

NASA TECHNICAL  
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N73-29715  
NASA TM X-2848

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GROUND AND FLIGHT EXPERIENCE WITH  
A STRAPDOWN INERTIAL MEASURING UNIT  
AND A GENERAL PURPOSE  
AIRBORNE DIGITAL COMPUTER

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1. Report No. NASA TM X-2848	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle GROUND AND FLIGHT EXPERIENCE WITH A STRAPDOWN INERTIAL MEASURING UNIT AND A GENERAL PURPOSE AIRBORNE DIGITAL COMPUTER		5. Report Date August 1973	
		6. Performing Organization Code	
7. Author(s) Thomas D. Wolf and Robert C. McCracken		8. Performing Organization Report No. H-735	
		10. Work Unit No. 501-09-03-00	
9. Performing Organization Name and Address NASA Flight Research Center P. O. Box 273 Edwards, California 93523		11. Contract or Grant No.	
		13. Type of Report and Period Covered Technical Memorandum	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		14. Sponsoring Agency Code	
		15. Supplementary Notes  NOBRI TSL YIN...	
16. Abstract  <p>Ground and flight tests were conducted to investigate the problems associated with using a strapdown inertial flight data system. The objectives of this investigation were to develop a three-axis inertial attitude reference system, to evaluate a self-alignment technique, and to examine the problem of time-sharing a general purpose computer for the several tasks required of it.</p> <p>The performance of the strapdown platform/computer system that was developed was sufficiently accurate for the tasks attempted. For flights on the order of 45 minutes duration, attitude angle errors of <math>\pm 0.035</math> radian (<math>\pm 2^\circ</math>) in all axes were observed. Laboratory tests of the self-alignment technique gave accuracies of <math>\pm 0.00075</math> radian (<math>\pm 0.043^\circ</math>) in pitch and roll axes and <math>\pm 0.0045</math> radian (<math>\pm 0.25^\circ</math>) in the yaw axis. Self-alignment flight results were inconsistent, since a stable solution was not obtained on windy days because of aircraft rocking motions. Software requirements arising from time-sharing were not excessive when considered within the context of the state-of-the-art airborne digital computers; less than half of the total computational time available was used by this program.</p>			
17. Key Words (Suggested by Author(s))  Inertial attitude system Inertial platforms		18. Distribution Statement  Unclassified - Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 33	22. Price* \$3.00

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# GROUND AND FLIGHT EXPERIENCE WITH A STRAPDOWN INERTIAL MEASURING UNIT AND A GENERAL PURPOSE AIRBORNE DIGITAL COMPUTER

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## INTRODUCTION

Inertial navigation systems have been widely applied to both civil and military aircraft in recent years. These systems have all utilized inertial measurement units which are mechanized with multiple gimbals to isolate the stable member from aircraft angular motion. On this stable member or platform is mounted an orthogonal triad of gyroscopes. The gyroscopes establish a reference system within which the outputs of accelerometers, similarly mounted in an orthogonal triad on the platform, may be integrated to determine vehicle velocity and position with respect to some initial conditions. The gyroscopes are part of a closed-loop control system that keeps the platform in the correct orientation to preserve the reference system.

Another mechanization, which has been widely considered, is hard mounting the inertial sensors (gyroscopes and accelerometers) to the body axis of the vehicle. In this mechanization, the gimbals are replaced by a computer transformation matrix, and this matrix is continually updated in real time to maintain the desired reference system. This mechanization, referred to as a strapdown system, has certain advantages over a gimballed system (ref. 1). These include simplicity of platform, lower initial cost, smaller size, simpler maintenance, and wider choice of inertial sensor location. Its main disadvantage, and the reason that this mechanization has been held back, is that maintaining a real-time update of the transformation matrix increases the computer's computational load. Several strapdown systems have been built and flown in both space and aeronautical vehicles, but these systems used special purpose computers with dedicated circuitry to overcome the real-time performance problem. Little information other than analytical studies has been available on the use of general purpose computers in strapdown system mechanizations. With the increasing utilization of general purpose computers in aircraft, it is of interest to determine the impact of including the inertial attitude equations on the computer's computational load.

This study consisted of mechanizing a strapdown attitude reference system with a general purpose digital computer and time-sharing that computer to do the attitude computation task and profile optimization simultaneously. In addition, a self-alignment technique was utilized to determine the initial orientation of the vehicle. The complete system was flight tested in an F-104 airplane to gain operational experience and to identify possible problems.

From the outset, the program approach was constrained by several conditions. The hardware configuration was fixed by the fact that only a certain type of computer and strapdown inertial measurement unit was available. Thus, the overall

system was limited and not optimized for this application. The limitations of the system hardware caused problems to be passed onto the software. The software was limited in that the majority of the computation time was reserved for tasks other than solution of the inertial attitude equations.

## SYMBOLS

Physical quantities in this report are given in the International System of Units (SI) and parenthetically in U.S. Customary Units. The measurements were taken primarily in Customary Units. Factors relating the two systems are presented in reference 2.

a	sensed acceleration, $\text{m/sec}^2$ ( $\text{ft/sec}^2$ )
$C_{i,j}$	ith row, jth column element of the direction cosine matrix
DCM	direction cosine matrix
g	acceleration due to gravity at test site, $9.79518 \text{ m/sec}^2$ ( $32.136353 \text{ ft/sec}^2$ )
h	net rotational rate, rad/sec
IMU	inertial measurement unit
$K_{i,j}$	ith row, jth column element of the filter gain matrix
LV	local vertical
MUI	acceleration-sensitive mass unbalance drift coefficient about gyro input axis, $(\text{rad/sec})/g$
MUS	acceleration-sensitive mass unbalance drift coefficient about gyro spin axis, $(\text{rad/sec})/g$
n	current iteration cycle
PT	pitch attitude angle, rad
RD	random gyro drift coefficient, rad/sec
RL	roll attitude angle, rad
UAM	updating algorithm matrix
YW	yaw attitude angle, rad
$\Delta t$	time interval between iterations, sec

$\epsilon_{a_X}$	normalized error for IMU X acceleration axis
$\epsilon_{\omega_X}$	normalized error for IMU X gyro
$\theta$	earth rotational rate about gyro axis, rad/sec
$\lambda$	latitude of test site, 0.60999 rad
$\varphi$	total non-acceleration- and acceleration-sensitive IMU gyro drift rotational rates, rad/sec
$\Omega$	total earth rotational rate, $0.72921511 \times 10^{-4}$ rad/sec
$\omega$	rotational rate, rad/sec

Subscripts:

E,N,V	right-hand cartesian coordinates along the earth's defined east, north, and vertical axes, respectively
X,Y,Z	right-hand cartesian coordinates along the IMU sensor-defined X, Y, and Z axes, respectively

## HARDWARE DESCRIPTION

### Inertial Measurement Unit

The inertial measurement unit (IMU) was originally designed and used in a different application (ref. 3). The IMU and its power supply are shown in figure 1. It contains three orthogonally mounted rate integrating gyroscopes, three linear pendulous accelerometers, gyro and accelerometer rebalance electronics, signal conditioning, a precision timing generator, temperature control elements and circuitry, precision voltage references, and digital-to-analog conversion modules. Its purpose is to furnish three-axis angular rate and linear acceleration information to the guidance computer. Some of the specifications for the IMU are given in table 1. Tables 2 and 3 show characteristics of the gyros and accelerometers.

The IMU power supply furnishes all the operating voltages and sequencing logic for the IMU. Some of the characteristics of the IMU power supply are shown in table 1.

The IMU and its power supply were designed for use in space, but no difficulty was encountered in using the components in the flight environment of the F-104 aircraft. The IMU was constructed of electronic modules epoxied into metal trays, which were then bolted together to form an almost solid unit.

The IMU operated on 28-Vdc power. It required approximately 16 amperes of current during the 20- to 30-minute warmup period and 8 amperes during normal operation. Its components did not require external cooling.

The system mechanization imposed several constraints on the operational procedures followed during this program. The primary limitation involved the angular rates that could be measured without exceeding the maximum rebalance rates of the gyroscopes. The maximum rate was 0.35 radian per second about any axis. The aircraft used is capable of exceeding this rate in the yaw axis during taxi, and of at least 10 times this rate in the roll axis during flight; consequently, the pilot was asked to limit his maneuvers.

### Computer

The computer used in this experiment was a general purpose high-speed digital airborne computer (ref. 4). The computer consists of three units - a central processing unit, a memory unit, and a power supply (fig. 2). Table 4 gives a description of the computer's physical characteristics. An ambient forced-air-cooled cold plate method was used to provide computer cooling. The computer utilizes small-scale integrated circuit components, parallel organization, and 24-bit word length, and it complies with standard MIL-E-5400 Class 2.

The central processing unit controls all the arithmetic and logical operations of the computer by utilizing two accumulator and six index registers. An instruction repertoire of 89 instructions is provided. Arithmetic operations are performed in fractional format with fixed-point hardware. Typical execution times, including memory access, are 6 microseconds for addition and subtraction, 14 microseconds for multiplication, and 32 microseconds for division. The central processing unit provides automatic power failure and power recovery interrupt processing.

The memory unit contains 12,288 randomly addressable words. Information is retained on toroidal cores arranged in a three-dimensional, three-wire configuration. Memory cycle time is 2 microseconds.

The computer has its own electrical power supply. The input to the computer power supply is 28 Vdc, which is converted to the voltages for the central processing and memory units. Electrical power sequencing, including power failure protection, is also provided.

### System Interface

The system interface unit (fig. 3) provides a communication link between the computer and the peripheral systems. A description of the system interface unit's physical characteristics is given in table 5. It was designed of solid-state components and modules, with standard MIL-E-5400 Class 2 used as a design goal. The functions performed by the system interface unit are the transfer of inertial data into the computer, the handling of input and output discrete signals, the analog-to-digital and digital-to-analog conversions, and the generation of real-time clock interrupts to the computer. Ambient air circulation provides adequate cooling.

