TWO-IMPULSE TRAJECTORY OPTIMIZATION FOR THE RAE-B ORBIT TRIM PROBLEM

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SUMMARY

This report documents the results of work accomplished by the authors on a task undertaken to suggest an appropriate approach to the solution of the optimum two-impulse transfer problem between orbits of specified inclination. The task included a literature search to identify the current state of the art and a definition of the suggested approach for the specific application of a lunar orbit trim. Some numerical data available at the start of the study prompted Mr. Pines to undertake some original research on the subject which was outside the scope of this task. Although the research study is not reported here, the applications of his results to the problem of interest is included as Appendix A of this report. The formulation for a computer program developed under this task following a more conventional approach is included as Appendix B.

I. The Problem

The first phase of this assignment was to conduct a literature search to discover what numerical and/or analytic techniques might be available for the solution of the following impulsive orbit transfer problem:

1. All orbital parameters are given for an initial elliptic orbit. The only assumption made on these parameters is that the eccentricity has an upper bound of .8.

2. The final orbit is to be circular, have specified inclination and radius specified to within ± 200 km.

The application is to a circular orbit about the moon at critical inclination. The initial orbit will be achieved by a retro-burn from the hyperbolic approach trajectory, and will be tracked long enough to establish its elements. One would hope that this initial orbit would be close to the desired final circular orbit, but techniques are desired for achieving the optimal insertion into the circular orbit whether or not this hope is realized.

The second phase of the assignment was to pursue analytic investigations suggested by the literature search.

II. Results of the Literature Search

The starting point for the search was a comprehensive survey by Gobetz and Doll[1] published in May 1969, which covers work on impulsive transfers carried out before mid-1968. An extensive bibliography (over 300 references) is included, and an outline of general results obtained is presented.

An effort was made to review all of Gobetz's references which appeared at all pertinent to the problem at hand. It was not possible to obtain copies of all such references; however, enough were reviewed to be reasonably sure that no general analytic conclusions are available in the literature for this problem - as, for example, the conclusion that if the orientation of one of the orbits

is left entirely unspecified, optimality of the transfer requires that this orbit be chosen coplanar and coaxial with the other orbit.

The papers actually examined from Gobetz's list of references are those numbered as follows:

9, 17, 19, 20, 35, 37, 54, 55,
56, 61, 104, 105, 106, 113, 123,
124, 125, 192, 193, 195, 213,
214, 216, 220, 221, 241, 251,
276, 277.

In addition, recent issues of Astronautica Acta, the AIAA Journal and the Journal of the Astronautical Sciences were scanned, and several additional references were examined [(2) - (8)]. Prior familiarity with the work of the groups at Huntsville (Hoelker et. al.), G. E. (Altman et. al.), North American (Bender, McCue et. al.), Stanford (Breakwell and his students) and the work of F. T. Sun made it unnecessary to look in detail at all of the papers by these authors listed by Gobetz.

It should be noted that my examination of these papers was primarily directed at looking at the results claimed by the authors in order to determine applicability of their work to the particular problem outlined above. No effort was made to follow all the analyses through in detail. Of all these papers, very few dealt with non-coplanar transfers and still fewer with transfers of more than two impulses. No useful analytic results were found. H. W. Small's program for optimal N-impulse transfers appears to have possibilities for our problem. This program is a culmination of the efforts of Breakwell and several of his students over some years. It is based on the indirect method, requires some preliminary numerical work, and requires guesses on the initial values of the adjoint variables. All trajectories generated are optimal and the iteration involved is to match the terminal conditions. Small has used the program for an

exhaustive study of coplanar transfers. Our problem is not, however, the problem of surveying all possible transfers, but rather of optimally matching certain terminal conditions. S. Pines has criticized procedures such as this on the ground that the numerical match obtained on the terminal conditions will not be adequate for practical implementation. Small has made considerable additional material (in the form of a very rough draft of his thesis) available. Some effort would be involved in understanding the procedure, if one wished to use his program on this problem. Its principal attraction appears to be that it would yield the global N-impulse optimum.

