DISPERSION OF TURBOJET ENGINE EXHAUST IN FLIGHT

by James D. Holdeman

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The dispersion of the exhaust of turbojet engines into the atmosphere is estimated by using a model developed for the mixing of a round jet with a parallel flow. The analysis is appropriate for determining the spread and dilution of the jet exhaust from the engine exit until it is entrained in the aircraft trailing vortices. Chemical reactions are not expected to be important and are not included in the flow model. Calculations of the dispersion of the exhaust plumes of three aircraft turbojet engines with and without afterburning at typical flight conditions are presented. Calculated average concentrations for the exhaust plume from a single engine jet fighter are shown to be in good agreement with measurements made in the aircraft wake during flight.
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SUMMARY

The dispersion of the exhaust of turbojet engines into the atmosphere is modeled as a round jet mixing with a parallel flow. The analysis is appropriate for determining the spread and dilution of the exhaust plume in the region of the aircraft wake from the engine exit plane until the exhaust is entrained into the aircraft trailing vortices. Chemical reactions are not considered since residence times in this region are too short for photochemical reactions and engine exhaust static temperatures and pressures at altitude flight conditions are below levels required for any other significant reactions. An eddy viscosity model appropriate for the jet engine exhaust problem was formulated. It is used in an integral momentum method to calculate the dispersion of the exhaust plumes of both large and small aircraft turbojet engines with and without afterburning at typical flight conditions.

Measurements of carbon monoxide, oxides of nitrogen, and ozone obtained in the wake of an F-104 aircraft flying at Mach 0.8 at an altitude of 11 kilometers are compared with exhaust plume concentrations calculated by using the turbulent diffusion model, estimated concentrations at the engine exhaust plane, and experimentally determined ambient concentrations. The calculated average concentrations are shown to be in good agreement with values measured in the wake of the source aircraft.

INTRODUCTION

A method for estimating the dispersion of turbojet engine exhaust in flight is presented. The method is appropriate for determining the dilution of exhaust products and the plume size in the region of the aircraft wake from the engine exit plane until the exhaust is entrained in the aircraft trailing vortices.

Public concern about the possible adverse impact of a fleet of high-altitude aircraft on the environment has motivated interest in aircraft wakes and stratospheric chemical kinetics. The significance of chemical reactions in aircraft wakes is expected to be
dependent on atmospheric conditions and on the concentrations of possible reactants. The latter, of course, are determined by the engine exhaust emissions and their dispersion into the atmosphere.

The jet wake behind an aircraft in flight may be considered in three regions as described in reference 1. These are (1) the jet region, which extends from the engine exhaust plane to the downstream location where the exhaust is entrained in the aircraft trailing vortices; (2) the vortex region, which extends from the jet region to vortex breakup; and (3) the wake dissipation region, which extends beyond vortex breakup. The extent of the jet region is dependent on aircraft configuration and flight speed. For the aircraft and conditions examined in reference 1, this region was reported to extend to distances of the order of 2 kilometers behind the aircraft.

Chemical reactions are not considered to be important in the jet region. Residence times are too short for any significant photochemical reactions. And static temperatures and pressures at the engine exhaust at altitude flight conditions, even with afterburning, are low enough to preclude any other significant chemical reactions in the jet. This assumption is supported by chemical kinetics calculations presented in reference 1 for the constant-property jet core of the GE4 turbojet engine at Mach 2.7 and 19.8 kilometers altitude with maximum afterburning. Similar unpublished calculations made at NASA Lewis Research Center for various altitudes, flight speeds, and power levels typical of the J85 afterburning turbojet engine also support this assumption.

