COORDINATE TRANSFORMATION
BY MINIMIZING CORRELATIONS
BETWEEN PARAMETERS

by
Muneendra Kumar

Prepared for
National Aeronautics and Space Administration
Washington, D.C.

Contract No. NGR 36-008-093
OSURF Project No. 2514

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and is under the technical direction of James P. Murphy, Special Programs, Code ES, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546

A revised version of this report has been submitted to the Graduate School of The Ohio State University in partial fulfillment of the requirements for the Master of Science degree.
ABSTRACT

The subject of this investigation is to determine the transformation parameters (three rotations, three translations and a scale factor) between two Cartesian coordinate systems from sets of coordinates given in both systems. The objective is the determination of well separated transformation parameters with reduced correlations between each other, a problem especially relevant when the sets of coordinates are not well distributed. The above objective is achieved by preliminarily determining the three rotational parameters and the scale factor from the respective direction cosines and chord distances (these being independent of the translation parameters) between the common points, and then computing all the seven parameters from a solution in which the rotations and the scale factor are entered as weighted constraints according to their variances and covariances obtained in the preliminary solutions.

Numerical tests involving two geodetic reference systems were performed to evaluate the effectiveness of this approach as follows:

(a) A non-constrained solution for general transformation for the seven parameters (including the three translations and scale factor).

(b) A constrained solution for general transformation for the seven parameters utilizing the three rotations with their statistics as constraints.

(c) A constrained solution for general transformation for the seven parameters using the three rotations and scale factor with their statistics as constraints.

The above schemes were then separately repeated for each of the following three cases:
(i) Using the full variance-covariance matrix between coordinates of the geodetic reference systems.

(ii) Using only a $(3 \times 3)$ banded diagonal variance-covariance matrix, thus assuming no correlation between coordinates of any two points within the system.

(iii) Using only variances for the coordinates, thereby further omitting the correlation between the three coordinates of any one point in the system.

In the case of seven parameter general transformation, the best estimates were obtained using full variance-covariance matrix and constraining three rotations and the scale factor, case (c) and (iii) above. The improvement in correlation between translations and rotations was more significant compared to between translation and scale factor.
ACKNOWLEDGMENTS

The writer of this report expresses his sincere gratitude to Dr. Ivan I. Mueller for his constant help, cooperation and most valuable guidance in the execution of this study.

Special indebtedness and thanks are owed by the author to Dr. R. H. Rapp, Dr. N. K. Saxena and Mr. J. P. Reilly for the valuable suggestions and comments.

The writer is also thankful to Mrs. Evelyn Rist and Ms. Michelle Neff for all the help and cooperation extended and for the excellent typing of this report.
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1. INTRODUCTION

During the last twenty-five years with the availability of computer technology and its phenomenal growth in basic hardware and core storage capacity and the exceptional increase in a computer's ability of solving problems in lesser and lesser time, a trend has set in to analyze the problems in geodesy and photogrammetry more and more in three dimensional space rather than to follow traditional concepts.

Further, the advent of artificial satellites and their subsequent use in geodesy made it possible to obtain Cartesian coordinates of points on earth surface.

Several projects involving satellite-networks of continental or global extent were begun and at present they are in varying stages of completion. Many new solutions have recently come out, each delineating its own reference system. These systems in reality should differ from each other only in having different origins, sets of axes or scale.

Thus, the relationship between any two such reference systems (e.g., UVW and XYZ) would generally consist of seven parameters—three translations ($\Delta X, \Delta Y, \Delta Z$) between the two origins, three rotations ($\omega, \psi, \epsilon$) of the Euler's angle type between the two sets of axes and the scale factor ($\Delta S$), if any (Figure 1).

![Figure 1](image-url)
The mathematical model to be used in the computations of the above seven parameters from a least squares solution may be written in the following form [Badekas, 1969; Bursa, 1965; Wolf, 1963]:

\[
\begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3
\end{bmatrix}
= 
\begin{bmatrix}
    X \\
    Y \\
    Z
\end{bmatrix}
- 
\begin{bmatrix}
    \Delta X \\
    \Delta Y \\
    \Delta Z
\end{bmatrix}
- 
\begin{bmatrix}
    1 & \omega & -\psi \\
    -\omega & 1 & \epsilon \\
    \psi & -\epsilon & 1
\end{bmatrix}
\begin{bmatrix}
    U \\
    V \\
    W
\end{bmatrix}
- 
\Delta s
\begin{bmatrix}
    U \\
    V \\
    W
\end{bmatrix}
= 0,
\]

(1)

where "i" denotes any point common to both the systems. The three angles \(\omega\), \(\psi\), and \(\epsilon\) of the Euler type correspond to small rotations about the Z, Y and X axes respectively—the positive direction of rotations taken in counter clockwise mode, when viewed from the end of the respective axes towards the origin. It may be worth while to mention here that the station coordinates in both the systems (\(U_i\), \(V_i\), \(W_i\) and \(X_i\), \(Y_i\), \(Z_i\)) are treated as observations in the above model.

The above equation written in matrix notation can then be modified into the observation equation below [Uotila, 1967]:

\[
BV + AX + W = 0,
\]

(2)

where

\[
\mathbf{B} = 
\begin{bmatrix}
    \frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} & \frac{\partial f_1}{\partial Z} & \frac{\partial f_1}{\partial U} & \frac{\partial f_1}{\partial V} & \frac{\partial f_1}{\partial W} \\
    \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} & \frac{\partial f_2}{\partial Z} & \frac{\partial f_2}{\partial U} & \frac{\partial f_2}{\partial V} & \frac{\partial f_2}{\partial W} \\
    \frac{\partial f_3}{\partial X} & \frac{\partial f_3}{\partial Y} & \frac{\partial f_3}{\partial Z} & \frac{\partial f_3}{\partial U} & \frac{\partial f_3}{\partial V} & \frac{\partial f_3}{\partial W}
\end{bmatrix}_i
\]

\[
= 
\begin{bmatrix}
    1 & 0 & 0 & -1 & 0 & 0 \\
    0 & 1 & 0 & 0 & -1 & 0 \\
    0 & 0 & 1 & 0 & 0 & -1
\end{bmatrix}
,\]

\[
\mathbf{A} = 
\begin{bmatrix}
    \frac{\partial f_1}{\partial \Delta X} & \frac{\partial f_1}{\partial \Delta Y} & \frac{\partial f_1}{\partial \Delta Z} & \frac{\partial f_1}{\partial \Delta S} & \frac{\partial f_1}{\partial \omega} & \frac{\partial f_1}{\partial \psi} & \frac{\partial f_1}{\partial \epsilon} \\
    \frac{\partial f_2}{\partial \Delta X} & \frac{\partial f_2}{\partial \Delta Y} & \frac{\partial f_2}{\partial \Delta Z} & \frac{\partial f_2}{\partial \Delta S} & \frac{\partial f_2}{\partial \omega} & \frac{\partial f_2}{\partial \psi} & \frac{\partial f_2}{\partial \epsilon} \\
    \frac{\partial f_3}{\partial \Delta X} & \frac{\partial f_3}{\partial \Delta Y} & \frac{\partial f_3}{\partial \Delta Z} & \frac{\partial f_3}{\partial \Delta S} & \frac{\partial f_3}{\partial \omega} & \frac{\partial f_3}{\partial \psi} & \frac{\partial f_3}{\partial \epsilon}
\end{bmatrix}_i
\]
while $V$ and $X$ represent the residuals to the observations and corrections to the parameter estimates, respectively. Hence, collecting all the matrices as above, pointwise in the systems, the observation equation becomes:

$$
\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
V_u \\
V_v \\
V_w
\end{bmatrix}_i
= \begin{bmatrix}
-1 & 0 & 0 & -U & -V & W & 0 \\
0 & -1 & 0 & -V & U & 0 & -W \\
0 & 0 & -1 & -W & 0 & -U & V
\end{bmatrix}_i
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z \\
\Delta s \\
\omega \\
\psi \\
\epsilon
\end{bmatrix}
= 0
$$

(3)

Defining the geodetic reference systems on the assumption that the Laplace-condition has been enforced throughout the network (which implies that the axes of the reference ellipsoid are parallel to the conventional earth-fixed axes), many experiments have been made in recent times to determine the seven transformation parameters in relating the different geodetic systems to each other using an observation equation of type (3) [Lambeck, 1971; Marsh et. al., 1971].
However, in the above general transformation, if the geodetic reference systems are properly oriented through the Laplace-condition, the three rotations arising due to the improper relative orientation of the systems are generally never more than a few seconds of arc, while translations may amount up to 200 to 300 meters. Also, due to the presence of high correlations between the rotations, the scale factor and the translations, satisfactory independent estimates for these parameters are difficult to obtain from a combined general solution using equation (3).

This investigation separates the determinations of the rotations and the scale factor (from that of the translations) for subsequent use as constraints in a combined general solution.

2. THE INDEPENDENT DETERMINATIONS OF ROTATIONAL AND SCALAR PARAMETERS

2.1 Determination of Rotations

2.1.1 Mathematical Model

The mathematical model used in this study is as follows [Bursa, 1966]:

\[ T_{i}^{(1)} - T_{i}^{(2)} + \omega + \psi \sin T_{i}^{(1)} \tan \delta_{i}^{(1)} - \epsilon \cos T_{i}^{(1)} \tan \delta_{i}^{(1)} = 0 \]

\[ \delta_{i}^{(1)} - \delta_{i}^{(2)} + \psi \cos T_{i}^{(1)} + \epsilon \sin T_{i}^{(1)} = 0 \]

where \( T_{i} \) and \( \delta_{i} \) are defined as the geodetic hour angle and declination of the \((i-k)\)th direction of the observed point at \( k \)th station and the observer at \( i \)th station. The indexes (1) and (2) denote the two systems with the transformation proceeding from system #1 to system #2.

If \( A_{ik}, B_{ik}, C_{ik} \) are taken to denote the direction cosines of the \((i-k)\)th line of length \( R_{ik} \), then for the first (UVW) system one gets:
\[ A_{ik} = \frac{U_k - U_i}{R_{ik}} = \frac{\Delta U_{ik}}{R_{ik}}, \]

\[ B_{ik} = \frac{V_k - V_i}{R_{ik}} = \frac{\Delta V_{ik}}{R_{ik}}, \]

\[ C_{ik} = \frac{W_k - W_i}{R_{ik}} = \frac{\Delta W_{ik}}{R_{ik}}, \]

and \[ T_{ik} = -\arctan \frac{B_{ik}}{A_{ik}}, \]

\[ \delta_{ik} = \arctan \frac{C_{ik}}{(A_{ik}^2 + B_{ik}^2)^{\frac{1}{2}}}. \]

In the above relations (4) through (6) the elements of translation do not enter the picture. A similar set of relations as per (5) and (6) can be established for the second (XYZ) system.

\[ \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_t \end{bmatrix} + \begin{bmatrix} 1 & \sin T_{ik}^{(1)} \tan \delta_{ik}^{(1)} & -\cos T_{ik}^{(1)} \tan \delta_{ik}^{(1)} \end{bmatrix} \begin{bmatrix} \omega \\ \cos T_{ik}^{(1)} \sin T_{ik}^{(1)} \end{bmatrix} + \begin{bmatrix} T_{ik}^{(1)} - T_{ik}^{(2)} \\ \delta_{ik}^{(1)} - \delta_{ik}^{(2)} \end{bmatrix} = 0 \]

Using the conventional weight matrix \( P \) for the coordinates of points included in the transformation (see section 2.1.3), and the principle of least squares by making \( V'PV \) as minimum, the equation (7) is then solved for correction vector \((\omega, \psi, \epsilon)\) and for the variance-covariance matrix \((\Sigma \omega \psi \epsilon)\) of the three parameters.
2.1.3 Weights

Using the variance-covariance matrices $\Sigma X$ and $\Sigma U$ in respect of $i^{th}$ and $k^{th}$ points for the XYZ and UVW systems, the variance-covariance matrices $\Sigma_{r5}$ for the two systems of coordinates can be computed through propagation of errors [Uotila, 1967].

