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AUTOMATED PROCEDURE FOR DESIGN OF WING STRUCTURES TO SATISFY STRENGTH AND FLUTTER REQUIREMENTS

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16. Abstract

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Computation times during the design process are reduced by three techniques. First, iterative analysis methods are used to reduce significantly reanalysis times compared with the original analysis of the structure. Second, the number of design variables is kept small by not using a one-to-one correspondence between finite elements and design variables. Third, a technique for using approximate second derivatives with Newton's method for the optimization is incorporated since it proves to be superior to the commonly used Davidon-Fletcher-Powell algorithm.

The program output is compared with previous published results. Reasonable agreement is obtained. In addition, it is found that some flutter characteristics, such as the flutter speed, can display discontinuous dependence on the design variables (which are the thicknesses of the structural elements). It is concluded that it is undesirable to use such quantities in the formulation of the flutter constraint.

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SUMMARY

A pilot computer program has been developed for the design of minimum mass wing structures under flutter, strength, and minimum gage constraints. The wing structure is idealized by finite elements, and second-order piston theory aerodynamics is used in the flutter calculation. Mathematical programing methods are used for the optimization.

Computation times during the design process are reduced by three techniques. First, iterative analysis methods are used to reduce significantly reanalysis times compared with the original analysis of the structure. Second, the number of design variables is kept small by not using a one-to-one correspondence between finite elements and design variables. Third, a technique for using approximate second derivatives with Newton's method for the optimization is incorporated since it proves to be superior to the commonly used Davidon-Fletcher-Powell algorithm.

The program output is compared with previous published results. Reasonable agreement is obtained. In addition, it is found that some flutter characteristics, such as the flutter speed, can display discontinuous dependence on the design variables (which are the thicknesses of the structural elements). It is concluded that it is undesirable to use such quantities in the formulation of the flutter constraint.

INTRODUCTION

The usual design process for aircraft wing structures consists of sizing, and perhaps optimizing, for strength (with the use, for example, of fully stressed design techniques, ref. 1), checking for flutter and, if required, determining a flutter fix by trial and error. In the past few years, there has been considerable interest in developing optimization procedures for the design of aircraft structures to satisfy aeroelastic constraints.

Work in this field is reviewed in references 2 and 3. When flutter is the only constraint (except for minimum gage) optimality criteria (refs. 4 to 6) or special flutter

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oriented optimization algorithms (ref. 7) can be used. When flutter is only one of many design conditions, the more general mathematical programing methods are more convenient. The first effort in this area, dealing with the design of an airfoil for minimum wing drag work during a given flight mission, is presented in reference 8 where flutter Mach number is one of several constraints included. A gradient method is used to optimize the two design variables which are the thickness and chord of the airfoil.

An automated design program called SWIFT is presented in reference 9, wherein a simple structural model of an aircraft wing is used to show the effects of strength and flutter requirements on the design of minimum mass aircraft wing structures. The wing is idealized as an isotropic sandwich plate with a specified variable depth between covers and a variable cover thickness distribution which is optimized. The sequence of unconstrained minimizations technique (SUMT, refs. 10 and 11) with an interior penalty function is used. The Davidon-Fletcher-Powell algorithm (ref. 12) is used for each unconstrained minimization. Reference 13 is similar to reference 9 in that it also uses a plate wing model but differs from reference 9 in that it employs the feasible directions method (ref. 14) to optimize both the structure and the configuration under a large number of constraints.

The plate model is not adequate for general built-up wings, for which finite element models are required; such models were used even in some of the earlier works (refs. 3, 4, 6, and 7). Finite element analysis, however, involves considerably more computation than plate analysis. In the past when finite element models have been used in conjunction with mathematical programing methods, computation times tended to become excessive. For example, the optimization of a finite element wing model under strength, buckling, frequency, and flutter constraints is discussed in reference 15 with the use of the same optimization algorithm as in reference 9. Computation times are long even for crude finite element models.

The present paper considers the structural optimization for minimum mass of built-up wings (ribs, spars, cover panels) under maximum stress, minimum gage, and flutter constraints. Particular attention is given to the problem of reducing the computing time required for executing hundreds of analyses of a finite element wing model, as required by mathematical programing techniques. Of course, computer time is not the only consideration in computer cost, but the other considerations, such as core storage, are installation dependent and therefore not discussed herein. It is shown that the use of appropriate analysis algorithms can reduce the computation time of a reanalysis of the wing to a small fraction of the time needed for the original analysis. In addition, a technique for using approximate second derivatives in conjunction with Newton's method is shown to be superior to the Davidon-Fletcher-Powell (DFP) algorithm. Attention is also given to the efficient choice of design variables that will reduce their total number.

A computer program using these ideas was written for the design of wings assumed to have symmetric cross-section profiles and to be clamped at the root. The flutter calculations are made at a fixed altitude with the use of second-order piston theory aerodynamics and natural vibration modes as generalized coordinates. Both Newton's method and the DFP algorithm were used in the present program which is called WIDOWAC (WIng Design Optimization With Aeroelastic Constraints). Analysis and sample results are presented along with a study of discontinuous behavior of the flutter speed as a function of design variables, which was found to occur under some circumstances.

SYMBOLS

The physical quantities used in this paper are given in both the International System of Units (SI) (ref. 16) and in the U.S. Customary System of Units. The measurements and calculations were made in U.S. Customary Units. Appendix A presents some factors relating these two systems of units.

A_{ij}	generalized aerodynamic forces (see eq. (13))
a	speed of sound
a_1,a_2,a_3	modal amplitudes
$[B]$	matrix defined by equation (21)
$f(\vec{v})$	object function for optimization
G	shear modulus
g	structural damping
g_j	structural damping associated with jth vibration mode
g_1,g_{2i},g_{3i},g_{4j}	constraint functions
h	total local depth of wing
$[I]$	unit matrix
$[K]$	stiffness matrix

$\{L\}$	load vector
M	Mach number
$[M]$	modal mass matrix
n	number of vibration modes
$P(r)$	sum of object function and penalty terms
Δp	amplitude of lifting pressure (see eq. (15))
Δp_j	Δp due to oscillation of wing in jth vibration mode
$\Delta\bar{p}(x,y,t)$	lifting pressure at point x,y on wing at time t, positive up
r	factor of penalty function (see eq. (5))
S	area
$[S]$	stress matrix
$\{s\}$	stress vector
t	time
t_{ej}	prescribed minimum thickness for jth element
t_i	thickness at ith node
t_j	thickness of jth element
$t_{s,min}$	minimum skin thickness
V	free-stream velocity
V_f	flutter speed
$V_{f,cr}$	critical flutter speed

$\vec{v}$	vector of design variables
v_i	ith design variable
W	amplitude of normal displacement (see eq. (15))
$\overline{\overline{W}}$	normal displacement at wing surface
$\overline{W}_i$	$\overline{W}$ for oscillation of wing in ith vibration mode
$\{w\}$	displacement vector
x,y,z	chordwise, spanwise, and normal coordinates, respectively
γ	ratio of specific heats
δ_{ij}	Kronecker delta
ν	Poisson's ratio
ρ	air mass density
σ_{cr}	critical value of stress
$\sigma_x,\sigma_y,\tau_{xy}$	components of normal and shearing stress in wing elements
ω	frequency, rad/s
ω_f	flutter frequency
ω_j	frequency of jth vibration mode

ANALYTICAL PROCEDURES

Problem Definition

The general optimization procedure is to determine a vector $\vec{v}$ that solves the problem:

Minimize the object function $f(\vec{v})$ subject to the constants $g_j(\vec{v}) \geqq 0$ $\quad (j = 1(1)m)$ (1)

Herein the object function is the structural mass of the wing, and the following three constraints are considered:

(1) Flutter constraint: The flutter speed V_f at a specified altitude is greater than a prescribed value $V_{f,cr}$. Thus,

$$g_1 \equiv 1 - \frac{V_{f,cr}}{V_f} \geqq 0 \tag{2}$$

(2) Stress constraints: The Von Mises stress σ at any point in the structure does not exceed a prescribed value σ_{cr}. Thus,

$$g_{2i} \equiv 1 - \frac{\sigma_i}{\sigma_{cr,i}} \geqq 0 \qquad \left(i = 1(1)n_{el}\right) \tag{3}$$

where σ_i and $\sigma_{cr,i}$ are the Von Mises stress and the critical stress for the ith element, respectively, and n_{el} is the number of finite elements in the structure.

