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NET RADIATION METHOD FOR ENCLOSURE SYSTEMS INVOLVING PARTIALLY TRANSPARENT WALLS

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16. Abstract

The net radiation method is developed for analyzing radiation heat transfer in enclosure systems involving partially transparent walls. One such system is an enclosure with windows in it. The conventional net radiation method was developed by Hottel, Poljak, and others for enclosures having opaque walls. If a partially transparent wall is present, it will permit radiation to enter and leave the enclosure. The net radiation equations are developed here for gray and semigray enclosures with one or more windows. Another system of interest, such as in a flat plate solar collector, consists of a series of parallel partially transparent layers. The transmission characteristics of such window systems are obtained by the net radiation method, and the technique appears to be more convenient than the ray tracing method which has been used in the past. Relations are developed for windows consisting of any number of parallel layers having differing absorption coefficients and differing surface reflectivities, and for systems composed of parallel transmitting layers and opaque plates.

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SUMMARY

The net radiation method is developed for analyzing radiation heat transfer in enclosure systems involving partially transparent walls. One such system is an enclosure with windows in it. The conventional net radiation method has been well developed in the literature for enclosures with opaque walls. When one or more of the enclosure surfaces are transparent or partially transparent, radiation can directly enter or leave the enclosure. The enclosure equations are developed here to account for these effects. Specifically treated are gray and semigray enclosures. Another system of interest is a series of parallel partially transparent layers such as used in flat-plate solar collectors. The characteristics of such a system are obtained here by the net radiation method which appears to be more convenient and systematic than the ray tracing technique that has been used in the past. Overall transmission, absorption, and reflection relations are developed for windows consisting of any number of parallel layers with or without spaces between them, and that have differing absorption coefficients and differing surface reflectivities. Also considered are the characteristics of combinations of parallel transparent layers and opaque walls.

INTRODUCTION

The net radiation method has been well developed in the literature, such as in the textbooks (refs. 1 to 3), for calculating radiation heat transfer in enclosures. The walls of these enclosures have been opaque. The object of this report is to show how the net radiation method can be modified and applied for systems involving walls that are partially[1] transparent for radiant energy. Such systems would include enclosures with one

[1]The term "partially" transparent means that there can be absorption of some of the radiation traveling within the wall with the rest being transmitted through the entire thickness.

or more windows, or devices like flat plate solar collectors that consist of a series of parallel partially transparent plates with an opaque absorber surface behind them.

The transmission characteristics of windows in the form of single or multiple glass plates have been derived by Whillier (ref. 4) using ray tracing techniques. It will be shown here how these results can be developed by use of net radiation concepts that provide a convenient systematic approach.

SYMBOLS

A	surface area
a	absorptance for path within layer
F	radiation configuration factor
$F_{0-\lambda_c T}$	fractional emission of blackbody in wavelength interval 0 to λ_c
m	number of opaque surfaces in enclosure
N	total number of surface areas in enclosure
q	heat flux
r	surface reflectivity r_{nm} at m^{th} surface of nth layer
T	absolute temperature
α	overall absorptance of layer or system of layers
$\alpha_{at\ plate}$	absorptance at plate in presence of transparent plate system
β	angle measured away from normal of surface
ϵ	emittance of partially transparent layer; emissivity of opaque surface
λ	wavelength of radiation
λ_c	cut-off wavelength for partially transparent material
ρ	overall reflectance of transparent plate system
σ	Stefan-Boltzmann constant
τ	overall transmittance of transparent plate system

Subscripts:

e	externally incident
k	k^{th} surface area
i	incoming radiation

l loss to outside

o outgoing radiation

s for entire system

Superscripts:

i for radiation incident on inside surface

l long wavelength radiation $\lambda > \lambda_c$

(n) for system of n layers

o for radiation incident on outside surface

s short wavelength radiation $\lambda < \lambda_c$

ANALYSIS

Radiative Characteristics of a Single Layer

To illustrate the transmission, absorption, and reflection characteristics of a window for incident radiation, consider a single layer as shown in figure 1. The reflectivity

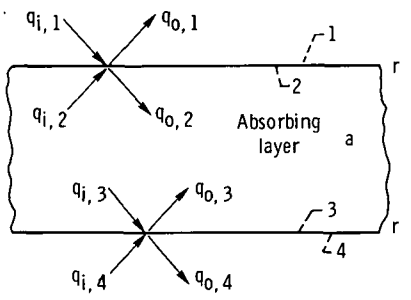

Figure 1. - Net radiation quantities for analyzing overall performance of partially transparent single layer.

at each of the interfaces is r and the absorptance of the layer is a. The relations between the outgoing and incoming energy fluxes at each interface can be expressed in terms of the reflection at the interface and the transmission across the interface. Using the net radiation concepts, the outgoing flux is written for each side of each interface to give

$$q_{o,1} = rq_{i,1} + (1 - r)q_{i,2} \tag{1a}$$

$$q_{o,2} = (1 - r)q_{i,1} + rq_{i,2} \tag{1b}$$

$$q_{o,3} = rq_{i,3} + (1 - r)q_{i,4} \tag{1c}$$

$$q_{o,4} = (1 - r)q_{i,3} + rq_{i,4} \tag{1d}$$

The internal q_i's and q_o's are related by using the absorptance of the layer to give

$$q_{i,2} = (1 - a)q_{o,3} \tag{2a}$$

$$q_{i,3} = (1 - a)q_{o,2} \tag{2b}$$

The overall radiative behavior can be studied by considering energy incident on only one side of the layer, and superposition used when radiation is incident on both sides. Since it is the fractions of incident radiation transmitted, absorbed, and reflected, that are desired, the incident flux can be set equal to unity without any loss of generality. Then any q in the calculation will be a fraction of the incident flux. Hence, using $q_{i,1} = 1$ and $q_{i,4} = 0$ along with equations (2), the q_i's are eliminated from equations (1) to yield

$$q_{o,1} = r + (1 - r)(1 - a)q_{o,3} \tag{3a}$$

$$q_{o,2} = 1 - r + r(1 - a)q_{o,3} \tag{3b}$$

$$q_{o,3} = r(1 - a)q_{o,2} \tag{3c}$$

$$q_{o,4} = (1 - r)(1 - a)q_{o,2} \tag{3d}$$

These are solved simultaneously to yield the reflectance ρ and transmittance τ of the entire layer as

$$\rho = q_{o,1} = \frac{r\left[1 + (1 - 2r)(1 - a)^2\right]}{1 - r^2(1 - a)^2} \tag{4a}$$

$$\tau = q_{o,4} = \frac{(1 - r)^2(1 - a)}{1 - r^2(1 - a)^2} \tag{4b}$$

The absorptance is then obtained from the overall heat balance

$$\alpha = 1 - \rho - \tau = \frac{a(1 - r)}{1 - r(1 - a)} \tag{4c}$$

Equations (4) can be directly applied when the incident energy is from a single direction. The r would be evaluated for that incidence angle and the a would be based on the path length traversed within the medium (allow for change in direction as radiation crosses interface of layer). If the incident flux is diffuse, the average ρ is obtained by averaging over all angles as (ref. 1)

$$\rho = 2 \int_{\beta=0}^{\pi/2} \rho(\beta) \cos \beta \sin \beta \, d\beta \tag{5}$$

where β is the incidence angle measured away from the surface normal. The $\rho(\beta)$ is given by equation (4a) with the r and a inserted as a function of β. The τ from equation (4b) can be averaged in the same manner and the average α then found as $1 - \rho - \tau$.

