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TRANSMISSION OF WAVE ENERGY IN CURVED DUCTS

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ABSTRACT

A formulation of wave energy flow has been developed for motion in curved ducts. A parametric study over a range of frequencies determined the ability of circular bends to transmit energy for the case of perfectly rigid walls.

INTRODUCTION

Propagation of waves in curved ducts and pipes is characterized by wave patterns totally different from those known in straight ducts or in unlimited space. The curvilinear boundaries are responsible for the appearance of a continuous standing radial wave which in turn affects the transmitted tangential waves. General theory for motion of long and acoustic waves in bends has been developed and published (refs. 1 and 2). However, no formulation was given for energy density or energy flux for motion of acoustic waves in circular bends. In 1956 Karpman (ref. 3) attempted to develop expressions for the kinetic and the potential energies of standing waves in torus. Because of mathematical difficulties, he limited himself to partial results in terms of acoustic energy density. An interesting analysis on reflection and transmission coefficients due to circular bends in rectangular wave guides was published in 1943 by Marshak (ref. 4). By approximate methods applicable to electromagnetic waves in gradual bends a qualitative formulation was given for motion of waves in bends in general. His transmission coefficient was defined as amplitude ratio of the first mode of the transmitted to incident vibrations. In 1955 Lippert (ref. 5) published experimental data on characteristic transmission factors of square and rounded mitered elbows. He defined the amplitude of the transmission factor as the ratio of sound pressure amplitude of the transmitted wave at the output plane to the sound pressure amplitude of the incident wave at the input plane.

There are both scientific and engineering reasons to study the ability of bends to transfer energy of waves. For example, calculation wave energy flow may be desired in acoustical studies of air conditioning ducts. The calculated energy flux does not give any measure of the ability of any given bend to transmit sound. It will be of interest, however, to compare energy flow

in a bend to energy flow in a straight infinite duct. See figure 1. The ratio will give transmissivity of a bend because motion in a straight duct is without reflections or diffraction and may be considered one-dimensional. To be successful in the analytical approach, the comparison must involve propagating waves only. The standing and evanescent waves in the vicinity of the junction of the two ducts must be excluded. In the series solution of the wave equation all terms pertaining to higher, evanescent modes will be dropped and only terms of the basic mode retained. From the energy balance viewpoint such a procedure is perfectly correct. The evanescent waves do not constitute a loss because eventually all energy in them is transferred to the steady uniform field some distance downstream from the discontinuity. From previous studies, references 1 and 2, general expressions for the motion of waves in curved ducts are available. For waves in the very low frequency range (wave length is at least one hundred times larger than the width of the duct) approximate expressions have been obtained by simplified series expansions of Bessel functions of fractional orders. For acoustic frequencies, (wave length is of the order of the width of the duct) the analysis was based on closed form solutions of Bessel functions of the order $\nu = (n + \frac{1}{2})$, $n = 0,1,2...10$. This paper presents a study of the ability of circular bends to transmit wave energy in a wide range of frequencies and for the case of perfectly rigid walls.

ANALYSIS

In a system with progressive waves, energy flux, called sound intensity, is given, in most general terms by

$$I = \frac{1}{\Omega}\frac{1}{T}\int^{T}\int^{\Omega} p\bar{v}\, dt\, d\Omega \tag{1}$$

The $\bar{v}$ designates the vector of particle velocity of the progressive wave and p is the acoustic pressure and Ω is the area of bend's cross section. The first integration, over a period of oscillation, is designed to replace instantaneous values with time averages. The second integration over area Ω is necessary in all cases where p or $\bar{v}$ depend on two coordinates as in the case of motion in bends. The equations for pressure $p = -\rho\partial\phi/\partial t$, ($\phi$ is the velocity potential and ρ the density), and velocity $v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$ have been derived in previous studies, references 1 and 2. The general solution for the velocity potential in cylindrical coordinates and without the axial dependence is

$$\phi = \sum_{n=0}^{\infty} e^{i(\omega t - \nu_n\theta + \alpha)}\left[A_{\nu_n} J_{\nu_n}(kr) + B_{\nu_n} Y_{\nu_n}(kr)\right] \tag{2}$$

where $k = \omega/c$, ν_n are the angular wave numbers to be determined for

different modes of motion. The phase angle α is an arbitrary constant. The three integration constants ν_n, A_{ν_n} and B_{ν_n} will be determined by the following boundary conditions:

(a) at R_1 and R_2 (the two cylindrical walls of the bend) $\partial\phi/\partial r = 0$

(b) at $\theta = 0$ (at bend's inlet) the tangential velocity is equal to axial velocity of a piston $v_o \exp(i\omega t)$

(c) the bend is infinite. This condition implies that there will be no backward reflected waves.*

Evaluation of the integral (1) requires solution of equation (2) for fractional ν_n in order to obtain a solution for propagation waves. Also determination of ν itself requires application of boundary condition $\partial\phi/\partial r = 0$ at R_1 and R_2 to the derivative with respect to the radius of equation (2)[1]

$$J'_{\nu_n}(akR_1)J'_{-\nu_n}(kR_1) - J'_{-\nu_n}(akR_1)J'_{\nu_n}(kR_1) = 0 \qquad (3)$$

For the case of long waves it was possible to solve equation (3) for ν directly. For shorter waves it was not possible to solve this cross product of Bessel functions for an arbitrary argument and obtain values for ν_n. The difficulty was by passed by use of discrete values of $\nu_n = (n + 1/2)$, $n = 0,1,2...10$, which correspond to Bessel functions characterized by closed form solutions. This basically the approach taken in reference 2. Figure 2 illustrates the results. The graph gives the angular wave numbers ν's for any arbitrary imposed wave number parameter kR_1. The calculated curves are for $a = R_2/R_1 = 1.5$, 2.0 and 2.5. The graph indicates that for $a = 2.0$ up to $kR_1 = 3.2$ only a single mode will be transmitted. In order to interpolate between $\nu = 0.5$, 1.5, etc. the fuction of equation (3) was formed using a general expansion of Bessel functions, limited to fifteen terms, for arbitrary, non-integer ν. By such general expansion ν's are calculable with high accuracy in the basic (zero'h) mode and in the first mode up to $\nu = 3.5$. For higher modes and higher kR_1 accuracy decreases rapidly.

In a straight duct the width of a duct has no influence on the character of the basic mode. In the curved duct the width of the channel has a significant effect. The wave number will change, and the distribution of velocities will be altered. In both the straight and the curved ducts, the narrower the duct is the less chances are that higher modes will be transmitted. A wide duct admits higher modes much more easily, see reference 2. with known characteristic numbers kR_1 corresponding to $\nu = 0.5$, 1.5... evaluation of the integral in equation (1) is possible. The obtained values for energy flux in the bends have been

*An infinite bend could be imagined as a tightly wound coil.

compared with energy flux in straight ducts which has a well known form $pv = \rho c v_o^2/2$ where v_o is the uniform particle velocity across the duct and ρc is the characteristic impedance and the specific acoustic impedance for plane propagating waves. Calculations have been done on a digital computer with the help of available subroutines for sine and cosine integrals and for the Fresnel integrals.

Figure 3 gives results of the analysis of the transmissivity of curved ducts. In this figure the independent variable is the nondimensional wave number kR_1, proportional to frequency. The three curves on this figure correspond to three ducts of different radii ratios $a = 1.5$, 2.0 and 2.5. Should R_1 be the same for some three ducts, the three curves would pertain to three ducts of different width. The parameter a strongly influences the transmissivity. The curves exhibit a maximum with progressively lower amplitude but are more prominent as parameter a increases. The three points calculated for $kR_1 = 0.01$ have been obtained using the theory of reference 1. It is interesting to note that the transmission at these low frequencies is not one hundred percent. The dotted extremities of the curves pertaining to $a = 2.0$ and 2.5 correspond to regions where two modes may be transmitted by the ducts. As the present analysis concerns just the basic mode, these parts of the curves may be altered by the coexistence of a second mode.