The system interface unit transfers six channels of digital information from the IMU to the computer. There are three channels of angular rotation data and three channels of acceleration data. The six channels of information are copies of a set of free-running 12-bit data registers in the IMU. Because these registers are free-running, it is not the numbers they contain that are important, but rather the change in the number since the last sample was taken. Successive channel readings must be subtracted to determine the number of pulses that occurred over the last sample interval. The number of pulses is proportional to the amount of rotation or acceleration sensed by the IMU during that period of time. Each channel requires 30 microseconds to read.

Capability for handling 10 input and 10 output discrete signals is provided by the system interface unit. The output discrete signals are converted to the 28-Vdc discrete signal voltage level required by aircraft systems, and the input discrete signals are converted to the 5-Vdc discrete signal voltage level used by the computer. The signals are connected to the aircraft cockpit control panel for software execution control and validation.

The analog-to-digital converter is a 12-channel multiplexed computer input system. The analog signals are in the range from -5 Vdc to 5 Vdc. Conversion of each signal into a 12-bit computer word is accomplished in 45 microseconds (including 25 microseconds of converter settling time).

The digital-to-analog converter is a 16-channel multiplexed computer output system. A 12-bit computer data word provides an analog signal in the range from -5 Vdc to 5 Vdc. Since the analog output voltages are stored in a capacitor, they must be refreshed frequently to prevent signal degradation within the frequency response range desired. Moderate voltage changes between channel refresh cycles require 100 microseconds (including 80 microseconds of converter settling time) for conversion. For full range changes (-5 Vdc to 5 Vdc or 5 Vdc to -5 Vdc), about 300 microseconds of conversion time is required.

One of the IMU's basic clocks (5333 hertz) is divided by 96 in the system interface unit to generate an 18-millisecond real-time clock interrupt to the computer. The real-time clock interrupt is used by the computer to control software timing.

### System Mechanization

The components described in the preceding sections were integrated to form a system that would take inputs from the aircraft, operate upon them, and provide suitable outputs to the pilot. The inputs from the aircraft used for profile optimization were angle of attack, pressure altitude, and airspeed. Within the system, the IMU provided incremental angles and accelerations. The system outputs were three-axis inertial aircraft attitudes and a pitch command for profile optimization which were displayed to the pilot.

A block diagram depicting the system is shown in figure 4.

## AIRCRAFT AND INSTALLATION

The F-104 test airplane used in this program (fig. 5) is a single-seat, high-performance fighter aircraft capable of speeds in excess of Mach 2 and altitudes above 18,288 meters (60,000 feet). The space available for the installation of the equipment was limited, and fuel-loading conditions were such that only one maximum performance maneuver (profile optimization) per flight was possible.

The computer, strapdown system, and interface electronics were mounted in an aluminum box structure, which was designed to retract as a unit into an ammunition bay located on top of the aircraft directly behind the canopy (figs. 6 and 7). This configuration enabled the test engineers to verify system operations in the laboratory prior to flight, as well as to do extensive software verification and development with the flight hardware in its airborne configuration.

A control panel and a display (fig. 8) were fabricated and installed in the vehicle for use in the program. This panel contained the necessary control switches and displays to operate the various components of the system, monitor IMU block temperature, and select and verify various modes in the computer program. The panel also contained the command attitude display. The command attitude display instrument is a three-axis ball type of device that displays pitch, roll, and yaw angles. It also provides pitch and roll commands by means of cross pointers. The vertical cross pointer (roll command) was not mechanized for this program, since lateral guidance was not used. The small meter pointer on the bottom of the instrument was used to display roll rate to the pilot.

## SOFTWARE DEVELOPMENT

A strapdown system uses a computer transformation matrix or direction cosine matrix (DCM) to express the relationship between the IMU-sensor-defined axis and the earth-defined reference axis. The earth axis used for this program was a local vertical reference system. Once the initial DCM was established, it was updated to maintain the correct relationship between the two axis systems.

### Self-Alinement

To establish the initial DCM, some type of alinement must be made. This can be accomplished in several ways, such as using an external reference, a known set of initial conditions, or self-alinement. One of the biggest potential advantages of a strapdown inertial system is its self-alinement capability. Because this alinement does not require the physical positioning of hardware required by gimballed inertial systems, the potential for a fast, accurate solution exists.

The following system of equations (eqs. (1) to (5)) applies only to a stationary base alinement. To determine the initial DCM a solution must be found which satisfies the equations:

$$\left. \begin{aligned}
 \begin{bmatrix} g_E \\ g_N \\ g_V \end{bmatrix}_{LV} &= \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}_{LV} = \begin{bmatrix} \text{DCM} \end{bmatrix} \begin{bmatrix} a_X \\ a_Y \\ a_Z \end{bmatrix}_{IMU} \\
 \begin{bmatrix} \omega_E \\ \omega_N \\ \omega_V \end{bmatrix}_{LV} &= \begin{bmatrix} 0 \\ \Omega \cos \lambda \\ \Omega \sin \lambda \end{bmatrix}_{LV} = \begin{bmatrix} \text{DCM} \end{bmatrix} \begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix}_{IMU}
 \end{aligned} \right\} \quad (1)$$

By multiplying both sides of equations (1) by the transpose of DCM, which because of the special matrix properties is equal to the inverse, and rearranging, equations (2) result. Because of problem symmetry only the X-axis is shown from here on.

$$\left. \begin{aligned}
 a_X &= -gC_{3,1} \\
 \omega_X &= C_{2,1}\Omega \cos \lambda + C_{3,1}\Omega \sin \lambda
 \end{aligned} \right\} \quad (2)$$

Since sensor outputs are not perfect, equations (2) may be used to form a set of normalized errors, equations (3).

$$\left. \begin{aligned}
 \epsilon_{\omega_X} &= C_{3,1} - \frac{a_X}{g} \\
 \epsilon_{\omega_X} &= -\frac{\omega_X}{\Omega} + C_{2,1} \cos \lambda + C_{3,1} \sin \lambda
 \end{aligned} \right\} \quad (3)$$

These errors were put into a suboptimal Kalman filter to obtain a set of updated coefficient estimates for the last two row vectors of the DCM as follows:

$$\left. \begin{aligned} C_{2,1}(n+1) &= C_{2,1}(n) + K_{1,1}\epsilon_{\omega_X}(n) + K_{1,2}\epsilon_{a_X}(n) \\ C_{3,1}(n+1) &= C_{3,1}(n) + K_{2,1}\epsilon_{\omega_X}(n) + K_{2,2}\epsilon_{a_X}(n) \end{aligned} \right\} \quad (4)$$

An extensive treatment of Kalman filters is contained in reference 5.

The gain matrix  $K_{i,j}$  is constant, which makes this filter suboptimal. The constant gain filter that was selected reduces the computer's computational load but sacrifices short-period transient response. In practice this tradeoff works out quite well. The first row is obtained by forming the vector cross product of the second and third rows as follows:

$$\left. \begin{aligned} C_{1,1} &= C_{2,2}C_{3,3} - C_{3,2}C_{2,3} \\ C_{1,2} &= C_{3,1}C_{2,3} - C_{2,1}C_{3,3} \\ C_{1,3} &= C_{2,1}C_{3,2} - C_{3,1}C_{2,2} \end{aligned} \right\} \quad (5)$$

These equations were solved on an iteration basis every 18 milliseconds. Because the alinement is stationary, only earth rates are sensed by the IMU. These sensed rates were time-averaged before the equations were solved. Once every 200 iterations the second and third rows of the DCM were normalized before computing the first row. This was done to help maintain an orthogonal matrix. Important assumptions made in developing the self-alinement equations were a stationary base and a Gaussian sensor input distribution. The constant filter gains used were:

$$\begin{aligned} K_{1,1} &= 0.18703 & K_{1,2} &= 0.07515 \\ K_{2,1} &= 0 & K_{2,2} &= -0.10344 \end{aligned}$$

### Transformation Update

In a strapdown inertial system the sensor axes are fixed to the vehicle at a known orientation. As a result, whenever the vehicle orientation changes, the strapdown IMU axis orientation also changes and sensor outputs are observed. These sensor outputs are used to mathematically update the previous value of the transformation matrix to its current value. The procedure for updating the transformation matrix is referred to as an updating algorithm. Many different updating algorithm forms have been developed. For this program a second-order Taylor series expansion form was used. A complete derivation of this form is contained in reference 6. The primary reasons for selecting the second-order Taylor series

expansion updating algorithm were the low iteration rate, which was used because it kept the computer's computational load low, and the fact that the hardware configuration resulted in parallel whole word data processing. In the opinion of the authors, the updating algorithm form chosen represented a reasonable compromise between system performance and test program constraints. A similar conclusion was reached in reference 7.

The system of matrix equations used to update the direction cosine (or transformation) matrix is

$$\left[ \text{DCM}(n) \right] = \left[ \text{DCM}(n-1) \right] \left[ \text{UAM}(n) \right] \quad (6)$$

where the updating algorithm matrix (UAM) is

$$\left[ \text{UAM}(n) \right] = \begin{bmatrix} \left(1 - \frac{h_Y^2}{2} - \frac{h_Z^2}{2}\right) \left(-h_Z + \frac{h_X h_Y}{2}\right) \left(h_Y + \frac{h_X h_Z}{2}\right) \\ \left(h_Z + \frac{h_X h_Y}{2}\right) \left(1 - \frac{h_X^2}{2} - \frac{h_Z^2}{2}\right) \left(-h_X + \frac{h_Z h_Y}{2}\right) \\ \left(-h_Y + \frac{h_X h_Z}{2}\right) \left(h_X + \frac{h_Y h_Z}{2}\right) \left(1 - \frac{h_X^2}{2} - \frac{h_Y^2}{2}\right) \end{bmatrix} \quad (7)$$

The term in equation (7) represents the net change in orientation between the IMU axis and the local vertical reference axis. As equation (8) shows, the net change is the combination of several effects.

$$\begin{bmatrix} h_X \\ h_Y \\ h_Z \end{bmatrix} = \begin{bmatrix} A \\ \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix} - \begin{bmatrix} B \\ \theta_X \\ \theta_Y \\ \theta_Z \end{bmatrix} - \begin{bmatrix} C \\ \varphi_X \\ \varphi_Y \\ \varphi_Z \end{bmatrix} \quad (8)$$

Term A of equation (8) is the amount of angular rotation sensed by the IMU gyros. Its effect is determined by taking the number of gyro pulses on each output and multiplying it by the appropriate gyro channel scale factor. The gyro output pulses represent the summation of all sensed rotational effects. This output includes the rotation of the earth. Since only the relative orientation of the two axis systems is of interest, the earth's rate must be subtracted from the DCM. Term B of

equation (8) makes this correction. The detailed computation of the earth rate term is:

$$\begin{bmatrix} \theta_X \\ \theta_Y \\ \theta_Z \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \Omega \cos \lambda \Delta t \\ \Omega \sin \lambda \Delta t \end{bmatrix} \quad (9)$$

It should be noted that in equation (9) the earth rate terms are multiplied by the inverse of the DCM. This transfers the correction into the axis system that the correction is for. Since only an attitude system was mechanized, equation (9) assumes a constant latitude angle. In reality, the latitude changes as the vehicle's position over the earth changes. Since the flight times and distances traveled were relatively short, however, it was felt that this assumption would not jeopardize the results of the program.