In the enclosure theory that follows, the ρ and τ for a layer refer to expressions such as equations (4a) and (4b), that is, relations that provide the overall performance of a window for incident energy. If the window is hot enough to radiate appreciably, the emittance of the window is also needed in the enclosure heat balances. According to Kirchhoff's law, the ϵ can be found by evaluating α for incident radiation having the same spectral distribution as blackbody emission at the temperature of the window. This assumes that the window configuration is such that a uniform temperature can be assigned to its entire volume.

Enclosure Theory Equations for Gray Surfaces and Windows

The net radiation method in the literature will now be generalized to include enclosure boundaries that are partially transparent to radiation. The usual restrictions of the enclosure theory apply which include diffuse emission and reflection, uniform fluxes over each surface area and a uniform temperature over each area. A window such as a smooth glass plate will reflect primarily in a specular fashion so that the assumption of diffuse reflection is violated. However, within an enclosure there are extensive multiple reflections and the directionality of each reflection loses its importance in contributing to the heat fluxes on the boundaries. Hence, the assumption of diffuse reflections is

usually satisfactory. The enclosure heat balances include possible emission from each window. The formulation here is only general enough to account for a window that can be characterized by a single temperature throughout its thickness. The formulation would require extension to account for a multilayered window with each layer at a different temperature. No extension is necessary, however, if the multilayered windows are kept cool by convection for example, so that emission from the layers is small relative to the transmission through them.

A typical enclosure containing one or more partially transparent windows is shown in figure 2. In general, let there be a total of N surface areas with the kth area being a typical area. There are m opaque surfaces and N - m partially transparent surfaces.

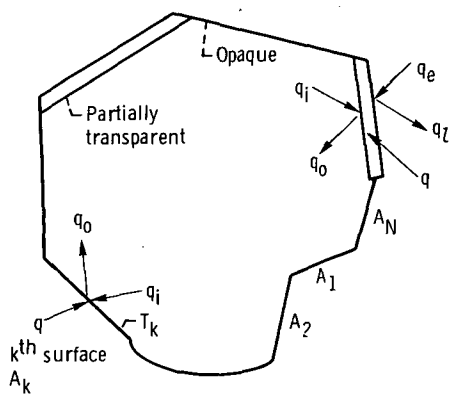

Figure 2. - Enclosure with N surface areas, some being partially transparent.

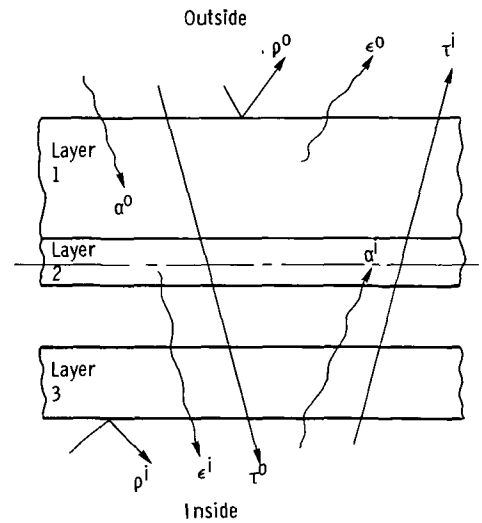

Figure 3. - Typical nonsymmetric window.

Each window can be nonsymmetric about its center plane as shown in figure 3. The superscripts o and i refer to the outside and inside surfaces. The α^o and ρ^o are the absorbed and reflected portions of the radiation incident from the outside. The ϵ^o is the emittance toward the outside and the τ^o is the fraction of the outside radiation that is transmitted all the way through the window. A similar notation is used for the quantities at the inside surface (it will be found later that, for composite windows, $\tau^o = \tau^i$).

For the m surface areas that are opaque the conventional net radiation equations can be written as:

Overall heat balance:

$$q_k = q_{o,k} - q_{i,k} \qquad (1 \le k \le m) \tag{6a}$$

Outgoing energy consists of emitted and reflected portions:

$$q_{o,k} = \epsilon_k \sigma T_k^4 + (1 - \epsilon_k) q_{i,k} \qquad (1 \le k \le m) \tag{6b}$$

Incoming energy in terms of outgoing fluxes and view factors:

$$A_k q_{i,k} = \sum_{j=1}^{N} q_{o,j} A_j F_{j-k} \qquad (1 \le k \le m)$$

$$q_{i,k} = \sum_{j=1}^{N} q_{o,j} F_{k-j} \qquad (1 \le k \le m) \tag{6c}$$

These equations can be combined in various ways to provide other interrelations between the variables that are sometimes useful. Combine equations (6a) and (6c) to relate q and q_o's,

$$q_k = q_{o,k} - \sum_{j=1}^{N} q_{o,j} F_{k-j} \qquad (1 \le k \le m) \tag{7a}$$

Combine equations (6a) and (6b) to relate q, T and q_o

$$q_k = \frac{\epsilon_k}{1 - \epsilon_k} \left(\sigma T_k^4 - q_{o,k}\right) \qquad (1 \le k \le m) \tag{7b}$$

Combine equations (6b) and (6c) to relate the q_o's and T,

$$q_{o,k} - (1 - \epsilon_k) \sum_{j=1}^{N} q_{o,j} F_{k-j} = \epsilon_k \sigma T_k^4 \qquad (1 \le k \le m) \tag{7c}$$

Now consider a partially transparent surface. Using the quantities in figure 2 an overall heat balance yields

$$q_k = q_{o,k} - q_{i,k} + q_{l,k} - q_{e,k} \tag{8a}$$

The q_k could be supplied for example by heating wires within the semitransparent material or by convection on the outside surface. For cooling the q_k is negative. The radiative flux leaving the inside surface of the window consists of emitted energy, reflected incoming energy, and transmitted externally incident flux,

$$q_{o,k} = \epsilon_k^i \sigma T_k^4 + \rho_k^i q_{i,k} + \tau_k^o q_{e,k} \tag{8b}$$

Similarly the flux lost from the outside surface of the window is given by

$$q_{l,k} = \epsilon_k^o \sigma T_k^4 + \rho_k^o q_{e,k} + \tau_k^i q_{i,k} \tag{8c}$$

The incoming flux $q_{i,k}$ is given by the same relation as equation (6c)

$$q_{i,k} = \sum_{j=1}^{N} q_{o,j} F_{k-j} \tag{8d}$$

Equations (8) are valid for $m + 1 \leq k \leq N$.