Two distinct cases would arise here. Firstly, when in addition to correlation between $X, Y, Z$-coordinates of any point, the correlation between the coordinates of one point to others is also considered. In such a case, the necessary relation will be

$$\begin{bmatrix} \Sigma (\hat{r}_{ij})_{a,2} \\ \Sigma (\hat{r}_{ij})_{a,2} \end{bmatrix} = G \begin{bmatrix} \Sigma U_{i} & \Sigma U_{ik} \\ \Sigma U_{ik} & \Sigma U_{k} \end{bmatrix} G'$$

(8)

where

$$G = \begin{bmatrix} \frac{\partial T_{(1)}^{(1)}}{\partial U_{1}} & \frac{\partial T_{(1)}^{(1)}}{\partial V_{1}} & \frac{\partial T_{(1)}^{(1)}}{\partial W_{1}} & \frac{\partial T_{(1)}^{(1)}}{\partial U_{k}} & \frac{\partial T_{(1)}^{(1)}}{\partial V_{k}} & \frac{\partial T_{(1)}^{(1)}}{\partial W_{k}} \\ \frac{\partial \delta_{(1)}}{\partial U_{1}} & \frac{\partial \delta_{(1)}}{\partial V_{1}} & \frac{\partial \delta_{(1)}}{\partial W_{1}} & \frac{\partial \delta_{(1)}}{\partial U_{k}} & \frac{\partial \delta_{(1)}}{\partial V_{k}} & \frac{\partial \delta_{(1)}}{\partial W_{k}} \end{bmatrix}$$

and

$$\frac{\partial T_{ik}}{\partial U_{1}} = \frac{\partial T_{ik}}{\partial U_{k}} = -\frac{\Delta V_{ik}}{\Delta U_{ik}^2 + \Delta V_{ik}^2},$$

$$\frac{\partial T_{ik}}{\partial V_{1}} = \frac{\partial T_{ik}}{\partial V_{k}} = -\frac{\Delta U_{ik}}{\Delta U_{ik}^2 + \Delta V_{ik}^2},$$

$$\frac{\partial T_{ik}}{\partial W_{1}} = \frac{\partial T_{ik}}{\partial W_{k}} = 0,$$

$$\frac{\partial \delta_{ik}}{\partial U_{1}} = \frac{\partial \delta_{ik}}{\partial U_{k}} = \frac{\Delta U_{ik} \Delta W_{ik}}{R_{ik}^{(1)} \Delta U_{ik}^2 + \Delta V_{ik}^2},$$

$$\frac{\partial \delta_{ik}}{\partial V_{1}} = \frac{\partial \delta_{ik}}{\partial V_{k}} = \frac{\Delta V_{ik} \Delta W_{ik}}{R_{ik}^{(1)} \Delta U_{ik}^2 + \Delta V_{ik}^2},$$
Secondly, ignoring the correlations between the coordinates of different points within a system, equation (8) can be modified as under:

\[
\frac{\partial \delta_{ik}}{\partial W_i} = \frac{\partial \delta_{ik}}{\partial W_k} = \frac{\Delta U_{ik}^2 + \Delta V_{ik}^2}{R_{ik}^{2(1)}}.
\]

\[
R_{ik}^{2(1)} = \Delta U_{ik}^2 + \Delta V_{ik}^2 + \Delta W_{ik}^2.
\]

In the equations (8) and (9), \( \Sigma U_i \) and \( \Sigma U_k \) correspond to \( i^{th} \) and \( k^{th} \) point of the first system and can be either full (3 X 3) matrices with covariances between the three coordinates of a point, or may contain variances for \( U, V \) and \( W \) in a diagonal form only. However, in the case of covariances \( \Sigma U_{ik} \) between the points being included, the matrix in equation (8) would be a full (6 X 6).

Obtaining similarly \( \Sigma_{T6}^{(2)} \), the combined variance-covariance matrix, to be used with equation (7), is given by:

\[
\begin{align*}
\begin{bmatrix}
\Sigma_{T6}^{(2)} & 0 \\
0 & \Sigma_{U_k}
\end{bmatrix}
\end{align*}
\]

It may be noted here that the matrix \( P \) is always in 2 X 2 banded diagonal form.

2.2 Determination of Scale Factor

2.2.1 Mathematical Model

The scale factor between the systems #1 and #2 would be given as follows:

\[
\Delta s_{ik} = \frac{R_{ik}^{(2)}}{R_{ik}^{(1)}} - 1
\]
where
\[ R_{1k}^{(2)} = (\Delta X_{1k}^2 + \Delta R_{1k}^2 + \Delta Z_{1k}^2)^{\frac{1}{2}} \]
\[ R_{1k}^{(1)} = (\Delta U_{1k}^2 + \Delta V_{1k}^2 + \Delta W_{1k}^2)^{\frac{1}{2}} \]

2.2.2. **Weights**

Using the variance-covariances matrices \( \Sigma X \) and \( \Sigma U \) for the coordinates of \( i \)th and \( k \)th points in the two systems included in the transformation (section 2.1.3), a variance \( \sigma_{\Delta s}^2 \) is established for the scale factor through error propagation. Two cases similar to equations (8) and (9) would arise according to the case when full variance-covariance matrix between different points within the system is considered or not.

The matrix \( G \) for the scale factor determination is

\[
G = \begin{bmatrix}
\frac{\partial \Delta s}{\partial U_1} & \frac{\partial \Delta s}{\partial V_1} & \frac{\partial \Delta s}{\partial W_1} & \frac{\partial \Delta s}{\partial U_k} & \frac{\partial \Delta s}{\partial V_k} & \frac{\partial \Delta s}{\partial W_k} & \frac{\partial \Delta s}{\partial X_1} & \frac{\partial \Delta s}{\partial Y_1} & \frac{\partial \Delta s}{\partial Z_1} & \frac{\partial \Delta s}{\partial X_k} & \frac{\partial \Delta s}{\partial Y_k} & \frac{\partial \Delta s}{\partial Z_k}
\end{bmatrix}.
\]

where
\[
\frac{\partial \Delta s}{\partial U_1} = -\frac{\partial \Delta s}{\partial U_k} = \frac{\Delta U_{1k} \cdot R_{1k}^{(2)}}{[R_{1k}^{(1)}]^{3/2}},
\]
\[
\frac{\partial \Delta s}{\partial V_1} = -\frac{\partial \Delta s}{\partial V_k} = \frac{\Delta V_{1k} \cdot R_{1k}^{(2)}}{[R_{1k}^{(1)}]^{3/2}},
\]
\[
\frac{\partial \Delta s}{\partial W_1} = -\frac{\partial \Delta s}{\partial W_k} = \frac{\Delta W_{1k} \cdot R_{1k}^{(2)}}{[R_{1k}^{(1)}]^{3/2}},
\]
\[
\frac{\partial \Delta s}{\partial X_1} = -\frac{\partial \Delta s}{\partial X_k} = -\frac{\Delta X_{1k}}{R_{1k}^{(1)} \cdot R_{1k}^{(2)}},
\]
\[
\frac{\partial \Delta s}{\partial Y_1} = -\frac{\partial \Delta s}{\partial Y_k} = -\frac{\Delta Y_{1k}}{R_{1k}^{(1)} \cdot R_{1k}^{(2)}},
\]
\[
\frac{\partial \Delta s}{\partial Z_1} = -\frac{\partial \Delta s}{\partial Z_k} = -\frac{\Delta Z_{1k}}{R_{1k}^{(1)} \cdot R_{1k}^{(2)}}.
\]
\[ \sigma_{\Delta s_{ik}}^2 = G \begin{bmatrix} \Sigma U_i & \Sigma U_{ik} \\ \Sigma U_{ik} & \Sigma U \end{bmatrix} G^T \begin{bmatrix} 0 & 0 \\ 0 & \Sigma X_i & \Sigma X_{ik} \\ \Sigma X_{ik} & \Sigma X_e \end{bmatrix} \]

(12)

where the full \((12 \times 12)\) matrix would become a \((3 \times 3)\) banded diagonal matrix in case \(\Sigma U_{ik}\) and \(\Sigma X_{ik}\) are zero, i.e., covariances are not considered. The complete \((12 \times 12)\) matrix would assume a diagonal pattern when only variances are used for station coordinates.

Using the value of \(\Delta s_{ik}\) and \(\sigma_{\Delta s_{ik}}^2\) from equations (11) and (12), the value for weighted mean and its variance for the transformation under investigation is established as given below [Hirvonen, 1971]:

\[ \Delta s_{ws} = \frac{\left[w_{ik} \cdot \Delta s_{ik}\right]}{\left[w_{ik}\right]} \]

(13)

\[ \sigma_{\Delta s_{ws}}^2 = \frac{\left[w_{ik} \cdot (\Delta s_{ik} - \Delta s_{ws})^2\right]}{\left[w_{ik}\right](n-1)} \]

(14)

where \(w_{ik} = 1/\sigma_{\Delta s_{ik}}\) and \(\sum w_{ik}\) denotes the sum of all such weights.

\(n\) = Total number of scale factor values used in the sample.

3. BRIEF DISCUSSION ON THE FORTRAN PROGRAM

Appendix I gives the complete computer program for obtaining the constrained or non-constrained solution for seven parameters. With appropriate coding non-constrained solutions for three parameters (\(\Delta X, \Delta Y\) and \(\Delta Z\)) and scale factor \(\Delta s\) can also be obtained.

The input coordinates can either be Cartesian or geodetic (ellipsoidal) with 35 as the maximum number of points in each system. However, the matrices can easily be re-dimensioned to accommodate more points when required. The
The program is self-explanatory with regard to definition of various option codes for input, type of solution and inclusion of correlation data, etc.

The broad basic divisions of the program are as under:

(a) **Main Program:** This section takes as input the various options in input/solutions, coordinates of points, rectangular or ellipsoidal, and semimajor axis and flattening of the ellipsoid used, if required. It then prints out the two sets of coordinates used for checking purposes.

The various options of input/solutions have been designated in the program as KCODE e.g., KCODE (1) refers to number of common points involved in the transformation. A complete list with necessary explanatory remarks has been included in the beginning of the program.

(b) **Subroutine "EULERS":** This subroutine first reads the variance-covariance matrices of the station coordinates, with or without correlation, and then sets up matrices A, W and P to be used for the solutions of three rotations through direction cosines (equation (7)).

The subroutine writes up the variance-covariance matrices for the coordinates on the disk and stores the estimates for $\omega$, $\psi$ and $\epsilon$, and their variance-covariance matrix \( \Sigma_{\omega\psi\epsilon} \) in the common block for subsequent use.

(c) **Subroutine "SCALE":** This subroutine computes the weighted mean value for scale factor \( \Delta s \) and its variance by direct chord comparison independent of other transformation parameters (equations (13) and (14)).

(d) **Subroutine "TFORM":** This subroutine solves for a general transformation (equation (3)), utilizing the common block core memory for coordinates of points and variance-covariance matrices from the disk.

The matrix \( M^2 \) to be utilized for generating normal equations is computed by calling another subroutine "SETUP".

**NOTE:** In case the solution is required ONLY for three translation or three translations and scale factor, KCODE (3) is coded as "0" and then subroutine "EULERS" is skipped by the program.
(e) **Subroutine "CSTRNT":** This subroutine uses the results of subroutines SCALE and EULERS as constraints with their appropriate statistics and computes for a constrained solution of seven parameters. The results are returned to subroutine TFORM for printout. KCODE (11) refers to the option whether 3 or 4 parameters are to be constrained.

(f) **Subroutine "RESIDU":** This subroutine computes the residuals vector V for observations i.e., the station coordinates used in the program. The residuals are printed station wise for both systems #1 and #2.

In the computer program, the storage mode used for major computation is in vector form for increased flexibility and saving of core storage.

Appendix II gives a typical set of Job Control Cards (JCL).
4. NUMERICAL EXAMPLE

The above transformation models were used to study the relationship between the transformation parameters and obtaining their best estimates by minimizing correlation for the following two reference systems:

(i) System MPS-7, [Mueller and Whiting, 1972].
(ii) System NA-9, [Mueller et. al., 1972].

Using the same set of thirty common stations of the above two systems, the following solutions were obtained during the investigation:

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>Type of Variance-Covariance Matrix Used</th>
<th>7-Parameter General Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Unconstrained Solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constraints: 3 Rotation</td>
</tr>
<tr>
<td>(i)</td>
<td>Only Variances</td>
<td>✓</td>
</tr>
<tr>
<td>(ii)</td>
<td>(3 x 3) Banded Diagonal Variance-Covariance Matrix</td>
<td>✓</td>
</tr>
<tr>
<td>(iii)</td>
<td>Full Variance-Covariance Matrix</td>
<td>✓</td>
</tr>
</tbody>
</table>

@Note: The constraints for these solutions (rotations and/or scale factor) with their statistics were computed independently of the translation parameters (subroutine EULERS and SCALE of the Fortran IV program).