Term C is added to equation (8) because the IMU gyros are not perfect sensors. The form of this correction is

$$\begin{bmatrix} \varphi_X \\ \varphi_Y \\ \varphi_Z \end{bmatrix} = \begin{bmatrix} RD_X(\Delta t) - MUI_X(a_Y) - MUS_X(a_X) \\ RD_Y(\Delta t) + MUI_Y(a_X) - MUS_Y(a_Y) \\ RD_Z(\Delta t) - MUI_Z(a_X) - MUS_Z(a_Z) \end{bmatrix} \quad (10)$$

The first term in equation (10) is the non-acceleration-sensitive drift and is a function of time only. The last two terms are acceleration-sensitive mass unbalance drifts about the gyro input and spin axis, respectively, and are functions of the accelerations present along those axes. The accelerations are computed by taking the accelerometer pulse outputs along the sensor axis, multiplying them by the appropriate accelerometer channel scale factors, and then accounting for the accelerometer bias. Because the earth rate and gyro drift corrections are of relatively small magnitude, these terms are accumulated until they total an amount equal to one gyro pulse scale factor before their influence is accounted for.

Shortly after beginning the laboratory tests, a tendency for the DCM to become nonorthogonal was observed. To remedy this, the first and third rows of the DCM were normalized to a value of 1 by requiring that the sum of the squares of each term equal 1. After normalizing the first and third row vectors, the second row vector is formed by taking the vector cross product of the first and third row vectors. This procedure, which is performed once per minute, forces the DCM to an orthogonal condition.

Attitude information is obtained by equating the DCM to an equivalent transformation matrix containing sines and cosines of the Euler angles between the two axis systems. Details of this expression can be found in almost any advanced dynamics reference. For the transformation defined here, the pitch (PT), roll (RL), and yaw (YW) angles are equal to:

$$PT = \arctan \frac{C_{2,3}}{C_{3,3}} \quad (11a)$$

$$RL = \arctan \frac{-C_{1,3}}{\sqrt{1 - C_{1,3}^2}} \quad (11b)$$

$$YW = \arctan \frac{C_{1,2}}{C_{1,1}} \quad (11c)$$

where the C term represents elements of the DCM.

These three attitude angles are put out as analog signals to the aircraft cockpit command attitude display instrument. The entire system of equations (eqs. (6) to (11)) is solved every 72 milliseconds.

### Software Organization

The software is organized in modular form. This approach was taken to ease the checkout, modification, and substitution of programs. Software timing is controlled by the 18-millisecond real-time clock interrupt from the IMU. Mode control for the software is provided by the sense switches located on the aircraft cockpit control panel. Figure 9 shows the major program elements and overall software flow.

The main program is entered at the start point, where initialization is performed. As part of initialization, the real-time clock interrupt counter and main program cycle indicator are set at zero. Upon completion of initialization, a wait loop is entered waiting for the main program cycle indicator to become nonzero. The program continues in the wait loop until a real-time clock interrupt is received and program control is transferred to the interrupt processing routine. This interrupt occurs once every 18 milliseconds.

The first test made by the interrupt processing routine is to determine whether the aline or attitude mode has been selected. If the aline mode has been selected, a jump is made to the read IMU data sequence. If the attitude mode has been selected, the real-time clock interrupt counter is checked to determine if four interrupts have been received. If four interrupts have not been received, the real-time clock interrupt counter is incremented and control is returned to the wait loop in the main program. If four interrupts have been received, the main program cycle indicator is set at 1, the read IMU data sequence is initiated, and the real-time clock interrupt counter is set at zero. The mode test for aline or attitude is again made. If the aline mode has been selected, the alinement computations are exercised, and the pitch, roll, and yaw

attitude angles are sent to the command attitude display in the aircraft cockpit through the digital-to-analog converter. Control is then returned to the wait loop of the main program. If the attitude mode has been selected, the attitude program is executed and control is returned to the wait loop of the main program.

Program control will remain in the wait loop of the main program until the main program cycle indicator is nonzero. This occurs every fourth interrupt when the attitude mode has been selected. The main program cycle indicator is set at zero for the next pass through the wait loop test. The analog-to-digital converter makes inputs to the computer on airplane flight path angle, Mach number, and altitude for the profile optimization program. If the profile optimization program is selected, the computations are executed. If it is not selected, the computations are bypassed. Outputs of attitude angle are made by the digital-to-analog converter at this point when computed by the attitude program. If the profile optimization program has been executed, outputs of guidance information are also made to the horizontal command needle on the command attitude display instrument in the aircraft cockpit. Before returning to the wait loop, a test is made to determine whether data are to be saved in the computer memory. Up to 8500 computer words can be saved. The number of parameters and the frequency at which data are saved are variable. Once control is returned to the wait loop, the sequence described above is repeated.

## TEST DESCRIPTION

### Laboratory Alinement Tests

To verify the results of the alinement program in operation and to study the effects of software changes, the system was run and measurements were taken in a controlled environment. The test involved fixing the IMU to an inertial test stand and measuring its actual orientation angles. These angles were then compared with the computed angles, the difference representing the error.

The test stand was a stable concrete block approximately 0.9 meter (3 feet) on each side. This cube had a 0.05-meter- (2-inch-) thick aluminum plate mounted on the top. While the surface of this plate was nominally level, no attempt was made to make it precisely so. The IMU was attached to a cooling plate, clamped to the top of the block, and hooked up to the remaining system components.

The orientation of the IMU in pitch and roll was measured by using a precise clinometer to measure the tilt of the cooling plate surface. This device is capable of measuring angles to an accuracy of  $\pm 0.000005$  radian. The orientation of the IMU in yaw was measured by using an optical theodolite to measure the angle between a surveyed north reference mirror and a porro prism mounted on the IMU. This prism is parallel with one axis of the IMU coordinate system to  $\pm 0.000015$  radian. This measurement then can be used to determine the yaw angle of the IMU.

After these angles were measured, the alinement program was executed and a continuous record of the results was stored in the computer memory. After approximately 9 minutes the runs were terminated, and the results stored in the

computer memory were printed out. Options as to what information was stored were available, but in general the attitude angles were the parameters of interest. In some cases, the angular rates in each axis were stored to check the long-term stability of the system.

The angles calculated by the computer were then compared with the measured angles as an index of performance.

### Laboratory Attitude Tests

The attitude tests were run in a similar fashion. The angle data calculated by the computer from the strapdown system were printed out in real time. The initial conditions for the attitude runs were printed out when the mode of operation was changed from aline to attitude. New data were then printed out at intervals chosen by the operator.

For fixed-base tests, the IMU was fastened to the block used for alinement tests. These runs were allowed to proceed for periods of a few minutes to 2 hours. At the end of the time period, the last computed values were used as end conditions and compared with the initial conditions to determine drifts. The difference between these values was then divided by the elapsed time to get drift rates.

Some runs were also made on a Scoresby table. This table is a device that introduces angular inputs in three axes simultaneously, simulating coning motion, one of the most difficult environments for an inertial platform to measure. Again, the alinement program furnished the starting conditions. At the end of the run, the table was brought to a stop, and the alinement program was used again to determine the final conditions.

### Flight Tests

A series of 28 flights was flown to verify the hardware and software in a real environment. The flights were made to provide operational experience with the strapdown hardware as well as to determine the integrity of the system.

Windy weather did not permit the self-alinement program to be used for most data flights, so initial aircraft attitude angles prior to taxi on the ramp were determined by a precomputed set of initial conditions. All the alinements were performed with the aircraft positioned as close as possible to the same point on the ramp. After alinement, the aircraft proceeded to the runway, took off, and flew a predetermined stored profile. An option for coarse system realinement in flight was also provided. This feature allowed the pilot to reestablish reasonable inertial attitude angles after the system's rate capability had been exceeded. The realinement was made by visually alining the aircraft in level flight, visually establishing a north or south heading along section lines on the ground, and selecting the appropriate conditions via a pushbutton on the cockpit control panel. The computer then initialized the direction cosine matrix with the proper values.

A recording of selected parameters was made within the airborne computer during the flight. This information was printed out after each flight. In addition, the pilots' comments were solicited in regard to attitude display, command sensitivity, and overall system performance.

## RESULTS AND DISCUSSION

### Laboratory Alinement Performance

Results of tests in the laboratory using a fixed-base configuration indicate that the system as configured for the test gave adequate results. Attitude angle errors were  $\pm 0.00075$  radian in the roll and pitch axes and  $\pm 0.0045$  radian in the yaw or heading axis. In general, the pitch and roll axes were more accurate than the yaw axis. This was expected, since the pitch and roll axes were calculated from accelerometer data alone, while calculations for the yaw axis used gyroscope data. When measuring earth rates for alinement, resolution of the IMU accelerometers was approximately 1600 times better than that of the gyroscopes.