(3) Minimum gage constraints: No element thickness t_j shall be less than a prescribed value t_{ej}. Also, at no node on the cover shall the skin thickness t_i be less than a prescribed value $t_{s,min}$. (The double constraint is needed because, on the cover element, thickness is the average of nodal thicknesses and a constraint on the element is not sufficient.) Thus,

$$\left.\begin{aligned} g_{3i} &\equiv 1 - \frac{t_{s,min}}{t_i} \geqq 0 \qquad \left(i = 1(1)n_{node}\right) \\ g_{4j} &\equiv 1 - \frac{t_{ej}}{t_j} \geqq 0 \qquad \left(j = 1(1)n_{el}\right) \end{aligned}\right\} \tag{4}$$

Optimization Technique

The mass is minimized subject to these three constraints by using the SUMT approach (refs. 10 and 11) with an interior penalty function. With this method the problem defined by equation (1) is replaced by

$$\text{Find } \lim_{r \to 0} \min \left(P(r)\right) \text{ where}$$

$$P(r) = f(\vec{v}) + r \sum_{i=1}^{m} \frac{1}{g_i(\vec{v})} \tag{5}$$

For each value of r an unconstrained minimization of P is carried out.

The DFP algorithm is among the most efficient methods for unconstrained optimization which are commonly used with the SUMT approach (refs. 9 and 15). Such methods require calculation of the first derivatives of the object function, and each unconstrained minimization is executed by about n one-dimensional searches, n being the number of design variables. For this investigation, however, Newton's method, which requires both first and second derivatives of the object function, is found to be more efficient than the DFP algorithms. It is shown in the following paragraphs that the first derivatives of the constraints (which are needed anyhow for calculation of the first derivatives of the P function) are sufficient to establish a good approximation to the second derivatives of the P function. With Newton's method, each unconstrained optimization requires only a small number of one-dimensional searches, a number which is independent of n. The second derivatives of P are obtained as follows:

Differentiating P twice, one has

$$\frac{\partial^2 P}{\partial v_j\, \partial v_l} = \frac{\partial^2 f}{\partial v_j\, \partial v_l} + r \sum_{i=1}^{m} \left(2 \frac{\partial g_i}{\partial v_j} \frac{\partial g_i}{\partial v_l} - g_i \frac{\partial^2 g_i}{\partial v_j\, \partial v_l} \right) g_i^{-3} \tag{6}$$

An approximation to $\frac{\partial^2 P}{\partial v_j\, \partial v_l}$ is available under the following assumptions:

(1) $\frac{\partial^2 f}{\partial v_j\, \partial v_l}$ is known

(2) Some of the constraints are nearly critical so that for some values of i, $g_i(\vec{v})$ is very small

(3) The second derivatives of the g's are not very large

Under these assumptions, the second term in the summation in equation (6) may be neglected so that

$$\frac{\partial^2 P}{\partial v_j\, \partial v_l} \approx \frac{\partial^2 f}{\partial v_j\, \partial v_l} + Q_{jl} \tag{7}$$

where

$$Q_{jl} = 2r \sum_{i=1}^{m} \left(\frac{\partial g_i}{\partial v_j} \frac{\partial g_i}{\partial v_l} \right) g_i^{-3}$$

In the present work, $\frac{\partial^2 f}{\partial v_j\, \partial v_l} = 0$ because the mass is a linear function of the design variables, and the first derivatives of the g's are obtained numerically by forward differences (with reanalysis of the structure). The matrix Q is positive semidefinite, but when the number of active constraints is small, it is singular or ill-conditioned. This is because when there are only a few constraints and they are linear with respect to the design variables, there is no unique optimum point (the nonlinearity of the constraints may create a unique optimum point). To guard against this possibility, Q_{ij} is replaced by

$$\overline{Q}_{ij} = Q_{ij}\left(1 + \epsilon\delta_{ij}\right)$$

A value of the constant ϵ of 0.01 has been found satisfactory to avoid the singularity or ill-conditioning.

Idealization of Structure and Design Variables

The wing is assumed to have a symmetric airfoil profile and to be clamped at the root. Constant strain triangular membrane elements are used to represent the wing cover panel, and quadrilateral shear-web elements represent the ribs and spars (see appendix B for additional details).

The natural choice for design variables in a structural design problem is the dimensions of the structural elements. When the analysis of the structure is carried out by using a finite element idealization, the thicknesses of the finite elements are often picked as design variables. This choice, however, is not always logical. For example, an area of stress concentration may require a fine mesh of finite elements, but it is not always reasonable to assign to the thickness of that area a large number of design variables. When the optimization is done by mathematical programing, it is often necessary to use far fewer design variables than finite elements and, hence, distinguish the finite-element model from the design variable model (ref. 17, for example). Typical finite element and design models are shown in figure 1. In WIDOWAC, the design variables do not describe the thickness of individual finite elements but of segments of the wing, each segment containing several finite elements. The wing planform is divided into a number of triangular or quadrilateral segments. The division is such that no finite element crosses the boundary of a segment. The design variables are the thicknesses of the cover panels at the vertices of the segments (indicated by the numbers in fig. 1). The thickness varies linearly in the triangular segments and is piecewise linear inside the quadrilateral ones (each quadrilateral is divided into two triangles and the thickness variation in each one is linear). It is optional to lump together several vertices and assign to them the same design variable. For example, if the same design variable is assigned to vertices 30, 37, 38, and 33 in figure 1, segment VI will be a constant-thickness segment. Similarly, all the shaded segments in the figure can be lumped together. On the other hand, it is optional to create discontinuities in thickness by assigning different design variables to the same point, if that point is a vertex of more than one segment.

The shear webs which represent the spars and ribs are assigned design variables by finite elements, but it is again possible to lump together several elements. For example, the thicknesses of all the spar elements may be lumped together as one design variable and the thicknesses of all the rib elements as another.

The methods just discussed for assigning design variables provide considerable flexibility. It is possible, for example, to first optimize the wing with a low number of design variables. This first optimization usually discloses that certain regions of the wing are more critical than others. It is possible as a next step to increase the number of design variables that describe these regions but continue with a crude representation of less important regions.

Two basic questions have to be answered regarding the usefulness of this method for reducing the number of the design variables. First, what is the difference in the final mass because of the smaller number of design variables? Second, how different are the resulting designs? To be useful for preliminary design it is important that basically the same design will emerge with the smaller number of design variables as with the larger one. These two questions have been investigated for the example problems and the results, presented in a subsequent section, are encouraging.

Flutter and Stress Analyses

The calculation of the flutter speed and stresses appearing in constraint equations (2) and (3) is given as follows. The displacement method of finite element analysis is used so that the displacements are the primary variables. The displacement vector $\{w\}$ is found from the following equation:

$$[K]\{w\} = \{L\} \tag{8}$$

where $[K]$ is the stiffness matrix and $\{L\}$ is a load vector. Stresses are calculated from the displacements for each element

$$\{s\} = [S]\{w\} \tag{9}$$

where $[S]$ is the stress matrix and $\{s\}$ is the stress vector which has the components σ_x, σ_y, and τ_{xy} for each element. The Von Mises stress σ which is used in constraint equation (3) is

$$\sigma = \left(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2\right)^{1/2} \tag{10}$$

For the flutter problem the load vector $\{L\}$ in equation (8) is replaced by inertia, structural damping, and aerodynamic forces. When harmonic motion is assumed, the flutter equation is

$$\Big[[K]\big(1 + i[G]\big) - \omega^2[M] - [A(\omega,V)]\Big]\{w\} = 0 \tag{11}$$

where $[A]$ is the aerodynamic matrix, which (for a fixed altitude) is a function of the frequency ω and speed V, $[G]$ is the structural damping matrix, and $[M]$ is the mass matrix. Equation (11) is an eigenvalue problem in the pair V,ω determined from the condition that the determinant of equation (11) should vanish.

