \]

\[ W_{P_{OMS}} = \text{orbital maneuvering system propellant weight} \]

\[ W_{PAYLOAD} = \text{payload weight} \]

\[ W_{TANK} = \text{orbiter fuel tank weight} \]

\[ \delta_{FPR} = \text{decimal fraction used in FPR calculation (typically .01)} \]

\[ \eta = \text{scale factor used in tank weight calculation (typically .08155)} \]

Note that the payload calculation is performed at all times and that if orbiter cutoff weight is desired rather than payload, the user should omit the input for: \( \delta_{FPR} \) (FPRFAC), \( \eta \) (SCALE), \( \Delta V_{OMS} \) (DVOMS), and \( W_{CONSTORB} \) (COKBWT).
Appendix E

FLYBACK FUEL CALCULATION

A flyback fuel calculation is included in RAGMOP in order to more realistically optimize the performance of reusable flyback space shuttle boosters. This calculation is based on the Breguet* range equation and is performed as follows:

The flyback range required is determined by subroutine ASIMP using a trivariant lookup. Flyback range is tabulated in terms of range, velocity, radius, and flight path angle at staging. The flyback fuel required in order to return to the launch site is the amount of fuel required to cover the flyback range, plus reserves. The jettisoned booster weight at staging includes the flyback requirements.

The Breguet range equation gives the vehicle range as a function of lift over drag at a constant cruise velocity with a constant thrust specific fuel consumption for jet engine powered vehicles:

\[ R = \frac{V}{C_t} \frac{C_L}{C_D} \ln \frac{W_0}{W_1} \]  

(E-1)

where

- \( R \) = range
- \( V \) = cruise velocity (ideally velocity at best \( C_L/C_D \))
- \( C_t \) = thrust specific fuel consumption
- \( \frac{C_L}{C_D} \) = best ratio of lift coefficient to drag coefficient for the vehicle (ideally at cruise velocity)
- \( W_0 \) = weight at beginning of flyback leg
- \( W_1 \) = weight at end of flyback leg.

The flyback fuel required, $W_p$, is given by

$$W_p = W_1 (e^{-\frac{R c_t CD}{v \frac{C_L}{C_D}}} - 1)$$

(E-2)

In order to include the effects of engine out and go-around fuel requirements, the approximation is made in RACMOP that each may be expressed as an input decimal fraction of the landed weight of the booster. The equation for flyback fuel used in the program is, then:

$$W_p = W_1 (1 + Pct_{EO} + Pct_{GA})(e^{-\frac{R c_t}{v \frac{C_L}{C_D}}} - 1)$$

where $Pct_{EO}$ = decimal fraction of booster landed weight required for engine out flyback

and $Pct_{GA}$ = decimal fraction of booster landed weight required for go-around capability at landing site.
Appendix F

VELOCITY LOSS EQUATIONS

The velocity loss equations result from the calculation of the change in the inertial velocity, $V_I$, during the time interval $t_a - t_b$.

It is obvious that

$$V_I(t_b) - V_I(t_a) = \int_{t_a}^{t_b} \frac{\dot{V}_I}{V_I} dt$$  \hspace{1cm} (F-1)

Since

$$V_I^2 = \ddot{V}_I \cdot \ddot{V}_I$$  \hspace{1cm} (F-2)

it follows that

$$\dot{V}_I = \frac{\ddot{V}_I \cdot \ddot{V}_I}{V_I}$$  \hspace{1cm} (F-3)

Therefore

$$\Delta V_I = V_I(t_b) - V_I(t_a) = \int_{t_a}^{t_b} \frac{\ddot{V}_I \cdot \ddot{V}_I}{V_I} dt$$  \hspace{1cm} (F-4)

Three different thrust levels are now defined. The first, $T_v$, is the total resultant thrust calculated as the square root of the sum of the squares of the thrust components. The second, $T_s$, is the scalar sum of the thrusts of all engines. The third, $T_{vac}^a$, is the scalar sum of the vacuum thrust of all engines.