Alinement runs were limited to approximately 9 minutes, since the computed attitude angles reached steady values by that time. A plot of one of the runs is shown in figure 10. The values of the computed attitude angles are shown as a function of time. The values in the pitch and roll axes took approximately 2 minutes to reach a reasonably steady value, those in the yaw axis considerably longer.

Some special runs were made in which the angular rates computed in the alinement program were stored. It is from these values that the yaw angle was derived. A typical plot of the normalized angular rates is shown in figure 11. It is apparent that the calculated values changed continuously at a slow rate in each axis, even though the IMU operating temperature had had time to stabilize at its nominal value. While the cause of this drift is not known, it was felt that the poor temperature control within the IMU due to its single heater element caused the values of the gyroscope scale factors or the drift terms, or both, to change.

### Laboratory Attitude Performance

The two types of attitude performance tests run in the laboratory on the system were Scoresby and steady-state drift. These tests provided data on system performance under conditions of both dynamic and static operation.

Unfortunately, no method of determining system attitude was available during the Scoresby test, so only the initial and final attitudes were used to measure performance. The results of the Scoresby test showed errors of less than  $\pm 0.03$  radian in yaw and less than  $\pm 0.013$  radian in pitch and roll angle. These values include the initial alinement error, its propagation throughout the 30-minute test period, and the final alinement error.

A typical steady-state-test time history of attitude angle error is shown in figure 12. This run, which was typical, shows the drift rates for all axes to be within

the desired 0.02-radian-per-hour attitude program performance. The repeatability and linearity of the errors suggest that they could be reduced with additional software refinement. However, no refinement was attempted, since the desired system performance had been achieved.

### Flight Testing

Many of the first flights were made to isolate problems in the aircraft/test package interface. When it was felt that the system was performing satisfactorily, a number of flights were made for data. All flights lasted approximately 45 minutes.

Because of limited instrumentation, all the data concerning system operation were derived from two sources - pilot comments and data stored in the computer. The most useful information was from the pilots' comments.

The alinement program was exercised to establish initial conditions on many occasions during the program. Because of the location of the equipment, it was not possible to measure the IMU attitude angles in the aircraft, so no accurate comparison could be made between true and computed alinement values. As long as the weather permitted the aircraft to be reasonably stable, the attitudes displayed on the command attitude display instrument indicated that the computed alinement results were reasonable, however. On windy days, when the aircraft was subject to rapid angular motion, the alinement program failed to arrive at a steady solution, and the attitudes displayed on the command attitude display were oscillatory. As stated previously, most data flights were begun by inserting precalculated values, representing known conditions at a spot on the ramp, into the direction cosine matrix.

In general, the pilots were satisfied with the attitude information displayed by the command attitude display instrument. It was reported that after initialization on the ramp prior to taxi, it was possible to proceed to the runway, take off, fly a stored profile, land, and return to the ramp with minimum observable attitude angle errors, believed to be on the order of  $\pm 0.035$  radian. The instrument seemed to provide steady data that were better than the information from the conventional vertical gyroscope and flux gate compass installed in the airplane. The pilots reported that it was not difficult to fly the airplane within the angular rate constraints imposed by the IMU, that is, 0.35 radian per second.

The flight data stored in the computer were printed out at the end of each flight. Stored values of altitude, Mach number, and attitude angle and computed values of flight path angle commands were examined to determine system performance. These data supported the pilots' opinions and also gave an indication of how well the pilot performed the tracking task required to fly the stored profile.

The computer time-sharing task was not a problem. The solution of the attitude equations took approximately 25 percent of the total computational time available, input and output approximately 10 percent, and the computation of the predetermined stored profile approximately 5 percent. This left 60 percent of the total available computational time unused.

It is felt that the system as mechanized performed well, in view of the limitations of its components. The tasks originally set for the program were satisfactorily accomplished.

Hardware problems.— The problems encountered during this experiment may well be typical of those to be expected in attempting to bring a strapdown system to operational status. Although some of these problems may be peculiar to the NASA system, they are recounted for the sake of record.

The IMU and its power supply suffered few malfunctions during the tests. The only significant occurrence was a change in the gyroscope scale factor in one of the gyros in an IMU. The problem arose in flight and became noticeable when the roll axis information drifted off rapidly. Because of time constraints, no effort was made to determine whether the malfunction was in the gyroscope or the rebalance electronics. Another IMU was available, and the program continued with it. There was a problem in the block temperature control electronics of another IMU, but when an attempt was made to isolate the difficulty, it cured itself. That unit was not subsequently used. The usual broken wires and so forth that occur in ground test sets plagued the experimenters, but in general the system's components were reliable. The second system, which was used for most of the flight tests, suffered no problems, and when it was last used it was still functioning properly although it had considerably exceeded the operating time it was designed for.

It was noticed early in the first flight that the computer was operating on marginal power. The computer contained circuitry to turn itself off under low power conditions, and this seemed to be occurring when almost any aircraft system using 28-Vdc power was activated. Considerable effort was expended in trying to increase the voltage at the computer. Subsequent investigation revealed that only two phases of the 115-Vac three-phase power were being carried over to the 28-Vdc transformer-rectifier units that furnish voltage for the 28-Vdc systems in the airplane. This resulted in poor voltage regulation under load. When this wiring error was corrected, no further power difficulties were encountered.

The computer itself was essentially trouble-free. The only difficulties occurred when the memory unit was modified to increase its capacity. The type of memory used is fairly common (see HARDWARE DESCRIPTION), and it was somewhat sensitive after being disassembled and reassembled. No actual computer failures occurred during the flight test program. Some problems with the peripheral equipment, specifically the input-output typewriter, caused delays in data printout after flights, but this type of hardware problem was not considered a computer malfunction and was readily correctable.

In general, it is felt that a general purpose computer and strapdown platform can be mechanized in a reliable cost-effective system and that failures should be no more frequent than in other electronic aircraft hardware.

Software problems.— The software problems encountered were not serious, although they did prolong checkout time and affect the efficiency of the final software system.

This program showed that a definite advantage exists in having a software-compatible commercial computer with good peripheral capability to conduct software development, for the following reasons. Airborne systems usually have minimal ground support equipment. As a result, they rarely have high-speed bulk data storage capability to assist in the detailed analysis of real-time software performance. Program assemblies are usually performed on other computers, causing long turn-around times. Commercial computers have more manufacturer-supplied system software. By using a software-compatible commercial computer for system checkout, operating time on expensive flight hardware can be minimized.

The system interface unit was built with very little communication between the hardware designers and software users. The primary considerations of the hardware designers were technical risk and economy. The shortcuts taken by the hardware designers meant that problems were passed on to the software users. As a result, programs required more storage and took longer to execute. This is not the best way to design hardware from the software user's viewpoint, but hardware design criteria will not change until the cost of this kind of inefficiency can be reliably predicted.

### CONCLUDING REMARKS

A program was conducted to explore the use of a system that combined a strap-down inertial platform with a general purpose digital computer. To investigate the software time-sharing capabilities of this mechanization, the computer calculated initial alinement angles, computed attitude angles in a dynamic environment, and did additional computational tasks.

The laboratory tests indicated that alinement accuracies of  $\pm 0.00075$  radian in the roll and pitch axes and  $\pm 0.0045$  radian in the yaw axis were attainable with the system as currently mechanized. After operation of the alinement program in the aircraft, the computed attitudes indicated that reasonable results were obtainable on calm days. On windy days, the program failed to reach a steady solution and provided an oscillatory result.

Attitude computations performed with a fixed base yielded a steady-state drift rate of less than 0.02 radian per hour in all axes. When the Scoresby table was used, the accuracy degraded to  $\pm 0.03$  radian in the yaw axis and less than  $\pm 0.013$  radian in the pitch and roll axes for a 30-minute run.

Flight results on an F-104 aircraft were obtained from a combination of data recorded in the computer and pilot observations. In flight the system provided angular data that were good enough to make possible 45-minute flights with minimum observable errors, believed to be on the order of  $\pm 0.035$  radian.

The computer easily handled the additional task of computing attitude angles. More than 50 percent of the total computational time available was not used by the flight program.

Overall, the system performed well, and the problems encountered were not unlike those that would be encountered in mechanizing any inertial attitude system. It is believed that the strapdown technique is a viable mechanization for future attitude reference systems, and that it can function satisfactorily using a general purpose digital computer for calculations. A software self-alinement technique like the one mechanized in this system can also be used, if a satisfactory method can be found to solve the alinement equations when the aircraft is subject to rocking motion.

