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SIMULTANEOUS OBSERVATION SOLUTIONS FOR NASA-MOTS AND SPEOPT STATION POSITIONS ON THE NORTH AMERICAN DATUM

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ABSTRACT

Simultaneous observations of the GEOS-I and II flashing lamps by the NASA MOTS and SPEOPT cameras on the North American Datum (NAD) have been analyzed using geometrical techniques to provide an adjustment of the station coordinates. Two separate adjustments have been obtained. An optical data-only solution has been computed in which the solution scale was provided by the Rosman-Mojave distance obtained from a dynamic station solution. In a second adjustment, scaling was provided by processing simultaneous laser ranging data from Greenbelt and Wallops Island in a combined optical-laser solution. Comparisons of these results with previous GSFC dynamical solutions indicate an rms agreement on the order of 4 meters or better in each coordinate. Comparison with a detailed gravimetric geoid of North America yields agreement of 3 meters or better for mainland U.S. stations and 7 and 3 meters, respectively, for Bermuda and Puerto Rico.

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SIMULTANEOUS OBSERVATION SOLUTIONS FOR NASA-MOTS AND SPEOPTS STATION POSITIONS ON THE NORTH AMERICAN DATUM

1. INTRODUCTION

The flashing lamps of GEOS-I and -II satellites tracked as part of the National Geodetic Satellite Program (NGSP) provided a large amount of precise simultaneous optical data from a relatively well-distributed network of tracking stations on the North American Datum (NAD). This report describes the results of two determinations of the relative positions of a twenty-one station NASA MOTS-SPEOPTS camera network on the NAD using geometrical adjustment techniques to process simultaneous observations of these satellites. One determination was obtained in which the solution scale was provided by the Rosman-Mojave distance derived from the dynamic station adjustment of Marsh et al (1971). A second determination was obtained in which the solution scale was determined by the inclusion of a small quantity of simultaneous laser tracking data from Greenbelt and Wallops Island.

A total of over 4000 two-, three-, and four-station optical observations and approximately eight passes (40 observations) of laser data were investigated in this work; the final results are based upon 90% of the observational data. This geometric solution will be used in conjunction with dynamic, gravimetric, astrogeodetic, and other types of geodetic information to provide the relation of the NAD to a unified world geodetic reference system.

2. DATA SELECTION AND PREPROCESSING

Table 1 presents the names and approximate locations of the MOTS, SPEOPTS, and laser tracking stations from which significant amounts of data were available.

The simultaneous observational data used consisted of two-, three-, and four-station events. Tables 2-5 indicate the approximate extent of the simultaneous data; the figures given are the total number of flashes observed. In evaluating the significance of the amount of optical data it should be kept in mind that flashes occur in sequences of seven, each flash being separated by four seconds in time. Normally, five to seven flashes of any given sequence are simultaneously observed during a pass. Errors in computed station-to-flash directions for all flashes of a sequence observed by any station are correlated. Also all flashes in a sequence occur within 24 seconds so that only a small amount of geometric strength is gained from increasing the number of flashes observed during a given sequence. For these reasons, the number of flashes given in the tables should be divided by about five to arrive at an estimate of the number of independent pieces of information along each line. However, the total number of flash observations does contribute to the reduction of errors due to shimmer, which is largely random from flash to flash.

The original observation data were available in the form of topocentric right ascensions and declinations for each flash, together with the instant of triggering of the flash in UTC. The right ascensions and declinations were referred to the true equator and equinox of date.

A number of corrections were applied to the right ascension and declination values in the course of plate reduction. These corrections were for the following effects:

1. Diurnal aberration

2. Proper motion

3. Precession of the star positions to the time of triggering of the flash

4. Nutation of the precessed star positions

5. Annual aberration

6. Radial and tangential lens distortion

7. Astronomical and parallactic refraction.

The methods used in making these corrections are described in Rawlinson and Oosterhout (1971) and Hotter (1967).

Two additional corrections were applied to the data at the outset of the work which included:

1. Correction to observations to account for the fact that the satellite is at a finite distance from the tracker, rather than the value of infinity used for analyzing stellar data. This correction compensates for the rotation of the earth between emission of the flash and reception at the camera.

2. Conversion of UTC time of triggering of the flash to UTC time of the instant of maximum light flux.

Preprocessing analysis of the data was performed in the following two-step process:

1. The right ascension and declination data in an inertial coordinate system were converted to station-to-satellite directions in a terrestrial coordinate system.