The model considered in reference 1 for the dilution and spreading of the exhaust in the jet region is an empirical formulation from experiments on low-speed, incompressible jets mixing with an annular coaxial stream at nearly common density and temperature (ref. 2). Since the turbojet engine exhaust is (at least) sonic and jet temperatures are of the order of 5 times the ambient temperature with afterburning, the relations of reference 2 are not appropriate. Furthermore, these relations do not reduce to the correct asymptotic jet spreading and decay relations for distances from the jet exit where the free-stream velocity is large with respect to the difference between the jet centerline velocity and the free-stream velocity (ref. 3).

To overcome some of the shortcomings of previous methods, the present investigation was undertaken to develop a model which would be appropriate for the initial conditions presented by the turbojet engine exhaust and which would yield the known asymptotic spreading and decay relations at large distances from the jet exhaust. The analysis is an integral momentum method for which the dimensionless velocity difference profile shape and the shear stress relation must be specified. The eddy viscosity model used in the shear stress relation was formulated by assuming that the eddy viscosity is directly proportional to the jet mass flow rate plus the absolute value of the entrainment mass flow rate and that it is inversely proportional to the jet width.

Calculations of exhaust plume dispersion for both large and small afterburning turbojet engines at typical flight conditions are presented. Finally, measurements of oxides
of nitrogen (NO\textsubscript{x}), carbon monoxide (CO), and ozone (O\textsubscript{3}) obtained in the wake of an F-104 aircraft flying at Mach 0.8 at an altitude of 11 kilometers are presented and compared with concentrations calculated by using the diffusion model, estimated exhaust concentrations, and experimentally determined ambient concentrations.

**ANALYSIS**

The exhaust of a turbojet engine is modeled as a round jet mixing with a parallel flow. This flow is shown schematically in figure 1. Free mixing of ideal gases is assumed to occur between an axially symmetric jet (subscript \(j\)) and an ambient gas stream (subscript \(\infty\)). The two streams are assumed to be chemically inert. The Prandtl and Schmidt numbers throughout the flow are assumed to be equal to 1; thus, the conservation equations for momentum, total enthalpy, and species concentration are identical. This permits the problem to be solved by considering only the continuity and momentum equations. The integral momentum method of Donaldson and Gray (ref. 4) for the mixing of an axisymmetric jet with a quiescent ambient gas of a different density has been extended to the case where the ambient gas may be moving at a velocity \(u_\infty\).

**Integral Relations**

The integral momentum equation evaluated at the half-value radius is

\[
\frac{d}{dx} \int_0^{r_5} \rho u_x^2 r \, dr = \left(\frac{u_\infty^2 + u_\infty}{2}\right) \frac{d}{dx} \int_0^{r_5} \rho u_x r \, dr + \tau_5 r_5^2
\]

where \(r_5\) is the radius at which the velocity is one-half the sum of the centerline and ambient velocities. A second relation, obtained by evaluating the integral momentum equation across the entire width of the flow, requires that the net thrust remains constant at all downstream locations. That is,

\[
\int_0^{\infty} \rho u_x (u_x - u_\infty) r \, dr = p_j u_j (u_j - u_\infty) \frac{r_j^2}{2}
\]

All symbols are defined in the appendix.
Velocity Profiles

To provide a relation between $u_x$ and $r$, velocity profiles are assumed. Those used in reference 4 are adopted here, except that now the exponential relations apply to the velocity difference ratio rather than to a velocity ratio. For the core region (see fig. 1),

$$\frac{u_x - u_\infty}{u_j - u_\infty} = \exp \left[ -\lambda \left( \frac{r^2 - r_i^2}{r_j^2 - r_i^2} \right) \right] \quad \text{for } r > r_i$$

and

$$\frac{u_x - u_\infty}{u_j - u_\infty} = 1 \quad \text{for } r \leq r_i$$

where $r_i$ is the radius of the constant-property core, $r_j$ is the half-value radius, and $\lambda = \ln 2$. For the developed region,

$$\frac{u_x - u_\infty}{u_j - u_\infty} = \left( \frac{u_c - u_\infty}{u_j - u_\infty} \right) \exp \left[ -\lambda \left( \frac{r}{r_j} \right)^2 \right]$$

where $u_c$ is the velocity on the jet centerline at the streamwise location of interest. Because the Prandtl and Schmidt numbers are assumed to be equal to 1, the concentration difference, total enthalpy difference, and velocity difference ratios are identical. Thus, equation (4) provides the relation for $(c - c_\infty)/(c_j - c_\infty)$ and $(h^0 - h_\infty)/(h_j^0 - h_\infty)$ in addition to $(u_x - u_\infty)/(u_j - u_\infty)$.