Flight Research Center  
National Aeronautics and Space Administration  
Edwards, Calif., February 14, 1973

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TABLE 1.- PERTINENT PHYSICAL CHARACTERISTICS AND DIMENSIONS  
OF THE INERTIAL MEASUREMENT UNIT AND ITS POWER SUPPLY

(a) Inertial measurement unit

Length, m (in.) . . . . .	0.23 (9.0)
Width, m (in.) . . . . .	0.23 (9.0)
Height, m (in.) . . . . .	0.15 (6.0)
Weight, kg (lb) . . . . .	10.30 (22.7)
Operating block temperature, °K (°F) . . . . .	358 (180)
Component/rebalance loop limits -	
Angular, rad/sec . . . . .	0.35
Acceleration, vertical, g . . . . .	-3 to 10
Acceleration, horizontal, two axes, g . . . . .	-3 to 3

(b) Inertial measurement unit power supply

Length, m (in.) . . . . .	0.24 (9.5)
Width, m (in.) . . . . .	0.13 (5.0)
Height, m (in.) . . . . .	0.10 (4.0)
Weight, kg (lb) . . . . .	4.77 (10.5)
Operating voltage, dc, nominal . . . . .	28
Current required -	
Warmup, A . . . . .	16
Operating, A . . . . .	8

TABLE 2.- GYROSCOPE CHARACTERISTICS

Type . . . . .	Single degree of freedom, rate integrating
Mechanization . . . . .	Heated fluid, paddle damping
Angular momentum, kg-cm <sup>2</sup> /sec (oz-in <sup>2</sup> /sec) . . . . .	1000 (546.45)
Damping (output axis), N-cm/rad/sec (oz-in/rad/sec) . . . . .	3.33 (15524.86)
Maximum time constant, sec . . . . .	0.0008
Speed, rpm . . . . .	24,000
Operating temperature, °K (°F) . . . . .	358 (180)
Input angular freedom, rad . . . . .	±0.0087
Analog torquer scale factor, rad/hr/mA . . . . .	14.39625
Continuous maximum torquing rate, rad/sec . . . . .	0.43625
30-day stability -	
Maximum g insensitivity, rad/hr . . . . .	0.0087
Maximum input axis mass unbalance, rad/hr . . . . .	0.0087
Maximum spin axis mass unbalance, rad/hr . . . . .	0.014

TABLE 3.—ACCELEROMETER CHARACTERISTICS

Type . . . . .	Linear
Mechanization . . . . .	Hinged pendulum, fluid filled
Current scale factor, mA/g . . . . .	4.0
Pendulous scale factor, N-cm/g (oz-in/g) . . . . .	0.002805 (0.00398)
Pendulum inertia, kg-cm <sup>2</sup> (oz-in <sup>2</sup> ) . . . . .	0.0071 (0.0388)
Damping coefficient, N-cm-sec (oz-in-sec) . . . . .	120 (17.04)
Hinge restraint, g/rad . . . . .	3.0
Operating temperature, °K (°F) . . . . .	358 (180)
Pendulum freedom, rad . . . . .	±0.004

TABLE 4.—PERTINENT PHYSICAL CHARACTERISTICS OF THE COMPUTER

(a) Central processing unit

Length, m (in.) . . . . .	0.497 (19.55)
Width, m (in.) . . . . .	0.128 (5.06)
Height, m (in.) . . . . .	0.193 (7.60)
Weight, kg (lb) . . . . .	10.4 (23)

(b) Memory unit

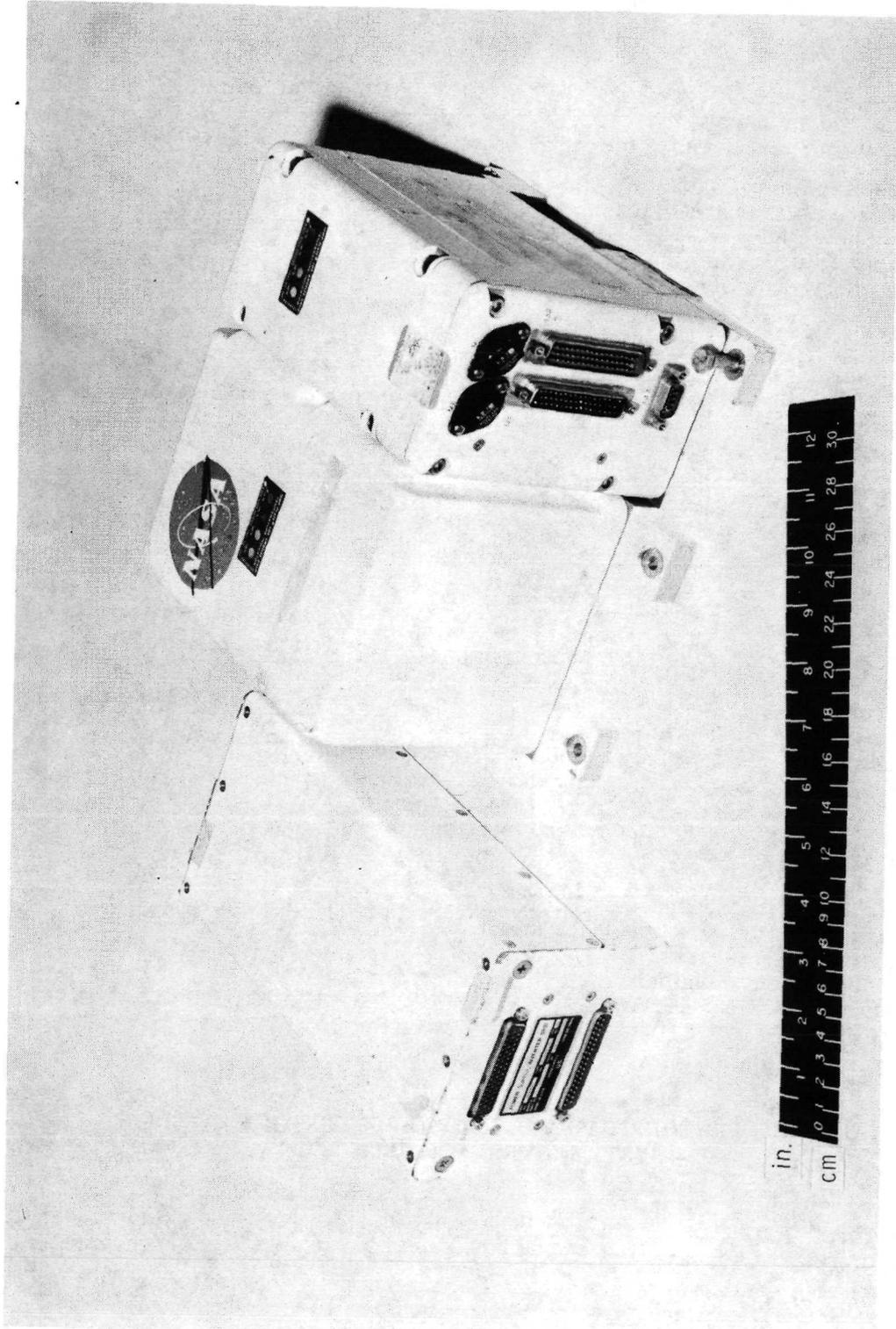
Length, m (in.) . . . . .	0.495 (19.50)
Width, m (in.) . . . . .	0.152 (6.00)
Height, m (in.) . . . . .	0.193 (7.60)
Weight, kg (lb) . . . . .	15.4 (34)

(c) Computer power supply

Length, m (in.) . . . . .	0.497 (19.55)
Width, m (in.) . . . . .	0.125 (4.93)
Height, m (in.) . . . . .	0.193 (7.60)
Weight, kg (lb) . . . . .	13.6 (30)
Operating voltage, dc, nominal . . . . .	28
Current required, A . . . . .	8

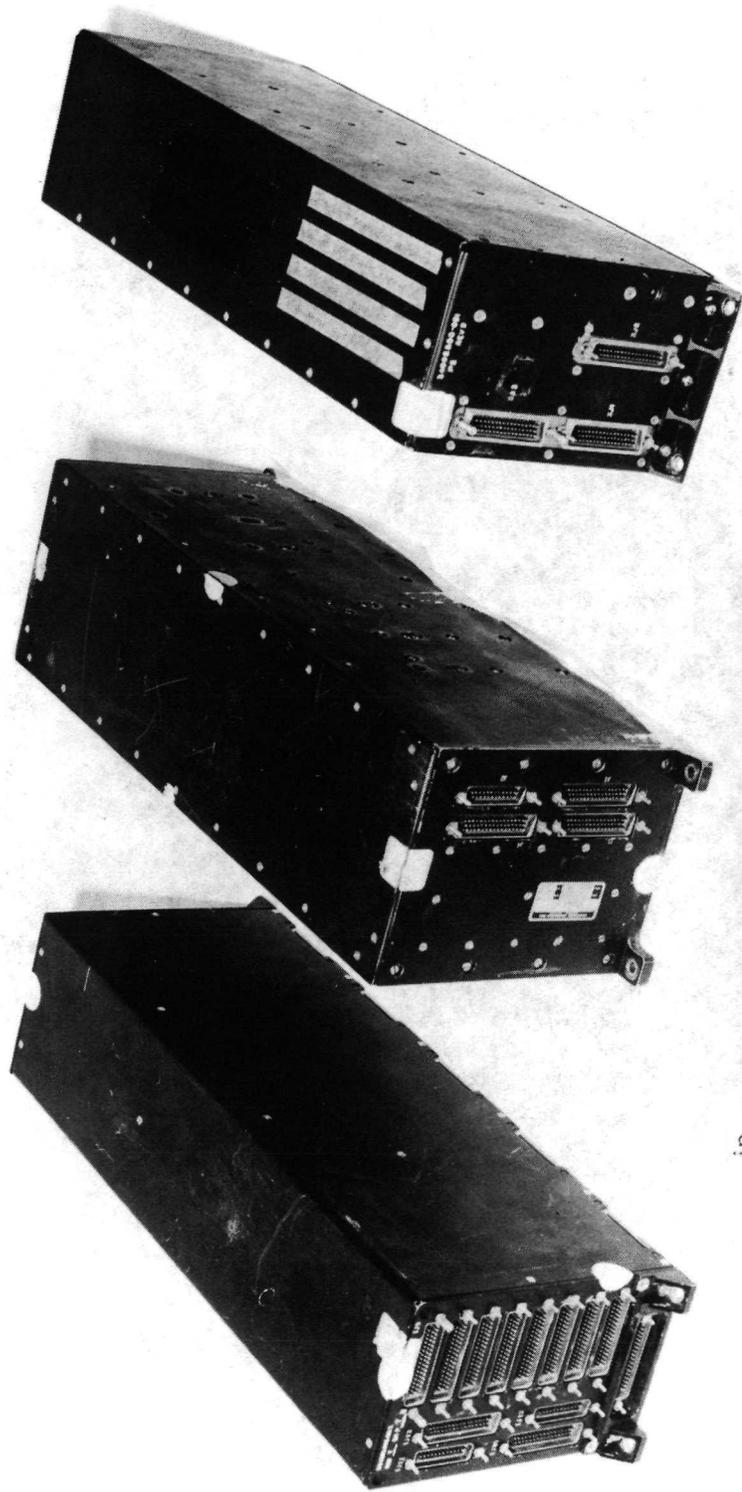
TABLE 5.—PERTINENT PHYSICAL CHARACTERISTICS OF THE SYSTEM INTERFACE UNIT

Length, m (in.) . . . . .	0.445 (17.50)
Width, m (in.) . . . . .	0.292 (11.50)
Height, m (in.) . . . . .	0.114 (4.50)
Weight, kg (lb) . . . . .	10.4 (23)
Operating voltage, dc, nominal . . . . .	28
Current required, A . . . . .	2