Now, adding and subtracting each of these divided by mass to the right hand side of (4) leads to
Since $T_{\text{vac}}_s$ is the largest of the three, we choose it to define ideal velocity so that everything else will represent losses. This gives

$$\Delta V_I = \int_{t_a}^{t_b} \left[ \frac{T_v}{m} - \frac{T_v}{m} + \frac{T_s}{m} - \frac{T_s}{m} + \frac{T_{\text{vac}}_s}{m} - \frac{T_{\text{vac}}_s}{m} - \frac{\ddot{V}_I \cdot \ddot{V}_I}{V_I} \right] \, dt \quad (F-5)$$

$$\Delta V_I = \int_{t_a}^{t_b} \left[ \frac{T_{\text{vac}}_s}{m} + \frac{T_s - T_{\text{vac}}_s}{m} + \frac{T_v - T_s}{m} + \frac{\ddot{V}_I}{V_I} - \frac{T_v}{m} \right] \, dt \quad (F-6)$$

In RAGMOP

$$\ddot{V}_I = \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} + \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \quad (F-7)$$

where

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \cos \chi_p & \cos \chi \sin \chi_p & -\sin \chi \sin \chi_p \\ \sin \chi_p & \cos \chi_p & -\sin \chi_p \\ 0 & \sin \chi_p & \cos \chi_p \end{bmatrix} \begin{bmatrix} T_{xs} + T_{xx} - F_{AN} \\ T_{yy} + T_{ys} - F_{AA} \\ T_{zz} + \text{SIDE} \end{bmatrix} \quad (F-8)$$

Taking the dot product of (7) and $\ddot{V}_I/V_I$ leads to
\[
\frac{\vec{v}_I \cdot \vec{V}_I}{V_I} = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} + \frac{1}{m} \left( (T_{xs} + T_{xx}) \cos \alpha_1 + (T_{yy} + T_{ys}) \cos \alpha_2 + T_{zz} \cos \alpha_3 \right) \\
- \frac{1}{m} (F_{AN} \cos \alpha_1 + F_{AA} \cos \alpha_2 - \text{SIDE} \cos \alpha_3) 
\]  

\[\text{(F-9)}\]

where

\[
\cos \alpha_1 = \frac{\vec{v}_I}{V_I} \begin{bmatrix} \cos \chi_p \\ -\sin \chi_p \\ 0 \end{bmatrix}
\]

\[\text{(F-10)}\]

\[
\cos \alpha_2 = \frac{\vec{v}_I}{V_I} \begin{bmatrix} \cos \chi_2 \sin \chi_p \\ \cos \chi_2 \cos \chi_p \\ \sin \chi_2 \end{bmatrix}
\]

\[\cos \alpha_3 = \frac{\vec{v}_I}{V_I} \begin{bmatrix} -\sin \chi_2 \sin \chi_p \\ -\sin \chi_2 \cos \chi_p \\ \cos \chi_2 \end{bmatrix}
\]

If there is a roll angle, \(\chi_r\), to be considered also, the variables in (10) are used to re-define \(\cos \alpha_1\), \(\cos \alpha_2\), and \(\cos \alpha_3\)

\[
\begin{bmatrix} \cos \alpha_1 & \cos \alpha_2 & \cos \alpha_3 \end{bmatrix} = \begin{bmatrix} \cos \chi_1 & \cos \chi_2 & \cos \chi_3 \end{bmatrix} \begin{bmatrix} \cos \chi_r & 0 & \sin \chi_r \\ 0 & 1 & 0 \\ -\sin \chi_r & 0 & \cos \chi_r \end{bmatrix}
\]

\[\text{(F-11)}\]

Rewriting (6) and using either (10) or (11) as desired, we now have

\[
\Delta V_I = \int \frac{t_b}{t_a} \begin{bmatrix} T_{vac} + \frac{T_s - T_{vac}}{m} + \frac{T_v - T_s}{m} + \frac{\vec{v}_I}{V_I} \cdot \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \\ \frac{1}{m} \left\{ (T_{xs} + T_{xx}) \cos \alpha_1 + (T_{yy} + T_{ys}) \cos \alpha_2 + T_{zz} \cos \alpha_3 - T_v \right\} \\ \frac{1}{m} \left\{ -F_{AN} \cos \alpha_1 - F_{AA} \cos \alpha_2 + \text{SIDE} \cos \alpha_3 \right\} \end{bmatrix} dt 
\]

\[\text{(F-12)}\]
With the exception of $\frac{T_{\text{vac}}}{m}$, each of the other terms in (12) is negative on the average and represents a loss. We can therefore define the following:

$$\text{ideal vel} \equiv \int_{t_a}^{t_b} \frac{T_{\text{vac}}}{m} \, dt$$  \hspace{1cm} (F-13)

