Reports of the Department of Geodetic Science
Report No. 184

COORDINATE TRANSFORMATION
BY MINIMIZING CORRELATIONS
BETWEEN PARAMETERS

by
Muneendra Kumar

Prepared for
National Aeronautics and Space Administration
Washington, D.C.

Contract No. NGR 36-008-093
OSURF Project No. 2514

The Ohio State University
Research Foundation
Columbus, Ohio 43212

July, 1972
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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and is under the technical direction of James P. Murphy, Special Programs, Code ES, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546

A revised version of this report has been submitted to the Graduate School of The Ohio State University in partial fulfillment of the requirements for the Master of Science degree.
ABSTRACT

The subject of this investigation is to determine the transformation parameters (three rotations, three translations and a scale factor) between two Cartesian coordinate systems from sets of coordinates given in both systems. The objective is the determination of well separated transformation parameters with reduced correlations between each other, a problem especially relevant when the sets of coordinates are not well distributed. The above objective is achieved by preliminarily determining the three rotational parameters and the scale factor from the respective direction cosines and chord distances (these being independent of the translation parameters) between the common points, and then computing all the seven parameters from a solution in which the rotations and the scale factor are entered as weighted constraints according to their variances and covariances obtained in the preliminary solutions.

Numerical tests involving two geodetic reference systems were performed to evaluate the effectiveness of this approach as follows:

(a) A non-constrained solution for general transformation for the seven parameters (including the three translations and scale factor).

(b) A constrained solution for general transformation for the seven parameters utilizing the three rotations with their statistics as constraints.

(c) A constrained solution for general transformation for the seven parameters using the three rotations and scale factor with their statistics as constraints.

The above schemes were then separately repeated for each of the following three cases:
(i) Using the full variance-covariance matrix between coordinates of the geodetic reference systems.

(ii) Using only a $(3 \times 3)$ banded diagonal variance-covariance matrix, thus assuming no correlation between coordinates of any two points within the system.

(iii) Using only variances for the coordinates, thereby further omitting the correlation between the three coordinates of any one point in the system.

In the case of seven parameter general transformation, the best estimates were obtained using full variance-covariance matrix and constraining three rotations and the scale factor, case (c) and (iii) above. The improvement in correlation between translations and rotations was more significant compared to between translation and scale factor.
ACKNOWLEDGMENTS

The writer of this report expresses his sincere gratitude to Dr. Ivan I. Mueller for his constant help, cooperation and most valuable guidance in the execution of this study.

Special indebtedness and thanks are owed by the author to Dr. R. H. Rapp, Dr. N. K. Saxena and Mr. J. P. Reilly for the valuable suggestions and comments.

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1. INTRODUCTION

During the last twenty-five years with the availability of computer technology and its phenomenal growth in basic hardware and core storage capacity and the exceptional increase in a computer's ability of solving problems in lesser and lesser time, a trend has set in to analyze the problems in geodesy and photogrammetry more and more in three dimensional space rather than to follow traditional concepts.

Further, the advent of artificial satellites and their subsequent use in geodesy made it possible to obtain Cartesian coordinates of points on earth surface.

Several projects involving satellite-networks of continental or global extent were begun and at present they are in varying stages of completion. Many new solutions have recently come out, each delineating its own reference system. These systems in reality should differ from each other only in having different origins, sets of axes or scale.

Thus, the relationship between any two such reference systems (e.g., UVW and XYZ) would generally consist of seven parameters—three translations ($\Delta X, \Delta Y, \Delta Z$) between the two origins, three rotations ($\omega, \psi, \epsilon$) of the Euler's angle type between the two sets of axes and the scale factor ($\Delta s$), if any (Figure 1).

![Figure 1.](image_url)
The mathematical model to be used in the computations of the above seven parameters from a least squares solution may be written in the following form [Badekas, 1969; Bursa, 1965; Wolf, 1963]:

\[
\begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3 \\
\end{bmatrix}
= \begin{bmatrix}
    X \\
    Y \\
    Z_i \\
\end{bmatrix}
- \begin{bmatrix}
    \Delta X \\
    \Delta Y \\
    \Delta Z_i \\
\end{bmatrix}
- \begin{bmatrix}
    1 & \omega & \psi \\
    -\omega & 1 & \epsilon \\
    \psi & -\epsilon & 1 \\
\end{bmatrix}
\begin{bmatrix}
    U \\
    V \\
    W_i \\
\end{bmatrix}
- \Delta s
\begin{bmatrix}
    U \\
    V \\
    W_i \\
\end{bmatrix}
= 0,
\]

(1)

where "i" denotes any point common to both the systems. The three angles \( \omega \), \( \psi \), and \( \epsilon \) of the Euler type correspond to small rotations about the Z, Y and X axes respectively—the positive direction of rotations taken in counter clockwise mode, when viewed from the end of the repsective axes towards the origin. It may be worth while to mention here that the station coordinates in both the systems (\( U_1, V_1, W_i \) and \( X_1, Y_1, Z_1 \)) are treated as observations in the above model.

The above equation written in matrix notation can then be modified into the observation equation below [Uotila, 1967]:

\[
BV + AX + W = 0,
\]

(2)

where

\[
B = \begin{bmatrix}
    \frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} & \frac{\partial f_1}{\partial Z} & \frac{\partial f_1}{\partial U} & \frac{\partial f_1}{\partial V} & \frac{\partial f_1}{\partial W} \\
    \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} & \frac{\partial f_2}{\partial Z} & \frac{\partial f_2}{\partial U} & \frac{\partial f_2}{\partial V} & \frac{\partial f_2}{\partial W} \\
    \frac{\partial f_3}{\partial X} & \frac{\partial f_3}{\partial Y} & \frac{\partial f_3}{\partial Z} & \frac{\partial f_3}{\partial U} & \frac{\partial f_3}{\partial V} & \frac{\partial f_3}{\partial W} \\
\end{bmatrix}_i
\]

\[
= \begin{bmatrix}
    1 & 0 & 0 & -1 & 0 & 0 \\
    0 & 1 & 0 & 0 & -1 & 0 \\
    0 & 0 & 1 & 0 & 0 & -1 \\
\end{bmatrix},
\]

\[
A = \begin{bmatrix}
    \frac{\partial f_1}{\partial \Delta X} & \frac{\partial f_1}{\partial \Delta Y} & \frac{\partial f_1}{\partial \Delta Z} & \frac{\partial f_1}{\partial \Delta U} & \frac{\partial f_1}{\partial \Delta V} & \frac{\partial f_1}{\partial \Delta W} \\
    \frac{\partial f_2}{\partial \Delta X} & \frac{\partial f_2}{\partial \Delta Y} & \frac{\partial f_2}{\partial \Delta Z} & \frac{\partial f_2}{\partial \Delta U} & \frac{\partial f_2}{\partial \Delta V} & \frac{\partial f_2}{\partial \Delta W} \\
    \frac{\partial f_3}{\partial \Delta X} & \frac{\partial f_3}{\partial \Delta Y} & \frac{\partial f_3}{\partial \Delta Z} & \frac{\partial f_3}{\partial \Delta U} & \frac{\partial f_3}{\partial \Delta V} & \frac{\partial f_3}{\partial \Delta W} \\
\end{bmatrix}_i
\]

\[-2-\]
while \( V \) and \( X \) represent the residuals to the observations and corrections to the parameter estimates, respectively. Hence, collecting all the matrices as above, pointwise in the systems, the observation equation becomes:

\[
\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 \\
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
\end{bmatrix}
+ \begin{bmatrix}
-1 & 0 & 0 & -U & -V & W & 0 \\
0 & -1 & 0 & -V & U & 0 & -W \\
0 & 0 & -1 & -W & 0 & -U & V \\
\end{bmatrix}_i
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z \\
\Delta s \\
\omega \\
\psi \\
\epsilon \\
\end{bmatrix}_i
= 0
\]

Defining the geodetic reference systems on the assumption that the Laplace-condition has been enforced throughout the network (which implies that the axes of the reference ellipsoid are parallel to the conventional earth-fixed axes), many experiments have been made in recent times to determine the seven transformation parameters in relating the different geodetic systems to each other using an observation equation of type (3) [Lambeck, 1971; Marsh et al., 1971].
However, in the above general transformation, if the geodetic reference systems are properly oriented through the Laplace-condition, the three rotations arising due to the improper relative orientation of the systems are generally never more than a few seconds of arc, while translations may amount up to 200 to 300 meters. Also, due to the presence of high correlations between the rotations, the scale factor and the translations, satisfactory independent estimates for these parameters are difficult to obtain from a combined general solution using equation (3).

This investigation separates the determinations of the rotations and the scale factor (from that of the translations) for subsequent use as constraints in a combined general solution.

2. THE INDEPENDENT DETERMINATIONS OF ROTATIONAL AND SCALAR PARAMETERS

2.1 Determination of Rotations

2.1.1 Mathematical Model

The mathematical model used in this study is as follows [Bursa, 1966]:

\[ T_{ik}^{(1)} - T_{ik}^{(2)} + \omega + \psi \sin T_{ik}^{(1)} \tan \delta_{ik}^{(1)} - \epsilon \cos T_{ik}^{(1)} \tan \delta_{ik}^{(1)} = 0 \]

\[ \delta_{ik}^{(1)} - \delta_{ik}^{(2)} + \psi \cos T_{ik}^{(1)} + \epsilon \sin T_{ik}^{(1)} = 0 \]

(4)

where \( T_{ik} \) and \( \delta_{ik} \) are defined as the geodetic hour angle and declination of the \((i-k)\)th direction of the observed point at \( k \)th station and the observer at \( i \)th station. The indexes (1) and (2) denote the two systems with the transformation proceeding from system #1 to system #2.

If \( A_{ik}, B_{ik}, C_{ik} \) are taken to denote the direction cosines of the \((i-k)\)th line of length \( R_{ik} \), then for the first (UVW) system one gets:
In the above relations (4) through (6) the elements of translation do not enter the picture. A similar set of relations as per (5) and (6) can be established for the second (XYZ) system.

2.1.2 Observation Equations

The mathematical model (4) then, for each \((i-k)\)th line, yields the following generalized form of observation equations [Uotila, 1967]:

\[
\begin{bmatrix}
\sin T_{ik}^{(1)} \tan \delta_{ik}^{(1)} & -\cos T_{ik}^{(1)} \tan \delta_{ik}^{(1)} & \omega \\
0 & \cos T_{ik}^{(1)} & \sin T_{ik}^{(1)} \\
\end{bmatrix}
\begin{bmatrix}
v_t \\
v_b \\
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
v_t \\
v_b \\
\end{bmatrix}
= 0
\]

(7)

Using the conventional weight matrix \(P\) for the coordinates of points included in the transformation (see section 2.1.3), and the principle of least squares by making \(V^T PV\) as minimum, the equation (7) is then solved for correction vector \((\omega, \psi, \epsilon)\) and for the variance-covariance matrix \((\Sigma \omega \psi \epsilon)\) of the three parameters.
### 2.1.3 Weights

Using the variance-covariance matrices \( \Sigma X \) and \( \Sigma U \) in respect of \( i \)th and \( k \)th points for the XYZ and UVW systems, the variance-covariance matrices \( \Sigma_{ij} \) for the two systems of coordinates can be computed through propagation of errors [Uotila, 1967].