2. Erroneous data were edited.

Conversion from inertial to terrestrial coordinates was performed using UT1 time values of the BIH, polar motion data of the IPMS, and standard precession and nutation data. The terrestrial directions were obtained in the form of two direction angles, analogous to geocentric latitude and longitude angles. These angles were referenced to a set of coordinate axes parallel to the geocentric coordinate axes but with an origin at an observing station. Then the two angles (ω, ϕ) defining the station-to-satellite direction with respect to this station-centered coordinate system were as indicated in Figure 1.

Elimination of obviously erroneous data was performed through the use of a geometric test using a quantity called the "skew distance". Skew distance is described as follows. If the coordinates of two camera stations are exactly known and the station-to-satellite directions from the two stations observing simultaneously are exactly known, the two camera-to-station rays intersect exactly at the satellite and the satellite position can be computed. In the actual case, camera station positions and observed station-to-satellite directions have small errors. If approximate station positions and station-to-satellite directions having small errors are used, the two station-to-satellite rays will, in general, fail to intersect in space. In this case the minimum distance between the two rays is a line segment in space, normal to both rays and intersecting them. Half of this minimum distance line segment is defined as the skew distance. The skew distance was computed for each set of simultaneous observations and observations were eliminated when the skew distance exceeded 60 meters. Use of this criteria resulted in the elimination of approximately 7% of the original observations.

3. METHOD OF ANALYSIS

The mathematical analysis leading to performance of the geometric adjustment of tracking station coordinates is based on the following observational configurations:

1. Two cameras observe the satellite simultaneously
2. Three cameras observe the satellite simultaneously
3. Four cameras observe the satellite simultaneously
4. Two cameras and one laser observe the satellite simultaneously.

For each observation analyzed, condition equations are developed in the following form:

$$\sum_{i}^{m} a_i v_i + \sum_{j}^{n} b_j x_j + c = 0 \qquad (1)$$

where a_i , b_j, and c are known constants

v_i are observational residuals

x_j are unknown parameters, i.e., corrections to station coordinates, to be estimated statistically

m is the number of observed quantities

n is the number of unknown coordinates.

Condition equations resulting from a given simultaneous observation are of two types:

1. Coplanarity equations, which require that the two observing stations and the satellite lie in the same plane.

2. Laser length equations, which require that the satellite observation satisfying the two-station coplanarity relationship also satisfies the laser range from a third station.

Additional condition equations are employed in the solution process which imposes constraints on the solution. Constraints may be imposed either on a statistical basis or on an absolute basis. Statistical constraints involve the specification of coordinate "residuals" and take the form of Equation 1. Absolute constraints fix certain relationships between solution variables in a specified fashion and are of the form:

$$\sum_{k=1}^{I} p_k x_k + e = 0 \qquad (2)$$

where p_k and e are known constants

x_k are the unknown corrections to station coordinates

I is the number of coordinates involved in the constraint.

Three types of constraint equations may be applied:

1. Coordinate equations, which require a given coordinate value to remain at or near a given value throughout the adjustment.
2. Distance equations, which require the distance between two stations to remain at or near a given value throughout the adjustment.
3. Coordinate shift equations, which require the coordinate differences between two stations to retain a specified differential relationship.

The adjustment is effected by processing observational data in the four observational categories cited at the introduction to this section to produce condition equations. In particular, the observational category/condition equation requirements are:

1. For a two-station event (two cameras observe simultaneously), one coplanarity equation is used.
2. For a three-station event, three coplanarity equations are used.
3. For a four-station event, five coplanarity equations are used.
4. For a three-station laser/optical event, one coplanarity equation and one laser length equation are used.

These equations lead to an equation in the form

$$A V + B X + C = 0 \tag{3}$$

Letting the aggregation of constraints be of the form

$$P X + E = 0$$

the function to be minimized is written

$$V^T W V - 2k^T (A V + B X + C) - 2 \lambda^T (P X + E) \tag{4}$$

where W is the weight matrix for observations

k and λ are Lagrangian multipliers.

