DOCUMENT NO. D5-17250

TITLE STATISTICAL DETERMINATION OF SPACE SHUTTLE COMPONENT DYNAMIC MAGNIFICATION FACTORS

MODEL NO. CONTRACT NO. NAS8-28650

PREPARED BY

F. Lehner

STRUCTURES AND PROPULSION

MAY 31, 1973

(NASA-CR-124337) STATISTICAL DETERMINATION OF SPACE SHUTTLE COMPONENT DYNAMIC MAGNIFICATION FACTORS (Boeing Co., Huntsville, Ala.) 81 p HC $6.25 CSCL 22B G3/31

N73-30842

Unclas 15179

ISSUE NO. ISSUED TO

THE BOEING COMPANY SPACE DIVISION LAUNCH SYSTEMS BRANCH

PREFACE

This document is submitted to the National Aeronautics and Space Administration, Huntsville, Alabama, in compliance with Contract NAS8-28650, "Statistical Determination of Space Shuttle Component Dynamic Magnification Factors."

The study was performed by The Boeing Company, Huntsville, Alabama. F. A. Lehner was principal investigator and W. C. Smith was technical advisor. J. B. Herring of Astronautics Laboratory, Analytical Mechanics Division of Marshall Space Flight Center acted as the Contract Officer Representative.

Questions concerning the document should be addressed to L. D. McTigue, who was the Boeing Study Manager. Mr. McTigue can be contacted via:

The Boeing Company
P. O. Box 1470
Huntsville, Alabama 35807
Phone: Area Code 205, 895-2020

TABLE OF CONTENTS

TABLE OF CONTENTS (CONTINUED)

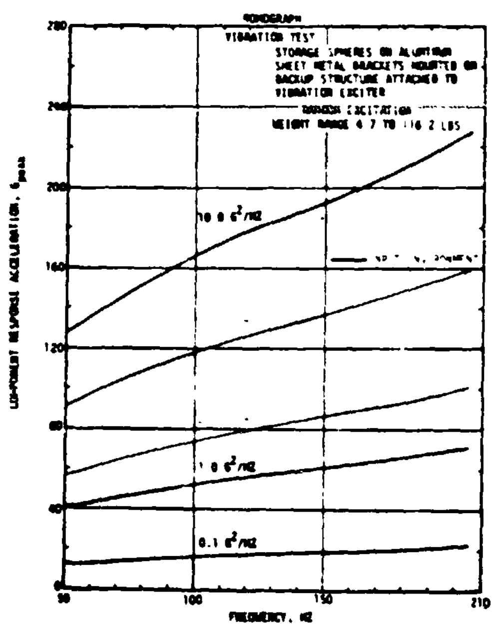
LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (CONTINUED)

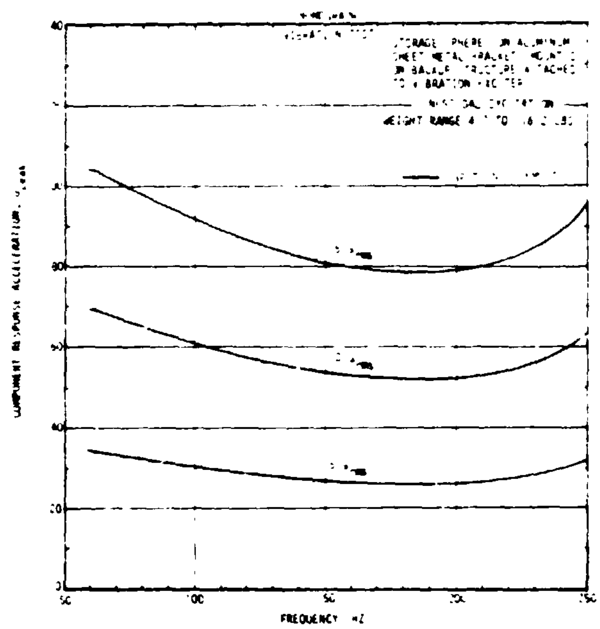
LIST OF ILLUSTRATIONS (CONTINUED)

SUMMARY

This report presents a method of obtaining vibration design loads for components and brackets. Dynamic Magnification Factors (Q) from applicable Saturn/Apollo qualification, reliability, and vibroacoustic tests have been statistically formulated into design nomographs. These design nomographs have been developed for different component and bracket types, mounted on backup structure or rigidly mounted and excited by sinusoidal or random inputs. Typical nomographs are shown below.



RANDOM INPUT NOMOGRAPH



SINUSOIDAL INPUT NOMOGRAPH

The cross-hatched lines in the table below show the specific areas for which data were available for development of this study.

TYPE OF BRACKET	RANDOM EXCITATION	SINUSOIDAL EXCITATION	BACKUP STRUCTURE MOUNTED	RIGIDLY MOUNTED
SHEET METAL BRACKETS	X	X	X	
STRUTS	X			X
SHEET METAL STRUT COMBINATION	X		X	

The data in each of these areas were categorized into statistical samples of type of components and into weight ranges defined by the specific components. The data points of Q versus component and bracket resonant frequency were assembled into these categories for development of the design nomographs. The limited data available for this study required that similar component and bracket types be assembled together to obtain a statistical meaningful sample size.

The design procedure is carried out as follows (refer to Figure 1 on page x).

1. Starting with a trial design, compute the frequency of the bracket/component system, f_n.

2. Select the appropriate nomograph from section 6.0 based on component weight, type of bracket, sinusoidal or random excitation.

3. Enter the design nomograph at the computed frequency; follow a vertical line until it intercepts the input environment curve at point P; read the design load D_L on the vertical scale.

4. Using the design load obtained from the nomograph, strength check the design and update as required.

5. Compute a new resonant frequency and repeat steps 3 and 4 as required to obtain an acceptable design.

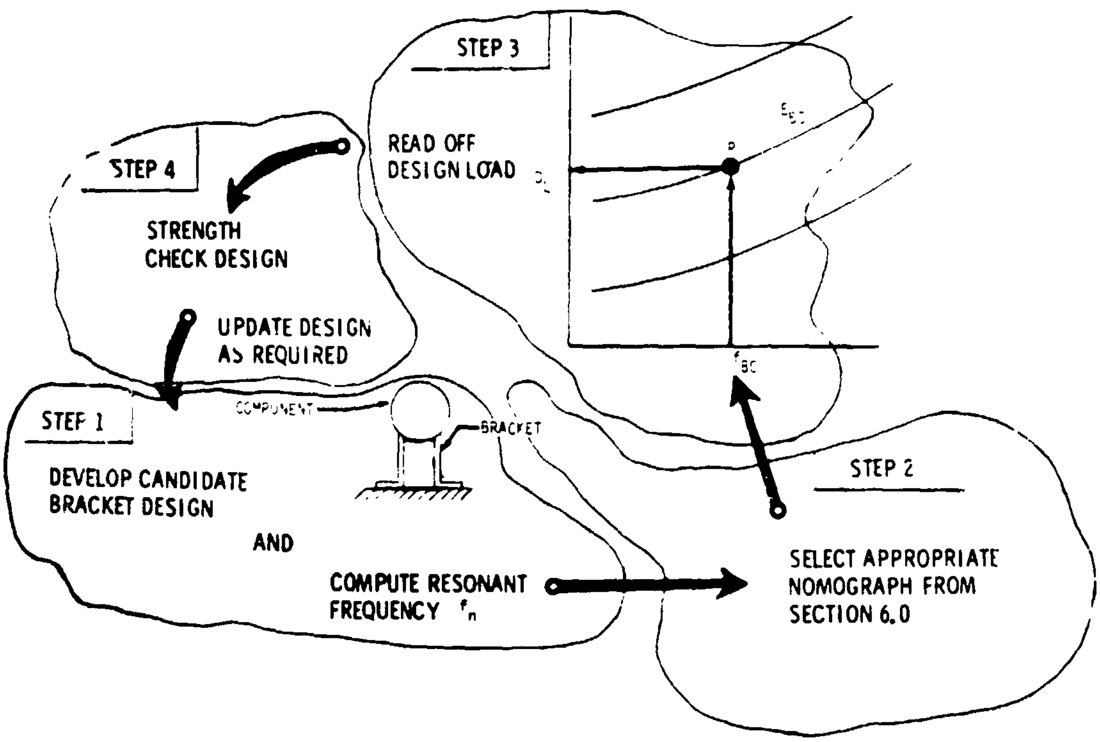


FIGURE 1 DESIGN PROCEDURE

The design nomographs were developed by statistical techniques from available data. The dynamic magnification factors for sinusoidal and random input were computed from the test data as follows:

SINUSOIDAL EXCITATION

$$Q_S = \frac{g_R}{g_I} = \frac{\text{response acceleration g}}{\text{input acceleration g}}$$

RANDOM EXCITATION

$$Q_R = \sqrt{\frac{G_R}{G_I}} = \sqrt{\frac{\text{response power spectral density } G^2/\text{Hz}}{\text{input power spectral density } G^2/\text{Hz}}}$$

These values were calculated for specific components on brackets and assembled into categories previously described.

Statistical analyses were conducted on the data points assembled into these categories. The principles of linear regression and correlation analyses were applied to the data in each category. A linear regression equation of the form

$$Q(V) = a + b \text{ VARIABLE}$$

was used to fit a line to the data scatter.

Correlation analyses were used on the data to measure the degree of association between the variable of Q versus component and bracket resonant frequency and Q versus component weight. Model regression equations and correlation coefficients were calculated for each category. Categories showing less than 0.5 correlation were eliminated.

Regression variance analyses were applied to each category to define confidence level limits for the linear regression lines developed for the design nomograph categories. A confidence level of 95% was selected as the limit in this study. This level was used to develop the design nomographs in this report.

SECTION 1

INTRODUCTION

1.0 GENERAL

This study provides a statistical determination of component Dynamic Magnification Factors (Q) that can be used in the design of components and their associated bracketry for a vibration environment. The dynamic environments associated with launch vehicles such as Saturn and Space Shuttle impose critical design requirements on components and supporting structure.

The design and verification of components and support structure to withstand vibration is a critical part of the design development phases of a launch vehicle. During the Saturn/Apollo development program numerous vibration and acoustic tests were conducted on flight hardware and support structure to demonstrate the adequacy of the design. These tests were required to assure the structural and functional integrity of the hardware when subjected to the flight environment. These tests constitute the vibration data base used to develope this study.

In conducting this study the vibration data obtained during vibration and acoustic laboratory tests, static firings and flights were reviewed in order to determine Q for significant parametric categories. The significant parametric categories investigated were type of test, component, bracket, mounting technique and excitation source. These categories were further subdivided into sub-categories which were considered to be significant in predicting Q for Space Shuttle components and associated

1.0 (Continued)

bracketry. The data scatter for each of these categories was statistically evaluated by predicting the mean Q for each category and calculating the degree of correlation of the data.

Design nomographs for each of the categories and sub-categories mentioned above were developed. Significant parameters necessary for designing components and bracketry are outlined on each nomograph to assist in selection of the appropriate nomograph. Design guidelines were also provided to assist in the initial design of space vehicle components and bracketry.

SECTION 2

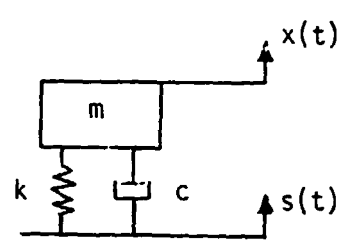
DYNAMIC MAGNIFICATION FACTOR

2.0 GENERAL

Mechanical systems of the type discussed in this report can be represented by either one or two degree of freedom systems. The equations of these systems are quite familiar and do not require detailed discussion. However, in order to define Dynamic Magnification Factor (Q) adequately, it is necessary to mathematically describe a single degree of freedom (SDF) mechanical system.

2.1 DYNAMIC MAGNIFICATION FACTOR FOR SINGLE DEGREE OF FREEDOM SYSTEM

The equation of motion for a SDF system is not limited to these systems, since in normal mode theory the differential equation of motion for a single normal mode of a multidegree of freedom system has the same form as that for a SDF system. The simplified mechanical model used in this evaluation is shown in Figure 2-1.



$$y(t) = x(t) - s(t)$$

FIGURE 2-1 SINGLE DEGREE OF FREEDOM (SDF) SYSTEM

where

m = mass (lb-sec^2/in) (Kg-sec^2/m)

c = viscous damping constant (lb-sec/in) (Kg-sec/m)

2.1 (Continued)

k = spring constant (lbs/in) (kg/m)

x = total displacement of the mass (m)

s = displacement of the base

y = x-s = relative motion of the mass

A base excitation system was chosen here because a majority of the component tests conducted on Saturn vehicles used base motion. For mathematical consideration we will be concerned with restrained vibration motion of the base to a single base excitation direction. Only steady state solution of the equations for sinusoidal excitation is necessary for defining Q. The differential equation of motion for a SDF system is:

$$m \frac{d^2x}{dt^2} + c\left(\frac{dx}{dt} - \frac{ds}{dt}\right) + k(x-s) = 0 \qquad (2\text{-}1)$$

and if related in terms of relative motion

$$m \frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = -m\frac{d^2s}{dt^2} \qquad (2\text{-}2)$$

The steady state solution of a mechanical system to sinusoidal input of the form $s(t) = Fo \sin \omega t$, where Fo and ω are the initial force and forcing frequency, has the following form:

$$\frac{X}{Fo} = T \sin(\omega t - \theta) \qquad (2\text{-}3)$$

and

$$\frac{Y}{Fo} = Q \sin(\omega t - \theta) \qquad (2\text{-}4)$$

2.1 (Continued)

where T and Q are defined as the transmissibility and dynamic magnification factor respectively. These terms can be expressed in their familiar forms as

$$T = \sqrt{\frac{1 - (2\zeta \cdot \omega/\omega_n)^2}{[1 - (\omega^2/\omega_n^2)]^2 + (2\zeta\omega/\omega_n)^2}} \qquad (2\text{-}5)$$

and

$$Q = \sqrt{\frac{1}{[1 - (\omega/\omega_n)^2]^2 + (2\zeta\omega/\omega_n)^2}} \qquad (2\text{-}6)$$

where

ζ = fraction of critical damping ($c/c_{critical}$)

ω_n = undamped natural frequency (radians/sec)

f_n = $\omega_n/2\pi$ undamped natural frequency in Hz

If the transmissibility in equation 2-5 is calculated for various values of damping ζ, a maximum steady state excitation will occur when the forcing frequency is equal to the undamped natural frequency ($\omega/\omega_n = 1$). Since this is true, we can express equation 2-6 as

$$Q_S = \frac{1}{2\zeta} = \frac{g_R}{g_I} \qquad (2\text{-}7)$$

where

g_R = response acceleration at f_n (g_{peak})

g_I = input acceleration at f_n (g_{peak})

2.1 (Continued)

For a random excitation the dynamic magnification factor can be expressed as the square root of the ratio (Reference 1).

$$Q_R = \sqrt{\frac{G_R}{G_I}} \tag{2-8}$$

where

G_R = response power spectral density at f_n (g^2/Hz)

G_I = input power spectral density at f_n (g^2/Hz)

The value Q is often expressed as the peak amplification or quality factor. It is also expressed as a measure of the sharpness of the resonant peak of a SDF system. As shown in Figure 2-2, which has obtained from Reference 2, $\Delta\omega$ is the bandwidth of the resonant peak at the half-power point (i.e., at a value of $R = R_{max}/\sqrt{2}$). The damping of the system can then be defined to a good approximation by

$$\frac{\Delta\omega}{\omega_n} = \frac{1}{Q} = 2\zeta \tag{2-9}$$

for values of damping ζ less than 0.1. From the expressions for a simple oscillator with sinusoidal excitation applied at the base we have developed the expression for Q_S and Q_R. These expressions will be used in this study to develop the design nomographs.

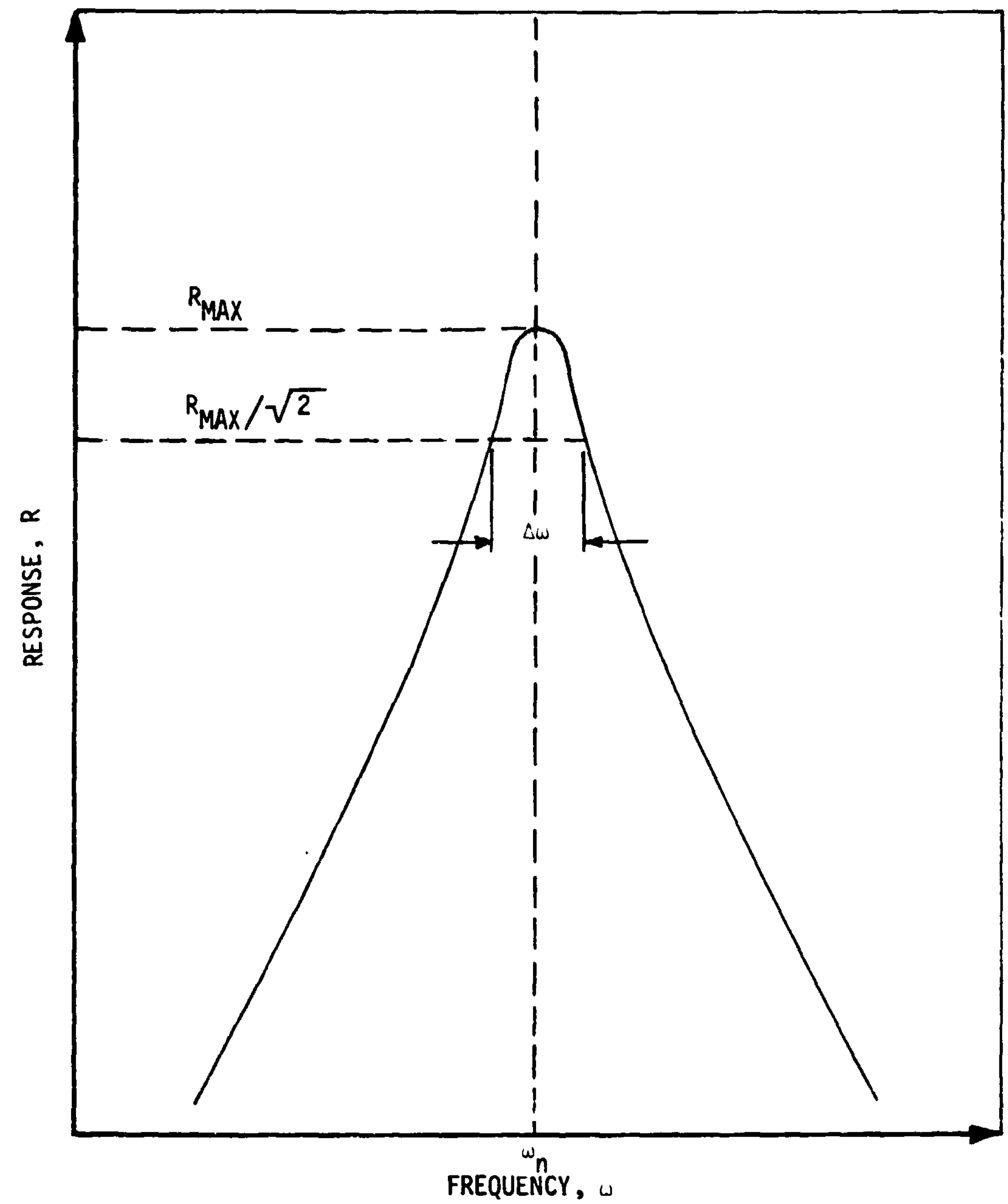


FIGURE 2-2 DAMPING IN A SYSTEM AS A FUNCTION OF SHARPNESS AND WIDTH AT THE RESONANT FREQUENCY ω_n

THIS PAGE INTENTIONALLY

LEFT BLANK

SECTION 3

DATA ACQUISITION AND EVALUATION

3.0 GENERAL

During the Saturn/Apollo developmental program, numerous vibration and acoustic tests were conducted to prove the adequacy of the flight hardware and supporting structure. These tests consisted of laboratory vibration and acoustic tests conducted on specific vehicle components, components attached to a portion of support structure, and vibroacoustic tests conducted on portions of the vehicle structure. The results of these tests were documented in qualification, reliability, acceptance and vibroacoustic test reports. A review of the documentation listing of Reference 3 was initially made to determine the number of reports which were considered applicable to this study. Approximately 1,010 reports were considered as applicable to this study. An initial screening of these reports was accomplished to eliminate those which were acceptable for use in this study. This initial screening of the documentation titles was done to eliminate tests conducted on piece parts such as relays, connectors, resistors, etc. This screening process deleted approximately 110 reports out of the 1,010 available reports. This initial evaluation of the adequacy of applicable reports produced approximately 900 reports as possible sources of vibration data necessary for determining Q. In addition, a review of the Quality and Reliability Laboratory, S&E-QUAL-ATF, reports was also conducted to supplement the reports obtained from Reference 3. A great majority of the reports stored there were available from the Reference 3. Therefore, only

3.0 (Continued)

approximately 40 additional reports were considered applicable to this study. Documentation sources at Johnson Spacecraft Center in Houston, Texas, were reviewed as another potential source of applicable reports. This review of contractor data and documentation produced three applicable reports.