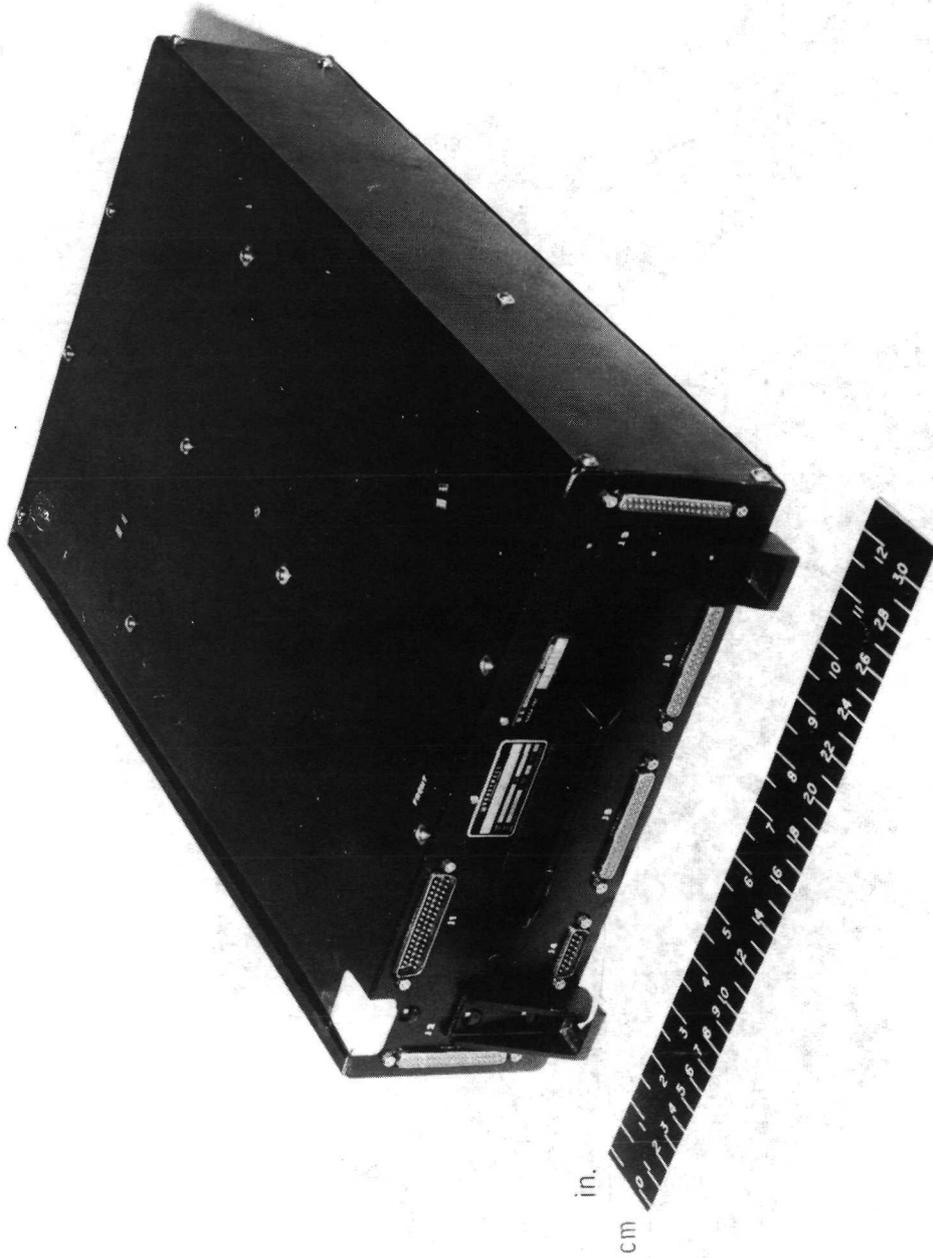
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Figure 1. Inertial measurement unit and its power supply.



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Figure 2. Central processor, memory unit, and power supply of the computer.



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Figure 3. System interface unit for the attitude reference/computer system.

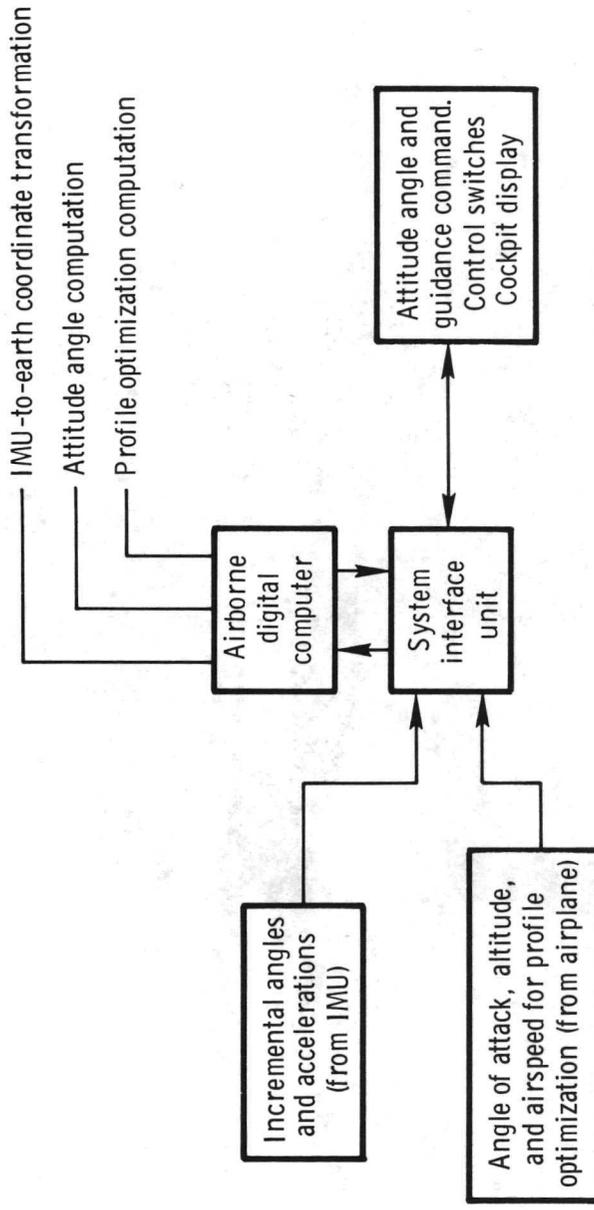


Figure 4. Block diagram of the system.