To reduce the order of the problem it is common to use the natural vibration modes as generalized coordinates for the flutter problem. In terms of the natural vibration modes (normalized to give a unit generalized mass matrix) the determinant of equation (11) is

$$\left|A_{ij} + \left(\omega^2 - \omega_j^2 - i\omega_j^2 g_j\right)\delta_{ij}\right| = 0 \qquad (i,j = 1(1)n) \tag{12}$$

where ω_j is the jth natural frequency, g_j is the structural damping associated with the jth mode, and A_{ij} is the generalized aerodynamic force

$$A_{ij} = \iint_S \Delta p_j \overline{W}_i \, dS \tag{13}$$

The pressure loading on the wing as predicted by second-order piston theory (refs. 18 and 19) is

$$\Delta\bar{p}(x,y,t) = -2\rho a\left(1 + \frac{\gamma + 1}{4} M \frac{\partial h}{\partial x}\right)\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)\overline{W} \tag{14}$$

When harmonic dependence on time is assumed so that

$$\left.\begin{aligned} \Delta\bar{p} &= \Delta p e^{i\omega t} \\ \overline{W} &= W e^{i\omega t} \end{aligned}\right\} \tag{15}$$

equation (14) becomes

$$\Delta p = -2\rho a\left(1 + \frac{\gamma + 1}{4} M \frac{\partial h}{\partial x}\right)\left(i\omega W + V \frac{\partial W}{\partial x}\right) \tag{16}$$

Equation (16) is substituted into equation (13) to calculate the generalized aerodynamic forces.

Solution Methods

The choice of computation algorithms for a design program is not motivated so much by their efficiency in the execution of one analysis but by their efficiency in executing many analyses. Specifically, iterative methods which may not be appropriate for a single analysis because they must be started with a very good guess of the solution become very attractive when many repeated analyses are to be executed. The choice of appropriate computation algorithms, described in the following sections, in WIDOWAC has reduced the computer time required for a reanalysis to a small fraction of the computer time required for the first analysis. The following steps are required for the computation of stresses and flutter speed:

(1) Stiffness matrix assembly

(2) Solution for displacement and stresses (eqs. (8) to (10))

(3) Calculation of vibration modes

(4) Flutter solution

The algorithms used in these steps are discussed in the following sections.

Stiffness matrix assembly.- The design variables describe the thicknesses of the structural elements, and because no bending elements are used, the stiffness matrix depends linearly on the design variables. This linear dependence is used to reduce the assembly time during the optimization process by computing the stiffness matrix as

$$[K] = [K]_0 + \sum_{i=1}^{n} v_i \left[\frac{\partial K}{\partial v_i}\right] \tag{17}$$

where $[K]$ is the stiffness matrix, v_i is the ith design variable, and $[K]_0$ is the residual stiffness matrix obtained when all the design variables are set to zero (i.e., the stiffness of the part of the structure which is not optimized). It should be noted that, although higher order finite elements are not presently available in WIDOWAC, this method of assembly permits the use of such elements with almost no penalty in computation time because the element matrices are computed only once.

Solution for displacements and stresses.- The matrix equations resulting from the finite element model are sparse. Efficient handling of operations with sparse matrices is achieved by using the node as the basic unit of the calculation rather than the degree of freedom. Each node has three degrees of freedom (translations in the x, y, and z directions), and this situation results in a stiffness matrix built up of 3×3 submatrices.

The use of 3×3 submatrices has two advantages. First, the storage requirements and amount of bookkeeping operations required for taking full advantage of the topology of a sparse matrix are greatly reduced. Second, operations on 3×3 matrices are programed without use of loops by writing out all the steps in the operations. As a result computation times with these matrices are reduced by a factor of 3 or 4.

Calculation of vibration modes.- The first step in the flutter analysis is the calculation of a number of the lowest vibration frequencies and the corresponding vibration modes. The eigenvalue problem has the following form:

$$\Big[[K] - \omega^2[M]\Big]\{w\} = \{0\} \tag{18}$$

By introducing

$$\{\bar{w}\} = [M]^{1/2}\{w\} \tag{19}$$

the eigenvalue problem (eq. (18)) may be transformed into

$$\left[[M]^{1/2}[K]^{-1}[M]^{1/2} - \frac{1}{\omega^2}[I]\right]\{\bar{w}\} = \{0\} \tag{20}$$

so that the problem is reduced to finding the largest eigenvalues and corresponding eigenvectors of the matrix $[B]$

$$[B] = [M]^{1/2}[K]^{-1}[M]^{1/2} \tag{21}$$

The eigenvalue problem is solved by a simultaneous iteration method (ref. 20) coded by Alfred F. Vachris, Jr., of the Grumman Aerospace Corporation. The essential feature of the method is that it is very efficient when a good approximation to the eigenvectors is available. If the relative error in the eigenvalue is $O(\epsilon)$ with $\epsilon < 1$, it is reduced in one iteration to $O(\epsilon^2)$. The method is, therefore, very suitable for the optimization process.

Solution of flutter determinant.- The flutter determinant (eq. (12)) is a function of V and ω. The usual way of finding values of V and ω which make the determinant vanish is to evaluate all eigenvalues as a function of the reduced frequency (ref. 21) leading to the familiar V-g diagram method.

In WIDOWAC, a different method is employed which is more efficient when a very good initial approximation to the solution is available. The method (same as ref. 9) uses a Newton iteration with V and ω to drive the real and imaginary parts of determinant to zero simultaneously. For simplicity, the calculation is done with zero structural damping g_j and for a constant air density (i.e., constant altitude).

For each step in the course of the optimization, the initial approximation for V and ω is based on extrapolation of the results from preceding steps. Only one of the flutter modes is followed and therefore there is a danger that during the design process a different mode will have a lower flutter speed. Because of this possibility, the final design should be checked for a wide range of V and ω.

The flutter point for the initial and final design are calculated by separate runs, in which the flutter determinant is computed at a grid of points in the V-ω plane, and points are found where both the real and imaginary parts of the determinant change sign. The method may be very inconvenient and inefficient for one analysis, but it is very efficient for design.

Computer Program

A pilot computer program has been developed for the design of minimum mass wing structures under flutter, strength, and minimum gage constraints. The wings are assumed to have symmetric cross-section profiles and to be clamped at the root. The flutter calculations are made at a fixed altitude with the use of second-order piston theory aerodynamics and natural vibration modes as generalized coordinates. Both Newton's method and the DFP algorithm are used in the present program which is called WIDOWAC (WIng Design Optimization With Aeroelastic Constraints). The pilot computer program was developed for use on the Control Data Series 6000 computer systems.

APPLICATIONS AND RESULTS

Full-Depth Sandwich Wing

Results obtained from WIDOWAC for the titanium clipped delta wing, defined in figure 2, are compared with similar results obtained from the SWIFT program of reference 9. SWIFT is a wing design program which served as the starting point for WIDOWAC development. It treats a wing as a sandwich plate with a core having infinite transverse shear stiffness. Plate theory is used for the structural representation. The design variables are the coefficients of a polynomial which expresses the thickness of the panel covers over the wing. In the numerical analysis, the spanwise behavior is discretized by a finite difference scheme; the chordwise behavior is handled by a parabolic approximation.

The SWIFT model had seven finite difference stations, and this modeling results effectively in 21 degrees of freedom. The finite element model used in WIDOWAC must consist of many more degrees of freedom than a plate wing model used in SWIFT to provide adequate accuracy; a model with 93 degrees of freedom was used in the calculations for the present paper. To simulate the rigid core of the SWIFT model, the shear webs in WIDOWAC were made very stiff and were not assigned any design variables. Only the thickness of the cover panels was optimized.

The division of the wing into segments is shown in figure 1. The optimization process was usually started by assigning one design variable to each segment. In this application, it was found that the shaded segments in figure 1 were governed by the minimum gage constraint. Subsequently, all nodes of the three shaded segments were assigned the same design variable and the rest of the design variables were distributed among the other three segments.

An initial configuration having uniform 0.127-cm-thick (0.05 in.) cover panels was used to start the design process both in SWIFT and WIDOWAC. Stress and flutter results for this initial configuration are given in table I. Because of the difference in the analytical models, the maximum stress under a uniform load in WIDOWAC is 613 MN/m^2 (89.0 ksi) compared with 690 MN/m^2 (100 ksi) in SWIFT. A more refined finite element model having 162 degrees of freedom was run with NASTRAN (ref. 22) and gave a maximum stress of 645 MN/m^2 (93.5 ksi). Flutter speed was 12 percent lower in WIDOWAC than in SWIFT. Because of these differences, the WIDOWAC model is more nearly critical in flutter and the SWIFT model is more nearly critical in stress.