Density

The density can be expressed in terms of the velocity difference ratios through the perfect-gas law and the assumption of identical velocity difference, total enthalpy difference, and concentration difference distributions. For economy, let $U = (u_x - u_\infty)/(u_j - u_\infty)$. The density expression is then
\[ p = \frac{\rho_\infty \left\{ \left[ \left( \frac{C_p}{C_p} \right)_i - 1 \right] U + 1 \right\}}{\left( \frac{m_\infty}{m_i} - 1 \right) U + 1 \left[ 1 + \left( \frac{h^0_{i} - h^0_{\infty} - u_1 u_{\infty}}{h_{\infty}} \right) U - \left( \frac{u_1^2}{2 h_{\infty}} \right) U^2 \right]} \]  

(5)

where \( C_p \) is the specific heat, \( m \) is the molecular weight, \( h \) is the enthalpy, and \( u_1 = (u_j - u_{\infty}) \).

**Eddy Viscosity**

A gradient diffusion model for the shear stress is assumed; thus,

\[ \tau_5 = \varepsilon \frac{\partial u_x}{\partial r} \bigg|_{r=r_5} \]  

(6)

where \( \varepsilon \) is the eddy viscosity. Numerous eddy viscosity models have been postulated in the literature. Those of particular relevance to the present problem are

Prandtl model (cited in ref. 5):

\[ \varepsilon = 0.025 \rho r_5 |u_5 - u_{\infty}| \]  

(7)

Ferri model (cited in ref. 5):

\[ \varepsilon = 0.025 r_0 (\rho u)_5 |(\rho u)_5 - (\rho u)_{\infty}| \]  

(8)

Zelazny model (ref. 5):

\[ \varepsilon = \frac{0.036}{r_5} \int_0^\infty |\rho u_x - \rho_{\infty} u_{\infty}| r \, dr }{r_5(1.0 + 0.6|M_C - M_{\infty}|) } \]  

(9)

where \( M \) is the Mach number.
The Prandtl model has been very successful in predicting the mean flow in nearly-
constant-density cases where $u_{\infty} \neq 0$ and in variable-density cases where $u_{\infty} = 0$.
However, this model predicts decreased shear and hence decreased mixing with decreasing $u_{j}/u_{\infty}$ for all density ratios. This result is consistent with the experimental data of reference 2 for mixing of a jet and an ambient stream at nearly common density, but it is contrary to the experimental results of Chriess (ref. 6) for the mixing of low-density (hydrogen) jets in air.

The Ferri model attempts to account for the density ratio effects and does predict the correct mixing trends for both the hydrogen-air and air-air experimental results. However, this model predicts segregation of the two streams (no mixing) when $(\rho u)_{j} = (\rho u)_{\infty}$. Unfortunately, the initial conditions posed by turbojet engines in flight are often near this condition; hence, the Ferri model is not appropriate.

The Zelazny model is one of the more recent models and has been shown to be effective for predicting the experimental results of most of the jet test cases considered during the Langley Working Conference on Free Turbulent Shear Flows (ref. 7). This model is based on the displacement thickness concept previously used in boundary layer analysis. However, this model, like the Ferri model, is inappropriate for $(\rho u)_{j} \neq (\rho u)_{\infty}$.