Two distinct cases would arise here. Firstly, when in addition to correlation between X, Y, Z-coordinates of any point, the correlation between the coordinates of one point to others is also considered. In such a case, the necessary relation will be

\[
\begin{bmatrix}
\Sigma_{ij}^G
\end{bmatrix}_a, z = G \begin{bmatrix}
\Sigma U_i & \Sigma U_{ik} \\
\Sigma U_{ik} & \Sigma U_k
\end{bmatrix} G'
\]

where

\[
G = \begin{bmatrix}
\frac{\partial T_{ik}^{(1)}}{\partial U_1} & \frac{\partial T_{ik}^{(1)}}{\partial V_1} & \frac{\partial T_{ik}^{(1)}}{\partial W_1} & \frac{\partial T_{ik}^{(1)}}{\partial U_k} & \frac{\partial T_{ik}^{(1)}}{\partial V_k} & \frac{\partial T_{ik}^{(1)}}{\partial W_k} \\
\frac{\partial \delta_{ik}^{(1)}}{\partial U_1} & \frac{\partial \delta_{ik}^{(1)}}{\partial V_1} & \frac{\partial \delta_{ik}^{(1)}}{\partial W_1} & \frac{\partial \delta_{ik}^{(1)}}{\partial U_k} & \frac{\partial \delta_{ik}^{(1)}}{\partial V_k} & \frac{\partial \delta_{ik}^{(1)}}{\partial W_k}
\end{bmatrix}
\]

and

\[
\frac{\partial T_{ik}}{\partial U_1} = -\frac{\partial T_{ik}}{\partial U_k} = -\frac{\Delta V_{ik}}{\Delta U_{ik}^2 + \Delta V_{ik}^2},
\]

\[
\frac{\partial T_{ik}}{\partial V_1} = -\frac{\partial T_{ik}}{\partial V_k} = -\frac{\Delta U_{ik}}{\Delta U_{ik}^2 + \Delta V_{ik}^2},
\]

\[
\frac{\partial T_{ik}}{\partial W_1} = -\frac{\partial T_{ik}}{\partial W_k} = 0,
\]

\[
\frac{\partial \delta_{ik}}{\partial U_1} = -\frac{\partial \delta_{ik}}{\partial U_k} = \frac{\Delta U_{ik} \Delta W_{ik}}{R_{ik}^{2(1)} \Delta U_{ik}^2 + \Delta V_{ik}^2},
\]

\[
\frac{\partial \delta_{ik}}{\partial V_1} = -\frac{\partial \delta_{ik}}{\partial V_k} = \frac{\Delta V_{ik} \Delta W_{ik}}{R_{ik}^{2(1)} \Delta U_{ik}^2 + \Delta V_{ik}^2},
\]
Secondly, ignoring the correlations between the coordinates of different points within a system, equation (8) can be modified as under:

\[
\frac{\partial \delta_{ik}}{\partial W_i} = -\frac{\partial \delta_{ik}}{\partial W_k} = -\frac{\Delta U_{ik}^2 + \Delta V_{ik}^2}{R_{1k}^{(1)}}.
\]

\[
R_{1k}^{(1)} = \Delta U_{ik}^2 + \Delta V_{ik}^2 + \Delta W_{ik}^2.
\]

In the equations (8) and (9), \( \Sigma U_i \) and \( \Sigma U_k \) correspond to \( i^{th} \) and \( k^{th} \) point of the first system and can be either full (3 x 3) matrices with covariances between the three coordinates of a point, or may contain variances for \( U, V \) and \( W \) in a diagonal form only. However, in the case of covariances (\( \Sigma U_{ik} \)) between the points being included, the matrix in equation (8) would be a full (6 x 6).

Obtaining similarly \( \Sigma_{1k}^{(2)} \), the combined variance-covariance matrix, to be used with equation (7), is given by:

\[
\begin{bmatrix}
\Sigma_{1k}^{(1)} \\
\Sigma_{1k}^{(2)}
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
= G
\begin{bmatrix}
\Sigma U_i \\
0
\end{bmatrix}
G'.
\]

(9)

It may be noted here that the matrix \( P \) is always in 2 x 2 banded diagonal form.

2.2 Determination of Scale Factor

2.2.1 Mathematical Model

The scale factor between the systems #1 and #2 would be given as follows:

\[
\Delta s_{ik} = \frac{R_{1k}^{(2)}}{R_{1k}^{(1)}} - 1.
\]

(11)
where \[ R_{ik}^{(2)} = (\Delta X_{ik}^2 + \Delta Y_{ik}^2 + \Delta Z_{ik}^2)^{\frac{1}{2}} \]
\[ R_{ik}^{(1)} = (\Delta U_{ik}^2 + \Delta V_{ik}^2 + \Delta W_{ik}^2)^{\frac{1}{2}} \]

2.2.2. Weights

Using the variance-covariances matrices \( \Sigma X \) and \( \Sigma U \) for the coordinates of \( i \)th and \( k \)th points in the two systems included in the transformation (section 2.1.3), a variance \( \sigma_{\alpha\nu}^2 \) is established for the scale factor through error propagation. Two cases similar to equations (8) and (9) would arise according to the case when full variance-covariance matrix between different points within the system is considered or not.

The matrix \( G \) for the scale factor determination is

\[
G = \left[ \frac{\partial \Delta s}{\partial U_i} \frac{\partial \Delta s}{\partial V_i} \frac{\partial \Delta s}{\partial W_i} \frac{\partial \Delta s}{\partial U_k} \frac{\partial \Delta s}{\partial V_k} \frac{\partial \Delta s}{\partial W_k} \frac{\partial \Delta s}{\partial X_i} \frac{\partial \Delta s}{\partial Y_i} \frac{\partial \Delta s}{\partial Z_i} \frac{\partial \Delta s}{\partial X_k} \frac{\partial \Delta s}{\partial Y_k} \frac{\partial \Delta s}{\partial Z_k} \right],
\]

where

\[ \frac{\partial \Delta s}{\partial U_i} = -\frac{\partial \Delta s}{\partial U_k} = \frac{\Delta U_{ik} \cdot R_{ik}^{(2)} \cdot R_{ik}^{(2)}}{[R_{ik}^{(1)}]^3 R_{ik}^{(2)}],} \]
\[ \frac{\partial \Delta s}{\partial V_i} = -\frac{\partial \Delta s}{\partial V_k} = \frac{\Delta V_{ik} \cdot R_{ik}^{(2)} \cdot R_{ik}^{(2)}}{[R_{ik}^{(1)}]^3 R_{ik}^{(2)}],} \]
\[ \frac{\partial \Delta s}{\partial W_i} = -\frac{\partial \Delta s}{\partial W_k} = \frac{\Delta W_{ik} \cdot R_{ik}^{(2)} \cdot R_{ik}^{(2)}}{[R_{ik}^{(1)}]^3 R_{ik}^{(2)}],} \]
\[ \frac{\partial \Delta s}{\partial X_i} = -\frac{\partial \Delta s}{\partial X_k} = -\frac{\Delta X_{ik} \cdot R_{ik}^{(2)} \cdot R_{ik}^{(2)}}{[R_{ik}^{(1)}] \cdot R_{ik}^{(2)}}, \]
\[ \frac{\partial \Delta s}{\partial Y_i} = -\frac{\partial \Delta s}{\partial Y_k} = \frac{\Delta Y_{ik} \cdot R_{ik}^{(2)} \cdot R_{ik}^{(2)}}{[R_{ik}^{(1)}] \cdot R_{ik}^{(2)}}, \]
\[ \frac{\partial \Delta s}{\partial Z_i} = -\frac{\partial \Delta s}{\partial Z_k} = \frac{\Delta Z_{ik} \cdot R_{ik}^{(2)} \cdot R_{ik}^{(2)}}{[R_{ik}^{(1)}] \cdot R_{ik}^{(2)}}. \]
Hence,

\[ \sigma^2_{\Delta s_{ik}} = G \begin{bmatrix} \Sigma U_1 & \Sigma U_{ik} \\ \Sigma U_{ik} & \Sigma U_{kk} \\ 0 & 0 \\ \Sigma X_1 & \Sigma X_{ik} \end{bmatrix} G^t \]  

where the full \((12 \times 12)\) matrix would become a \((3 \times 3)\) banded diagonal matrix in case \(\Sigma U_{ik}\) and \(\Sigma X_{ik}\) are zero, i.e., covariances are not considered. The complete \((12 \times 12)\) matrix would assume a diagonal pattern when only variances are used for station coordinates.

Using the value of \(\Delta s_{ik}\) and \(\sigma^2_{\Delta s_{ik}}\) from equations (11) and (12), the value for weighted mean and its variance for the transformation under investigation is established as given below [Hirvonen, 1971]:

\[ \Delta s_n = \frac{[w_{ik} \cdot \Delta s_{ik}]}{[w_{ik}]} \]  

\[ \sigma^2_{\Delta s_n} = \frac{[w_{ik} \cdot (\Delta s_{ik} - \Delta s_n)^2]}{[w_{ik}](n-1)} \]  

where \(w_{ik} = 1/\sigma_{\Delta s_{ik}}\) and \([w_{ik}]\) denotes the sum of all such weights.

\[ n = \text{Total number of scale factor values used in the sample.} \]

3. BRIEF DISCUSSION ON THE FORTRAN PROGRAM

Appendix I gives the complete computer program for obtaining the constrained or non-constrained solution for seven parameters. With appropriate coding non-constrained solutions for three parameters (\(\Delta X, \Delta Y\) and \(\Delta Z\)) and scale factor \(\Delta s\) can also be obtained.

The input coordinates can either be Cartesian or geodetic (ellipsoidal) with 35 as the maximum number of points in each system. However, the matrices can easily be re-dimensionalized to accommodate more points when required. The
program is self-explanatory with regard to definition of various option codes for input, type of solution and inclusion of correlation data, etc.

The broad basic divisions of the program are as under:

(a) **Main Program**: This section takes as input the various options in input/solutions, coordinates of points, rectangular or ellipsoidal, and semimajor axis and flattening of the ellipsoid used, if required. It then prints out the two sets of coordinates used for checking purposes.

The various options of input/solutions have been designated in the program as KCODE e.g., KCODE (1) refers to number of common points involved in the transformation. A complete list with necessary explanatory remarks has been included in the beginning of the program.

(b) **Subroutine "EULERS"**: This subroutine first reads the variance-covariance matrices of the station coordinates, with or without correlation, and then sets up matrices \(A, W\) and \(P\) to be used for the solutions of three rotations through direction cosines (equation (7)).

The subroutine writes up the variance-covariance matrices for the coordinates on the disk and stores the estimates for \(\omega, \psi\) and \(\epsilon\), and their variance-covariance matrix \([\Sigma \omega \psi \epsilon]\) in the common block for subsequent use.

(c) **Subroutine "SCALE"**: This subroutine computes the weighted mean value for scale factor \(\Delta s\) and its variance by direct chord comparison independent of other transformation parameters (equations (13) and (14)).

(d) **Subroutine "TFORM"**: This subroutine solves for a general transformation (equation (3)), utilizing the common block core memory for coordinates of points and variance-covariance matrices from the disk.

The matrix \(M^2\) to be utilized for generating normal equations is computed by calling another subroutine "SETUP".

**NOTE**: In case the solution is required ONLY for three translation or three translations and scale factor, KCODE (3) is coded as "0" and then subroutine "EULERS" is skipped by the program.
(e) **Subroutine "CSTRNT":** This subroutine uses the results of subroutines SCALE and E ULERS as constraints with their appropriate statistics and computes for a constrained solution of seven parameters. The results are returned to subroutine TFORM for printout. KCODE (11) refers to the option whether 3 or 4 parameters are to be constrained.

(f) **Subroutine "RESIDU":** This subroutine computes the residuals vector $V$ for observations i.e., the station coordinates used in the program. The residuals are printed station wise for both systems #1 and #2.

In the computer program, the storage mode used for major computation is in vector form for increased flexibility and saving of core storage.

Appendix II gives a typical set of Job Control Cards (JCL).
4. NUMERICAL EXAMPLE

The above transformation models were used to study the relationship between the transformation parameters and obtaining their best estimates by minimizing correlation for the following two reference systems:

(i) System MPS-7, [Mueller and Whiting, 1972].
(ii) System NA-9, [Mueller et. al., 1972].

Using the same set of thirty common stations of the above two systems, the following solutions were obtained during the investigation:

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>Type of Variance-Covariance Matrix Used</th>
<th>7-Parameter General Transformation</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Unconstrained Solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constrained Solution @</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constraints: 3 Rotation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constraints: 3 Rotations and Scale Factor</td>
</tr>
<tr>
<td>(i)</td>
<td>Only Variances</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>(ii)</td>
<td>(3 x 3) Banded Diagonal Variance-Covariance Matrix</td>
<td>√</td>
</tr>
<tr>
<td>(iii)</td>
<td>Full Variance-Covariance Matrix</td>
<td>√</td>
</tr>
</tbody>
</table>

@Note: The constraints for these solutions (rotations and/or scale factor) with their statistics were computed independently of the translation parameters (subroutine EULERS and SCALE of the Fortran IV program).

Two solutions in full have been appended in the report as specimens in Tables 1 and 2 as under:

Table 1: Sample printout of the solution for three rotations (ω,ψ,ε) and scale factor (Δs), using full variance-covariance matrix.