III. Theoretical Work

Construction of a direct theory for a two-impulse solution to the problem was started, but the time limitations did not permit its completion. The parameters tentatively selected were:

1. Argument of latitude of the departure point on the initial orbit.

2. A parameter α related to the radius of the circular orbit by

$$r = a + \rho \cos \alpha \qquad a \text{ given}$$
$$\rho = 200 \text{ km}$$

3. Longitude of the ascending node of the circular orbit.

4. Argument of latitude of the arrival point on the circular orbit.

These four parameters define the initial and final position vectors on the transfer orbit, and one or two transfer-orbit parameters are required as follows:

5. Any inplane parameter of the transfer orbit: semimajor axis, eccentricity, argument of latitude of one end or the other, magnitude of the angular momentum, or transit time.

6. If the initial and final position vectors of the transfer orbit are collinear, an out-of-plane parameter: either inclination or longitude of the

ascending node.

The formulation of the problem is such that any choice of parameters which define the geometry of the transfer is easy to implement. As noted earlier the theory is not complete; further detail does not seem warranted for reasons which follow immediately.

About midway through the study, the conjecture (partly based on numerical results of C. Uphoff) was put forward by S. Pines, that the optimal two-impulse solution to the open-time problem is a 180° transfer chosen such that the changes in angular momentum resulting from the two impulses are collinear. The proof of this will be published independently; however, the application of this solution to the problem at hand is outlined in Appendix A.

To permit the study of fixed transfer-time transfers, a two-impulse trajectory optimization program based on the indirect technique was constructed. A general iterator was incorporated in the program which also permits optimization by direct methods. Although the formulation for this program is sufficiently general to include open-time problems, it will not yield the 180° transfer (which appears to be the globally optimal solution) because such a transfer represents a singular solution for the formulation employed. The formulation is described in Appendix B.

IV. Conclusions

1. The general two-impulse trajectory optimization program is available for fixed transfer-time problems.

2. Pines' two-impulse 180° analytic solution is available for open-time problems.

3. For multi-impulse solutions, Small's program is a possibility. As noted above, considerable preliminary study is required. The detailed

material supplied by Small did not arrive in time to carry out such a study, nor were computer facilities available for familiarization with the program.

4. Because of the last comment, no assessment of the relative merits of Small's program compared to the first two items will be attempted.

V. References

(1) F. W. Gobetz and J. R. Doll, "A Survey of Impulsive Trajectories", AIAA Jour. 7, p. 801, May 1969.

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(3) T. L. Vincent and E. M. Cliff, "Maximum-Minimum Sufficiency and Lagrange Multipliers", AIAA Jour. 8, p. 171, January 1970.

(4) H. G. Moyer, "A Computer Survey of Impulsive Ellipse-Ellipse Transfer", AIAA Jour. 9, p. 321, February 1971.

(5) J. V. Breakwell and W. Heine, "Minimum Time-free Transfer between Neighboring Noncoplanar Elliptical Orbits with Major Axes Perpendicular to the Line of Nodes", AIAA Jour. 7, p. 1236, July 1969.

(6) H. G. Moyer, "Integrals for Impulsive Orbit Transfer from Noether's Theorem", AIAA Jour. 7, p. 1232, July 1969.

(7) D. J. Jezewski and H. I. Rozendaal, "An Efficient Method for Calculating Optimal Free-Space N-Impulse Trajectories", AIAA Jour. 6, p. 2160, November 1968.

(8) F. T. Sun, "Analysis of Optimum Two-Impulse Orbital Transfer Under Arbitrary Terminal Conditions", AIAA Jour. 6, p. 2145, November 1968.

APPENDIX A

OPTIMAL TWO-IMPULSE TRIM FOR RAE-B

INTRODUCTION

The equations for the optimal two-impulse trim for transferring from an elliptic conic with given a, e, i, Ω, ω, and t_p to a required circular orbit of given inclination are described herein. The final circular radius is designated by $\bar{r}$. A test is included for the condition where a nonoptimal three- or four-impulse trim may prove superior for conditions of large changes in the angular momentum vectors. The results are given without derivation.

NOTATION

i_o	initial inclination
i_f	required final inclination
H_o	initial angular momentum vector
$\hat{k}$	unit vector in the direction of the equatorial pole of the planet
h_o	magnitude of H_o
h_f	magnitude of the final circular angular momentum vector
$\hat{\Omega}_o$	unit vector in the direction of the initial ascending node
r_1^+	180° transfer radius on original conic
r_1^-	radius opposite r_1^+ on original conic
h_{Ti}	magnitude of the transfer angular momentum vector
e_o	eccentricity on initial orbit

DESCRIPTION OF THE SOLUTION

The optimal two-impulse solution is characterized by a 180° transfer. The direction of the two-impulses are anti-parallel. The initial, transfer, and final angular momentum vectors all lie in a plane and the two vector changes in angular momentum are collinear.