The Zelazny model includes an empirical factor for compressibility and an empirical function for variation of $\epsilon$ with radial position. This latter factor is not included here since only the eddy viscosity at the half-value radius is of interest. Compressibility corrections have been applied by others also; for example, the eddy viscosity model used successfully by Donaldson and Gray (ref. 4) is the Prandtl model with an empirical compressibility correction applied.

Because of the inapplicability of previous models to the engine exhaust problem, a new eddy viscosity model was developed. The displacement thickness concept (eq. (9)) suggests that the eddy viscosity is proportional to the entrainment or detrainment of ambient fluid. Since segregation of the streams is not expected even for cases with zero net entrainment, it is postulated in this report that the eddy viscosity should be directly proportional to the jet mass flow rate plus the absolute value of the entrainment mass flow rate and inversely proportional to the half-value radius. In addition, Zelazny's compressibility factor (eq. (9)) is adopted. The resultant eddy viscosity relation is

$$\epsilon = \frac{0.036 \left( \rho_{j} u_{j} \frac{r_{j}^{2}}{2} + \int_{r_{0}}^{r_{\infty}} (\rho u_{x} - \rho_{\infty} u_{\infty}) r \, dr - (\rho_{j} u_{j} - \rho_{\infty} u_{\infty}) \frac{r_{j}^{2}}{2} \right)}{r_{5}(1.0 + 0.6|M_{c} - M_{\infty}|)}$$

(10)
The expression analogous to equation (10), which is appropriate to the core region, is

\[
\epsilon = \frac{0.038 \left( \frac{r_j^2 - r_i^2}{2} \right) + \int_{r_i}^{\infty} \left( \rho u_x - \rho_{\infty} u_{\infty} \right) r \, dr - \left( \rho_j u_j - \rho_{\infty} u_{\infty} \right) \frac{r_j^2 - r_i^2}{2} }{(r_5 + r_i)(1.0 + 0.6|M_j - M_{\infty}|)}
\]

(11)

The relations given provide a closed set of equations for the solution of the spreading and dilution of the jet from a uniform (slug profile) condition at the jet exit plane to any desired downstream location. The solution parallels that given by Donaldson and Gray (ref. 4) except that, because of the inclusion of a moving ambient stream, additional integral terms appear and many of the constants are different.

The integrals in equations (1) and (2) can be evaluated in closed form at any streamwise location. For the constant-property core region, \( r_5 \) is obtained in terms of \( r_i \) with equation (2), whence the streamwise distance corresponding to \( r_i \) is obtained with equation (1). The entire solution for the core region is obtainable in closed form. For the developed region, \( r_5 \) is obtained in terms of \( u_c \) with equation (2). The incremental changes in streamwise distance corresponding to selected incremental changes in centerline velocity are obtained with equation (1). The streamwise distance corresponding to any desired \( u_c \) must then be obtained by numerical integration.

The method described has been used to calculate the mixing for the jet test cases in reference 7. For all cases where the jet dynamic pressure was greater than the free-stream dynamic pressure, the agreement between calculated and experimental results was very good.

RESULTS AND DISCUSSION

Dispersion of Jet Exhaust Plumes

Calculations of exhaust plume dispersion have been made for both small and large aircraft turbojet engines at typical flight conditions. Exhaust conditions for the J85-GE-13 were chosen to represent a small afterburning turbojet engine. The estimated exhaust conditions for the GE4 engine which was proposed for the Boeing-2707 SST were used to represent a large afterburning turbojet engine. Calculations were also made for the dispersion of the exhaust plume of a J79 engine for comparison with flight data obtained in the wake of an F-104 aircraft. Both maximum afterburning and maximum power without afterburning were considered for each engine and each flight condition. The following
starting conditions were used: flight Mach number, ambient temperature, jet exhaust pressure ratio, jet total temperature, and primary nozzle diameter. An isentropic expansion was assumed from the sonic primary nozzle to obtain the pressure-matched condition which was assumed to exist at the engine exit. This expansion yielded the jet-to-free stream density and velocity ratios and the exhaust jet diameter which provided the required initial conditions for the plume dispersion calculation. These initial conditions for each of the engines and flight conditions examined are shown in table I.