Equations (8a) and (8c) are combined to eliminate q_l to give

$$q_k - \epsilon_k^o \sigma T_k^4 + q_{e,k}\left(1 - \rho_k^o\right) = q_{o,k} - q_{i,k}\left(1 - \tau_k^i\right) \tag{9}$$

Some additional relations are found by combining some of the previous equations. Combine equations (8d) and (9) to yield

$$q_k - \epsilon_k^o \sigma T_k^4 + q_{e,k}\left(1 - \rho_k^o\right) = q_{o,k} - \left(1 - \tau_k^i\right)\sum_{j=1}^{N} q_{o,j} F_{k-j} \qquad (m + 1 \leq k \leq N) \tag{10}$$

Combine equations (8b) and (9) to yield

$$q_k = \frac{1}{\rho_k^i}\left\{\left[\epsilon_k^o \rho_k^i + \left(1 - \tau_k^i\right)\epsilon_k^i\right]\sigma T_k^4 - \epsilon_k^i q_{o,k} - \left[\rho_k^i\left(1 - \rho_k^o\right) - \tau_k^o\left(1 - \tau_k^i\right)\right]q_{e,k}\right\} \tag{11}$$

where the relation $1 - \tau_k^i - \rho_k^i = \alpha_k^i = \epsilon_k^i$ has been used for gray surfaces. Combine equations (8b) and (8d) to yield

$$q_{o,k} - \rho_k^i \sum_{j=1}^{N} q_{o,j} F_{k-j} = \epsilon_k^i \sigma T_k^4 + \tau_k^o q_{e,k} \tag{12}$$

The previous relations relate the q_k, T_k, $q_{e,k}$, and $q_{o,k}$ for each of the enclosure surfaces. By writing these equations for each surface there are sufficient equations to solve for the unknown values. For an enclosure with opaque walls this is discussed in detail in references 1 to 3. One convenient approach is to eliminate the $q_{o,k}$'s thereby obtaining a set of equations relating the surface temperatures T_k's, the surface heat flux additions q_k's, and the externally incident heat fluxes q_e's.

As a first step the $q_{o,k}$ is obtained as a function of T_k and q_k from equations (7b) and (11)

$$q_{o,k} = \sigma T_k^4 - \frac{1 - \epsilon_k}{\epsilon_k} q_k \qquad (1 \le k \le m) \tag{13a}$$

$$q_{o,k} = \frac{1}{\epsilon_k^i} \left\{ \left[\epsilon_k^o \rho_k^i + \left(1 - \tau_k^i\right) \epsilon_k^i \right] \sigma T_k^4 - \rho_k^i q_k - \left[\rho_k^i \left(1 - \rho_k^o\right) - \tau_k^o \left(1 - \tau_k^i\right) \right] q_{e,k} \right\} \qquad (m + 1 \le k \le N) \tag{13b}$$

Equations (13) are then inserted into equation (7a) and into equation (10) to eliminate the $q_{o,k}$'s. The summation $\sum_{j=1}^{N}$ has to be made in two parts and equations (13a) and (13b) used in their appropriate ranges. After simplification this yields for the opaque surfaces,

$$\sum_{j=1}^{m}\left(\frac{\delta_{jk}}{\epsilon_j} - \frac{1-\epsilon_j}{\epsilon_j}F_{k-j}\right)q_j - \sum_{j=m+1}^{N}\frac{\rho_j^i}{\epsilon_j^i}F_{k-j}q_j = \sum_{j=1}^{m}\left(\delta_{jk} - F_{k-j}\right)\sigma T_j^4$$

$$- \sum_{j=m+1}^{N}\frac{\epsilon_j^o\rho_j^i + \left(1-\tau_j^i\right)\epsilon_j^i}{\epsilon_j^i}F_{k-j}\sigma T_j^4$$

$$+ \sum_{j=m+1}^{N}\frac{\rho_j^i\left(1-\rho_j^o\right) - \tau_j^o\left(1-\tau_j^i\right)}{\epsilon_j^i}F_{k-j}q_{e,j}$$

$$(1 \le k \le m) \tag{14a}$$

and for the partially transparent surfaces;

$$-\sum_{j=1}^{m}\frac{1-\epsilon_j}{\epsilon_j}F_{k-j}q_j + \sum_{j=m+1}^{N}\left(\delta_{jk} - \rho_j^i F_{k-j}\right)\frac{q_j}{\epsilon_j^i} = -\sum_{j=1}^{m}F_{k-j}\sigma T_j^4$$

$$+ \sum_{j=m+1}^{N}\left\{\frac{\left(\epsilon_j^o + \epsilon_j^i\right)\delta_{jk} - \left[\epsilon_j^o\rho_j^i + \left(1-\tau_j^i\right)\epsilon_j^i\right]F_{k-j}}{\epsilon_j^i}\right\}\sigma T_j^4 \tag{14b}$$

$$- \sum_{j=m+1}^{N}\left\{\frac{\epsilon_j^o\delta_{jk} - \left[\rho_j^i\left(1-\rho_j^o\right) - \tau_j^o\left(1-\tau_j^i\right)\right]F_{k-j}}{\epsilon_j^i}\right\}q_{e,j} \qquad (m+1 \le k \le N)$$

Writing equation (14) for each k yields the final set of N simultaneous equations relating the N surface temperatures, the N surface heat flux additions, and the $N - m$ external fluxes on the windows. If $2N - m$ of these quantities are specified, the equations can be solved for the remaining unknowns.

The enclosure theory formulation contains within it the transmittance, reflectance, and emittance of each window. If a window is kept cool by convection so it does not emit

appreciably, then the window emission is not of significance in the analysis. The window can then be multilayered as the overall transmittance and reflectance can be obtained for such a system as will be shown in subsequent sections. If a window is hot as a result of absorbing radiation or being convectively or internally heated and is emitting significantly, then the emission term involves the window temperature which is specified as uniform. This would restrict the formulation to simple windows for which a uniform temperature can be assigned. The net radiation method can be applied for more complex situations than those discussed here, including enclosures with multilayered windows, with each layer emitting and at a different temperature.

Semigray (Two Band) Method for Enclosure Analysis

The transmission of most window materials is very wavelength dependent having high transmittance in the visible and a portion of the near infrared and low transmittance at longer wavelengths. There is often a rather sharp transition between the high and low transmission regions. The cut-off wavelength at which this transition occurs can be typically at about 2.5 micrometers for ordinary window glass or 25 micrometers for a sodium chloride crystal.

In the semigray method (ref. 5) or two-band approximation the property variations are accounted for by using two sets of property values, one uniform value for wavelengths shorter than the cut-off and a second value for wavelengths longer than the cut-off. The notation ϵ^s, ρ^s, and so forth, will be used for the short wavelength region $0 < \lambda < \lambda_c$ and ϵ^l, ρ^l, and so forth, will be used for the long wavelength region $\lambda_c < \lambda < \infty$. A common application of the semigray method is in dealing with devices exposed to solar energy. The incident solar energy is predominantly at short wavelengths characteristic of visible radiation, while the reradiation is at long wavelengths characteristic of the device surface temperature. In the present discussion it will be assumed that the externally incident fluxes and their spectral distributions are known; hence, q_e^s and q_e^l are known.