EQUATIONS DEFINING THE SOLUTION

Let the minimum possible change in inclination be Δi_M, then

$$\Delta i_M = i_f - i_o \tag{1}$$

The initial unit ascending-node vector is given by

$$\hat{\Omega}_o = \frac{\hat{k} \times \hat{H}_o}{\sin i_o} \tag{2}$$

where

$$\hat{H}_o = H_o / h_o \tag{2a}$$

The minimum possible change in angular momentum, corresponding to Δi_M, is given by rotation of $\hat{H}_o$ about the initial ascending node $\hat{\Omega}_o$ through the angle Δi_M.

$$H_{fM} = h_f \left[\left(\cos \Delta i_M + \frac{\sin \Delta i_M \cos i_o}{\sin i_o} \right) \hat{H}_o - \frac{\sin \Delta i_M}{\sin i_o} \hat{k} \right] \tag{3}$$

where

$$h_f = \sqrt{\mu \bar{r}}$$

Introducing the single-degree-of-freedom variable, λ, as the angle between the initial and final ascending nodes, we have

$$\hat{\Omega}_f = \cos\lambda\, \hat{\Omega}_o + \sin\lambda\, \hat{k} \times \hat{\Omega} \tag{4a}$$

The final angular momentum vector is given by a rotation of H_{fM} about $\hat{k}$ through λ.

$$H_f = \cos\lambda\, H_{fM} + (1 - \cos\lambda)\cos i_f\, \hat{k} + \sin\lambda\, \hat{k} \times H_{fM} \tag{4b}$$

The corresponding increment in inclination from the initial angular momentum vector to the final angular momentum vector is given by

$$\Delta i_f = \sin^{-1} \frac{|H_o \times H_f|}{h_o\, h_f} \tag{5}$$

The 180° transfer takes place along the unit position vector on the original conic and is given by

$$\hat{R} = \frac{H_o \times H_f}{h_o\, h_f \sin \Delta i_f} \tag{6}$$

The central angle between the original ascending node and $\hat{R}$ is given by

$$\alpha = \cos^{-1} (\hat{\Omega}_o \cdot \hat{R}) \tag{7}$$

The conic radial distance in the positive direction of $\hat{R}$ is given by

$$r_1^+ = \frac{h_o^2}{\mu[1 + e_o \cos(\alpha - \omega_o)]} \tag{8}$$

The radial distance along the negative direction of $\hat{R}$ is

$$r_1^- = \frac{h_o^2}{\mu[1 - e_o \cos(\alpha - \omega_o)]} \tag{9}$$

The first impulse takes place along the larger of the two radii. If $r_1^+ > r_1^-$, then the radius vector for the first impulse is

$$R_1 = r_1^+ \hat{R} \tag{10a}$$

and the second impulse takes place at the radius vector

$$R_2 = -\bar{r}\,\hat{R} \tag{10b}$$

If $r_1^- > r_1^+$, then

$$\left.\begin{aligned} R_1 &= -r_1^-\,\hat{R} \\ R_2 &= \bar{r}\,\hat{R} \end{aligned}\right\} \tag{11}$$

Figure 1 is convenient to describe the various angles and momenta magnitudes required for the solution.

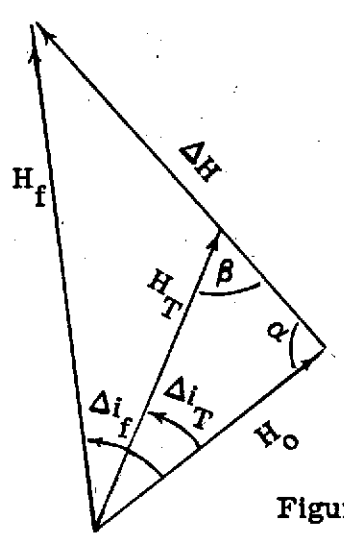

Figure 1

Let the vector change in angular momentum be ΔH

$$\Delta H = H_f - H_o \tag{12}$$

and its magnitude be

$$\delta h = \sqrt{h_f^2 - 2 h_f h_o \cos \Delta i_f + h_o^2} \tag{13}$$

From the law of sines, we have, for the angle between H_o and ΔH,

$$\sin \alpha = \frac{h_f}{h_o} \sin \Delta i_f \tag{14a}$$

and

$$\cos \alpha = \frac{h_o - h_f \cos \Delta i_f}{\delta h} \tag{14b}$$

The transfer angular momentum has a magnitude given by

$$h_T = \sqrt{\frac{2\mu(\bar{r}\, r_1^+)}{\bar{r} + r_1^+}} \qquad \text{if } r_1^+ > r_1^- \tag{15a}$$