Table II shows the results of the optimization with seven design variables of the same initial design by the two programs.[1] Results are shown for the strength-only optimum design, the flutter-only optimum design, and the combined strength-flutter optimum design. Running times for WIDOWAC are shorter in spite of the extra complexity of the wing model. (A faster version of SWIFT is now available (as used in ref. 23) which takes only 295 CPU seconds for the combined strength-flutter design.) Because of the small number (seven) of design variables used in the present example the run times with Newton's method are only lower by about 50 percent than run times with the DFP method.

[1]In WIDOWAC, it was not possible to model the full-depth sandwich wing up to the leading and trailing edges where the thickness is zero. A narrow strip was therefore cut from both edges. The mass of the two strips, based on minimum gage thickness (compatible with SWIFT results), is 104 kg (230 lb) and was added to WIDOWAC results.

Built-Up Wing

A second program which was used for comparison with WIDOWAC is DAWNS (ref. 24). DAWNS (Design of Aircraft WiNg Structures) can be used for the fully stressed design of wings under stress, buckling, and minimum gage constraints. It generates its own loads for a symmetric pull-up maneuver using Mach box supersonic aerodynamics. A built-up delta wing with 65° sweep optimized by DAWNS (ref. 25) was used for comparison with WIDOWAC. The wing planform and the final distribution of the skin thickness is shown in figure 3; the broken line surrounds the part of the wing and carry-through structure that was modeled for the structural idealization. The finite element model used in DAWNS was modified by replacing each quadrilateral membrane element by two triangular elements. The applied loads were taken from the DAWNS run where they were calculated for a 3g maneuver at Mach 3. Many of the spars and ribs are critical in buckling; hence, buckling stress rather than yield stress governs their design. WIDOWAC does not calculate buckling stresses or loads; therefore, to obtain a meaningful comparison with DAWNS, the buckling stresses calculated by DAWNS were used as maximum stress constraints whenever they were lower than the yield stress. The wing was modeled by 86 triangular skin elements and 101 shear web elements totaling 156 degrees of freedom and was designed under stress and minimum gage constraints. A few runs with 6 to 10 design variables were made to find the critical areas of the wing and then the wing was optimized with 23 design variables allocated according to the information obtained by the earlier runs. Using the DFP algorithm, about 2300 analyses were required to converge the design which took 2400 seconds of CDC 6600 CPU time; run time with Newton's method was only about 400 seconds for the 333 analyses required for convergence. Sixteen of the design variables were allocated to the skin panels and 7 to the shear webs. The final mass was 9550 kg (21 030 lbm) compared with a final mass of 8080 kg (17 800 lbm) in DAWNS. The skin thickness distribution was similar as is shown in figures 3 and 4. To investigate the reason for the big differences in mass, the DAWNS final design was analyzed by using WIDOWAC. Although the difference between the maximum displacement was only 2.5 percent, the maximum stress in WIDOWAC was 28 percent higher. These higher stresses made the WIDOWAC design heavier. The difference in the stresses is attributed to the different finite elements used in the two programs. DAWNS used a quadrilateral linear strain membrane element while WIDOWAC uses a triangular constant strain membrane element. The use of quadrilateral elements for this wing is not appropriate because some of these quadrilaterals are not planar. As shown in reference 26, the error induced by the use of nonplanar elements can be large. (The wing analyzed in ref. 26 is the same one as used in this paper.)

Number of Design Variables

As was noted before, it is impractical to assign each finite element a design variable when using mathematical programing methods for the optimization. The technique used in this work for reducing the number of design variables is to divide the cover panels into segments and assign design variables to corners. A study was conducted to find the dependence of the final mass on the number of design variables. It should be noted, however, that when the number of the design variables is smaller than that of the finite elements there are many ways to assign the design variables. (As a result the curves shown in figs. 5 and 6 are not unique.)

The first study was on the full-depth sandwich wing divided into segments as shown in figure 1. Figure 5 shows the dependence of the final mass on the number of design variables. The division into segments was held constant. Because the shaded segments in figure 1 always came out as minimum gage they were lumped together and assigned one design variable. The other three segments could have a maximum of 11 design variables. It is seen from figure 5 that for the given segmentation there is no need to use the full number of design variables. The result of designating the thickness of each finite element as a design variable was an additional 5 percent reduction in mass as shown by the minimum mass line in figure 5. This 5 percent represents the constraint of freezing the segmentation. It should be noted that, as the number of design variables was increased, the changes in the final design had the character of refinement rather than changes in design concept.

The second study was on the built-up wing optimized for stress constraints where both the cover panels and the rib and spars are optimized. The strategy in assigning design variables to the rib and spar elements was according to the level of stress. Elements with similar stress margins (σ_{cr}/σ) were grouped together. As the number of design variables was increased the stress margins of previous designs were used.

Figure 6 shows the dependence of the mass of the built-up wing on the number of design variables. Of the 23 design variables used for the last point on the curve, 16 were cover panel design variables and 7 were rib and spar design variables. The final design with 23 design variables as shown in figure 3 is not much different from the DAWNS final, fully stressed design with 160 design variables (one for each element). Some of the difference may be attributed to the different finite elements used in DAWNS and WIDOWAC.

From these two studies, it appears that it may be possible to get a good approximation to the optimum design with a number of design variables which is much smaller than the number of finite elements. The two studies were done on relatively crude finite element models, and for more refined models, the ratio between the number of finite elements and the number of design variables can be bigger. Because working with a relatively small number of design variables does involve a penalty in the final mass, these procedures for reducing the number of design variables may be satisfactory for preliminary design but not for the final design process.

Discontinuity of Flutter Speed

It is known (ref. 27) that the flutter speed may be a discontinuous function of air density or Mach number. A similar discontinuous behavior as a function of structural properties was revealed during an investigation of the utility of a high-modulus—low-density material (beryllium) being used in wing design. In this supplemental study, segment III of the full-depth sandwich wing (fig. 1) was assumed to be made of beryllium and the rest of the wing was assumed to be titanium. Although no benefit was found from this use of beryllium (possibly because buckling was not considered), it was found that the flutter speed is discontinuous as a function of the patch thickness.

Figure 7(a) shows the dependence of the lowest flutter speed on the beryllium patch thickness. The rest of the cover panels are uniform 0.254-cm-thick (0.1 in.) titanium. The discontinuity is the result of the appearance of a new flutter mode when the patch thickness exceeds 0.371 cm (0.15 in.). The flutter frequency and the first three natural frequencies are shown in figure 7(b). The flutter eigenvector for both modes is a combination of primarily the first three natural vibration modes. For a patch thickness of 0.42 cm (0.165 in.) and unit amplitude of a 6-mode flutter vector the amplitudes of the first three modes in the higher speed mode are

$$a_1 = 0.594 \qquad a_2 = 0.431 \exp(-0.411i) \qquad a_3 = 0.677 \exp(-0.304i)$$

The amplitudes of the lower speed mode are

$$a_1 = 0.673 \qquad a_2 = 0.280 \exp(-2.22i) \qquad a_3 = 0.684 \exp(-0.468i)$$

The amplitudes of the fourth and higher natural vibration modes are small. Note that all three vibration modes participate significantly in both flutter modes. Actual V-g plots were not calculated, but on the basis of past experience, the switch between the two flutter modes may have taken place as shown in figure 8.

In the present work, the flutter constraint is defined in terms of the flutter speed, and, therefore, the discontinuity in the flutter speed causes the breakdown of the optimization procedure; the flutter constraint, however, may be formulated in terms of continuous parameters of the flutter phenomenon. For example, instead of equation (2) the following expression may be used:

$$g_1 = \left[\int_{V_{min}}^{V_{f,cr}} (g_{cr} - g)^{-1} \, dV\right]^{-1} \tag{22}$$

where g_{cr} is the critical damping value and V_{min} is a lower bound on the flutter speed. Other continuous constraints are possible (ref. 28); however, such constraints require more computation than the constraint of equation (2).