Ability of bends to transmit acoustic energy strongly depends on frequency, the bend's radii ratio and the inner radius R_1. These parameters, along with the product ρc, determine the specific acoustic impedance of the bend. Narrow rectangular ducts, whose mean radius of curvature of the bend is large compared to the width of the duct, transmit energy very well over a wide range of frequencies. With $a = 1.5$ and up to $kR_1 = 4.4$ transmission is at least 0.993. Consequently, tubing of a regular orchestra trumpet ($a = 1.2$, R_1=5 cm) is indeed transmitting energy very well over a frequency range in excess of 5000 Hz. In bends characterized by radii ratio $a = 2$ or more the transmissivity decreases rapidly at higher frequencies. To account for the difference between the rated energy flux in bends and the rate of energy flux in straight ducts (infinite, to consider only waves going in one direction) it is necessary to recall that the power radiated by the piston at $\theta = 0$ in the first mode is proportional to the strength of the source (which is measured by v_o) and the acoustic impedance at the inlet. Since the specific acoustic impedance in a bend in the case of zeroth mode of the progressing wave is, by reference 6,

$$z = \frac{p}{v_\theta} = \rho c(kR_1/\nu_o)(r/R_1)$$

while for a straight duct it is just ρc, the radiated power (the rate of energy flux) in the two cases is different.

CONCLUDING REMARKS

Ability of circular bends to transmit acoustic energy flux has been examined. Transmissivity of bends strongly depends on frequency, the bend's radii ratio and the inner radius R_1. The well known fact that bends may act as low pass filter has been analytically confirmed. The analysis included bends of radii ratio $a = R_2/R_1 = 1.5$, 2.0 and 2.5 and was not restricted in the inner radius R_1. The range of acoustical frequencies studied was from wave number parameter $kR_1 = 0.01$ to 4.4. The basic, zeroth mode was the only mode considered.

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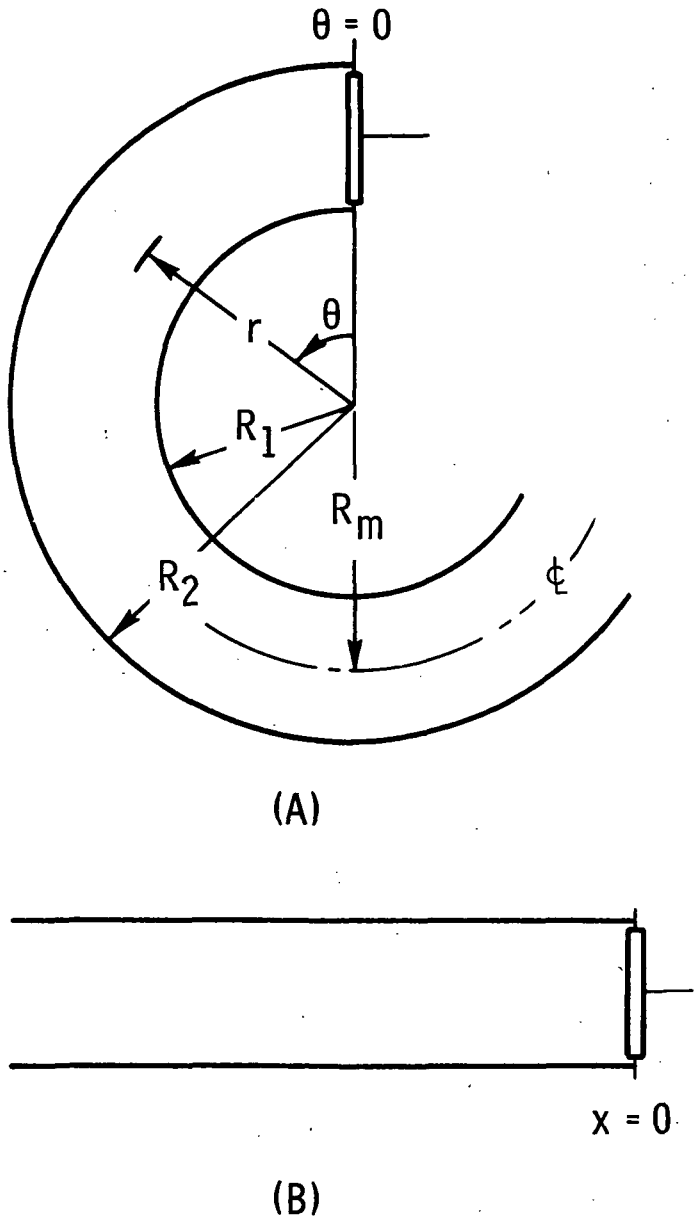