Assuming the existence of a reduced normal equation matrix

$$J = B^T (A W^{-1} A^T)^{-1} B \tag{5}$$

The value of the solution vector minimizing Equation 4 is

$$X = -J^{-1}\{I - P^T[PJ^{-1}P^T]^{-1}PJ^{-1}C + P^T[PJ^{-1}P^T]^{-1}E\} \quad (6)$$

The reduced normal equation matrix is obtained in a step-wise manner as each event is processed. Since the matrix $AW^{-1}A^T$ is quasi-diagonal (i.e., $AW^{-1}A^T$ is comprised of symmetric submatrices located along the principal diagonal with each submatrix of the order of the number of equations in the event), it is easily inverted. Each symmetric submatrix is inverted as it occurs and the inverse is placed in the appropriate position in the $(AW^{-1}A^T)^{-1}$ matrix. This leads to formation of the reduced normal equations $(B^T(AW^{-1}A^T)^{-1}B)$ by forming and summing partial normal equations. The matrix $AW^{-1}A^T$ required for a large solution may be on the order of 20,000-by-20,000, but the largest submatrix requiring inversion is 5-by-5.

4. RESULTS OBTAINED

The results obtained in this investigation consist of two sets of coordinates for the MOTS-SPEOPTS stations. One set of coordinates resulting from an optical data adjustment is shown in Table 6. A set of coordinates resulting from an optical-laser data adjustment is shown in Table 7.

The following constraints were applied to the optical data solution:

1. The position of station 1042 was held fixed at

   x = 647,516 m. y = -5,177,918 m. z = 3,656,704 m.

2. The distance of station 1030 from station 1042 was held fixed at 3051442 m.

3. Station 1037 was held fixed relative to station 1042. In this solution

this was equivalent to holding station 1037 fixed at

x = 647,523 m.　　y = -5,177,918 m.　　z = 3,656,704 m.

4. Stations 1034 and 7034 were constrained to have the same location.

5. Stations 7072, 7073, and 7074 were constrained to maintain the same relative positions.

Constraints 1, 3, and 4 above were also applied to the optical-laser data solution. In addition, the following constraints were applied:

1. Stations 1022, 7071, 7072, 7073, and 7074 were constrained to maintain the same relative positions, i.e., the differences in coordinates were constrained as follows:

	Δx (meters)	Δy (meters)	Δz (meters)
1022-7071	168410	50582	46733
1022-7072	168414	50589	46744
1022-7073	168421	50589	46748
1022-7074	168421	50592	46748

2. The station pairs (1021, 7043), (7043, 7077), (7043, 7050), (7043, 7078), and (7043, 7052) were constrained to maintain relative positions equal to the a priori coordinate differences. The constraints applied were:

	Δx (meters)	Δy (meters)	Δz (meters)
7043-1021	12682.	-44985.	-51160.
7077-7043	-653.	-1711.	1878.
7050-7043	-39.	37.	-32.
7078-7043	130867.	-50026.	-100694.
7052-7043	130836.	50257.	-100970.

5. ANALYSIS OF RESULTS

The results of this investigation have been evaluated by comparison with results obtained from dynamic analyses of MOTS-SPEOPTS data and from estimates of station positions derived from ground survey. Coordinate comparisons and inter-site distance comparisons are presented and analyzed. A scale parameter is derived from the laser-optical geometric solution and subjected to error analysis as a function of uncertainty in the laser range data.

A comparison of the coordinate solutions of this paper with the dynamic analysis of Lerch et al (1972) and Marsh et al (1971) is presented in Table 8. Datum shifts were applied to all four solutions so that the coordinate values for station No. 1042 agreed exactly with the NAD survey values. Table 8 presents the differences between the shifted coordinates and the NAD values after removal of the mean difference from each value. This comparison indicates that the degree of agreement among the optical geometrical and dynamic results is excellent. The rms difference in each coordinate between the geometric solution and the dynamic solutions as shown in Table 8 is less than 4 meters, exclusive of station 7039 (Bermuda).

A comparison of the geoid heights derived from the geometric solutions with the detailed gravimetric geoid heights of Vincent et al (1973) is given in Table 9. The accuracy of the detailed gravimetric geoid is on the order of 2 meters rms. The differences between the gravimetric geoid heights and the geoid heights derived from the geometric solutions are generally on the order of 3 meters or less for all stations except the two island stations: Bermuda (7039), and Jamaica (7076). The difference at Jamaica may be attributed in part to a possible survey error since comparisons with dynamic solutions offer good agreement.