Table 2: Sample printout of the constrained seven parameter general solution between NA-9 and MPS-7 with three rotations and
TABLE 1

Sample Printout of the solutions for three rotations as parameters and the scale factor, using full variance-covariance matrix.
**TABLE 1**

**SOLUTION FOR '3' ROTATION PARAMETERS**

*(FROM DIRECTION COSINES -- UNITS SECONDS OF ARC)*

*(USING FULL VARIANCE-COVARIANCE MATRIX)*

<table>
<thead>
<tr>
<th>OMEGA</th>
<th>PSI</th>
<th>EPSILON</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1693791D+00</td>
<td>-0.3520145D-01</td>
<td>-0.2173630D+00</td>
</tr>
</tbody>
</table>

**VARIANCE - COVARIANCE MATRIX**

\[
\begin{array}{ccc}
0.16753861D-02 & 0.40623287D-03 & -0.93767764D-03 \\
0.40622287D-03 & 0.12317991D-02 & -0.48803740D-03 \\
-0.93767764D-03 & -0.48803740D-03 & 0.27191935D-02 \\
\end{array}
\]

**COEFFICIENT OF CORRELATION**

\[
\begin{array}{ccc}
0.10000000D+01 & 0.28277933D+00 & -0.43931501D+00 \\
0.28277933D+00 & 0.10000000D+01 & -0.26666321D+00 \\
-0.43931501D+00 & -0.26666321D+00 & 0.10000000D+01 \\
\end{array}
\]

**SOLUTION FOR SCALE FACTOR**

*(FROM CHORD COMPARISON)*

<table>
<thead>
<tr>
<th>SCALE FACTOR (10.D+5)</th>
<th>VARIANCE (10.D+11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.16</td>
<td>0.06</td>
</tr>
</tbody>
</table>

-14-
TABLE 2

Sample printout of the constrained seven parameters general solution, using full variance-covariance matrix (case (c)/(iii)).
## TABLE 2

### SCALE FACTOR AND ROTATION PARAMETERS CONstrained

**SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS**

*(USING FULL VARIANCE-COVARIANCE MATRIX)*

<table>
<thead>
<tr>
<th></th>
<th>DX</th>
<th>DY</th>
<th>DZ</th>
<th>DL</th>
<th>OMEGA</th>
<th>PSI</th>
<th>EPSILON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meters</td>
<td>Meters</td>
<td>Meters</td>
<td>(10.4*5)</td>
<td>Seconds</td>
<td>Seconds</td>
<td>Seconds</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-45.38</td>
<td>171.94</td>
<td>187.44</td>
<td>5.14</td>
<td>0.17</td>
<td>-0.04</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

**VARIANCE-COVARIANCE MATRIX**

\[
\begin{bmatrix}
M_{02} & 0.84 \\
0.176D+01 & 0.250D+00 & 0.453D+00 & 0.310D-07 & 0.126D-06 & 0.778D-07 & -0.652D-07 \\
0.250D+00 & 0.228D+01 & -0.322D-01 & 0.243D-06 & 0.551D-07 & 0.238D-07 & -0.124D-06 \\
0.453D+00 & -0.322D-01 & 0.206D+01 & -0.144D-06 & 0.615D-07 & 0.222D-07 & -0.177D-06 \\
-0.310D-07 & 0.243D-06 & -0.149D-06 & 0.441D-13 & -0.325D-17 & -0.298D-16 & -0.127D-16 \\
0.126D-06 & 0.551D-07 & 0.615D-07 & -0.325D-17 & 0.225D-13 & 0.525D-14 & -0.125D-13 \\
0.778D-07 & 0.238D-07 & 0.222D-07 & -0.298D-16 & 0.525D-14 & 0.167D-13 & -0.654D-14 \\
-0.852D-07 & -0.124D-06 & -0.177D-06 & -0.127D-16 & -0.125D-13 & -0.654D-14 & 0.364D-13 \\
\end{bmatrix}
\]

**COEFFICIENTS OF CORRELATION**

\[
\begin{bmatrix}
0.100D+01 & 0.125D+00 & 0.238D+00 & -0.111D+00 & 0.635D+00 & 0.454D+00 & -0.337D+00 \\
0.125D+00 & 0.100D+01 & -0.149D-01 & 0.765D+00 & 0.244D+00 & 0.122D+00 & -0.429D+00 \\
0.238D+00 & -0.149D-01 & 0.100D+01 & -0.493D+00 & 0.286D+00 & 0.122D+00 & -0.648D+00 \\
-0.111D+00 & 0.765D+00 & -0.493D+00 & 0.100D+01 & -0.103D-03 & 0.110D-02 & -0.317D-03 \\
0.635D+00 & 0.244D+00 & 0.286D+00 & -0.103D-03 & 0.100D+01 & 0.271D+00 & -0.436D+00 \\
0.454D+00 & 0.122D+00 & 0.122D+00 & -0.110D-02 & 0.271D+00 & 0.100D+01 & -0.265D+00 \\
-0.337D+00 & -0.429D+00 & -0.648D+00 & -0.317D-03 & 0.436D+00 & 0.265D+00 & 0.100D+01 \\
\end{bmatrix}
\]
scale factor as constraints, using full variance-covariance matrix (case (c)/(iii)).

A summary of the results for cases (a) through (c) and (i) through (iii) are presented in the following tables:

**TABLE 3** gives the results for three rotations, as obtained independently of translations and scale factor from direction cosines, for cases (i) through (iii).

**TABLE 4** gives the results for the scale factor, as obtained by direct chord comparisons independent of other transformation parameters, for cases (i) through (iii).

**TABLE 5** gives the results for the constrained and non-constrained seven parameters general transformation solutions (cases (a) through (c) and (i) through (iii)).

**TABLE 6** gives the comparative study of the results for seven parameters general transformation solutions as regards correlation between translations and rotations/scale factor, using different variance-covariance matrices (cases (i) through (iii)).

**TABLE 7** gives the comparative study of the results for seven parameters general transformation solutions as regards correlation between translations and rotations/scale factor, using different constraints (cases (a) through (c)).
### TABLE 3

Three Rotation Parameters from Direction Cosines

**NA-9~MPS-7**

<table>
<thead>
<tr>
<th>Case</th>
<th>Using Variances Only</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix</th>
<th>Using full Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega ) (°)</td>
<td>0.17 ± 0.05</td>
<td>0.17 ± 0.04</td>
<td>0.17 ± 0.04</td>
</tr>
<tr>
<td>( \psi ) (°)</td>
<td>0.04 ± 0.04</td>
<td>-0.02 ± 0.04</td>
<td>-0.04 ± 0.04</td>
</tr>
<tr>
<td>( \epsilon ) (°)</td>
<td>-0.20 ± 0.06</td>
<td>-0.24 ± 0.05</td>
<td>-0.22 ± 0.05</td>
</tr>
<tr>
<td>( \sigma_0^2 )</td>
<td>1.15</td>
<td>1.30</td>
<td>1.36</td>
</tr>
</tbody>
</table>

### TABLE 4

Scale Factor From Chord Comparison

**NA-9~MPS-7**

<table>
<thead>
<tr>
<th>Case</th>
<th>Using Variances Only</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix</th>
<th>Using full Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta s \times 10^5 )</td>
<td>5.46 ± 0.24</td>
<td>5.37 ± 0.24</td>
<td>5.18 ± 0.24</td>
</tr>
<tr>
<td>Case</td>
<td>Non-Constrained Solutions</td>
<td>Constrained Solutions</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>--------------------------</td>
<td>----------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a)/(i)</td>
<td>(a)/(ii)</td>
<td>(a)/(iii)</td>
</tr>
<tr>
<td>ΔX (m)</td>
<td>-44.5 ± 5.2</td>
<td>-44.9 ± 3.6</td>
<td>-44.9 ± 3.6</td>
</tr>
<tr>
<td>ΔY (m)</td>
<td>171.5 ± 5.1</td>
<td>170.3 ± 4.7</td>
<td>170.3 ± 4.7</td>
</tr>
<tr>
<td>ΔZ (m)</td>
<td>190.4 ± 5.5</td>
<td>190.4 ± 4.3</td>
<td>190.4 ± 4.3</td>
</tr>
<tr>
<td>ω (&quot;')</td>
<td>0.15 ± 0.16</td>
<td>0.17 ± 0.12</td>
<td>0.17 ± 0.12</td>
</tr>
<tr>
<td>ψ (&quot;')</td>
<td>0.04 ± 0.14</td>
<td>-0.03 ± 0.11</td>
<td>-0.03 ± 0.11</td>
</tr>
<tr>
<td>ε (&quot;')</td>
<td>-0.30 ± 0.20</td>
<td>-0.28 ± 0.15</td>
<td>-0.28 ± 0.15</td>
</tr>
<tr>
<td>Δθ (10^6)</td>
<td>4.9 ± 0.7</td>
<td>4.7 ± 0.7</td>
<td>4.7 ± 0.7</td>
</tr>
<tr>
<td>σ_e</td>
<td>0.95</td>
<td>0.83</td>
<td>0.83</td>
</tr>
</tbody>
</table>
### TABLE 6

Comparative Study of Correlation Coefficients
Between Transformation Parameters
(Using Different Variance-Covariance Matrices)

**Case (i): USING VARIANCES ONLY**

<table>
<thead>
<tr>
<th>Case</th>
<th>Non-Constrained Solution</th>
<th>Constrained Solutions</th>
<th>3 Rotations</th>
<th>3 Rotations and Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td></td>
<td>Translations</td>
<td>Translations</td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>Rotations and Scale Factor</td>
<td></td>
<td></td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>ω</td>
<td>0.88</td>
<td>0.40</td>
<td>0.43</td>
<td>0.68</td>
</tr>
<tr>
<td>ψ</td>
<td>0.63</td>
<td>0.19</td>
<td>0.13</td>
<td>0.49</td>
</tr>
<tr>
<td>ε</td>
<td>-0.47</td>
<td>-0.67</td>
<td>-0.88</td>
<td>-0.38</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.10</td>
<td>0.74</td>
<td>-0.40</td>
<td>-0.29</td>
</tr>
</tbody>
</table>

**Case (ii): USING (3 x 3) BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX**

<table>
<thead>
<tr>
<th>Case</th>
<th>Non-Constrained Solution</th>
<th>Constrained Solutions</th>
<th>3 Rotations</th>
<th>3 Rotations and Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td></td>
<td>Translations</td>
<td>Translations</td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>Rotations and Scale Factor</td>
<td></td>
<td></td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>ω</td>
<td>0.83</td>
<td>0.27</td>
<td>0.33</td>
<td>0.58</td>
</tr>
<tr>
<td>ψ</td>
<td>0.54</td>
<td>0.11</td>
<td>0.13</td>
<td>0.38</td>
</tr>
<tr>
<td>ε</td>
<td>-0.45</td>
<td>-0.51</td>
<td>0.80</td>
<td>-0.32</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.15</td>
<td>0.84</td>
<td>-0.56</td>
<td>-0.36</td>
</tr>
</tbody>
</table>
TABLE 6 (Continued)

Case (iii): USING FULL VARIANCE-COVARIANCE MATRIX

<table>
<thead>
<tr>
<th>Case</th>
<th>Non-Constrained Solution</th>
<th>Constrained Solutions</th>
<th>Constrained Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rotations</td>
<td>Translations</td>
<td>3 Rotations</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td></td>
<td>Rotations and Scale Factor</td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>ω</td>
<td>0.83</td>
<td>0.27</td>
<td>0.33</td>
</tr>
<tr>
<td>β</td>
<td>0.54</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>ε</td>
<td>-0.45</td>
<td>-0.51</td>
<td>-0.80</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.15</td>
<td>0.84</td>
<td>-0.56</td>
</tr>
</tbody>
</table>
### TABLE 7

Comparative Study of Correlation Coefficients Between Transformation Parameters
(Using Different Constraints)

#### Case (a): NON-CONSTRAINED SOLUTION

<table>
<thead>
<tr>
<th>Case</th>
<th>Using Variances Only</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix</th>
<th>Using Full Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
</tr>
<tr>
<td>Rotations and Scale Factor</td>
<td>Translations</td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>ω</td>
<td>0.88</td>
<td>0.40</td>
<td>0.43</td>
</tr>
<tr>
<td>ψ</td>
<td>0.63</td>
<td>0.19</td>
<td>0.13</td>
</tr>
<tr>
<td>ε</td>
<td>-0.47</td>
<td>-0.67</td>
<td>-0.88</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.10</td>
<td>0.74</td>
<td>-0.40</td>
</tr>
</tbody>
</table>

#### Case (b): CONSTRAINED SOLUTIONS
(CONSTRAINTS: 3 ROTATIONS)

<table>
<thead>
<tr>
<th>Case</th>
<th>Using Variances Only</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix</th>
<th>Using Full Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
</tr>
<tr>
<td>Rotations and Scale Factor</td>
<td>Translations</td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>ω</td>
<td>0.68</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>ψ</td>
<td>0.49</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>ε</td>
<td>-0.38</td>
<td>-0.23</td>
<td>-0.45</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.29</td>
<td>0.95</td>
<td>-0.83</td>
</tr>
</tbody>
</table>
**TABLE 7 (Continued)**

Case (c): CONSTRAINED SOLUTIONS

(CONSTRAINTS: 3 ROTATIONS AND SCALE FACTOR)

<table>
<thead>
<tr>
<th>Rotations and Scale Factor</th>
<th>Using Variances Only (i)</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix (ii)</th>
<th>Using Full Variance-Covariance Matrix (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ΔX</td>
<td>ΔY</td>
<td>ΔZ</td>
</tr>
<tr>
<td>ω</td>
<td>0.71</td>
<td>0.32</td>
<td>0.35</td>
</tr>
<tr>
<td>ψ</td>
<td>0.51</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>ε</td>
<td>-0.40</td>
<td>-0.51</td>
<td>-0.73</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.10</td>
<td>0.72</td>
<td>-0.44</td>
</tr>
</tbody>
</table>

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5. CONCLUSIONS

The comparison between different columns of Table 3 shows that the estimates for three rotation parameters remain more or less the same, but that their standard deviations show some improvement as we proceed from column 1 (variances only) to column 3 (full variance-covariance matrix). However, in the case of scale factor (Table 4) the estimates for $\Delta s$ indicate a definite trend while standard deviation remains constant.