Figure 1. - The two physical systems considered: (a) infinite bend (b) infinite straight duct.

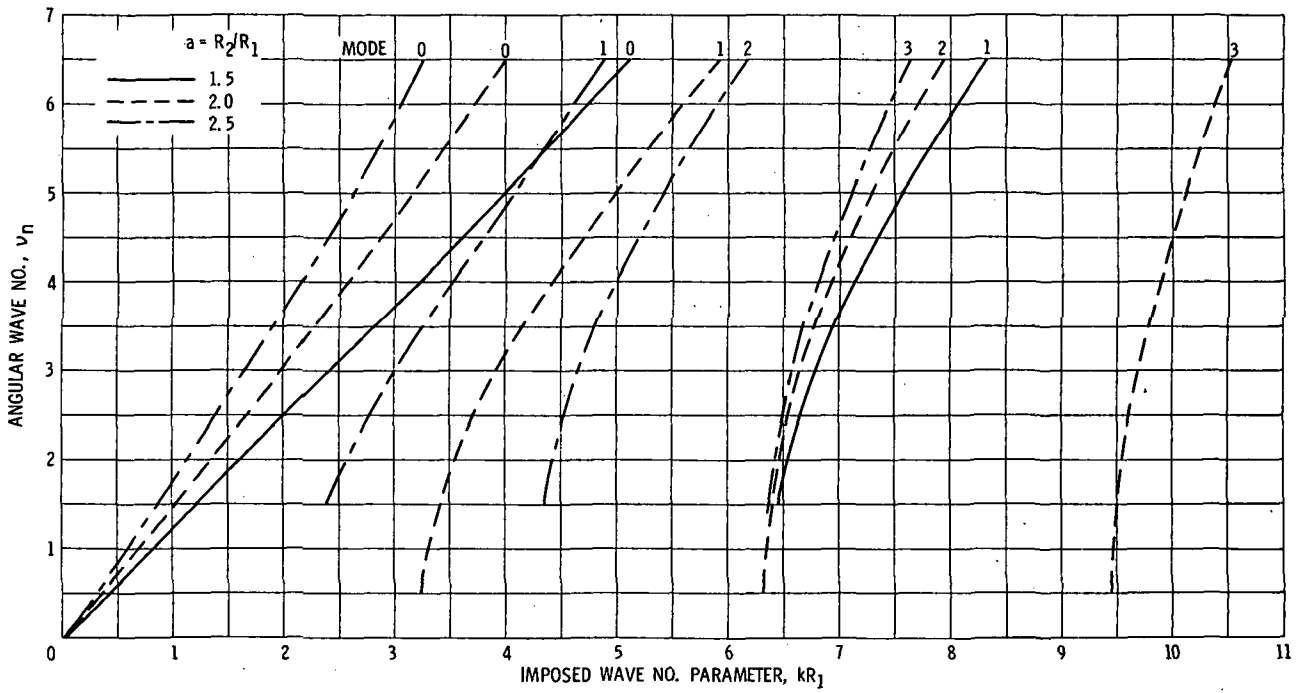

Figure 2. - Characteristics of motion in bends for three bends of different widths.