CONCLUSIONS

The present work explored methods for reducing the amount of computation involved in the automated design of complex structures for stress and flutter requirements. A pilot finite element program for the minimum mass design of wing structures under stress, flutter, and minimum gage constraints was developed. The following conclusions were reached:

1. The use of appropriate analysis algorithms can reduce the computation effort required for a reanalysis of a structure to a small fraction of the computation effort required for the analysis of the original structure.

2. When constraints are represented by a penalty function, Newton's method with approximate second derivatives may be much more efficient than the commonly used quasi-Newton methods (such as Davidon-Fletcher-Powell algorithm).

3. The number of design variables can be significantly less than the number of finite elements without excessive mass penalties.

4. The flutter speed may not be a continuous function of structural stiffness. Therefore, in optimization under flutter constraint, the constraint should be formulated in terms of other parameters of the flutter phenomenon which are continuous.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., May 22, 1973.

APPENDIX A

CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

The International System of Units (SI) was adopted by the Eleventh General Conference on Weights and Measures, Paris, October 1960 (ref. 16). Conversion factors for the units herein are given in the following table:

Physical quantity	U.S. Customary Unit	Conversion factor (*)	SI Unit
Force	lbf	4.44822	newtons (N)
Length	ft	0.3048	meters (m)
	in.	0.0254	
Mass	lbm	0.45359	kilograms (kg)
Stress, modulus . . .	ksi	6.895×10^6	newtons/meter^2 (N/m^2)
Density	lbm/in^3	27.68×10^6	grams/meter^3 (g/m^3)
Pressure	lbf/ft^2	47.88	newtons/meter^2 (N/m^2)
Speed	ft/sec	0.3048	meters/second (m/s)

*Multiply value given in U.S. Customary Unit by conversion factor to obtain equivalent value in SI Unit.

Prefixes to indicate multiples of units are as follows:

Prefix	Multiple
giga (G)	10^9
mega (M)	10^6
kilo (k)	10^3
deci (d)	10^{-1}
centi (c)	10^{-2}
milli (m)	10^{-3}

APPENDIX B

FINITE ELEMENT ANALYSIS OF SYMMETRIC WING

WIDOWAC assumes that the wing has a symmetric profile and symmetric behavior; that is, the lower and upper surfaces of the wing are assumed to have x,y displacements of equal magnitude but opposite sign and the same z displacements. Therefore, only nodes on the upper surface have to be considered. The shear web element in the program is derived from the corresponding NASTRAN element (ref. 22, p. 5.3-1). It is defined by two nodes on the upper surface of the wing. The other two nodes are assumed to be on the $z = 0$ surface, to be tied rigidly to the upper surface nodes in the z direction, and to be constrained from any movement in the x,y directions.

If the upper two nodes have the coordinates $(x_1,y_1,z_1), (x_2,y_2,z_2)$, then the stiffness matrix for the element $[K_e]$ of thickness t_e is given by

$$[K_e] = \frac{1}{Z}\{c_g\}\{c_g\}^T \tag{B1}$$

where

$$Z = \frac{(z_1 + z_2)L}{Gt_e}\left[1 + \frac{2}{3(1+\nu)}\left(\frac{z_1 - z_2}{L}\right)^2\right] \tag{B2}$$

and

$$\left.\begin{aligned}
\{c_g\} &= \frac{1}{2}\left[\lambda_x L, \lambda_y L, -(z_1+z_2), \lambda_x L, \lambda_y L, z_1+z_2\right]^T \\
L &= \left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]^{1/2} \\
\lambda_x &= \frac{x_2 - x_1}{L} \\
\lambda_y &= \frac{y_2 - y_1}{L}
\end{aligned}\right\} \tag{B3}$$

The shear flow q_1 is given by

$$q_1 = \frac{1}{Z}\{c_g\}^T\{w\} \tag{B4}$$

where $\{w\}$ is the displacement vector containing the three displacements at point 1 followed by the three displacements at point 2. The shear stress $\bar{s}$ in the element is given as

$$\bar{s} = \frac{q_1}{t} \tag{B5}$$

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TABLE I.- COMPARISON OF RESULTS OBTAINED FROM SWIFT AND WIDOWAC FOR INITIAL CONFIGURATION OF FULL-DEPTH SANDWICH WING

[Uniform skin thickness, 0.127 cm (0.05 in.); skin mass, 4000 kg (9042 lbm); altitude, 7620 m (25 000 ft)]

	SWIFT (ref. 9)	WIDOWAC	NASTRAN
Flutter speed	1362 m/s (4470 ft/sec)	1211 m/s (3980 ft/sec)	--------------------
Maximum stress	690 MN/m^2 (100 ksi)	613 MN/m^2 (89 ksi)	645 MN/m^2 (93.5 ksi)
Degrees of freedom	3 chordwise × 7 spanwise	93	162

TABLE II.- COMPARISON OF FINAL DESIGNS BY SWIFT AND WIDOWAC FOR FULL-DEPTH SANDWICH WING

Quantity	Method	Results		
		Flutter only	Strength only	Strength and flutter
Mass	SWIFT	1450 kg (3193 lbm)	1832 kg (4037 lbm)	1837 kg (4048 lbm)
	WIDOWAC	1424 kg (3136 lbm)	1822 kg (4014 lbm)	1822 kg (4014 lbm)
CPU*	SWIFT	470 s	380 s	690 s
	WIDOWAC-DFP	214 s	110 s	201 s
	WIDOWAC-Newton	87 s	64 s	114 s

*CPU = CDC 6600 Central Processor time.

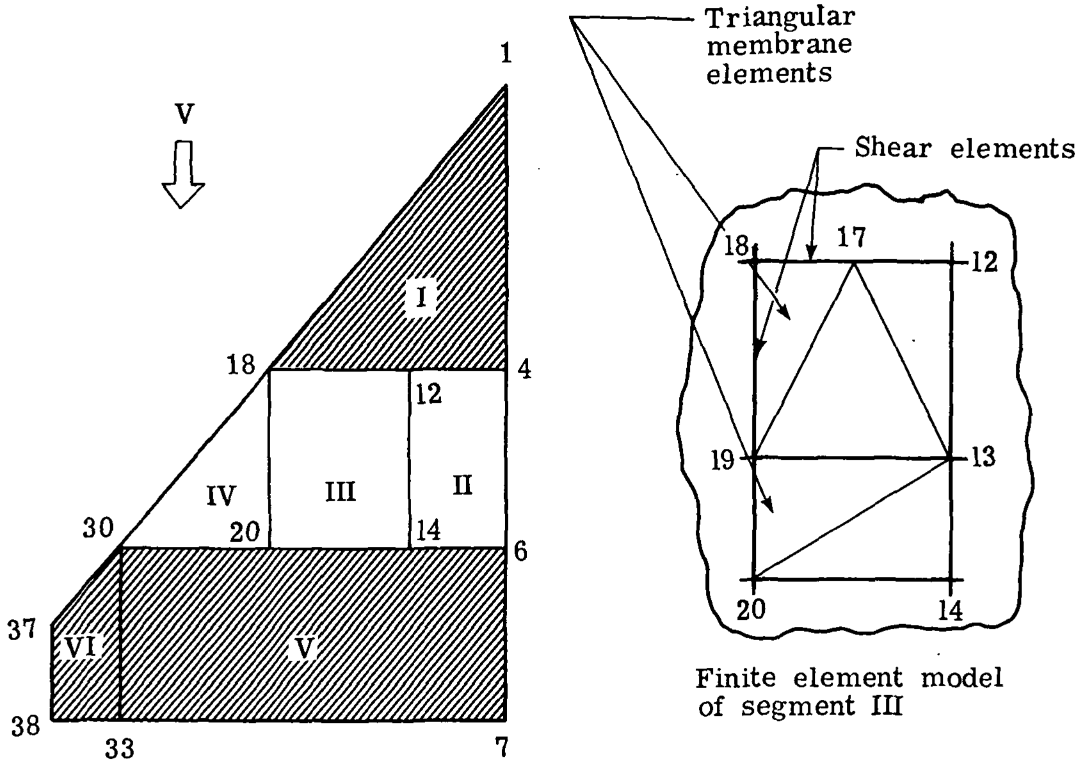
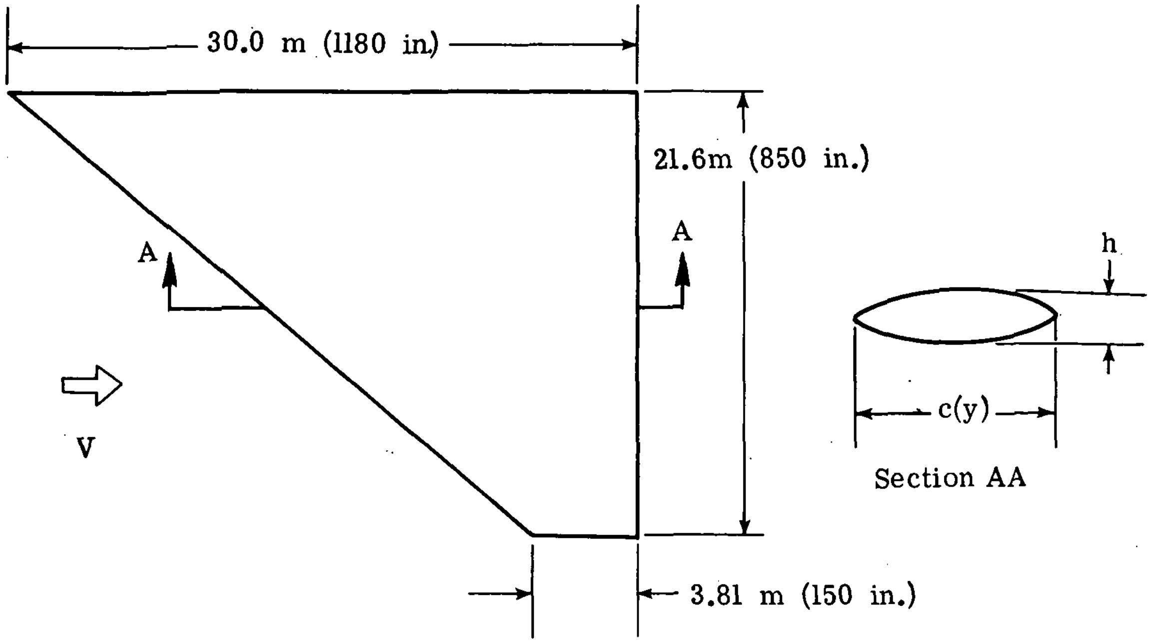


