AN IMPROVED METHOD FOR DESIGN OF EXPANSION-CHAMBER MUFFLERS WITH APPLICATION TO AN OPERATIONAL HELICOPTER

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An improved method for the design of expansion-chamber mufflers is described and applied to the task of reducing exhaust noise generated by a helicopter. The method is an improvement of standard transmission-line theory in that it accounts for the effect of the mean exhaust-gas flow on the acoustic-transmission properties of a muffler system, including the termination boundary condition. The method has been computerized, and the computer program includes an optimization procedure that adjusts muffler component lengths to achieve a minimum specified desired transmission loss over a specified frequency range. A printout of the program is included together with a user-oriented description.

A field test of a muffler designed with the aid of this method was conducted on a helicopter with a known exhaust-noise problem. When the exhaust noises of the helicopter with a standard exhaust system and a similar helicopter with a muffler system installed were compared for hover flight conditions, the muffler system was found to reduce the exhaust noise by approximately 11 dB(A). No significant degradation in the engine performance was observed.
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SUMMARY

An improved method for the design of expansion-chamber mufflers is described and applied to the task of reducing exhaust noise generated by a helicopter. The method is an improvement of standard transmission-line theory in that it accounts for the effect of the mean exhaust-gas flow on the acoustic-transmission properties of a muffler system, including the termination boundary condition. The method has been computerized, and the computer program includes an optimization procedure that adjusts muffler component lengths to achieve a minimum specified desired transmission loss over a specified frequency range. A printout of the program is included together with a user-oriented description.

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INTRODUCTION

There is an increasing awareness of noise pollution on the part of the general public, especially noise caused by aircraft. In particular, users of certain types of general aviation aircraft may be compelled to restrict operations and/or modify their aircraft to comply with existing or forthcoming noise legislation that will specify upper limits on external noise levels (ref. 1). In addition, measurements have indicated that internal noise levels of 11 light, twin-engine, fixed-wing aircraft exceed the currently accepted damage-risk criterion for hearing loss. Analysis of this noise indicated that engine-exhaust noise was the primary cause of both unacceptably high cabin-noise levels and radiated far-field noise (ref. 2).
The commonly accepted solution for excessive reciprocating-engine exhaust noise is the installation of an appropriate reactive-muffler system. Reactive-muffler design efforts in the past have relied upon acoustic-transmission-line theory as a design guide. However, these efforts have been characterized, for the most part, by trial-and-error techniques. Since trial-and-error methods tend to be expensive and inflexible, there is a need to provide a more rational basis for muffler design.

The aircraft-muffler designer must achieve a number of goals. The primary goal is to achieve a specified exhaust-noise reduction over a selected frequency range. Other goals include the minimization of power loss due to back pressure and the muffler volume. Factors of importance are the total weight, weight distribution, geometrical layout of the muffler and exhaust system, and the service life. Thus, to produce a viable muffler system requires not only a knowledge of applicable acoustic theory but also a knowledge of various operational constraints and their interrelationships.

This paper describes an improved analytical design method for expansion-chamber mufflers and indicates how the method is applied. The method, which was developed by Alfredson (ref. 3), includes the effect of mean exhaust-gas flow. In the present paper the theory has been formulated as an extension of the standard transmission-line theory whose primary shortcoming has been thought to be the failure to account for mean-flow effects (refs. 3, 4, 5, and 6).

The second objective of this paper is to describe a computer program that was originally developed by Alfredson. A discussion is also given of the application of the computer program to the problem of designing an expansion-chamber muffler for an aircraft engine. The paper concludes by presenting the results of this application to a helicopter.

SYMBOLS

Values are given both in SI Units and in U.S. Customary Units. Measurements and calculations were made in U.S. Customary Units.

\[
\begin{align*}
A_{k,j}, B_{k,j'} \\
C_{k,j}, D_{k,j}
\end{align*}
\] elements of impedance-parameter matrix

\[D_1, D_2\] denominators of elements of impedance-parameter matrix

c average sound speed in exhaust gas

\[f_i\] ith harmonic of engine firing frequency

2
h enthalpy

I acoustic intensity

j imaginary unit, \( \sqrt{-1} \)

k wave number, \( 2\pi/\lambda \)

L length of expansion-chamber component

M average Mach number

N final component of expansion-chamber muffler system

\( N_1, N_2, \) numerators of elements of impedance-parameter matrix

\( N_3, N_4, \) numerators of elements of impedance-parameter matrix

p average total or fluctuating pressure over cross section

\( p_i \) fluctuating pressure at station \( i \)

R reflection factor

S cost function (eq. (29))

\( S_i \) cross-sectional area at station \( i \)

s entropy

T temperature

t time

V average total velocity over duct cross section

\( V_i \) fluctuating velocity at station \( i \)

\( X_i \) transformed variable corresponding to component lengths
\( Z \)  
specific acoustic impedance

\( \gamma \)  
ratio of specific heats

\( \delta \)  
irreversible pressure loss

\( \lambda \)  
wavelength

\( \Pi \)  
series multiplication

\( \rho \)  
mass density of exhaust medium

\( \Sigma \)  
series summation

\( \phi \)  
phase angle between incident and reflected wave

\( \omega \)  
circular frequency

Subscripts:

\( b \)  
branch pipe

\( f \)  
presence of mean flow

\( i \)  
station designation, matrix-element index

\( k, j \)  
identification of impedance-parameter element

\( o \)  
stagnation value

\( t \)  
termination value

Superscripts:

\( (+) \)  
wave propagating in forward direction

\( (-) \)  
wave propagating in negative direction
MUFFLER DESIGN METHOD

Review of Transmission-Line Theory

An improved muffler design method has been developed which is an extension of transmission-line theory to include the effects of flow, termination impedance, extended inlet and outlet pipes, and yielding walls. The details underlying the development of this method are given in references 3, 4, and 5. In this section the method is described through the use of the concepts of transmission-line theory. Then, in the following section a description of the computer program and its use will be given. The reader concerned only with the application of the computer program may proceed to the section "Computer Program" without loss of continuity.

The application of the electrical transmission-line analogy to acoustic filter design was successfully accomplished in this country by Stewart in 1923 (as discussed in ref. 7). The applicability of this design tool to mufflers for reciprocating engines used in aircraft was investigated experimentally in the early fifties by Davis, Stokes, Moore, and Stevens (ref. 6). Original credit for the development of the transfer-matrix approach to four-terminal (or two-port) physical systems is due to Strecker and Feldtkeller (ref. 8). The widespread use of the transfer-matrix approach in mechanical, fluid, and thermal systems is largely due to the contributions of Pipes (ref. 9). In Japan this approach was extended to engine-exhaust muffler systems by Igarashi, Toyama, Miwa, and Arai (refs. 10, 11, and 12). Although transmission-line theory is well developed, it is of
interest to review briefly the fundamental ideas and assumptions of this theory to place
in better perspective the improved design method discussed in this paper.

The key assumptions underlying transmission-line theory in its simplest applica-
tion to muffler design are

(1) Fluctuating pressures are small relative to the average pressure in the system.
(2) The duct and muffler walls are rigid.
(3) Only plane waves are propagated (i.e., \( p(\pm) = \pm\rho c V(\pm) \)).
(4) The temperature of the medium is constant with time and spatially uniform
throughout the muffler.
(5) The average velocity of the medium is zero.
(6) Viscosity effects at the duct and muffler walls are negligible.
(7) Fluctuating quantities are decomposable into sinusoidal components of the form
\( \tilde{p} e^{j\omega t} \).

Operating principle.- Reactive mufflers, of which expansion chambers are a special
case, function by causing reflected waves to be propagated back toward the acoustic
source. In the case of expansion chambers, these reflected waves are generated by
abrupt changes of cross-sectional area. The objective in design of expansion-chamber
mufflers is to arrange combinations of duct area changes in the most propitious way for
reflecting acoustic energy back into the source over the desired frequency range. The
simplest practical expansion-chamber muffler is shown in figure 1. It consists of two
cross-sectional-area changes separated by a length \( L \). The amount of acoustic energy
associated with the reflected pressure wave \( p_1(\cdot) \) is a function of the cross-sectional
area ratio \( S_2/S_1 \), whereas the length \( L \) determines the acoustic frequency at which the
maximum reflection occurs.

Muffler performance.- The ratio of fluctuating pressure to fluctuating volume veloc-
ity of a forward-going plane wave averaged over a duct cross section through which the
wave is propagating is a useful quantity for describing the noise-reduction performance
of a reactive-muffler system and is called the analogous characteristic acoustic impedance
of the duct, or simply the characteristic impedance when the meaning is clear from the
context. At an isolated discontinuity in duct area, the change in characteristic impedance
of the duct is related to the amount of acoustic energy reflected at the discontinuity. The
total impedance change associated with a series of discontinuities may be calculated from
an appropriate set of boundary conditions relating fluctuating quantities across the dis-
continuities, or it may be determined from laboratory tests. When all the impedance
changes in a muffler system are known, the input impedance at the muffler inlet may be
calculated by starting at the tailpipe and working systematically backward to the inlet. If
the source impedance is known, the total radiated power of the system may be determined and compared with the radiated power before the muffler system is attached. This comparison is accomplished by taking the ratio of the radiated power before the muffler is attached to that after the muffler is attached. This ratio is called the insertion loss of the muffler system, in analogy with the electrical filter theory. The insertion loss, as a function of frequency, completely characterizes a muffler system insofar as its noise-reducing ability is concerned.

**Source-impedance effects.** If the amplitude of the incident wave arriving at the muffler inlet from the source is equal to the amplitude of the incident wave when the muffler system is absent, then the source is said to be nonreflecting and the effective source impedance is equal to the duct impedance. For reciprocating-engine exhaust systems in general, this condition seems unlikely to hold for all frequencies of interest; however, experimental and/or analytical studies to date have not provided reliable information on source impedance. In its absence, the muffler-system designer is forced to rely on transmission loss as a measure of muffler-system performance.

**Transmission loss.** Transmission loss is the ratio of the incident acoustic power to the transmitted power through the muffler. It is a single number (for each frequency of interest) and does not permit the prediction of acoustic performance of a complete engine and exhaust muffler system. Furthermore, transmission loss is not directly measurable and is influenced by the termination boundary condition. Its advantages are that it is conceptually simple and does provide a figure of merit when comparing muffler systems with identical terminations, hence the traditional propensity for its use. Also, to the extent that the source can be considered nonreflecting, the transmission loss is an indication of the insertion loss to be expected.