The vibration measurements located on the flights and static firings of the Saturn launch vehicles were reviewed for possible cases where both input and response data might be obtained for components installed on brackets. The major objective of flight and static firing measuring programs was to determine the vibration input to flight hardware and not the response of the hardware. Therefore, no applicable data were available.

3.1 ACQUISITION OF SOURCE TEST DATA

3.1.1 RELIABILITY AND QUALIFICATION TEST DATA

The major contributing source to the effectual completion of this study was the results of reliability and qualification tests conducted on the S-IB, S-IV, and S-IC, S-II, S-IVB and IU flight hardware. However, it must be noted that most of the reliability and qualification vibration test measurements used during testing were not analyzed. This severely reduced the number of data samples available for this study. Approximately 94% of the reliability and qualification test reports reviewed contained no vibration data which could be used to determine Q.

3.1.1 (Continued)

Of the remaining 6%, approximately one-half of these documents were the results of tests conducted on components which were not considered to be representative of conventional component and bracket configuration. Those components included such items as components on isolators, components on panels, and fuel feedlines with bellows and expansion joints.

The usable data acquired from the reliability and qualification tests consisted of 21 data samples. The maximum component response was recorded for the first mode of the component and bracket for each of three mutually perpendicular axes (longitudinal, radial, and tangential). This mode was considered as the natural frequency of the system. Additional peak response amplitudes were recorded at various other modes in order to be certain that the maximum Q would be obtained for the component and bracket. These response amplitudes were recorded for both the sinuosoidal and random excitation in the form of acceleration amplitude (g_{peak}) versus frequency and power spectral density (g^2/Hz) versus frequency, respectively. The input levels to the bracket were also recorded for the frequencies established by the response accelerometer. The maximum Q values for the natural frequency of the systems were calculated from equations 2-7 and 2-8 for each axis of excitation. These values along with significant parameters associated with the component, bracket and support structure were recorded.

3.1.2 VIBROACOUSTIC TEST DATA

Laboratory acoustic tests conducted during the Saturn/Apollo program were reviewed for applicable component and bracket input and response vibration data. These tests revealed that very few components had both input and response measurements. A majority of the components that were instrumented were not mounted on flight bracketry. Only five applicable data samples were obtained for this study. The Q values were calculated and the significant parametric information related to the component, bracket and supporting structure were recorded.

3.1.3 FLIGHT AND STATIC FIRING TEST DATA

During the developmental stages of the Saturn program, flight and static firings were heavily instrumented with vibration measurements. These measurements were normally located at the input to the component and did not measure the response. An evaluation was made of all the vibration locations for the Saturn flight vehicles and stage static firings to determine those locations which had input and response vibration data for components on brackets. Neither test program produced components and bracket categories which could be used in this study.

3.2 EVALUATION OF THE SOURCE TEST DATA

The source test data as described in Section 3.1 represent the total accumulation of test data applicable to the development of specific parametric categories. The test data for various components on brackets

3.2 (Continued)

consisted of 21 data points obtained from the excitation categories of Qualification and Reliability Tests and five data points from Vibro-acoustic Tests. These 26 data points were assembled into the categories outlined in Section 4.0 and constitute the data base used in this study.

SECTION 4

DEVELOPMENT OF DYNAMIC MAGNIFICATION FACTOR CATEGORIES FOR STATISTICAL ANALYSES

4.0 GENERAL

The vibration data obtained during the Saturn/Apollo development program and described in the flow chart of Figure 4-1 was ass d into categories defined and described below. These catego were:

Type of Test

Type of Component

Type of Bracket

Type of Mounting

Type of Excitation Source

Type of Test - Defined under this category are Qualification and Reliability and Vibroacoustic tests.

Type of Components - These components as defined by the data were valves, spheres, modules, solid retro-rocket motors and batteries.

Type of Brackets - The brackets defined by the data were sheet metal brackets, forged brackets, machined brackets and struts. All brackets used in this study were aluminum.

Type of Mounting - The methods of mounting were defined as components on brackets mounted on backup structure and components on brackets mounted rigidly. All of the vibroacoustic tests had backup structure.

Type of Excitation Source - Under this category were sinusoidal and random sources.

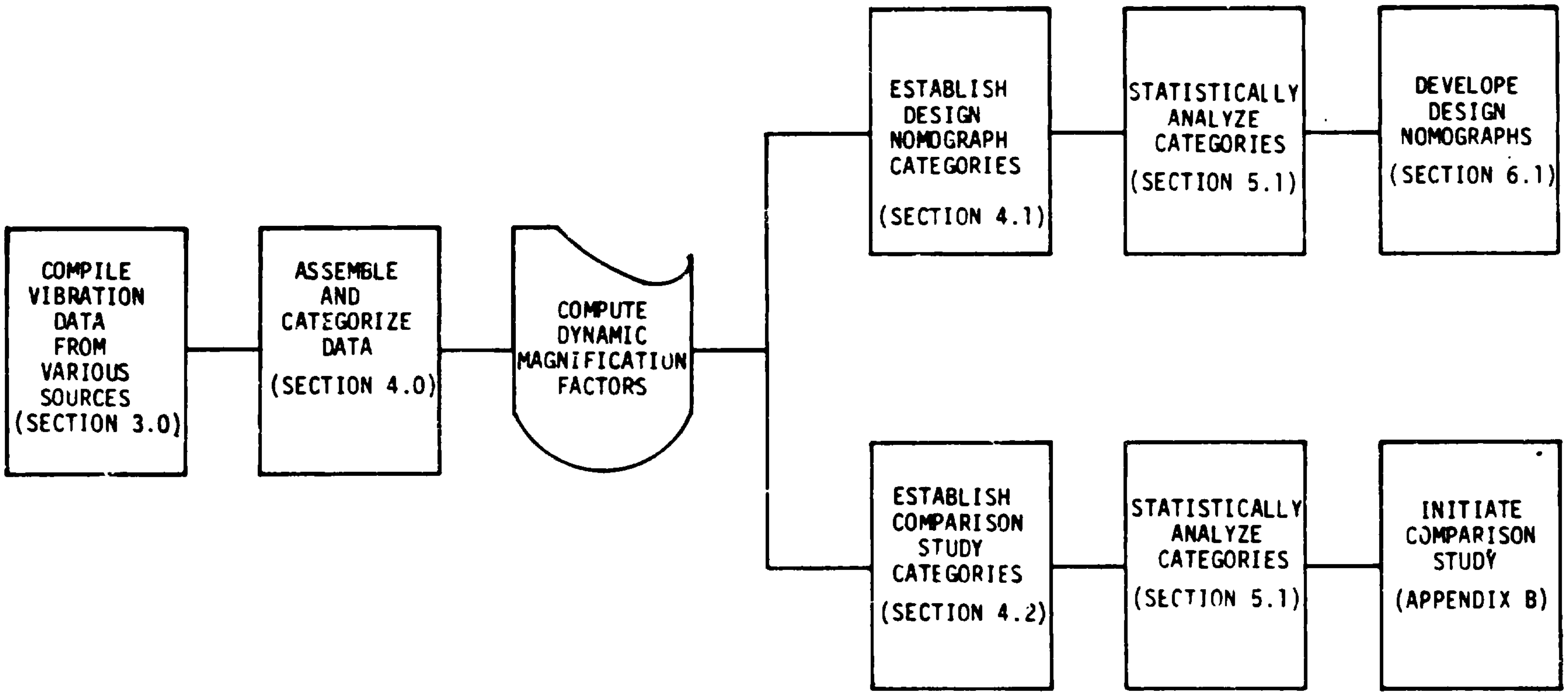


FIGURE 4-1: DATA DEVELOPMENT FLOW CHART

4.0 (Continued)

Statistically meaningful results were developed for aluminum sheet metal brackets and aluminum struts. Lack of available data did not permit evaluating other bracket types.

The input and response vibration data obtained at the first mode of the specific component and bracket were applied to equations 2-7 and 2-8 to calculate the Q. These Q values as a function of the fundamental resonant frequency (first mode) of the individual components and brackets were assembled into the specific categories established above. The component weights were recorded for each component and bracket, along with the resonant frequency and Q values in order to develop the design nomograph categories and comparison study categories described in the following sections.

4.1 CATEGORIZATION OF DATA FOR DEVELOPMENT OF DESIGN NOMOGRAPHS

The specific data categories defined in Section 4.0 were evaluated and reassembled into categories which could be used to develop design nomographs. This reassembling was necessary due to the lack of sufficient data to develop all of the categories listed in that section. The data were assembled into the categories defined below for development of design nomographs.

I. Qualification and Reliability Tests
   - A. Components on Aluminum Sheet Metal Brackets Mounted on Backup Structure and Attached to Vibration Exciter
   - B. Storage Spheres on Aluminum Sheet Metal Brackets Mounted on Backup Structure and Attached to Vibration Exciter

4.1 (Continued)

C. Solid Retro-Rocket Motors on Aluminum Sheet Metal Brackets and Struts Mounted on Backup Structure and Attached to Vibration Exciter

D. Components on Aluminum Struts Rigidly Mounted to Vibration Exciter

II. Vibro-Acoustic Test

A. Storage Spheres on Aluminum Sheet Metal Brackets Mounted on Backup Structure

The data points in these design nomograph categories were subdivided into type of excitation source (sinusoidal or random). After a thorough evaluation of the data points in each of these categories listed above, it was determined that component and bracket resonant frequency would be the variable used to define Q. This variable was chosen because most bracket designers optimize their designs based on the natural frequency of the system. These categories were also divided into weight ranges depending upon component weights in each category. These categories as defined were statistically analyzed as outlined in Section 5.0.

4.2 CATEGORIZATION OF DATA FOR DEVELOPMENT OF COMPARISON STUDY

In addition to development of design nomographs, a significant part of this study was to define significant parameters which contribute to changes in Q. The method chosen to define these parameters was a comparison study. The comparison study consisted of comparing data trends for random versus sinusoidal excitation and rigidly mounted versus backup-structure mounted

4.2 (Continued)

components on brackets. Initially, it was decided that component and bracket resonant frequency would be the variable used to define Q, and to develop this portion of the study. However, no comparison could be made between rigidly mounted and backup structure mounted components on brackets, since all of the design nomograph categories were related to specific types of components (see Section 4.1). Using component weight as the variable defining Q, a comparison could be made between components and brackets mounted on backup structure and rigidly mounted. The comparison study categories developed using component weight as the parameter to define Q were:

(1) Components on Aluminum Sheet Metal Brackets Mounted on Backup Structure Attached to Vibration Exciter

(2) Components on Aluminum Sheet Metal Brackets Rigidly Mounted to Vibration Exciter.

The data assembled into the categories defined in Section 4.0 were reassembled into these categories. These comparison study categories were subdivided into the weight ranges specified in Appendix B. These categories were also divided into specific type of excitation source (sinusoidal and random).

As can be seen the two categories listed above are a considerable reduction in effective categories when compared to those defined in Section 4.1. The number of data points available for this portion of the study severely limited the number of categories developed. When using frequency as the variable to define Q, all three orthogonal axes of excitation were

4.2 (Continued)

assembled together to give approximately 72 data points. However, using component weight as the variable limited the data points to 26. These 26 data points in the form of Q versus component weight were assembled into the categories defined above for statistical analyses outlined in Section 5.0.

SECTION 5

STATISTICAL ANALYSES OF DYNAMIC MAGNIFICATION FACTOR CATEGORIES

5.0 GENERAL

Statistical analyses were conducted on the data points assembled into the specific categories presented in Sections 4.1 and 4.2. The statistical methods chosen to determine the best functional relationship between the variables of Q, (component weight and component and bracket resonant frequency) are linear regression and correlation analyses. As specified in the Sections 4.1 and 4.2 the vibration data in the form of Q versus component and bracket resonant frequency and component weight were the related variables to be defined by regression analyses. Correlation analyses were used to measure the degree to which the different variables are associated.

5.1 REGRESSION AND CORRELATION ANALYSES

The principles of regression and correlation analyses are widely used statistical techniques for predicting or estimating parametric relationships. In regression analyses these estimates or predictions require that a functional relationship be found between two or more related variables. It is also desirable to know the strength of this relationship. Regression methods are used in this study to establish the best functional relationship between variables. Correlation methods are used to measure the degree to which these variables are associated. These methods are well known and will be briefly explained in the following paragraphs.

The method of least squares is the most common approach in fitting a regression line to a set of data. The sufficiency of the data population

5.1 (Continued)

in each category is initially evaluated to determine if there is a sufficient number of data points in a category. Less than three data points would not be enough justification for consideration as a significant category. As specified earlier in this report, Q will be the dependent variable which will be defined individually by the independent variables, component and bracket resonant frequency and component weight. A mean Q as a function of frequency was calculated for the component and bracket resonant frequency range. Mean Q as a function of weight was also calculated for component weight ranges of the categories developed for the comparison study. Regression and correlation analyses were performed on Q(f) and Q(w) over the frequency and weight ranges of interest. It was determined that the equation that could define Q for development of design nomographs would be a linear regression equation of the form

$$Q(f) = a + b \text{ FREQUENCY} \qquad 5\text{-}1$$

and the equation necessary for development of the comparison study would be

$$Q(w) = c + d \text{ WEIGHT} \qquad 5\text{-}2$$

where f and w are the component and bracket resonant frequencies and component weight respectively. In these equations, frequency and weight are the independent variables while, a and b, and c and d, are the least square estimators of the regression coefficients. Model regression equations are developed for each significant category defined and are presented in Appendices A and B.

5.1 (Continued)

Correlation analysis by its nature is closely associated with the concepts of regression analyses. Correlation analyses can be used to determine the degree of association between variables in order to define how well the regression equation fits the data sets. For the purpose of this report these values will be denoted as correlation coefficient (r) and are expressed as

$$r = \frac{\Sigma XY}{\left[(\Sigma X^2)\ (\Sigma Y^2)\right]^{1/2}} \qquad 5\text{-}3$$

where X and Y are defined as $X = X - \overline{X}$ and $Y = Y - \overline{Y}$. Here X is used to define the component weight and natural frequency and Y is the Q value. In equation 5-3, r will assume the sign of ΣXY and hence the same sign as b and d of equations 5-1 and 5-2. Further, r will assume a value between $-1 \leq r \leq 1$, where -1 represents perfect negative linear association in the sample and +1 represents perfect positive association in the sample. A value of zero is interpreted to mean no linear association between X and Y in the sample. The individual correlation coefficient for each data set is presented on the plots presented in Appendices A and B.

SECTION 6

DEVELOPMENT OF DESIGN NOMOGRAPHS AND DESIGN GUIDELINES

6.0 GENERAL

The design of vehicle components (valves, spheres, retro-rocket motors and modules) and secondary structure (bracketry) usually require complex analysis procedures. It is therefore important to supply to the design engineer a simplified yet reasonably accurate method of developing optimum designs for components and their bracketry. The design nomographs which are developed in this section have been developed to give the designer a simplified method of determining the maximum loads (G_{peak}) for specific types of components mounted on brackets. The design nomograph categories were statistically analyzed by regression variance method to define the 95% confidence level Q. These confidence levels are presented in Appendix A. The confidence level trends were applied to well known equations to determine the maximum loads (G_{peak}) which are presented in the design nomographs of Section 6.2.

In addition to the design nomographs there are some design guidelines which have been expressed as specific concepts which can be applied in the design of new components and brackets. These guidelines are listed in Section 6.3 of this report.

6.1 DEVELOPMENT OF DESIGN NOMOGRAPHS

In the past, numerous authors have sought to develop quantitative values for development of design factors. These design factors were expressed as constants which could guarantee the life of a particular component and

6.1 (Continued)

bracket. These quantitative values were obtained from extensive vibration tests and analyses conducted on simplified component and bracket combinations. However, these values are difficult to define for complex components and brackets such as those which were used on the Saturn/Apollo vehicles and spacecraft. With this in mind, major emphasis in this study was given to the development of vibration test data obtained during the Saturn/Apollo program. Those data as defined in the preceding sections have been formulated into mean Q as a function of component and bracket resonant frequency. The data are plotted in Figures A-1 through A-9 in the form of mean linear regression lines. The model equations as denoted on the plots for each category was further analyzed by Regression Variance methods defined in Reference 4. This analysis method is used to determine the confidence intervals for the regression coefficients of equation 5-1. The coefficients are expressed as estimates in the linear regression equations presented on the figures. These estimates which are expressed in equation 5-1 as coefficients a and b are subject to error. It is therefore necessary to determine with some degree of confidence that the data points be below a defined limit. The limits of the resulting confidence intervals for the estimated coefficients defined in the model regression equations expressed on these figures are based on the Student "t" distribution of statistical analyses. A confidence level of 95% was selected as the limits for the coefficients. The 95% confidence level band has been defined as a positive and negative excursion about the mean Q. For this study we are concerned with the maximum Q as a function of frequency and, therefore, only the positive excursion will be used to

6.1 (Continued)

develop the design nomographs in Section 6.2. The correlation coefficient (r) defined in Section 5.1 was calculated for each design nomograph category and is presented on the plots of Appendix A as a reference to indicate how well the linear regression line fits the data scatter. As can be seen, all of the correlation coefficients were above 0.50 which indicates that the mean regression line fits the data scatter. A correlation coefficient of zero would indicate no linear association between the variables used in the regression analyses.

As can be readily seen in Figures A-1 through A-9, the estimated trends are comparatively flat for the total frequency range and, when comparing random and sinusoidal trends, very little difference exists between the mean estimated levels.

6.2 APPLICATION OF DESIGN NOMOGRAPHS

A nomograph as presented in this report is a graphical solution in cartesian coordinates of the relationship among three or more variables. The variables for this study are expressed on each design nomograph as the predicted or measured input acceleration or power spectral density, frequency and Dynamic Magnification Factor (Q). Since both sinusoidal and random excitation sources have been defined by nomographs, either input acceleration (G_{rms}) or power spectral density (G^2/HZ) can be used to obtain new component response acceleration in G_{peak} for the categories and weight ranges defined on the nomographs. The design nomographs are presented in Figures 6-1 through 6-9 along with the necessary information required to select the appropriate nomograph. The nomographs developed

6.2 (Continued)

for the sinusoidal excitation source were developed from equation 2-7. The random excitation source nomographs were developed from equations presented in Reference 1. It was determined that most designers require the peak response acceleration rather than the root mean square acceleration. Therefore, equation 2-7 was multiplied by the constant 1.414 and the equation presented in Reference 1 was multiplied by the constant 2.2. This constant has been stated in numerous references as the 2.2σ value or 2.2 times the root mean square acceleration. So this value is used to define a limiting random vibration level, and states with 97.5% confidence that the equivalent peak acceleration of sinusoidal vibration falls within this limiting level. The equation necessary for determining the new component response acceleration in G_{peak} is presented on each nomograph as a reference. The nomographs developed for the sinusoidal excitation are plotted for incremental inputs of 5, 10, 15 G_{rms} while the random excitation is 0.1, 1.0, 2.0, 5.0 and 10.0 G^2/HZ. By selecting the appropriate input acceleration or power spectral density, the component response acceleration can be obtained. This response acceleration along with component weight can be used to determine the maximum allowable stress which could be used to design components and brackets.