Two solutions in full have been appended in the report as specimens in Tables 1 and 2 as under:

Table 1: Sample printout of the solution for three rotations (ω, ψ, ε) and scale factor (Δs), using full variance-covariance matrix.

Table 2: Sample printout of the constrained seven parameter general solution between NA-9 and MPS-7 with three rotations and
Table 1

Sample Printout of the solutions for three rotations as parameters and the scale factor, using full variance-covariance matrix.
### TABLE 1

**SOLUTION FOR '3' ROTATION PARAMETERS**

*(FROM DIRECTION COSINES — UNITS SECONDS OF ARC)*

*(USING FULL VARIANCE-COVARIANCE MATRIX)*

<table>
<thead>
<tr>
<th>OMEGA</th>
<th>PSI</th>
<th>EPSILON</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1693791D+00</td>
<td>-0.3520145D-01</td>
<td>-0.2173630D+00</td>
</tr>
</tbody>
</table>

**VARIANCE - COVARIANCE MATRIX**

<table>
<thead>
<tr>
<th></th>
<th>OMEGA</th>
<th>PSI</th>
<th>EPSILON</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1675386D-02</td>
<td>0.4062328D-03</td>
<td>-0.9376776D-03</td>
<td></td>
</tr>
<tr>
<td>0.4062228D-03</td>
<td>0.1231799D-02</td>
<td>-0.4880374D-03</td>
<td></td>
</tr>
<tr>
<td>-0.9376776D-03</td>
<td>-0.4880374D-03</td>
<td>0.271919350D-02</td>
<td></td>
</tr>
</tbody>
</table>

**COEFFICIENT OF CORRELATION**

<table>
<thead>
<tr>
<th></th>
<th>OMEGA</th>
<th>PSI</th>
<th>EPSILON</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.160000000D+01</td>
<td>0.2827793D+00</td>
<td>-0.4393150D+00</td>
<td></td>
</tr>
<tr>
<td>0.2827793D+00</td>
<td>0.100000000D+01</td>
<td>-0.2666632D+00</td>
<td></td>
</tr>
<tr>
<td>-0.4393150D+00</td>
<td>-0.2666632D+00</td>
<td>6.100000000D+01</td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION FOR SCALE FACTOR**

*(FROM CHORD COMPARISON)*

<table>
<thead>
<tr>
<th>SCALE FACTOR (10.D+5)</th>
<th>VARIANCE (10.D+11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.16</td>
<td>0.06</td>
</tr>
</tbody>
</table>
TABLE 2

Sample printout of the constrained seven parameters general solution, using full variance-covariance matrix (case (c)/(iii)).
TABLE 2

SCALE FACTOR AND ROTATION PARAMETERS CONstrained

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING FULL VARIANCE-COVARIANCE MATRIX)

<table>
<thead>
<tr>
<th>DX</th>
<th>DY</th>
<th>DZ</th>
<th>DL (10^-4)</th>
<th>OMEGA</th>
<th>PSI</th>
<th>EPSILON</th>
</tr>
</thead>
<tbody>
<tr>
<td>METERS</td>
<td>METERS</td>
<td>METERS</td>
<td>SECONDS</td>
<td>SECONDS</td>
<td>SECONDS</td>
<td></td>
</tr>
<tr>
<td>-45.38</td>
<td>171.94</td>
<td>187.44</td>
<td>5.14</td>
<td>0.17</td>
<td>-0.04</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

VARIANCE-COVARIANCE MATRIX

<table>
<thead>
<tr>
<th></th>
<th>0.176D+01</th>
<th>0.250D+00</th>
<th>0.453D+00</th>
<th>-0.310D-07</th>
<th>0.126D-06</th>
<th>0.778D-07</th>
<th>-0.652D-07</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250D+00</td>
<td>0.228D+01</td>
<td>-0.322D-01</td>
<td>0.243D-06</td>
<td>0.551D-07</td>
<td>0.238D-07</td>
<td>-0.124D-06</td>
<td></td>
</tr>
<tr>
<td>0.453D+00</td>
<td>-0.322D-01</td>
<td>0.206D+01</td>
<td>-0.144D-06</td>
<td>0.615D-07</td>
<td>0.222D-07</td>
<td>-0.177D-06</td>
<td></td>
</tr>
<tr>
<td>-0.310D-07</td>
<td>0.243D-06</td>
<td>-0.149D-06</td>
<td>0.441D-13</td>
<td>-0.325D-17</td>
<td>-0.298D-16</td>
<td>-0.127D-16</td>
<td></td>
</tr>
<tr>
<td>0.126D-06</td>
<td>0.551D-07</td>
<td>0.615D-07</td>
<td>-0.325D-17</td>
<td>0.225D-13</td>
<td>0.525D-14</td>
<td>-0.125D-13</td>
<td></td>
</tr>
<tr>
<td>0.778D-07</td>
<td>0.238D-07</td>
<td>0.222D-07</td>
<td>-0.298D-16</td>
<td>0.525D-14</td>
<td>0.167D-13</td>
<td>-0.654D-14</td>
<td></td>
</tr>
<tr>
<td>-0.852D-07</td>
<td>-0.124D-06</td>
<td>-0.177D-06</td>
<td>-0.127D-16</td>
<td>-0.125D-13</td>
<td>-0.654D-14</td>
<td>0.364D-13</td>
<td></td>
</tr>
</tbody>
</table>

COEFFICIENTS OF CORRELATION

<table>
<thead>
<tr>
<th></th>
<th>0.100D+01</th>
<th>0.125D+00</th>
<th>0.238D+00</th>
<th>-0.111D+00</th>
<th>0.635D+00</th>
<th>0.654D+00</th>
<th>-0.337D+00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125D+00</td>
<td>0.100D+01</td>
<td>-0.149D-01</td>
<td>0.765D+00</td>
<td>0.244D+00</td>
<td>0.122D+00</td>
<td>-0.429D+00</td>
<td></td>
</tr>
<tr>
<td>0.238D+00</td>
<td>-0.149D-01</td>
<td>0.100D+01</td>
<td>-0.493D+00</td>
<td>0.286D+00</td>
<td>0.120D+00</td>
<td>-0.648D+00</td>
<td></td>
</tr>
<tr>
<td>-0.111D+00</td>
<td>0.765D+00</td>
<td>-0.493D+00</td>
<td>0.100D+01</td>
<td>-0.103D-03</td>
<td>-0.116D-02</td>
<td>-0.317D-03</td>
<td></td>
</tr>
<tr>
<td>0.635D+00</td>
<td>0.244D+00</td>
<td>0.286D+00</td>
<td>-0.103D-03</td>
<td>0.100D+01</td>
<td>0.271D+00</td>
<td>-0.436D+00</td>
<td></td>
</tr>
<tr>
<td>0.654D+00</td>
<td>0.122D+00</td>
<td>0.120D+00</td>
<td>-0.116D-02</td>
<td>0.271D+00</td>
<td>0.100D+01</td>
<td>-0.265D+00</td>
<td></td>
</tr>
<tr>
<td>-0.337D+00</td>
<td>-0.429D+00</td>
<td>-0.648D+00</td>
<td>-0.317D-03</td>
<td>-0.436D+00</td>
<td>-0.265D+00</td>
<td>0.100D+01</td>
<td></td>
</tr>
</tbody>
</table>

-16-
scale factor as constraints, using full variance-covariance matrix (case (c)/(iii)).

A summary of the results for cases (a) through (c) and (i) through (iii) are presented in the following tables:

TABLE 3 gives the results for three rotations, as obtained independently of translations and scale factor from direction cosines, for cases (i) through (iii).

TABLE 4 gives the results for the scale factor, as obtained by direct chord comparisons independent of other transformation parameters, for cases (i) through (iii).

TABLE 5 gives the results for the constrained and non-constrained seven parameters general transformation solutions (cases (a) through (c) and (i) through (iii)).

TABLE 6 gives the comparative study of the results for seven parameters general transformation solutions as regards correlation between translations and rotations/scale factor, using different variance-covariance matrices (cases (i) through (iii)).

TABLE 7 gives the comparative study of the results for seven parameters general transformation solutions as regards correlation between translations and rotations/scale factor, using different constraints (cases (a) through (c)).
### TABLE 3

Three Rotation Parameters from Direction Cosines

**NA-9~MPS-7**

<table>
<thead>
<tr>
<th>Case</th>
<th>Using Variances Only (i)</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix (ii)</th>
<th>Using full Variance-Covariance Matrix (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega (^\circ)$</td>
<td>$0.17 \pm 0.05$</td>
<td>$0.17 \pm 0.04$</td>
<td>$0.17 \pm 0.04$</td>
</tr>
<tr>
<td>$\psi (^\circ)$</td>
<td>$0.04 \pm 0.04$</td>
<td>$-0.02 \pm 0.04$</td>
<td>$-0.04 \pm 0.04$</td>
</tr>
<tr>
<td>$\epsilon (^\circ)$</td>
<td>$-0.20 \pm 0.06$</td>
<td>$-0.24 \pm 0.05$</td>
<td>$-0.22 \pm 0.05$</td>
</tr>
<tr>
<td>$\sigma_0^2$</td>
<td>$1.15$</td>
<td>$1.30$</td>
<td>$1.36$</td>
</tr>
</tbody>
</table>

### TABLE 4

Scale Factor From Chord Comparison

**NA-9~MPS-7**

<table>
<thead>
<tr>
<th>Case</th>
<th>Using Variances Only (i)</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix (ii)</th>
<th>Using full Variance-Covariance Matrix (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s (x 10^5)$</td>
<td>$5.46 \pm 0.24$</td>
<td>$5.37 \pm 0.24$</td>
<td>$5.18 \pm 0.24$</td>
</tr>
<tr>
<td>Case</td>
<td>Non-Constrained Solutions</td>
<td>Constrained Solutions</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>--------------------------</td>
<td>-----------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Using Variances Only</td>
<td>Using (3x3) Banded</td>
<td>Using Full Variance-Covariance Matrix</td>
</tr>
<tr>
<td></td>
<td>(a)/(i)</td>
<td>(a)/(ii)</td>
<td>(a)/(iii)</td>
</tr>
<tr>
<td>ΔX (m)</td>
<td>-44.5 ± 5.2</td>
<td>-44.9 ± 3.6</td>
<td>-44.9 ± 3.6</td>
</tr>
<tr>
<td>ΔY (m)</td>
<td>171.5 ± 5.1</td>
<td>170.3 ± 4.7</td>
<td>170.3 ± 4.7</td>
</tr>
<tr>
<td>ΔZ (m)</td>
<td>190.4 ± 5.5</td>
<td>190.4 ± 4.3</td>
<td>190.4 ± 4.3</td>
</tr>
<tr>
<td>ω (°)</td>
<td>0.15 ± 0.16</td>
<td>0.17 ± 0.12</td>
<td>0.17 ± 0.12</td>
</tr>
<tr>
<td>Ψ (°)</td>
<td>0.04 ± 0.14</td>
<td>-0.03 ± 0.11</td>
<td>-0.03 ± 0.11</td>
</tr>
<tr>
<td>ε (°)</td>
<td>-0.30 ± 0.20</td>
<td>-0.28 ± 0.15</td>
<td>-0.28 ± 0.15</td>
</tr>
<tr>
<td>Δσ(x10³)</td>
<td>4.9 ± 0.7</td>
<td>4.7 ± 0.7</td>
<td>4.7 ± 0.7</td>
</tr>
<tr>
<td>σ²</td>
<td>0.95</td>
<td>0.83</td>
<td>0.83</td>
</tr>
</tbody>
</table>
### TABLE 6

**Comparative Study of Correlation Coefficients Between Transformation Parameters**

(Using Different Variance-Covariance Matrices)

**Case (i): USING VARIANCES ONLY**

<table>
<thead>
<tr>
<th>Case</th>
<th>Non-Constrained Solution</th>
<th>Constrained Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>Rotations and Scale Factor</td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>Translations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω</td>
<td>0.88</td>
<td>0.40</td>
</tr>
<tr>
<td>ψ</td>
<td>0.63</td>
<td>0.19</td>
</tr>
<tr>
<td>ε</td>
<td>−0.47</td>
<td>−0.67</td>
</tr>
<tr>
<td>Δs</td>
<td>−0.10</td>
<td>0.74</td>
</tr>
</tbody>
</table>