$$\text{back pressure loss} \equiv \int_{t_a}^{t_b} \left( T_s - T_{\text{vac}} \right) \left( \frac{1}{m} \right) \, dt$$  \hspace{1cm} (F-14)

$$\text{gimbal loss} \equiv \int_{t_a}^{t_b} \left( T_v - T_s \right) \left( \frac{1}{m} \right) \, dt$$  \hspace{1cm} (F-15)

$$\text{gravity loss} \equiv \int_{t_a}^{t_b} \left( \ddot{v}_I - \frac{\ddot{v}_I}{V_I} \right) \cdot \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \, dt$$  \hspace{1cm} (F-16)

$$\text{turning loss} \equiv \int_{t_a}^{t_b} \frac{1}{m} \left[ \left( T_{xs} + T_{xx} \right) \cos \alpha_1 + \left( T_{yy} + T_{ys} \right) \cos \alpha_2 + \right.$$

$$T_{zz} \cos \alpha_3 - T_v \right] dt$$  \hspace{1cm} (F-17)

$$\text{aero loss} \equiv \int_{t_a}^{t_b} \frac{1}{m} \left[ -F_A \cos \alpha_1 - F_A \cos \alpha_2 + \text{SIDE} \cos \alpha_3 \right] \, dt$$  \hspace{1cm} (F-18)

characteristic vel. = ideal vel - back pressure loss - gimbal loss

These losses are such that

$$\Delta V_f = \text{Ideal velocity} - \text{pressure loss} - \text{gimbal loss} - \text{gravity loss} - \text{turning loss} - \text{aero loss}.$$  

F-4
The definition of ideal velocity used here has the added advantage of being analytic in the atmosphere.

However, if other definitions of characteristic velocity are desired, they can be calculated easily from (13) through (15), i.e.

\[ \int_{t_a}^{t_b} \frac{T_s}{m} \, dt = \text{ideal velocity - pressure loss} \]

\[ \int_{t_a}^{t_b} \frac{T_v}{m} \, dt = \text{ideal velocity - pressure loss - gimbal loss} \]

It is suggested therefore that (13) through (18) be used in RAGMOP for the ideal velocity and losses integration.
Appendix G

PARTIAL DERIVATIVES OF THE PAYOFF AND CONSTRAINTS WITH RESPECT TO THE FINAL STATE*

The partial derivatives of the various payoff and terminal constraint quantities with respect to the final state (position xyz, velocity xyz, mass m) are sometimes required to determine the influence coefficient of Paragraph 3.3.4. These derivative calculations are presented in this appendix.

Mass $M$ (final cutoff weight)

\[
\frac{\partial m}{\partial x} = \frac{\partial m}{\partial y} = \frac{\partial m}{\partial z} = \frac{\partial m}{\partial x} = \frac{\partial m}{\partial y} = \frac{\partial m}{\partial z} = 0
\]

\[\frac{\partial m}{\partial m} = 1\]

Inertial Velocity $V_I$

\[
\frac{\partial V_I}{\partial x} = \frac{\partial V_I}{\partial y} = \frac{\partial V_I}{\partial z} = \frac{\partial V_I}{\partial m} = 0
\]

\[\frac{\partial V_I}{\partial x} = \frac{x}{V_I}\]

\[\frac{\partial V_I}{\partial y} = \frac{y}{V_I}\]

\[\frac{\partial V_I}{\partial z} = \frac{z}{V_I}\]

*Extracted in large part from reference 1 (see Section IV).
Inertial Flight Path Angle $\gamma$

\[
\frac{\partial \gamma}{\partial x} = \left( N_{2,1} - \frac{x \sin \gamma}{v_1} \right) / v_1 \cos \gamma
\]

\[
\frac{\partial \gamma}{\partial y} = \left( N_{2,2} - \frac{y \sin \gamma}{v_1} \right) / v_1 \cos \gamma
\]

\[
\frac{\partial \gamma}{\partial z} = \left( N_{2,3} - \frac{z \sin \gamma}{v_1} \right) / v_1 \cos \gamma
\]

\[
\frac{\partial \gamma}{\partial x} = \frac{\partial \gamma}{\partial y} = \frac{\partial \gamma}{\partial z} = \frac{\partial \gamma}{\partial m} = 0
\]