Figure 1.- Typical model for defining design variables.



Constraints:

1. Maximum stress 862 MN/m^2 (125 ksi) for uniform loading 6.89 kN/m^2 (1 psi)
2. Flutter speed at 7620 m (25 000 ft) not less than 762 m/s (2500 ft/sec)
3. Minimum skin thickness, 0.0381 cm (0.015 in.)

Figure 2.- Definition of full-depth sandwich wing and design constraints.

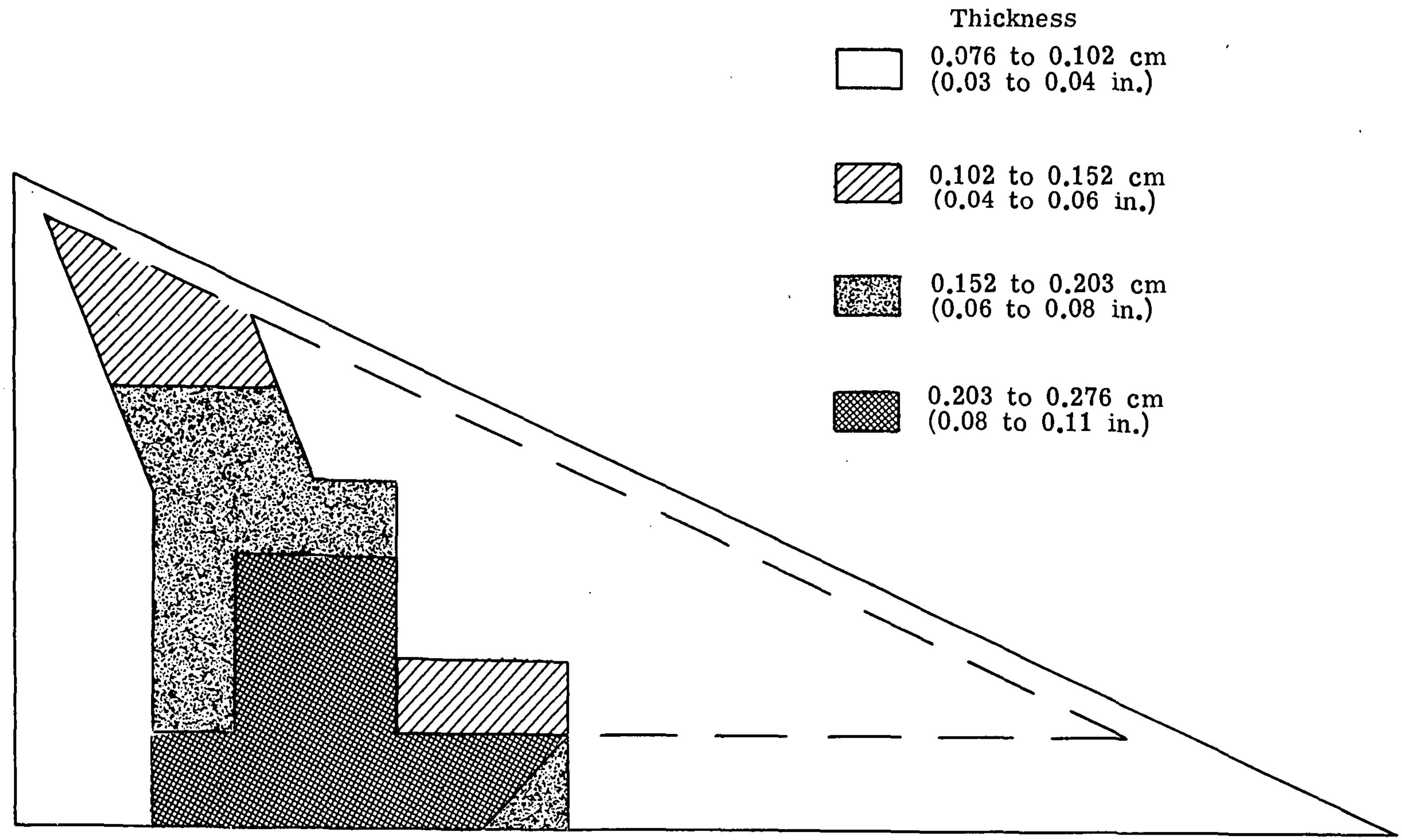


Figure 3.- Cover panel thickness distribution obtained with WIDOWAC for built-up 65° delta wing.