A review of the equations presented on each design nomograph will reveal that the component response acceleration for sinusoidal input will be proportional to the increase in input acceleration. The component response acceleration for a random input will be proportional to the square root of the input power spectral density.

6.2 (Continued)

The following procedure should be used to determine the maximum response acceleration for development of new vehicle component and bracket designs.

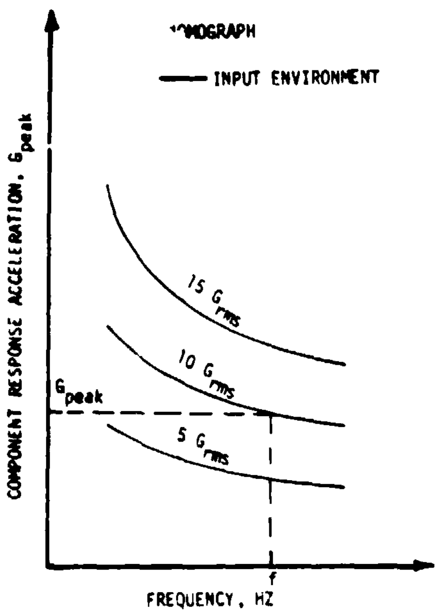
1. Select the specific type of component and bracket category desired from the information located on Figures 6-1 through 6-9.
2. Selections should be made based upon type of component, bracket and weight range of interest.
3. Select the appropriate excitation source required. Sinusoidal and random graphs are presented for most of the categories in Figures 6-1 through 6-9. It is suggested that, when making the selection between sinusoidal and random nomographs, the nomograph giving the highest component response be used.
4. If the predicted or measured input acceleration (G_{rms}) or power spectral density (G^2/HZ) is known along with component and bracket resonant frequency, the design nomographs can be used, as shown on the following page, to determine the component response acceleration (G_{peak}) for sinusoidal or random excitation.

It must be realized that these acceleration values are representative of the specific bracket design for components on the Saturn/Apollo program and the component response is limited by these specific types of designs.

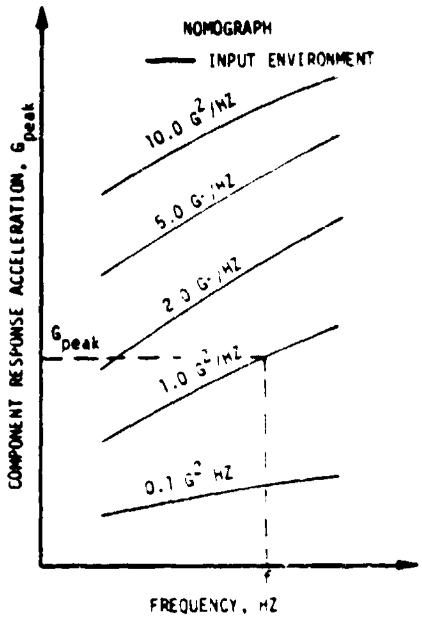
6.3 DESIGN GUIDELINES

The repetitive loads such as produced by vibration and acoustic environments are the major contributing factors in the design of components and brackets. In order to adequately design components and brackets to these

6.3 (Continued)

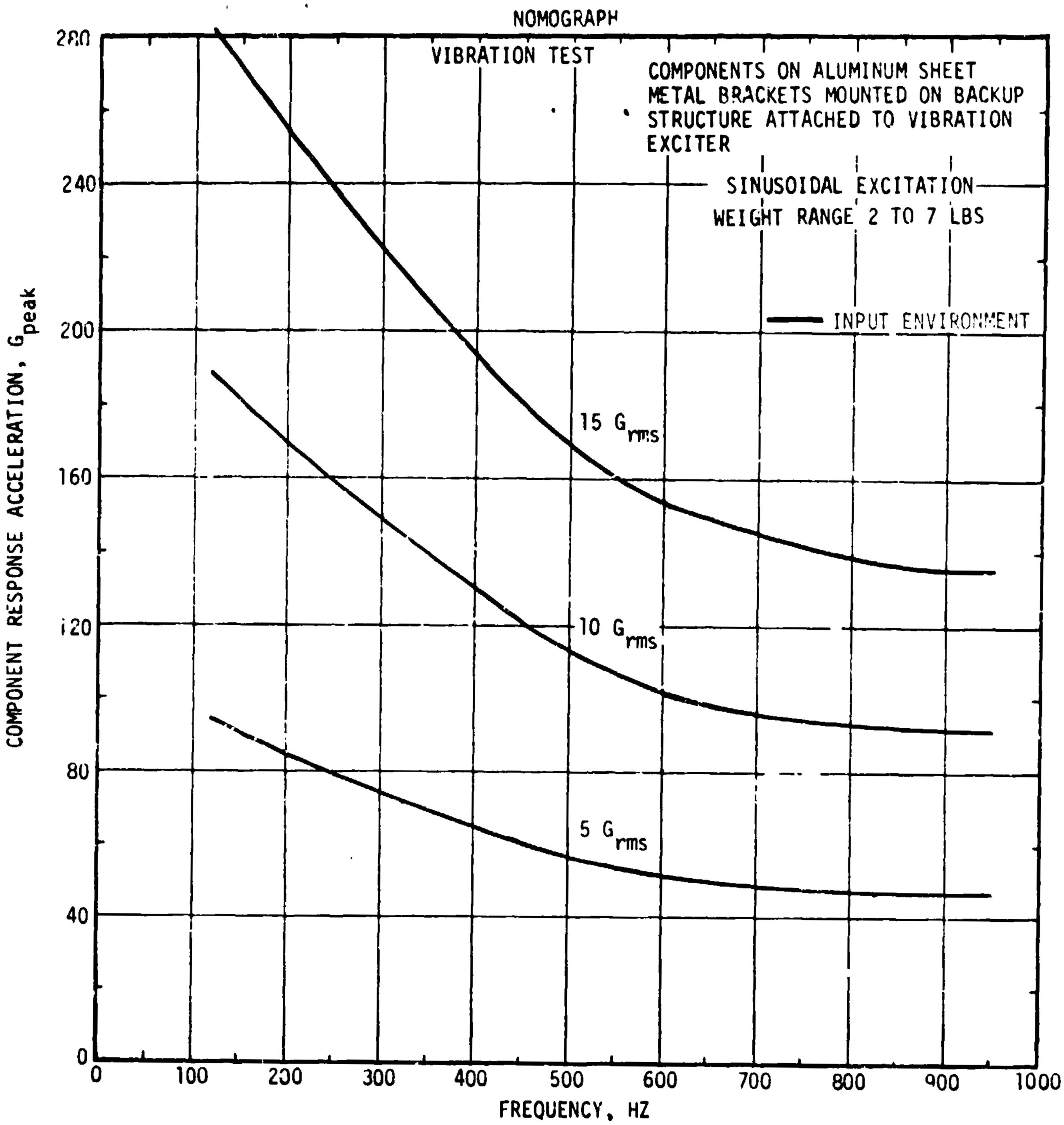


SINUSOIDAL EXCITATION



RANDOM EXCITATION

loads a designer must have prior knowledge of the parameters necessary to define these loads. The various differences in loads, materials, stresses and environments associated with the operational phases of a vehicle make it an almost impossible task to define these parameters with any degree of reliability. Design nomographs presented in Section 6.2 were developed in order to predict with good reliability the maximum loads for designing new vehicle components and brackets. These design nomographs can be more advantageous if some design concepts could be defined to assist the design engineer in the initial design of a new component and bracket. These concepts have been developed by numerous authors in the past and are presented below as specific design guides.



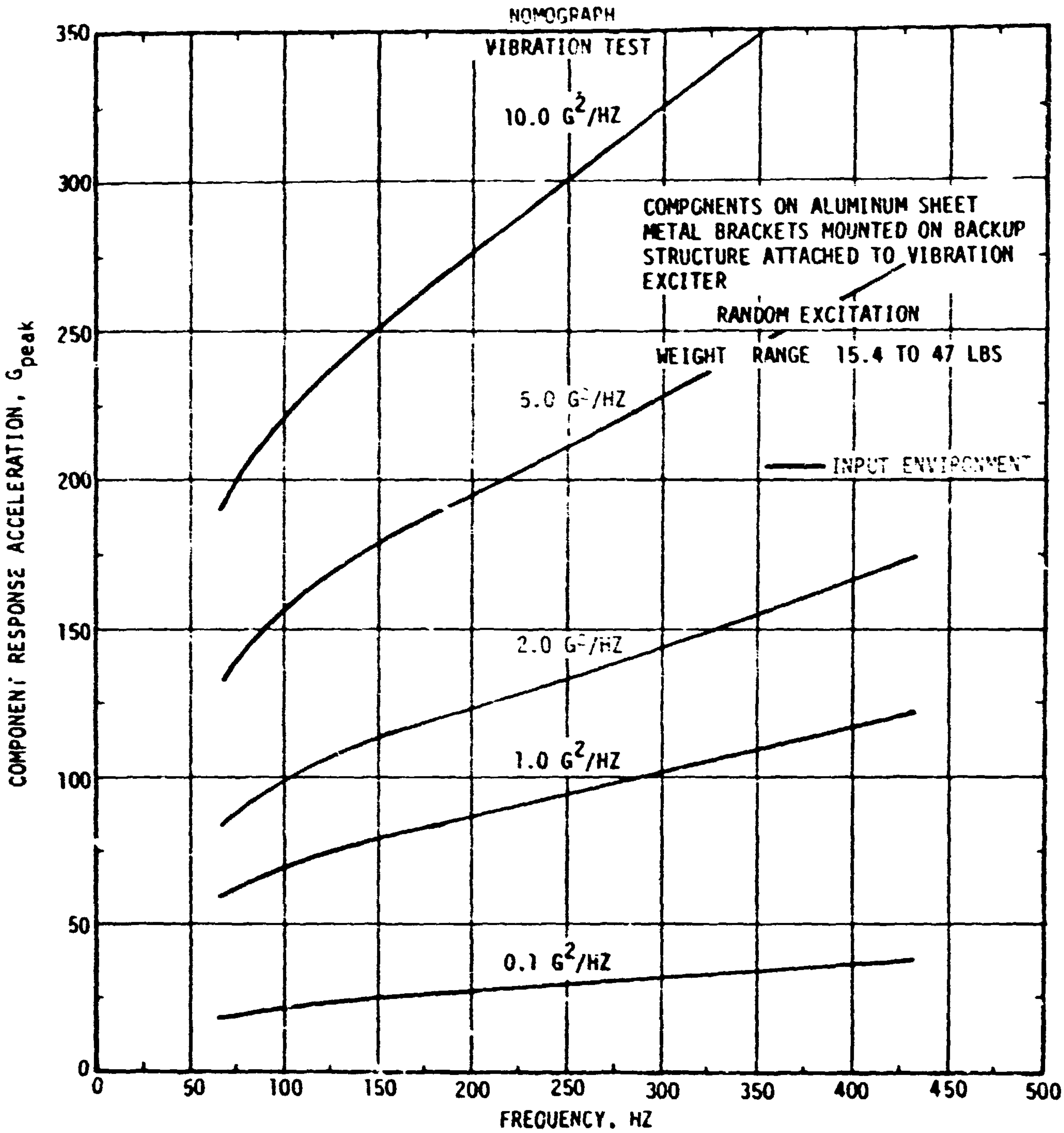
$G_{peak}(R) = Q(f) \times G_{rms}(I) \times 1.414$

WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

$Q(f)$ - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR AS A FUNCTION OF FREQUENCY

$G_{rms}(I)$ - PREDICTED OR MEASURED INPUT ACCELERATION (G_{rms})

FIGURE 6-1: VIBRATION TEST DESIGN NOMOGRAPH FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 2 TO 7 POUNDS



$$G_{peak}(R) = 2.2\sqrt{\pi/2 \times Q \times f_0 \times PSD(I)}$$

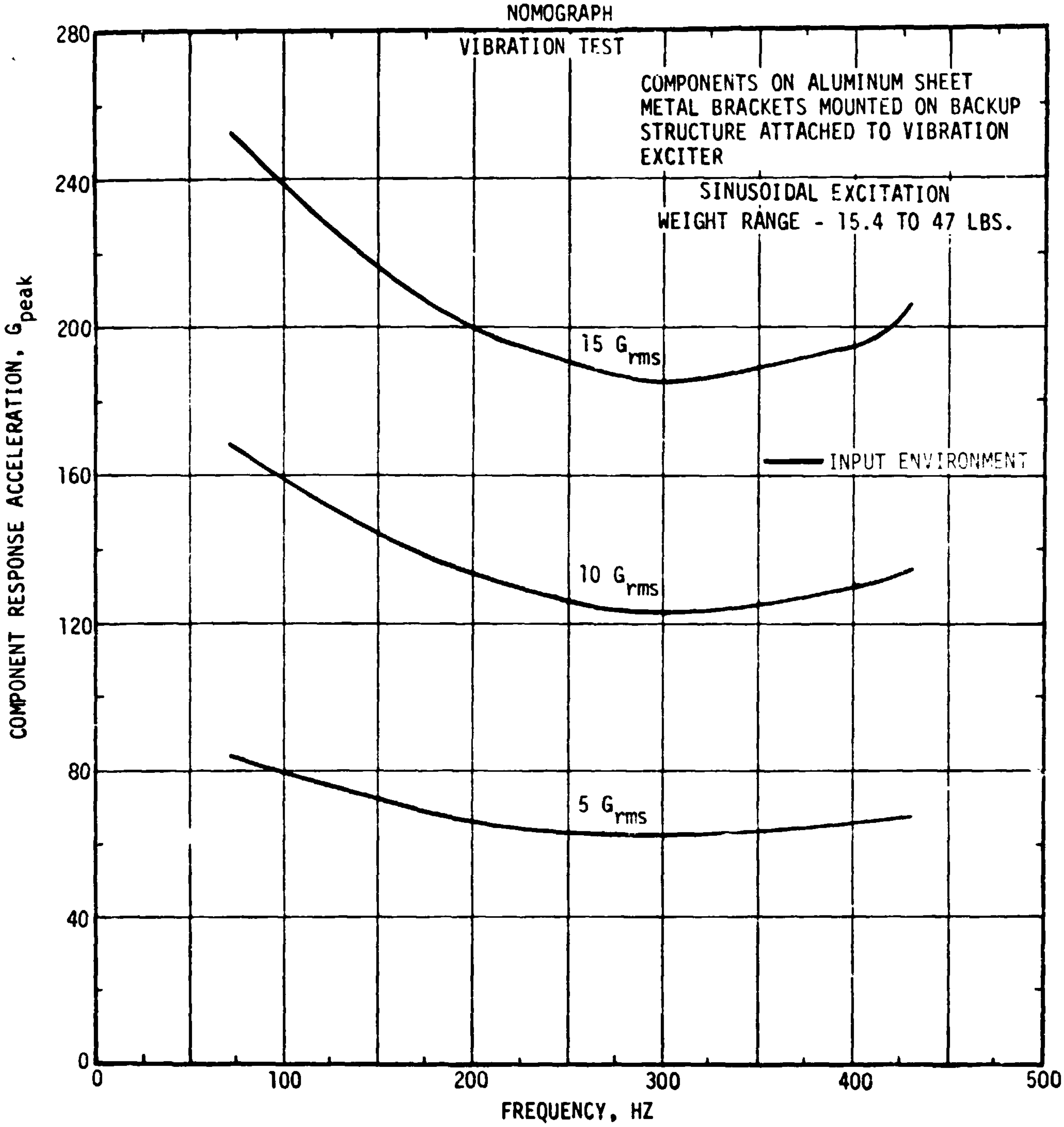
WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

Q - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR

f_0 - RESONANT FREQUENCY OF NEW COMPONENT AND BRACKET, HZ

$PSD(I)$ - PREDICTED OR MEASURED INPUT POWER SPECTRAL DENSITY (G^2/HZ)

FIGURE 6-2: VIBRATION TEST DESIGN NOMOGRAPH FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO RANDOM EXCITATION, WEIGHING 15.4 TO 47 POUNDS



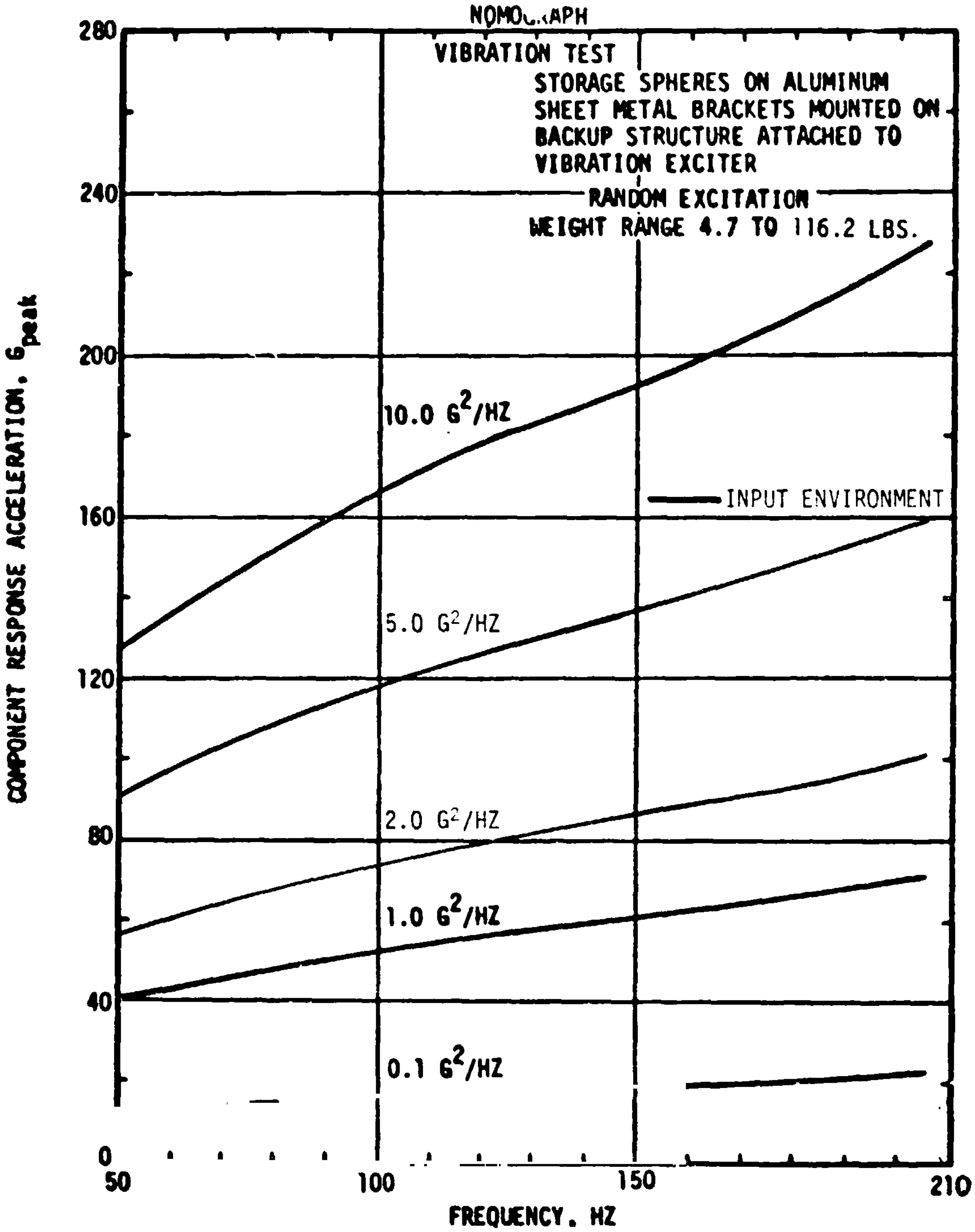
$G_{peak}(R) = Q(f) \times G_{rms}(I) \times 1.414$

WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

$Q(f)$ - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR AS A FUNCTION OF FREQUENCY

$G_{rms}(I)$ - PREDICTED OR MEASURED INPUT ACCELERATION (G_{rms})

FIGURE 6-3: VIBRATION TEST DESIGN NOMOGRAPH FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 15.4 TO 47 POUNDS



$$G_{peak}(R) = 2.2\sqrt{\pi/2 \times Q \times f_0 \times PSD(I)}$$

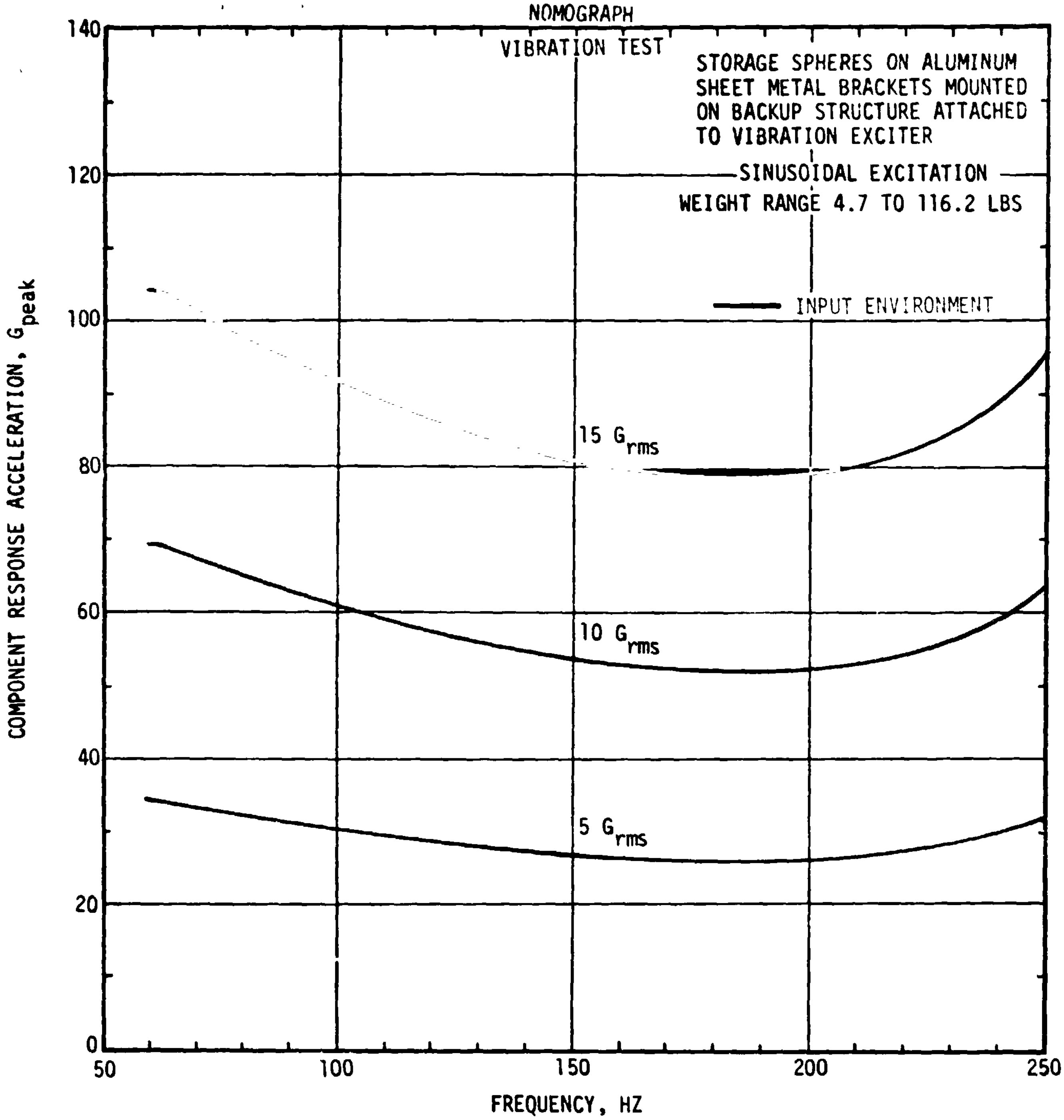
WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

Q - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR

f_0 - RESONANT FREQUENCY OF NEW COMPONENT AND BRACKET, HZ

PSD(I) - PREDICTED OR MEASURED INPUT POWER SPECTRAL DENSITY (G^2/HZ)

FIGURE 6-4: VIBRATION TEST DESIGN NOMOGRAPH FOR STORAGE SPHERES ON SHEET METAL BRACKETS EXPOSED TO RANDOM EXCITATION, WEIGHING 4.7 TO 116.2 POUNDS



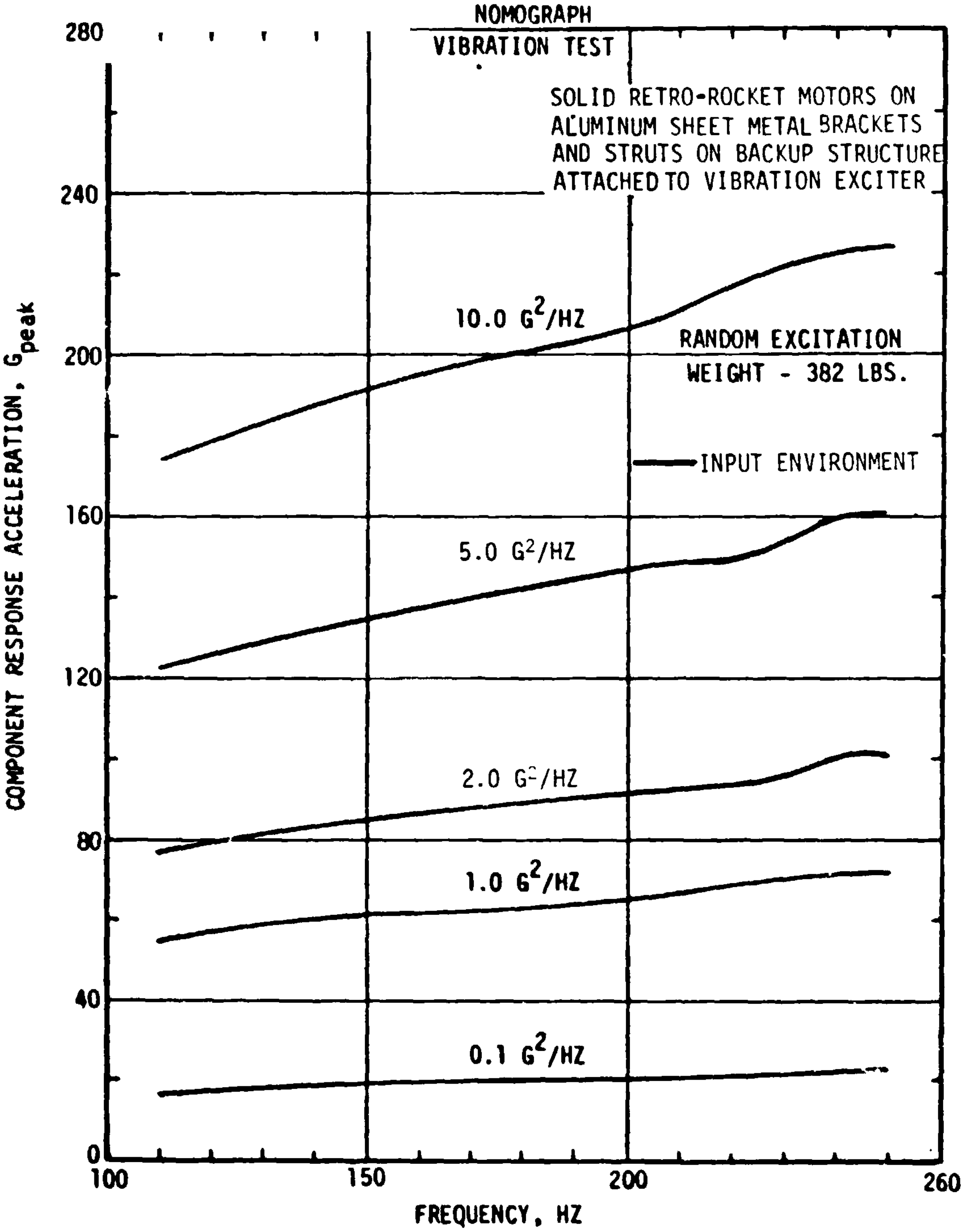
$G_{peak}(R) = Q(f) \times G_{rms}(I) \times 1.414$

WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

$Q(f)$ - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR AS A FUNCTION OF FREQUENCY

$G_{rms}(I)$ - PREDICTED OR MEASURED INPUT ACCELERATION (G_{rms})

FIGURE 6-5: VIBRATION TEST DESIGN NOMOGRAPH FOR STORAGE SPHERES ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 4.7 TO 116.2 POUNDS



$$G_{peak}(R) = 2.2\sqrt{\pi/2 \times Q \times f_0 \times PSD(I)}$$

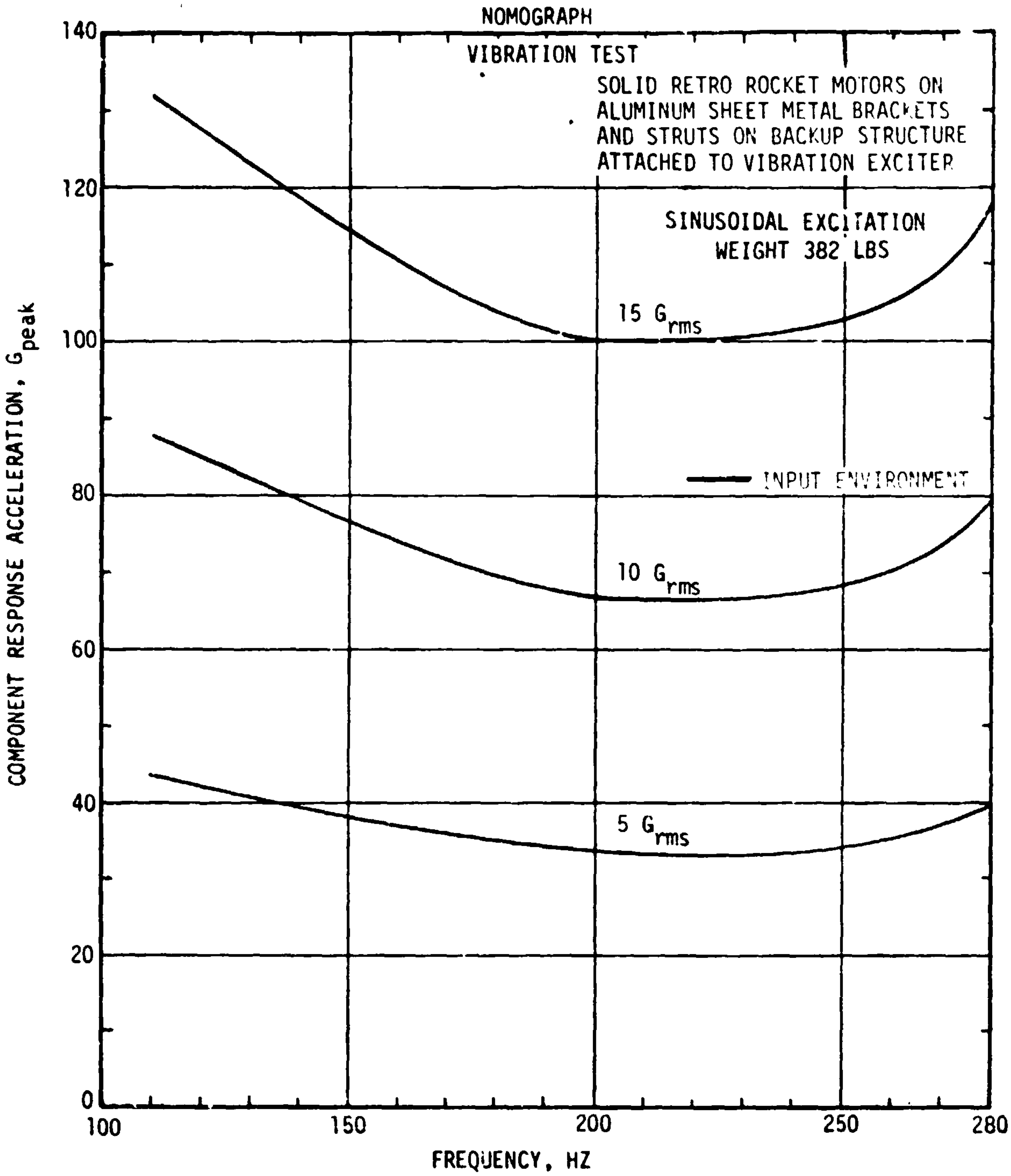
WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

Q - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR

f_0 - RESONANT FREQUENCY OF NEW COMPONENT AND BRACKET, HZ

PSD(I) - PREDICTED OR MEASURED INPUT POWER SPECTRAL DENSITY (G^2/HZ)

FIGURE 6-6: VIBRATION TEST DESIGN NOMOGRAPH FOR SOLID RETRO-ROCKET MOTORS ON SHEET METAL BRACKETS AND STRUTS, EXPOSED TO RANDOM EXCITATION, WEIGHING 382 POUNDS



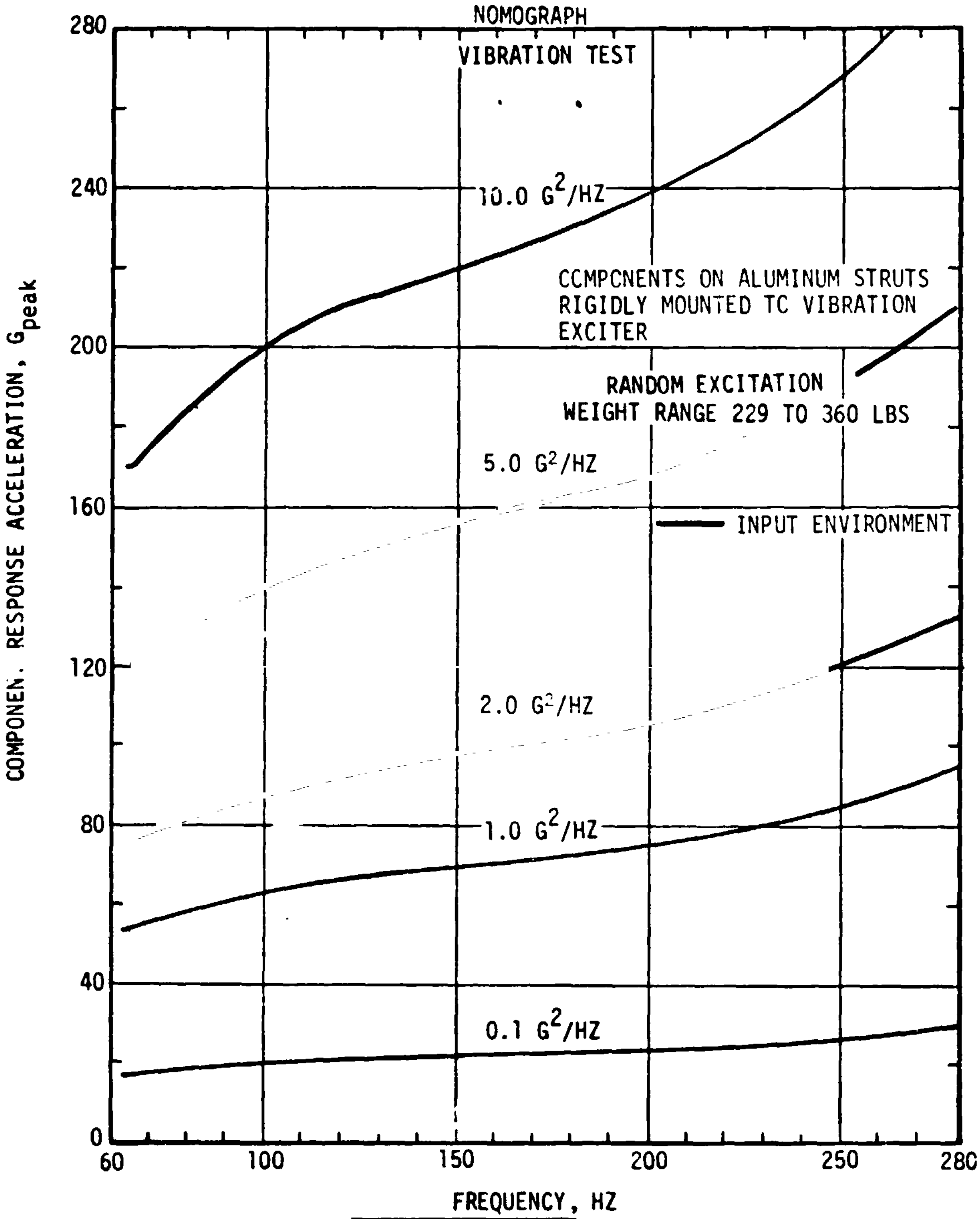
$G_{peak}(R) = Q(f) \times G_{rms}(I) \times 1.414$

WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

$Q(f)$ - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR AS A FUNCTION OF FREQUENCY

$G_{rms}(I)$ - PREDICTED OR MEASURED INPUT ACCELERATION (G_{rms})