**Matrix analysis.** The algebraic complexities involved in the analysis of muffler performance make the use of matrix methods attractive. These methods have been taken over from transmission-line theory and can be easily adapted to high-speed computers. Also, the concept of the "impedance-parameter matrix" provides a better understanding of system component interaction. The impedance-parameter matrix relates the total fluctuating pressure and velocity at one station in a muffler system to the total fluctuating pressure and velocity at a different station in the system. For example, the inlet and outlet total fluctuating pressures and velocities for the expansion chamber of figure 1 (i.e., the sum of the respective forward- and backward-going wave pressures) can be related by a matrix equation of the form

\[
\begin{bmatrix}
  p_k \\
  v_k
\end{bmatrix} =
\begin{bmatrix}
  A_{k,j} & B_{k,j} \\
  C_{k,j} & D_{k,j}
\end{bmatrix}
\begin{bmatrix}
  p_j \\
  v_j
\end{bmatrix}
\]  

(1)
The elements of the square matrix can be related to the impedances of the system at the stations where the fluctuating pressures and velocities are observed. Such a matrix is built up from similar matrices characterizing the behavior of the muffler elements. For instance, continuity of pressure and volume velocity for the simple area discontinuity shown in figure 2(a) gives

\[
\begin{bmatrix}
p_2 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & \frac{S_1}{S_2}
\end{bmatrix}
\begin{bmatrix}
p_1 \\
v_1
\end{bmatrix}
\]

(2)

and for a section of constant-diameter pipe of length \( L \), the appropriate relationship is

\[
\begin{bmatrix}
p_2 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
\cos kL & j\rho c \sin kL \\
\frac{j}{\rho c} \sin kL & \cos kL
\end{bmatrix}
\begin{bmatrix}
p_1 \\
v_1
\end{bmatrix}
\]

(3)

For a branch component (see fig. 2(b)) that can be treated as a lumped branch impedance \( Z_b \), the fluctuating pressures and velocities are related by

\[
\begin{bmatrix}
p_3 \\
v_3
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
\frac{1}{\rho c} \frac{S_2}{S_3} & \frac{S_1}{S_3} \frac{1}{Z_b}
\end{bmatrix}
\begin{bmatrix}
p_1 \\
v_1
\end{bmatrix}
\]

(4)

If there are \( N \) sections in series, then the fluctuating pressure and velocity at the inlet section \( N \) can be related to the pressure and velocity at the outlet section 1 by an impedance-parameter matrix which is equal to the product of all the component matrices as follows:

\[
\begin{bmatrix}
A_{N,1} & B_{N,1} \\
C_{N,1} & D_{N,1}
\end{bmatrix} = \prod_{i=1}^{N-1} \begin{bmatrix}
A_{i+1,i} & B_{i+1,i} \\
C_{i+1,i} & D_{i+1,i}
\end{bmatrix}
\]

(5)

Transmission-loss calculation.- For transmission-loss calculations it is convenient to express the fluctuating pressure and velocities throughout the system in terms of the fluctuating pressure amplitude associated with the incident wave at the tailpipe termina-
tion \( p_1^{(+)} \). By using the boundary condition for the radiation impedance at the tailpipe exit

\[
p_1 = \rho c \bar{Z}_t V_1
\]  

(6)

the fluctuating pressure and velocity at the exit may be expressed as

\[
\begin{bmatrix}
\begin{bmatrix} p_1 \\ V_1 \end{bmatrix} & = \frac{2}{\bar{Z}_t + 1} \begin{bmatrix} \bar{Z}_t \\ 1/\rho c \end{bmatrix} \end{bmatrix} p_1^{(+)}
\]  

(7)

The radiation impedance \( \bar{Z}_t \) may be calculated (ref. 13) or measured; however, assuming \( \bar{Z}_t \) is known, equation (7), together with the impedance-parameter matrix given by equation (5), implies a knowledge of the fluctuating pressure and velocity at the muffler system inlet from which the pressure associated with the incident wave at the muffler inlet can be shown to be simply

\[
p_N^{(+)} = \frac{1}{2} \left( p_N + \rho c V_N \right)
\]  

(8)

The transmission loss is given in terms of the areas and incident-wave pressures at the muffler system inlet and termination as

\[
TL = 10 \log_{10} \frac{S_N \left[ \hat{p}_N^{(+)} \right]^2}{S_1 \left[ \hat{p}_1^{(+)} \right]^2} = 20 \log_{10} \frac{\hat{p}_N^{(+)}}{\hat{p}_1^{(+)}} + 10 \log_{10} \frac{S_N}{S_1}
\]  

(9)

Since \( p_N^{(+)} \) is proportional to \( \hat{p}_1^{(+)} \), the transmission loss is independent of \( p_1^{(+)} \) and any convenient numerical value, say unity, may be assumed for \( \hat{p}_1^{(+)} \) for computational purposes.

Inadequacies of transmission-line theory.- The laboratory data of reference 6 indicated good agreement between measured and calculated transmission losses for expansion chambers applicable to aircraft engines. These data were taken at room temperature, with no mean flow, and with a loudspeaker capable of producing a pure-tone sound pressure level of approximately 140 dB in the muffler inlets. Also, nonreflecting terminations
were used for the muffler outlets. When similar mufflers were tested on an aircraft engine, the exhaust-noise reduction fell short of that predicted from the transmission-loss calculations (ref. 6). The authors of reference 6 believed that the discrepancies were related to mean-flow effects and the violation of the small-amplitude assumption of linear acoustics. The recent work reported in references 3, 4, 5, and 14 suggested that for expansion-chamber mufflers, the nonlinear effect due to large acoustic pressures was negligible for design purposes. Further investigations reported in references 3, 4, and 5 suggested that the effect of mean gas flow on the termination boundary condition and other impedance discontinuities was the most significant factor neglected in the standard transmission-line theory. Consequently, mean-flow effects were included in the analysis of expansion-chamber performance. The results of this analysis will be summarized in the following section.

Effect of Mean Gas Flow on Muffler Performance

The improved muffler design method described herein involves the effect of the mean flow on the impedance changes at duct-area and branch discontinuities. Also, the effect on the tailpipe termination impedance is included. The effect of flow on the internal impedance changes is a straightforward extension of transmission-line theory using the linearized equations for energy, mass, and in some cases momentum conservation across area discontinuities. The single most significant effect of the flow is associated with the radiation characteristics of the tailpipe.

Tailpipe radiation. - In a duct with a termination characterized by the unflanged-pipe reflection factor \( R \), the relation between the incident-wave pressure amplitude \( \hat{p}^{(+)} \) and the reflected-wave pressure amplitude \( \hat{p}^{(-)} \) is given by

\[
\hat{p}^{(-)} = \text{Re}^j \phi \hat{p}^{(+)}
\]

where \( \phi \) is the phase angle between the incident and reflected wave. Thus the net intensity in the pipe is given by

\[
I = \frac{\left[ \hat{p}^{(+)} \right]^2}{2 \rho c} \left( 1 - R^2 \right)
\]

where \( \hat{p}^{(+)} \) is the pressure amplitude of the forward-going wave. For a mean flow with Mach number \( M \), the acoustic intensity becomes

\[
I_f = \frac{\left[ \hat{p}^{(+)} \right]^2}{2 \rho c} \left[ (1 + M)^2 - (1 - M)^2 R^2(M) \right]
\]
In this equation (see ref. 3) the reflection factor becomes a function of Mach number. An increase in acoustic radiation occurs as a result of the mean flow. To appreciate this effect, consider the ratio of $I_f$ to $I$ expressed in decibels:

$$\text{Increase in PWL} = 10 \log \frac{I_f}{I} = 10 \log \frac{(1 + M)^2 - (1 - M)^2 R^2(M)}{1 - R^2(0)} \quad (13)$$

where $R(M)$ is now a reflection factor which depends upon the Mach number of the mean flow. The measurements of reference 12 suggest a 3- to 5-percent increase in the reflection factor and no change in phase angle for Mach number and wave-number ranges typical of exhaust systems. Thus $R(M)$ in the numerator of equation (13) can be replaced by the reflection factor for zero mean flow $R(0)$ with a maximum error of approximately 2 dB. Thus, to a good approximation, the increase in the radiated acoustic power level relative to that for zero mean flow is given by

$$\text{Increase in PWL} \approx 10 \log \frac{(1 + M)^2 - (1 - M)^2 R^2(0)}{1 - R^2(0)} \quad (14)$$

This equation is plotted in figure 3 with the Mach number as a parameter. From this plot it is seen that the radiated power increases significantly relative to that for zero mean flow when the reflection factor approaches 1 and as the mean-flow Mach number increases. Typical exhaust-pipe diameters, exhaust-gas sound speeds, and exhaust-noise frequencies are such that the reflection factor is greater than about 0.975. For a typical exhaust-flow Mach number of 0.1, the plot of figure 3 implies an increase of 8 dB or more in the radiated acoustic power over that for zero mean flow.

Area contraction.- References 3 and 5 indicated that a one-dimensional description of wave propagation on either side of a duct discontinuity would be adequate for muffler design provided that the mean-flow effect was included. For flow area contractions the isentropic energy-balance equation was linearized to obtain one equation relating fluctuating pressure and velocity on either side of the discontinuity. This resulting equation is given as

$$p_2 + \rho c M_2 V_2 = p_1 + \rho c M_1 V_1 \quad (15)$$

where the subscripts 1 and 2 denote the downstream and upstream stations, respectively, as indicated in the schematic of figure 4(a). From mass conservation across the area discontinuity a second linearized equation is found to be
If these equations are formulated in a matrix format and solved for the upstream fluctuating pressure and velocity, there results

\[
\begin{bmatrix}
  p_2 \\
  V_2
\end{bmatrix}
= \begin{bmatrix}
  1 & \rho c M_1 \\
  0 & \frac{S_1}{S_2}
\end{bmatrix}
\begin{bmatrix}
  1 - \left(\frac{S_1}{S_2}\right)^2 \\
  1 - M_1^2 \left(\frac{S_1}{S_2}\right)^2
\end{bmatrix}
\begin{bmatrix}
  p_1 \\
  V_1
\end{bmatrix}
\]

Thus, the effect of the mean flow is to introduce a new element into the impedance-parameter matrix and a correction for the element which relates the velocities. Clearly, when the Mach number becomes zero, the matrix reduces to the form of that in equation (2).