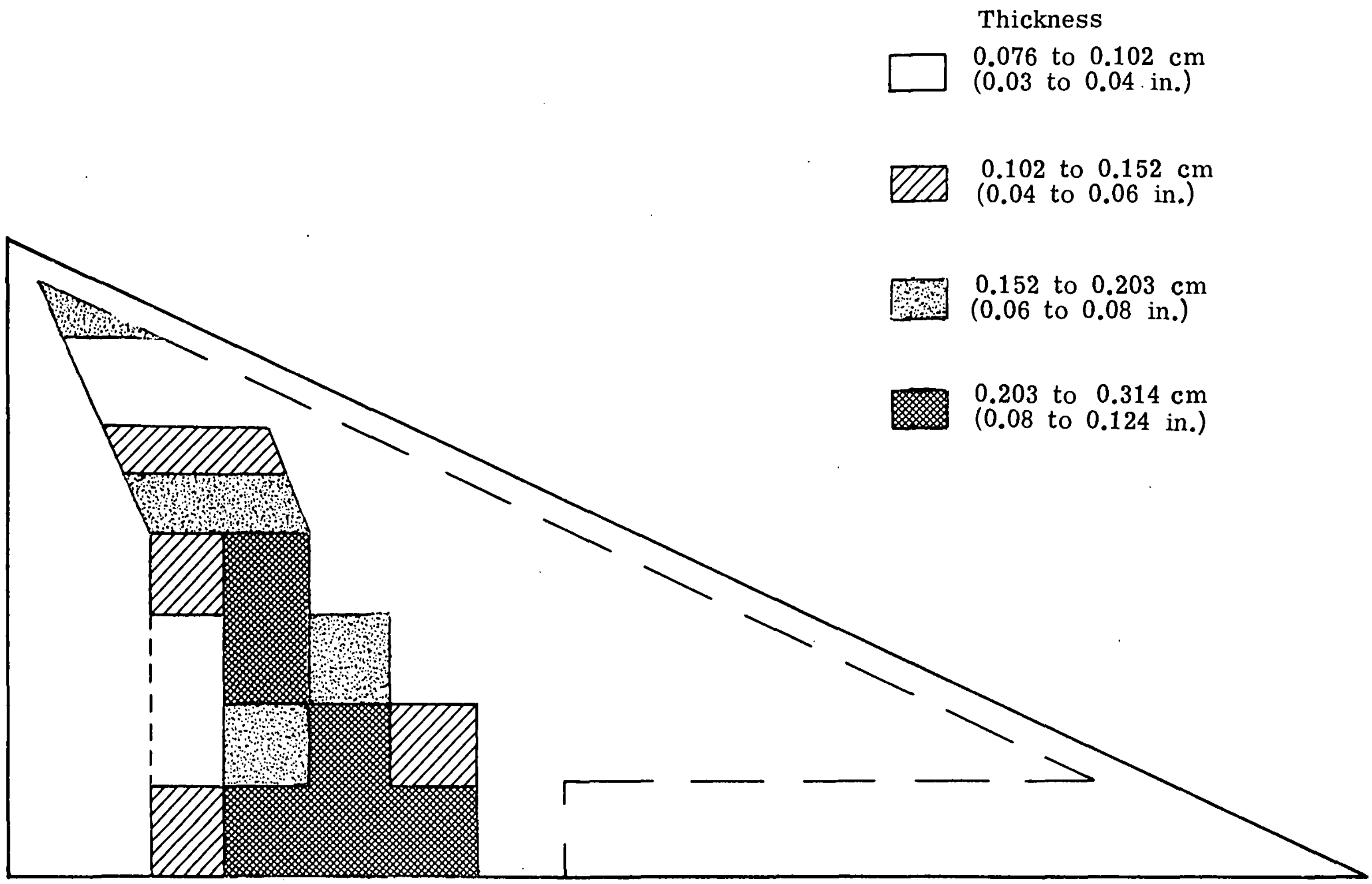


Figure 4.- Cover-panel thickness distribution obtained with DAWNS for built-up 65° delta wing (ref. 25).

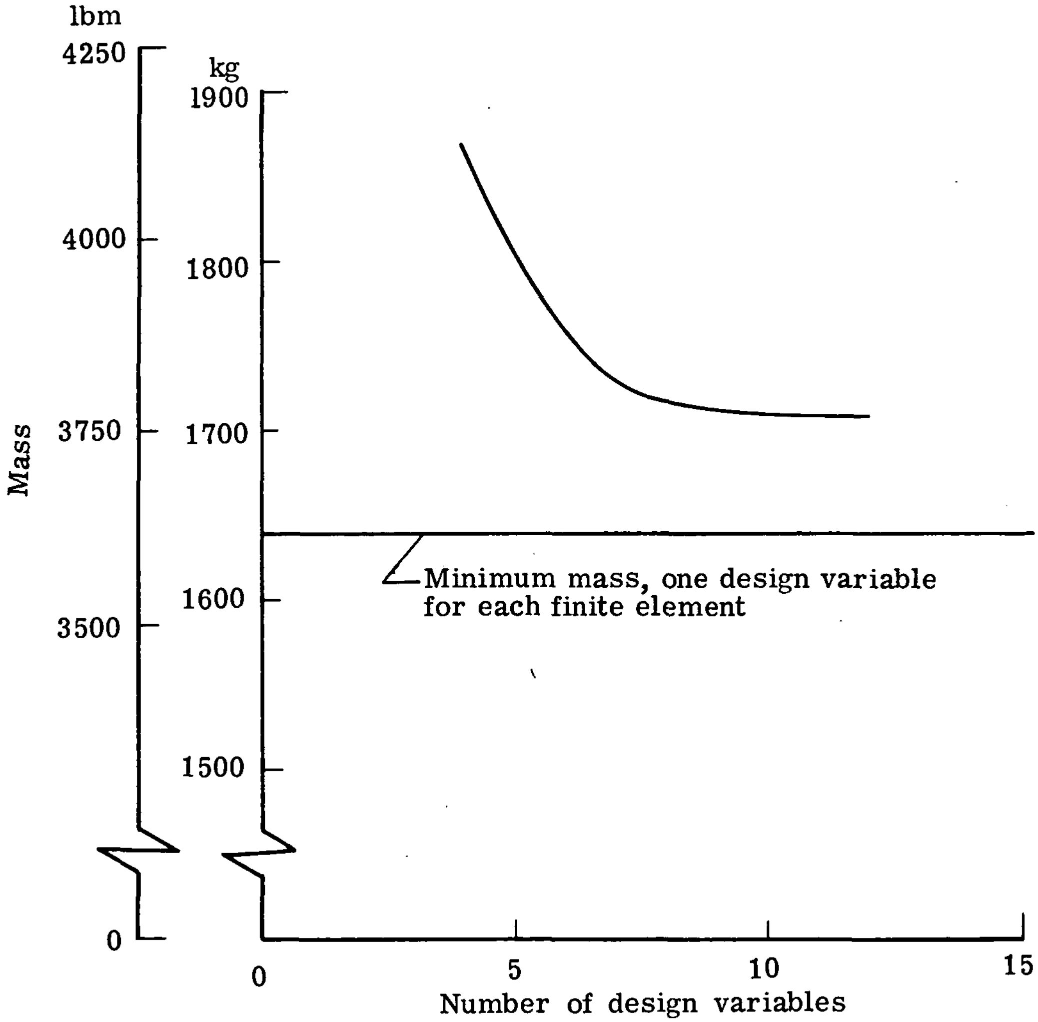


Figure 5.- Mass as a function of number of design variables for full-depth sandwich wing (51 elements per cover).