Radius $R$

\[
\frac{\partial R}{\partial x} = \frac{x}{R}
\]

\[
\frac{\partial R}{\partial y} = \frac{y}{R}
\]

\[
\frac{\partial R}{\partial z} = \frac{z}{R}
\]

\[
\frac{\partial R}{\partial x} = \frac{\partial R}{\partial y} = \frac{\partial R}{\partial z} = \frac{\partial R}{\partial m} = 0
\]

Energy $C_3$

\[
\frac{\partial C_3}{\partial x} = 2x
\]

\[
\frac{\partial C_3}{\partial y} = 2y
\]
\[ \frac{\partial C_3}{\partial z} = 2z \]

\[ \frac{\partial C_3}{\partial x} = \frac{2ux}{R^3} \]

\[ \frac{\partial C_3}{\partial y} = \frac{2uy}{R^3} \]

\[ \frac{\partial C_3}{\partial z} = \frac{2uz}{R^3} \]

\[ \frac{\partial C_3}{\partial m} = 0 \]

**Angular Momentum \( C_1 \)**

\[ \frac{\partial C_1}{\partial x} = \frac{(z^* x - xz z - xy y + y^2 x)}{C_1} \]

\[ \frac{\partial C_1}{\partial y} = \frac{(x^* y - xy x - yz z + z^2 y)}{C_1} \]

\[ \frac{\partial C_1}{\partial z} = \frac{(y^* z - yz y - xz x + x^2 z)}{C_1} \]

\[ \frac{\partial C_1}{\partial x} = \frac{(x y^2 - y xy - z xx + x z^2)}{C_1} \]

\[ \frac{\partial C_1}{\partial y} = \frac{(y z^2 - z yz - x xy + y x^2)}{C_1} \]

\[ \frac{\partial C_1}{\partial z} = \frac{(z x^2 - x xz - y yz + z y^2)}{C_1} \]
\[
\frac{\partial C_{m}}{\partial m} = 0
\]

**Inertial Longitude \( \phi \)**

\[
\frac{\partial \phi}{\partial x} = N_{4,1}
\]

\[
\frac{\partial \phi}{\partial y} = N_{4,2}
\]

\[
\frac{\partial \phi}{\partial z} = N_{4,3}
\]

\[
\frac{\partial \phi}{\partial x} = N_{4,4}
\]

\[
\frac{\partial \phi}{\partial y} = N_{4,5}
\]

\[
\frac{\partial \phi}{\partial z} = N_{4,6}
\]

\[
\frac{\partial \phi}{\partial m} = 0
\]

**Inertial Heading Angle \( \theta \)**

\[
\frac{\partial \theta}{\partial x} = \frac{V_{S} N_{14} - W_{S} N_{34}}{W_{S}^{2} + V_{S}^{2}}
\]

\[
\frac{\partial \theta}{\partial y} = \frac{V_{S} N_{15} - W_{S} N_{35}}{W_{S}^{2} + V_{S}^{2}}
\]
\[
\varphi = \frac{V_S \ N_{16} - W_S \ N_{36}}{W_S^2 + V_S^2}
\]

\[
\varphi = \frac{V_S \ N_{11} - W_S \ N_{21}}{W_S^2 + V_S^2}
\]

\[
\varphi = \frac{V_S \ N_{12} + W_S \ N_{32}}{W_S^2 + V_S^2}
\]

\[
\varphi = \frac{V_S \ N_{13} - W_S \ N_{33}}{W_S^2 + V_S^2}
\]

\[
\varphi = 0
\]

**Colatitude \(\theta\)**

\[
\varphi \frac{\partial}{\partial x} = N_{64}
\]

\[
\varphi \frac{\partial}{\partial y} = N_{65}
\]

\[
\varphi \frac{\partial}{\partial z} = N_{66}
\]

\[
\varphi \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} = 0
\]

**Inclination: \(i\)**

\[
\varphi \frac{\partial}{\partial x} = \frac{\sin \theta \cos \theta}{\sin i \ N_{64}} - \frac{\cos \theta \sin \theta}{\sin i} \frac{\partial}{\partial x}
\]