If all the surface temperatures are known, then equations (14a) and (14b) can be solved after small modifications. Two equations are written for each surface, one for $\lambda < \lambda_c$ and the second for $\lambda > \lambda_c$. In each of these equations the σT^4 is replaced by the blackbody emission at T within the respective wavelength intervals $0 - \lambda_c$ and $\lambda_c - \infty$. These quantities are given by $\sigma T^4 F_{0-\lambda_c T}$ and $\sigma T^4(1 - F_{0-\lambda_c T})$ where $F_{0-\lambda_c T}$ is the fractional emission of a blackbody in the range $0 - \lambda_c$ for a blackbody at T; the function $F_{0-\lambda T}$ depends only on the product λT.

Since all quantities on the right of each equation are given, this yields $2N$ equations that can be solved directly for the unknown N values of q_k^s and N values of q_k^l.

Then the heat flux being supplied to each of the k surfaces or windows by means other than radiation is

$$q_k = q_k^s + q_k^l \tag{15}$$

The set of equations for two wavelength bands is given in the appendix.

A complication arises in the simultaneous solution when it is the equilibrium temperature of a surface that is to be calculated when the q_k for that surface is specified. In that instance the q_k^s and q_k^l are not known individually, and it is only known that their sum is q_k. This introduces an additional unknown in the simultaneous solution since the T_k must be found with the separate values of q_k^s and q_k^l still being involved in the equations as unknowns as it is only their sum that is known. The equations given in the appendix still apply. There are equations (14a) and (14b) written for each λ range which gives 2N equations, and equation (15) written for each of the N surfaces. In general, let there be p surfaces that have q_k specified; hence N - p surfaces have T_k specified. The total of 3N simultaneous equations can then be solved for the following unknowns: p values of T_k, N - p values of q_k, N values of q_k^s and N values of q_k^l. These equations contain, however, the term $\sigma T^4 F_{0-\lambda_c T}$ and the temperature is not contained in $F_{0-\lambda_c T}$ in a simple way. Hence, an iterative procedure is required to determine the unknown T values.

If the property variations with wavelength are such that many more than two wavelength regions are required to adequately account for the property variations, then the net radiation equations can be written in spectral form and the fluxes then integrated over wavelength, or a summation made over a finite number of wavelength bands. This is an extension of the two band procedure and has been described in textbooks such as reference 1 (chapter 10); hence, the procedure will not be further detailed here.

Transmission Relations for Various Types of Windows

Single layer with different surface reflectivities. - A geometry that provides a useful building block to obtain results for more complex windows is a single layer with a different reflectivity at each surface. A dual subscript notation will be used for the surface reflectivities: For a group of several parallel layers, the first subscript designates the layer number and the second designates the upper or lower surface of that layer (the layers will be drawn horizontally). Thus, r_{k2} would refer to the second or lower surface of the kth layer. Referring to figure 4 the net radiation equations for

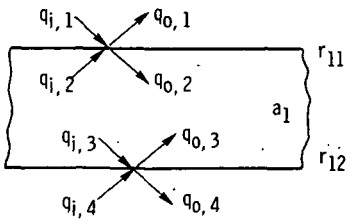

Figure 4. - Single absorbing layer with different reflectivity at each surface.

the first of several possible layers are written as

$$q_{o,1} = r_{11}q_{i,1} + (1 - r_{11})q_{i,2} \tag{16a}$$

$$q_{o,2} = (1 - r_{11})q_{i,1} + r_{11}q_{i,2} \tag{16b}$$

$$q_{o,3} = r_{12}q_{i,3} + (1 - r_{12})q_{i,4} \tag{16c}$$

$$q_{o,4} = (1 - r_{12})q_{i,3} + r_{12}q_{i,4} \tag{16d}$$

The absorptance of the medium is used to relate the internal q_i's and q_o's to yield

$$q_{i,2} = (1 - a_1)q_{o,3} \tag{17a}$$

$$q_{i,3} = (1 - a_1)q_{o,2} \tag{17b}$$

To consider the interaction of the layer with radiation incident on the upper surface, let $q_{i,1} = 1$ and $q_{i,4} = 0$. Using these relations and equations (17) to eliminate the q_i's from equations (16) give

$$q_{o,1} = r_{11} + (1 - r_{11})(1 - a_1)q_{o,3} \tag{18a}$$

$$q_{o,2} = (1 - r_{11}) + r_{11}(1 - a_1)q_{o,3} \tag{18b}$$

$$q_{o,3} = r_{12}(1 - a_1)q_{o,2} \tag{18c}$$

$$q_{o,4} = (1 - r_{12})(1 - a_1)q_{o,2} \tag{18d}$$

The solution yields the overall transmittance and reflectance for radiation incident on the upper surface as

$$\tau_{11} = q_{o,4} = \frac{(1 - r_{11})(1 - r_{12})(1 - a_1)}{1 - r_{11}r_{12}(1 - a_1)^2} \tag{19a}$$

$$\rho_{11} = q_{o,1} = \frac{r_{11} + r_{12}(1 - 2r_{11})(1 - a_1)^2}{1 - r_{11}r_{12}(1 - a_1)^2} \tag{19b}$$

The subscript 11 indicates that the τ_{11} and ρ_{11} are for radiation incident on the first surface of the first of several possible layers. The overall absorptance for the layer is obtained from the energy balance

$$\alpha_{11} = 1 - \rho_{11} - \tau_{11}$$

Substituting the previous expressions for τ_{11} and ρ_{11}, this simplifies to

$$\alpha_{11} = \frac{a_1(1 - r_{11})\left[1 + r_{12}(1 - a_1)\right]}{1 - r_{11}r_{12}(1 - a_1)^2} \tag{19c}$$

If radiation is now considered incident from below (that is on surface 12), the only change in the analysis is that r_{11} and r_{12} are interchanged. The overall transmittance, reflectance, and absorptance are designated by τ_{12}, ρ_{12}, α_{12} and they are given by

$$\tau_{12} = \frac{(1 - r_{12})(1 - r_{11})(1 - a_1)}{1 - r_{12}r_{11}(1 - a_1)^2} \tag{20a}$$

$$\rho_{12} = \frac{r_{12} + r_{11}(1 - 2r_{12})(1 - a_1)^2}{1 - r_{12}r_{11}(1 - a_1)^2} \tag{20b}$$

$$\alpha_{12} = \frac{a_1(1 - r_{12})\left[1 + r_{11}(1 - a_1)\right]}{1 - r_{12}r_{11}(1 - a_1)^2} \tag{20c}$$

This yields the relation $\tau_{11} = \tau_{12}$ so that the second subscript for τ can be omitted and the overall transmittance of the layer called τ_1. The emissivity of the layer can be found according to Kirchhoff's law by evaluating α using window properties for incident radiation that has the same spectral distribution as blackbody radiation at the window temperature.