The inter-site distances obtained from the geometric solutions and the inter-site distances derived from dynamic station adjustments have been compared with ground survey distances. The extent to which the satellite solutions agree with one another and with the survey is given by the implied scale differences:

(Optical-Laser Geometric) - (Survey) = .6 ± 2.3 parts per million (ppm)

(Marsh et al, 1971) - (Optical-Laser Geometric) = .6 ± 2.5 ppm

(Lerch et al, 1972) - (Optical-Laser Geometric) = -.7 ± 2.9 ppm.

Analysis of the coordinate differences between the optical geometric solution and the optical-laser geometric solution shows the solutions to be in excellent agreement.

Analysis of the inter-site distances obtained show that scale is determined by the sparse laser observations to an accuracy of three parts per million or better. In addition, an error analysis was performed by perturbing all laser ranges and re-computing the site coordinates. It has been determined that one unit of uncertainty in laser ranges produces approximately two units of uncertainty in inter-site distances.

6. CONCLUSIONS

Coordinates have been derived for fifteen MOTS and SPEOPTS tracking stations in North America using geometric techniques to process simultaneous observations of the flashing lights on the GEOS-I and -II satellites. Comparisons with independent results derived using dynamic techniques and with gravimetric data indicate that an accuracy of three meters or better in each coordinate has been achieved for stations in the continental United States and an accuracy of five meters or better has been achieved for the islands of Bermuda, Puerto Rico, and Jamaica.

The results of the solution employing simultaneous optical and laser data have shown that a satisfactory scaling of the MOTS-SPEOPTS NAD network has been obtained which is independent of either survey or dynamic solution data.

ACKNOWLEDGMENTS

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TABLE 1

TRACKING SYSTEMS ANALYZED

MOTS-SPEOPT Camera Stations

Station No.	Station Name	Approximate Latitude	Approximate Longitude
1021	Blossom Point, Maryland	38° 25' 50"	282° 54' 49"
1022	Fort Myers, Florida	26° 32' 53"	278° 08' 04"
1030	Goldstone, California	35° 19' 48"	243° 05' 59"
1034 (7034)	East Grand Forks, Minnesota	48° 01' 22"	262° 59' 20"
1042 (1037)	Rosman, North Carolina	35° 12' 07"	277° 07' 41"
7036	Edinburg, Texas	26° 22' 47"	261° 40' 07"
7037	Columbia, Missouri	38° 53' 36"	267° 47' 41"
7039	Bermuda	32° 21' 50"	295° 20' 35"
7040	San Juan, Puerto Rico	18° 15' 29"	294° 00' 29"
7043 (7077)	Greenbelt, Maryland	39° 01' 15"	283° 10' 20"
7045	Denver, Colorado	39° 38' 48"	255° 23' 38"
7075	Sudbury, Canada	46° 27' 22"	279° 03' 10"
7076	Kingston, Jamaica	18° 04' 34"	283° 11' 27"
7072	Jupiter, Florida*	27° 01' 13"	279° 53' 12"
7078	Wallops Island, Virginia	37° 51' 13"	284° 29' 27"
	Laser Systems		
7050	Greenbelt, Maryland	39° 01' 14"	283° 10' 18"
7052	Wallops Island, Virginia	37° 51' 35"	284° 29' 23"

*Data from Jupiter stations 7071, 7073, and 7074 were analyzed also.

TABLE 2

AVAILABLE TWO-STATION SIMULTANEOUS OPTICAL OBSERVATIONS

Two Stations Observing	Number of Flashes Observed	Two Stations Observing	Number of Flashes Observed
1021-1022	20	1030-7075	8
1021-1034	16	1034-1042	41
1021-1042	10	1034-7036	33
1021-7036	5	1034-7037	220
1021-7037	29	1034-7045	100
1021-7039	21	1034-7075	60
1021-7040	16	1042-7036	34
1021-7045	1	1042-7037	25
1021-7075	81	1042-7039	9
1022-1030	58	1042-7040	13
1022-1034	16	1042-7045	24
1022-1042	64	1042-7075	19
1022-7036	109	7036-7037	92
1022-7037	124	7036-7045	94
1022-7039	48	7036-7076	50
1022-7040	106	7037-7039	35
1022-7045	44	7037-7045	168
1022-7076	151	7037-7075	91
1030-1034	134	7037-7076	16
1030-1042	6	7039-7040	95
1030-7036	254	7039-7075	25
1030-7037	114	7045-7075	14
1030-7045	358		