**Area expansion.** Experimental evidence discussed in reference 3 suggests that the energy relation connecting fluid state properties across a sudden flow area expansion is nonisentropic. However, an adiabatic condition can be assumed to hold. Hence the stagnation enthalpy would be conserved across such a discontinuity; that is,

\[
dh_0 = T \, ds + \frac{dp}{\rho} + V \, dV = 0
\]

The entropy term \( T \, ds \) has been shown in reference 15 to be given by

\[
T \, ds \approx \frac{d\delta}{\rho(\gamma - 1)}
\]
\[ \frac{\rho}{c^2} = \frac{p + \delta}{c^2} \]  

(20)

By using the relation (19), equation (18) can be integrated across the discontinuity from the downstream to the upstream side, as indicated in the schematic of figure 4(b). There results after linearization

\[ p_2 + \rho c M_2 V_2 = p_1 + \rho c M_1 V_1 - \frac{\delta}{\gamma - 1} \]  

(21)

From mass conservation and the use of equation (20) to eliminate the fluctuating density,

\[ S_2 M_2 p_2 + \rho c S_2 V_2 = S_1 M_1 p_1 + \rho c S_1 V_1 + S_1 M_1 \delta \]  

(22)

Because of the introduction of the unknown quantity \( \delta \), a third equation must be derived. This can be obtained from momentum conservation across the discontinuity, which gives after linearizing and the use of equation (20)

\[ \left( S_1 + S_2 M_2^2 \right) p_2 + 2 \rho c S_2 M_2 V_2 = \left( S_1 + S_1 M_1^2 \right) p_1 + 2 \rho c S_1 M_1 V_1 + S_1 M_1^2 \delta \]  

(23)

where the momentum source term due to the discontinuity has been taken to be \( p_2 (S_1 - S_2) \), which is based on experimental evidence indicated in reference 3. When this equation is solved for \( \delta \) and substituted into equations (21) and (22), the upstream fluctuating pressure and velocity can again be related to the corresponding downstream quantities by an impedance-parameter matrix whose elements are given by

\[ A_{k,j} = \frac{1 + \left[ \frac{S_1}{S_2} \right]^2 (\gamma - 1) - 2 \gamma \frac{S_1}{S_2} + \gamma}{D_1} \]  

(24a)

\[ B_{k,j} = \frac{\rho c \left\{2 \left(1 - \frac{S_1}{S_2}\right) M_1 + (\gamma - 1) \left[ \frac{S_1}{S_2} \right]^2 - 2 \frac{S_1}{S_2} + 1 \right\} M_1^3}{D_1} \]  

(24b)
\[
C_{k,j} = \frac{1}{\rho c} \left[ \frac{S_1}{S_2} \left( \frac{S_1}{S_2} - 1 \right) \right] M_1^3
\]
where

\[
D_1 = 1 + \left\{ (\gamma - 1) \left[ 1 + \left( \frac{S_1}{S_2} \right)^2 + \frac{S_1}{S_2} \left( \frac{S_1}{S_2} - 2 \gamma \right) \right] M_1^2 + \left[ (\gamma - 1) \left( \frac{S_1}{S_2} \right)^2 \left( \frac{S_1}{S_2} - 1 \right) \right] M_1^4 \right\}
\]

Again, these equations are seen to reduce to a form equivalent to equation (2) when the Mach number approaches zero.

**Area contraction with a branch.** Practical expansion-chamber mufflers usually have extended inlet and outlet pipes, as shown in the schematic of figures 4(c) and (d). Such geometrical arrangements perform acoustically as a side branch with impedance \( Z_b \). The side branch is indicated as region (2) in figures 4(c) and (d). The value of \( Z_b \), which is specified at the junction of regions (1), (2), and (3), is controlled by the rigidity of the end wall and by the branch length. The branch impedance \( \bar{Z}_b \) is related to the reflection factor \( R_b \) and phase angle \( \phi_b \) at the branch entrance by the equation

\[
\bar{Z}_b = \frac{1 + R_b e^{j\phi_b}}{1 - R_b e^{j\phi_b}}
\]

If the reflection factor and phase angle at the end cap are denoted by \( R \) and \( \phi \), respectively, then \( R_b = R \) for no energy dissipation between the entrance and end cap. Also, \( \phi_b = \phi - 2kL \), where \( L \) is the branch length and the coordinate direction is that shown in figures 4(c) and (d). By the appropriate selection of the various branch lengths in a muffler system, the transmission-loss characteristic can be tailored to specifications within certain limits. For a branch impedance associated with a flow area contraction shown schematically in figure 4(c), the impedance-parameter matrix for the simple area contraction can be generalized to include the branch impedance. The result is
Note that if $Z_b$ approaches infinity (i.e., the end wall becomes rigid and the branch length becomes zero), then the impedance-parameter matrix reduces to that of equation (17). Furthermore, if the mean-flow Mach number approaches zero, then the result becomes equivalent to that of equation (4).

**Area expansion with a branch.** - If the muffler inlet pipe is extended, then a branch impedance becomes associated with an area expansion. When this impedance is included in the boundary condition for an area expansion, the elements for the impedance-parameter matrix relating upstream and downstream fluctuating pressures and velocities become

$$A_{k,j} = \frac{N_1 + \frac{S_1 S_2}{Z_b S_3 S_3} \left[ M_1^2 + (\gamma - 1)M_1^3 + \frac{S_1}{S_3} M_1^4 \right]}{D_1 + D_2} \quad (27a)$$

$$B_{k,j} = \frac{N_2 - \frac{\rho c}{Z_b S_3 S_3} \left[ 2M_1^2 - (\gamma - 1)M_1^4 \right]}{D_1 + D_2} \quad (27b)$$

$$C_{k,j} = \frac{N_3 - \frac{1}{\rho c Z_b S_3} \left( 1 + \gamma M_1^2 \right)}{D_1 + D_2} \quad (27c)$$
where \( N_1, N_2, N_3, \) and \( N_4 \) are the respective numerators of equations (24a, b, c, and d), and \( D_1 \) is given by equation (24e). The quantity \( D_2 \) is associated with the branch impedance \( Z_b \) and is given by

\[
D_2 = \frac{1}{Z_b} \frac{S_2}{S_3} \left[ \frac{S_1}{S_3} - 2 \right] M_1 + \left( \frac{S_1}{S_3} \right)^2 M_1^3
\]

Again, when \( Z_b \) becomes large, these equations are seen to reduce to equations (24), as expected. It was found by the authors of references 4, 5, and 6 that yielding walls could significantly affect muffler performance; consequently, in references 4 and 5 the impedance of the end caps was altered in a trial-and-error procedure to force the mathematical models for the branched elements to conform with measurements. For a flow area contraction the reflection factor and phase angle of the branch impedance at the end cap were taken as 0.8 and 0.01 radian, respectively, whereas for flow area expansion the reflection factor was taken to be 1.0 and the phase angle as 0.01 radian. The lower reflection factor for area contraction was used because the isentropic flow condition across that type of discontinuity does not allow the occurrence of acoustic energy dissipation, which in fact is known to exist but to a much lesser extent than for an expansion type of discontinuity. This completes the formulations of the impedance-parameter matrix for the various discontinuities encountered in a muffler system consisting of a series of cascaded expansion chambers.

Finite-length section.- To relate the fluctuating pressure and velocity at the upstream side of a discontinuity to the fluctuating pressure and velocity at the downstream side of the next discontinuity joined by a constant-diameter duct (see fig. 5) carrying a mean flow with Mach number \( M_3 \), equation (3) is modified as follows:

\[
\begin{bmatrix}
  p_4 \\
  V_4
\end{bmatrix} = \begin{bmatrix}
  \cos \left( \frac{kL}{1 - M_3^2} \right) & j \rho c \sin \left( \frac{kL}{1 - M_3^2} \right) \\
  \frac{j}{\rho c} \sin \left( \frac{kL}{1 - M_3^2} \right) & \cos \left( \frac{kL}{1 - M_3^2} \right)
\end{bmatrix} \begin{bmatrix}
  p_3 \\
  V_3
\end{bmatrix}
\]

(28)
Clearly, for the typical mean-flow Mach numbers of 0.1 encountered in exhaust systems, this correction is relatively insignificant.

**Model comparisons.** - Figure 6 shows a comparison of transmission loss calculated by the theoretical model described in this paper with the standard transmission-line theory discussed in reference 6. The muffler configuration chosen for illustrative purposes is shown in the sketch in the figure along with relevant geometrical data. Note that there is a significant reduction of transmission loss in the frequency range 200 to 350 Hz due to both the mean-flow effects and the yielding end cap (reflection factor 0.8 for flow area contraction). On the other hand there is an increase in transmission loss above 350 Hz over that for no mean flow.

**COMPUTER PROGRAM**

A computer program called EXRSIL has been written that incorporates the effects of the mean flow as well as estimated reflection factors for the muffler end walls associated with flow area contractions. This program will enable a user to design expansion-chamber mufflers of up to four stages. To increase the usefulness of the program as a design tool, an optimization subroutine has been included that will adjust all muffler component lengths to approach a specified minimum transmission-loss characteristic within imposed component-length constraints. The physical meanings of the inputs are described in this section and the detailed formats are described in appendix A. Appendix B contains a listing of the program.

**Program Inputs**

Input quantities for the computer program consist of the following:

1. Desired minimum transmission loss at three wavelengths
2. Acoustic sound speed and wavelength in the exhaust system
3. Gas-flow Mach number in exhaust tailpipe
4. Optimization attempts
5. Number of expansion chambers
6. Length constraints and initial lengths for all expansion-chamber components
7. Cross-sectional areas of expansion chamber, annular region, and tailpipe

**Desired transmission loss.** - The desired minimum transmission-loss characteristic in decibels is deduced with the help of narrow-band analysis of far-field exhaust noise over the relevant operating range of engine load and rotational speed. The engine should preferably be equipped with an exhaust pipe of the same length as the final com-
plete muffler system. A representative narrow-band spectrum of exhaust-noise-dominated helicopter noise is shown in figure 7. Prominent engine-bank firing frequencies are labeled in the figure as $f_1$, $f_2$, and so forth. The test conditions for this spectrum will be discussed in the section "Application to Helicopter." The frequency range over which the muffler is to be effective and the amount of transmission loss desired at each frequency are largely a matter of engineering judgment. Since the low-frequency noise components are less objectionable, it is important to transform the narrow-band noise levels to A-weighted levels to reduce the required minimum transmission losses at the lower frequencies. To see how the minimum desired transmission loss is specified, the bar graph of figure 8 should be studied. This bar graph represents the important exhaust noise components of the spectrum shown in figure 7.

In figure 8, the shaded bars represent the abstracted narrow-band levels associated with the engine firing frequencies shown in the measurement represented in figure 7. The overall level associated with the exhaust noise was found to be 96 dB. The solid bars represent the A-weighted levels for which the overall level was found to be 82 dB(A). The arrows represent the desired levels to which the lowest, intermediate, and highest frequency A-weighted components are to be reduced. These desired exhaust noise levels are decided upon from a determination of the levels of other dominating noise sources and, possibly, other considerations such as volume and weight. The computer program constructs by linear interpolation the desired transmission-loss characteristic as a function of $\log_{10} \lambda$, as shown by the solid line in figure 9. This interpolation is based upon three pairs of numbers consisting of transmission losses and wavelengths corresponding to the firing frequencies indicated by the arrows in figure 8. The dashed line in figure 9 is the computed transmission loss for an optimized muffler which will be discussed in the section "Application to Helicopter."