or

$$h_T = \sqrt{\frac{2\mu(\bar{r}\, r_1^-}{\bar{r} + r_1^-}} \qquad \text{if } r_1^- > r_1^+ \tag{15b}$$

In either case, we have, for the angle between H_T and ΔH,

$$\sin \beta = \sin \alpha \frac{h_o}{h_T} \tag{16a}$$

$$\cos\beta = \text{sign}\,(h_T - h_o \cos\alpha)\sqrt{1-\sin^2\beta} \tag{16b}$$

The angle between $\hat{H}_o$ and H_T is

$$\Delta i_T = \pi - (\alpha+\beta)$$

The transfer orbit angular momentum vector is given by a rotation of $\hat{H}_o$ about the unit vector R_1 / r_1, where r_1 is the larger of r_1^+ and r_1^-, through the angle Δi_T.

$$H_T = h_T \left\{ -\cos(\alpha+\beta)\,\hat{H}_o + \frac{\sin(\alpha+\beta)}{r_1} R_1 \times \hat{H}_o \right\} \tag{17}$$

The magnitude of the change in angular momentum after the first impulse is given by

$$\delta h_1 = |H_T - H_o| = \frac{h_T}{\sin\alpha}\left[\sin(\alpha+\beta)\right] \tag{18}$$

Let the vector dot product of the position and velocity prior to the first impulse be

$$d_1^- = R_1 \cdot \dot{R}_1^- \tag{19}$$

Then, the vector dot product after the first impulse is given by

$$d_1^+ = \frac{d_1^-}{1 - \dfrac{\delta h_1}{(\delta h - \delta h_1)}\,\dfrac{\bar{r}}{r_1}} \tag{20}$$

B

The velocity vector after the first impulse is given by

$$\dot{R}_1^+ = \frac{H_T \times R_1}{r_1^2} + \frac{d_1^+ R_1}{r_1^2} \tag{21}$$

The velocity prior to the second impulse is given by

$$\dot{R}_2^- = \frac{H_T \times R_2}{\bar{r}^2} - \frac{d_1^+ R_2}{\bar{r}^2} \tag{22}$$

The final circular velocity vector after the second impulse is given by

$$\dot{R}_2^+ = \frac{H_T \times R_2}{h_f \bar{r}} \sqrt{\frac{\mu}{\bar{r}}} \tag{23}$$

We may now compute the sum of the two impulses and carry out the one-dimensional search for the λ which yields the minimum sum of the two impulses.

$$\delta V_1 = |\dot{R}_1^+ - \dot{R}_1^-| \tag{24}$$

$$\delta V_2 = |\dot{R}_2^+ - \dot{R}_2^-| \tag{25}$$

This is the optimal two-impulse trim unless the condition in the next section is violated.

TEST FOR MULTI-IMPULSE NONOPTIMAL TRIMS

More than two trims will be better than the two-trim program outlined herein if

$$\delta h > \left|\left(\sqrt{\frac{2}{1+e_o}} - 1\right)\right| h_o + \left(\sqrt{2} - 1\right) h_f \tag{26}$$

For such an event, there is no unique minimum and much better three- or four-impulse trims can be found for longer transfer times.

APPENDIX B

GENERAL FORMULATION FOR OPTIMAL 2-IMPULSE TRANSFER BETWEEN INCLINED, ELLIPTICAL ORBITS

Consider a spacecraft in an initial orbit with elements a_o, e_o, i_o, Ω_o, ω_o, and t_{po} which denote the semimajor axis, eccentricity, inclination, longitude of ascending node, argument of periapse and time of periapse passage, respectively. Suppose it is desired to transfer to a final orbit of specified semimajor axis a_f, eccentricity e_f and inclination i_f using two impulses, the magnitudes of which we denote v_o and v_f, so as to minimize the sum $\pi = v_o + v_f$. The motion is described by the vector equation