FIGURE 6-7: VIBRATION TEST DESIGN NOMOGRAPH FOR SOLID RETRO-ROCKET MOTORS ON SHEET METAL BRACKETS AND STRUTS, EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 382 POUNDS



$$G_{peak}(R) = 2.2\sqrt{\pi/2 \times Q \times f_0 \times PSD(I)}$$

WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

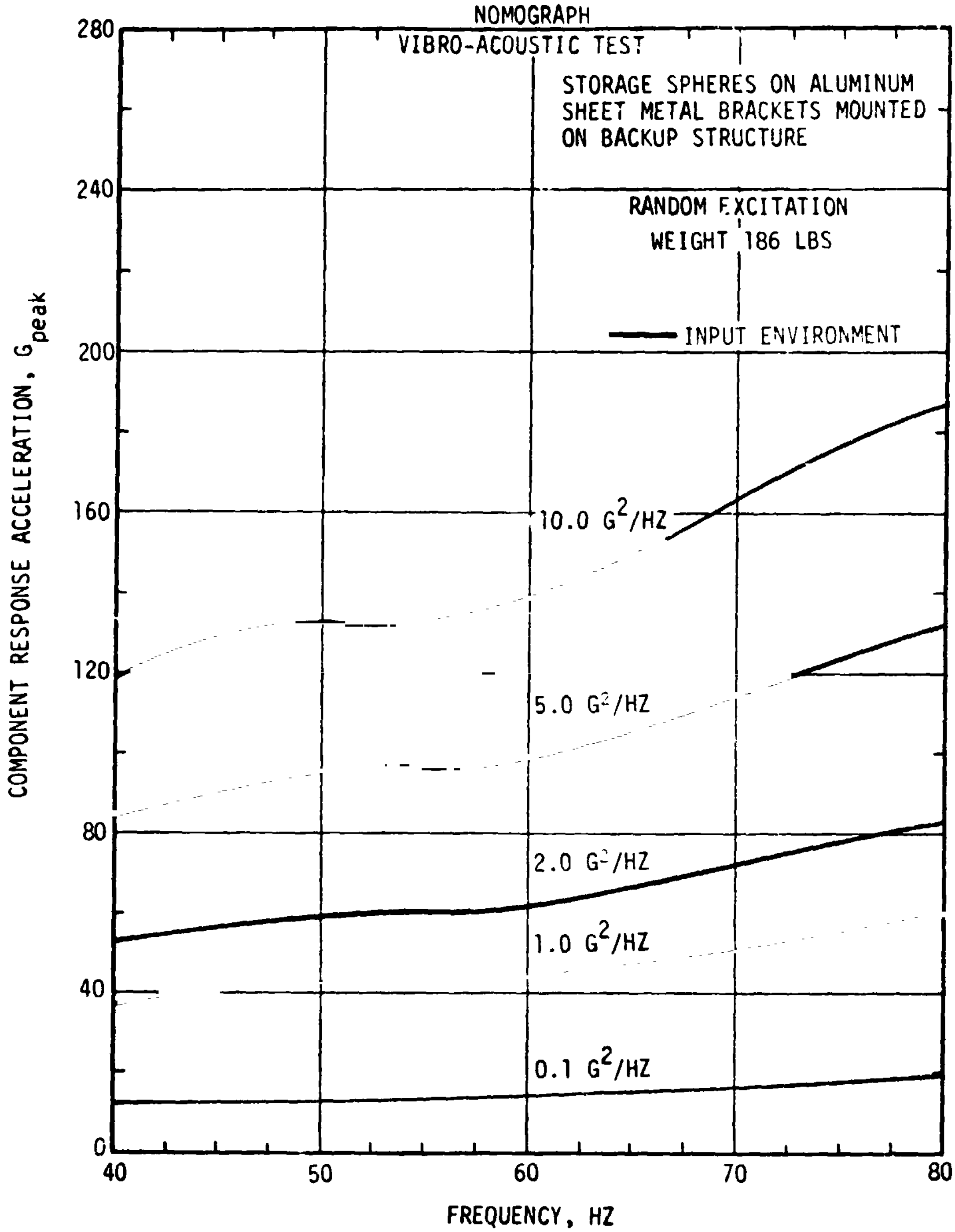
Q - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR

f_0 - RESONANT FREQUENCY OF NEW COMPONENT AND BRACKET, HZ

PSD(I) - PREDICTED OR MEASURED INPUT POWER SPECTRAL DENSITY (G^2/HZ)

FIGURE 6-8: VIBRATION TEST DESIGN NOMOGRAPH FOR COMPONENTS ON STRUTS EXPOSED TO RANDOM EXCITATION, WEIGHING 229 TO 360 POUNDS

NOMOGRAPH



$$G_{peak}(R) = 2.2\sqrt{\pi/2 \times Q \times f_0 \times PSD(I)}$$

WHERE: $G_{peak}(R)$ - NEW COMPONENT RESPONSE ACCELERATION (G_{peak})

Q - 95% CONFIDENCE LEVEL DYNAMIC MAGNIFICATION FACTOR

f_0 - RESONANT FREQUENCY OF NEW COMPONENT AND BRACKET, HZ

PSD(I) - PREDICTED OR MEASURED INPUT POWER SPECTRAL DENSITY (G^2/HZ)

FIGURE 6-9: VIBRO-ACOUSTIC DESIGN NOMOGRAPHS FOR STORAGE SPHERES ON SHEET METAL BRACKETS, WEIGHING 186 POUNDS

6.3 (Continued)

1. The design should be simple.
2. The design should provide for multiple load paths.
3. Special design consideration should be given to components with tension loaded fittings.
4. Factor of safety should be applied to stresses around holes.
5. Laboratory test should be conducted on newly designed joints.
6. Materials with longitudinal grain direction should be utilized in design.
7. Generous fillets and radii should be provided.
8. Break all sharp edges.
9. Protect all parts from corrosion.
10. Design structural reinforcements to give a gradual rather than abrupt change in cross section.
11. Design parts for minimum mismatch on installation; this will result in lower preload tensile strains.
12. Select component and bracket configuration with high structural damping.
13. Optimize bracket and component resonant frequencies considering both service environment and equipment fragility.
14. Reduce the number of coupled resonances between component and structural assembly.
15. Mismatch impedances of mounted item on its bracket.
16. When selecting the right material, consider the cost, strength allowables, how well it can be fabricated, and environmental effects.

6.3 (Continued)

17. For optimum forming of components, consider fabrication techniques.
18. Select welding techniques in which there is some reliability in reproducing point strengths.

These design guides are representative of a selective list of guides that could be used in conjunction with the design nomographs. The results of applying these design nomographs and design guidelines to the design of new vehicle components and brackets should be an efficient design, with weight savings, low cost design, long service life, good reliability and an easy method of fabrication of the component and bracket.

SECTION 7

CONCLUSIONS AND RECOMMENDATIONS

7.0 CONCLUSIONS AND RECOMMENDATIONS

The nomographs that were developed herein are the results of an exhaustive study of the various vibration and acoustic tests conducted during the Saturn/Apollo program. A total of 1,010 different test reports were reviewed, yielding 72 useable Q factor data points. This number of data points limited the total number of categories developed and reduced the number of design nomographs presented in this report. In conclusion, the results presented in the form of design nomographs of specific component and bracket categories represents an effective method of determining the response of components due to a predicted or measured input acceleration. It is recommended that, when using the design nomographs of this report, one must remember that these new component response accelerations are best estimates based on specific Saturn/Apollo component and bracket design and should be used accordingly.

REFERENCES

1. Harris, C. M., and Crede, C. E., "Shock and Vibration Handbook, McGraw-Hill Book Company, Volume 3, PP 24-7, 1961.

2. Curtis, A. J., Tinling, N. G., and Abstein, Jr., H. T., "Selection and Performance of Vibration Tests," Shock and Vibration Information Center, SVM-8, 1971.

3. "Repository Index of Saturn Apollo Engineering Reports and Procedures," NASA File Number 001, September 1971.

4. Bartee, E. M., "Statistical Methods in Engineering Experiments," Charles E. Merrill Books, Inc., 1961.

APPENDIX A

This appendix provides the results of the regression, correlation and regression variance analyses conducted on the data categories assembled for development of design nomographs. The data points of Q versus component and bracket resonant frequency were assembled into the categories listed in Section 4.1 and statistically analyzed as outlined in Section 5.1. The results of the statistical analyses are presented in Figures A-1 through A-9 as mean and 95% confidence level regression lines. On each figure are the model regression equations and correlation coefficients for defining the categories. These figures were used in Section 6.0 to develop the design nomographs.

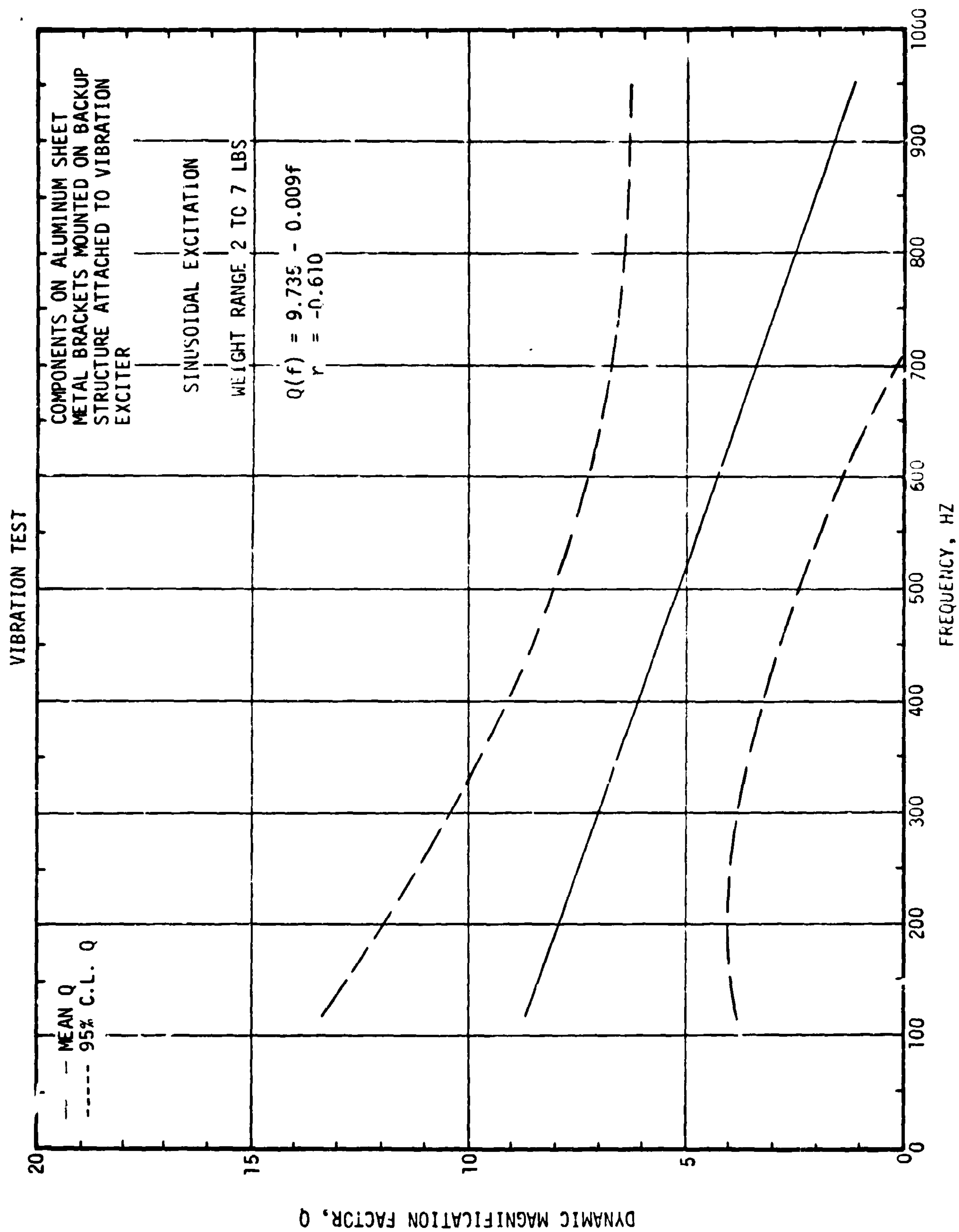


FIGURE A-1: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 2 TO 7 POUNDS

PRECEDING PAGE BLANK NOT FILMED

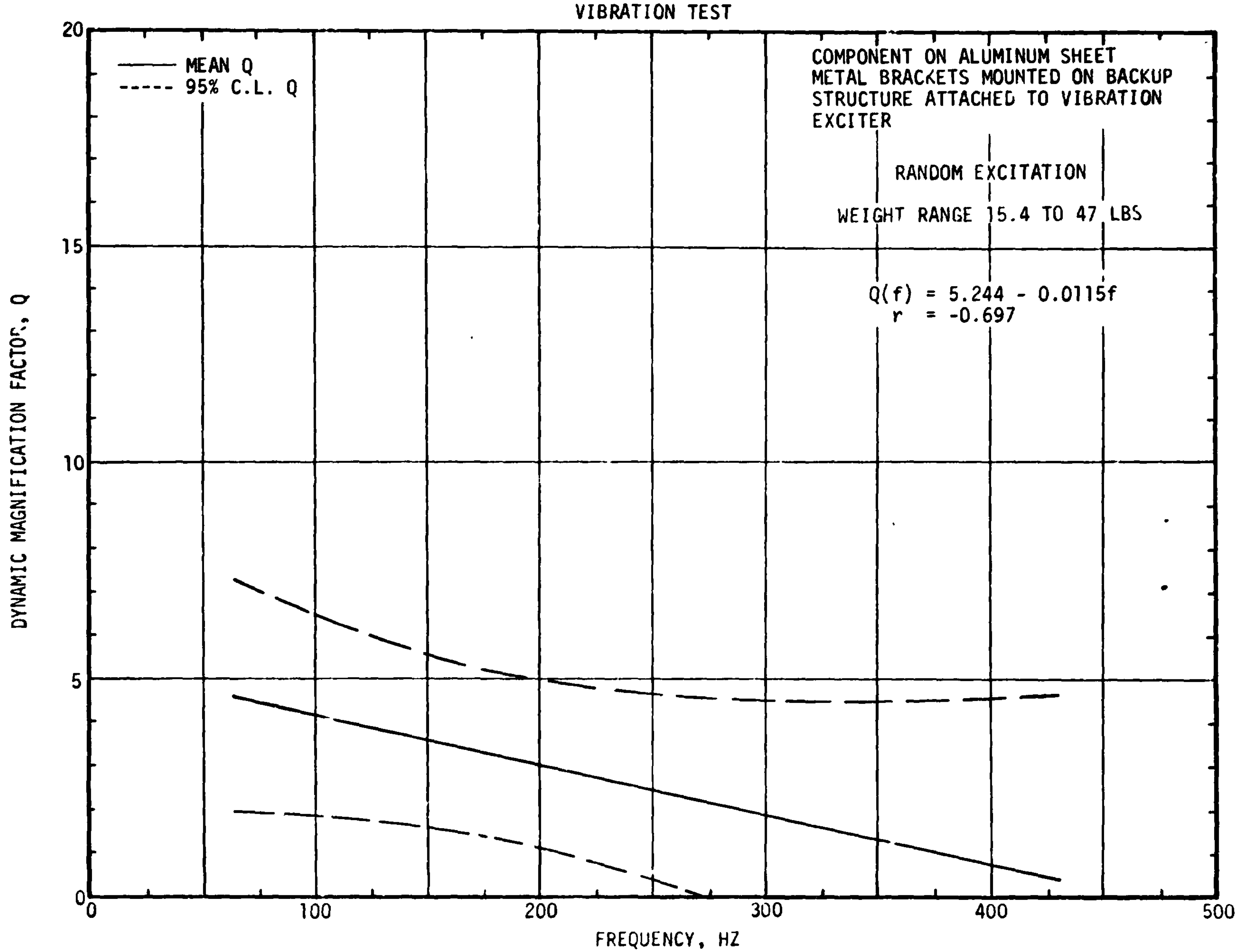


FIGURE A-2: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO RANDOM EXCITATION, WEIGHING 15.4 TO 47 POUNDS

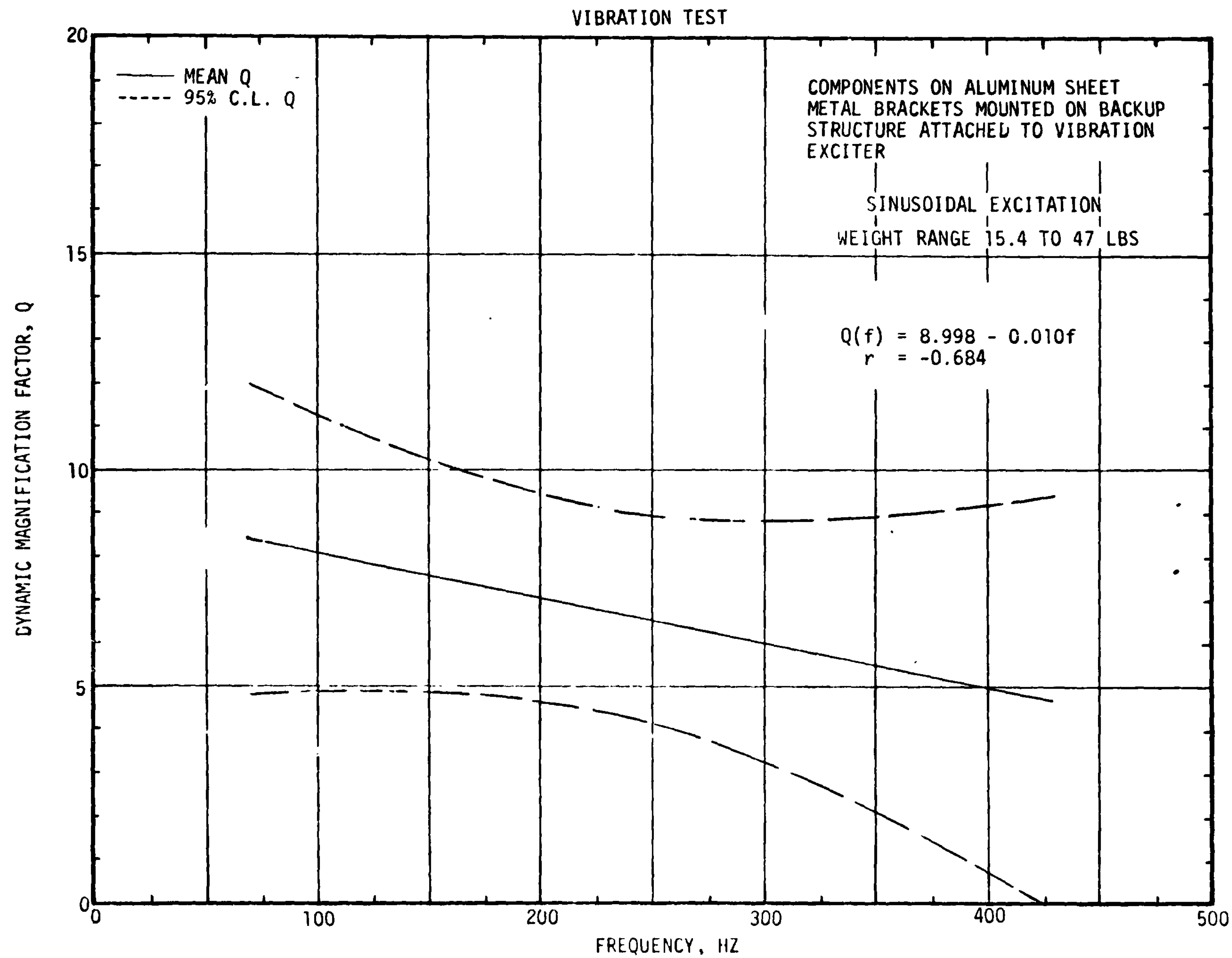


FIGURE A-3: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 15.4 TO 47 POUNDS