Wavelength and Mach number.- The wavelengths at which the designer specifies a desired transmission loss must be calculated from the speed of sound in the exhaust gas estimated from the mean temperature distribution throughout the proposed muffler system. Gas-flow Mach number must likewise be measured or estimated for the tailpipe of the proposed muffler system.

Optimization attempts.- This input specifies the number of different directions of the gradient vector taken in cost-function space (see section "Optimization Subroutine") to achieve a minimum cost function. If a zero value of the cost function is achieved, then the number of optimization attempts should be increased and/or adjustments to geometric constraints considered.

Geometrical inputs.- The remainder of the inputs concern the geometry of the muffler system. The volume and shape constraints are specified by the number of expansion chambers, the muffler stage component lengths, and the cross-sectional areas of the tail-
pipe, chamber, and annulus. It should be noted that the cross-sectional areas cannot be varied within a stage or between stages. There are four component lengths per muffler stage as indicated in figure 10. Constraints on these lengths are specified independently for each stage together with an initial length for each stage component.

Program Outputs

Program outputs are the following:

1. Input data listing
2. Computer-generated desired minimum transmission-loss characteristic
3. Record of optimization attempts
4. Optimized component lengths
5. Computed transmission-loss characteristic

Input data listing. - This section of output simply tabulates the minimum, initial, and maximum values of all component lengths working from the tailpipe to the inlet. Also given are the cross-sectional areas and tailpipe Mach number.

Desired transmission loss. - The minimum desired transmission-loss characteristic is listed along with the frequency and wavelength as determined from the mean temperature of the exhaust gas.

Optimization attempts. - The next listing shows a record of the optimization attempts which consists of the cost functions and the corresponding component lengths at a particular point in the optimization process. The message NEW POINT indicates the change of direction of the gradient vector in the cost-function space.

Optimized component lengths. - This listing shows the component lengths for the final configuration together with the final cost function.

Computed transmission loss. - This section of output lists the computed transmission loss of the final configuration at the same wavelengths and frequencies as was done for the desired transmission loss.

Engineering Judgments

Reduction of low-frequency exhaust noise generally deteriorates with decreasing expansion-chamber length. Thus it is mandatory to establish the maximum muffler length available if low-frequency noise is a problem. Most practical muffler systems will incorporate flow-reversing bends to achieve necessary compactness, as shown in the schematic of figure 10. The transmission loss of such bends is estimated to be less than 2 dB when transmitting into a nonreflecting termination over the frequency range of interest in
reactive-muffler design. Thus, the effect of smooth bends should be relatively unimportant for design purposes.

The muffler geometry also affects the back pressure at the engine exhaust ports. Since an increase in mean back pressure has been found to correlate roughly with degradation of multicylinder-engine performance, mean back pressure is conventionally used as an indicator of possible loss in engine performance. Mean back pressure is minimized in purely expansion-chamber mufflers by making bends as smooth as possible and by aligning inlet and outlet pipes, as shown in figure 10. No attempt is made in this program to compute magnitude of mean back pressure.

All component-length constraints should be mutually consistent and compatible with the space requirements. If some component lengths must be held constant, constraints must still be specified to bracket the initial length so that a significant variation cannot occur. Maximum lengths of internal pipes should not be such that their separation is less than 1 pipe diameter and preferably 1 chamber diameter.

Optimization Subroutine

The transmission loss of a muffler system is treated by the optimization subroutine as a function of \(4N\) variables, where \(N\) is the number of muffler stages. By systematically changing the various component lengths and comparing the resulting transmission losses with the desired minimum transmission losses at 151 equally spaced values of the logarithm of the wavelength, the subroutine attempts to minimize a cost function. This cost function is defined by

\[
S(L_1, L_2, \ldots, L_{4N}) = \sum_{i=1}^{151} \left[ (\text{TL})_d(\lambda_i) - (\text{TL})_c(\lambda_i) \right]^2
\]

where \((\text{TL})_d(\lambda_i)\) is the desired transmission loss at the wavelength \(\lambda_i\) and \((\text{TL})_c(\lambda_i)\) is the computed transmission loss at the same wavelength for some given configuration. If \((\text{TL})_d(\lambda_i) \leq (\text{TL})_c(\lambda_i)\) for a particular wavelength, then that term is omitted.

The cost-function independent variables \(L_i\) must all have boxlike constraints imposed on them, that is,

\[
L_{\text{min}} < L_i < L_{\text{max}}
\]

On the other hand, the optimization procedure must operate with independent variables that are unconstrained. This condition is fulfilled by defining a new set of variables \(X_i\) by means of the transformation
The details of the process by which the procedure systematically alters the component lengths are discussed in reference 3 and will not be given here; however, the process is basically a modification and extension of the steepest-descent method that results in an improved rate of convergence. From the user's standpoint, he can regard the subroutine as a minimization process that operates by following a prescribed direction vector in the cost-function space until a minimum is achieved; then the direction vector is changed or updated whereupon a search for a new minimum will begin. This event is signified in the output listing by the heading NEW POINT. The number of new points (or optimization attempts) to be taken is set by the user by reading in a value of the integer variable NO, which is usually taken to be between 5 and 10. It is the nature of the optimization procedure to find the minimum cost function which is nearest the initial configuration of component lengths. Since other more significant or preferable minima may exist, several different sets of initial configurations should be tried. At the end of the last minimum search, a normal return of control to the main program will be indicated by the message SEARCH COMPLETED, NORMAL RETURN.

This completes the description of the computer program and its use. In the remainder of this paper an evaluation of the results obtained upon applying the program to the design of a muffler for a helicopter will be given.

APPLICATION TO HELICOPTER

The purpose of this section is to describe the design and to present the measured noise-reduction performance of a muffler for a helicopter engine exhaust. The muffler design was accomplished with the aid of the previously described computer program.

Apparatus and Methods

Test helicopter.—A photograph of the test helicopter for which a muffler system was designed is shown in figure 11. The physical characteristics are given in table I. The aircraft was powered by a six-cylinder, horizontally opposed engine. The exhaust gases from each bank of three cylinders are routed through a manifold and then out a straight stack to the side and rear of the helicopter, as shown in the photograph of figure 12. The engine fundamental firing frequency is defined to be \[ N_c \times \text{rot. speed} \over 120 \] for a four-stroke operating cycle, where \( N_c \) is the number of cylinders. For the manifold

\[
sin^2 x_i = \frac{L_i - L_{\text{min}}}{L_{\text{max}} - L_{\text{min}}} \quad (31)
\]
geometry of this particular engine, it was convenient to correlate the exhaust noise components with harmonics of the engine-bank fundamental firing frequency, in which case $N_c$ was taken as 3.

**Acoustic measurements.** Initial base-line acoustic measurements were conducted for the helicopter with the engine equipped with the standard exhaust manifold system shown by the photograph of figure 12 and hereinafter called the standard helicopter. The acoustic signals emitted by the helicopter were measured with commercially available, piezoelectric, ceramic-type microphones and were recorded by a multichannel, frequency-modulated tape recorder at a tape speed of 0.76 m/sec (30 in/sec) and a center frequency of 54 kHz. The frequency response of the complete measurement system was within 3 dB over the frequency range 12 Hz to 12 kHz.

The microphones were positioned at a radius of 30.5 m (100 ft) from the aircraft every 18° over one quadrant. A complete 360° noise survey of the helicopter noise was obtained by hovering the helicopter first at 0°, then 90°, 180°, and finally at 270°. The measurements were conducted over flat terrain at the approach end of runway 17 at Langley Air Force Base.

**Muffler Design and Installation**

**Design procedure.** The computerized analytical muffler design method and required engineering judgments have already been described elsewhere in this report. In this section specific design considerations for the test helicopter will be discussed.

Weight and volume limitations indicated that a single muffler that would handle the exhausts from both engine banks would be preferable to a muffler for each bank. Also, it was anticipated that the Y-connector used for combining the two streams of engine exhaust gas would provide some noise reduction of the engine odd-harmonic firing frequencies and some amplification of the engine even-harmonic firing frequencies. This possibility permitted a muffler design with a less demanding transmission-loss performance at the engine-bank fundamental firing frequency. This relaxed performance requirement is evidenced by the 9 dB desired minimum transmission loss at the engine-bank fundamental firing frequency indicated by the arrow in figure 8.

**Back-pressure considerations.** Engine back pressure produced by the muffler system was not computed or measured; however, an effort was made to minimize back pressure by making the total exhaust-pipe area as large as was practical. Also, the inlet and outlet pipes in all the chambers were arranged so that the center lines coincided insofar as was practical. Finally, sudden flow-direction changes were avoided as much as possible.
Muffler configuration.- The computer-generated component lengths and a sketch of the corresponding geometrical configuration are shown in figure 13. It is important to understand that the general configuration is determined by the designer. The designer must specify constraints on the overall lengths, cross-sectional areas, relative arrangement and number of chambers, and the handling of the internal flow. The computer program adjusts the various component lengths within imposed constraints to achieve as nearly as possible a desired transmission-loss characteristic. The configuration shown in figure 13 produces the transmission-loss characteristic shown by the dashed line in figure 9, which compares favorably with the minimum desired transmission-loss characteristic shown by the solid line in figure 9. The muffler system was designed to produce an exhaust-noise reduction of 15 to 20 dB(A).

Installation.- The installation of the Y-connector and muffler is shown in figures 14 and 15, respectively. The exhaust gases were combined with the Y-connector then led into the muffler inlet, as shown in figure 15. This particular mounting arrangement allowed minimum disturbance of the aircraft weight balance and prevented operating problems relative to the high-temperature exposed surfaces. The muffler volume was 0.057 m$^3$ (2.0 ft$^3$) and the weight of the entire system was 21.3 kg (47 lb).

Results and Discussion

Exhaust-noise reduction.- Total radiated noise from the helicopter was measured with the muffler system installed (hereinafter called the modified helicopter) as was done for the standard helicopter configuration. The narrow-band (4-Hz) spectrum of the noise from the modified helicopter is shown in figure 16 by the solid curve, and the narrow-band spectrum for the standard helicopter from figure 7 is shown superimposed in figure 16 by the dashed line to facilitate evaluation of the muffler-system effectiveness. Clearly, the dominant components of the exhaust noise have been eliminated. At the low frequencies, rotor noise now appears to dominate the spectrum. In the higher frequency range, 320 to 520 Hz, there is a 3 to 5 dB increase in the spectrum floor which may be attributable to self-noise (see ref. 16) generated by the muffler system. The overall exhaust-noise reduction obtained was approximately 11 dB(A), whereas the desired reduction was 15 to 20 dB(A).