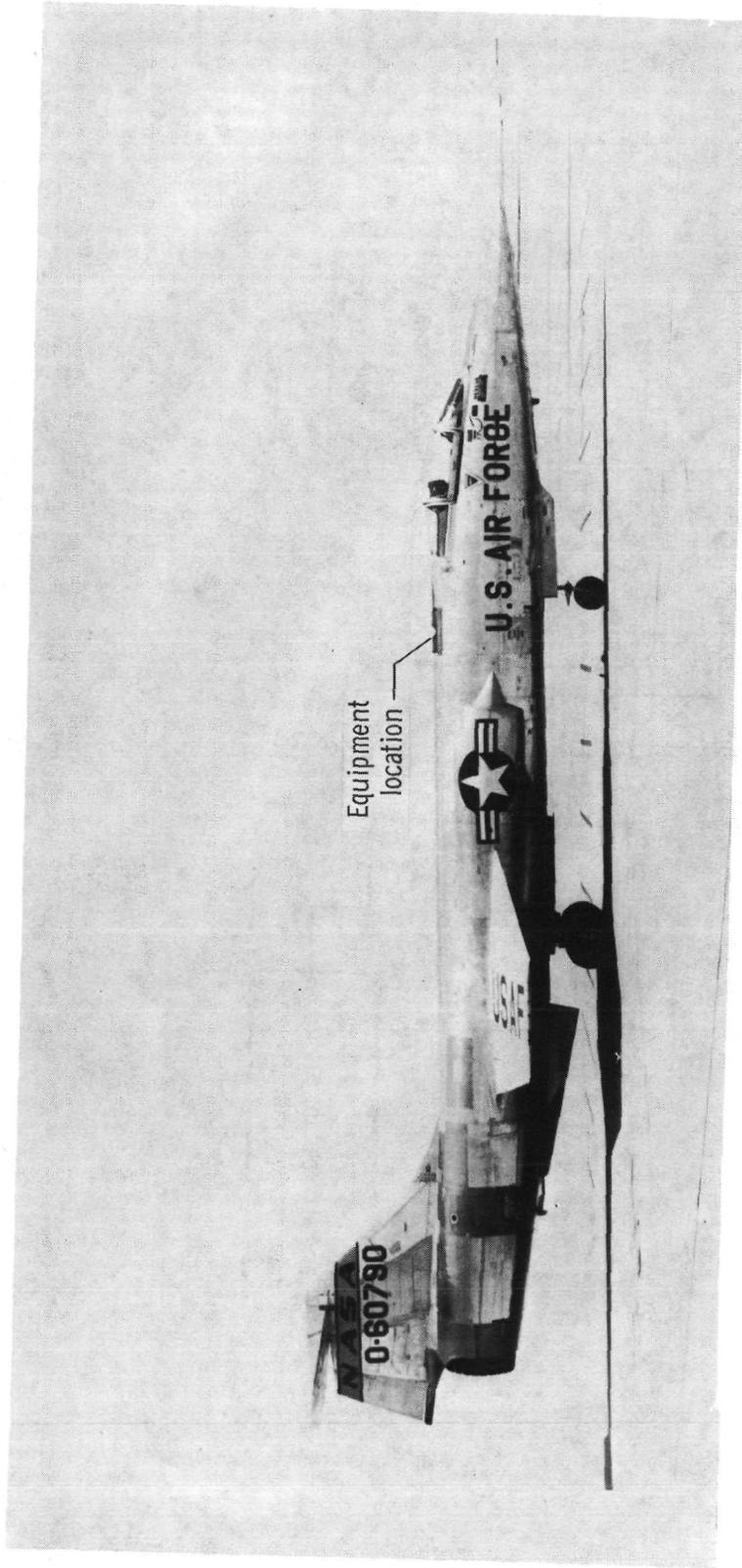
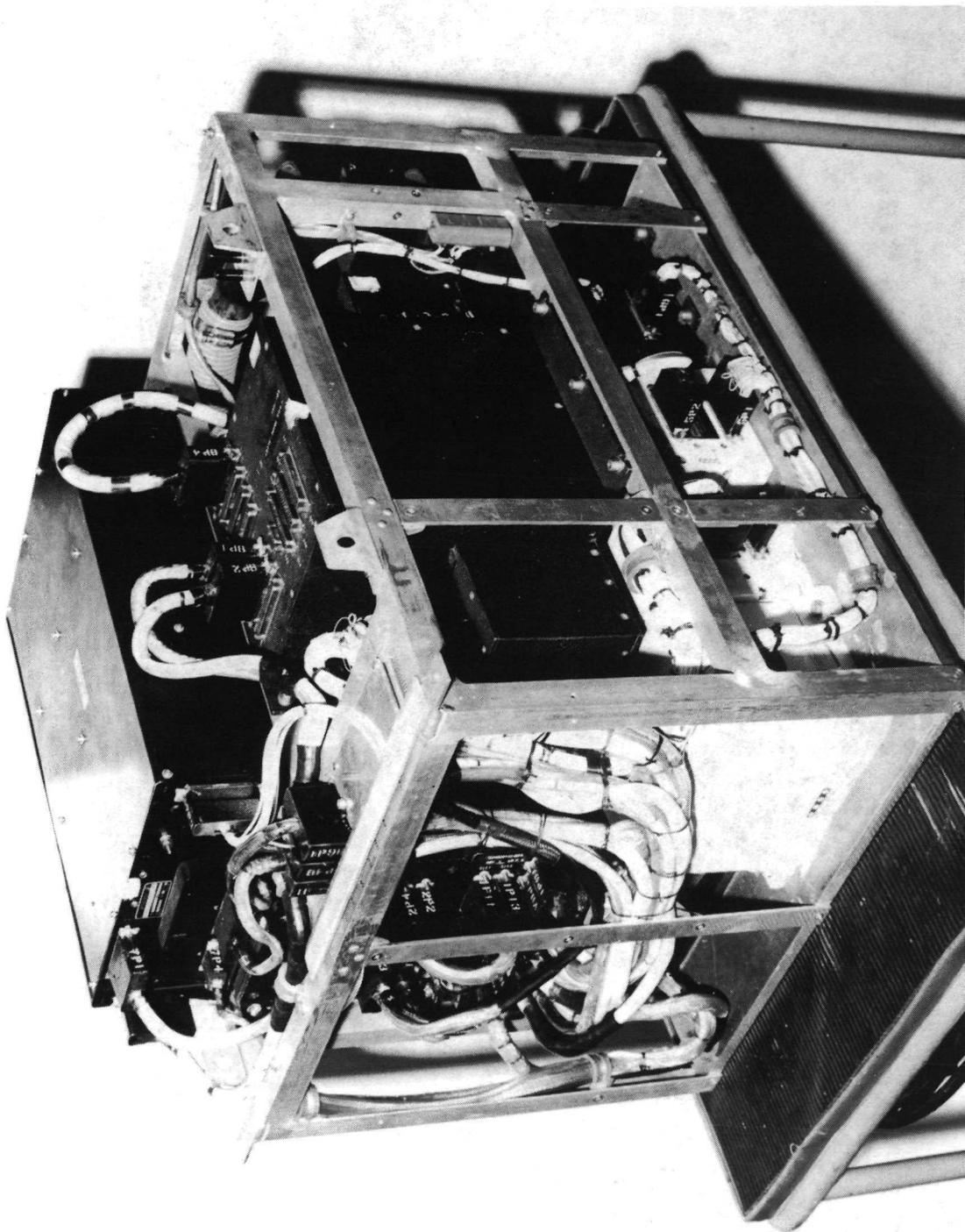


Figure 5. F-104 test aircraft and the location of the equipment.



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Figure 6. Airborne hardware package.



Figure 7. Airborne package installed in the aircraft.

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E-22798

Figure 8. Cockpit control panel and command attitude display.

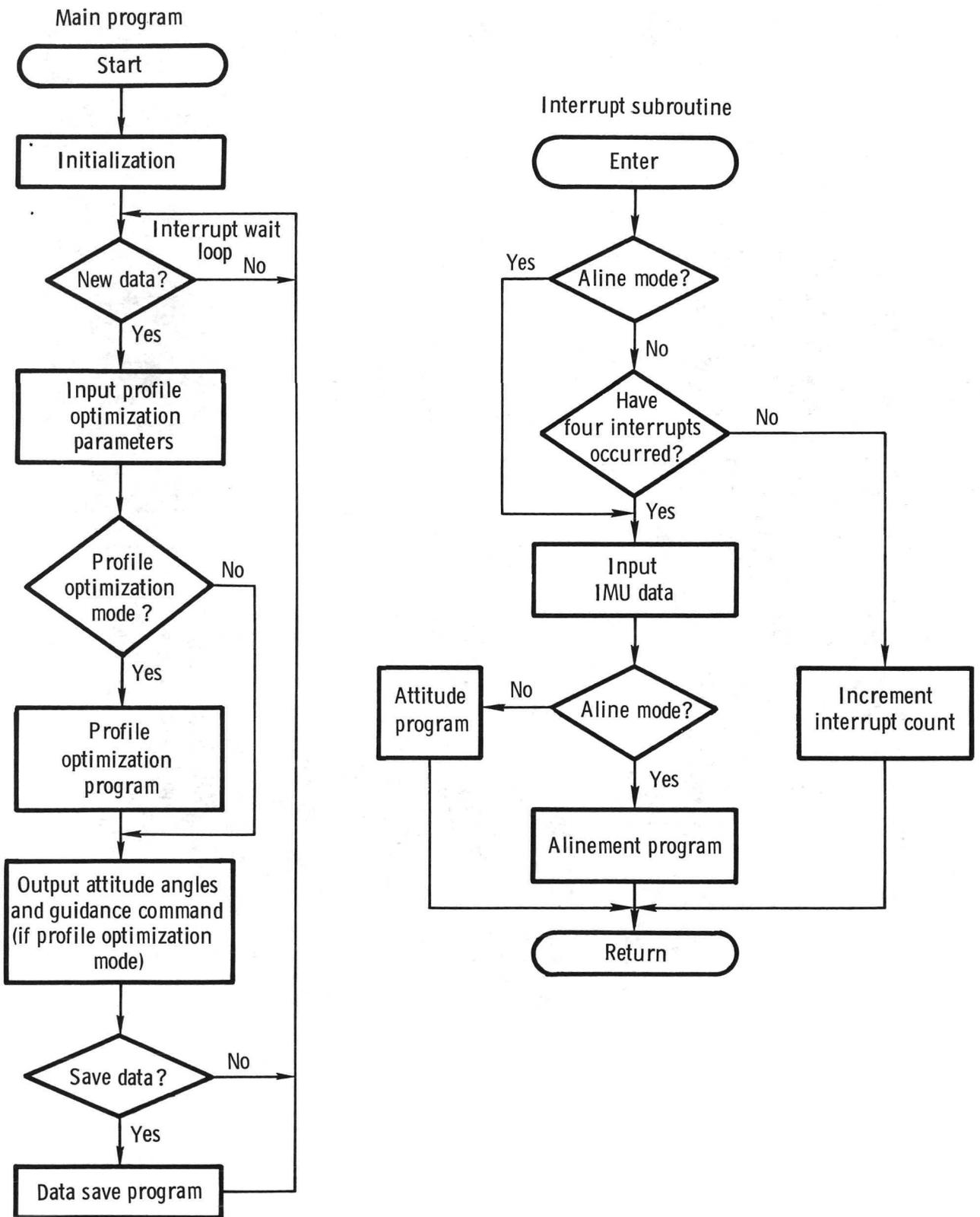


Figure 9. Block diagram of the software.