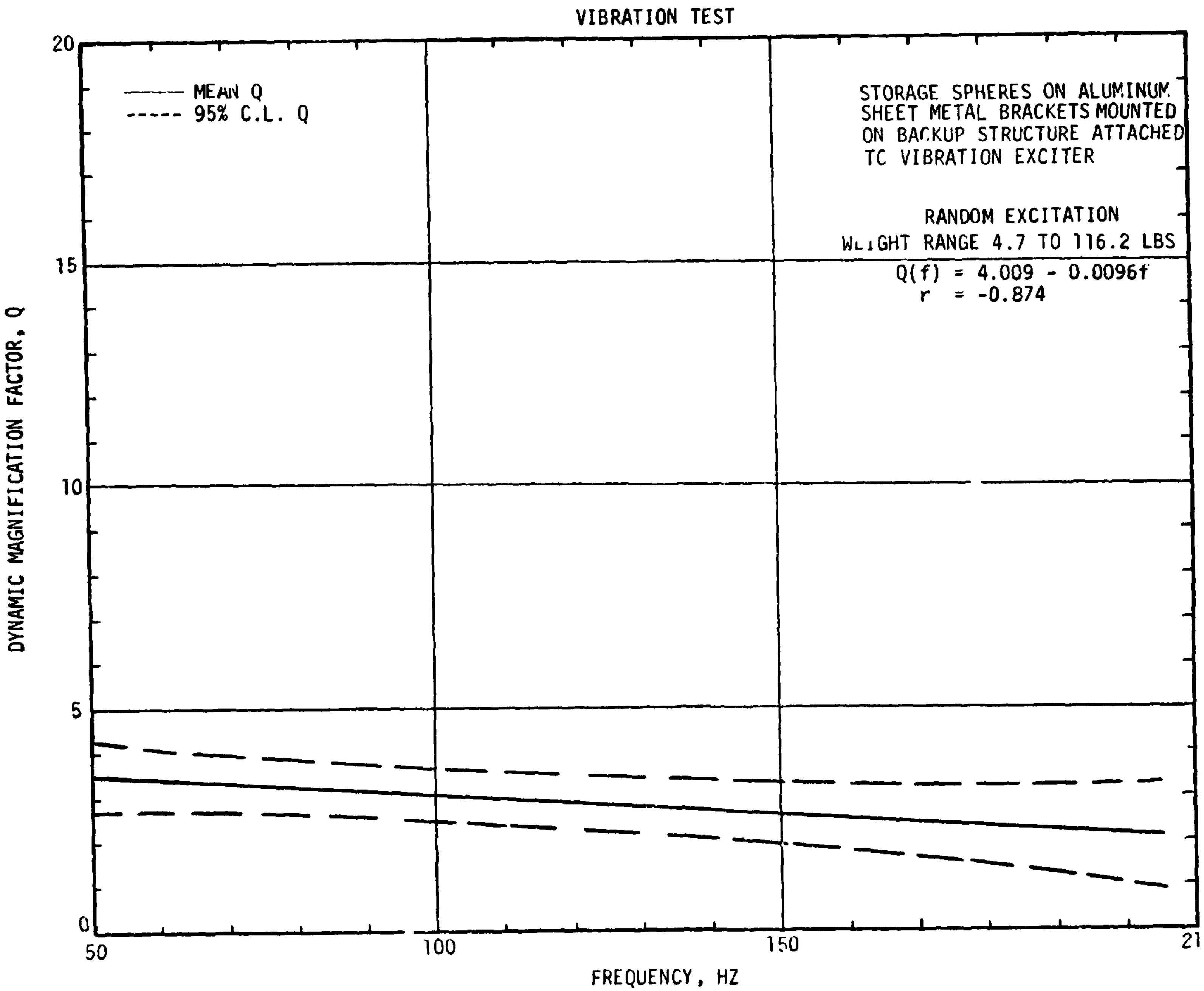


FIGURE A-4: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR STORAGE SPHERES ON SHEET METAL BRACKETS EXPOSED TO RANDOM EXCITATION, WEIGHING 4.7 TO 116.2 POUNDS

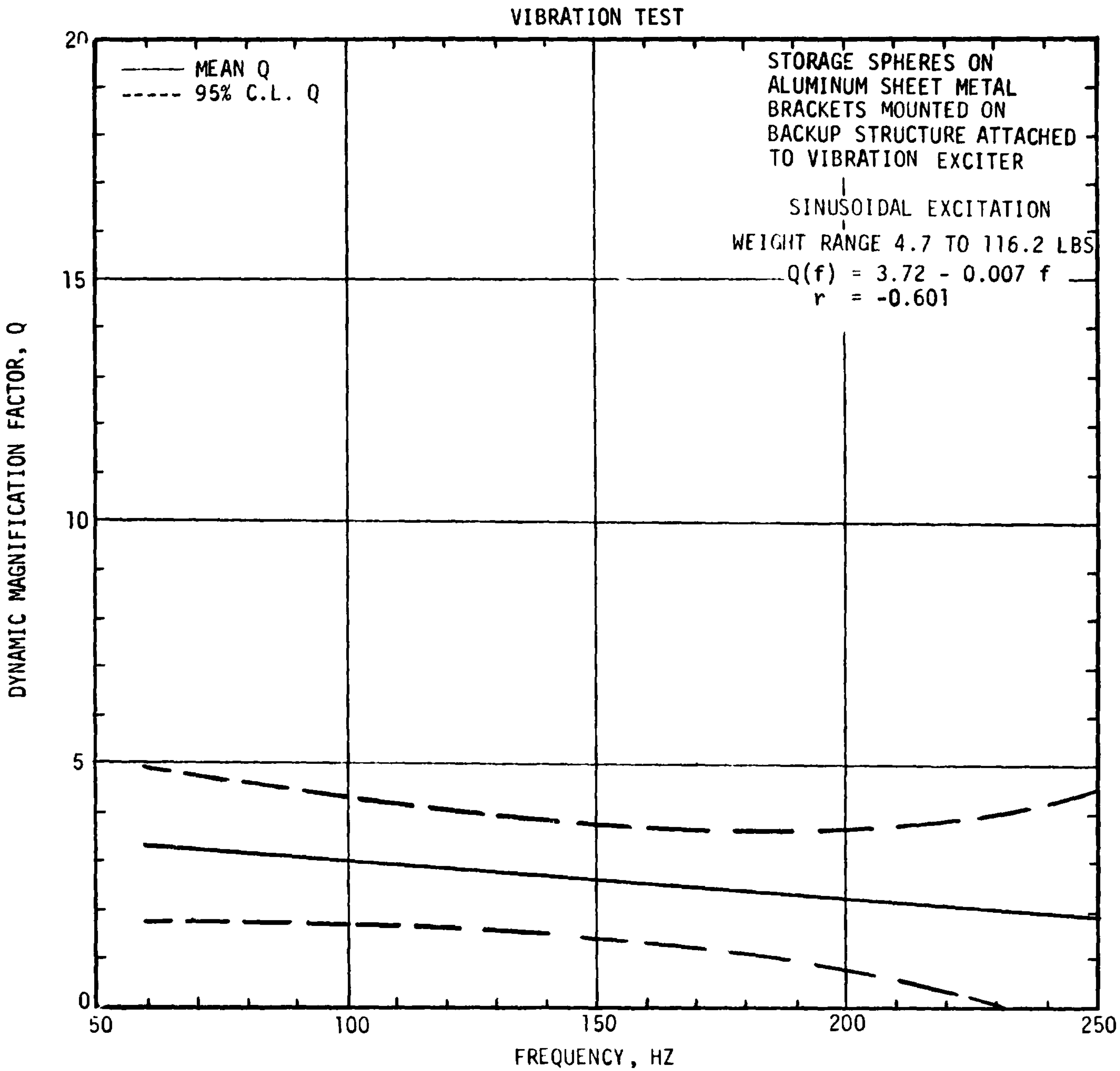


FIGURE A-5: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR STORAGE SPHERES ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 4.7 TO 116.2 LBS

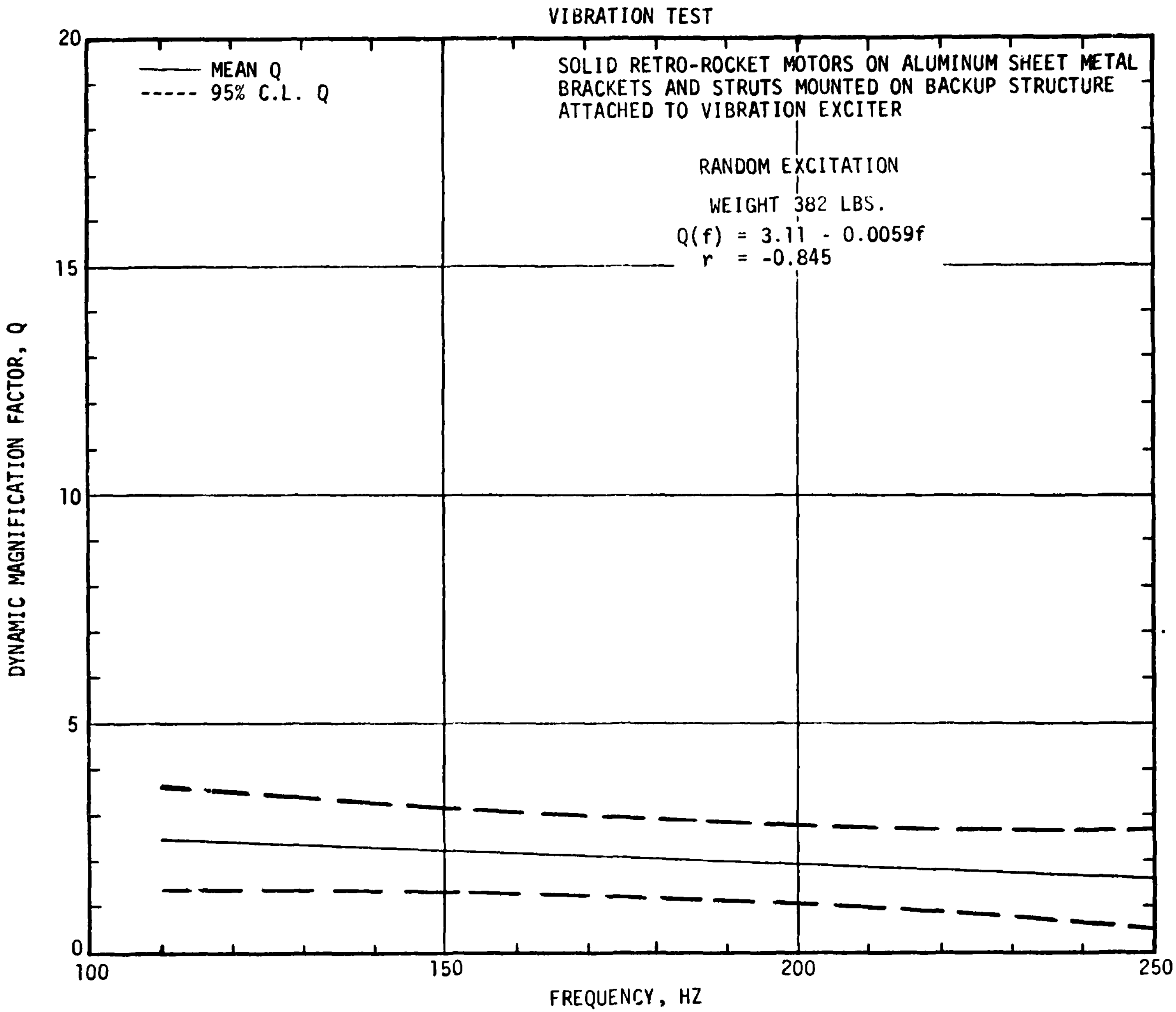


FIGURE A-6: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR SOLID RETRO-ROCKET MOTORS ON SHEET METAL BRACKETS AND STRUTS EXPOSED TO RANDOM EXCITATION, WEIGHING 382 POUNDS

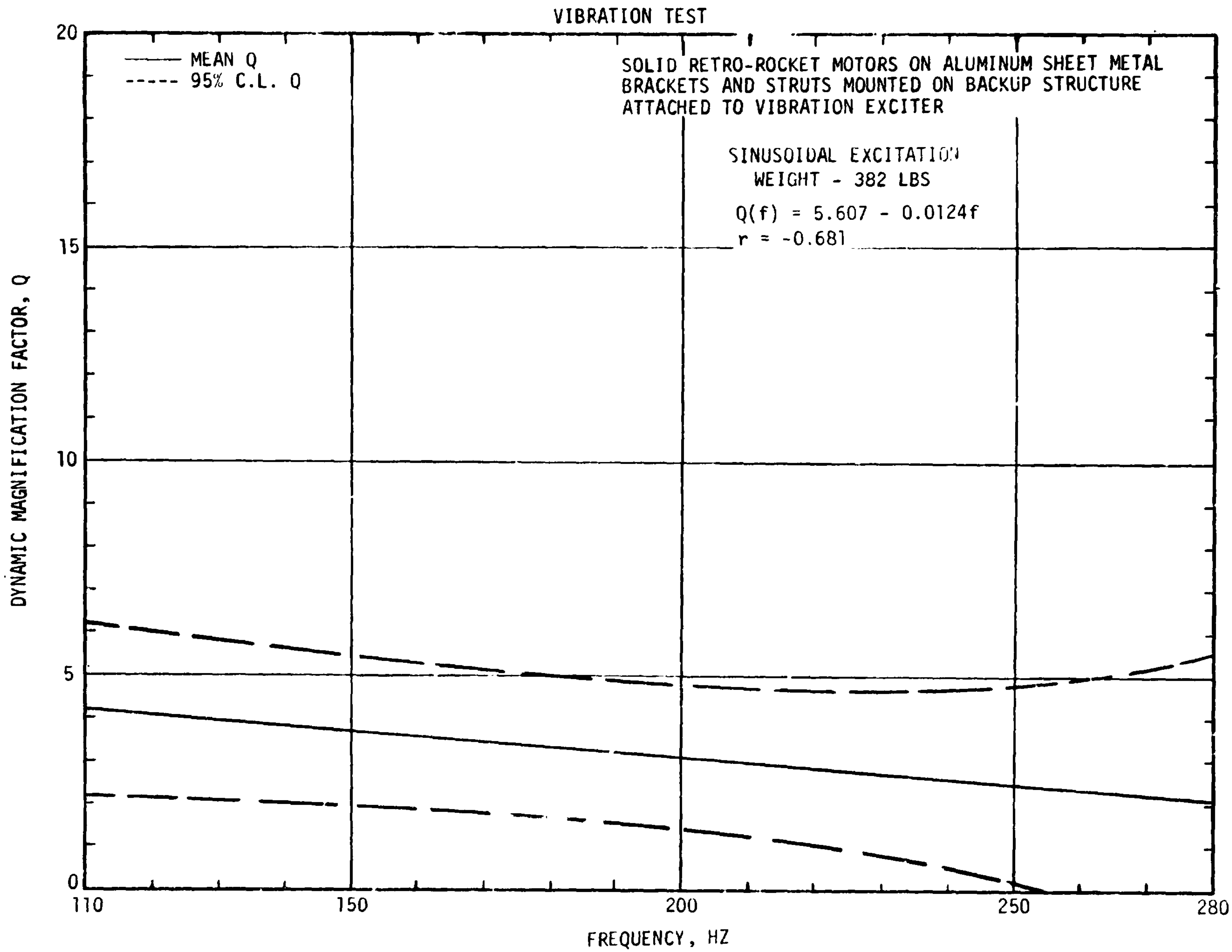


FIGURE A-7: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR SOLID RETRO-ROCKET MOTORS ON SHEET METAL BRACKETS AND STRUTS EXPOSED TO SINUSOIDAL EXCITATION, WEIGHING 382 POUNDS

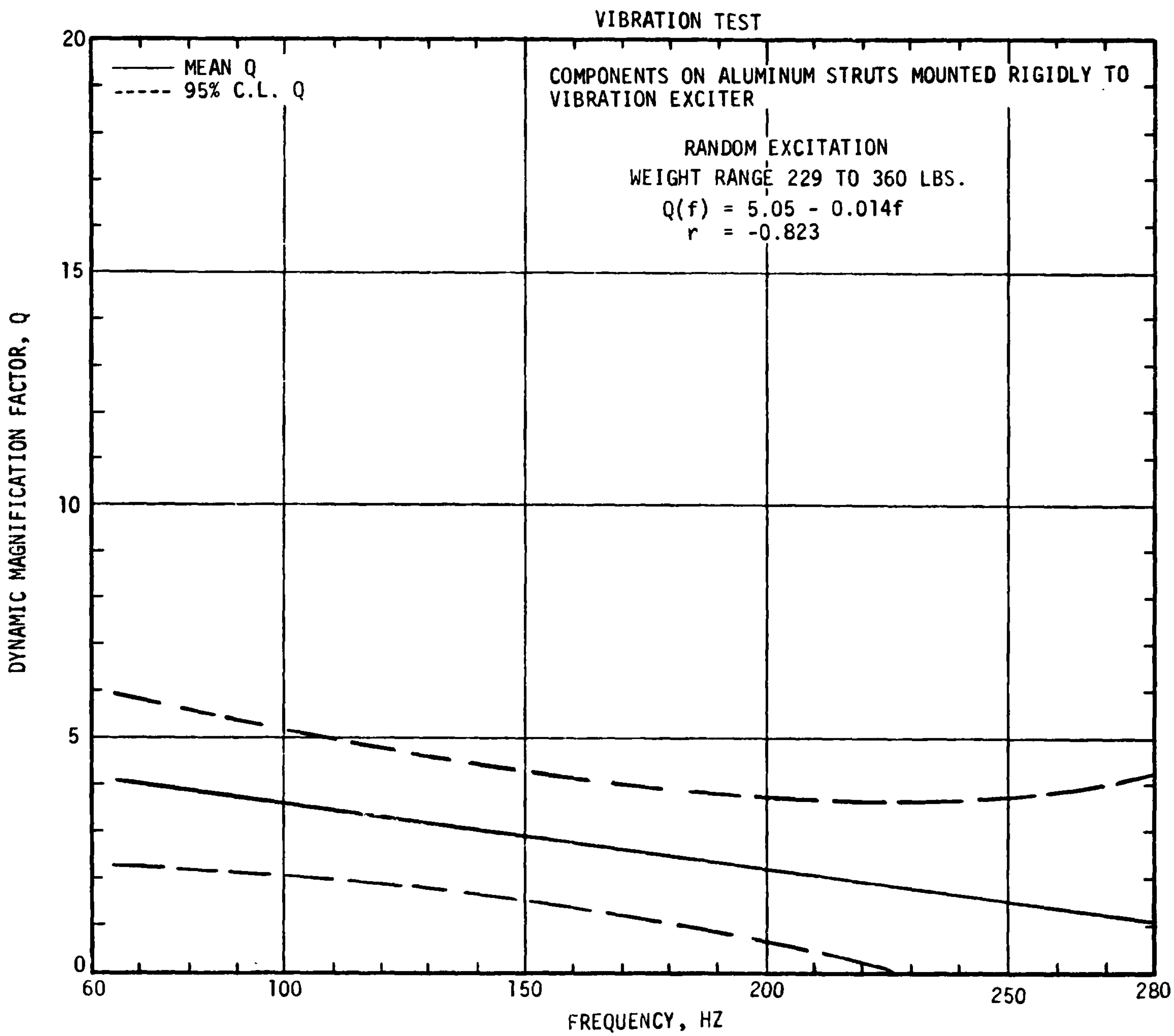


FIGURE A-8: VIBRATION TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR COMPONENTS ON STRUTS EXPOSED TO RANDOM EXCITATION, WEIGHING 229 TO 360 POUNDS