Estimated muffler insertion losses.- The spectrum information shown in figure 7 and similar data with the Y-connector only installed were used to estimate the insertion losses of the muffler at the engine-bank firing frequencies. These insertion losses are listed in the last column of table II. Also listed are the measured sound pressure levels under comparable test conditions corresponding to the standard exhaust system, Y-connector only, and Y-connector with muffler. From these tabulations the estimated muffler insertion losses were obtained. It is important to note that the listed insertion
losses are lower bounds on the actual muffler insertion losses because of possible contributions from other discrete-frequency noise sources radiating at approximately the same frequencies as the tailpipe exit.

**Estimated transmission losses.** - If at a particular frequency the amplitude of the incident wave in the exhaust pipe outlet is not changed by the addition of the muffler, then the insertion loss is equal to the transmission loss at that frequency. Assuming this condition to hold, the estimated insertion losses of table II were equated to muffler transmission losses and plotted on the computed transmission-loss curve for the muffler. The results for the first four engine-bank firing frequencies are shown in figure 17. For frequencies 2, 3, and 4 the computed transmission losses are high by 5 to 15 dB. For the higher engine-bank firing frequencies, the estimated insertion losses are negative, indicating that the muffler is amplifying at these frequencies by 9 to 14 dB. Plausible explanations for this behavior include both source-impedance effects and self-generated noise by the muffler at discrete frequencies (ref. 16). The mathematical model described in this paper fails to account for these effects.

**Engine performance loss.** - Accurate back-pressure measurements for the modified exhaust system were not obtained; however, the aircraft operator did not observe any significant degradation of engine performance for hover flight conditions.

**CONCLUDING REMARKS**

An improved design method for expansion-chamber mufflers has been described and applied to a helicopter exhaust-noise problem. The resulting three-stage expansion-chamber muffler together with the exhaust-pipe Y-connector for combining the exhaust gases from all cylinders reduced the exhaust noise by 11 dB(A). Experimental comparisons between the noise levels for the standard and modified exhaust systems indicated that the muffler was generating self-noise and/or interacting with the source impedance to result in an amplification of 9 to 14 dB at the fifth and sixth engine-bank firing frequencies.

The muffler volume was 0.057 m\(^3\) (2.0 ft\(^3\)) and the weight was 21.3 kg (47 lb), which were within the limits for a flightworthy system. There was no significant loss of engine performance as judged by the aircraft operator.

Langley Research Center,
National Aeronautics and Space Administration,
APPENDIX A

COMPUTER PROGRAM EXRSIL

Language: FORTRAN

Purpose: To compute transmission loss at 151 values of wavelength for multiple-stage expansion-chamber mufflers. The program makes a specified number of attempts to optimize the transmission loss in accordance with a desired minimum transmission-loss characteristic. The effect of mean flow is included in the program.

Use: Data cards are prepared to be read by statements shown below:

READ 8, NQ, C
READ 10, S1, S2, S3
READ 10, AMACH
READ 10, XYZ(1,1),XYZ(1,2)
READ 22, NUMBER
READ 10, G, (1),X(1),H(1)
8 FORMAT (I10,F10.3)
10 FORMAT (3F10.5)
22 FORMAT (15)

Input variables are

NQ       Number of optimization attempts
C       Sound speed in exhaust gas of proposed muffler, ft/sec
S1      Tailpipe cross-sectional area, ft²
S2      Annular cross-sectional area, ft²
S3      Chamber cross-sectional area, ft²
AMACH    Mach number of mean flow in tailpipe of muffler
XYZ(I,1) = I=1, 2, 3  Wavelengths, ft (minimum, intermediate, maximum)
XYZ(I,2)I=1, 2, 3  Desired minimum transmission losses at above wavelengths, dB
NUMBER    Number of stages in muffler system (should not exceed 4)
G(I),X(I),H(I) I=1,2,3,4 Minimum, initial, and maximum lengths of muffler-stage components starting at tailpipe and working toward inlet, ft (note that there will be four lengths per chamber)

Restrictions: Tailpipe Mach number should be in the approximate range 0.05 to 0.15.
The ratio of expansion-chamber cross-sectional area to tailpipe cross-sectional area should not exceed approximately 6. The number of expansion-chamber stages should not exceed 4.
APPENDIX A – Concluded

**Method:** The theoretical basis used in this program is given in references 3, 4, 5 and described briefly in this paper.

**Accuracy:** Within the limitations of the theoretical model, the accuracy of the computed transmission losses is for practical purposes a function only of the accuracy of the input data.

**References:** See references 3, 4, and 5 of this report.

**Run time:** 200 to 600 sec

**Storage:** 42,0008 (CDC 6000 series)
APPENDIX B

PROGRAM LISTING

PROGRAM EXRSIL (INPUT, OUTPUT)
C THIS PROGRAM WAS ORIGINATED BY DR. R. J. ALFREDSON AND IS BASED ON
C EMPIRICAL DATA FOR A MACH NO RANGE .05 TO .15 AND EXPANSION RATIOS
C UP TO 6.
COMMON/ONE/NS, FS,T, JP, TST, STP
COMMON/TWO/W1,W2,W3
COMMON/THREE/G(20),X(20),H(20),AL(20,20),DO(20,20),D(20),U(20)
COMMON/FOUR/NO, IROCT, TUL
COMMON/FIVE/S1, S2, S3, SVEL, AM(20), VAL(20,3), NUMBER
COMMON/SIX/XYZ(1,2), AIM(151), AAA
C READ NUMBER OF OPTIMIZATION ATTEMPTS AND EXHAUST GAS SOUND SPEED
   READ 8,NO,C
      8 FORMAT(I10, F10.3)
C READ AREAS OF TAILPIPE, ANNULUS, AND EXPANSION CHAMBER
   READ 10, S1, S2, S3
C READ MACH NUMBER OF GAS FLOW IN TAILPIPE
   READ 10, AMACH
C READ THREE SETS OF WAVELENGTHS AND TRANSMISSION LOSSES
   DO 23 I=1, 3
      23 READ 10, XYZ(I,1), XYZ(I,2)
C FORM IN ARRAY AIM DESIRED MINIMUM TRANSMISSION LOSSES AT 151 VALUES OF
C WAVELENGTH
   ABC=ALG10(XYZ(1,1))
   AAB=ALG10(XYZ(2,1))
   BCA=ALG10(XYZ(2,1)/XYZ(1,1))
   CAB=ALG10(XYZ(3,1)/XYZ(2,1))
C VARIABLE #AAA# IS AN INCREMENTAL LOGARITHMIC WAVELENGTH GOTTED BY
C DIVIDING WAVELENGTH RANGE INTO AN EQUAL NUMBER OF PARTS
   AAA=0.0066667*ALG10(XYZ(2,1)/XYZ(1,1))
   RAB=ABC
   DO 25 I=1, 151
      25 AIM(I)=XYZ(I,2)+(RAB-ABC)/BCA*(XYZ(2,2)-XYZ(1,2))
C NEXT STATEMENT CAUSES SHIFT FROM POSITIVE SLOPE SIDE OF DESIRED
C TRANSMISSION LOSS CURVE TO NEGATIVE SLOPE OF CURVE
   IF(RAB.GT.AAB) GC TO 26
   26 RAB=RAB+AAA
C THIS DO LOOP STORES VALUES IN AIM FROM NEGATIVE SLOPE SIDE OF
C TRANSMISSION LJSS CURVE
26 DO 27 N=1,151
   RAB=RAB+AAA
27 AIM(N)=XYZ(2,2)*(XYZ(3,2)-XYZ(2,2))*(RAB-AAB)/CA3
   RAB=AAB
C READ IN NUMBER OF CHAMBERS NOT TO EXCEED FOUR
C NN=NUMBER OF AXIAL DIMENSIONS FOR SILENCER (4 PER CHAMBER)
C NN=4*NUMBER
C NS=NUMBER OF VARIABLES ENTERING THE COST FUNCTION
C NS=NN
C READ DIMENSION CONSTRAINTS (MINIMUM, INITIAL VALUE, MAXIMUM)
20 DO 20 I=1,NN
   10 FORMAT(3F10.5)
      AM(I)=AMACF
      AM(3)=AMACF*S1/S3
C PRINT DETAILS OF INITIAL CONFIGURATION
PRINT 12
12 FORMAT(1HO,25X*RANGES AND INITIAL LENGTHS OF SILENCER COMPONENTS*)
PRINT 11
11 FORMAT(1HO,12X*TAILPIPE EXTENDED Outlet
1 EXPANSION CHAMBER EXPANDED INLET*/1HO,4X*MIN
2 INITIAL MAX MIN INITIAL MAX MIN MAX*)
   PRINT 5,(G(I),X(I),H(I),I=1,NN)
   5 FORMAT(12F10.4)
   PRINT 14,S1,S2,S3
14 FORMAT(1HO,14HTAILPIPE AREA=F10.5,18H
1 EXPANSION CHAMBER AREA=F10.5)
   PRINT 3,AMACH
   3 FORMAT(1HO,21HTAILPIPE MACH NUMBER=F10.5)
PRINT 50
50 FORMAT(/3dH DECREASE MINIMUM TRANSMISSION LOSS/1HO,33H*WAVELENG
1 DECREASES FREQUENCY)
C THIS DO LOOP PRINTS THE DESIRED MINIMUM TRANSMISSION LOSS
28 DO 28 I=1,151
   FRED=10.0**RAB
   SPEED OF SOUND ESTIMATED FOR PROPOSED MUFFLER SYSTEM IS USED HERE
   FRED=C/FRED
   PRINT 17,FRED,AIM(I),FREQ
28 RAB=RAB+AAA
   CALL OPTIM
C PRINT DETAILS OF OPTIMUM SILENCER
PRINT 15
15 FORMAT(39X*OPTIMUM LENGTHS OF SILENCER COMPONENTS IN FEET*)
PRINT 13, (X(I),I=1,NN)

13 FORMAT(1H9,9HTAILPIPE=F10.4,24H       EXTENDED OUTLET(1)=F10.4,26H
     1 EXPANSION CHAMBER(1)=F10.4,23H       EXTENDED INLET(1)=F10.4/1H0,
     29HCON PIPE=F10.4,24H       EXTENDED OUTLET(2)=F10.4,26H EXPANSION
     3N CHAMBER(2)=F10.4,23H       EXTENDED INLET(2)=F10.4/1H0,3HCON PIPE=
     4F10.4,24H       EXTENDED OUTLET(3)=F10.4,26H EXPANSION CHAMBER(3)
     5)=F10.4,23H       EXTENDED INLET(3)=F10.4)
PRINT 21,FS
21 FORMAT(5x20HFINAL COST FUNCTION=F10.3)
PRINT 52
52 FORMAT(1HO*FREQUENCY IS COMPUTED FOR SCNIC VELOCITY ESTIMATED FOR
1EXHAUST GAS TEMPERATURE*)
PRINT 51
51 FORMAT(1H25H       TRANSMISSION LOSS/1H0,33H*WAVELENGTH   DECIBELS
1  FREQUENCY)
DO 16 I=1,151
   FRED=10.0**VAL(I,1)
   FREQ=C/FRED
16 PRINT 17,FRED,VAL(I,2),FREQ
17 FORMAT(3F10.3,2X)
STOP
END