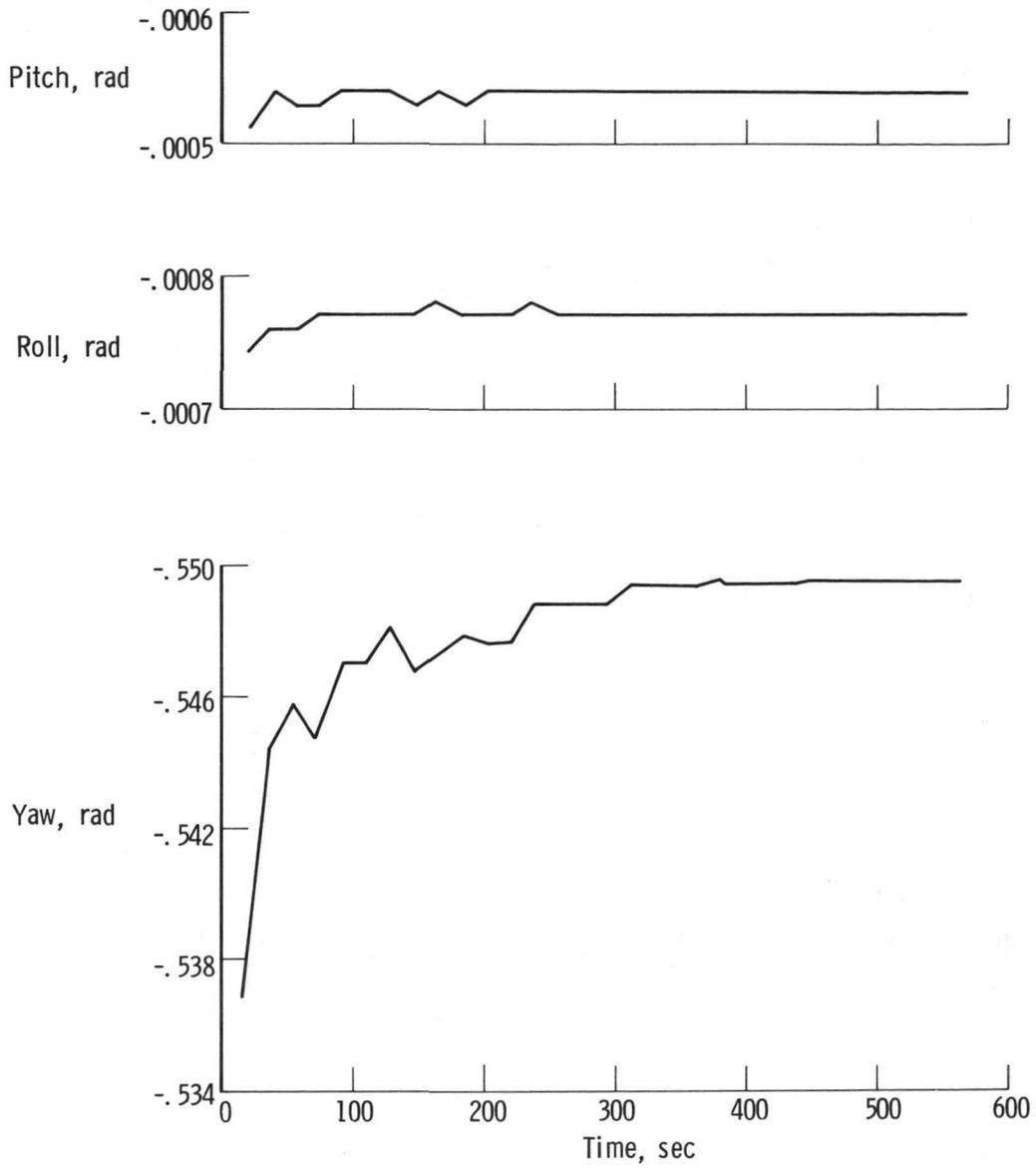


Figure 10. Time history of computed self-alinement attitude angles for a typical laboratory run.

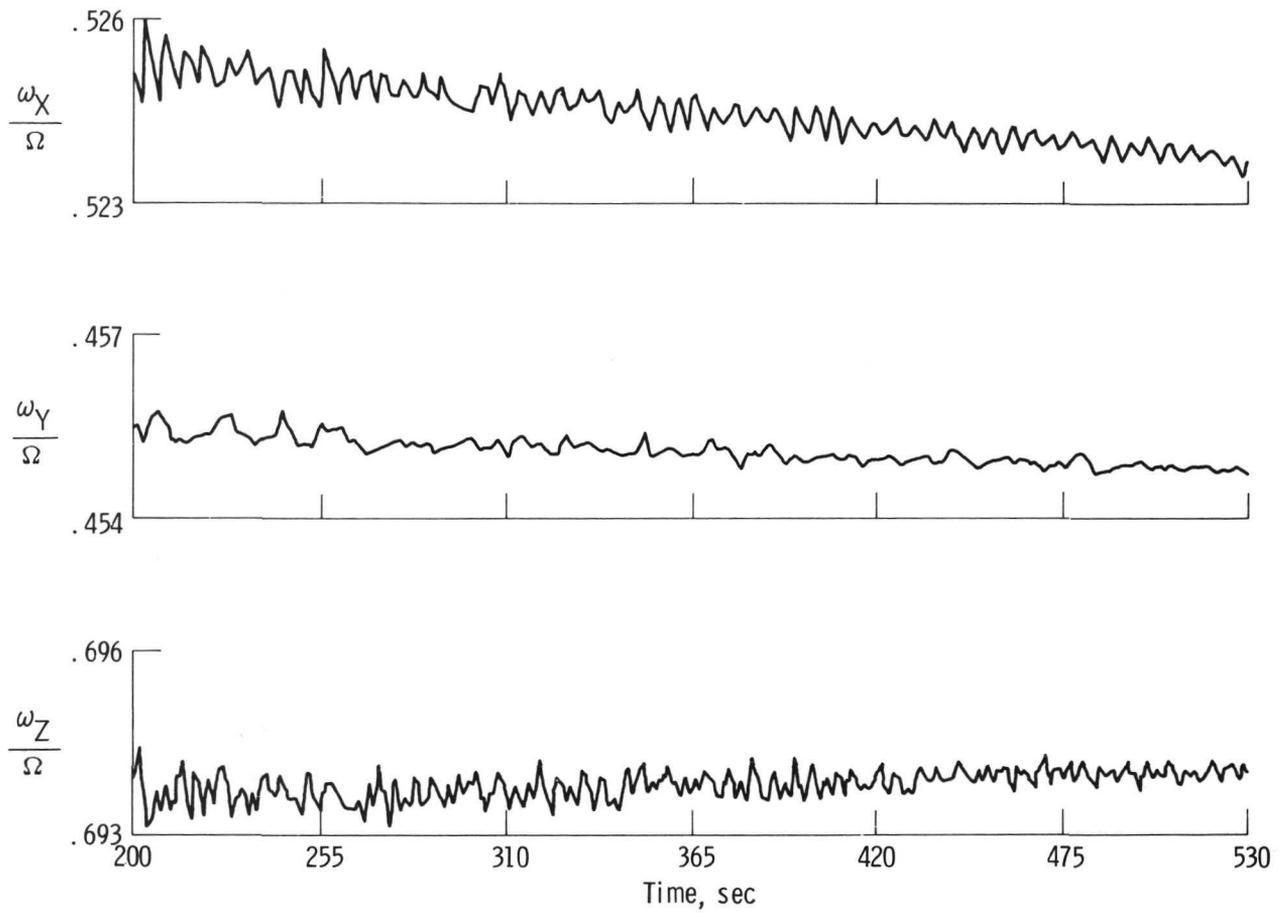


Figure 11. Time history of computed normalized angular rates during a laboratory alinement run.

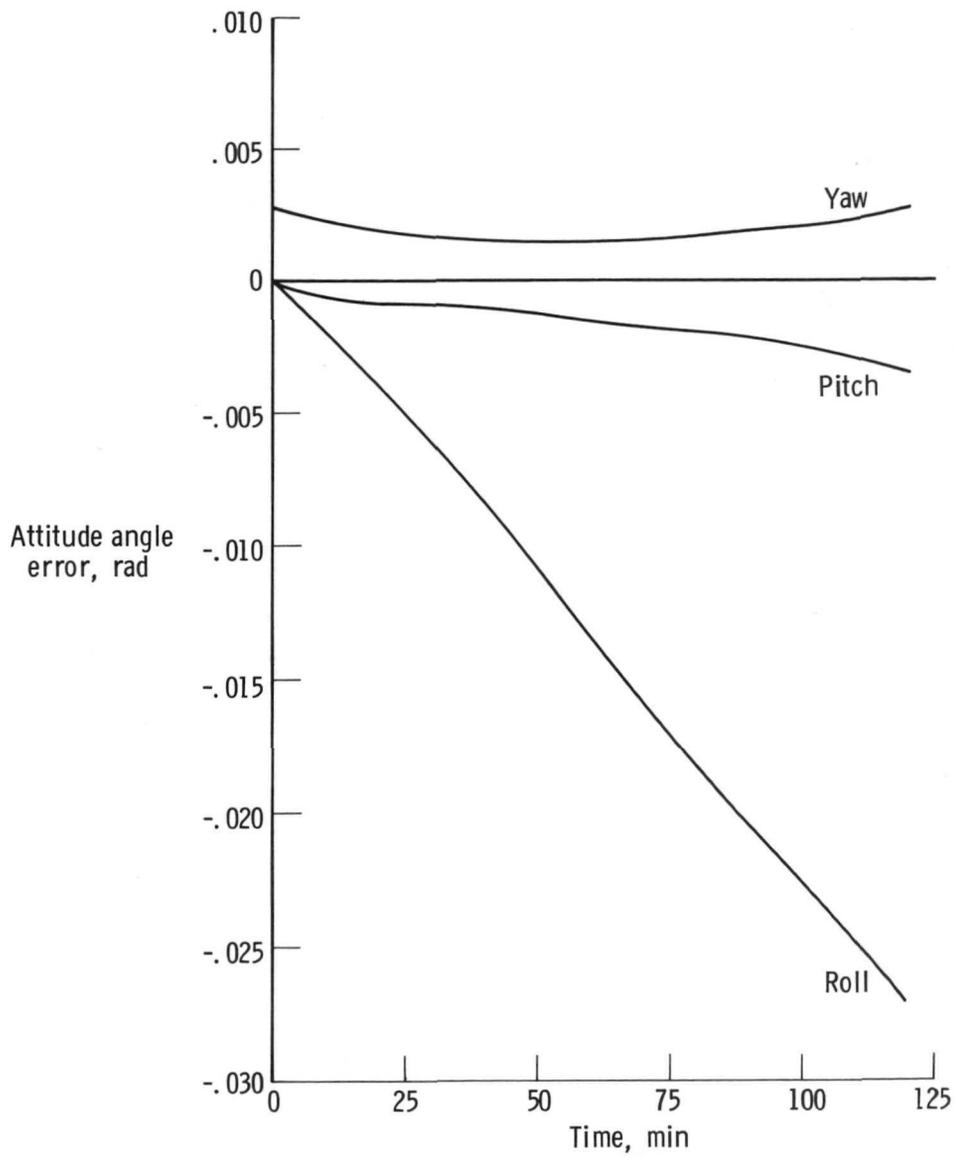


Figure 12. Time history of attitude angle errors for a typical laboratory run.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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