G-5
\[ \frac{\partial i}{\partial y} = -\frac{\sin \theta \cos \phi}{\sin \phi} N_{65} - \frac{\cos \theta \sin \phi}{\sin \phi} \frac{\partial \phi}{\partial y} \]

\[ \frac{\partial i}{\partial z} = -\frac{\sin \theta \cos \phi}{\sin \phi} N_{66} - \frac{\cos \theta \sin \phi}{\sin \phi} \frac{\partial \phi}{\partial z} \]

\[ \frac{\partial i}{\partial x} = -\frac{\sin \theta \cos \phi}{\sin \phi} N_{61} - \frac{\cos \theta \sin \phi}{\sin \phi} \frac{\partial \phi}{\partial x} \]

\[ \frac{\partial i}{\partial y} = -\frac{\sin \theta \cos \phi}{\sin \phi} N_{62} - \frac{\cos \theta \sin \phi}{\sin \phi} \frac{\partial \phi}{\partial y} \]

\[ \frac{\partial i}{\partial z} = -\frac{\sin \theta \cos \phi}{\sin \phi} N_{63} - \frac{\cos \theta \sin \phi}{\sin \phi} \frac{\partial \phi}{\partial z} \]

\[ \frac{\partial i}{\partial m} = 0 \]

Line of Nodes \( \omega \)

Defining \( A = \frac{w_s \cdot v_s \cdot \sin \theta}{v_s^2 + w_s^2 \cdot \cos^2 \theta} \)

and \( B = \frac{(v_s^2 + w_s^2) \cdot \cos \theta}{v_s^2 + w_s^2 \cdot \cos^2 \theta} \)

we have

\[ \frac{\partial \omega}{\partial x} = (N_{44} - A N_{64}) + B \frac{\partial \phi}{\partial x} \]

\[ \frac{\partial \omega}{\partial y} = (N_{45} - A N_{65}) + B \frac{\partial \phi}{\partial y} \]

\[ \frac{\partial \omega}{\partial z} = (N_{46} - A N_{66}) + B \frac{\partial \phi}{\partial z} \]

G-6
\[ \frac{\partial \omega}{\partial x} = B \frac{\partial \alpha}{\partial x} \]

\[ \frac{\partial \omega}{\partial y} = B \frac{\partial \alpha}{\partial y} \]

\[ \frac{\partial \omega}{\partial z} = B \frac{\partial \alpha}{\partial z} \]

\[ \frac{\partial \omega}{\partial m} = 0 \]

**Semi-latus Rectum \( l \)**

\[ \frac{\partial l}{\partial x} = \frac{2R^2}{\mu} (x - U_s N_{21}) \]

\[ \frac{\partial l}{\partial y} = \frac{2R^2}{\mu} (y - U_s N_{22}) \]

\[ \frac{\partial l}{\partial z} = \frac{2R^2}{\mu} (z - U_s N_{23}) \]

\[ \frac{\partial l}{\partial x} = \frac{2R^2}{\mu} \left[ \left( \left( \frac{W_s^2 + V_s^2}{R^2} \right) \frac{x}{R^2} - U_s N_{24} \right) \right] \]

\[ \frac{\partial l}{\partial y} = \frac{2R^2}{\mu} \left[ \left( \left( \frac{W_s^2 + V_s^2}{R^2} \right) \frac{y}{R^2} - U_s N_{25} \right) \right] \]

\[ \frac{\partial l}{\partial z} = \frac{2R^2}{\mu} \left[ \left( \left( \frac{W_s^2 + V_s^2}{R^2} \right) \frac{z}{R^2} - U_s N_{26} \right) \right] \]

\[ \frac{\partial l}{\partial m} = 0 \]
Eccentricity \( e \)

\[
\frac{\partial e}{\partial x} = \frac{C_3}{2\mu} \frac{\partial t}{\partial x} + \frac{t x}{\sqrt{1 + \frac{C_3 t}{\mu} \frac{\partial x}{\partial x}}}
\]

\[
\frac{\partial e}{\partial y} = \frac{C_3}{2\mu} \frac{\partial t}{\partial y} + \frac{t y}{\sqrt{1 + \frac{C_3 t}{\mu} \frac{\partial y}{\partial y}}}
\]

\[
\frac{\partial e}{\partial z} = \frac{C_3}{2\mu} \frac{\partial t}{\partial z} + \frac{t z}{\sqrt{1 + \frac{C_3 t}{\mu} \frac{\partial z}{\partial z}}}
\]

\[
\frac{\partial e}{\partial m} = 0
\]

Burn Time \( T \)

\[
\frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = \frac{\partial T}{\partial x} = \frac{2T}{\partial y} = \frac{2T}{\partial z} = 0
\]

\[
\frac{\partial T}{\partial m} = \frac{1}{m}
\]