SUBROUTINE AUX(V)
C CALCULATES TL FOR 151 VALUES OF WAVELENGTH AND THE COST FUNCTION
CUMPLEX PI,PR,E
DIMENSION V(20),X(20)
COMMON/ONE/NS,FS,T,J,P,T1,STP
COMMON/THREE/G(20),X(20),AL(20,20),DG(20,20),D(20),U(20)
COMMON/FOU/S1,S2,S3,SVEL,AM(20),VAL(200,3),NUMBER
COMMON/TEN/XYZ(3,2),AIM(151),AAA
C COORDINATE TRANSFORMATION
DO 10 I=1,NS
10  W(I)=G(I)*H(I)-G(I)**SIN(V(I))*SIN(V(I))
RAD=SQR(T(S1/2,1412))
RAD=ALOG10(XYZ(1,1))
C CALCULATE TL AT 151 VALUES OF WAVELENGTH
DO 11 I=1,151
   WLTH=10.0**RAD
   AMACH=AM(I)
C RADIATION CONDITION
CALL RAUPLAN(WLTH,RAD,AMACH,PI,PR,R,TH)
II=0
C TAIL PIPE OR PIPE SEPARATING SILENCERS
DO 12 IJ=1,NUMBER
AMACH=AM(I)
II=II+1
AL1=W(IJ)
CALL PIPE(WLTH,AL1,AMACH,PI,PR)
C EXTENDED OUTLET
II=II+1
AL1=W(IJ)
CALL EXOUTLET(WLTH,S1,S2,S3,AMACH,AL1,PI,PR)
C PIPE BETWEEN INLET AND OUTLET
AMACH=AM(3)
II=II+1
AL1=W(IJ)-W(IJ-1)-W(IJ+1)
CALL PIPE(WLTH,AL1,AMACH,PI,PR)
C EXTENDED INLET
II=II+1
AL1=W(IJ)
CALL EXINLET(WLTH,S3,S2,S1,AMACH,AL1,PI,PR)
C CALC TL FOR EACH WAVELENGTH
12 CONTINUE
CALL FINDCUT(PI,NA,WW,WWW,WWW)
VAL(I,1)=RAB
VAL(I,2)=WWW
VAL(I,2)=10.0*ALOG10(VA(I,2))
C GAS FLOW NOISE LIMITS MAX TL TO ABOUT 45 DB
IF(VA(I,2).LT.45.0)GO TO 11
VAL(I,2)=45.0
11 RAB=RAB+AAA
C FORM COST FUNCTION----SUM OF SQUARES LESS THAN AIM
FS=0.0
DO 15 I=1,151
IF(VA(I,2).GT.AIM(I)) GO TO 15
FS=FS+(AIM(I)-VA(I,2))*(AIM(I)-VA(I,2))
15 CONTINUE
RETURN
END
SUBROUTINE RADFLAN(WLTH,RAD,AMACH,PI,PR,R,TH)
C       THIS S/R CALCULATES REF. AND PHASE ANGLE, R, TH.
C       INCIDENT WAVE, PI=1.0000, 0.0000, REFLECTED WAVE= R * PI * EIC
COMPLEX PI,PR
PI=(1.0000,0.0000)
P=6.2832*RAD/WLTH
P1=P
P2=P1*P
P3=P2*P
P4=P3*P
P5=P4*P
P6=P5*P
TH=3.0885347-0.726735573*P1-0.1681905*P2+0.0631405174566*P3
PR=CMPLX(COS(TH),SIN(TH))
RZERO=0.9925+0.05142*P1-0.9360*P2+1.0654*P3-0.6833*P4+0.2289*P5-P6
R=RZERO
PR=R*PR
RETURN
END

SUBROUTINE PIPE(WLTH,ALength,AMACH,PI,PR)
COMPLEX E,PI,PR
C       THIS CALCULATES PI AND PR AT ONE OF A PIPE GIVEN PI AND PR AT
C       THE OTHER END. PI AND PR REPLACE ORIGINAL VALUES
AK=6.2832/(WLTH*(1.0+AMACH))
AKA=-6.2832/(WLTH*(1.0-AMACH))
E=CMPLX(COS(AK*ALength),SIN(AK*ALength))
PI=P1*E
E=CMPLX(COS(AKA*ALength),SIN(AKA*ALength))
PR=PR*E
RETURN
END

SUBROUTINE EXTCLET(WLTH,S1,S2,S3,AMACH,ALength,PI,PR)
COMPLEX PI,PR,PP1,PP2,PR,TRAN,REFL,E
C       THIS S/R CALCULATES INCIDENT AND REFLECTED WAVES IN CHAMBER AT
C       PIPE END GIVEN PI AND PR IN PIPE AT CHAMBER END
AAMACH=AMACH*S1/S3
CALL PIPE(WLTH,ALength,AMACH,PI,PR)
AK=6.2832/WLTH
TH=0. 
E=CMPLX(COS(TH-2.0*AK*ALENGTH),SIN(TH-2.0*AK*ALENGTH))
PPI=(1.0-E)/(1.0+E)
TRANS=S2*(1.0+AMACH+PR/PI*(1.0-AMACH))*PPI
1+(S1+S3)*(1.0+AMACH)
1+(S3-S1)*PR/PI*(1.0-AMACH)
TRANS=TRANS/(2.0*S3*(1.0+AMACH))
REFL=(1.0+AMACH+PR/PI*(1.0-AMACH))/TRANS-1.0-AMACH
REFL=REFL/(1.0-AMACH)
PI=PI*TRANS
PK=PI*REFL
RETURN
END

SUBROUTINE EXTNLET(WLTH,S1,S2,S3,M1,AL1,PI,PR)
C THIS SUBROUTINE CALCULATES PI AND PR IN PIPE AT END OF CHAMBER
C GIVEN PI AND PR IN CHAMBER AT PIPE END. PI AND PR REPLACE
C ORIGINAL VALUES , ** , ADIABATIC
COMPLEX A(4,4),E(4,1),DETERM
COMPLEX PI,PR,TRANS,REFL,EE,GG,GH
DIMENSION IPIVC(4),INDEX(4,2)
REAL M1,M3
M3=M1*S1/S3
E=PR/PI
TH=0.
AK=6.2832/WLTH*2.0*AL1
EE=CMPLX(COS(TH+AK),SIN(TH+AK))
N=4
M=1
MAX=4
DO 100 1=1,4
B(I,1)=(0.0,0.0)
DO 100 1=1,4
100 A(I,J)=(0.0,0.0)
A(1,1)=CMPLX(1.0+M3,0.0)
A(1,2)=CMPLX(1.0-M3,0.0)
A(1,4)=(2.5,0.0)
A(2,1)=CMPLX(S3*(1.0+M3),0.0)
A(2,2)=CMPLX(-S3*(1.0-M3),0.0)
A(2,3)=S2*(1.0-EE)
A(2,4)=CMPLX(-S1*M1,0.0)
A(3,1)=(1.0,0.0)
A(3,2)=(1.0,0.0)
A(3,3)=-(1.0+EE)
A(4,1) = CMPLX((S1 + S3 * (2.0 * M3 + M3 * M3)), 0.0)
A(4,2) = CMPLX((S1 + S3 * (M3 * M3 - 2.0 * M3)), 0.0)
A(4,4) = CMPLX(-PI * M1, 0.0)
B(1,1) = 1.0 * M1 + PR / PI * (1.0 - M1)
B(2,1) = S1 * (1.0 * M1) - PR / PI * S1 * (1.0 - M1)
B(4,1) = S1 * (1.0 + M1) * PR / PI * (1.0 - M1) * S1
CALL CAINV(A, N, B, M, DETERM, IPIVOT, INDEX, MAX, ISCALE)
PI = PI * B(2,1)
CALL PIPE(MLTH, AL1, M3, PI, PR)
RETURN
END

SUBROUTINE FINDOUT(E, H, WW, WWN, WWNN)
C THIS S/R CALCULATES THE MODULUS(WWN) AND PHASE(WWNN) OF A COMPLEX
C NUMBER, E, WHOSE REAL AND IMPLIED PARTS ARE H AND WW
C RESPECTIVELY
COMPLEX E
H = REAL(E)
WW = AIMAG(E)
WWN = SQRT(W * H + WWN * WWN)
WWNN = ATAN2(HH, W)
RETURN
END

SUBROUTINE OPTIM
DIMENSION D1(20), D2(20), D3(20), X1(20), X2(20), X3(20), C2(20), C(20)
COMMON/ONE/S, FS, T, JP, TST, STP
COMMON/THU/h, h1, h2, h3
COMMON/THREE/G(20), X(20), H(20), AL(20, 20), D(20, 20), U(20)
COMMON/FOUR/NUMBER, IROOT, TOL
IRA = 0
PK = 0.01
EPS = 0.1E-6
JP = 0
JK = 0
T = 0.0
JS = -1
STP = 0.01
W = 0.1E-9
ITER = 0
W1 = 0.1E-11
w2=0.1E-9
w3=0.1E-8
DO 5 I=1,NS
5 X(I)=ASIN(SQRT(((X(I)-G(I))/(H(I)-G(I)))))
DO 20 I=1,NS
DO 20 J=1,NS
AL(I,J)=0.0
20 D(I)=0.0
DO 25 I=1,NS
AL(I,I)=1.0
25 DO(I,I)=1.0
PRINT 30
30 FORMAT(2d11) OPTIMISATION OF SYSTEM
PRINT 35
35 FORMAT(10H,COST FUNCTION TP ED EC EI 1CP EC EI CP EU EC EI*))
CALL AUX(X)
CALL WRITE(X)
IF(FS .NE. 0.0) GO TO 3d
PRINT 36
DO 18 I=1,NS
18 X(I)=G(I)+H(I)-G(I))*SIN(X(I))*SIN(X(I))
36 FORMAT(54H INITIAL MUFFLER MEETS DESIRED TRANSMISSION LOSS)
RETURN
38 CONTINUE
U1=FS
DO 45 I=1,NS
STC=X(I)
X(I)=X(I)+100.0*W
CALL AUX(X)
U2=FS
U(I)=(U2-U1)/(100.0*W)
45 X(I)=STC
ST=0.0
DO 50 I=1,NS
50 ST=ST+U(I)*U(I)
ST=SQRT(ST)
DO 55 I=1,NS
55 D(I)=-U(I)/ST
DO 60 I=1,NS
60 U(I)=-D(I)
DO 65 I=1,NS
65 X(I)=X(I)+STP*C(I)
   CALL AUX(X)
   CALL WRITE(X)
   CALL UPDATE
   W=W1
70 DO 75 I=1,NS
   D1(I)=D(I)
75 Q(I)=X(I)
80 DO 85 I=1,NS
85 D(I)=0.0
   DO 90 I=1,NS
   DO 90 J=1,NS
90 D(I)=D(I)+AL(I,J)*U(J)
   DO 95 I=1,NS
95 D(I)=D(I)*JS
   S1=0.0
   DO 100 I=1,NS
100 S1=S1+D(I)*U(I)
   IF(S1.LE.0.1E-7) GO TO 110
   DO 105 I=1,NS
   DO 105 J=1,NS
   AL(I,J)=0.0
   AL(I,I)=1.0
105 D6(I,I)= 1.0
   GO TO 80
110 CONTINUE
   S2=0.0
   DO 120 I=1,NS
120 S2=S2+U(I)*D(I)
   S2=SQR(S2)
   IF(IROT.GT.0) GO TO 145
   IF(S2*GT.0.01*EPS) GO TO 130
   PRINT 125
125 FORMAT(2TH SMALL DIRECTION VECTOR)
   IRA=IRA+1
   IF(IRA.NE.3) GO TO 130
   DO 17 I=1,NS
17 X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
   RETURN
130 CONTINUE
   I2=0
   DO 135 I=1,NS
135 IF(TST*ABS(U(I)).LT.EPS)*I2=I2+1
   IF(I2.NE.NS) GO TO 145
   PRINT 140
   CONTINUE