Since the shape of the radial distribution is assumed to be invariant downstream of the core region and because the velocity difference and concentration difference ratios are assumed to be identical, the exhaust dispersion is completely defined by relations for the centerline concentration difference ratio and the half-value radius as a function of streamwise distance from the engine exhaust plane. Because Gaussian profiles are assumed for the radial distributions, an arbitrary choice for the radial extent of the plume region must be made in order to define an average concentration. The jet edge radius \( r_e \) is defined as the radius to the point where the concentration difference ratio is 0.001 times the centerline value. Thus,

\[
r_e = 3.157 r_5
\]

and

\[
\left( \frac{c_{sV} - c_\infty}{c_j - c_\infty} \right) = 0.145 \left( \frac{c_c - c_\infty}{c_j - c_\infty} \right)
\]

In the far downstream region, where \( \rho \approx \rho_\infty \) and \( u_\infty >> (u_x - u_\infty) \), the net thrust within the radius \( r_e \) is 0.999 times the initial thrust at the engine exhaust.

Dilution in the exhaust plume. - The centerline dilution as a function of distance downstream from the engine is shown for the J85, GE4, and J79 engines in figures 2(a), 3(a), and 4(a), respectively. The concentration difference ratio is proportional to \( x^{-2/3} \) for centerline dilutions greater than 100:1 (concentration difference ratio < 0.01) for all engines and conditions examined. This is the asymptotic decay expected for axysymmetric coflowing streams at large streamwise distances. The distance for which \( (c_c - c_\infty)/(c_j - c_\infty) = 1 \) is the length of the constant-property jet core region. The jet core lengths and the distances to 100:1 centerline and 1000:1 average dilutions are tabulated in table I.

The downstream distance corresponding to a given dilution varies significantly with power level. For the flight conditions and engines examined, the distance required to achieve a given dilution with maximum afterburning is from 1/4 to 1/2 of the distance required to achieve the same dilution at maximum power without afterburning. The more
rapid mixing with afterburning is caused primarily by the larger shear stress calculated for the higher thrust jet which occurs with afterburning.

For the same flight conditions and power levels, the variation in downstream distance to a given dilution for different engines would be expected to be directly related to the variation in engine exhaust diameter. However, since nominally equal power levels yield different exhaust conditions for different engines, engine operating characteristics are also significant. For example for Mach 2 flight with maximum afterburning, the distance to 100:1 centerline dilution for the GE4 is 4.5 times the corresponding distance for the J85, while the engine exhaust diameter for the GE4 is 3.5 times that for the J85. The remaining difference is caused by variations in engine exhaust conditions (i.e., density ratio and velocity ratio).

Because for centerline dilutions greater than 100:1 the concentration difference ratio varies as $x^{-2/3}$ and since the average concentration difference ratio is 0.145 times the centerline value at any location, the distance to an average 1000:1 dilution is approximately 75 percent greater than the distance to a 100:1 centerline dilution, as shown in table I. Because the regions of the aircraft wake are often discussed in terms of time since aircraft passage, where $t = x/u_\infty$, the times corresponding to the distances for 100:1 centerline and 1000:1 average dilution are also given in table I. For all of the conditions examined, the time after aircraft passage in which a 1000:1 average dilution is achieved is less than 15 seconds, as shown in table I.