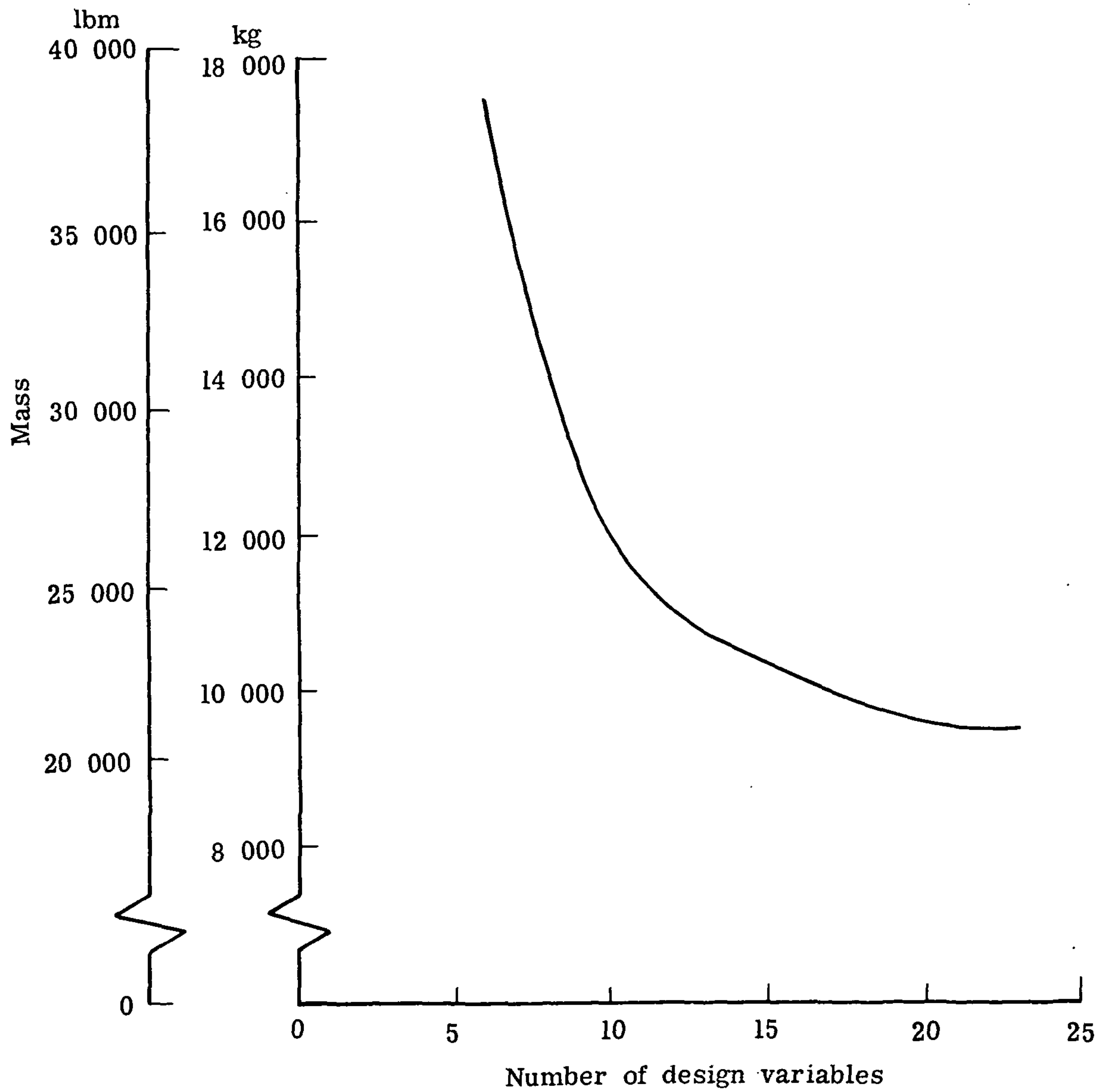


Figure 6.- Mass as a function of number of design variables for built-up 65° delta wing.

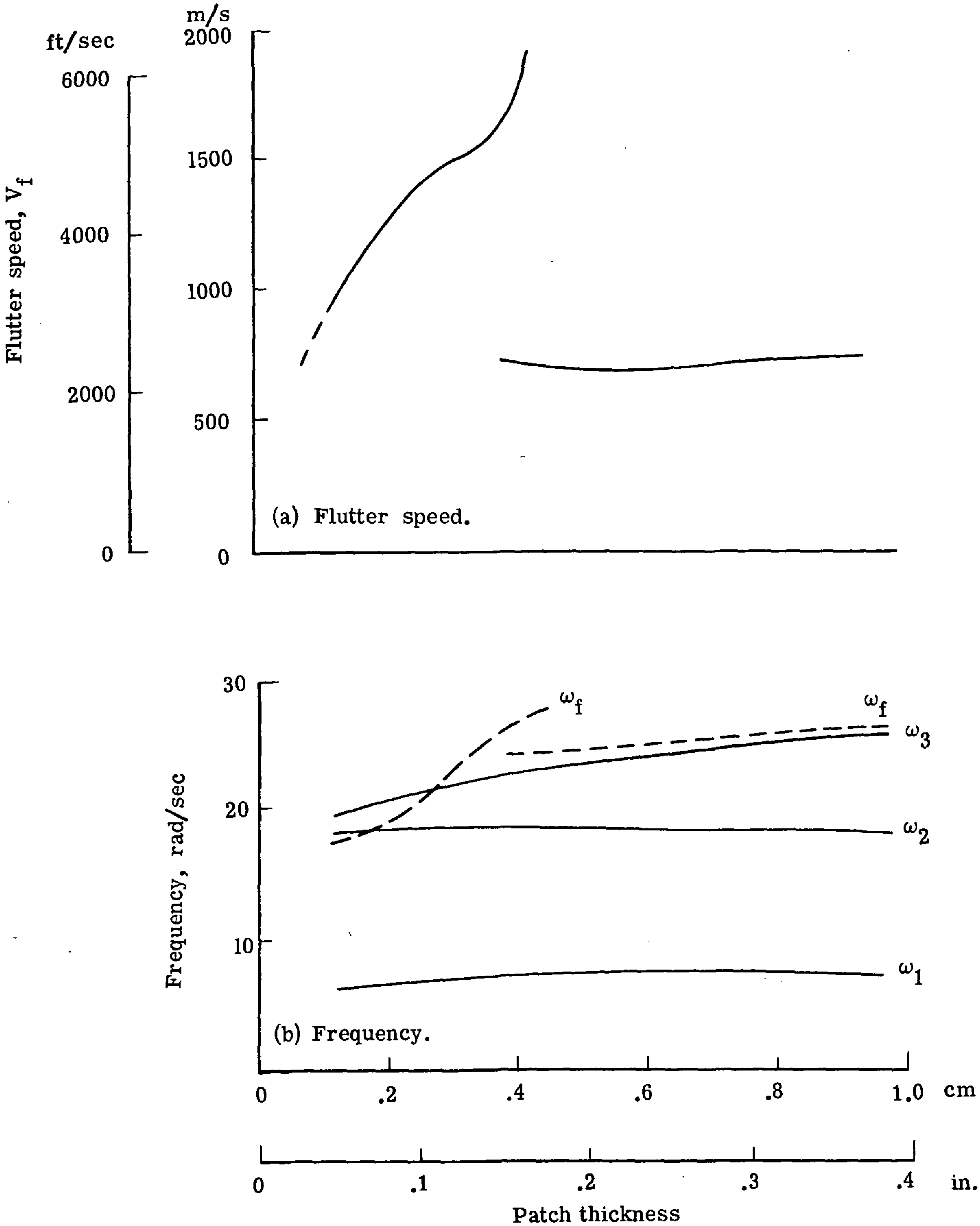


Figure 7.- Natural frequency, flutter frequency, and flutter speed as a function of patch thickness for titanium-beryllium sandwich wing.

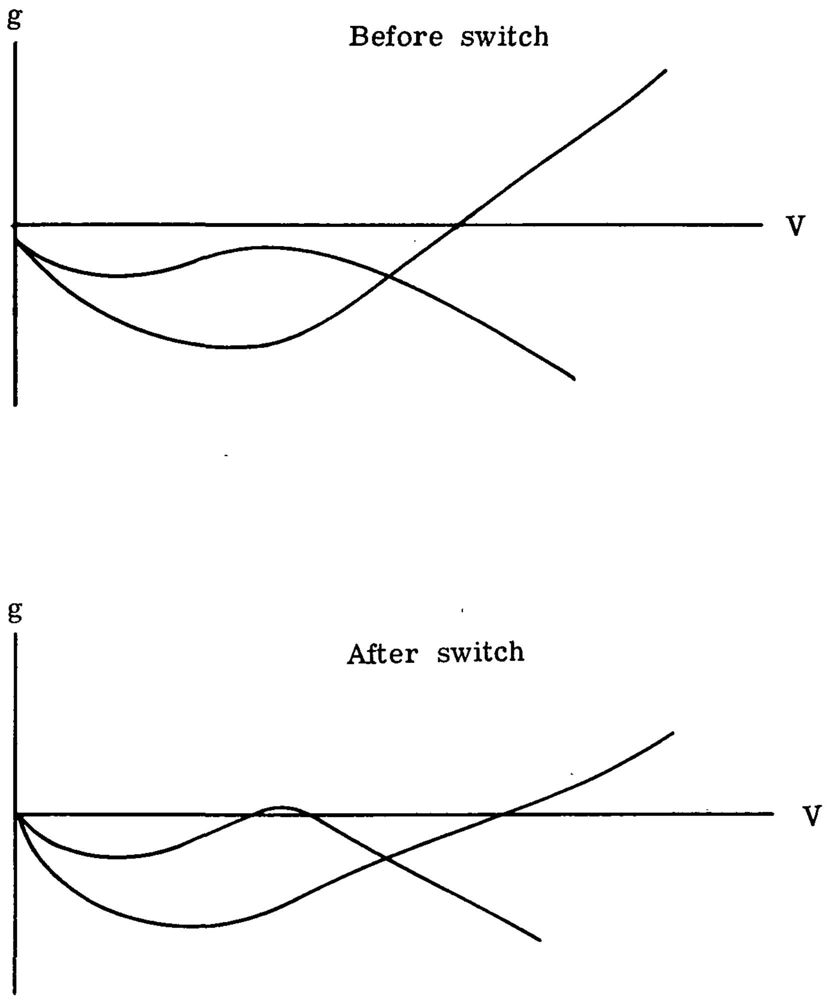


Figure 8.- A sketch showing discontinuity of flutter speed on a V-g diagram.

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