140 FORMAT(19H SMALL GRADIENT)
IRA=IRA+1
IF(IRA.NE.3) GO TO 145
DO 8 I=1,NS
8 X(I)=G(I)*(H(I)-G(I))*SIN(X(I))*SIN(X(I))
RETURN
145 CONTINUE
STP=STP
U1=FS
TS1=0.0
TS2=0.0
DO 150 I=1,NS
150 TS1=TS1+U1(I)*C1(I)
TS1=SQR(TS1)
TS2=S2
YS=0.6667*STP*TS1/TS2
ICT=0
ICOUNT=0
DO 160 I=1,NS
160 X1(I)=X1(I)
160 X2(I)=X1(I)+YS*C(I)
CALL AUX(X2)
CALL WRITE(X2)
U2=FS
IF(U2.LT.U1) GO TO 235
DO 170 I=1,NS
170 X3(I)=X2(I)
170 X2(I)=0.6667*X1(I)+0.3333*X3(I)
U3=U2
ICOUNT=ICOUNT+1
IF(ICOUNT.EQ.8) 230,175,175
175 ICT=ICT+1
ICOUNT=0
IF((ICT.EQ.2).AND.(NS.EQ.1)) GO TO 210
GO TO (220,176,220,210),ICT
176 DO 180 I=1,NS
180 D2(I)=-D(I)
180 C2(I)=0.0
IF(ABS(X1(I)).GT.0.1E-8) GO TO 185
C2(I)=X1(I)+0.100
GO TO 190
185 C2(I)=X1(I)
190 DO 195 I=1,NS
195 D3(I)=C2(I)-D2(I)*C2(I)*C2(I)
   TP=0.0
   DO 200 I=1,NS
200   TP=TP+D3(I)*D3(I)
   TP=SQRT(TP)
   DO 205 I=1,NS
205   D(I)=D3(I)*S2/TP
   GO TO 155
210 PRINT 215
215 FORMAT(43H       EXCESSIVE NUMERICAL ERRORS IN GRADIENT)  
      FS=U1
      DO 16 I=1,NS
16   X(I)=G(I)+(H(I)-G(I))*/SIN(X(I))*/SIN(X(I))
      RETURN
220 DO 225 I=1,NS
225   D(I)=-D(I)
   GU TO 155
230 CALL AUX(X2)
   CALL WRITE(X2)
   U2=FS
      IF(U2-U1)260,260,165
235 DO 240 I=1,NS
240   X3(I)=3.0*X2(I)-2.0*X1(I)
   CALL CHECK(X3,C,JUMP)
      IF(JUMP.EQ.0) GO TO 250
245 DO 245 I=1,NS
245   X(I)=X2(I)
   FS=U2
   GU TO 280
250 CALL ALX(X3)
   U3=FS
   CALL WRITE(X3)
      IF(U3.GE.U2) GO TO 260
255 DO 255 I=1,NS
255   X1(I)=X2(I)
   U1=U2
   U2=U3
   GU TO 235
260 CONTINUE
   S1=0.0
   S2=0.0
   DO 265 I=1,NS
   FRED=X2(I)-X1(I)
   SAM=X3(I)-X1(I)
   S1=S1+FRED*FRED
   265 S2=S2+SAM*SAM
   A=SQRT(S1)
   B=SQRT(S2)
   ZNUM=B*B*U2-A*A*U3-(B*B-A*A)*U1
   DENOM=B*U2-A*U3-(B-A)*U1
   STP=0.5*ZNUM/DENOM
   DO 270 I=1,NS
270 X(I)=X1(I)+(STP/TS2)*D(I)
   CALL AUX(X)
   IF(FS.LT.U2) GO TO 280
   DO 275 I=1,NS
275 X(I)=X2(I)
   FS=U2
280 CONTINUE
   PRINT 285
285 FORMAT(13H NEW POINT/)
   IF(FS.NE.0.0) GO TO 286
   DO 287 I=1,NS
287 X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
   RETURN
286 CALL WRITE(I)
   ITER=ITER+1
   IF(ITER.NE.NUMBER) GO TO 295
   PRINT 290
290 FORMAT(36H SEARCH COMPLETED, NORMAL RETURN)
   DO 14 I=1,NS
14 X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
   RETURN
295 IF(INST.LT.0) GO TO 305
   IF(FS.GT.TOL) GO TO 305
   PRINT 300
300 FORMAT(35H FUNCTION SMALLER THAN TOLERANCE)
   DO 12 I=1,NS
12 X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
   RETURN
305 CONTINUE
I1=0
DO 310 I=1,NS
PT=ABS(SIN(Q(I)))*SIN(Q(I))-SIN(X(I))*SIN(X(I))
1+PT.LT.0.000025) I1=11+1
310 CONTINUE
IF(I1.NE.NS) GO TO 320
PRINT 315
315 FORMAT(5+H, STEP SIZE LESS THAN 0.01 PERCENT OF VARIABLE RANGE)
IRA=IRA+I
IF(IRA.LT.30) GO TO 320
DO 10 I=1,NS
10 X(I)=G(I)*(H(I)-G(I))*SIN(X(I))*SIN(X(I))
RETURN
320 CONTINUE
STP=0.0
DO 325 I=1,NS
FRED=X(I)-Q(I)
325 STP=STP+FRED*FRED
STP=SQRST(STP)/TS2
JK=JK+1
IF(JK.LT.3) GO TO 330
JP=JP+1
JK=0
CALL ALTER4
330 T=T+STP*TS2
CALL UPDATE
GO TO 70
END

SUBROUTINE GRAD
DIMENSION DEL(20)
COMMON/ONE/NS,FS,T,JP,TST,STP
COMMON/TW2/W1,W1,W2,W3
COMMON/THRE/G(20),X(20),H(20),AL(20,20),UG(20,20),U(20),UL(20)
W=ABS(W)
TK=0.01
IF(ABS(FS).GT.0.1E-7) GO TO 2
DO 1 I=1,NS
1 DEL(I)=0.000001
GO TO 4
2 DO 3 I=1,NS
DEL(I)=2.0*SQRST(W*ABS(FS)/UG(I,I))
IF(DEL(I).GT.0.000001) DEL(I)=0.000001
39
3 CONTINUE 
   U4=FS
4 U1=FS
   DO 7 I=1,NS
   STO=X(I)
   X(I)=X(I)+DEL(I)
   CALL AUX(X)
   U2=FS
   U(I)=(U2-U1)/DEL(I)
   IF(ABS(U(I)).LT.0.1E-10) GO TO 5
   FRED=2.0*DEL(I)*DG(I,I)/ABS(U(I))
   IF(FRED.LT.TK) GO TO 6
5 X(I)=STO-DEL(I)
   CALL AUX(X)
   U3=FS
   U(I)=(U2-U3)/(2.0*DEL(I))
6 X(I)=STO
7 CONTINUE
   FS=U1
   TST=0.0
   DO 8 I=1,NS
8 TST=TST+U(I)*U(I)
   TST=SQRT(TST)
   DO 9 I=1,NS
9 U(I)=U(I)/TST
   RETURN
END

SUBROUTINE UPDATE
DIMENSION P(20),Y(20),T(20,20),G(20,20),S(20,20),F(20)
COMMON/ONE/NS,FS,Q,JP,TST,STP
COMMON/THREE/G(20),X(20),H(20),AL(20,20),DG(20,20),D(20),U(20)
DO 1 I=1,NS
1 P(I)=U(I)
   CALL GRAD
   IF(NS.EQ.1) GC TO 14
   DO 2 I=1,NS
2 Y(I)=U(I)-P(I)
   R=0.0
   B=0.0
   DO 3 I=1,NS
   B=B+Y(I)*U(I)
3 CONTINUE
\[ R = R + P(I) \cdot D(I) \]
\[ S_1 = 1.0 / (B \ast STP) \]
\[ S_2 = R / (B \ast B) \]
\[ S_3 = STP / B \]
\[ DO \ 5 \ I = 1, NS \]
\[ DO \ 5 \ J = 1, NS \]
\[ T(I, J) = S_3 \ast D(I) \ast D(J) \]
\[ DO \ 6 \ I = 1, NS \]
\[ DO \ 6 \ J = 1, NS \]
\[ C(I, J) = 0.0 \]
\[ S(I, J) = 0.0 \]
\[ F(I) = 0.0 \]
\[ DO \ 7 \ I = 1, NS \]
\[ DO \ 7 \ J = 1, NS \]
\[ F(I) = F(I) + A(I, J) \ast Y(J) \]
\[ S_4 = 0.0 \]
\[ DO \ 8 \ I = 1, NS \]
\[ S_4 = S_4 + Y(I) \ast F(I) \]
\[ DO \ 9 \ I = 1, NS \]
\[ DO \ 9 \ J = 1, NS \]
\[ S(I, J) = F(I) \ast Y(J) \]
\[ DO \ 10 \ I = 1, NS \]
\[ DO \ 10 \ K = 1, NS \]
\[ DO \ 10 \ J = 1, NS \]
\[ C(I, K) = C(I, K) + S(I, J) \ast A(K, J) \]
\[ DO \ 11 \ I = 1, NS \]
\[ DO \ 11 \ J = 1, NS \]
\[ A(I, J) = A(I, J) + T(I, J) \ast S(I, J) / S_4 \]
\[ DO \ 12 \ I = 1, NS \]
\[ O(I, I) = O(I, I) \ast Y(I) \ast Y(I) \ast (S_1 - S_2) + 2.0 \ast P(I) \ast Y(I) / B \]
\[ DO \ 13 \ I = 1, NS \]
\[ O(I, I) = \text{Abs}(O(I, I)) \]
\[ \text{RETURN} \]
\[ A(I, I) = 1.0 \]
\[ O(I, I) = 1.0 \]
\[ \text{RETURN} \]
\[ \text{END} \]