Relation between dilution and spreading. - The half-value radius $r = r_5$, where $(c - c_\infty)/(c_c - c_\infty) = 0.5$ is related to the centerline concentration difference ratio (velocity difference ratio) by the conservation of momentum (eq. (2)). For large distances from the exhaust plane, $u_\infty >> (u_x - u_\infty)$, $\rho \approx \rho_\infty$, and equation (2) reduces to

$$\frac{r_5^2}{r_j^2} = \frac{(\rho u_x)}{(\rho_\infty u_\infty)} \frac{(c_c - c_\infty)}{(c_j - c_\infty)}$$

(14)

Since in this region of the flow $(c_c - c_\infty)/(c_j - c_\infty) \sim x^{-2/3}$, equation (14) gives the expected result that the half-value radius is proportional to $x^{1/3}$. The growth of the half-value radius with downstream distance for the engines and conditions examined is shown in figures 2(b), 3(b), and 4(b).

In the far downstream region, the half-value radius of the plume corresponding to any specified centerline dilution can be obtained directly, by using equation (14), from the initial diameter of the jet and the initial mass-velocity ratio.
For most turbojet engines, engine airflow remains nearly constant from maximum power without afterburning through maximum afterburning. Thus, equation (14) provides the result that the half-value radius corresponding to a chosen centerline dilution does not vary significantly with power level, although the downstream distance to the specified dilution ratio decreases as power level increases.

From the relations between the jet edge radius and the half-value radius (eq. (12)) and the average and centerline concentration difference ratios (eq. (13)), the relation between \( r_e \) and \( \frac{c_{jw} - c_\infty}{c_j - c_\infty} \) in the downstream region becomes

\[
\frac{r_e^2}{r_j^2} = 0.999 \left( \frac{\rho_j u_j}{\rho_\infty u_\infty} \right) \left( \frac{c_{jw} - c_\infty}{c_j - c_\infty} \right)
\]

The plume radii corresponding to 1000:1 average dilution and the half-value radii corresponding to 100:1 centerline dilution for the engines and flight conditions examined are given in Table I.

Estimated range of plume concentrations for \( \text{NO}_2, \text{CO}, \) and \( \text{O}_3 \). Table II compares estimated plume and background concentrations for afterburning and nonafterburning conditions for 1000:1 dilution in the plume. The engine exhaust concentrations expressed in parts per million by volume (ppmv) represent the range of values obtained from tests of various engines at simulated altitude flight conditions (refs. 8 to 11). The ambient atmospheric concentrations (ref. 12) and diluted concentrations in the plume are expressed in parts per billion by volume (ppbv). Ambient ozone levels near the lower end of the range shown would be expected in the troposphere, while the higher concentrations would be expected in the stratosphere.

**Calculated and Measured Pollution Levels in an Aircraft Jet Wake**

As part of a study being conducted to assess the feasibility of locating and sampling engine exhaust products in an aircraft wake, a rendezvous flight between the NASA Ames Research Center's CV-990 and the NASA Flight Research Center's F-104 was conducted. The flight experiment was performed at an altitude of approximately 11 kilometers at nominal flight speeds of Mach 0.8. The F-104 as it rendezvoused with the CV-990 is shown in Figure 5(a). Because of favorable weather conditions, a clearly visible contrail was produced by the source aircraft, as shown in Figure 5(b). This visible contrail was used to position the sensing aircraft (CV-990) in the F-104 wake. The separation
between the two aircraft was varied from approximately 2 to 10 kilometers during each of two encounters, as shown in figure 6(a). For the first encounter, the F-104 was flown without afterburning of the J79 engine. During the second encounter, after turnaround, the J79 afterburner was operated.

The CO, NO\textsubscript{x}, and O\textsubscript{3} data obtained during the rendezvous flight with instruments on board the CV-990 are shown in figures 6(b) to (d). The decrease in both CO and NO\textsubscript{x} levels as aircraft separation distance was increased shows the relative mixing and dispersion of the engine exhaust products with distance. The CO data clearly show the higher concentration of this species expected during afterburner operation. The maximum measured NO\textsubscript{x} during the afterburning encounter was approximately equal to that measured with no afterburning. This result would also be expected, since engine altitude emission tests have shown that afterburning has only a small effect on the NO\textsubscript{x} concentration (ref. 10). The CO data do not show any variations which can be related to the F-104 wake.