TOTAL Two-Station Optical Observations — 3062

TABLE 3

AVAILABLE THREE-STATION SIMULTANEOUS OPTICAL OBSERVATIONS

Three Stations Observing	Number of Flashes Observed
1021-1034-7037	12
1021-1034-7075	12
1021-1042-7037	14
1022-1034-1042	39
1022-1034-7045	10
1022-1034-7075	14
1022-1042-7037	35
1022-7037-7039	21
1022-7039-7040	26
1022-7040-7076	47
1022-7045-7076	13
1030-1034-7036	10
1030-1034-7037	39
1030-1034-7045	14
1030-7036-7037	43
1030-7036-7045	45
1030-7037-7045	76
1034-1042-7037	17
1034-1042-7045	13
1034-7037-7045	44
1034-7037-7075	40
7036-7037-7045	30

TOTAL Three-Station Optical Observations — 614

TABLE 4

AVAILABLE FOUR-STATION SIMULTANEOUS OPTICAL OBSERVATIONS

Four Stations Observing	Number of Flashes Observed	Four Stations Observing	Number of Flashes Observed
1021 1022 1042 7043	6	1022 1034 1042 7075	6
1021 1022 7034 7040	1	1022 1034 7036 7039	5
1021 1022 7040 7043	4	1022 1034 7036 7072	5
1021 1022 7043 7045	3	1022 1034 7037 7075	6
1021 1030 1034 7037	6	1022 1034 7039 7074	3
1021 1034 1042 7037	8	1022 1034 7072 7074	3
1021 1034 1042 7045	5	1022 1034 7072 7076	4
1021 1034 7037 7043	7	1022 1037 7034 7036	8
1021 1034 7037 7075	6	1022 1037 7037 7040	2
1021 1034 7043 7045	6	1022 1037 7037 7075	2
1021 1042 7036 7037	6	1022 1037 7039 7075	2
1021 1042 7037 7045	14	1022 1037 7075 7077	5
1021 1042 7040 7043	6	1022 1037 7076 7077	5
1021 7036 7037 7039	1	1022 1042 7036 7037	5
1021 7043 7072 7074	1	1022 1042 7036 7045	3
1021 7045 7072 7076	1	1022 1042 7037 7072	10
1022 1030 1037 7037	5	1022 1042 7039 7072	1
1022 1030 7034 7037	7	1022 1042 7043 7076	1
1022 1030 7036 7037	7	1022 1042 7071 7072	4
1022 1030 7036 7045	7	1022 1042 7072 7076	5
1022 1030 7045 7072	5	1022 7036 7037 7039	6
1022 1034 1042 7036	7	1022 7036 7037 7045	17
1022 1034 1042 7043	5	1022 7036 7037 7076	3
1022 1034 1042 7045	7	1022 7037 7039 7040	7

TABLE 4 (Continued)

Four Stations Observing	Number of Flashes Observed	Four Stations Observing	Number of Flashes Observed
1022 7037 7039 7043	7	1034 1042 7037 7043	6
1022 7037 7040 7072	4	1034 1042 7037 7045	2
1022 7037 7040 7077	4	1034 1042 7037 7075	1
1022 7037 7043 7045	7	1034 1042 7039 7045	1
1022 7037 7045 7072	7	1034 1042 7045 7075	7
1022 7039 7040 7076	14	1034 7036 7037 7043	5
1022 7071 7072 7073	5	1034 7037 7039 7075	13
1022 7071 7072 7074	2	1034 7037 7045 7075	4
1022 7072 7073 7074	13	1037 7034 7036 7037	7
1030 1034 7036 7037	23	1037 7034 7037 7045	20
1030 1034 7036 7045	12	1037 7034 7039 7045	7
1030 1034 7037 7045	33	1037 7036 7037 7045	3
1030 1034 7037 7075	6	1037 7036 7037 7076	6
1030 1037 7034 7036	11	1037 7036 7076 7078	3
1030 1037 7034 7045	1	1037 7037 7039 7045	5
1030 1037 7036 7045	6	1042 7036 7037 7075	7
1030 1042 7036 7075	1	1042 7036 7043 7045	7
1030 7034 7036 7037	6	1042 7040 7043 7076	2
1030 7034 7036 7045	2	7034 7036 7037 7045	7
1030 7034 7037 7045	35	7034 7036 7037 7077	1
1030 7034 7045 7075	7	7036 7037 7043 7076	7
1030 7036 7037 7045	20	7036 7039 7075 7076	1
1030 7045 7071 7072	4	7039 7040 7071 7072	4
1034 1042 7037 7039	1		