SUBROUTINE ALTER
COMMON/UNE/NS,F,S,T,JP,TST,STP
COMMON/THO/TH,W1,W2,W3
GO TO (L,2,3),JP
1 T1=T
   W=W2
RETURN
2. T2=T
w=w3
RETURN
3. T3=T
IF(T1.LT.T2) GO TO 4
T2=T1
w=w1
4. w=w2
5. IF(T2.LT.T3) GC TO 6
w2=w
w1=0.01*w2
w3=100.0*w2
w=w1
JP=0
T=0.0
RETURN
6. w2=w3
w1=0.01*w2
w3=100.0*w2
w=w1
JP=0
T=0.0
RETURN
END

SUBROUTINE CHECK(X3,Q,JUMP)
DIMENSION X3(20),Q(20),V(20),W(20),Z(20)
COMMON/ONE/NS,FS,T,JP,TST,STP
COMMON/THREE/G(20),X(20),H(20),AL(20),DG(20),D(20),U(20)
DO 1 I=1,NS
V(I)=G(I)+H(I)-G(I))*SIN(Q(I))*SIN(Q(I))
1 W(I)=G(I)+H(I)-G(I))*SIN(X3(I))*SIN(X3(I))
JUMP=0
DO 2 I=1,NS
Z(I)=0.20*(H(I)-G(I))
2 IF(ABS(W(I)-V(I)).GT.Z(I)) JUMP=JUMP+1
RETURN
END
SUBROUTINE write(V)
DIMENSION V(20),I(20)
COMMON/UNE/NS,FS,T,JP,TST,STP
COMMON/THRE/G(20),X(20),H(20),AL(20,20),DG(20,20),U(20),U(20)
DO 1 I=1,NS
1 W(I)=G(I)*(H(I)-G(I))*SIN(V(I))*SIN(V(I))
PRINT 2,FS,(W(I),I=1,NS)
2 FORMAT(20,4,2OF9.4)
RETURN
END

RANGES AND INITIAL LENGTHS OF SILENCER COMPONENTS

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<th>TAILPIPE</th>
<th>EXTENDED OUTLET</th>
<th>EXPANSION CHAMBER</th>
<th>EXTENDED INLET</th>
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TAILPIPE AREA= .02190 ANNUAL AREA= .10410 EXPANSION CHAMBER AREA= .12190
TAILPIPE MACH NUMBER= .10000

DESIRABLE MINIMUM TRANSMISSION LOSS

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**Search Completed; Normal Return**

**Optimum Lengths of Silencer Components in Feet**

- Tailpipe: 2.0533
- Extended Outlet(1): .2120
- Expansion Chamber(1): .7963
- Extended Inlet(1): .1827
- Con Pipe: .0829
- Extended Outlet(2): .2865
- Expansion Chamber(2): 1.6630
- Extended Inlet(2): .5602
- Con Pipe: 1.9811
- Extended Outlet(3): .3779
- Expansion Chamber(3): 2.5054
- Extended Inlet(3): 1.5696

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<td>--------</td>
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</tr>
<tr>
<td>91.000</td>
<td>21.000</td>
<td>22.000</td>
</tr>
</tbody>
</table>

APPENDIX B - Concluded
REFERENCES


<table>
<thead>
<tr>
<th>Main rotor:</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Diameter, m (ft)</td>
<td>11.3</td>
</tr>
<tr>
<td>Number of blades</td>
<td>2</td>
</tr>
<tr>
<td>Blade chord, m (in.)</td>
<td>0.28</td>
</tr>
<tr>
<td>Airfoil</td>
<td>NACA 0015</td>
</tr>
<tr>
<td>Blade area, m² (ft²)</td>
<td>3.18</td>
</tr>
<tr>
<td>Disk area, m² (ft²)</td>
<td>100.8</td>
</tr>
<tr>
<td>Solidity</td>
<td>0.0314</td>
</tr>
<tr>
<td>Tip speed, m/sec (ft/sec)</td>
<td>210.3</td>
</tr>
<tr>
<td>Design operating speed, rpm</td>
<td>356</td>
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</table>

<table>
<thead>
<tr>
<th>Tail rotor:</th>
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</tr>
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<tbody>
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<td>Diameter, m (ft)</td>
<td>1.7</td>
</tr>
<tr>
<td>Number of blades</td>
<td>2</td>
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<tr>
<td>Blade chord, m (in.)</td>
<td>0.10</td>
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<tr>
<td>Blade area, m² (ft²)</td>
<td>0.209</td>
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<td>Disk area, m² (ft²)</td>
<td>2.35</td>
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<td>Design operating speed, rpm</td>
<td>1920</td>
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<th>General:</th>
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<tr>
<td>Normal gross weight, kg (lb)</td>
<td>1111</td>
</tr>
<tr>
<td>Empty weight, kg (lb)</td>
<td>716.6</td>
</tr>
<tr>
<td>Overall length, m (ft)</td>
<td>13.29</td>
</tr>
<tr>
<td>Power, kW (hp)</td>
<td>194</td>
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<tr>
<td>Maximum level airspeed, knots</td>
<td>85</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Gear ratios:</th>
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<tbody>
<tr>
<td>Main rotor mast</td>
<td>0.111</td>
</tr>
<tr>
<td>Tail rotor</td>
<td>0.6</td>
</tr>
<tr>
<td>Cooling fan</td>
<td>1.5</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Engine characteristics:</th>
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<tbody>
<tr>
<td>Number of cylinders</td>
<td>6</td>
</tr>
<tr>
<td>Operating cycle</td>
<td>4</td>
</tr>
<tr>
<td>Exhaust-gas temperature at MAP of 48 cm Hg, K (°F)</td>
<td>1090</td>
</tr>
<tr>
<td>Gas-flow Mach number at MAP of 48 cm Hg and area of 0.0045 m²</td>
<td>0.1</td>
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</table>
TABLE II.- ESTIMATED INSERTION LOSS OF MUFFLER OBTAINED FROM MEASURED SOUND PRESSURE LEVELS OF STANDARD AND MODIFIED EXHAUST SYSTEM

<table>
<thead>
<tr>
<th>Engine-bank firing-frequency harmonic</th>
<th>Sound pressure level, dB, for -</th>
<th>Estimated insertion loss of muffler, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard exhaust system</td>
<td>Y-connector only</td>
</tr>
<tr>
<td>1</td>
<td>94.5</td>
<td>81</td>
</tr>
<tr>
<td>2</td>
<td>83.0</td>
<td>94</td>
</tr>
<tr>
<td>3</td>
<td>88.5</td>
<td>65</td>
</tr>
<tr>
<td>4</td>
<td>77.0</td>
<td>74</td>
</tr>
<tr>
<td>5</td>
<td>66.6</td>
<td>54</td>
</tr>
<tr>
<td>6</td>
<td>72.5</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>76.2</td>
<td>---</td>
</tr>
<tr>
<td>8</td>
<td>70.0</td>
<td>---</td>
</tr>
</tbody>
</table>
Area expansion ratio $\frac{S_2}{S_1}$
Forward-going waves $p^{(+)}$
Backward-going waves $p^{(-)}$
Total fluctuating pressure $p = p^{(+)} + p^{(-)}$

Figure 1.- Simple expansion-chamber muffler with descriptive notation.

(a) Area expansion or contraction discontinuity.
(b) Side branch discontinuity.

Figure 2.- Simple duct discontinuities.
Figure 3.—Increase in radiated power level from tailpipe due to transport effect of mean gas flow, with Mach number as a parameter.
(a) Flow area contraction.

(b) Flow area expansion.

(c) Flow area contraction with branch impedance.

(d) Flow area expansion with branch impedance.

Figure 4.- Schematic diagrams of area discontinuities.
Figure 5.- Expansion-chamber muffler with extended inlet and outlet pipes.
Figure 6.- Comparison of transmission loss with and without flow.

c = 610 m/sec (2000 ft/sec)
S₂/S₁ = 4
L = 0.61 m (2 ft)
Figure 7.- Narrow-band (1 Hz) spectrum of helicopter noise emission at 30.5 m (100 ft), an angle of 90° to nose of helicopter, and a heaver height of 1.83 m (6 ft).
Figure 8. – Bar graph of significant exhaust noise components.
Figure 10. - Schematic of three-stage muffler with bends to conserve space.
Figure 11 - Test helicopter for which muffler system was designed.
<table>
<thead>
<tr>
<th>Component</th>
<th>Length, m</th>
<th>Area, m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Tailpipe</td>
<td>0.60</td>
<td>0.002</td>
</tr>
<tr>
<td>2. Extended outlet</td>
<td>0.06</td>
<td>0.002</td>
</tr>
<tr>
<td>3. First chamber</td>
<td>0.25</td>
<td>0.019</td>
</tr>
<tr>
<td>4. Extended inlet</td>
<td>0.04</td>
<td>0.002</td>
</tr>
<tr>
<td>5. Connector</td>
<td>0.01</td>
<td>0.002</td>
</tr>
<tr>
<td>6. Extended outlet</td>
<td>0.08</td>
<td>0.002</td>
</tr>
<tr>
<td>7. Second chamber</td>
<td>0.51</td>
<td>0.019</td>
</tr>
<tr>
<td>8. Extended inlet</td>
<td>0.30</td>
<td>0.002</td>
</tr>
<tr>
<td>9. Connector</td>
<td>0.61</td>
<td>0.002</td>
</tr>
<tr>
<td>10. Extended outlet</td>
<td>0.13</td>
<td>0.002</td>
</tr>
<tr>
<td>11. Third chamber</td>
<td>0.76</td>
<td>0.019</td>
</tr>
<tr>
<td>12. Extended inlet</td>
<td>0.44</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Figure 13.- Schematic of muffler configuration.
Figure 14.- Y-connector installation.
Figure 16. - Narrow-band (4 Hz) spectra of noise emitted by standard and modified helicopter.
Figure 17. - Comparison of calculated transmission losses with estimated transmission losses.
(Arrows denote firing-frequency harmonic.)
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