The oxides of nitrogen were measured with a chemiluminescence monitor with a sensitivity of 10 ppbv. Carbon monoxide was measured with a fluorescent NDIR instrument with a sensitivity of 200 ppbv. Ozone was measured with both an electrochemical total oxidant meter and an ultraviolet absorption monitor. The sensitivities of the ozone instruments were, respectively, 5 and 3 ppbv. From manufacturers' specifications, the response time of the CO monitor to 90 percent of a step input is about 30 seconds, while the response time of the NO\textsubscript{x} monitor to 90 percent of a step input is less than 2 minutes. However, because of the turbulence in the F-104 wake, the CV-990 was intermittently pitched in and out of the visible contrail, and the sampling instrumentation was subject to a constantly varying input signal. Because of this, the time response of the instruments can only be approximated. And since a correction is not required for interpretation of the data obtained in the wake, no correction has been applied to the data shown in figure 6.

The instrumentation used was on board the CV-990 for flight evaluation as part of the N\textsuperscript{ASA} Global Air Sampling Program (GASP). This program is discussed in reference 12. Calculated dilution and plume spreading for the J79 engine at Mach 0.8 with and without afterburning are shown in figure 4. In addition to the centerline dilution and half-value radius relations, the average dilution and the corresponding jet edge radius r\textsubscript{e} are shown. Also the half-wingspan of the F-104 and the CV-990 are indicated in figure 4(b) for comparison with the plume radius. The concentration of any nonreacting species in the plume at any desired downstream distance can be calculated from the dilution relation (fig. 4(a)) and known or estimated concentrations at the engine exhaust and in the ambient atmosphere.

For the separation distances between the CV-990 and F-104 during the flight experiment (fig. 6(a)), average NO\textsubscript{x} concentrations were calculated for engine exhaust concentrations of 20 and 40 ppmv without afterburning and 40 and 80 ppmv with afterburning.
The ambient NO\textsubscript{x} level was assumed to be 1 ppbv. Carbon monoxide average concentrations were calculated for engine exhaust concentrations of 100 and 300 ppmv without afterburning and 1000 and 3000 ppmv with afterburning. An ambient CO level of 50 ppbv was assumed. The engine exhaust was assumed to contain no ozone. A mean ambient O\textsubscript{3} level of 52 ppbv was assumed, based on experimental data obtained preceding and following the rendezvous flight experiment. The average concentrations of CO, NO\textsubscript{x}, and O\textsubscript{3} calculated for the conditions of the experiment are shown in figures 6(b) to (d). The separation distance - flight time relation has been used to present the calculated concentrations as a function of time for comparison with the measured emission levels in the plume. Because the CV-990 was traversing the plume in an unknown pattern, the gas sampling probe was exposed to a constantly varying input. Also, as discussed previously, the sampling instrumentation does not have instantaneous response. Consequently, concentrations of the order of the centerline concentrations were not expected and not observed. The good agreement between the measured emissions and the calculated average concentrations should not be interpreted as a calibration of the J79 tailpipe emissions, which were not known; nor should the interpretation be made that the sample probe yielded an integrated emission level as defined by equation (13). Rather, the comparison is intended to show that concentrations estimated by assuming that the exhaust diffuses as a round jet in a parallel flow with no chemical kinetics are of the same order of magnitude as the measured concentration levels.

**SUMMARY OF RESULTS**

The dispersion of the exhaust of turbojet engines into the atmosphere was estimated by using a model developed for the mixing of a round jet with a parallel flow. The analysis is appropriate for determining the spread and dilution of the engine exhaust from the engine exit until it is completely entrained in the aircraft trailing vortex pair. In this region, chemical reactions are not expected to be important and were not considered in the flow model.