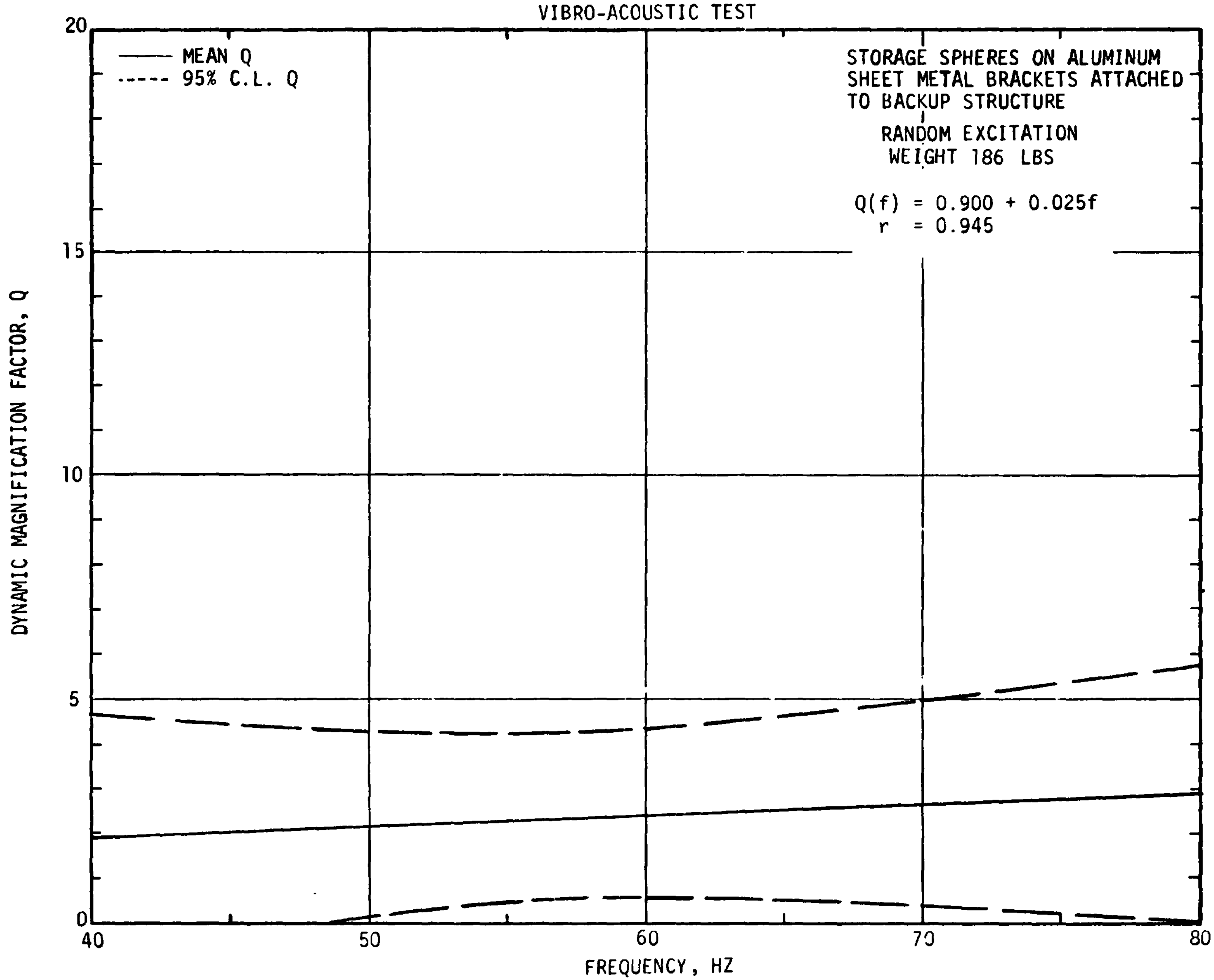


FIGURE A-9: VIBRO-ACOUSTIC TEST REGRESSION AND CONFIDENCE LEVEL LINES FOR STORAGE SPHERES ON SHEET METAL BRACKETS WEIGHING 186 POUNDS

APPENDIX B

The objectives of this appendix are to perform a comparison study between significant parameters and to determine what parameters affect the Dynamic Magnification Factor (Q) of components on brackets. The most significant parameters were determined to be component weight, component and bracket resonant frequency, type of excitation (random and sinusoidal) and mounting technique (rigidly mounted or backup structure mounted). The first two parameters, component weight and component and bracket resonant frequency, were determined to be the most significant. These parameters as discussed in Sections 4.1 and 4.2 were the major variables chosen to develop the design nomographs in Section 6.2 and the comparison study in this appendix. The remaining two parameters, type of excitation and type of mounting techniques, will be studied by comparing random versus sinusoidal testing and rigidly mounted versus backup structure mounted components and brackets, respectively. The data in the form of Q versus component weight were assembled in categories listed in Section 4.2. These categories are:

1. Components on Aluminum Sheet Metal Brackets Mounted on Backup Structure Attached to Vibration Exciter.
2. Components on Aluminum Sheet Metal Brackets Rigidly Mounted to Vibration Exciter.

The data were assembled into types of excitation categories established earlier and also into weight ranges for the categories listed above. It was determined that, due to the limited amounts of data points, it would be necessary to assemble the data points (Q versus component weight) into

weight ranges in order to compare estimated trends. The data points for the specific type of excitation and category above were subdivided to weight ranges of 2 to 7, 15.4 to 47. and 68 to 382 pounds and statistically analyzed as outlined in Section 5.1. The results of the statistical analyses in the form of linear regression lines are shown in Figures B-1 through B-6. As can be seen, the mean Q decresed with increase in frequency for random and sinusoidal excitation. This decreasing trend was also evident for the total component weight range of 2 to 382 pounds as shown in Figures B-7 and B-8. In the low weight range, the change in component weight does not significantly affect Q for either random or sinusoidal excitation source. A comparison of the mean regression lines for the excitation sources in thé weight range of 2 to 7 pounds shows similar amplitudes and trends. This would indicate that testing of very small components to either random or sinusoidal excitation sources has no effect on the response characteristics. Also this would indicate that tne component bracket design was such that it limited the component response characteristics extremely well. It must be assumed throughout this study that each test produces the same damage on a second-order system. The mean trends in the weight ranges of 15.4 to 47 pounds and 68 to 382 pounds are shown in Figures B-3 and B-4. No analogy could be drawn between different excitation sources due to the lack of data.

A comparison was made between components on brackets mounted to backup structure (Figure B-1) and components on brackets rigidly mounted to vibration exciter (Figure B-6). This comparison showed considerable difference in Q for the weight range of 2 to 7 pounds. The components on

brackets which were mounted rigidly to vibration exciter experienced a Q of 3 times greater than those experienced by components on brackets mounted to backup structure. For components on brackets mounted rigidly to vibration exciter, great pains are taken to restrict the vibration motion of the shaker and fixture to a single direction and to make the test fixture as massive as possible to eliminate any cross axes sensitivity and fixture contributions to the resonant behavior of the component and bracket. The addition of a backup structure will change the interface characteristics between the component bracket and backup structure and will limit the response as can be seen when comparing Figures B-1 and B-6. It can be concluded based on the limited data obtained that the sinusoidal test of components on brackets rigidly mounted to vibration exciter represents a more severe test for components weighing from 2 to 7 pounds. Due to lack of data for components on brackets in the weight range above 7 pounds, no comparison could be made between these parameters. It is anticipated, however, that tests conducted on components mounted rigidly to vibration exciter for any component weight does expose the component to a more severe environment.

The mean regression lines for components in the weight range of 15.4 to 47 pounds are presented in Figure B-3 for components on sheet metal brackets mounted to backup structure and tested to a random environment. Unfortunately, no other category for either sinusoidal or random excitation sources could be developed due to lack of data points. However, it should be noted that in this weight range the mean Q values are approximately twice as high as the Q values recorded for the weight range of 2 to 7

pounds. The brackets in this weight range (15.4 to 47 pounds) were apparently not limiting the response acceleration as effectively as they do in other weight ranges. This could be due to the particular bracket design for this weight range.

In the much higher weight range of 68 to 382 pounds, as shown in Figures B-4 and B-5 for random and sinusoidal excitation, the mean Q values were relatively constant for the total weight range. The comparatively low Q values of approximately 4.5 are comparable to those recorded for components consisting of heavy spheres and solid rocket motors on very intricate bracketry. It would appear that the complex bracket design contributed to the low Q obtained for this weight range.

In conclusion, based upon the mean regression trends presented, sinusoidal excitation exposed the component to a higher acceleration at the component and bracket resonant frequency. Also, the linear regression analyses conducted on components on brackets mounted rigidly to the vibration exciter represented a more severe environment than components on brackets mounted on backup structure.

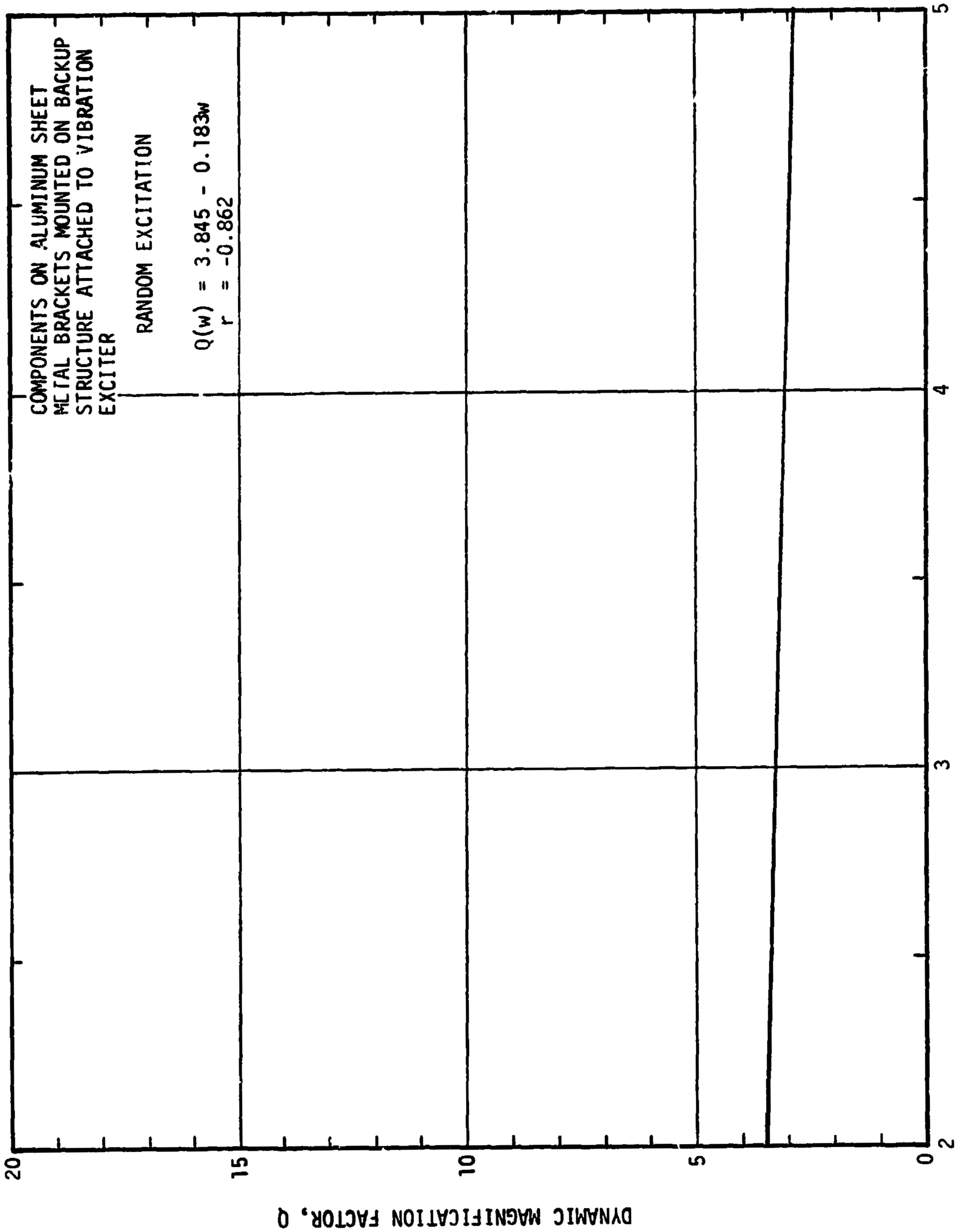


FIGURE B-7: LINEAR REGRESSION LINE OF DYNAMIC MAGNIFICATION FACTOR VERSUS COMPONENT WEIGHT FOR COMPONENTS ON SHEET METAL BRACKETS, EXPOSED TO RANDOM EXCITATION, 2 TO 5 POUNDS

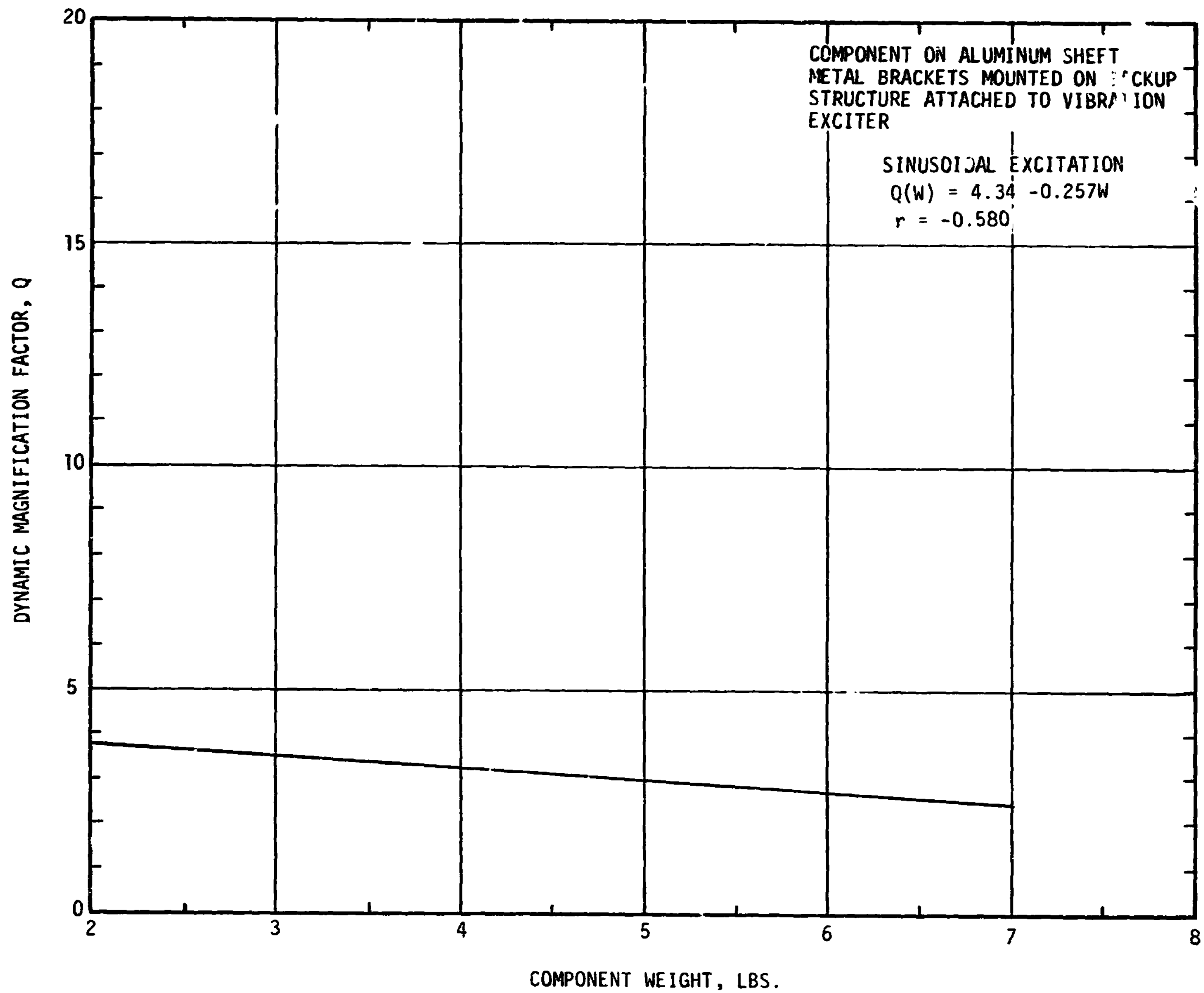


FIGURE B-2: LINEAR REGRESSION LINE OF DYNAMIC MAGNIFICATION FACTOR VERSUS COMPONENT WEIGHT FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, 2 TO 7 POUNDS

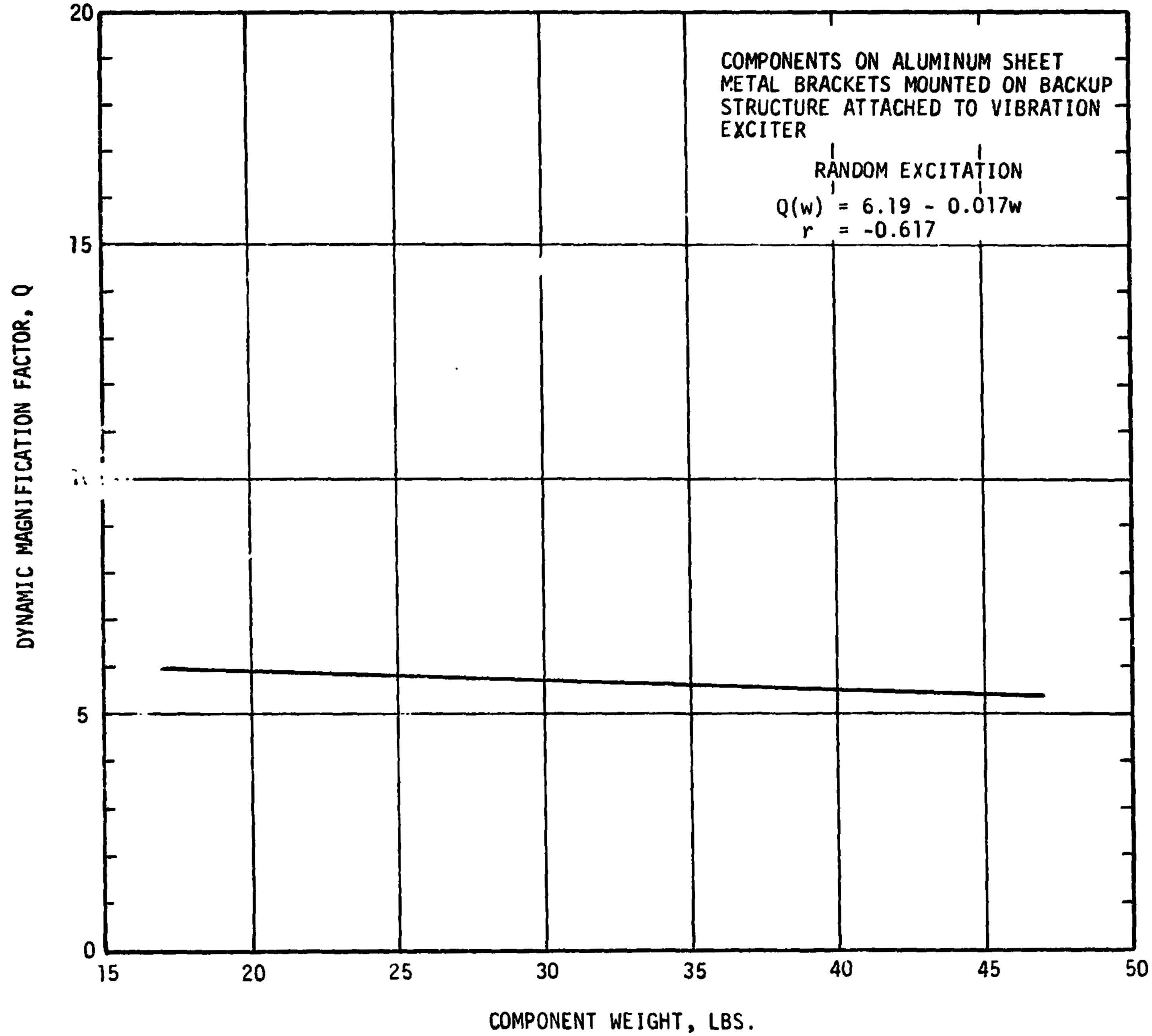


FIGURE B-3: LINEAR REGRESSION LINE OF DYNAMIC MAGNIFICATION FACTOR VERSUS COMPONENT WEIGHT FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO RANDOM EXCITATION, 15.4 TO 47 POUNDS