TABLE 5

AVAILABLE THREE-STATION SIMULTANEOUS
OPTICAL-LASER OBSERVATIONS

Three Stations Observing	Number of Flashes Observed
7040-7077-7050	1
7075-7077-7050	2
1037-7077-7050	19
1037-7075-7050	7
1022-7034-7052	4
1037-7034-7052	6
1037-7078-7052	1

TOTAL Three-Station Optical/Laser Observations — 40

TABLE 6

CARTESIAN COORDINATES FOR OPTICAL GEOMETRIC SOLUTION

Station Number	x (meters)	y (meters)	z (meters)
1021	1118032	-4876309	3942972
1022	807863	-5651972	2833503
1030	-2357242	-4646316	3668307
1034	-521702	-4242043	4718720
1037	647523	-5177918	3656704
1042	647516	-5177918	3656704
7036	-828487	-5657446	2816814
7037	-191285	-4967270	3983257
7039	2308226	-4873593	3394570
7040	2465063	-5534911	1985516
7043	1130714	-4831324	3994132
7045	-1240470	-4760218	4048979
7072	976277	-5601383	2880247
7075	692623	-4347062	4600479
7076	1384161	-5905659	1966540
7078	1261581	-4881350	3893438

TABLE 7

CARTESIAN COORDINATES FOR OPTICAL-LASER GEOMETRIC SOLUTION

Station Number	x (meters)	y (meters)	z (meters)
1021	1118032	-4876309	3942972
1022	807863	-5651972	2833502
1030	-2357245	-4646316	3668307
1034	-521703	-4242042	4718721
1037	647523	-5177918	3656704
1042	647516	-5177918	3656704
7036	-828488	-5657446	2816814
7037	-191286	-4967270	3983257
7039	2308227	-4873593	3394570
7040	2465063	-5534911	1985515
7043	1130714	-4831324	3994132
7045	-1240472	-4760218	4048980
7072	976277	-5601383	2880246
7075	692623	-4347062	4600479
7076	1384161	-5905660	1966540

TABLE 8

COMPARISONS OF SATELLITE SOLUTIONS WITH NAD SURVEY

Station	Coordinate Differences (meters)											
	x				y				z			
	ΔG[1]	ΔG(L)[2]	ΔM[3]	ΔL[4]	ΔG	ΔG(L)	ΔM	ΔL	ΔG	ΔG(L)	ΔM	ΔL
1021	5	4	2	5	0	0	3	1	-5	-5	2	2
1022	-4	-5	-8	-8	-2	-2	3	2	-1	0	0	-1
1030	4	6	4	5	3	3	9	-4	2	2	-2	1
1034	-1	-1	-1	3	8	7	0	5	-2	-3	-7	-1
1037	0	-1	0	-5	-2	-2	-1	2	4	4	4	2
1042	0	-1	0	-6	-2	-2	-1	2	4	4	4	2
7036	-1	-1	2	-1	5	5	4	5	0	0	3	1
7037	-1	0	0	0	4	4	0	3	2	2	1	3
7039	-25	-25	-25	-18	-5	-5	-14	-11	-13	-13	-20	-13
7040	3	2	7	3	-8	-8	-4	-7	4	4	4	2
7043	5	4	2	7	0	0	3	-8	-5	-5	2	-7
7045	-3	-2	0	1	2	2	-2	3	-1	-2	-6	-1
7072	-4	-5	-8	-5	-2	-2	3	-2	-1	0	0	8
7075	-1	-2	2	2	-2	-2	-10	-1	-6	-6	-11	-5
7076	3	2	-2	-1	-5	-4	-7	-4	2	2	3	1

RMS Differences	x (meters)	y (meters)	z (meters
ΔG(L) vs ΔM	2.6	4.0	3.6
ΔG(L) vs ΔL	2.8	3.5	3.2

1. Optical Geometric Minus Survey
2. Optical-Laser Geometric Minus Survey
3. Marsh et al (1971) Minus Survey
4. Lerch et al (1972) Minus Survey

TABLE 9

GEOMETRIC/GRAVIMETRIC GEOID COMPARISON (METERS)

(1) Station Number	(2) Geometric Geoid Height[a]	(3) Gravimetric Geoid Height	(4) (2) - (3)	(5) (4) + 9 m
1021	-46	-34	-12	-3
1022	-38	-31	-7	2
1030	-44	-35	-9	0
1034	-35	-28	-7	2
1042	-47	-32	-15	-6
7036	-36	-25	-11	-2
7037	-43	-34	-9	0
7039	-41	-39	-2	7
7040	-56	-50	-6	3
7045	-31	-18	-12	-3
7050	-47	-34	-13	-4
7072	-42	-36	- 6	3
7075	-45	-37	-8	1
7076	-26	-32	6	15

a. Referenced to an ellipsoid with semimajor axis = 6378142 m and an inverse flattening of 298.255.