In the case of seven parameters general transformation (Table 5) the comparisons among different columns indicate a definite overall improvement in all parameter estimates. The best estimates were obtained in the solution using full variance-covariance matrix and three rotations ($\omega, \psi, \epsilon$) and scale factor ($\Delta s$) as constraints (column 10). In this case the standard deviations for all the parameters are smaller (or at the most, equal) compared to those in any other column of Table 5.

Further, it is also noticeable that the improvement from a non-constrained solution to a constrained solution, both with three or four constraints, is more significant compared to the improvement from a constrained solution using variances only to a constrained solution using $(3 \times 3)$ banded diagonal or full variance-covariance matrix. The improvement from the solution using $(3 \times 3)$ banded diagonal to the solution using full variance-covariance matrix is, however, marginal.

A study of Table 6 indicates in all the three cases an overall improvement in correlation from a non-constrained to a constrained solution with four constraints (three rotations and one scale factor). The improvement in correlation between translations and rotations is quite significant while the same in not reflected between translations and scale factor. However, the improvement pattern from Table 7 is not straightforward. The correlations between translations and rotations show a downward trend from the solutions using variances only to the solutions using full variance-covariance matrix in all the three cases while the correlations between translations and $\Delta s$ show an upward trend.
REFERENCES


Mueller, Ivan I. and Marvin C. Whiting (1972). "Free Adjustment of a Geometric Global Satellite Network (Solution MPS-7)," Reports of the Department of Geodetic Science, No. 188, The Ohio State University, Columbus.

Uotila, Urho A. (1967). "Introduction to Adjustment Computation with Matrices," Department of Geodetic Science, The Ohio State University, Columbus.

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TRANSFORMATION OF AXES

PROGRAM WORKS ON TWO SETS OF COORDINATES -- EITHER
SET CAN BE INPUT AS ELLIPSOIDAL COORDINATES, BOTH IN
DEGREES AND METERS OR IN G60S FORMAT. IN SUCH CASE
SEMI-MAJOR AXIS 'A' AND ECCENTRICITY 'E' ARE NEEDED.

UVW MATRIX TAKES COORDINATES IN THE FIRST SYSTEM
--- ( IN FORMAT I5,3F15.5 )

XYZ MATRIX TAKES COORDINATES IN THE SECOND SYSTEM
--- ( IN FORMAT I4,5X,OF16.5 )

MAXIMUM NUMBER OF INPUT POINTS FOR EACH SYSTEM --- 35

SUBROUTINE -- 'CSTRNT'

SOLVES FOR TRANSFORMATION CASE WHEN CONSTRAINTS ARE
TO BE APPLIED FOR THREE ROTATIONS. NECESSARY COUNTER
--- KC ode(11) -- IS TO BE CODED AS ' 4 '

TWO SOLUTIONS ARE OBTAINED WITH THE SAME DATA ---
FIRST WITHOUT CONSTRAINTS AND SECOND WITH CONSTRAINTS.

INPUT CONSTRAINTS ARE OBTAINED FROM SUBROUTINE 'EULERS'
AND SUBROUTINE 'SCALE'.

SUBROUTINE -- 'TFORM'

TRANSFORMATION PARAMETERS SOLVED UNDER THREE CASES.
REFER K CODE(3) ALSO.
SCALE FACTOR BETWEEN SYSTEM #1 AND SYSTEM #2 IS COMPUTED BY COMPARISON OF (I-K)TH CHORDS IN THE TWO SYSTEMS. THE WEIGHT FOR EACH ESTIMATE OF SCALE FACTOR CORRESPONDING TO (I-K)TH CHORD IS COMPUTED USING VARIANCE - COVARIANCE MATRICES OF THE TH AND KTH POINTS OF SYSTEM #1 AND SYSTEM #2 THROUGH ERROR PROPAGATION.

EULERS ANGLES ARE COMPUTED FROM DIRECTION COSINES. WEIGHT MATRIX 'P' FOR 'TIK AND DIK' IS COMPUTED USING VARIANCE - COVARIANCE MATRICES OF THE POINT COORDINATES OF THE SYSTEMS BY ERROR PROPAGATION.

SETS UP AND STORES WEIGHT MATRIX 'P' IN VECTOR FORM.

INPUT --- KCODES TO BE CODED WITH EACH DATASET
KCODE(1) = 'TOTAL NUMBER OF POINTS -- IN (12) FIELD.'

KCODE(2) = 'PARAMETERS REQUIRED IN THE SOLUTION'
  3 DENOTES ONLY TRANSLATIONS OR
     ROTATIONS -- SEE KCODE(14)
     TO SEPARATE THESE SOLUTIONS.
  4 DENOTES TRANSLATIONS AND SCALE.
  7 DENOTES TRANSLATION, SCALE AND
     THREE ROTATIONS.

KCODE(3) = 'WHETHER CONSTRAINED SOLUTION IS NEEDED'
  0 DENOTES NO SUCH SOLUTION
  1 DENOTES CONSTRAINED SOLUTION.

KCODE(4) = 'FIRST SYSTEM IN ELLIPSOIDAL COORDINATES
     IN DEGREES FOR PHI AND LEMDA --
     AND METERS FOR HEIGHTS'
  0 DENOTES NO SUCH CASE
  1 DENOTES SUCH INPUT

KCODE(5) = 'INPUT FOR FIRST SYSTEM IN GEOS FORMAT'
  0 DENOTES NO SUCH CASE
  1 DENOTES SUCH INPUT

KCODE(6) = 'SECOND SYSTEM IN ELLIPSOIDAL COORDINATES
     IN DEGREES FOR PHI AND LEMDA --
     AND METERS FOR HEIGHTS'
  0 DENOTES NO SUCH CASE
  1 DENOTES SUCH INPUT

KCODE(7) = 'INPUT FOR SECOND SYSTEM IN GEOS FORMAT'
  0 DENOTES NO SUCH CASE
  1 DENOTES SUCH INPUT

KCODE(8) = 'VARIANCE - COVARIANCE MATRIX AS DIAGONAL
     IN FORMAT '3F10.2' -- 1 CARDS PER STN.'
  0 DENOTES NO SUCH CASE
  1 DENOTES SUCH INPUT

KCODE(9) = 'VARIANCE - COVARIANCE MATRIX IN 3X3 FORM'
  0 DENOTES NO SUCH CASE
  1 DENOTES SUCH INPUT

KCODE(10) = 'VARIANCE - COVARIANCE MATRIX IN FULL AS
     UPPER TRIANGLE (ROW-WISE) IN VECTOR FORM'
  0 DENOTES NO SUCH CASE
  1 DENOTES SUCH INPUT

-30-
KCODE(11) = 'FOR CONSTRAINED SOLUTION TO BE CODED AS'
"3" ROTATIONS ARE CONSTRAINED.
"4" ROTATIONS AND SCALE ARE CONSTRAINED.
KCODE(12) = 'TO BE CODED '1' TO OMIT CORRELATION
WITH KCODE(10) AS AN ALTERNATE SOLUTION.
KCODE(13) = 'TOTAL NUMBER OF TRANSFORMATIONS
TO BE PERFORMED ' --- TO BE CODED
WITH THE LAST DATA SET IN (12) FIELD.
KCODE(14) = '3 PARAMETER SOLUTION ONLY'
0 SOLUTION FOR TRANSLATIONS
1 SOLUTION FOR ROTATIONS

HOW TO SETUP INPUT DATA

FIRST CARD --- TO CONTAIN ALL KCODES
CARDS CONTAINING COORDINATES FOR THE FIRST SYSTEM
CARDS CONTAINING COORDINATES FOR THE SECOND SYSTEM
CARDS CONTAINING VARIANCE - COVARIANCE MATRIX
FOR THE FIRST SYSTEM.
CARDS CONTAINING VARIANCE - COVARIANCE MATRIX
FOR THE SECOND SYSTEM.
IMPLICIT REAL *8(A-H, O-Z)
PI = 3.1415926535897935
RHO = 180.DO/PI
RHO = 180.DO/PI
RHO = 180.DO/PI
RHO*3600.DO
******** REAP IN VARIOUS CODES INVOLVED
READ (5, 1) (KCODE(I), I = 1, 15), (NAME1(I), I = 1, 3)
READ (5, 1) (KCODE(I), I = 1, 15), (NAME2(I), I = 1, 3)
FOPMAT (12, 1111.12, 211, 3X, 3A4, 3X, 3A4)
WRITE (6, 2) (KCODE(I), I = 1, 15)
NO = KCODE(1)
IF (KCODE(4).EQ.0.AND.KCODE(5).EQ.0) GO TO 12
******** READ IN DATA FOR THE FIRST SYSTEM
READ (5, 3) AF1, F
F = 1.DO/F
E2 = 2.DO*F - F*F
IF (KCODE(5).EQ.1) GO TO 6
******** READ IN ELLIPSOIDAL COORDINATES IN DEGREES AND HEIGHT
DO 5 I = 1, NO
READ (5, 4) NSTA(I), PHI, LEMDA, HT
5 CONTINUE
GO TO 15
******** READ IN ELLIPSOIDAL COORDINATES IN GEOS FORMAT
6 DO 11 I = 1, NO
READ (5, 7) NSTA(I), 1SN, IPH, MPH, SPH, ILM, MLM, SLM, HT
11 CONTINUE

IF (ISN .EQ. MINUS) GO TO 8
PHI = (IPH+((MPH*(5PHI/60.00))/60.00))/RHO
GO TO 10

8 PHI = -(IPH+((MPH*(5PHI/60.00))/60.00))/RHO

10 WW = (1.00-E2*DSIN(PHI)*DSIN(PHI))**0.5DO
UVW(I,1) = (AE1/WW+HT)*DCOS(PHI)*DCOS(LEMDA)
UVW(I,2) = (AE1/WW+HT)*DCOS(PHI)*DSIN(LEMDA)
UVW(I,3) = (((AE1*(1.00-E2))/WW+HT)*DSIN(PHI))

11 CONTINUE
GO TO 15

******** READ IN RECTANGULAR COORDINATES ( U, V, W ) IN METERS

12 DO 14 I = 1, NO
READ (5, 13) NSTA(I), UVW(I,J), J=1,3
13 FORMAT (I4,5X,3F16.5)
14 CONTINUE

15 IF (KCODE(6).EQ.1.OR .KCODE(7).EQ.1) GO TO 20

16 FORMAT (I4,5X,3F16.9)

17 CONTINUE
GO TO 40

20 RFAD (5, 22) AE2 , F
22 FORMAT (2F15.10)
F = 1.00/F
E2 = 2.00*F - F*F
IF (KCODE(7).EQ.1) GO TO 25

******** READ IN ELLIPSOIDAL COORDINATES IN DEGREES AND HEIGHT

23 FORMAT (I4,5X,3F16.9)
PHI = PHI / RHO
LEMDA = LEMDA / RHO
WW = (1.00-E2*DSIN(PHI)*DSIN(PHI))**0.5DO
XYZ(I,1) = (AE2/WW+HT)*DCOS(PHI)*DSIN(LEMDA)
XYZ(I,2) = (AE2/WW+HT)*DCOS(PHI)*DSIN(LEMDA)
XYZ(I,3) = (((AE2*(1.00-E2))/WW+HT)*DSIN(PHI))