Two differing layers with all reflectivities different. - The behavior for a single layer can be used to obtain the performance for two layers as shown in figure 5. The

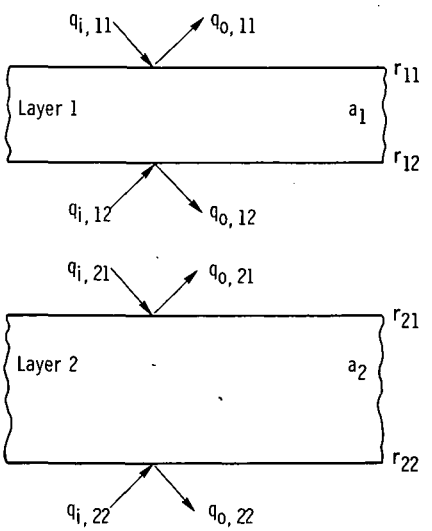

Figure 5. - Two differing absorbing layers with all surface reflectivities different.

q_o's are written in terms of the q_i's using the overall characteristics of single layers,

$$q_{o,11} = q_{i,11}\rho_{11} + q_{i,12}\tau_1 \tag{21a}$$

$$q_{o,12} = q_{i,12}\rho_{12} + q_{i,11}\tau_1 \tag{21b}$$

$$q_{o,21} = q_{i,21}\rho_{21} + q_{i,22}\tau_2 \tag{21c}$$

$$q_{o,22} = q_{i,22}\rho_{22} + q_{i,21}\tau_2 \tag{21d}$$

To obtain the overall characteristics of the two layer system for radiation incident from above, let $q_{i,11} = 1$ and $q_{i,22} = 0$. Also, from figure 5

$$q_{i,12} = q_{o,21}$$

and

$$q_{i,21} = q_{o,12}$$

These relations are used to eliminate the q_i's from equations (21) to yield

$$q_{o,11} = \rho_{11} + q_{o,21}\tau_1 \tag{22a}$$

$$q_{o,12} = q_{o,21}\rho_{12} + \tau_1 \tag{22b}$$

$$q_{o,21} = q_{o,12}\rho_{21} \tag{22c}$$

$$q_{o,22} = q_{o,12}\tau_2 \tag{22d}$$

Now introduce the notation τ_{s1}, τ_{s2} and ρ_{s1}, ρ_{s2}. The subscript s refers to the entire system. Hence, ρ_{s1} and ρ_{s2} are, respectively, the reflectances for radiation incident on the first and second surfaces of the entire system. The simultaneous equations (22) are solved to yield

$$\tau_{s1} = q_{o,22} = \frac{\tau_1\tau_2}{1 - \rho_{12}\rho_{21}} \tag{23a}$$

$$\rho_{s1} = q_{o,11} = \rho_{11} + \tau_1^2 \frac{\rho_{21}}{1 - \rho_{12}\rho_{21}} \tag{23b}$$

$$\alpha_{s1} = 1 - \tau_{s1} - \rho_{s1} = 1 - \rho_{11} - \frac{\tau_1}{1 - \rho_{12}\rho_{21}}(\tau_2 + \tau_1\rho_{21}) \tag{23c}$$

By changing subscripts to reverse the order in which the radiation passes through the layer surfaces, the overall system characteristics are obtained for radiation incident from below on the two layer system,

$$\tau_{s2} = \frac{\tau_2\tau_1}{1 - \rho_{21}\rho_{12}} \tag{24a}$$

$$\rho_{s2} = \rho_{22} + \frac{\tau_2^2 \rho_{12}}{1 - \rho_{21}\rho_{12}} \tag{24b}$$

$$\alpha_{s2} = 1 - \rho_{22} - \frac{\tau_2}{1 - \rho_{21}\rho_{12}} (\tau_1 + \tau_2\rho_{12}) \tag{24c}$$

A comparison of (23a) and (24a) reveals that $\tau_{s1} = \tau_{s2}$ so that the system transmittance can be called simply τ_s.

Two layers with no space between them. - Two adjacent layers with differing properties are shown in figure 6. The reflectivity at the common interface between the two

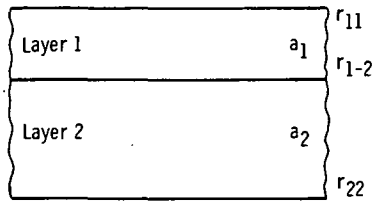

Figure 6. - Two differing adjacent absorbing layers.

media is called r_{1-2}. The configuration in figure 5 will correspond to this case if we let $r_{12} = r_{1-2}$ and $r_{21} = 0$ (or using $r_{21} = r_{1-2}$ and $r_{12} = 0$ will yield the same final result). The overall transmittance of the system can be computed by using equation (24a)

$$\tau_s = \frac{\tau_1\tau_2}{1 - \rho_{21}\rho_{12}} \tag{25}$$

where from equations (19) and (20),

$$\tau_1 = \frac{(1 - r_{11})(1 - r_{1-2})(1 - a_1)}{1 - r_{11}r_{1-2}(1 - a_1)^2} \tag{26a}$$

$$\tau_2 = (1 - r_{22})(1 - a_2) \tag{26b}$$

$$\rho_{12} = \frac{r_{1-2} + r_{11}(1 - 2r_{1-2})(1 - a_1)^2}{1 - r_{11}r_{1-2}(1 - a_1)^2} \tag{26c}$$

$$\rho_{21} = r_{22}(1 - a_2)^2 \tag{26d}$$

The expressions in equations (26) can also be used in equations (23b and c) and (24b and c) to compute ρ_{s1}, α_{s1}, ρ_{s2}, and α_{s2} for the system of two immediately adjacent layers.

A general system of N layers. - The overall characteristics of one layer were used to build up the overall behavior of a two layer system. This procedure can be continued and the behavior for one layer and two layers can be combined to yield the behavior for three layers; two layers and two layers can be combined to yield the behavior of four layers; etc. To accomplish this it is desired to have relations for determining the performance of a combined system of n layers and k layers, where $n + k = N$.