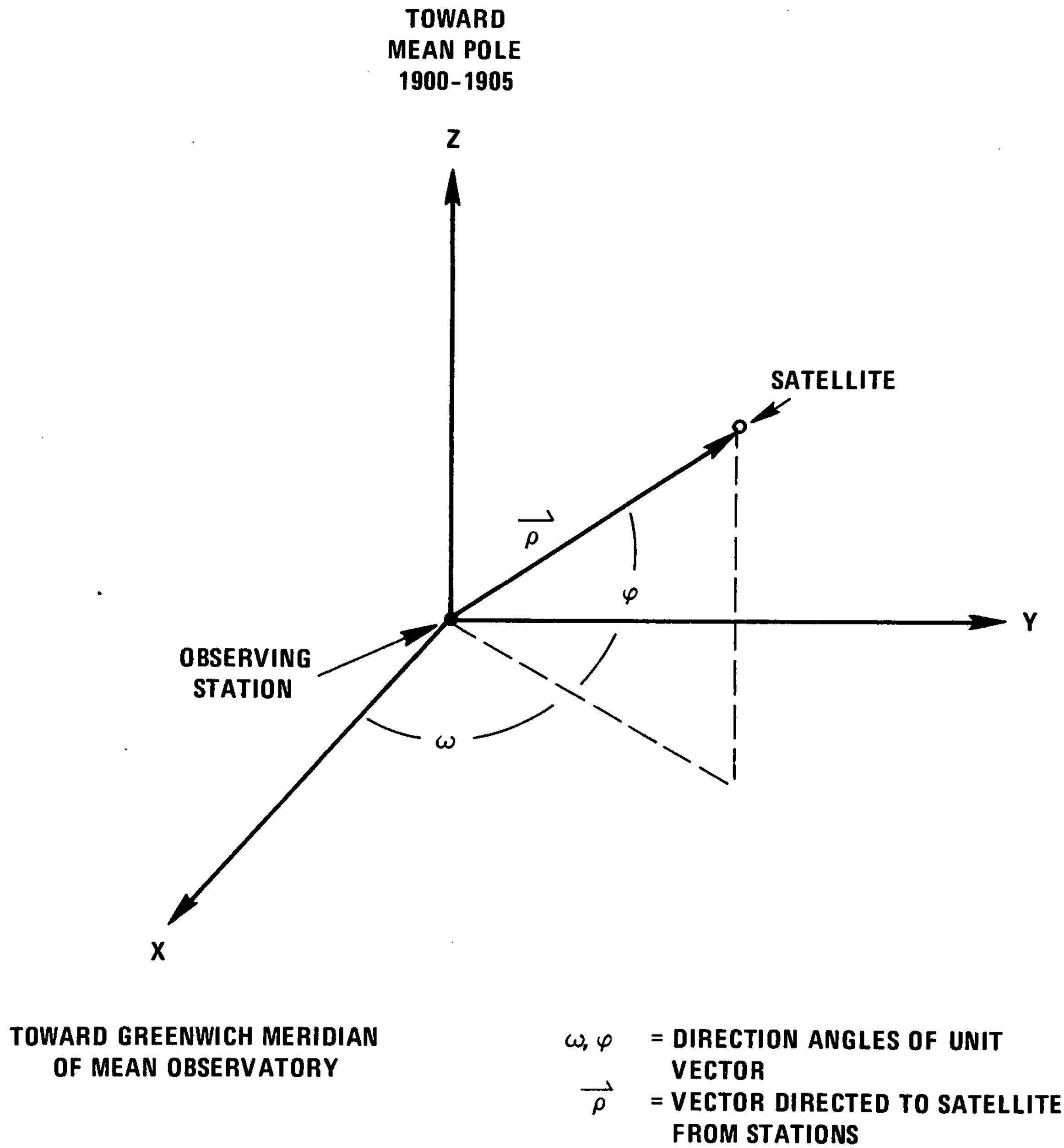

Figure 1. Optical Data Reference System