24 CONTINUE
GO TO 40
****** READ IN ELLIPSOIDAL COORDINATES IN GECS FORMAT

25 DO 31 I = 1, NO
   READ (5, 26) KSTA(I), ISN, IPH, MPH, SPH, ILM, MLM, SLM, HT
26 FORMAT (14, 20X, A1, 213, F8.3, 213, F8.3, F10.2)
   LEMDA  = (ILM+((LM+(SLM/60.DO))/60.DO))/RHO
   IF (ISN .EQ. MINUS) GO TO 28
   PHI    = (IPH+((MPH+(SPH/60.DO))/60.DO))/RHO
   GO TO 30
28 PHI    = -(IPH+((MPH+(SPH/60.DO))/60.DO))/RHO
30 WW     = (1.00-E2*DSIN(PHI)*DSIN(PHI))**0.5
   XYZ(I, 1) = ((AE2/WW+HT)*DCOS(PHI)*DSIN(LEMDA))
   XYZ(I, 2) = ((AE2/WW+HT)*DSIN(PHI)*DSIN(LEMDA))
   XYZ(I, 3) = (((E2*(1.DO-F2))/WW+HT)*DSIN(PHI))
31 CONTINUE

****** WRITING OF READ IN DATA FOR THE TWO SYSTEM IN RECTANGULAR COORDINATES

40 WRITE(6, 42)
42 FORMAT('1,///,25X,'RECTANGULAR COORDINATES FOR FIRST SYSTEM',///)
   WRITE(6, 43)
43 FORMAT ('1,13X,'STN.NO.',12X,'U',13X,'V',16X,'W',/)
   DG 4A I = 1, NO
   WRITE(6, 44) NSTA(I), (UWFIJ, J=1,3)
44 FORMAT ('1,13X,15,F20.4,2F16.4,(14X,15,F20.4,2F16.4))
46 CONTINUE
   WRITE(6, 50)
50 FORMAT('1,///,25X,'RECTANGULAR COORDINATES FOR SECOND SYSTEM',///)
   WRITE(6, 52)
52 FORMAT ('1,13X,'STN.NO.',12X,'X',13X,'Y',16X,'Z',/)
   DG 60 I = 1, NO
   WRITE(6, 58) KSTA(I), (XYZ(I,J), J=1,3)
58 FORMAT ('1,13X,15,F20.4,2F16.4,(14X,15,F20.4,2F16.4))
60 CONTINUE

****** SEPARATING THE TYPE OF SOLUTION REQUIRED

   KPAM  = KCODE(11)
   IF (KCODE(8) .NE. 1) GO TO 62
   KPAM  = 1
   GO TO 75
62 IF (KCODE(9) .NE. 1) GO TO 64
   KPAM  = 2
   GO TO 75
64 IF (KCODE(10) .EQ. 1) KPR = 3
   NM    = NO - 1
   NN    = NO * NM
   NNN   = 3*NO
   IF (KCODE(14) .EQ. 0) GO TO 85
   CALL FULERS (ND, NNN, AA, BB)

-14-
IF (KCODE(14).EQ.1.AND.KCODE(2).EQ.3) GO TO 95
85 CALL TFORM (NO,NNN)
   IF (KOUNT .EQ. KCODE(13)) GO TO 95
   KOUNT = KOUNT + 1
95 CALL TFORM (NO,NNN)
GO TO 1000
95 STOP
END
SUBROUTINE EULERS (NO, NNN, AA, BB)

IMPLICIT REAL *8 (A-H, O-Z)

RFAL *8 NI, N7, M02

DIMENSION UVW(35,3), XYZ(35,3), A(3600), W(1200), NAME1(3),
     2 P11(6,6), G1(2,6), GP(2,6), GT(6,2), PP(2,2), KY(2), KY(2), NAME2(3),
     3 R(2,4), BT(4,2), PZ(6,6), INDX(40), INV(40), OXYZ(4500), NZ(4,4),
     4 P(2400), RS(2,4), KSTA(35), NSTA(35), QUW(4500), AA(3, NNN), RB(3, NNN),
     5 PG(2,2), PR(4,4), NI(3,3), DX(13), U(3), VAR(3,3), KPR, KPARM,
     6

COMMON / WEIGHT/ P

COMMON / CODE/ KCODF

COMMON / ANGLE/ S, DX, NZ

COMMON / INAME/ NAME1, NAME2

COMMON / SFAC/ DW, DS, DA1, DB1, DC1,

COMMON NSTA, KSTA, NN, NM, UVW, XYZ, A, W, KPR, KPAR

PII  = 3.14159265358979320

RHO  = 180.00/ PII

RHOS = RHO*3600.00

MW  = 0.00

DS  = 0.00

S  = 0.00

VSF = 0.00

WT  = 0.00

LL  = 1

**** SETTING UP OF MATRIX 'B' -- COMMON TO ALL SOLUTION

B(1,1)  = -1.00
B(1,2)  = 0.00
B(1,3)  = 1.00
B(1,4)  = 0.00
B(2,1)  = 0.00
B(2,2)  = -1.00

-36-
\begin{verbatim}
E(2,3) = 0.00
VS = NN/2
E(2,4) = 1.00
DC 1 I = 1, 2
DC 1 J = 1, 4
PT(J,J) = E(I,J)
1 CONTINUE
DC 2 I = 1, 4
DC 2 J = 1, 4
PR(I,J) = 0.00
2 CONTINUE
IF (KCODE(8).EQ.1.0 .OR. KCODE(9).EQ.1) GO TO 10

*** FULL VARIANCE-COVARIANCE CASE

*** READING IN VARIANCE-COVARIANCES FOR 'FIRST SYSTEM'

JK = 1
DC 6 I = 1, NNN
JL = JK + NNN - 1
READ (5, 3) (QUVW(J), J = JK,JL)
2 FORMAT (8F10.4)
DC 4 L = LL + 3
PI(LL,L) = QUVW(JK+L-LL)
4 PI(LL,L) = PI(LL,L)
WRITE (1) (PI(LL,M), M = 1, 3)
LL = LL + 1
IF (LL .EQ. 4) LL = 1
6 JK = JL + 1
REWIND 1

*** READING IN VARIANCE-COVARIANCES FOR 'SECOND SYSTEM'

LL = 1
JK = 1
DC 9 I = 1, NNN
JL = JK + NNN - 1
READ (5, 7) (OXYZ(J), J = JK,JL)
7 FORMAT (8F10.4)
DC 8 L = LL + 3
P2(LL,L) = OXYZ(JK+L-LL)
8 P2(LL,L) = P2(LL,L)
WRITE (2) (P2(LL,M), M = 1, 3)

-37-
\end{verbatim}
**DIAGONAL OR 3X3 BANDED CASE**

**READING IN VARIANCE-COVARIANCE FOR FIRST SYSTEM**

10 DO 17 J = 1, NO
     KK = (I-1)*3 + 1
     KM = KK + 2
     IF (KCODE(J) .EQ. 1) GO TO 13

**VARIANCE - COVARIANCE MATRIX IN 3X3 BANDED FORM**

DO 12 J = 1, 3
   READ (5,11) (AA(J,K), K = KK,KM)
11 FORMAT(3F5.2)
   WRITE(1) (AA(J,K), K=KK,KM)
   GO TO 17

**VARIANCE - COVARIANCE MATRIX IN DIAGONAL FORM (ONLY VARIANCES)**

DO 14 J = 1, 3
   DO 15 K = KK , KM
14   AA(J,K) = 0.DO
15   FORMAT (3F10.2)
   READ (5,15) (AA(K,(K+KK-1)), K = 1,3)
   CONTINUE
   WRITE(1) (AA(J,K), K=KK,KM)

**READING IN VARIANCE-COVARIANCE FOR SECOND SYSTEM**

DO 23 I = 1, NO
     KK = (I-1)*3 + 1

** PLAIN **
KM = KK + 2
IF (KCONF(8) .NE. 1) GO TO 20
C C
**** VARIANCE - COVARIANCE MATRIX IN 3X3 BANDED FORM
C C
DO 19 J = 1, 3
READ (5,18) (BB(J,K), K=KK,KM)
18 FORMAT (3F5.2)
WRITE(2) (BB(J,K), K=KK,KM)
DO 20 J = 1, 3
C C
**** VARIANCE - COVARIANCE MATRIX IN DIAGONAL FORM (ONLY VARIANCES)
C C
20 DO 21 J = 1, 3
IF 21 K = KK,KM
READ (5,15) (BB(K,(K+KK-1)), K = 1,3)
DO 22 J = 1, 3
WRITE(2) (BB(J,K), K=KK,KM)
CONTINUE
C C
***** FORMING MATRICES 'A', 'W', AND 'P' FOR THE ENTIRE SYSTEM
***** BY COMPUTING DIRECTION COSINES FOR EACH LINE BETWEEN
***** ANY ONE SET OF TWO GIVEN POINTS.
C C
MKR = 1
MKM = 1
INDEX(1) = 1
MM1 = NNN + 1
DO 25 I = 1, NO
25 INV(I) = 3*(I - 1)
DO 26 J = 1, 6
IF 26 K = 1, 6
P1(J,K) = 0.00
P2(J,K) = 0.00
CONTINUE
IF (KCODE(10) .NE. 1) GO TO 28
DO 27 J = 1, 3
DO 27 L = 1, 3
LL = (I-1) * 3 + L
P1(J,L) = AA(J,L)
P2(J,L) = BB(J,L)
**CONTINUE**

GO TO 32

28 LL = INDEX(I)
DO 30 J = 1, 3
DO 29 L = J, 3
LLL = LL + L - J
PI(J,L) = QUVW(LLL)
29 P2(J,L) = QXYZ(LLL)
MM1 = MM1 - 1
30 LL = LL + MM1
32 JJ = J + 1
INDEX(JJ) = LL
MM2 = MM1
DO 50 K = J, NO
IF (KCODE(8),EQ.1.OR.KCODE(9),EQ.1) GO TO 43
LL = INDEX(K)
DO 34 J = 4, 6
DO 33 L = J, 6
LLL = III + L - J
P1(J,L) = QUVW(LLL)
33 P2(J,L) = QXYZ(LLL)
MM2 = MM2 - 1
34 LL = LL + MM2
KP = K + 1
INDEX(KP) = LL
III = INDEX(I) + INV(K-1)
IF (KCODE(12),EQ.1) GO TO 41
DO 38 J = 1, 3
DO 36 L = 4, 6
LLL = III + L - 3
P1(J,L) = QUVW(LLL)
36 P2(J,L) = QXYZ(LLL)
38 III = III + (NNN -(3*(I-1))-J)
41 DO 42 J = 1, 6
DO 42 L = 1, 6
P1(L,J) = P1(J,L)
42 P2(L,J) = P2(J,L)
GO TO 45
43 DO 44 L = 4, 6
JKL = L - 3
DO 44 M = 4, 6
KLM = (K-2)*M + 1
P1(L,M) = AA(JKL,KLM)
P2(L,M) = BR(JKL,KLM)
44 CONTINUE
45 KSM = MKR + NN
KMS = MKR + (2*NN)

****** COMPUTING DIRECTION COSINES FOR FIRST SYSTEM**

**DA1 = UVW(K,1) - UVW(I,1)**
**DB1 = UVW(K,2) - UVW(I,2)**
**DC1 = UVW(K,3) - UVW(I,3)**
**DK2 = DSQRT(DA1*DA1+DB1*DB1+DC1*DC1)**
**DK1 = DA1/RIK1**
**RIK1 = DA1/RIK1**
CIK1 = DC1/RIK1
TIK1 = -DATAN2(BIK1, AIK1)
IF (TIK1.LT.0.) TIK1 = (360.0+TIK1*RHO)/RHO
AB1 = DSORT(AIK1*AIK1+BIK1*BIK1)
DIK1 = DATAN2(CIK1, AB1)

**** COMPUTING DIRECTION COSINES FOR SECOND SYSTEM

DA2 = XYZ(K,1) - XYZ(I,1)
DB2 = XYZ(K,2) - XYZ(I,2)
DC2 = XYZ(K,3) - XYZ(I,3)
RIK2 = DSQRT(DA2*DA2+DB2*DB2+DC2*DC2)
AIK2 = DA2/RIK2
BIK2 = DB2/RIK2
CIK2 = DC2/RIK2
TIK2 = -DATAN2(BIK2, AIK2)
IF (TIK2.LT.0.) TIK2 = (360.0+TIK2*RHO)/RHO
AP2 = DSQRT(AIK2*AIK2+BIK2*BIK2)
DIK2 = DATAN2(CIK2, AB2)

**** SETTING UP MATRICES 'A' AND 'W' -- COMMON TO ALL SOLUTION

A(MKR) = 1.00
A(MKR+1) = 0.00
A(KSM) = DSIN(TIK2)*DTAN(DIK2)
A(KSM+1) = DCOS(TIK2)
A(KMS) = -DCOS(TIK2)*DTAN(DIK2)
A(KMS+1) = DSIN(TIK2)
W(MKR) = TIK1 - TIK2
W(MKR+1) = DIK1 - DIK2

**** FORMING VAR-COVARIANCE MATRIX FOR 'TIK' AND 'DIK' THROUGH PROPAGATION OF ERRORS -- WHERE 'TIK' AND ARE GEODETIC HOUR ANGLE AND DECLINATION.