**Case (ii): USING (3 × 3) BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX**

<table>
<thead>
<tr>
<th>Case</th>
<th>Non-Constrained Solution</th>
<th>Constrained Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>Rotations and Scale Factor</td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>Translations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω</td>
<td>0.83</td>
<td>0.27</td>
</tr>
<tr>
<td>ψ</td>
<td>0.54</td>
<td>0.11</td>
</tr>
<tr>
<td>ε</td>
<td>−0.45</td>
<td>−0.51</td>
</tr>
<tr>
<td>Δs</td>
<td>−0.15</td>
<td>0.84</td>
</tr>
</tbody>
</table>
### Table 6 (Continued)

Case (iii): **Using Full Variance-Covariance Matrix**

| Case                | | Non-Constrained Solution | | Constrained Solutions | | Constrained Solutions |
|---------------------|-----------------|----------------------------|-----------------|----------------------------|-----------------|
|                     | Rotations and Scale Factor |  | 3 Rotations | 3 Rotations and Scale Factor |  |
|                     | Translations    | ΔX | ΔY | ΔZ | ΔX | ΔY | ΔZ | ΔX | ΔY | ΔZ |ΔX | ΔY | ΔZ |
| ω                   | 0.83            | 0.27 | 0.33 | 0.60 | 0.09 | 0.15 | 0.64 | 0.24 | 0.29 |
| ψ                   | 0.54            | 0.11 | 0.13 | 0.43 | 0.04 | 0.07 | 0.45 | 0.12 | 0.12 |
| ε                   | -0.45           | -0.51 | -0.80 | -0.32 | -0.16 | -0.34 | -0.34 | -0.43 | -0.65 |
| Δs                  | -0.15           | 0.84 | -0.56 | -0.36 | 0.97 | -0.89 | -0.11 | 0.76 | -0.49 |
TABLE 7
Comparative Study of Correlation Coefficients
Between Transformation Parameters
(Using Different Constraints)

Case (a): NON-CONSTRAINED SOLUTION

<table>
<thead>
<tr>
<th>Rotations and Scale Factor</th>
<th>Using Variances Only</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix</th>
<th>Using Full Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
</tr>
<tr>
<td>Translations</td>
<td>ΔX</td>
<td>ΔY</td>
<td>ΔZ</td>
</tr>
<tr>
<td>ω</td>
<td>0.88</td>
<td>0.40</td>
<td>0.43</td>
</tr>
<tr>
<td>ψ</td>
<td>0.63</td>
<td>0.19</td>
<td>0.13</td>
</tr>
<tr>
<td>ε</td>
<td>-0.47</td>
<td>-0.67</td>
<td>-0.88</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.10</td>
<td>0.74</td>
<td>-0.40</td>
</tr>
</tbody>
</table>

Case (b): CONSTRAINED SOLUTIONS
(CONSTRAINTS: 3 ROTATIONS)

<table>
<thead>
<tr>
<th>Rotations and Scale Factor</th>
<th>Using Variances Only</th>
<th>Using (3x3) Banded Diagonal Variance-Covariance Matrix</th>
<th>Using Full Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
</tr>
<tr>
<td>Translations</td>
<td>ΔX</td>
<td>ΔY</td>
<td>ΔZ</td>
</tr>
<tr>
<td>ω</td>
<td>0.68</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>ψ</td>
<td>0.49</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>ε</td>
<td>-0.38</td>
<td>-0.23</td>
<td>-0.45</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.29</td>
<td>0.95</td>
<td>-0.83</td>
</tr>
</tbody>
</table>
TABLE 7 (Continued)

Case (c): CONSTRAINED SOLUTIONS

(CONRAINTS: 3 ROTATIONS AND SCALE FACTOR)

<table>
<thead>
<tr>
<th>Rotations and Scale Factor</th>
<th>Translations</th>
<th>Using Variances Only</th>
<th>Using (3×3) Banded Diagonal Variance-Covariance Matrix</th>
<th>Using Full Variance-Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
<td></td>
</tr>
<tr>
<td>ω</td>
<td>ΔX 0.71  ΔY 0.32  ΔZ 0.35</td>
<td>ΔX 0.62  ΔY 0.24  ΔZ 0.27</td>
<td>ΔX 0.64  ΔY 0.24  ΔZ 0.29</td>
<td></td>
</tr>
<tr>
<td>ψ</td>
<td>ΔX 0.51  ΔY 0.14  ΔZ 0.13</td>
<td>ΔX 0.40  ΔY 0.12  ΔZ 0.13</td>
<td>ΔX 0.45  ΔY 0.12  ΔZ 0.12</td>
<td></td>
</tr>
<tr>
<td>ε</td>
<td>ΔX -0.40  ΔY -0.51  ΔZ -0.73</td>
<td>ΔX -0.34  ΔY -0.44  ΔZ -0.66</td>
<td>ΔX -0.34  ΔY -0.43  ΔZ -0.65</td>
<td></td>
</tr>
<tr>
<td>Δs</td>
<td>ΔX -0.10  ΔY 0.72  ΔZ -0.44</td>
<td>ΔX -0.11  ΔY 0.76  ΔZ -0.49</td>
<td>ΔX -0.11  ΔY 0.76  ΔZ -0.49</td>
<td></td>
</tr>
</tbody>
</table>
5. CONCLUSIONS

The comparison between different columns of Table 3 shows that the estimates for three rotation parameters remain more or less the same, but that their standard deviations show some improvement as we proceed from column 1 (variances only) to column 3 (full variance-covariance matrix). However, in the case of scale factor (Table 4) the estimates for $\Delta s$ indicate a definite trend while standard deviation remains constant.

In the case of seven parameters general transformation (Table 5) the comparisons among different columns indicate a definite overall improvement in all parameter estimates. The best estimates were obtained in the solution using full variance-covariance matrix and three rotations ($\omega, \psi, \epsilon$) and scale factor ($\Delta s$) as constraints (column 10). In this case the standard deviations for all the parameters are smaller (or at the most, equal) compared to those in any other column of Table 5.

Further, it is also noticeable that the improvement from a non-constrained solution to a constrained solution, both with three or four constraints, is more significant compared to the improvement from a constrained solution using variances only to a constrained solution using $(3 \times 3)$ banded diagonal or full variance-covariance matrix. The improvement from the solution using $(3 \times 3)$ banded diagonal to the solution using full variance-covariance matrix is, however, marginal.

A study of Table 6 indicates in all the three cases an overall improvement in correlation from a non-constrained to a constrained solution with four constraints (three rotations and one scale factor). The improvement in correlation between translations and rotations is quite significant while the same in not reflected between translations and scale factor. However, the improvement pattern from Table 7 is not straightforward. The correlations between translations and rotations show a downward trend from the solutions using variances only to the solutions using full variance-covariance matrix in all the three cases while the correlations between translations and $\Delta s$ show an upward trend.
REFERENCES


Mueller, Ivan I. and Marvin C. Whiting (1972). "Free Adjustment of a Geometric Global Satellite Network (Solution MPS-7)," Reports of the Department of Geodetic Science, No. 188, The Ohio State University, Columbus.

Uotila, Urho A. (1967). "Introduction to Adjustment Computation with Matrices," Department of Geodetic Science, The Ohio State University, Columbus.

APPENDIX I

Fortran IV Program with Subroutines
PROGRAM WORKS ON TWO SETS OF COORDINATES — EITHER
SET CAN BE INPUT AS ELLIPSOIDAL COORDINATES, BOTH IN
DEGREES AND METERS OR IN GEOS FORMAT. IN SUCH CASE
SEMI-MAJOR AXIS 'A' AND ECCENTRICITY 'E' ARE NEEDED.

UVW MATRIX TAKES COORDINATES IN THE FIRST SYSTEM
---( IN FORMAT 15.3F15.5 )

XYZ MATRIX TAKES COORDINATES IN THE SECOND SYSTEM
---( IN FORMAT 14.5X,0F16.5 )

MAXIMUM NUMBER OF INPUT POINTS FOR EACH SYSTEM ---35

SUBROUTINE -- 'CSTRNT'

SOLVES FOR TRANSFORMATION CASE WHEN CONSTRAINTS ARE
TO BE APPLIED FOR THREE ROTATIONS. NECESSARY COUNTER
--- KCODE(11) -- IS TO BE CODED AS ' 4 '.

TWO SOLUTIONS ARE OBTAINED WITH THE SAME DATA ---
FIRST WITHOUT CONSTRAINTS AND SECOND WITH CONSTRAINTS.

INPUT CONSTRAINTS ARE OBTAINED FROM SUBROUTINE 'EULERS'
AND SUBROUTINE 'SCALE'.

SUBROUTINE -- 'TFORM'

TRANSFORMATION PARAMETERS SOLVED UNDER THREE CASES.
REFER KCODE(3) ALSO.
SUBROUTINE — 'SCALE'

SCALE FACTOR BETWEEN SYSTEM #1 AND SYSTEM #2 IS COMPUTED BY COMPARISON OF (I-K)TH CHORDS IN THE TWO SYSTEMS. THE WEIGHT FOR EACH ESTIMATE OF SCALE FACTOR CORRESPONDING TO (I-K)TH CHORD IS COMPUTED USING VARIANCE-COVARIANCE MATRICES OF ITH AND KTH POINTS OF SYSTEM #1 AND SYSTEM #2 THROUGH ERROR PROPAGATION.

SUBROUTINE — 'EULERS'

EULERS ANGLES ARE COMPUTED FROM DIRECTION COSINES. WEIGHT MATRIX 'P' FOR 'TIK AND 'DIK' IS COMPUTED USING VARIANCE-COVARIANCE MATRICES OF THE POINT COORDINATES OF THE SYSTEMS BY ERROR PROPAGATION.

SUBROUTINE — 'SETUP'

SETS UP AND STORES WEIGHT MATRIX 'P' IN VECTOR FORM.

INPUT — KCODES TO BE CODED WITH EACH DATASET
KCODE(1) = 'TOTAL NUMBER OF POINTS -- IN (12) FIELD.

KCODE(2) = 'PARAMETERS REQUIRED IN THE SOLUTION'
3 DENOTES ONLY TRANSLATIONS OR ROTATIONS -- SEE KCODE(14)
4 DENOTES TRANSLATIONS AND SCALE.
7 DENOTES TRANSLATION, SCALE AND THREE ROTATIONS.

KCODE(3) = 'WHETHER CONSTRAINED SOLUTION IS NEEDED'
0 DENOTES NO SUCH SOLUTION
1 DENOTES CONSTRAINED SOLUTION.

KCODE(4) = 'FIRST SYSTEM IN ELLIPSOIDAL COORDINATES
IN DEGREES FOR PHI AND LEMDA --
AND METERS FOR HEIGHTS'
0 DENOTES NO SUCH CASE
1 DENOTES SUCH INPUT

KCODE(5) = 'INPUT FOR FIRST SYSTEM IN GEOS FORMAT'
0 DENOTES NO SUCH CASE
1 DENOTES SUCH INPUT

KCODE(6) = 'SECOND SYSTEM IN ELLIPSOIDAL COORDINATES
IN DEGREES FOR PHI AND LEMDA --
AND HEIGHTS IN METERS'
0 DENOTES NO SUCH CASE
1 DENOTES SUCH INPUT

KCODE(7) = 'INPUT FOR SECOND SYSTEM IN GEOS FORMAT'
0 DENOTES NO SUCH CASE
1 DENOTES SUCH INPUT

KCODE(8) = 'VARIANCE - COVARIANCE MATRIX AS DIAGONAL'
IN FORMAT '13F10.2' -- 1 CARDS PER STN.
0 DENOTES NO SUCH CASE
1 DENOTES SUCH INPUT

KCODE(9) = 'VARIANCE - COVARIANCE MATRIX IN 3X3 FORM'
0 DENOTES NO SUCH CASE
IN FORMAT '3F5.2' -- 3 CARDS PER STN.
1 DENOTES SUCH INPUT

KCODE(10) = 'VARIANCE - COVARIANCE MATRIX IN FULL AS
UPPER TRIANGLE (ROW-WISE) IN VECTOR FORM'
IN FORMAT '8F10.4' -- EACH NEW ROW TO
BEGIN ON A NEW CARD FROM COLUMN 1.
SEE KCODE(12) ALSO.
0 DENOTES NO SUCH CASE
1 DENOTES SUCH INPUT
KCODE(11) = 'FOR CONSTRAINED SOLUTION TO BE CODED AS'
'3' ROTATIONS ARE CONSTRAINED.
'4' ROTATIONS AND SCALE ARE CONSTRAINED.