$$\ddot{R} = -\mu R/r^3$$

which represents free flight in an inverse-square force field where R is the position of the spacecraft, $r = |R|$, and μ is the gravitational constant. Clearly, the position at the time of the first impulse R_o and the velocity before adding the impulse $\dot{R}_o^-$ are functions only of the time at which the impulse is applied, i.e.,

$$R_o = R_o(t_o)$$

$$\dot{R}_o^- = \dot{R}_o^-(t_o)$$

and, thus

$$\dot{R}_o^+ = \dot{R}_o^-(t_o) + V_o$$

where the plus sign denotes the limit evaluated just after the impulse and V_o is the impulse velocity vector. With a_f, e_f and i_f specified, the position and velocity after insertion in the final orbit may be considered to be functions only of the final longitude of ascending node Ω_f and the argument of latitude β_f, i.e.,

$$R_f = R_f(\Omega_f, \beta_f)$$

$$\dot{R}_f^+ = \dot{R}_f^+(\Omega_f, \beta_f)$$

and therefore

$$\dot{R}_f^- = \dot{R}_f^+ (\Omega_f, \beta_f) - V_f$$

where V_f is the final velocity impulse.

Proceeding formally with the application of the Maximum Principle, one may write the adjoint equations for the problem which define the behavior of the primer vector Λ.

$$\ddot{\Lambda} = \frac{3\mu}{r^5} (R \cdot \Lambda) R - \frac{\mu}{r^3} \Lambda$$

This equation is known to possess the analytic solution

$$\begin{bmatrix} \Lambda(t) \\ \dot{\Lambda}(t) \end{bmatrix} = \Phi(t_o, t) \begin{bmatrix} \Lambda_o \\ \dot{\Lambda}_o \end{bmatrix}$$

where $\Phi(t_o, t)$ is the state transition matrix for the two-body transfer from time t_o to time t. The transversality conditions for the problem are obtained from the Maximum Principle by applying the general equation

$$d\pi + \left[\Lambda \cdot d\dot{R} - \dot{\Lambda} \cdot dR - h_V dt \right]_o^f = 0$$

where $h_V = \Lambda \cdot \ddot{R} - \dot{\Lambda} \cdot \dot{R}$ is the variational Hamiltonian which is a constant of the motion. Note that the differentials $d\dot{R}$ are evaluated at the terminals of the <u>transfer</u> trajectory; hence, $d\dot{R}_o^+$ and $d\dot{R}_f^-$ must be used. Forming the appropriate differential, one obtains

$$dR_o = \dot{R}_o^- dt_o$$

$$d\dot{R}_o^+ = \ddot{R}_o dt_o + dV_o$$

$$dR_f = (\bar{k} \times R_f)\, d\Omega_f + (\bar{h}_f \times R_f)\, d\beta_f$$

$$d\dot{R}_f^{-} = (\bar{k} \times \dot{R}_f^{+})\, d\Omega_f + (\bar{h}_f \times \dot{R}_f^{+})\, d\beta_f - dV_f$$

where $\bar{k}$ is a unit vector in the direction of the North pole of the central body and $\bar{h}_f$ is a unit vector in the direction of the angular momentum of the final orbit. Let each of the velocity impulses be defined in terms of a magnitude and two angles which uniquely define angular orientation. We arbitrarily select one angle, α, to be a rotation about the (initial or final) unit angular momentum vector $\bar{h}$ and the other, γ, to be a rotation about the unit vector $\bar{j}$ defined by $j = (V \times \bar{h})/|V \times \bar{h}|$. Then

$$dV_o = \frac{V_o}{v_o}\, dv_o + (\bar{h}_o \times V_o)\, d\alpha_o + (\bar{j}_o \times V_o)\, d\gamma_o$$

$$dV_f = \frac{V_f}{v_f}\, dv_f + (\bar{h}_f \times V_f)\, d\alpha_f + (\bar{j}_f \times V_f)\, d\gamma_f$$

Finally, writing

$$d\pi = \frac{\partial \pi}{\partial v_o}\, dv_o + \frac{\partial \pi}{\partial v_f}\, dv_f = dv_o + dv_f\,,$$

substituting into the general transversality condition, collecting coefficients of the remaining independent differentials and setting to zero, one obtains the following transversality conditions