FIRST SYSTEM

DAB1 = DA1*DA1+DB1*DB1
DAB = DSORT(DAB1)
G(1,1) = -DB1/DAB1
G(1,2) = DA1/DAB1
G(1,3) = 0.00
G(1,4) = -G(1,1)
G(1,5) = -G(1,2)
G(1,6) = 0.00
G(2,1) = DA1*DC1/(DBA*RIK1*RIK1)
G(2,2) = DB1*DC1/(DBA*RIK1*RIK1)
G(2,3) = -DBA/(RIK1*RIK1)
G(2,4) = -G(2,1)
G(2,5) = -G(2,2)
G(2,6) = -G(2,3)
DO 46 L = 1, 2
DO 46 M = 1, 6
GT(M,L) = G(L,M)
46 CONTINUE
CALL DGMPRD(G,P1,GP,2,6,6)
CALL DGMPRD(GP,GT,PP,2,6,2)

****
SECOND SYSTEM
****

DA2 = DA2*DA2+DB2*DB2
DBE = DSQRT(DBA2)
G(1,1) = -DB2/DA2
G(1,2) = DA2/DA2
G(1,3) = 0.00
G(1,4) = -G(1,1)
G(1,5) = -G(1,2)
G(1,6) = 0.00
G(2,1) = DA2*DC2/(DA2*RIK2*RIK2)
G(2,2) = DB2*DC2/(DA2*RIK2*RIK2)
G(2,3) = -DBA/(RIK2*RIK2)
G(2,4) = -G(2,1)
G(2,5) = -G(2,2)
G(2,6) = -G(2,3)
DO 47 L = 1, 2
DO 47 M = 1, 6
GT(M,L) = G(L,M)
47 CONTINUE
CALL DGMPRD(G,P2,GP,2,6,6)
CALL DGMPRD(GP,GT,PP,2,6,2)

**** FORMING MATRIX 'MI' FOR THE COMBINED SYSTEM

DO 48 L = 1, 2
J = L + 2
DO 48 M = 1, 2
N = M + 2
PP(L,M) = PQ(L,M)
PR(J,N) = PP(L,M)
48 CONTINUE
CALL DGMPRD(B,PR,BS,2,4,4)
CALL DGMPRD(ES,BT,PP,2,4,2)
CALL DMINV(PP,2,DT,KK,KY)
P(KMT) = PP(1,1)
P(KMT+1)= PP(2,1)
**KMT+2)= PP(1,2)
P(KMT+3)= PP(2,2)
MKR = MKR + 2
KMT = KMT + 4

** IF (KCODE(11), FO. 3) GO TO 50
CALL SCALE (NS, MK, S, VSF, WT)
MK = MK + 1

** FIndING WEIGHTED MEAN AND VARIANCE FOR
** SCALE FACTOR BY COMPARISON OF CHORDS IN
** THE TWO SYSTEMS BY CALLING SUBROUTINE 'SCALE'.

50 CONTINUE
VSF = VSF * 10.0
DO 75 J = 1, 3
 IJK = (J-1)*NN + 1
 JKL = IJK + NN - 1
75 WRITE(3) (A(I), I = IJK, JKL)
REWIND 3
WRITE(4) (W(K), K = 1, NN)
REWIND 4

**** FORMING MATRIX 'N' AND INVVERTING THE SAME

DO 80 I = 1, 3
READ (3) (W(J), J = 1, NN)
K1 = (I-1)*NN + 1
K2 = K1 + NN - 1
MMM = 0
DO 78 K = K1, K2
A(K) = 0.0
L1 = ((K-K1)/2)*2 + 1
L2 = L1 + 1
DO 78 L = L1, L2
MMM = MMM + 1
78 A(K) = A(K) + W(L)*P(MMM)
CONTINUE
DC 84 I = 1, 4
DO 84 J = 1, 4
84 NZ(I, J) = 0.0
REWIND 3
DO 86 I = 1, 3
READ (3) (W(L), L = 1, NN)
DC R5 J = 1, 3
DC R6 K = 1, NN
DC 8E K = (J-1)*NN + K
95 NI(J,1) = NI(J,1) + A(III)*W(K)
88 CONTINUE
PFWIND 3
15 (KCODE(II), EQ. 3) GO TO 89
50 DD 91 I = 2, 4
DO 91 J = 2, 4
91 NI(J,1) = NI(I-1,J-1)
CALL DMINV(NI,3,DF1,KQ,LO)
C
C
C
*************** *** COMPUTING SOLUTION VECTOR * DX * FOR 3 ROTATION PARAMETERS *******
C
C
C
READ(4) (W(I), I=1, NN)
REWIND 4
DO 92 J = 1, 3
U(J) = 0.0
DC 92 I = 1, NN
KKK = (J-1)*NN + 1
U(J) = U(J) - A(KKK)*W(I)
92 CONTINUE
CALL DGMPRD(NI,U,DX,3,3,1)
DC 95 I = 1, 3
JK = (I-1)*NN + 1
JM = JK + NN - 1
95 READ(3) (A(I), J= JK , JM)
REWIND 3
C
C
C
C
C
C
C
C
DC 96 I = 1, NN
W(1) = 0.0
DC 97 J = 1, 3
K = (J-1)*NN + 1
W(1) = W(1) - A(K)*DX(J)
96 CONTINUE
READ(4) (A(I), I= 1, NN)
REWIND 4
DO 97 K = 1, NN
W(K) = W(K) - A(K)
97 CONTINUE
MMX = 0
DC 98 K = 1, NN
A(K) = 0.00
L1 = ((K-1)/2)*2 + K
L2 = L1 + 2
L3 = L1 + L2 + 2
LMM = ((L-1)/2) + 1
READ(4) (W(I), I = 1, NN)
REWIND 4
VPV = 0.00
DO 99 K = 1, NN
99 VPV = VPV - A(K)*W(K)
M2 = VPV/(NN - 3)

C

C **** COMPUTING VARIANCE-COVARIANCE MATRIX 'VAR'
C

DO 100 I = 1, 3
DO 100 J = 1, 3
VAR(I,J) = M2*Rhos*Rhos*N(I,J)
100 CONTINUE

DO 105 I = 1, 3
105 DX(I) = DX(I)*Rhos

C

C **** COMPUTING COEFFICIENTS OF CORRELATIONS FOR PARAMETERS
C

DO 110 I = 1, 3
IF (I.EQ.3) GO TO 107
J = I + 1
DO 106 J = J, 3
N(I,J) = VAR(I,J)/(DSORT(VAR(I,I))*DSORT(VAR(J,J)))
106 N(I,J) = N(I,J)
107 N(I,I) = 1.00
110 CONTINUE

C

C ***************************************************************

C

C **** WRITING OF FINAL SOLUTION VECTOR AND VARIANCE-COVARIANCE MATRIX
C

WRITE(6,6025)
6025 FORMAT('I1', '///')
WRITE(6,6028) (NAMEK(I), I = 1, 3), (NAME2(I), I = 1, 3)

6028 FORMAT('I1,5X,3A4,' -TO- ',3A4,'//)

76X, '***************************************************************

WRITE(6,6030)
6030 FORMAT('I1,30X, 'SOLUTION FOR '3' ROTATION PARAMETERS' ,//
23IX,'------------------------------------',//,
325X,'(FROM DIRECTION COSINES -- UNITS SECONDS OF ARC)' ,//)
GO TO (112,114,116), KPR
112 WRITE(6,6031)
 FORMAT (• 7X,'(USING VARIANCES ONLY)'),//) GO TO 120
114 WRITE(6,6032)
6032 FORMAT(• 7X, 'USING 3X3 BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX'),//) GO TO 120
116 WRITE(6,6033)
6033 FORMAT(• 7X,'(USING FULL VARIANCE-COVARIANCE MATRIX'),//) 120 WRITE(6,6035)
6035 FORMAT(• 7X,'OMEGA',19X,'EPSILON'),//) WRITE(6,6040)(DX(I), I=1,3)
6040 FORMAT(• 7X,5X,3D24.7,/) WRITE(6,6045)
6045 FORMAT(• 7X,'VARIANCE-COVARIANCE MATRIX'),//) WRITE(6,6048)(MOD2,F6.2,/) 6048 FORMAT(• 7X, 'M02=',F6.2,/) WRITE(6,6050)((VAR(I,J), J=1,3), I=1,3)
6050 FORMAT(• 7X,3X,30D25.8,/) WRITE(6,6075)
6075 FORMAT(• 7X,'COEFFICIENT OF CORRELATION'),//) WRITE(6,6085)((N(I,J), J=1,3), I=1,3)
6085 FORMAT(• 7X,3X,30D25.8,/) IF (KCODE(11) .LE. 3) GO TO 150 WRITE (6,7000)
7000 FORMAT(• 7X,'SOLUTION FOR SCALE FACTOR'),//) 234X,'-----------------------------',//) WRITE(6,7004)
7004 FORMAT(• 7X,'SCALE FACTOR',27X,'VARIANCE'),//) 203X,'(10.0+5)',29X,'(10.0+11)',//) WRITE (6,7010) S , VSF
7010 FORMAT(• 7X,'RETURN',//) 150 KCODE(11) = 4 RETURN
*TFCRM*

SUBROUTINE TFCRM (NO, NC)
IMPLICIT REAL * 8 (A-H, O-Z)

DIMENSION XYZ(35,3), UVW(35,3), SIGMAX(7,7), NAME1(3),
2A(3600), W(1200), VAR(7,7), DX(7), NI(49), NSTA(35), NAME2(3),
3M(4), U(7), LT(7), KCODE(15), CNT(7,4), TT(7,4), CN(4,7), ZP(4,4),
4MT(7), KSTA(35), VR(7,7), XO(7), KL(150), KK(150), MI(2400), ROT(4,4)

COMMON /WEIGHT/ MI
COMMON / CODF / KCODE
COMMON / ANGLE / ANG, ROT
COMMON / INAME / NAME1, NAME2
COMMON / CRNT / VPV, DX, SO2, XO, SIGMAX
COMMON NSTA, KSTA, NN, NM, UVW, XYZ, A, W, KPR, KPARM

PI = 3.141592653589793
DO = 180.00/PI
RHO = RHO*3600.00
IPARA = KCODE(2)
IC = KCODE(11)
MUNT = 1
DO 5 I = 1, 4
DO 5 J = 1, 7
CN(I,J) = 0.00
5 TT(J,I) = 0.00

C CNT(J,I) = 0.00
DO 10 I = 1, 4
DO 10 J = 1, 4
10 ZP(I,J) = 0.00

**** SENDING UP MATRIX 'A' -- COMMON TO ALL SOLUTION

NNN = 6*NO
NNZ = NO*IPARA
NC 13 I = 1, NNN
A(I) = G.DO
CONTINUE
DO 15 I = 1, NO
KKK = (3*1-2)
LLL = KKK+NO+1
MMM = LLL+NO+1
A(KKK) = 1.DO
A(LLL) = 1.DO
A(MMM) = 1.DO

C

**** SETTING UP MATRIX W WHICH IS COMMON TO ALL SOLUTION

C

W(KKK) = (UVW(I,1)-XYZ(I,1))
W(KKK+1) = (UVW(I,2)-XYZ(I,2))
W(KKK+2) = (UVW(I,3)-XYZ(I,3))