KCODE(12) = 'TO BE CODED '1' TO OMIT CORRELATION WITH KCODE(10) AS AN ALTERNATE SOLUTION.

KCODE(13) = 'TOTAL NUMBER OF TRANSFORMATIONS TO BE PERFORMED ' --- TO BE CODED WITH THE LAST DATA SET IN (12) FIELD.

KCODE(14) = '3 PARAMETER SOLUTION ONLY'
0 SOLUTION FOR TRANSLATIONS
1 SOLUTION FOR ROTATIONS

FIRST CARD --- TO CONTAIN ALL KCODES
CARDS CONTAINING COORDINATES FOR THE FIRST SYSTEM
CARDS CONTAINING COORDINATES FOR THE SECOND SYSTEM
CARDS CONTAINING VARIANCE-COVARIANCE MATRIX
CARDS CONTAINING VARIANCE-COVARIANCE MATRIX FOR THE FIRST SYSTEM.
CARDS CONTAINING VARIANCE-COVARIANCE MATRIX FOR THE SECOND SYSTEM.

-31-
IMPLICIT REAL *(A-H, O-Z)
IMPLICIT REAL *8(A-H, O-Z)
*
SUBROUTINE PNAL *8 LEMDA,NI,M02
DIMENSION XYZ(35,3),RANGLE(4),VROT(4,4),NAME1(3),
AA(3,105),B&<3,105),NSTA(35),KSTA(35),KCODE(15)
COMMON /WEIGHT/ P
COMMON /CODE/ KCODE
COMMON /NAME/ NAME1,NAME2
COMMON NSTA,KSTA,NN,NM,UVW,XYZ,A,W,KPR,KPAM
COMMON /ANGLE/ RANGLF,VROT
DATA MINUS/IN-/
PI = 3.141592653589793
DO
RHO = 180.DO/PI
RHOS = RHO*3600.DO
KOUNT = 1
C
C ******** READ IN VARIOUS CODES INVOLVED
C
C
RHO = 180.DO/P11
RHOS = RHO*3600.DO
KOUNT = 1
C
C ******** READ IN VARIOUS CODES INVOLVED
C
C
1000 READ (5, 1) (KCODE(I), I = 1,15) , (NAME1 ( I ) , 1=1,3 ) ,
( NAME2(I),I=1,3)
1 FORMAT (12,1111.12,211,3X,3A4,3X,3A4)
WRITE (6, 2) (KCODf(I), I = 1,15)
2 FORMAT (•!«, //////, 25X, 'KCODE INPUT', //, 20X,1512, //)
NO = KCODE(1)
IF (KCODE(4).EQ.0.AND.KCODE(5).EQ.0) GO TO 12
C
C ******** READ IN DATA FOR THE FIRST SYSTEM
C
C
READ (5, 3) AF1,F
2 FORMAT (2F15.10)
F = 1.DO/F
E2 = 2.DO*F - F*F
IF ( KCODE(5) .EQ. 1 ) GO TO 6
C
C ******** READ IN ELLIPSOIDAL COORDINATES IN DEGREES AND HEIGHT
C
C
DO 5 I = 1 , NO
READ (5, 4) NSTA(I),PHI,LEMDA,HT
4 FORMAT (14,5X,3F16.9)
PHI = PHI / RHO
LEMDA = LEMDA / RHO
W = (1.DO-E2)*DSIN(PHI)*DSIN(PHI)**0.5
UVW(I,1)= (AF1/WW+HT)*DCOS(PHI)*DCOS(LEMDA)
UVW(I,2)= (AF1/WW+HT)*DCOS(PHI)*DSIN(LEMDA)
UVW(I,3)= (((AF1*(1.DO-E2 ))/WW)+HT)*DSIN(PHI)
5 CONTINUE
GO TO 15
C
C ******** READ IN ELLIPSOIDAL COORDINATES IN GEOS FORMAT
C
C
6 DO 11 I = 1 , NO
READ (5, 7) NSTA(I),15N,1PH,MHP,SPH,ILM,LM,SLM,HT
7 FORMAT (14,20X,A1,213,F8.3,213,F8.3,F10.2)
LEMDA = (ILM*(MLM/SLM/60.DO))/RHO
GO TO 15
C
C
-32-
IF (ISN .EQ. MINUS) GO TO 8

PHI = (IPH+((MPH+(SPH/60.0))/60.0))/RHO
GO TO 10

8 PHI = -(IPH+((MPH+(SPH/60.0))/60.0))/RHO

10 WW = (1.00-E2*DSIN(PHI)*DSIN(PHI))**0.5DO
UVW(J,1) = (AE1/WW+HT)*DCOS(PHI)*DCOS(LEMDA)
UVW(J,2) = (AE1/WW+HT)*DCOS(PHI)*DSIN(LEMDA)
UVW(J,3) = (((AE1*(1.00-E2))/WW+HT)*DSIN(PHI)

11 CONTINUE
GO TO 15

******** READ IN RECTANGULAR COORDINATES ( U, V, W ) IN METERS

12 DO 14 I = 1, NO
READ (5, 13) NSTA(I),(UVW(I,J),J=1,3)
13 FORMAT(14,5X,3F16.5)
14 CONTINUE

**** READ IN COORDINATES OF THE SECOND SYSTEM

15 IF (KCODE(6).EQ.1.OR KCODE(7).EQ.1) GO TO 20

******** READ IN RECTANGULAR COORDINATES ( X, Y, Z ) IN METERS

DO 18 I = 1, NO
READ (5, 16) KSTA(I),(XYZ(I,J), J=1,3)
16 FORMAT(14,5X,3F16.9)
18 CONTINUE
GO TO 40

20 RFAD (5, 22) AE2,F
22 FORMAT (2F15.10)
F = 1.00/F
E2 = 2.00*F - F*F
IF ( KCODE(7).EQ.1 ) GO TO 25

******** READ IN ELLIPSOIDAL COORDINATES IN DEGREES AND HEIGHT

DO 24 I = 1, NO
READ (5, 23) KSTA(I),PHI,LFMDA,HT
23 FORMAT(14,5X,3F16.9)
PHI = PHI / RHO
LFMDA = LFMDA / RHO
WW = (1.00-E2 *DSIN(PHI)*DSIN(PHI))**0.5DO
XYZ(I,1) = (AE2/WW+HT)*DCOS(PHI)*DCOS(LEMDA)
XYZ(I,2) = (AE2/WW+HT)*DCOS(PHI)*DSIN(LEMDA)
XYZ(I,3) = (((AE2*(1.00-E2))/WW+HT)*DSIN(PHI)

24 CONTINUE
GO TO 40
EAD IN ELLIPSOIDAL COORDINATES IN GECS FORMAT

25 DO 21 I = 1, NO
READ (5, 26) KSTA(I), ISN, IPH, MPH, SPH, ILM, MLM, SLM, HT
26 FORMAT (14, 20X, A1, 213, F8.3, 213, F8.3, F10.2)
LEMDA = (ILM+((MLM+(SLM/60.DO))/60.DO))/RHO
IF (ISN .EQ. MINUS) GO TO 28
PHI = (IPH+((MPH+(SPH/60.DO))/60.DO))/RHO
GO TO 30
28 PHI = -(IPH+((MPH+(SPH/60.DO))/60.DO))/RHO
30 WW = (1.00-E2*DSIN(PHI)*DSIN(PHI))**0.5DO
XY7(I, 1) = (AE2/WW+HT)*DCOS(PHI)*DCOS(LEMDA)
XY2(I, 2) = (AE2/WW+HT)*DCOS(PHI)*DCOS(LEMDA)
XY2(I, 3) = (AE2/((1.DO-F2)/WW)+HT)*DSIN(PHI)
31 CONTINUE

**** WRITING OF READ IN DATA FOR THE TWO SYSTEM IN RECTANGULAR COORDINATES

40 WRITE(6, 42)
42 FORMAT(1',//, 25X, 'RECTANGULAR COORDINATES FOR FIRST SYSTEM',//)
WRITE(6, 43)
DO 44 I = 1, NO
WRITE(6, 44) KSTA(I), (UWF(I, J), J = 1, 3)
44 CONTINUE
WRITE(6, 50)
50 FORMAT(1',//, 25X, 'RECTANGULAR COORDINATES FOR SECOND SYSTEM',//)
WRITE(6, 52)
DO 58 I = 1, NO
WRITE(6, 58) KSTA(I), (XYZ(I, J), J = 1, 3)
58 CONTINUE

**** SEPARATING THE TYPE OF SOLUTION REQUIRED

KPARM = KCODE(11)
IF (KCODE(8) .NE. 1) GO TO 62
KPR = 1
GO TO 75
62 IF (KCODE(9) .NE. 1) GO TO 64
KPR = 2
GO TO 75
64 KPR = 3
IF (KCODE(10).EQ.1.AND.KCODE(12).EQ.1) KPR = 2
75 NM = NO - 1
NN = NO * NM
NNN = 3*NM
IF (KCODE(14) .EQ. 1) GO TO 85
CALL FULERS (ND, NNN, AA, BB)
IF (KCODE(14).EQ.1.AND.KCODE(2).EQ.3) GO TO 95

85 CALL TFORM (NO, NNN)
IF (KOUNT .EQ. KCODE(13)) GO TO 95
KOUNT = KOUNT + 1
GO TO 1000

95 STOP
END
SUBROUTINE EULERS (NO, NNN, AA, BB)
IMPLICIT REAL *8 (A-H,O-Z)
RFAL *8 NI,NN,M02
DIMENSION UVW(35,3),XYZ(35,3),A(3600),W(1200),NAME1(3),
2P1(6,6),G(2,6),GP(2,6),GT(6,2),PP(2,2),XY(2),KY(2),NAME2(3),
3B(2,4),BT(4,2),P2(6,6),INDEX(40),INV(40),OXYZ(4500),NZ(4,4),
4P(2400),RS(2,4),KSTA(35),NSTA(35),QUVW(4500),AA(3,NNN),BB(3,NNN),
5P0(2,2),P1(4,4),NI(3,3),DX(3),U(3),VAR(3,3),KO(3),KCOOE(15),LO(3)
COMMON /WEIGHT/ P
COMMON /CODE/ KCODE
COMMON /ANGLE/ S,DX,NZ
COMMON /NAME/ NAME1,NAME2
COMMON /SFAC/ DW,DS,DA1,DA2,DC1,DC2
COMMON NSTA,KSTA,NM,NM,UVW,XYZ,A,W,KPR,KPARM
PI = 3.141592653589793D0
RHO = 180.D0/PI
RHOS = RHO*3600.D0
MW = 0.D0
DS = 0.D0
S = 0.D0
VSF = 0.D0
WT = 0.D0
LL = 1

**** SETTING UP OF MATRIX 'B' -- COMMON TO ALL SOLUTION

R(1,1) = -1.D0
R(1,2) = 0.D0
R(1,3) = 1.D0
R(1,4) = 0.D0
R(2,1) = 0.D0
R(2,2) = -1.D0

-36-
6(2,3) = 0.00
VS = NN/2
B(2,4) = 1.00
DC 1 I = 1, 2
DC 1 J = 1, 4
RT(J, I) = B(I, J)
1 CONTINUE
DC 2 I = 1, 4
DC 2 J = 1, 4
PR(I, J) = 0.00
2 CONTINUE
IF (KCODE(8).EQ.1 .OR. KCODE(9).EQ.1) GO TO 10

************
****
************

**** FULL VARIANCE-COVARIANCE CASE

************

************

**** FADING IN VARIANCE-COVARIANCES FOR 'FIRST SYSTEM'

JK = 1
DC 6 I = 1, NNN
JL = JK + NNN - 1
READ (5, 3) (OUVW(J), J = JK, JL)
2 FORMAT (8F10.4)
DC 4 L = LL + 3
P1(LL+L) = OUVW(JK+L-LL)
4 P1(L+LL) = P1(LL+L)
WRITE (1) (P1(LL,M), M = 1, 3)
LL = LL + 1
IF (LL .EQ. 4) LL = 1
6 JK = JL + 1
REWIND 1

**** FADING IN VARIANCE-COVARIANCES FOR 'SECOND SYSTEM'

LL = 1
JK = 1
DC 9 I = 1, NNN
JL = JK + NNN - 1
READ (5, 7) (OXYZ(J), J = JK, JL)
7 FORMAT (8F10.4)
DC 8 L = LL + 3
P2(LL+L) = OXYZ(JK+L-LL)
8 P2(LL+LL) = P2(LL+L)
WRITE (2) (P2(LL,M), M = 1, 3)