(a) $\bar{k} \cdot (F + V_f \times \Lambda_f) = 0$

(b) $\bar{h}_f \cdot (F + V_f \times \Lambda_f) = 0$

(c) $1 - (\Lambda_f \cdot V_f)/v_f = 0$

(d) $\bar{h}_f \cdot (V_f \times \Lambda_f) = 0$

(e) $\bar{j}_f \cdot (V_f \times \Lambda_f) = 0$

(f) $1 - (\Lambda_o \cdot V_o)/v_o = 0$

(g) $-\bar{h}_o \cdot (V_o \times \Lambda_o) = 0$

(h) $-\bar{j}_o \cdot (V_o \times \Lambda_o) = 0$

(i) $-h_V = 0$

(j) $-\Lambda_o \cdot \ddot{R}_o + \dot{\Lambda}_o \cdot \dot{R}_o^{-} + h_V = 0$

where F is a vector constant of the motion defined as follows

$$F = \dot{R} \times \Lambda - R \times \dot{\Lambda}$$

The equations (d), (e), (g) and (h) along with the definition of $\bar{j}$ are sufficient to conclude that the optimal directions of V_o and V_f are collinear with Λ_o and Λ_f, respectively. Equations (c) and (f) indicate that the velocity increments must also have the same sense as the associated primer and that

$$|\Lambda_o| = |\Lambda_f| = 1.$$

Due to the collinearity of the velocity impulse and primer at the final time, equations (a) and (b) reduce to

(a') $\bar{k} \cdot F = 0$

(b') $\bar{h}_f \cdot F = 0$

which means that F will be aligned with the line of nodes of the final orbit. Equations (i) and (j) arise if final time and initial time, respectively, are left completely open. If, instead the flight time is fixed, then the two equations (i) and (j) are replaced in favor of the single equation represented by their sum, i.e.,

(j') $\dot{\Lambda}_o \cdot \dot{R}_o^- - \Lambda_o \cdot \ddot{R}_o = 0$

The above formulation may then be implemented in the following manner.

(1) Estimate the initial time, final time and the longitude of ascending node of the final orbit and the argument of latitude at insertion into the final orbit. Along with the specified orbital parameters, these are sufficient to define the state and time in the initial orbit at departure and in the final orbit at arrival.

(2) Solve Lambert's problem for the transfer trajectory between the two terminals. Simultaneously, evaluate the state transition matrix for the transfer.

(3) Evaluate the velocity impulses by differencing the velocities at the terminals of the transfer orbit and the corresponding velocities in the initial and final orbits, i.e.,

$$V_o = \dot{R}_o^+ - \dot{R}_o^- \quad ; \qquad V_f = \dot{R}_f^+ - \dot{R}_f^-$$

(4) Define the initial and final primer vectors

$$\Lambda_o = V_o/v_o \quad ; \qquad \Lambda_f = V_f/v_f$$

(5) Partition the state transition matrix into four 3 x 3 elements

$$\Phi = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

and solve for $\dot{\Lambda}_o$ and $\dot{\Lambda}_f$ as follows

$$\dot{\Lambda}_o = B^{-1}(\Lambda_f - A\Lambda_o)$$

$$\dot{\Lambda}_f = C\Lambda_o + D\dot{\Lambda}_o$$

and evaluate the constants of the motion F and h_V.

(6) Solve for the left-hand side of equations (a'), (b') and (i), (j) or (j') and Δt.

(7) Initiate an iteration of steps (1) - (6) to solve the boundary value problem in which t_o, t_f, Ω_f and β_f are the independent parameters and the parameters evaluated in step (6) are the dependent parameters.

A computer program which successfully implements the above steps has been developed and delivered to the Principal Investigator. Limited exercising of the program prior to delivery indicated that the approach exhibited strong convergence properties. A major limitation of the program is that it cannot handle a 180° transfer trajectory which appears to be the globally optimal solution for an open-time transfer. The open-time solution which the program yields is a locally optimum solution which, for the specific cases investigated, yielded a velocity impulse sum about 30-40 percent greater than the 180° transfer solution. Care should also be exercised in solving for fixed-time solutions with the program because the existence of multiple local solutions to such problems are common. The particular local solution which the program converges to is totally dependent upon the initial guess of the independent parameters. Therefore, if the existence of multiple solutions is suspected, one should input estimates of the independent parameters in the neighborhoods of each suspected solution to find the global solution. The alternative is to map solutions throughout the admissible ranges of the independent parameters.