Referring to figure 7 let $\tau_s^{(n+k)}$ be the system transmittance for the entire system and let $\tau_s^{(n)}$, $\tau_s^{(k)}$ be the transmittances for n and k plate systems, respectively. The quantity $\rho_{s2}^{(n)}$ is the reflectance for a system of n layers by itself for energy incident on the second (lower) surface of that entire n layer system. Similarly, $\rho_{s1}^{(k)}$

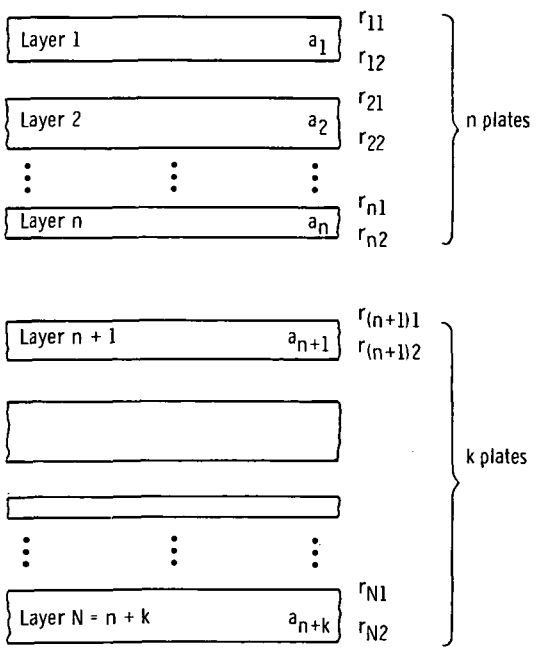

Figure 7. - System of N = n + k plates.

is the overall reflectance for a system of k layers by itself for energy incident on the first (upper) surface of that entire system.

Comparing figures 7 and 5 indicates that the results for the two layer system can be applied if we use the overall properties of the separate n and k plate systems in figure 7 to replace, respectively, the properties of the first and second single plates in figure 5. Then from equations (23a), (23b), and (24b),

$$\tau_s^{(n+k)} = \frac{\tau_s^{(n)}\tau_s^{(k)}}{1 - \rho_{s2}^{(n)}\rho_{s1}^{(k)}} \tag{27a}$$

$$\rho_{s1}^{(n+k)} = \rho_{s1}^{(n)} + \tau_s^{(n)^2} \frac{\rho_{s1}^{(k)}}{1 - \rho_{s2}^{(n)}\rho_{s1}^{(k)}} \tag{27b}$$

$$\rho_{s2}^{(n+k)} = \rho_{s2}^{(k)} + \tau_s^{(k)^2} \frac{\rho_{s2}^{(n)}}{1 - \rho_{s1}^{(k)}\rho_{s2}^{(n)}} \tag{27c}$$

If the spacing between two plates is zero, say the mth and $m + 1^{st}$ layers are in perfect contact, then let $r_{m2} = r_{m-(m+1)}$ and let $r_{(m+1)1} = 0$.

As an example, compute τ_s for a three layer system. Letting $n = 2$ and $k = 1$ in equation (27a) gives

$$\tau_s^{(3)} = \frac{\tau_s^{(2)}\tau_s^{(1)}}{1 - \rho_{s2}^{(2)}\rho_{s1}^{(1)}}$$

where

$$\tau_s^{(2)} = \frac{\tau_1\tau_2}{1 - \rho_{12}\rho_{21}}$$

$$\tau_s^{(1)} = \tau_3$$

$$\rho_{s2}^{(2)} = \rho_{22} + \frac{\tau_2^2\rho_{12}}{1 - \rho_{21}\rho_{12}}$$

$$\rho_{s1}^{(1)} = \rho_{31}$$

Then

$$\tau_s^{(3)} = \frac{\tau_1\tau_2\tau_3}{(1 - \rho_{12}\rho_{21})\left(1 - \rho_{22}\rho_{31} - \frac{\tau_2^2\rho_{12}\rho_{31}}{1 - \rho_{21}\rho_{12}}\right)} = \frac{\tau_1\tau_2\tau_3}{(1 - \rho_{12}\rho_{21})(1 - \rho_{22}\rho_{31}) - \tau_2^2\rho_{12}\rho_{31}} \tag{28}$$

where the required characteristics for single layers are given by equations (19) and (20) as

$$\left.\begin{aligned}
\tau_m &= \frac{(1 - r_{m1})(1 - r_{m2})(1 - a_m)}{1 - r_{m1}r_{m2}(1 - a_m)^2} \\
\rho_{m1} &= \frac{r_{m1} + r_{m2}(1 - 2r_{m1})(1 - a_m)^2}{1 - r_{m1}r_{m2}(1 - a_m)^2} \\
\rho_{m2} &= \frac{r_{m2} + r_{m1}(1 - 2r_{m2})(1 - a_m)^2}{1 - r_{m2}r_{m1}(1 - a_m)^2}
\end{aligned}\right\} \quad m = 1, 2, 3$$

As a special case if there is no absorption in any of the plates, $a_m = 0$ and, if in addition, all the surface reflectivities are equal, then equation (28) reduces to

$$\tau_s^{(3)} = \frac{\tau^3}{(1 - \rho^2)\left(1 - \rho^2 - \frac{\tau^2\rho^2}{1 - \rho^2}\right)}$$

where

$$\tau = \frac{(1 - r)^2}{1 - r^2} = \frac{1 - r}{1 + r}$$

$$\rho = \frac{r + r(1 - 2r)}{1 - r^2} = \frac{2r}{1 + r}$$

If τ and ρ are inserted into the expression for $\tau_s^{(3)}$ it simplifies to

$$\tau_s^{(3)} = \frac{1 - r}{1 + 5r}$$

This agrees with the general result in reference 6 for a series of n layers with a space between each of them and with all surfaces having the same reflectivity r

$$\tau_s^{(n)} = \frac{1 - r}{1 + (2n - 1)r} \tag{30}$$

Systems of Parallel Partially Transparent Layers and Opaque Plates

A system of partially transparent layers with an absorbing surface below them. - This is a typical solar collector geometry and is illustrated in figure 8. It is desired to know how much of the incident solar radiation is absorbed at the surface N. For a unit flux incident on the first surface of the first layer, $q_{i,11} = 1$, the fraction absorbed at the plate can be obtained from

$$\alpha_{\text{at plate}} = q_{i,N1} - q_{o,N1} \tag{31}$$

The net radiation method with $q_{i,11} = 1$ yields the following relations:

$$q_{o,(N-1)2} = q_{i,(N-1)2}\rho_{s2}^{(n)} + \tau_s^{(n)} \tag{32a}$$

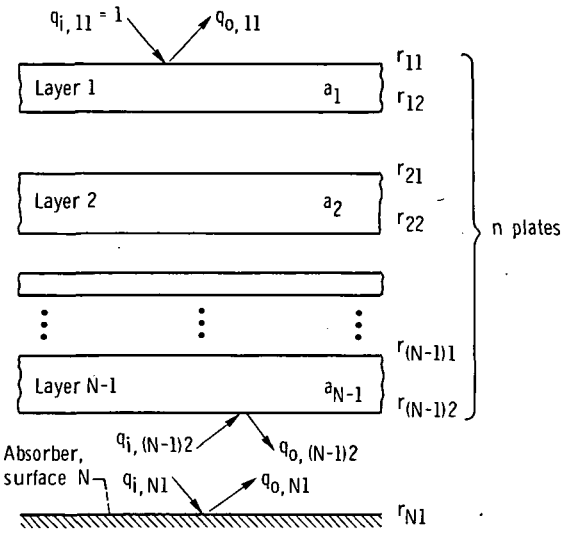

Figure 8. - System of n absorbing layers followed by absorber plate.