G-8
Maximum Dynamic Pressure $Q_{\text{max}}$

\[
\frac{\partial Q_{\text{max}}}{\partial x} = 0 \quad \text{(not dependent on final state)}
\]

True Anomaly $\eta$

Defining

\[
A = - \left( 1 + (w_S^2 + v_S^2) \frac{R}{\mu} \right) \sin \eta / e(w_S^2 + v_S^2)
\]

\[
B = \frac{v_1^2}{I_p} \frac{(w_S^2 + v_S^2) R}{\mu} \left( (w_S^2 + v_S^2) \frac{R}{\mu} - 1 + 2w_S^2 v_S^2 \right) / e(w_S^2 + v_S^2)
\]

and

\[
\zeta = \tan^{-1} \left[ \frac{1}{\sqrt{w_S^2 + v_S^2} \left( 1 - \frac{\mu}{(w_S^2 + v_S^2)R} \right)} \right]
\]

we have

\[
\frac{\partial \eta}{\partial x} = A^x + B^x N_{21}
\]

\[
\frac{\partial \eta}{\partial y} = A^y + B^y N_{22}
\]

\[
\frac{\partial \eta}{\partial z} = A^z + B^z N_{23}
\]

\[
\frac{\partial \eta}{\partial \xi} = - \frac{\sin \xi}{\xi^2} \cdot x + B^x N_{24}
\]

\[
\frac{\partial \eta}{\partial \eta} = - \frac{\sin \eta}{\eta^2} \cdot y + B^y N_{25}
\]

G-9
\[ \frac{3n}{\partial z} = -\frac{\sin \epsilon}{\epsilon^2} z + B_N \]

\[ \frac{3n}{\partial m} = 0 \]

**Argument of Perigee \( \epsilon \)**

**Defining**

\[ \rho = \tan^{-1} \frac{W_S}{V_S} \]

\[ \xi = \cos^{-1} (\sin \theta \sin \phi) \]

\[ A = -\frac{\cos \rho}{\sin^2 \xi} \]

\[ B = \frac{\cos \theta \cos \xi}{\cos \rho} \frac{(W_S^2 + V_S^2) \sin^2 \xi}{(W_S^2 + V_S^2) \sin^2 \xi} \]

**Then**

\[ \frac{\partial \xi}{\partial x} = A_N^{61} + B(V_S N_{11} - W_N N_{31}) + \frac{3n}{\partial x} \]

\[ \frac{\partial \xi}{\partial y} = A_N^{62} + B(V_S N_{12} - W_N N_{32}) + \frac{3n}{\partial y} \]

\[ \frac{\partial \xi}{\partial z} = A_N^{63} + B(V_S N_{13} - W_N N_{33}) + \frac{3n}{\partial z} \]

\[ \frac{\partial \xi}{\partial x} = A_N^{64} + B(V_S N_{14} - W_N N_{34}) + \frac{3n}{\partial x} \]

\[ \frac{\partial \xi}{\partial y} = A_N^{65} + B(V_S N_{15} - W_N N_{35}) + \frac{3n}{\partial y} \]

G-10
\[
\frac{\partial \mathbf{e}}{\partial z} = A N_{66} + B (V S N_{16} - W S N_{36}) + \frac{\partial \mathbf{n}}{\partial z}
\]

\[
\frac{\partial \mathbf{e}}{\partial m} = 0
\]

**Flyback Range R_F**

\[
\frac{\partial R_F}{\partial x} = \frac{\partial R_F}{\partial y} = \frac{\partial R_F}{\partial z} = \frac{\partial R_F}{\partial x} = \frac{\partial R_F}{\partial y} = \frac{\partial R_F}{\partial z} = \frac{\partial R_F}{\partial m} = 0
\]

The \( U_s, V_s, \) and \( W_s \) used in the above derivatives are the velocity components in the spherical inertial coordinate system.

The \( N_{11} \) used above are defined as follows:

\( N \) is a 6 x 6 matrix which can be partitioned into four 3 x 3 submatrices

\[
N = \begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\]

The four submatrices are presented below.