The analysis is an integral momentum method for which the dimensionless local velocity difference profile shapes and the shear stress relation must be specified. The eddy viscosity model used in the shear stress relation was formulated by assuming that the eddy viscosity is directly proportional to the jet mass flow rate plus the absolute value of the entrainment mass flow rate and inversely proportional to the jet width.

Calculations of the dilution of the exhaust for large and small aircraft turbojet engines with and without afterburning at typical flight conditions are presented. These results indicate that a 1000:1 average dilution may be expected in less than 15 seconds after aircraft passage. This dilution corresponds to distances from 0.5 to 7 kilometers.
downstream of the engine exhaust for the engines and conditions examined. The large range depends on both engine size and power level.

Measurements of carbon monoxide, oxides of nitrogen, and ozone made in the wake of an F-104 aircraft flying at Mach 0.8 at an altitude of 11 kilometers at distances from 2 to 10 kilometers behind the F-104 are presented. Calculated concentrations for the J79 turbojet engine plume at conditions of the flight experiment are in good agreement with the measured pollutant levels.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 31, 1973,
APPENDIX - SYMBOLS

\begin{align*}
C_p & \quad \text{specific heat at constant pressure} \\
c & \quad \text{concentration} \\
h & \quad \text{enthalpy} \\
h^0 & \quad \text{total enthalpy} \\
M & \quad \text{Mach number} \\
m & \quad \text{molecular weight} \\
r & \quad \text{radius} \\
r_e & \quad \text{jet edge radius} \\
r_i & \quad \text{jet core radius} \\
r_j & \quad \text{jet exit radius} \\
r_5 & \quad \text{half-value radius, where } (u - u_\infty)/(u_c - u_\infty) = 0.5 \\
r_{(\rho u)_5} & \quad \text{radius where } (\rho u - (\rho u)_\infty)(\rho u)_c - (\rho u)_\infty = 0.5 \\
u_x & \quad \text{streamwise velocity} \\
u_1 & \quad u_j - u_\infty \\
x & \quad \text{streamwise distance} \\
\epsilon & \quad \text{eddy viscosity} \\
\lambda & \quad \ln 2 \\
p & \quad \text{gas density} \\
\tau & \quad \text{shear stress} \\
\text{Subscripts:} \\
a & \quad \text{average} \\
c & \quad \text{centerline} \\
j & \quad \text{jet exit} \\
f & \quad \text{free stream} \\
5 & \quad \text{evaluated at } r_5
\end{align*}
REFERENCES


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<td>(e)</td>
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<td>3.0</td>
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*Maximum afterburning.

*Maximum nonafterburning.


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<th>Estimated plume concentrations</th>
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<td>With afterburning</td>
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<td>1000:1 dilution, ppbv</td>
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<td>500 to 5000</td>
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<sup>a</sup>Parts per billion by volume.

<sup>b</sup>Parts per million by volume.

---

**Figure 3.** - Round jet in a parallel flow.
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<th>Power level</th>
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<td>Maximum afterburning</td>
</tr>
<tr>
<td>1.8</td>
<td>11.0</td>
<td>Maximum nonafterburning</td>
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</table>

**Figure 2.** J-85 exhaust plume dispersion.
Figure 3. - GE4 exhaust plume dispersion.
Figure 4. 179 exhaust plume dispersion for Mach 0.8 and 11-kilometer altitude.
Figure 5. Rendezvous flight experiment at Mach 0.8 and 11 kilometers altitude.

- Encounters 1 and 2: Separation distance between F-104 and CV-998.
  - Encounter 1: No afterburning.
  - Encounter 2: Afterburning.

- Concentration data:
  - Carbon monoxide: Free-stream concentration, 50 ppm.
  - Concentration, parts per billion by volume.
  - Calculated vs. Flight data.

- Ozone: Free-stream concentration, 32 ppm; Jet exit concentration, 8.

Figure 6. Measured and calculated concentration in F-104 wake. Altitude, 11 kilometers.
END
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