-37-
** Diagonal or 3x3 Banded Case **

- ** VARIANCE - COVARIANCE MATRIX IN 3X3 BANDED FORM **

- ** VARIANCE - COVARIANCE MATRIX IN DIAGONAL FORM (ONLY VARIANCES) **

- ** READING IN VARIANCE-COVARIANCE FOR SECOND SYSTEM **
**VARIAACE - COVARIANCE MATRIX IN 3X3 BANDED FORM**

```fortran
KM = KK + 2
IF (KCONF(8) .EQ. 1) GO TO 20
C
C **** VARIANCE - COVARIANCE MATRIX IN 3X3 BANDED FORM
C
DO 19 J = 1 , 3
READ (5,18) (BB(J,K), K=KK,KM)
18 FORMAT (3F5.2)
19 WRITE(2) (BB(J,K), K=KK,KM)
GO TO 23
C
C **** VARIANCE - COVARIANCE MATRIX IN DIAGONAL FORM (ONLY VARIANCES)
C
20 DO 21 J
21 WRITE(2) (BB(J,K), K=KK,KM)
CONTINUE
C
**FORMING MATRICES 'A', 'W', AND 'P' FOR THE ENTIRE SYSTEM**
**BY COMPUTING DIRECTION COSINES FOR EACH LINE BETWEEN**
**ANY ONE SET OF TWO GIVEN POINTS.**

```
27 CONTINUE
28  GO TO 32
29  DO 30 J = 1, 3
30  DC 29 L = J, 3
31  LLL = LL + L - J
32  P1(J,L) = QUVW(LLL)
33  P2(J,L) = QXYZ(LLL)
34  MM1 = MM1 - 1
35  LL = LL + MM1
36  JJ = JJ + 1
37  INDEX(JJ) = LL
38  MM2 = MM1
39  DO 50 K = JJ, NO
40  IF (KCODE(8), EQ. 1, OR KCODE(9), EQ. 1) GO TO 43
41  IF (KCODE(12), EQ. 1) GO TO 41
42  DO 34 J = 4, 6
43  DC 36 L = J, 6
44  LLL = III + L - 3
45  P1(J,L) = QUVW(LLL)
46  P2(J,L) = QXYZ(LLL)
47  MM2 = MM2 - 1
48  LL = LL + MM2
49  KP = K + 1
50  INDEX(KP) = LL
51  III = INDEX(I) + INV(K-1)
52  IF (KCODE(12), EQ. 1) GO TO 41
53  DO 38 J = 1, 3
54  DC 38 L = J, 3
55  III = III + (JNN - (3*(I-1))-J)
56  LLL = III + L - 3
57  P1(J,L) = QUVW(LLL)
58  P2(J,L) = QXYZ(LLL)
59  GO TO 45
60  DO 44 L = 4, 6
61  JKL = L - 3
62  DC 44 M = 4, 6
63  KLM = (K-2)*3 + M
64  P1(L,M) = AA(JKL,KLM)
65  P2(L,M) = BB(JKL,KLM)
66  CONTINUE
67  CONTINUE
68  KSM = MKR + NN
69  KMS = MKR + (2*NN)

C **** COMPUTING DIRECTION COSINES FOR FIRST SYSTEM
C
C
C DA1 = UVW(K,1) - UVW(I,1)
C DB1 = UVW(K,2) - UVW(I,2)
C DC1 = UVW(K,3) - UVW(I,3)
C
C D1K1 = DSQRT(DA1*DA1+DB1*DB1+DC1*DC1)
C AK1 = DA1/D1K1
C BK1 = DB1/D1K1

-40-
CIK1 = DC1/RIK1
TIK1 = -DATAN2(BIK1, AIK1)
IF (TIK1.LT.0.) TIK1 = (360.0+TIK1*RHO)/RHO
AB1 = DSORT(AIK1•AIK1+BIK1•BIK1)
DIK1 = DATAN2(CIK1, AB1)

**** COMPUTING DIRECTION COSINES FOR SECOND SYSTEM

DA2 = XYZ(K,1) - XYZ(I,1)
DR2 = XYZ(K,2) - XYZ(I,2)
DC2 = XYZ(K,3) - XYZ(I,3)
RIK2 = DSORT(DA2*DA2+DB2•DB2+DC2•DC2)
AIK2 = DA2/RIK2
BIK2 = DB2/RIK2
CIK2 = DC2/RIK2
TIK2 = -DATAN2(BIK2, AIK2)
IF (TIK2.LT.0.) TIK2 = (360.0+TIK2*RHO)/RHO
AB2 = DSORT(AIK2•AIK2+BIK2•BIK2)
DIK2 = DATAN2(CIK2, AB2)

**** SETTING UP MATRICES 'A' AND 'W'- COMMON TO ALL SOLUTION

A(MKR) = 1.0D0
A(MKR+1) = 0.0D0
A(KSM) = DSIN(TIK2)•OTAN(DIK2)
A(KSM+1) = DCOS(TIK2)
A(KMS) = -DCOS(TIK2)•OTAN(DIK2)
A(KMS+1) = DSIN(TIK2)
W(MKR) = TIK1 - TIK2
W(MKR+1) = DIK1 - DIK2

*************** FORMING VAR-COVARIANCE MATRIX FOR 'TIK' AND 'DIK' ***************

FIRST SYSTEM

DAB1 = DA1•DA1+DB1•DB1
DBA = DSORT(DAB1)
G(1,1) = -DB1/DAB1
G(1,2) = DA1/DAB1
G(1,3) = 0.0D0
G(1,4) = -G(1,1)

-41-
G(1,5) = -G(1,2)  
G(1,6) = 0.D0  
G(2,1) = DA1*DC1/(DBA*RIK1*RIK1)  
G(2,2) = DB1*DC1/(DBA*RIK1*RIK1)  
G(2,3) = -DBA/(RIK1*RIK1)  
G(2,4) = -G(2,1)  
G(2,5) = -G(2,2)  
G(2,6) = -G(2,3)  
DO 46 L = 1, 2  
DO 46 M = 1, 6  
GT(M,L) = G(L,M)  
46 CONTINUE  
CALL DGMPRD(G,P1,GP,2,6,6)  
CALL DGMPRD(G,P,GT,PP,2,6,2)  

**** FORMING MATRIX 'MI' FOR THE COMBINED SYSTEM  

C  
C  
C  
DO 48 L = 1, 2  
J = L + 2  
DO 48 M = 1, 2  
N = M + 2  
PQ(L,M) = PQ(L,M)  
PP(L,M) = PP(L,M)  
48 CONTINUE  
CALL DGMPRD(B,PR,BS,2,4,4)  
CALL DGMPRD(BS,BT,PP,2,4,2)  
CALL DMINV(PP,2,DT,KX,KY)
\[ P(KMT) = PP(1,1) \]
\[ P(KMT+1) = PP(2,1) \]
\[ P(KMT+2) = PP(1,2) \]
\[ P(KMT+3) = PP(2,2) \]
\[ MKR = MKR + 2 \]
\[ KMT = KMT + 4 \]