Defining

\[
\theta_1 = \frac{\pi}{2} - \theta_0
\]

where

\( \theta_0 = \) launch site geodetic latitude

and

\[
\theta = \cos^{-1} \frac{x \cos \theta_1 \sin \theta_1 + y \cos \theta_1 - z \sin \theta_1 \sin \theta_1}{r}
\]

where

\( \theta_1 = \) launch azimuth

G-11
we have

\[
N_1 = \begin{bmatrix}
\frac{z \cos \theta + y \sin \alpha \sin \theta}{r \sin \theta} & \frac{-x \sin \alpha \sin \theta - z \cos \alpha \sin \theta}{r \sin \theta} & \frac{y \cos \alpha \sin \theta - x \cos \theta}{r \sin \theta} \\
\frac{x}{r} & \frac{y}{r} & \frac{z}{r} \\
\frac{x \cos \theta - \cos \alpha \sin \theta}{r \sin \theta} & \frac{y \cos \theta - \cos \theta}{r \sin \theta} & \frac{x \cos \theta + \sin \alpha \sin \theta}{r \sin \theta}
\end{bmatrix}
\]

\[
N_{II} = \begin{bmatrix}
\frac{\partial W_S}{\partial x} & \frac{\partial W_S}{\partial y} & \frac{\partial W_S}{\partial z} \\
\frac{\partial U_S}{\partial x} & \frac{\partial U_S}{\partial y} & \frac{\partial U_S}{\partial z} \\
\frac{\partial V_S}{\partial x} & \frac{\partial V_S}{\partial y} & \frac{\partial V_S}{\partial z}
\end{bmatrix}
\]

where

\[
\frac{\partial W_S}{\partial x} = \frac{a_{32} u - a_{22} v - W_S (N_S 31 \cos \theta + N_{21} \sin \theta)}{r \sin \theta}
\]

\[
\frac{\partial W_S}{\partial y} = \frac{a_{12} v - a_{32} w - W_S (N_S 32 \cos \theta + N_{22} \sin \theta)}{r \sin \theta}
\]

\[
\frac{\partial W_S}{\partial z} = \frac{a_{22} w - a_{12} u - W_S (N_S 33 \cos \theta + N_{23} \sin \theta)}{r \sin \theta}
\]

\[
\frac{\partial U_S}{\partial x} = \frac{(W - N_S 21 U_S)}{r}
\]

\[
\frac{\partial U_S}{\partial y} = \frac{(U - N_S 22 U_S)}{r}
\]
\[
\begin{align*}
\frac{\partial V}{\partial x} &= (V - N_{11} U_S)/r \\
\frac{\partial V}{\partial y} &= (N_{12} W_S \cot \theta - N_{31} U_S)/r \\
\frac{\partial V}{\partial z} &= (N_{13} W_S \cot \theta - N_{33} U_S)/r \\
\frac{\partial W}{\partial x} &= (N_{21} W_S \cot \theta - N_{31} V_S)/r \\
\frac{\partial W}{\partial y} &= (N_{22} W_S \cot \theta - N_{32} V_S)/r \\
\frac{\partial W}{\partial z} &= (N_{23} W_S \cot \theta - N_{33} V_S)/r \\
\end{align*}
\]

and

\[
A = \begin{bmatrix} \sin A_z & \cos A_z \sin \theta & -\cos A_z \cos \theta \\ 0 & \cos \theta & \sin \theta \\ \cos A_z & -\sin A_z \sin \theta & \sin A_z \cos \theta \end{bmatrix}
\]

\[
N_{III} = 0
\]

\[
N_{IV} = \begin{bmatrix} \frac{N_{11}}{r \sin \theta} & \frac{N_{12}}{r \sin \theta} & \frac{N_{13}}{r \sin \theta} \\ \frac{N_{21}}{r} & \frac{N_{22}}{r} & \frac{N_{23}}{r} \\ \frac{N_{31}}{r} & \frac{N_{32}}{r} & \frac{N_{33}}{r} \end{bmatrix}
\]

Thus, \( N \) is the matrix of first partial derivatives

\[
N = \frac{\partial S}{\partial P}
\]

where \( S \) is the 6 x 1 vector of spherical coordinate system state components
\[
S = \begin{bmatrix}
W_S \\
U_S \\
V_S \\
\phi \\
r \\
\theta
\end{bmatrix}
\]

and \( P \) is the \( 6 \times 1 \) vector of plumpline components

\[
P = \begin{bmatrix}
w \\
u \\
v \\
x \\
y \\
z
\end{bmatrix}
\]