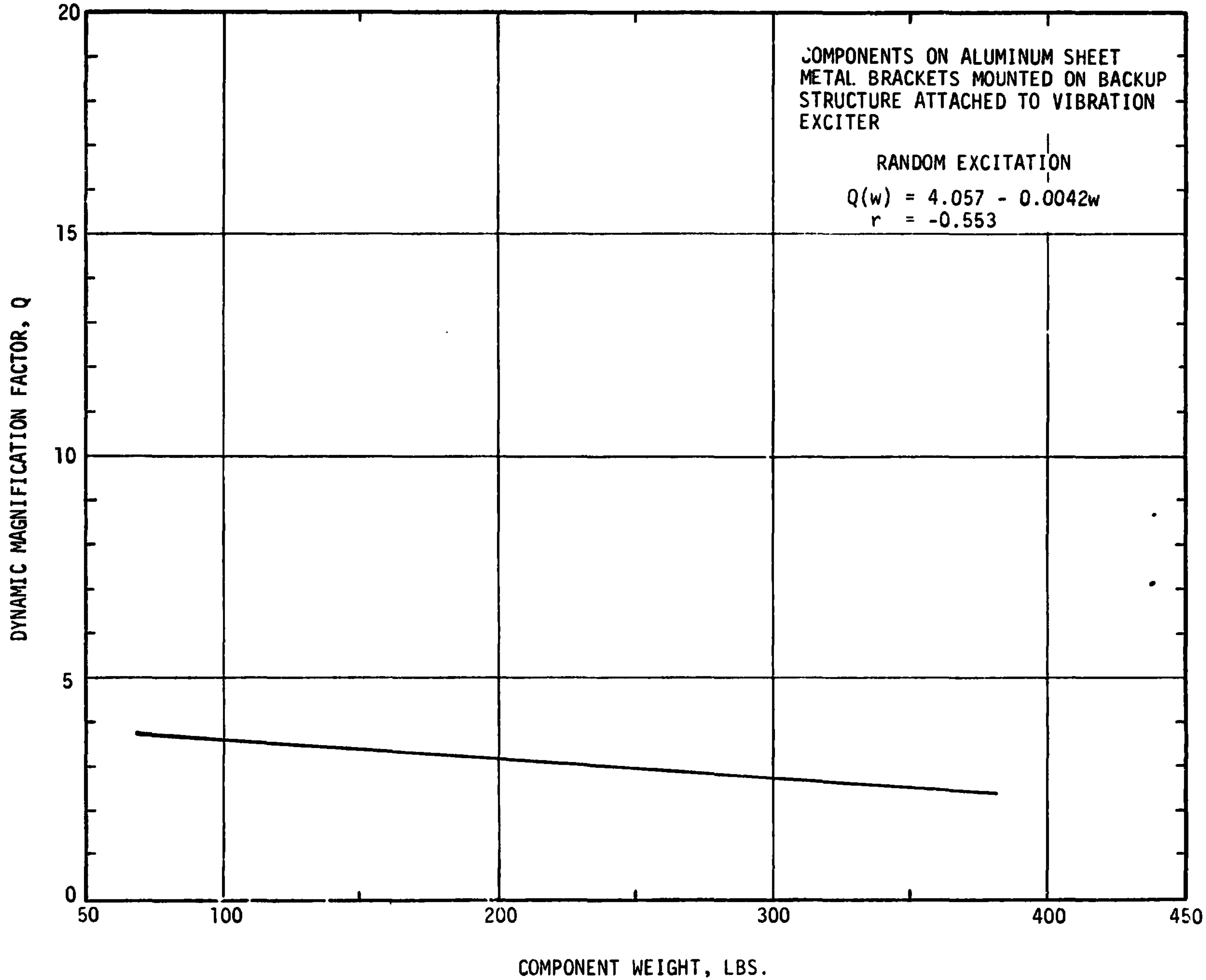


FIGURE B-4: LINEAR REGRESSION LINE OF DYNAMIC MAGNIFICATION FACTOR VERSUS COMPONENT WEIGHT FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO RANDOM EXCITATION, 68 TO 382 POUNDS

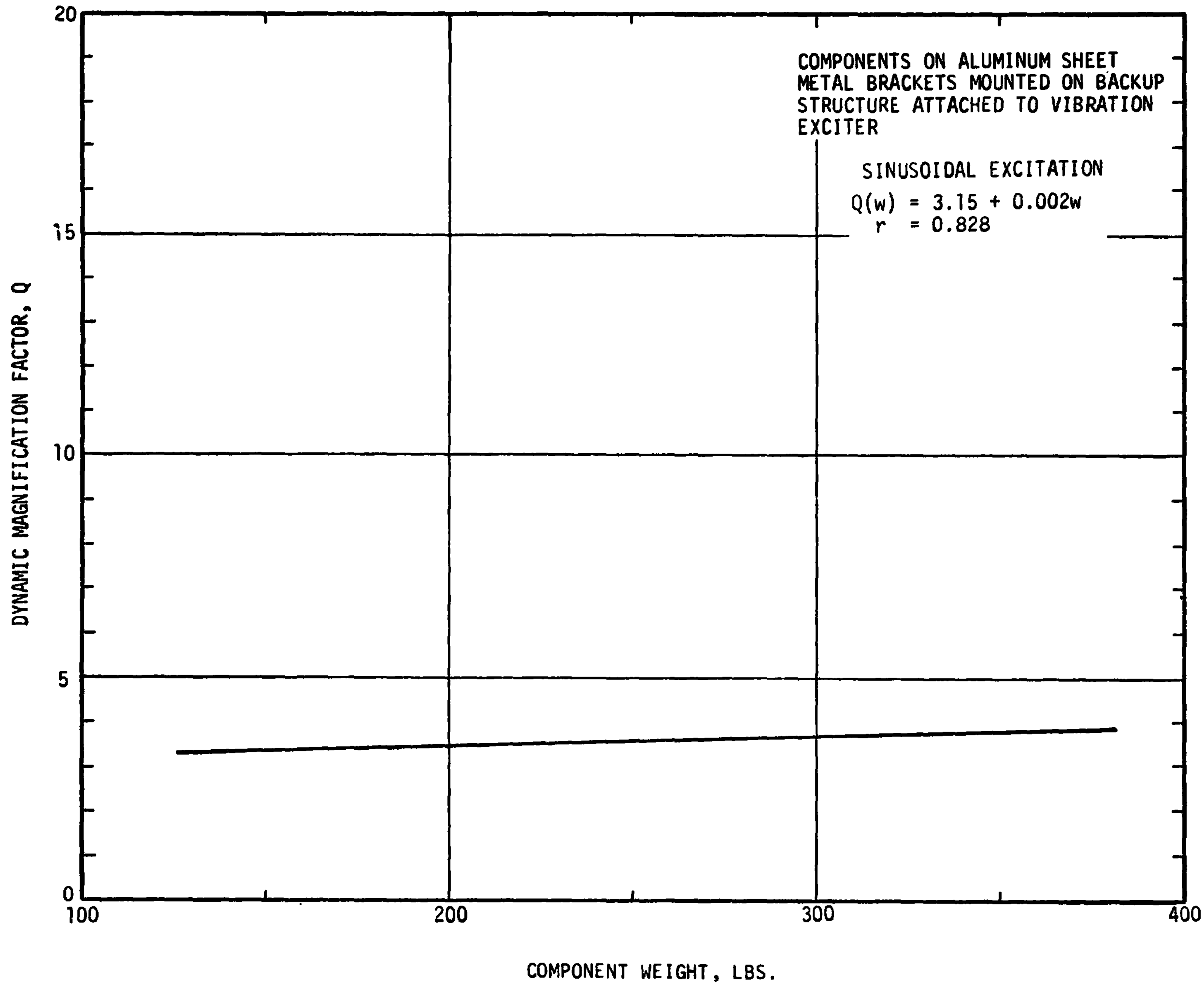


FIGURE B-5: LINEAR REGRESSION LINE OF DYNAMIC MAGNIFICATION FACTOR VERSUS COMPONENT WEIGHT FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, 116.2 TO 382 POUNDS

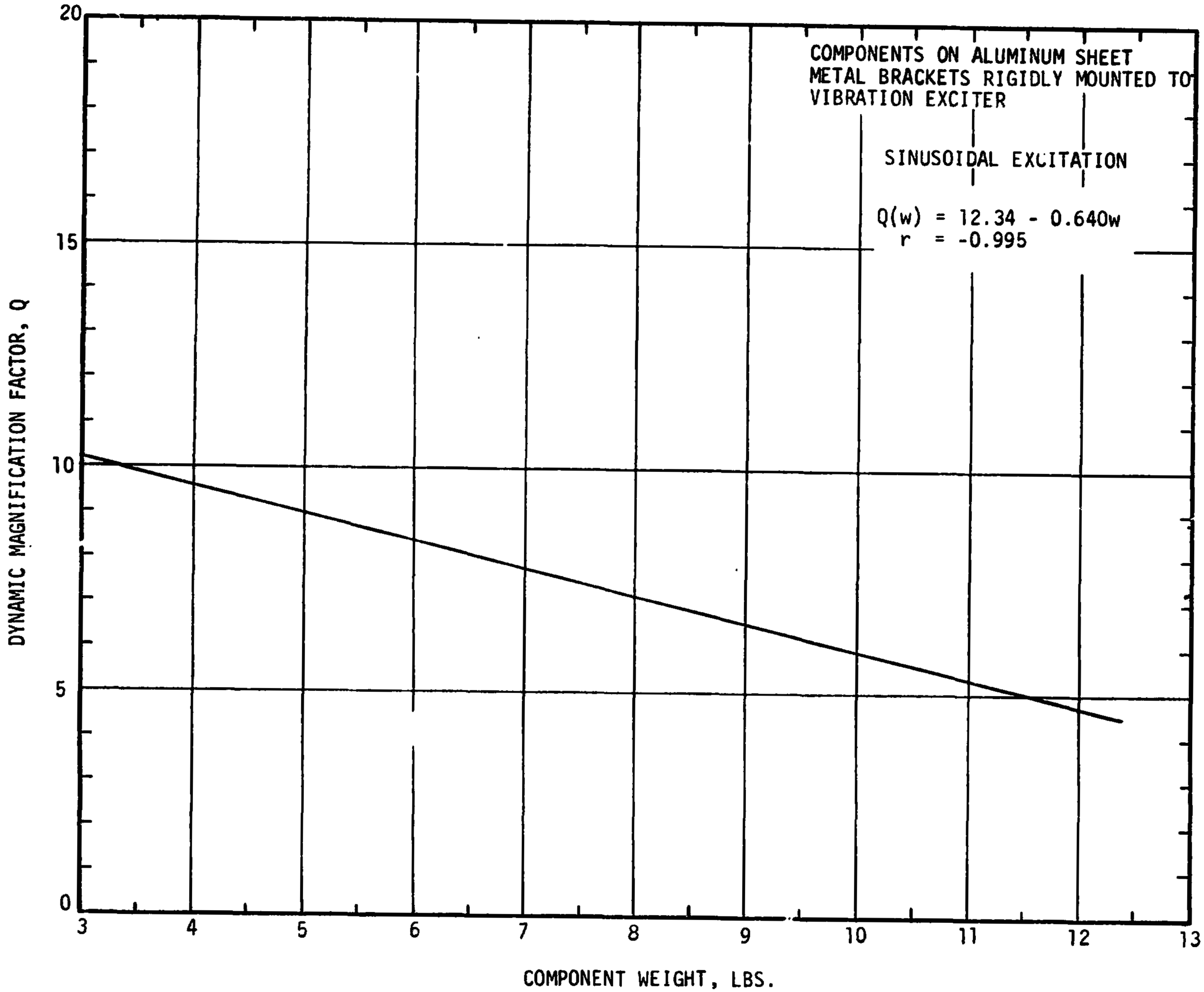


FIGURE B-6: LINEAR REGRESSION LINE OF DYNAMIC MAGNIFICATION FACTOR VERSUS COMPONENT WEIGHT FOR COMPONENTS ON SHEET METAL BRACKETS EXPOSED TO SINUSOIDAL EXCITATION, 3 TO 12.4 POUNDS

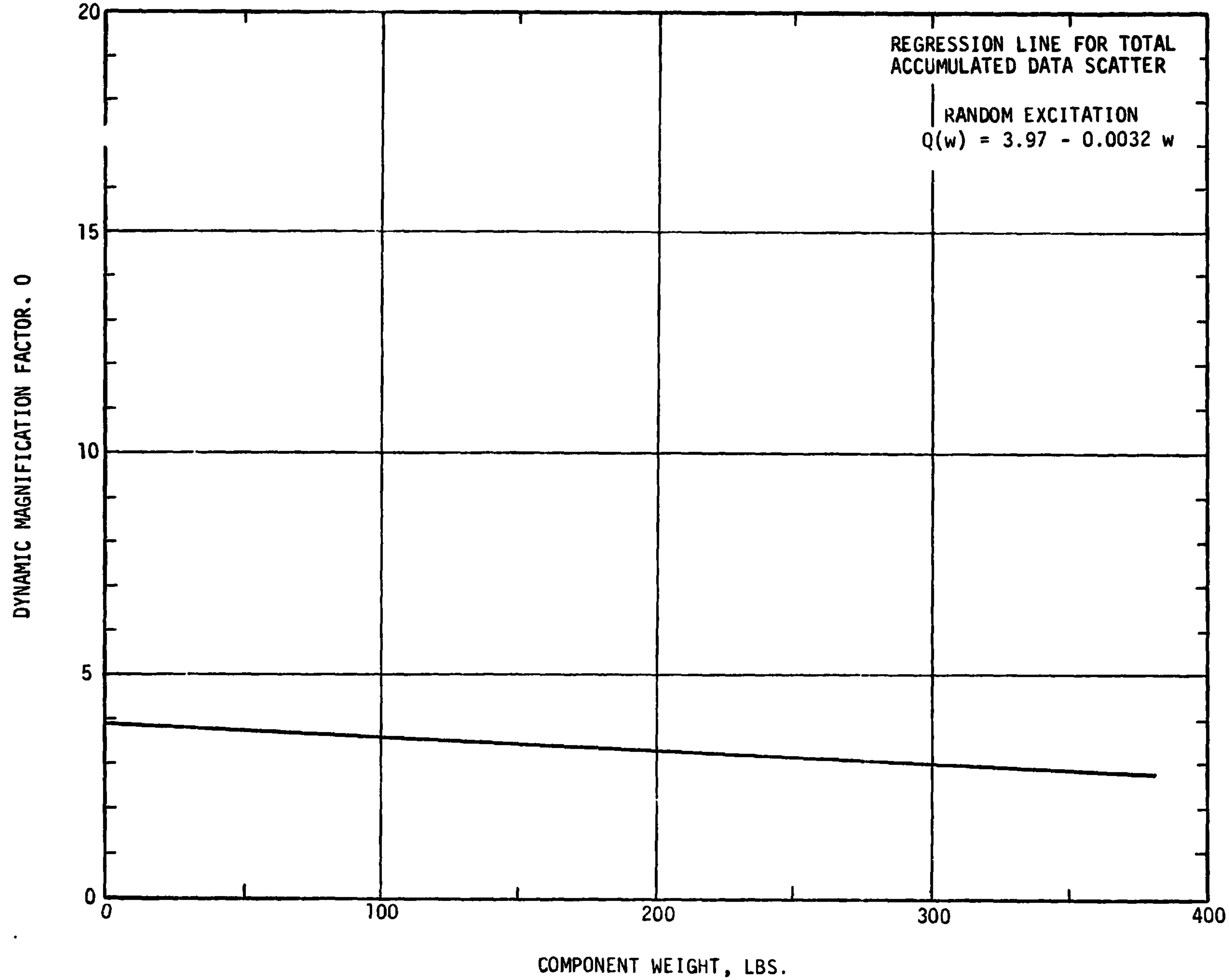


FIGURE B-7: LINEAR REGRESSION LINE FOR TOTAL DATA SCATTER FOR COMPONENTS EXPOSED TO RANDOM EXCITATION

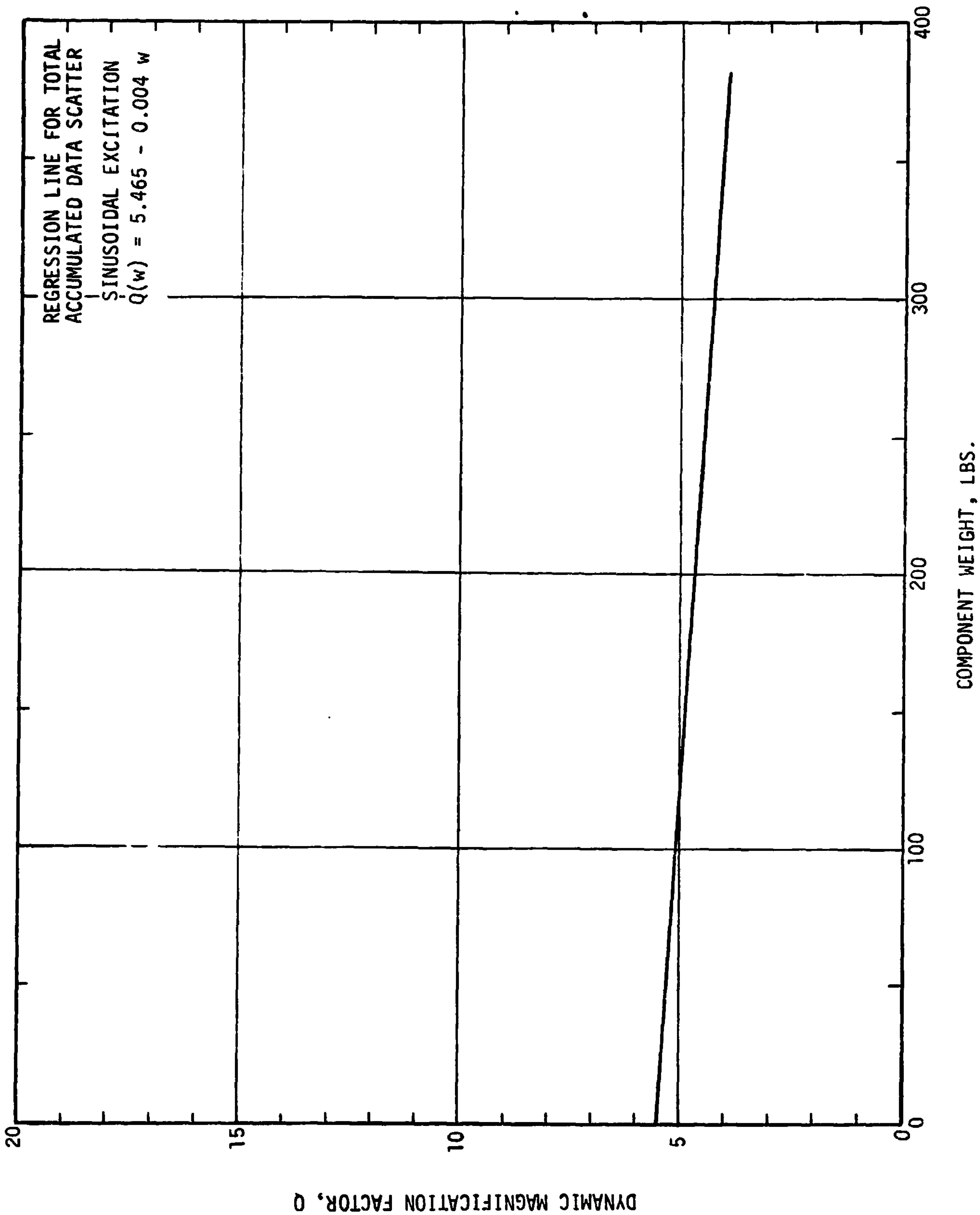


FIGURE B-8: LINEAR REGRESSION LINE FOR TOTAL DATA SCATTER FOR COMPONENTS EXPOSED TO SINUSOIDAL EXCITATION