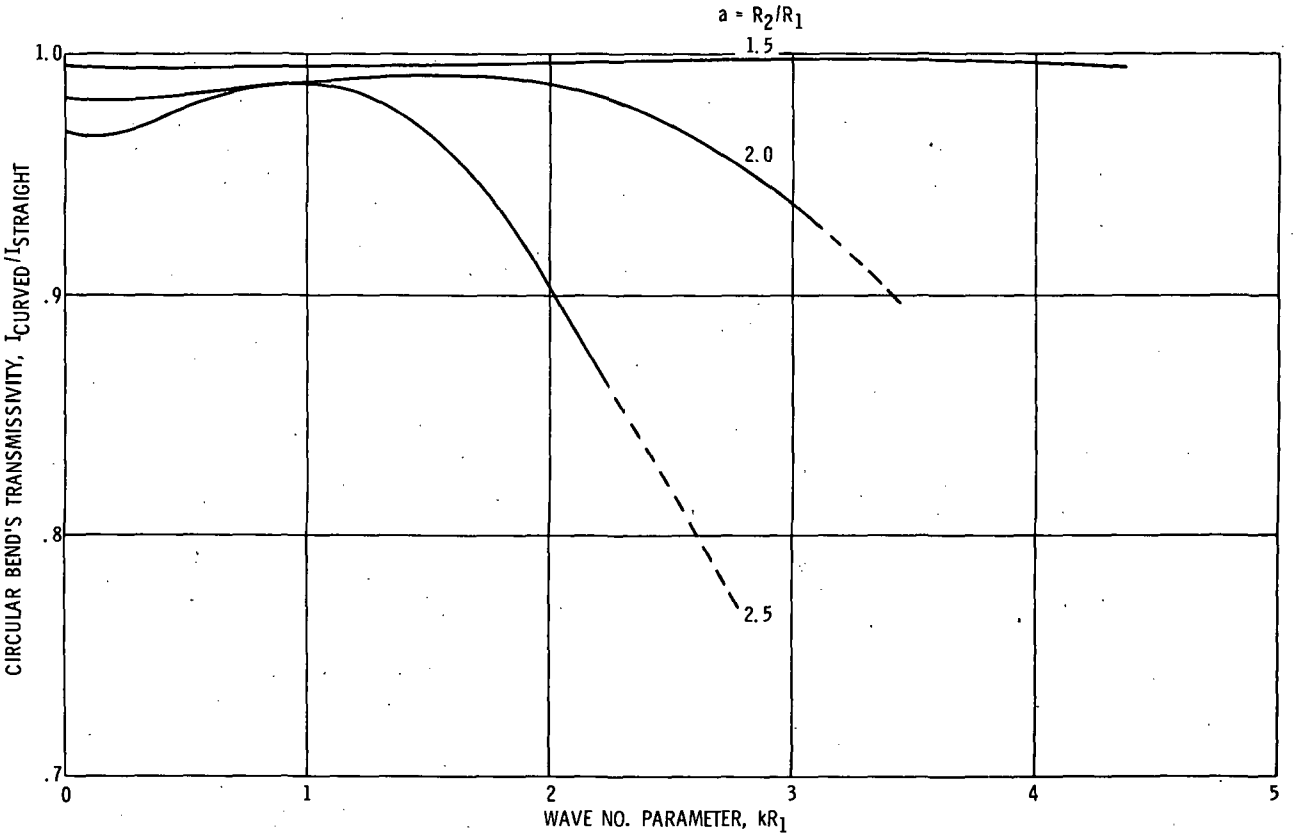

Figure 3. - Transmission of wave energy in curved ducts.