**C **** FORMING MATRIX 'N' AND INVERTING THE SAME**

**C**

```
DO 80 I = 1 ; 3
READ (3) (W(J), J=1,NN)
K1 = (I-1)*NN + 1
K2 = K1 + NN - 1
MMM = 0
DO 78 K = K1 , K2
A(K) = 0.000
L1 = (((K-K1)/2)*2) + 1
L2 = L1 + 1
DO 78 L = L1 , L2
MMM = MMM + 1
78 A(K) = A(K) + W(L)*P(MMM)
F0 CONTINUE
```

```
DC 84 I = 1 , 4
DO 84 J = 1 , 4
64 NZ(I,J) = 0.000
REWIND 3
DC 86 I = 1 , 3
REWIND 3
```

```
-43-```
DO 85 J = 1, 3
N(J,1) = 0.00
DO 85 K = 1, NN
III = (J-1)*NN + K
85 N(J,1) = N(J,1) + A(III)*W(K)
88 CONTINUE
REWRITE
3
IS
MT(1,1) = MT
90 DO 91 I = 2, 4
90 DO 91 J = 2, 4
91 MT(I,J) = MT(I-1,J-1)
CALL DMINV(N1,3,DF1,KQ,LO)
C
C*****************************************************************************
C **** COMPUTING SOLUTION VECTOR ' DX ' FOR 3 ROTATION PARAMETERS ******
C*****************************************************************************
C
READ(4) (W(I), I=1, NN)
REWIND 4
DO 92 J = 1, 3
U(J) = 0.00
DO 92 I = 1, NN
KKK = (J-1)*NN + I
U(J) = U(J) - A(KKK)*W(I)
92 CONTINUE
CALL DMMPRO(N1,U,DX,3,3,1)
DO 95 I = 1, 3
JK = (I-1)*NN + 1
JM = JK + NN - 1
95 READ(3) (A(J), J= JK, JM)
REWIND 3
C
C*****************************************************************************
C **** COMPUTING VARIANCE OF UNIT WEIGHT ' MD2 ' ******
C*****************************************************************************
C
DO 96 I = 1, NN
W(I) = 0.00
DO 96 J = 1, 3
K = (J-1)*NN + 1
W(I) = W(I) - A(K)*DX(J)
96 CONTINUE
READ(4) (A(I), I= 1, NN)
REWIND 4
DO 97 K = 1, NN
W(K) = W(K) - A(K)
97 CONTINUE
MMX = 0
DO 98 K = 1, NN
98 CONTINUE
A(K) = 0.00
L1 = ((K-1)/2)*2 + K
L2 = L1 + 2
DO 98 K = 1, NN
98 A(K) = A(K) + P(L)*W(MM)
READ(4) (W(I), I = 1, NN)
REWIND 4
VPV = 0.00
DO 99 K = 1, NN
99 VPV = VPV - A(K)*W(K)
M02 = VPV/(NN - 3)

C
C

**** COMPUTING VARIANCE-COVARIANCE MATRIX **** VAR ****
C
C
DO 100 I = 1, 3
DO 100 J = 1, 3
VAR(I,J) = M02*RHOS*RHOS*NI(I,J)
100 CONTINUE
DO 105 I = 1, 3
105 DX(I) = DX(I)*RHOS

C
C
**** COMPUTING COEFFICIENTS OF CO-RELATIONS FOR PARAMETERS ****
C
C
DO 110 I = 1, 3
IF(I.EQ.3) GO TO 107
JL = I + 1
DO 106 J = JL, 3
NI(I,J) = VAR(I,J)/(DSORT(VAR(I,I))*DSORT(VAR(J,J)))
106 NI(I,J) = NI(I,J)
107 NI(I,I) = 1.00
110 CONTINUE

C
C
************ WRITING OF FINAL SOLUTION VECTOR AND VARIANCE-COVARIANCE MATRIX ****

C

WRITE(6,6025)
6025 FORMAT('11,///')
WRITE(6,6028) (NAME(I), I = 1,3), (NAME2(I), I = 1,3)
6025 FORMAT('11,5X,3A4,8-TC-,3A4,///')
76X,'*************'///)
WRITE(6,6030)
6030 FORMAT('11,30X,'SOLUTION FOR '3' ROTATION PARAMETERS',///
221X,'------------------------',///
325X,'(FROM DIRECTION COSINES -- UNITS SECONDS OF ARC')///
GO TO (112,114,116), KPR
112 WRITE(6,6031)
**PROGRAM TO TRANSFORM ONE RECTANGULAR COORDINATES SYSTEM TO SECOND RECTANGULAR COORDINATES SYSTEM AND VICE-VERSA**

**SUPROUTINE TFORM (NO,NC)**

IMPLICIT REAL * 8 (A-H, O-Z)

DIMENSION XYZ(35,3),UVW(35,3),SIGMA(7,7),NAME1(3),
2A(3600),W(1200),VAR(7,7),DX(7),NI(49),NSTA(35),NAME2(3),
3M(4),V(7),L(7),KCODE(15),CNT(7,4),T(7,4),CN(4,7),ZP(4,4),
4MT(7),KSTA(35),VPV(7,7),XO(7),KL(150),KK(150),MI(2400),ROT(4,4)

COMMON /WEIGHT/ MI
COMMON /CODE/ KCODE
COMMON /ANGLE/ ANG,ROT
COMMON /INAME/ NAME1,NAME2
COMMON /CRNT/ VPV,DX,SO,XO,SIGMA
COMMON NSTA,KSTA,NN,NN,XYZ,UVW,VAR,VPV,KPAR

PI*1 = 3.141592653589793

COMMON COMMON COMMON COMMON COMMON COMMON COMMON COMMON COMMON COMMON

RHO = RHO*3600.00
IPARA = KCODE(2)
IC = KCODE(11)
KOUNT = 1
DO 5 I = 1 , 4
DO 5 J = 1 , 7
CNI(I,J) = 0.00
TT(J,I) = 0.00
5 CNT(J,I) = 0.00
DO 10 I = 1 , 4
DO 10 J = 1 , 4
10 ZP(I,J) = 0.00

**** SETTING UP MATRIX 'A' -- COMMON TO ALL SOLUTION

NIN = 6*NO
NNZ = NO*IPARA
DC 13 I = 1 , NNZ
CONTINUE
DO 15 I = 1, NO
KKK = (3*I-2)
LLL = KKK+NQ+1
MMM = LLL+NQ+1
A(KKK) = 1.00
A(LLL) = 1.00
A(MMM) = 1.00

**** SETTING UP MATRIX *W* WHICH IS COMMON TO ALL SOLUTION

W(KKK) = (UVW(I,1)-XYZ(I,1))
W(KKK+1) = (UVW(I,2)-XYZ(I,2))
W(KKK+2) = (UVW(I,3)-XYZ(I,3))

15 CONTINUE
IF (KCODE(2) .NE. 3) GO TO 50

**** SOLUTION FOR 3 TRANSLATION PARAMETERS

N = 3
ICASF = 1
GO TO 81

**** SOLUTION FOR 3 TRANSLATION AND 1 SCALE PARAMETERS

50 N = 4
DO 60 I = 1, NO
KKK = 3*(NQ+1)-2
A(KKK) = UVW(I,1)
A(KKK+1) = UVW(I,2)
A(KKK+2) = UVW(I,3)

60 CONTINUE
IF (KCODE(2) .NE. 4) GO TO 70
ICASF = 2
GO TO 81

**** SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

70 N = 7
ICASF = 3
DO 80 I = 1, NO
KKK = 4*NQ*(3*I-2)
LLL = KKK + NO
MMM = LLL + NO + 1
A(KKK) = UVW(I,2)
A(KKK+1) = UVW(I,1)
A(LLL) = UVW(I,3)
A(LLL+2) = UVW(I,1)
A(MMM) = UVW(I,3)
A(MMM+1) = UVW(I,2)

CONTINUE

DO 65 I = 1, N
KKK = (I-1)*NO+1
LLL = KKK+NO-1
WRITE(3) (A(J), J=KKK,LLL)

CONTINUE
RE.WIND 3
WRITE(4) (W(I), I=1,NO)
RE.WIND 4

********************************************

************ FORMING NORMAL EQUATIONS -- MATRICES 'N' AND 'U' ************

********************************************

100 CALL SETUP (NG,NG,IPAR)
DC 118 I = 1, N
READ(3) (W(J), J=1,NO)
K1 = (I-1)*NO+1
K2 = K1+NO-1
MMM = 0
DO 116 K = K1, K2
A(K) = 0.0D0
L1 = (((K-K1)/3)*3)+1
L2 = L1 + 2
DO 116 L = L1, L2
MMM = MMM + 1
116 A(K) = A(K) + W(L)*M1(MMM)

CONTINUE
RE.WIND 3
DO 120 I = 1, N
READ(3) (W(L), L = 1,NO)
JK = (I-1)*N+1
JL = JK+N-1
DO 119 J = JK, JL
NI(J) = 0.0D0
DO 119 K = 1, NO
II = (J-JK)*NO + K
119 NI(J) = NI(J) + A(II)*W(K)
CONTINUE
RE.WIND 3
DO 121 I = 1, N
DC 121 J = 1, N
K = (I-1)*N + J
121 SIGMAX(I,J) = NI(K)
READ(4) (W(I), I=1,NO)
RE.WIND 4
DO 122 J = 1, N

-49-
U(J) = 0.DO
DO 122 I=1, NO
   KKK = (J-1)*NO+1
   U(J) = U(J) - A(KKK)*W(I)
122 CONTINUE

******************************************************************************
C
**** COMPUTING SOLUTION VECTOR *DX* FOR TRANSFORMATION PARAMETERS
C
******************************************************************************
C
CALL DMINV(NI,N,LT,MT)
CALL DARRAY(1,N,N,7,7,NI,VR)
CALL DGMPRT(NI,W,DX,N,Nil)
DO 123 I = 1, N
   JK = (J-1)*NO + 1
   JM = JK + NO -1
123 READ(3) (A(J), J=JK, JM)
PEWIND 3
C
C
**** COMPUTING VARIANCE OF UNIT WEIGHT *W02*
C
DO 125 I = 1, NO
   W(I) = 0.DO
DO 125 J = 1, N
   KZX = (J-1)*NC+I
   W(I) = W(I) - A(KZX)*DX(J)
125 CONTINUE
PEAD(4) (W(I), I= 1, NO)
PEWIND 4
DO 126 K = 1, NO
   W(K) = W(K) - A(K)
126 CONTINUE
M = 0
DO 128 K = 1, NO
   A(K) = 0.DO
   L1 = ((K-1)/3)*6 + K
   L2 = L1 + 6
   L = L1+L2+3
   M = ((L-1)/3) +1
128 A(K) = A(K) + M(W(MM))
CALL RESIDU (NO,N,NN)
PEAD(4) (W(I), I= 1,NO)
PEWIND 4
VPV = 0.DO
DO 130 K = 1, NO
130 VPV = VPV - A(K)*W(K)
MC? = VPV/(NO-N)
C
C
**** COMPUTING VARIANCE-COVARIANCE MATRIX *VAR*
C
DO 122 I = 1, N
   -50-
DO J = 1, N
VAR(I,J) = M02*VR(I, JJ
132 CONTINUE
IF (KCODE(2),EQ, 3) GO TO 140
DX(4) = DX(4) * 10.05
IF (KCODE(2),EQ, 4) GO TO 140
DO I = 5 , 7
DX(I) = DX(I) * RHOS
135 CONTINUE
C
C
C
**** COMPUTING COEFFICIENTS OF CORRELATIONS FOR PARAMETERS

140 DO I = 1,N
IF(I,NEQ,N) GO TO 144
JJ = I + 1
DO J = JJ , N
VR(I,J) = VAR(I,J)/(DSORT(VAR(I,I))*DSORT(VAR(J,J))
142 VR(J, I) = VR(I,J)
144 VR(I, I) = 1.00
145 CONTINUE
C
C
C
C **** WRITING OF FINAL SOLUTION VECTOR AND VARIANCE-COVARIANCE MATRIX

500 WRITE(6,6025)
6025 FORMAT(' ',//)
WRITE(6,6030)
6030 FORMAT(' ',21X,'SOLUTION FOR 3 TRANSLATION PARAMETERS',//, 232X,'(UNITS - METERS)',//)
GO TO (512,514,516), KPR
512 WRITE(6,6032)
6032 FORMAT(' ',29X,'(USING VARIANCES ONLY)',//)
GO TO 520
514 WRITE(6,6034)
6034 FORMAT(' ',15X, 2*(USING 3X3 BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX)',//)
GO TO 520
516 WRITE(6,6036)

-51-
FORMAT(*'22X,'(USING FULL VARIANCE-COVARIANCE MATRIX)'),//)
WRITE(6,6038)
FORMAT(*'16X,'(USING FULL VARIANCE-COVARIANCE MATRIX)'),//)
WRITE(6,6040)(DX(I), I=1,3)
WRITE(6,6048) MD2
FORMAT(*'26X,'(USING VARIANCES ONLY)'),//)
WRITE(6,6050)((VR(I,J), J=1,N), I=1,N)
WRITE(6,6050) (DX(I), 1=1,4)
WRITE(6,6600)
FORMAT(*'26X, 'VARIVANCE - COVARIANCE MATRIX'),//)
WRITE(6,6625) MD2
FORMAT(*'8X, 'MD2=', F6.2, //)
WRITE(6,6650)(VAR(I,J), J=1,N), I=1,N)
FORMAT(*'8X, 'MD2=', F6.2, //)
WRITE(6,6675)(VR(I,J), J=1,N), I=1,N)
WRITE(6,6650) (DX(I), 1=1,4)
WRITE(6,6675)(VR(I,J), J=1,N), I=1,N)
WRITE(6,6650) (DX(I), 1=1,4)
WRITE(6,6675)(VR(I,J), J=1,N), I=1,N)
GO TO 1000
GO TO (710,705), Kount
IF (KPARM .EQ. 4) GO TO 708
WRITE(6,7002)
FORMAT(*'13X,'SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION
2 PARAMETERS'),//)
WRITE(6,7010)
GO TO (712,714,716), KPR

712 WRITE(6,7012)
7012 FORMAT(•,34X,'USING VARIANCES ONLY'),//)
GO TO 720

714 WRITE(6,7014)
7014 FORMAT(•,16X,
2*(USING 3X3 BANDED DIAGONAL VARIANCE-COVARIANCE MATRIX'),//)
GO TO 720

716 WRITE(6,7016)
7016 FORMAT(•,24X,'USING FULL VARIANCE-COVARIANCE MATRIX'),//)
720 WRITE(6,7020)
7020 FORMAT(•,16X,'5X,'OMEGA*,
2(X,5X,'SIGMA',1X,'(10.045)',1X,'SECONDS*,
4T57,'SECONDS*',2X,'SECONDS*/,
WRITE(6,7030) DX
7030 FORMAT(•,12X,F7.2,2F8.2,F8.2,T59,'PSC',4X,'EPSILON*,/
315X,'METERS',2X,'METERS',2X,'METERS',1X,'SECONDS*,
4T57,'SECONDS*',2X,'SECONDS*/,