$$q_{o,N1} = r_{N1} q_{i,N1} \tag{32b}$$

$$q_{i,N1} = q_{o,(N-1)2} \tag{32c}$$

$$q_{i,(N-1)2} = q_{o,N1} \tag{32d}$$

where $n = N - 1$ layers. Solving for $q_{i,N1}$ and $q_{o,N1}$ and substituting into equation (31) yield

$$\alpha_{\text{at plate}} = \frac{(1 - r_{N1})\tau_s^{(n)}}{1 - r_{N1}\rho_{s2}^{(n)}} \tag{33}$$

As a simplified case, if there are n nonabsorbing layers all having the same surface reflectivities r, then using equation (30) and the fact that $\rho = 1 - \tau$ for a nonabsorbing system, yields,

$$\tau_s^{(n)} = \frac{1 - r}{1 + (2n - 1)r}$$

$$\rho_{s2}^{(n)} = \rho_{s1}^{(n)} = 1 - \tau_s^{(n)} = \frac{2nr}{1 + (2n - 1)r}$$

If these relations are substituted into equation (33), the fraction absorbed by the plate becomes

$$\alpha_{\text{at plate}} = \frac{(1 - r_{N1})(1 - r)}{1 + (2n - 1)r - r_{N1}2nr} = \frac{(1 - r_{N1})(1 - r)}{(1 - r) + 2nr(1 - r_{N1})} \tag{34}$$

where $n = N - 1$ layers.

Some additional results can be obtained quite easily for the systems shown in figure 9. For the situation in figure 9(a), $\tau_s^{(n)} = \tau_{12} = \tau_{21}$ from equations (19a) or (20a)

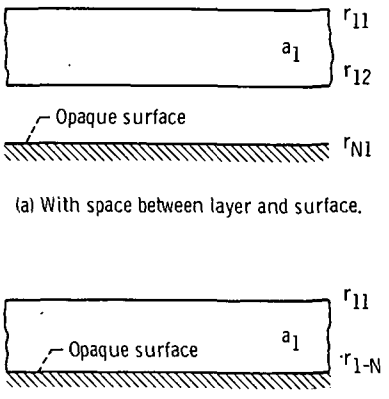

Figure 9. - Single absorbing layer interacting with opaque surface.

and $\rho_{s2}^{(n)} = \rho_{12}$ from equation (20b). Making the substitution in equation (33) yields

$$\alpha_{\text{at plate}} = \frac{(1 - r_{N1})(1 - r_{12})(1 - r_{11})(1 - a_1)}{1 - r_{12}r_{11}(1 - a_1)^2 - r_{N1}r_{12} - r_{N1}r_{11}(1 - 2r_{12})(1 - a_1)^2} \tag{35}$$

If $r_{11} = r_{12} = r$ as a special case, equation (35) reduces to

$$\alpha_{\text{at plate}} = \frac{(1 - r_{N1})(1 - r)^2(1 - a_1)}{1 - r^2(1 - a_1)^2 - r_{N1}r - r_{N1}r(1 - 2r)(1 - a_1)^2} \tag{36}$$

If in addition the layer is nonabsorbing, $a_1 = 0$, the result simplifies to

$$\alpha_{\text{at plate}} = \frac{(1 - r_{N1})(1 - r)}{1 + r - 2r_{N1}r} \tag{37}$$

For the situation in figure 9(b) the $\alpha_{\text{at plate}}$ can be obtained from equation (35) by letting $r_{12} = r_{1-N}$ and $r_{N1} = 0$. This yields

$$\alpha_{\text{at plate}} = \frac{(1 - r_{1-N})(1 - r_{11})(1 - a_1)}{1 - r_{1-N}r_{11}(1 - a_1)^2} \tag{38}$$

Transparent layer between two opaque plates. - As a further illustration of the application of the net radiation technique, consider the effect on the heat exchange of placing a transparent nonabsorbing layer between two parallel plates as shown in figure 10.

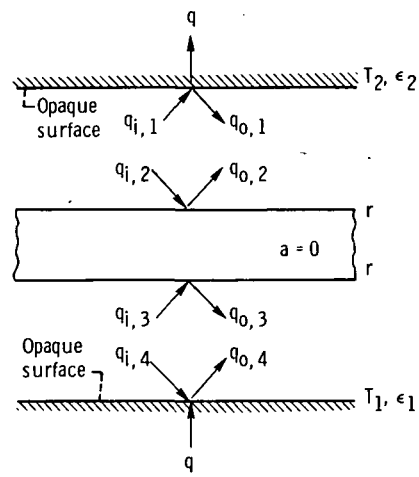

Figure 10. - Effect of intermediate transparent layer on exchange between two opaque parallel plates.

The net radiation equations are written as follows:

At plate 1

$$q_{i,4} + q = q_{o,4} \tag{39a}$$

$$q_{o,4} = \epsilon_1 \sigma T_1^4 + (1 - \epsilon_1)q_{i,4} \tag{39b}$$

At plate 2

$$q_{i,1} = q + q_{o,1} \tag{40a}$$

$$q_{o,1} = \epsilon_2 \sigma T_2^4 + (1 - \epsilon_2) q_{i,1} \tag{40b}$$

For the transparent plate

$$q_{o,2} = q_{i,3}\tau + q_{i,2}\rho \tag{41a}$$

$$q_{o,3} = q_{i,2}\tau + q_{i,3}\rho \tag{41b}$$

where ρ and τ are the overall reflectance and transmittance of the transparent plate. The connection between the q_i's and q_o's is given by

$$\left.\begin{array}{ll} q_{i,3} = q_{o,4} & q_{i,2} = q_{o,1} \\ q_{i,1} = q_{o,2} & q_{i,4} = q_{o,3} \end{array}\right\} \tag{42}$$

The relations in equations (42) are used to eliminate the q_i's from equations (39) to (41) to yield

$$q_{o,3} + q = q_{o,4}$$

$$q_{o,4} = \epsilon_1 \sigma T_1^4 + (1 - \epsilon_1) q_{o,3}$$

$$q_{o,2} = q + q_{o,1}$$

$$q_{o,1} = \epsilon_2 \sigma T_2^4 + (1 - \epsilon_2) q_{o,2}$$

$$q_{o,2} = q_{o,4}\tau + q_{o,1}\rho$$

$$q_{o,3} = q_{o,1}\tau + q_{o,4}\rho$$

The $q_{o,2}$ and $q_{o,3}$ are eliminated to yield

$$q_{o,4} = \epsilon_1 \sigma T_1^4 + (1 - \epsilon_1)(q_{o,4} - q)$$

$$q_{o,1} = \epsilon_2 \sigma T_2^4 + (1 - \epsilon_2)(q + q_{o,1})$$

$$q + q_{o,1} = q_{o,4}\tau + q_{o,1}\rho$$

$$q_{o,4} - q = q_{o,1}\tau + q_{o,4}\rho$$

These relations are solved for q which is the heat flux being transferred from the lower plate maintained at T_1 to the upper plate at T_2. The result is simplified by using the relation $\rho = 1 - \tau$ valid for a nonabsorbing layer to yield

$$q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{1}{\tau} - 2} \tag{43}$$

For a single transparent layer with surface reflectivities r, equation (4b) yields

$$\tau = \frac{1 - r}{1 + r}$$

so that by substituting into equation (43) the final result is obtained that

$$q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 + \frac{2r}{1 - r}}$$

The $2r/(1 - r)$ term is the contribution provided by the transparent plate.