15 CONTINUE
IF (KCODE(2) .NE. 3) GO TO 50

C

**** SOLUTION FOR 3 TRANSLATION PARAMETERS

C

N = 3
ICASF = 1
GO TO 81

C

**** SOLUTION FOR 3 TRANSLATION AND 1 SCALE PARAMETERS

C

50 N = 4
DO 60 I = 1, NO
KKK = 3*(NO+1)-2
A(KKK) = UVW(I,1)
A(KKK+1) = UVW(I,2)
A(KKK+2) = UVW(I,3)
CONTINUE
IF (KCODE(2) .NE. 4) GO TO 70
ICASF = 2
GO TO 81

C

**** SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

C

70 N = 7
ICASF = 3
DO 80 I = 1, NO
KKK = 4*NO+(3*I-2)

C

**** SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

C

80 CONTINUE
GO TO 50

C

**** SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

C

-48-
LLL = KKK + NO
MMM = LLL + NO + 1
A(KKK) = UWW(I,2)
A(KKK+1) = UWW(I,1)
A(LLL) = UWW(I,3)
A(LLL+2) = UWW(I,1)
A(MMM) = UWW(I,3)
A(MMM+1) = -UWW(I,2)

CONTINUE
81 DO 65 I = 1, N
     KKK = (I-1)*NO+1
     LLL = KKK+NO-1
     WRITE(5) (A(J), J=KKK,LLL)
65 CONTINUE
REWRITE 3
WRITE(4) (W(I), I=1,NO)
REWRITE 4

C **************************************************
C **** FORMING NORMAL EQUATIONS — MATRICES 'N' AND 'U'
C **************************************************
C
100 CALL SETUP (NG,NG,KPARA)
   DC 118 I = 1, N
   READ(W) (W(J), J=1,NO)
   K1 = (I-1)*NO+1
   K2 = K1+NO-1
   MMM = 0
   DC 116 K = K1, K2
   A(K) = 0.00
   L1 = (((K-K1)/3)*3)+1
   L2 = L1 + 2
   DC 116 L = L1, L2
   MMM = MMM + 1
116 A(K) = A(K) + W(L)*M1(MMM)
118 CONTINUE
REWRITE 3
DO 120 I = 1, N
   READ(W) (W(L), L=1,NO)
   JK = (I-1)*N+1
   JL = JK+N-1
   DO 119 J = JK, JL
   NI(J) = 0.00
   DO 119 K = 1, NO
   II = (J-JK)*NO + K
   119 NI(J) = NI(J) + A(II)*W(K)
120 CONTINUE
REWRITE 3
DO 121 I = 1, N
   DO 121 J = 1, N
      K = (I-1)*N + J
   121 SIGMAX(I,J) = NI(K)
REWRITE 4
DO 122 J = 1, N
   122 -49-


```
U(1) = 0.0D0
DO 122 J = 1, NO
    KKK = (J-1)*NG+1
    U(J) = U(J) - A(KKK)*W(I)
122 CONTINUE

C************************************************************************
C  SLOTS COMPUTING SOLUTION VECTOR DX FOR TRANSFORMATION PARAMETERS
C************************************************************************

CALL DMINV(NI,N,DT,LT,MT)
CALL DARRAY(1,N,N,7,7,NI,VR)
CALL DGMPRm(NI,U,DX,N,NI)
DO 123 I = 1, N
    JK = (I-1)*NG + 1
    JM = JK + NO - 1
123 READ3(3) {A(I),J=JK,JM}
CONTINUE

C************************************************************************
C  SLOTS COMPUTING VARIANCE OF UNIT WEIGHT M02
C************************************************************************

DO 125 I = 1, NO
    W(I) = 0.0D0
125 DO 125 J = 1, N
    KZX = (J-1)*NC+I
    W(I) = W(I) - A(KZX)*DX(J)
125 CONTINUE

C************************************************************************
C  SLOTS COMPUTING VARIANCE-COVARIANCE MATRIX VAR
C************************************************************************

DO 130 K = 1, NO
    A(K) = 0.0D0
    L1 = (((K-1)/3)*6 + K
    L2 = L1 + 6
    DO 130 L = L1, L2, 3
        MMM = (((L-1)/3) + 1
        A(K) = A(K) + M(L)*W(MMM)
130 CONTINUE

CALL RESIDU(NO,NNN)
CONTINUE

VPV = 0.0D0
DO 130 K = 1, NO
    VPV = VPV - A(K)*W(K)
130 VPV = VPV/(-NO-N)
```
DO 132 J = 1, N
VAR(I,J) = MO2*VR(I,J)
132 CONTINUE
IF (KCODE(2) .LE. 3) GO TO 140
DX(4) = DX(4) * 10.05
IF (KCOF(2) .LE. 4) GO TO 140
DO 135 I = 5, 7
DX(I) = DX(I) * RHOS
135 CONTINUE

**** COMPUTING COEFFICIENTS OF CORRELATIONS FOR PARAMETERS

DO 145 I = 1, N
IF (I.EQ. N) GO TO 144
JJ = I + 1
DO 142 J = JJ, N
VR(I,J) = VAR(I,J)/(DSORT(VAR(I,I))*DSORT(VAR(J,J)))
142 CONTINUE
VR(I,1) = 1.00
145 CONTINUE
200 WRITE(6,250)
250 FORMAT('*** WRITING OF FINAL SOLUTION VECTOR AND VARIANCE-COVARIANCE MATRIX ***

500 WRITE(6,6025)
6025 FORMAT(' ',/)
6030 FORMAT(' ',/)
WRITE(6,6030)
6030 FORMAT(' ',/)
WRITE(6,6030)
6032 FORMAT(' ',/)
512 WRITE(6,6032)
6032 FORMAT(' ',/)
GO TO 520
514 WRITE(6,6034)
6034 FORMAT(' ',/)
2*('USING 3X3 BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX'),/)
GO TO 520
516 WRITE(6,6036)
-51-
6036 FORMAT(' ',22X,'(USING FULL VARIANCE-COVARIANCE MATRIX)',//)
520 WRITE(6,6038)
6038 FORMAT(' ',16X,'DX',20X,'DY',22X,'DZ',//)
WRITE(6,6040)(DX(I), I=1,3)
6040 FORMAT(' ',1X,'3D23.8', /////)
WRITE(6,6045)
6045 FORMAT(' ',26X,'VARIANCE - COVARIANCE MATRIX',//)
WRITE(6,6048) MD2

6046 FORMAT(' ',14X,'M02=',F6.2, //)
WRITE(6,6050)((VAR(I,J), J=1,3), I=1,3)
6050 FORMAT(' ',1X,'3D23.8',/(2X,3D23.8,/) )
WRITE(6,6075)
6075 FORMAT(' ',27X,'COEFFICIENTS OF CORRELATION',///)
WRITE(6,6085)(VR(I,J), J=1,N), I=1,N)
6085 FORMAT(' ',1X,'3D23.8',/(2X,3D23.8,/) )
GO TO 1000
600 WRITE(6,6500)
6500 FORMAT(' ',26X,'VARIANCE-COVARIANCE MATRIX',//)
WRITE(6,6625) M02
6625 FORMAT(' ',8X,'M02=',F6.2, //)
WRITE(6,6650)((VAR(I,J), J=1,4), I=1,4)
6650 FORMAT(' ',1X,'4D20.8',/(2X,4D20.8,/) )
WRITE(6,6675)
6675 FORMAT(' ',27X,'COEFFICIENTS OF CORRELATION',///)
WRITE(6,6685)((VR(I,J), J=1,N), I=1,N)
6685 FORMAT(' ',1X,'4D20.8',/(2X,4D20.8,/) )
GO TO 1000
700 GO TO (710,705), KOUNT
705 IF (KPARM.EQ.4) GO TO 708
WRITE(6,7002)
7002 FORMAT(' ',28X,'SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED',//)
22DX, '---------------------------------------------',//)
GO TO 710
708 WRITE(6,7005)
7005 FORMAT(' ',20X,'SCALE FACTOR AND ROTATION PARAMETERS CONSTRAINED',//)
27X, '---------------------------------------------',//)
710 WRITE(6,7010)
7010 FORMAT(' ',13X,'SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION
PARAMETERS',//),14X,'---------------------------------------------')
**VARIANCE-COVARIANCE MATRIX**

\[ DZ, DL \]

\[ \Omega \]

GO TO (712, 714, 716), KPR

WRITE(6, 7012)

7012 FORMAT(13X, 'USING VARIANCES ONLY')

GO TO 720

WRITE(6, 7014)

7014 FORMAT(13X, 'USING 3X3 BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX')

GO TO 720

WRITE(6, 7016)

7016 FORMAT(13X, 'USING FULL VARIANCE-COVARIANCE MATRIX')

720 WRITE(6, 7020)

7020 FORMAT(13X, 'USING 3X3 BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX')

WRITE(6, 7030)

7030 FORMAT(13X, 'USING FULL VARIANCE-COVARIANCE MATRIX')

WRITE(6, 7040)

7040 FORMAT(13X, 'USING FULL VARIANCE-COVARIANCE MATRIX')

WRITE(6, 7045)

7045 FORMAT(13X, 'USING FULL VARIANCE-COVARIANCE MATRIX')

WRITE(6, 7050)

7050 IF (1.EG.0) GO TO 1000

WRITE(6, 7060)

7060 IF (1.EG.0) GO TO 1000

WRITE(6, 7070)

7070 IF (1.EG.0) GO TO 1000

WRITE(6, 7080)

7080 IF (1.EG.0) GO TO 1000

WRITE(6, 7090)

7090 IF (1.EG.0) GO TO 1000

WRITE(6, 7100)

7100 IF (1.EG.0) GO TO 1000

WRITE(6, 7110)

7110 IF (1.EG.0) GO TO 1000

WRITE(6, 7120)

7120 IF (1.EG.0) GO TO 1000

WRITE(6, 7130)

7130 IF (1.EG.0) GO TO 1000

WRITE(6, 7140)

7140 IF (1.EG.0) GO TO 1000

WRITE(6, 7150)

7150 IF (1.EG.0) GO TO 1000

WRITE(6, 7160)

7160 IF (1.EG.0) GO TO 1000

WRITE(6, 7170)

7170 IF (1.EG.0) GO TO 1000

WRITE(6, 7180)

7180 IF (1.EG.0) GO TO 1000

WRITE(6, 7190)

7190 IF (1.EG.0) GO TO 1000

WRITE(6, 7200)

7200 IF (1.EG.0) GO TO 1000

CALL CSTRT(N, NO, IC, U, CN, CNT, TT, ZP)

**OBTAINING CONSTRAINED SOLUTION FOR ROTATION PARAMETERS**

CALL CSTRT(N, NO, IC, U, CN, CNT, TT, ZP)

KCODE(3) = 0

725 CONTINUE

725 CONTINUE

CONTINUE
JJ = 1 + 1
DO 735 J = JJ, N
   VR(I,J) = VAR(I,J)/DSQRT(VAR(I,I)*DSQRT(VAR(J,J)))
735 VR(J,1) = VR(I,J)
740 VR(I,1) = 1.D0
750 CONTINUE
   KOUNT = 2
   MG2 = 502
   GO TO 200
1000 RETURN
END
SUBROUTINE SCALE

FINDING WEIGHTED MEAN AND VARIANCE FOR 'SCALE FACTOR' BY COMPARISON OF CHORDS IN THE TWO SYSTEMS BY CALLING SUBROUTINE 'SCALE'.