WRITE(6,7040) M02
7040 FORMAT(•,16X,'VARIANCE-COVARIANCE MATRIX'),//)
WRITE(6,7045) M02
7045 FORMAT(•,10X,'M02=»,F6.2,/) 
WRITE(6,7050) ((VAR(I,J), J=1,7), I = 1,7)
7050 FORMAT(•,2X,7D11.3,//(3X,7D11.3,/))
WRITE(6,7075)

7075 FORMAT(•,/, 29X,'COEFFICIENTS OF CORRELATION'),//)
WRITE(6,7065) ((VR(I,J), J=1,N), I=1,N)
7065 FORMAT(•,2X,7D11.3,\//(3X,7D11.3,//))
IF(1.EQ.0) GO TO 1000
WRITE(6,7090)
7090 FORMAT('1',////,36X,'RESIDUALS V',/,36X,'---------',///,
212X,'FIRST SYSTEM',33X,'SECOND SYSTEM'),//)
KSM = NNN + 1
KMR = NNN - 1 *KSM
WRITE(6,8000) (A(I), I = KSM,KMR)
8000 FORMAT(*,4X,3F8.3,22X,3F8.3,\//(5X,3F8.3,22X,3F8.3))
IF (KCODE(3).EQ.0) GO TO 1000

C
**OBTAINING CONSTRAINED SOLUTION FOR ROTATION PARAMETERS**
C
C
CALL CSTRNT(N,NQ,IC,U,CN,CNT,TT,ZP)
KCODE(3) = 0
DC 725 I = 1 , 7
DX(I) = XD(I)
DC 725 J = 1 , 7
VAR(I,J) = SIGMAX(I,J)
725 CONTINUE
DC 750 I = 1 ,N
IF(I.EQ.N) GO TO 740

-53-
JJ = I + 1
DO 735 J = JJ, N

VR(I,J) = VAR(I,J)/DSQRT(VAR(I,I)*DSQRT(VAR(J,J)))

735 VR(I,1) = VR(I,J)

740 CONTINUE

KOUNT = 2
M02 = 502
GO TO 200

1000 RETURN
END
SUBROUTINE SCALE

FINDING WEIGHTED MEAN AND VARIANCE FOR 'SCALE FACTOR' BY COMPARISON OF CHORDS IN THE TWO SYSTEMS BY CALLING SUBROUTINE 'SCALE'.

SUBROUTINE SCALE (NO,N,S,VSF,WT)
IMPLICIT REAL * 8 (A-H, O-Z)
DIMENSION P(12,12),H(12),PF(6,6),PS(6,6),
2 HI(12),DL(600),VI(600),WI(600)
COMMON /SFAC/ SW,VSF,DU,DV,DW,DX,DY,DZ,R1,R2,PF,PS
RR = R1 * R2
RT = R2 /(R1**3)

-- SETTING UP OF VARIANCES FOR EACH CHORD THROUGH ERROR PROPAGATION

H(1) = DU * RT
H(2) = DV * RT
H(3) = DW * RT
H(7) = -DX/RR
H(8) = -DY/RR
H(9) = -DZ/RR
DO 10 I = 1, 3
H(I+3) = -H(I)
H(I+9) = -H(I+6)
10 CONTINUE

DO 15 J = 1, 12
DO 20 I = 1, 6
L = I + 6
DO 20 J = 1, 6
M = J + 6
P(I,J) = PF(I,J)
20 P(L,M) = PS(I,J)
CALL DGMPRD (H,P,HI,1,12,12)
-55-
CALL DGMPRD (H1,H,WS,1,12,1)

FINDING WEIGHTED MEAN FOR SCALE FACTOR OF THE GIVEN SAMPLE

WS = 1.DO/WS
WI(N) = WS
SF1 = R2/R1 - 1
DL(N) = SF1
SF = SF + SF1 *WS
SW = SW + WS
S = SF/SW

FINDING VARIANCE FOR THE WEIGHTED MEAN OF THE SCALE FACTOR

IF (N .NE. NO) GO TO 500
PVV = 0.DO
DO 50 K = 1,NO
VI(K) = ((S-DL(K))**2)*WI(K)
50 PVV = PVV + VI(K)
VSF = PVV/(SW*(NO-1))
S = S * 10.05
WT = 1.DO / VSF
500 RETURN
END
**** SUBROUTINE SOLVES FOR CONSTRAINED CASE IN RESPECT OF 3 ✂️
**** ROTATION PARAMETERS. CONSTRAINTS ARE CODED FOR ALL THE
**** PARAMETERS — BLANKS CARDS ARE NEEDED FOR NON-CONSTRAINTS.
**** TMPUT CONSTRAINTS FOR ROTATION PARAMETERS ARE IN SECONDS OF ARC.

SUBROUTINE CSTRNT(N, NN, IC, WS, CN, CNT, TT, ZP)
IMPLICIT REAL * 6 (A-H, O-Z)
REAL * 8 M02, KC
DIMENSION XD(7), WS(7), WX(7), KC(4),
WC(4), WC(7), MM(7), PZ(4, 4), CN(7, IC), ZP(1C, IC)
COMMON /ANGLE/ WC, PZ

PII = 3.141592653589793*D0
RHO = 180.00/PII

20 WC(1) = WC(1) / 10.00
25 CONTINUE

** SETUP CONSTRAINTS MATRIX 'CN' REQUIRED FOR SOLUTION

25 CONTINUE
IF (IC .EQ. 4) GO TO 100
50 CONTINUE
IF (IC .EQ. 4) GO TO 100
50 CONTINUE
ZP(I, J) = PZ(I+1, J+1)
GO TO 200

CONTINUE

-57-
C ******************************************************************************
C **** SOLVE FOR EFFECTS OF CONSTRAINTS ON THE SOLUTION VECTOR 'DX'
C **************************** Obtained From Non-Constraint Solution ****************************
C ******************************************************************************
C
DO 520 I = 1 , IC
DO 520 J = 1 , N
CNT(J,1) = CN(1, J)
520 CONTINUE
CALL MTPY(CNT, ZP, N, IC, IC, TT)
CALL MTPY(TT, CN, N, IC, N, GG)
DO 522 I = 1 , N
DO 522 J = 1 , N
GG(I, J) = SIGMAX(I, J) + GG(I, J)
CALL DMINV(GG, N, DTT, LM, MM)
CALL MTPY(TT, WC, N, IC, 1, WX)
DO 525 I = 1 , N
WC(I) = (WS(I) - WX(I))
525 CONTINUE
CALL MTPY(GG, WS, N, N, 1, XD)
C ******************************************************************************
C **************************** Compute New Variance of Unit Weight And New Variance - Covariance Matrix ****************************
C ******************************************************************************
C
CALL MTPY(CN, XD, IC, N, 1, KC)
DO 535 I = 1 , IC
DO 535 J = 1 , N
KC(I) = -KC(I) - WC(I)
CALL MTPY(PZ, KC, IC, IC, 1, DX)
SUM = 0.0
DO 540 I = 1 , IC
SUM = SUM + DX(I) * WC(I)
540 CONTINUE
PVV = VPV - SUM
SO2 = PVV/(NN - N + IC)
DO 550 I = 1 , N
DO 550 J = 1 , N
SIGMAX(I, J) = SO2*GG(I, J)
550 CONTINUE
XD(1) = XD(1) * 10.05
DO 560 I = 5 , 7
XD(I) = XD(I) * PHOS
560 CONTINUE
RETURN
END
-58-
SUBROUTINE SETUP (NO, NN, PARA)

IMPLICIT REAL *8 (A-H, O-Z)
REAL *8 MINK
DIMENSION R(3,6), RT(6,3), PI(6,6), PK(3,3),
2XM(3,3), XK(3,6), MINK(2400), LM(3), MM(3), KCODE(15)
COMMON /RES/ RT
COMMON /WIGHT/ MINK
COMMON /CODE/ KCODE

C
C ** SFTTJNG UP MATRIX 'B', WHICH WILL BE SAME FOR ALL SOLUTION
C
C
NV = NN*3
DO 8 I = 1, NV
MINK(I) = 0.00
8 CONTINUE
DO 10 J = 1, 6
PI(I, J) = 0.00
XK(I, J) = 0.00
10 CONTINUE
B(1,1) = -1.00
B(2,2) = -1.00
B(3,3) = -1.00
B(4,4) = 1.00
B(5,5) = 1.00
B(6,6) = 1.00
DO 12 J = 1, 3
DO 12 I = 1, NV
B(I, J) = B(I, J)
12 CONTINUE
DO 15 J = 1, 6
DO 15 I = 1, NV
PI(I, J) = 0.00
15 CONTINUE
DO 20 J = 1, 3
DO 20 I = 1, NV
XM(I, J) = 0.00
20 CONTINUE

KWS = 0
IF (KCONF(14), EQ, 1) GO TO 65

-59-
IF (KCODE(8) .EQ. 1) GO TO 54
DO 40 L = 1, NO
DO 39 J = 1, 3
READ(5,38) (PI(J,K), K=1,3)
38 FORMAT(3F5.2)
39 WRITE (2) (PI(J,K), K = 1, 3)
40 CONTINUE
DO 52 M = 1, NO
DO 44 J = 4, 6
READ(5,42) (PI(J,K), K= 4, 6)
42 FORMAT(3F5.2)
44 CONTINUE
DO 52 M = 1, NO
DO 45 J = 1, 3
READ (5,55) (PI(I,J), I= 1,3)
55 FORMAT (3F10.2)
DO 56 I = 1, 3
56 WRITE (2) (PHI.J), J = l,3)
58 CONTINUE
DO 64 M = 1, NO
READ (5,55) (PI(I,J), I=4,6)
DO 60 J = 1, 3
60 WRITE(1) (PK(I,J), J = 1,3)
62 CONTINUE
DO 100 I = 1, NO
KMS = (I-1)*9 + 1
C **** READ IN VARIANCE - COVARIANCE MATRIX AS BLOCK DIAGONALS
C **** (6,6) MATRICES FOR EACH POINT USED IN TRANSFORMATION.
C **** MATRIX 'PI' IS BUILT UP POINTWISE - FIRST (3,3) BLOCK
C **** REFERS TO SECOND COORDINATE SYSTEM AND SECOND (3,3) BLOCK
C **** THEN CORRESPONDS TO FIRST COORDINATE SYSTEM.
C C
DO 70 J = 1, 3
READ(2) (PI(J,K), K= 1,3)
70 CONTINUE
DO 74 L = 4, 6
READ(1) (PI(L,M), M=4,6)
74 CONTINUE
CALL MTPY(B,PI,3,6,6,XK)
CALL MTPY(XK,BT,3,6,3,XM)
CALL DMINV(XM,3,DET,LM,MM)
MINK(KMS ) = XM(1,1)
MIN(KMS+1) = XM(2,1)
MIN(KMS+2) = XM(3,1)
MIN(KMS+3) = XM(1,2)
MIN(KMS+4) = XM(2,2)
MIN(KMS+5) = XM(3,2)
MIN(KMS+6) = XM(1,3)
MIN(KMS+7) = XM(2,3)
MIN(KMS+8) = XM(3,3)

100 CONTINUE
REWIND 1
REWIND 2
PFTUPN
FND
SUBROUTINE 'RESIDUE'

THIS SUBROUTINE COMPUTES RESIDUALS FOR EACH SYSTEM COORDINATES (USED AS OBSERVATIONS).

READ (1) (PI(L,M), M = 4,6)

READ (2) (PI(J,K), K = 1,3)

10 CONTINUE
CALL DGMPRD (PI,BT,BS,6,6,3)

C ********
C *******************COMPUTING RESIDUALS********************
C ********
C ************
C *******************DO 15 K = 1 , 6
C KK = JJ + K
C A(KK) = 0,0D0
C KM = (I-1) * 3
C DO 15 L = 1 , 3
C KM = KM + 1
C A(KK) = A(KK) + BS(K,L) * A(KM)
C DO 20 L = 1 , 3
C LL = JJ + L
C KM = LL + 3
C W(L) = A(LL)
C A(LL) = A(KM)
C 20 A(KM) = W(L)
C CONTINUE
C RFIND 1
C REWIND 2
C RETURN
C END
**SUBROUTINE DARRAY(MODE,I,J,N,M,S,D)**

**IMPLICIT REAL*8(A-H,O-Z)**

**DIMENSION S(1),D(1)**

**DIMENSION S(I),D(I)**

**NIM=N-1**

**IF(MODE-1) 100,100,120**

**100**

**IJ=IJ+1**

**NM=NM+1**

**DO 110 K=1,J**

**NM=NM-N1**

**DO 110 L=1,I**

**IJ=IJ-1**

**NM=NM-1**

**110**

**D(NM)=S(IJ)**

**GO TO 140**

**120**

**IJ=0**

**NM=0**

**DO 130 K=1,J**

**DO 130 L=1,I**

**IJ=IJ+1**

**NM=NM+1**

**130**

**S(IJ)=D(NM)**

**140**

**RETURN**

**END**
SUBROUTINE MTPY(AMT,BMT,M1,M2,M3,CMT)
IMPLICIT REAL *8 (A-H,O-Z)
DIMENSION AMT(M1,M2),BMT(M2,M3),CMT(M1,M3)
DO 10 I = 1, M1
  DO 10 J = 1, M3
    CMT(I,J) = 0.0
  10   CONTINUE
  DO 10 L = 1, M2
    CMT(I,J) = CMT(I,J) + AMT(I,L) * BMT(L,J)
  10   CONTINUE
RETURN
END
APPENDIX II

Job Control Cards
APPENDIX II

// (2500,100), CLASS=C
//STEP1 EXEC PROC=FORTRANG, PARM='MAP, ID', TIME, CMP=(0, 30)
//CMP. SYSIN DD *

FORTRAN PROGRAM DECK

/*
//STEP2 EXEC PROC=RUNFORT, PARM=LKED='OVLY, LIST, MAP', TIME, LKED=(0, 20),
// TIME.GO=(3, 10), REGION.GO=252K
//LKED.SYSLIB DD DSN=SYS1.FORTLIB, DISP=SHR
// DD DSN=SYS2.FORTSSP, DISP=SHR
//LKED.SYSLIN DD DSN=*,STEP1.CMP.SYSLIN, DISP=(OLD, DELETE)
// DD *
OVERLAY ALPHA
INSERT EULERS, SCALE
OVERLAY BETA
INSERT TFORM, RESIDU, MTPY, SETUP, CSTRMT, DARRAY
*/
//GO.FT01F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW, DELETE),
// DCR=(RECFM=VRS, LRECL=600, BLKSIZE=604)
//GO.FT02F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW, DELETE),
// DCR=(RECFM=VRS, LRECL=600, BLKSIZE=604)
//GO.FT03F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW, DELETE),
// DCR=(RECFM=VRS, LRECL=600, BLKSIZE=604)
//GO.FT04F001 DD UNIT=SYSDA, SPACE=(CYL,(1,1)), DISP=(NEW, DELETE),
// DCR=(RECFM=VRS, LRECL=600, BLKSIZE=604)
//GO.FT07F001 DD SYSPUT=P
//GO.SYSIN DD *

DATA DECK

/*
//  -68-