CONCLUSIONS

The net radiation method has been generalized to situations where partially transparent layers are present, such as an enclosure having one or more windows. The utility of the method is further illustrated by applying it to determine the overall transmission and reflection characteristics of systems of parallel plates with or without par-

allel opaque plates. These configurations arise in solar collectors, solar stills, and in coated surfaces where the coatings are thick relative to the wavelength of the radiation so that interference effects do not have to be included. The method is found convenient to apply and provides a systematic approach that is not difficult even for complex systems. Most partially transparent plates have spectrally dependent properties. The net radiation equations can be applied for various wavelength regions and the energy quantities then summed over wavelength. The semigray (two band) approximation has been specifically discussed here which utilizes two spectral regions in each of which the properties are constant. The utilization of multiple wavelength bands proceeds in the same fashion as has been well documented in the literature for systems with opaque walls.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 17, 1973,
770-18.

APPENDIX - ENCLOSURE EQUATIONS FOR TWO BAND APPROXIMATION

The equations for the short wavelength region are for the opaque surfaces,

$$\sum_{j=1}^{m}\left(\frac{\delta_{jk}}{\epsilon_j^s} - \frac{1-\epsilon_j^s}{\epsilon_j^s}F_{k-j}\right)q_j^s - \sum_{j=m+1}^{N}\frac{\rho_j^{i,s}}{\epsilon_j^{i,s}}F_{k-j}q_j^s = \sum_{j=1}^{m}(\delta_{jk} - F_{k-j})\sigma T_j^4 F_{0-\lambda_c T_j}$$

$$- \sum_{j=m+1}^{N}\frac{\epsilon_j^{o,s}\rho_j^{i,s} + (1-\tau_j^{i,s})\epsilon_j^{i,s}}{\epsilon_j^{i,s}}F_{k-j}\sigma T_j^4 F_{0-\lambda_c T_j}$$

$$+ \sum_{j=m+1}^{N}\frac{\rho_j^{i,s}(1-\rho_j^{o,s}) - \tau_j^{o,s}(1-\tau_j^{i,s})}{\epsilon_j^{i,s}}F_{k-j}q_{e,j}^s$$

$$(1 \le k \le m,\ \lambda < \lambda_c) \tag{A1}$$

and for the partially transparent surfaces,

$$-\sum_{j=1}^{m}\frac{1-\epsilon_j^s}{\epsilon_j^s}F_{k-j}q_j^s + \sum_{j=m+1}^{N}(\delta_{jk} - \rho_j^{i,s}F_{k-j})\frac{q_j^s}{\epsilon_j^{i,s}} = -\sum_{j=1}^{m}F_{k-j}\sigma T_j^4 F_{0-\lambda_c T_j}$$

$$+ \sum_{j=m+1}^{N}\left\{\frac{(\epsilon_j^{o,s} + \epsilon_j^{i,s})\delta_{jk} - \left[\epsilon_j^{o,s}\rho_j^{i,s} + (1-\tau_j^{i,s})\epsilon_j^{i,s}\right]F_{k-j}}{\epsilon_j^{i,s}}\right\}$$

$$\times\sigma T_j^4 F_{0-\lambda_c T_j} - \sum_{j=m+1}^{N}\left\{\frac{\epsilon_j^{o,s}\delta_{jk} - \left[\rho_j^{i,s}(1-\rho_j^{o,s}) - \tau_j^{o,s}(1-\tau_j^{i,s})\right]F_{k-j}}{\epsilon_j^{i,s}}\right\}q_{e,j}^s \tag{A2}$$

$$(m+1 \le k \le N,\ \lambda < \lambda_c)$$

The equations for the long wavelength region are for the opaque surfaces,

$$\sum_{j=1}^{m}\left(\frac{\delta_{jk}}{\epsilon_j^l} - \frac{1-\epsilon_j^l}{\epsilon_j^l}F_{k-j}\right)q_j^l - \sum_{j=m+1}^{N}\frac{\rho_j^{i,l}}{\epsilon_j^{i,l}}F_{k-j}q_j^l = \sum_{j=1}^{m}(\delta_{jk} - F_{k-j})\sigma T_j^4\left(1 - F_{0-\lambda_c T_j}\right)$$

$$- \sum_{j=m+1}^{N}\frac{\epsilon_j^{o,l}\rho_j^{i,l} + (1-\tau_j^{i,l})\epsilon_j^{i,l}}{\epsilon_j^{i,l}}$$

$$\times F_{k-j}\sigma T_j^4\left(1 - F_{0-\lambda_c T_j}\right)$$

$$+ \sum_{j=m+1}^{N}\frac{\rho_j^{i,l}(1-\rho_j^{o,l}) - \tau_j^{o,l}(1-\tau_j^{i,l})}{\epsilon_j^{i,l}}$$

$$\times F_{k-j}q_{e,j}^l \qquad (1 \le k \le m,\ \lambda < \lambda_c) \tag{A3}$$

and for the partially transparent surfaces,

$$-\sum_{j=1}^{m}\frac{1-\epsilon_j^l}{\epsilon_j^l}F_{k-j}q_j^l + \sum_{j=m+1}^{N}(\delta_{jk} - \rho_j^{i,l}F_{k-j})\frac{q_j^l}{\epsilon_j^{i,l}} = -\sum_{j=1}^{m}F_{k-j}\sigma T_j^4\left(1 - F_{0-\lambda_c T_j}\right)$$

$$+ \sum_{j=m+1}^{N}\left\{\frac{(\epsilon_j^{o,l} + \epsilon_j^{i,l})\delta_{jk} - \left[\epsilon_j^{o,l}\rho_j^{i,l} + (1-\tau_j^{i,l})\epsilon_j^{i,l}\right]F_{k-j}}{\epsilon_j^{i,l}}\right\}\sigma T_j^4\left(1 - F_{0-\lambda_c T_j}\right)$$

$$- \sum_{j=m+1}^{N}\left\{\frac{\epsilon_j^{o,l}\delta_{jk} - \left[\rho_j^{i,l}(1-\rho_j^{o,l}) - \tau_j^{o,l}(1-\tau_j^{i,l})\right]F_{k-j}}{\epsilon_j^{i,l}}\right\}q_{e,j}^l \qquad (m+1 \le k \le N,\ \lambda < \lambda_c) \tag{A4}$$

The total heat fluxes are found by summing the fluxes for the two wavelength regions,

$$q_{1_k} = q_{1_k}^{s} + q_{1_k}^{l} \qquad (1 \leq k \leq N) \tag{A5}$$

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