SUBROUTINE SCALE(NO,N,S,VSF,WT)
IMPLICIT REAL * 8 (A-H, O-Z)
DIMENSION P(12,12),HI(12),DL(600),VI(600),WU(600),PF(6,6),PS(6,6)
COMMON /SFAC/ SW, SF, DU, DV, DW, DX, DY, DZ, R1, R2, PF, PS
RR = R1 * R2
RT = R2 / (R1**3)

C SETTING UP OF VARIANCES FOR EACH CHORD THROUGH ERROR PROPOGATION

DO 10 I = 1,3
H(I*3) = -HI(I)
H(I+9) = -HI(I*3) + 6
10 CONTINUE
DO 15 J = 1,12
P(I,J) = 0.00
DO 20 I = 1,6
L = I + 6
DO 20 J = 1,6
M = J + 6
P(I,J) = PF(I,J)
20 CALL DGMPRD (H,P,H(1,12,12)

-55-
CALL DGMPRD (H1, H, WS, 1, 12, 1)

**FINDING WEIGHTED MEAN FOR SCALE FACTOR OF THE GIVEN SAMPLE**

```
WS = 1.0D0/WS  
WI(N) = WS  
SF1 = R2/R1 - 1  
DL(N) = SF1  
SF = SF + SF1 * WS  
SW = SW + WS  
S = SF/WS
```

**FINDING VARIANCE FOR THE WEIGHTED MEAN OF THE SCALE FACTOR**

```
IF (N .NE. NO) GO TO 500
PVV = 0.0D0  
DO 50 K = 1, NO  
VI(K) = ((S-DSL(K))**2)*WI(K)  
50 PVV = PVV + VI(K)  
VSF = PVV/(SW*(NO-1))  
S = S * 10.05  
WT = 1.0D0 / VSF  
500 RETURN  
END
```
SUBROUTINE CSTRNT(N,NN,IC,WS,CN,CNT,TT,ZP)
IMPLICIT REAL * 6 (A-H, O-Z)
REAL * 8 M02,KC
DIMENSION XD(7),WS(7),WX(7),KC(4),
WC(4),LM(7),MM(7),PZ(4,4),CNT(7,IC),GG(7,7),
SIGMAX(7,7),DX(7),TK(7),TT(7,IC),CN(1C,7),ZP(1C,1C)
COMMON /ANGLE/ WC,PZ
PII = 3.141592653589793DO
RHO = 180.DO/PII
RHO*3600.DO
WC(1) = WC(1) / 10.DO
WC(1) = WC(1) / 1C
CONTINUE

C C **** SETUP CONSTRAINTS MATRIX 'CN' REQUIRED FOR SOLUTION
C C
DO 25 I = 1, IC
DO 25 J = 1, N
CN(I,J) = 0.DO
TT(J,I) = 0.DO
CONTINUE
IF (IC .EQ. 4) GO TO 100
DO 50 I = 1, IC
DO 50 J = 1, IC
ZP(I,J) = PZ(I+1,J+1)
GO TO 200
100 DO 150 I = 1, IC
DO 150 J = 1, IC
ZP(I,J) = PZ(I,J)
200 CONTINUE
CONTINUE

-57-
C  ***********************************************************************
C  *** SOLVE FOR EFFECTS OF CONSTRAINTS ON THE SOLUTION VECTOR 'DX'  
C  *** OBTAINED FROM NON-CONSTRAINT SOLUTION                           
C  ***********************************************************************
C
DO 620  I = 1 , N
      CNT(I,J) = CN(I,J)
620  CONTINUE
      CALL MTPY(CNT,ZP,N,IC,IC,TT)
      CALL MTPY(TT,CN,N,IC,N,GG)
      DO 522  I = 1 , N
      DO 522  J = 1 , N
      GG(I,J) = SIGMAX(I,J) + GG(I,J)
      CALL DMINV(GG,N,DTT,LM,MM)
      CALL MTPY(TT,W,C,N,IC,WX)
      DO 525  I = 1 , N
      WC(I) = (WS(I) - WX(I))
525  CONTINUE
      CALL MTPY(GG,WS,N,IC,IC)
C  C  **** COMPUTE NEW VARIANCE OF UNIT WEIGHT AND
C  C  **** NEW VARIANCE - COVARIANCE MATRIX
C
      CALL MTPY(CN,XD,N,IC,KC)
      DO 535  I = 1 , N
      KC(I) = -KC(I) - WC(I)
      CALL MTPY(PZ,KC,IC,IC,DX)
      SUM = 0.0
      DO 540  I = 1 , N
      SUM = SUM + DX(I) * WC(I)
540  CONTINUE
      PPV = VPV - SUM
      SIGMAX(I,J) = PPV * GG(I,J)
      DO 550  I = 1 , N
      DO 550  J = 1 , N
      SIGMAX(I,J) = SIGMAX(I,J) / (IC(NM-N+1))
550  CONTINUE
      XN(4) = XN(4) * 10.05
      BC 560  1 = 5 , 7
      XN(1) = XN(1) * RHOS
560  CONTINUE
1000  RETURN
      END
SETUP

***** SETUP MATRIX B**TP -- MI -- AFTER READING VARIANCE
***** COVARIANCE MATRIX FOR EACH POINT SEPARATELY AND
***** THEN STORING THE ELEMENTS SO FORMED IN THE PROPER PLACE IN 'MI'.

SUBROUTINE SETUP (NO, integers, IPARA)
IMPLICIT REAL *8(A-H,O-Z)
REAL * 8 MINK
DIMENSION R(3,6),RT(6,3),PI(6,6),PK(3,3),
2XM(3,3),XX(3,6),MINK(2400),LM(3),MM(3),KCODE(15)
COMMON /RES/ RT
COMMON /MFRIGHT/ MINK
COMMON /CODE/ KCODE
C
C **** SETTING UP MATRIX 'B', WHICH WILL BE SAME FOR ALL SOLUTION
C
C NV = NN*3
DO 1 I = 1 , NV
MINK(I) = 0,00
8 CONTINUE
DO 10 J = 1 , 6
PI(I,J) = 0,00
XX(I,J) = 0,00
10 CONTINUE
B(1,1) = -1,00
B(2,2) = -1,00
B(3,3) = -1,00
B(4,4) = 1,00
B(5,5) = 1,00
B(6,6) = 1,00
DO 12 J = 1 , 3
DO 12 J = 1 , 6
B(I,J) = B(I,J)
12 CONTINUE
DO 15 J = 1 , 6
PI(I,J) = 0,00
15 CONTINUE
DO 20 J = 1 , 3
XX(I,J) = 0,00
20 CONTINUE

-59-
IF (KCODE(8) .EQ. 1) GO TO 54
DO 40 L = 1, NO
DO 39 J = 1, 3
READ(5,38) (PI(J,K), K=1,3)
WRITE (2) (PI(J,K), K = 1 , 3)
CONTINUE
DO 40 M = 1, NO
DO 44 J = 4, 6
READ(5,42) (PI(J,K), K = 4 , 6)
CONTINUE
DO 52 M = 1, NO
DO 44 J = 4, 6
READ(5,42) (PK(J,K), K = 4 , 6)
CONTINUE
DO 52 M = 1, NO
DO 56 I = 1, 3
WRITE (2) (PK(I,J), J = 1, 3)
CONTINUE
RETURN
RETIM 1
RETIM 2
GO TO 65
DO 58 M = 1, NO
READ (5,55) (PI(I,1) , I = 1,3)
CONTINUE
READ (5,55) (PI(I,1) , I = 4,6)
DO 56 I = 1, 3
WRITE (2) (PK(I,J), J = 1, 3)
CONTINUE
RETURN
RETIM 1
RETIM 2
DO 60 I = 1, 3
CONTINUE
RETURN
RETIM 1
RETIM 2
DO 64 M = 1, NO
READ (5,55) (PI(I,1) , I = 1,3)
CONTINUE
READ (5,55) (PI(I,1) , I = 4,6)
DO 60 J = 1, 3
WRITE (1) (PK(I,J), J = 1, 3)
CONTINUE
RETURN
RETIM 1
RETIM 2
DO 65 M = 1, NO
WRITE (1) (PK(I,J), J = 1, 3)
CONTINUE
RETURN
RETIM 1
RETIM 2
DO 70 J = 1, 3
READ(2) (PI(J,K), K = 1,3)
CONTINUE
DO 74 L = 4, 6
READ(1) (PI(L,M), M=4,6)
CONTINUE
CALL MTPY(8,PI,3,6,6,XK)
CALL MTPY(XK,BT,3,6,3,XM)
CALL DMINV(XM,3,DET,LM,MM)
MINK(KMS ) = XM(1,1)
C
C **** READ IN VARIANCE - COVARIANCE MATRIX AS BLOCK DIAGONALS
C **** 6x6 MATRICES FOR EACH POINT USED IN TRANSFORMATION.
C **** MATRIX 'PI' IS BUILT UP POINTWISE - FIRST (3x3) BLOCK
C **** REFERS TO SECOND COORDINATE SYSTEM AND SECOND (3x3) BLOCK
C **** THEN CORRESPONDS TO FIRST COORDINATE SYSTEM.
C
C DO 70 J = 1, 3
READ(2) (PI(J,K), K = 1,3)
CONTINUE
DO 74 L = 4, 6
READ(1) (PI(L,M), M=4,6)
CONTINUE
CALL MTPY(8,PI,3,6,6,XK)
CALL MTPY(XK,BT,3,6,3,XM)
CALL DMINV(XM,3,DET,LM,MM)
MINK(KMS ) = XM(1,1)
MIN(KMS+1) = XM(2,1)
MIN(KMS+2) = XM(3,1)
MIN(KMS+3) = XM(1,2)
MIN(KMS+4) = XM(2,2)
MIN(KMS+5) = XM(3,2)
MIN(KMS+6) = XM(1,3)
MIN(KMS+7) = XM(2,3)
MIN(KMS+8) = XM(3,3)

100 CONTINUE
REWIND 1
REWIND 2
PETUPN
FND
**SUBROUTINE 'RESIDUE'**

This subroutine computes residuals for each system coordinates (used as observations).

**READING VARIANCE - COVARIANCE MATRICES FOR THE FIRST SYSTEM -- POINTWISE -- AS (3X3).**

READ 
(1) (PI(I,J), I = 1,6)

**READING VARIANCE - COVARIANCE MATRICES FOR THE SECOND SYSTEM -- POINTWISE -- AS (3X3).**

READ 
(2) (PI(J,K), J = 1,3)
CALL DGMPRD (PI, BT, BS, 6, 6, 3)

**COMPUTING RESIDUALS**

```
DO 15 K = 1, 6
   KK = JJ + K
   A(KK) = 0.00
   KM = (I-1) * 3
   DO 15 L = 1, 3
      KM = KM + 1
      A(KK) = A(KK) + BS(K, L) * A(KM)
   15 CONTINUE
   DO 20 L = 1, 3
      LL = JJ + L
      KM = LL + 3
      W(L) = A(LL)
      A(LL) = A(KM)
   20 CONTINUE
RWIND 1
REWORD 2
RETURN
END
```
**SUBROUTINE DARRAY(MORE,I,J,N,M,S,D)**

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION S(1),D(1)
N1=N-1
IF(MORE-1) 100,100,120
100 IJ=I+J+1
NM=N+J+1
DO 110 K=1,J
NM=NM-N1
DO 110 L=1,I
IJ=IJ-1
NM=NM-1
110 D(NM)=S(IJ)
GO TO 140
120 IJ=0
NM=0
DO 130 K=1,J
DO 130 L=1,I
IJ=IJ+1
NM=NM+1
130 S(IJ)=D(NM)
140 RETURN
END
MTPY

SUBROUTINE MTPY(AMT,BMT,M1,M2,M3,CMT)
IMPLICIT REAL *8 (A-H,O-Z)
DIMENSION AMT(M1,M2),BMT(M2,M3),CMT(M1,M3)
DO 10 I = 1 , M1
DO 10 J = 1 , M3
CMT(I,J) = 0.0
DO 10 L = 1 , M2
10 CMT(I,J) = CMT(I,J) + AMT(I,L) * BMT(L,J)
RETURN
END
APPENDIX II

Job Control Cards
APPENDIX II

// (2500,100),CLASS=C
//STEP1 EXEC PROC=FPRTRANG,PARM='MAP, ID', TIME.CMP=(0,30)
//CMP.SYSIN DD *

FORTRAN PROGRAM DECK

/*
//STEP2 EXEC PROC=RUNFORT,PARM.LKED='OVLY, LIST, MAP', TIME.LKED=(0,20),
// TIME.GO=(3,10), REGION.GO=252K
//LKED.SYSLIB DD DSNAME=SYS1.FORTLIB,DISP=SHR
// DD DSNAME=SYS2.FORTSSP,DISP=SHR
//LKED.SYSLIN DD DSNAME=*.STEP1.CMP.SYSLIN,DISP=(OLD,DELETE)
// DD *
OVERLAY ALPHA
INSERT EULERS, SCALE
OVERLAY BETA
INSERT TFORM, RESIDU, NTPY, SETUP, CSTRMT, DARRAY
/*
//GO.FT01F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW,DELETE),
// DCR=(RECFM=VRS,LRECL=600,BLKSIZE=604)
//GO.FT02F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW,DELETE),
// DCR=(RECFM=VRS,LRECL=600,BLKSIZE=604)
//GO.FT03F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW,DELETE),
// DCR=(RECFM=VRS,LRECL=600,BLKSIZE=604)
//GO.FT04F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW,DELETE),
// DCR=(RECFM=VRS,LRECL=600,BLKSIZE=604)
//GO.FT07F001 DD SYSOUT=P
//GO.SYSIN DD *

DATA DECK

/*
//