PROBLEMS OF EXCESS CAPACITY

by George Douglas
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Abstract

This lecture discusses the problems of excess capacity in the airline industry and focuses on the following topics: load factors; "fair" rate of return on investment; service-quality rivalry among airlines; pricing (fare) policies; aircraft production; and the impacts of excess capacity on operating costs. The lecture also will include a discussion of the interrelationships among these topics.

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Excess Capacity, Service Quality and the Structure of Airline Fares

by George W. Douglas*

I. INTRODUCTION

A characteristic common to most scheduled transportation systems, is that "demand" only rarely equals "supply." Because of the discrete nature of the "supply," or capacity offered, and the stochastic nature of demand, the equilibrium of any scheduled transportation system is characterized over time by "excess" capacity. A measure commonly used to denote this excess capacity in the airline industry is the average load factor, the ratio of the number of passengers carried to the number of seats available. Moreover, since the costs of a scheduled transportation system are largely determined by the capacity offered, the cost per passenger is quite sensitive to the average load factor.

The average load factor in the scheduled airline industry has, in the past, been implicitly regarded as an exogenous parameter, characteristic of the nature of the industry and not subject to control by the airlines or the regulators. Following that assumption, average and long run marginal costs per passenger can be defined, with respect to the costs of capacity and the given average load factor. One might describe in this manner the costs and fare determination procedure as followed by the C.A.B. in the past.

It can be shown, however, that the system's equilibrium average load factor, rather than being exogenous, is determined endogenously by the market, given the costs and fares facing the carriers. In competitive markets, the existence of scheduling competition tends to bring about an equilibrium ALF at or near the "break-even" ALF defined by the costs of production and the fare level chosen. Similarly, the average load factors in non-competitive markets are higher, ceteris paribus, but their level is also related to the costs and the fare level chosen by the regulators. Most airline markets, moreover, can operate over a significant range of prices, or fare, each price level defining, in equilibrium, the average load factor of the system. Only recently has the C.A.B. recognized that by setting fares it implicitly determines the average load factor of the system, and that the setting of explicit load factor standards for use in computing fares is desirable and proper.

We will seek to describe in this paper the issues relevant to the selection

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*Assistant Professor of Economics, University of North Carolina at Chapel Hill. The author wishes to thank James C. Miller III, of the U.S. Department of Transportation, with whom many of these concepts are shared, and which were in part developed jointly. The author bears sole responsibility, however, for the views expressed here.

See C.A.B. Order 71-4-54, April 12, 1971. In this decision on the "Load Factor Phase" of the General Fares Investigation, the Board's decision reversed the Examiner's opinion and established for the first time desirable load factor standards for ratemaking purposes of 56% for Trunks, and 44.4% for the Local Service Carriers.
of load factor standards, and by analyzing the implications of the ALF for the system's level of service quality, suggest various characteristics of an efficient price structure.

II. SERVICE QUALITY AND THE AVERAGE LOAD FACTOR

Although a scheduled transportation system can feasibly operate over a wide range of average utilization, we should expect that the quality of service provided to be closely related to the excess capacity offered. The aspect of quality of crucial importance for us in this regard, relates to levels of delays incurred by passengers using the system. These delays arise from two sources: (1) that a departure is not scheduled at the time a passenger desires to depart, and (2) that the preferred flight might be filled, causing the traveler to take another, less desirable flight. From the first source, we might compare the scheduled departure times with the daily profile of desired departure times, and compute the absolute values of the time differentials. The mean absolute difference between the travelers' desired departure times and the scheduled departure time we denote as "frequency delay." The expected frequency delay should be a function then of the pattern of desired departure times for the route, and the number of flights scheduled. As the daily frequency of flights increases, we would expect frequency delay to be decreased.

The second source of delay encountered is a queuing phenomenon, generated by the fixed scheduled capacity faced by the stochastic demand. We would expect that as additional flights (capacity) are offered, the probability of being delayed and the expected time of the delay would be decreased.

The sum of these two kinds of delay we denote as expected "schedule delay," measuring the expected absolute difference between a traveler's desired departure time and the actual departure. The level of expected schedule delay can be considered a characteristic of service quality, and is a significant determinant of air travel demand, particularly in short to intermediate distance markets, where substitution among modes is feasible. As the capacity is increased by increasing the flight frequency (of a given aircraft type), we would expect the stochastic delay and frequency delay to both decrease, thereby decreasing schedule delay. However, as frequencies are increased, the average load factor would decline (in spite of the additional travel induced by the better service), thereby increasing the average cost per passenger.

We have simulated these delay processes (described in the appendix) and can approximate the level of frequency delay by:

(1) \[ T_f = 92F - 466 \]

The stochastic delay is approximated by:

(2) \[ T_s = 0.445 \left( \frac{N}{\sigma} \right) - 665 \left( \frac{S-N}{\sigma} \right) - 1.799 \]

Ideally, we might expect that the flights would be scheduled so as to minimize \( T_f \) for any given number of flights. In practice, constraints on scheduling flights over a route, and potential "clustering" effects of competition may prevent the actual scheduling pattern from being locally efficient.
THE STRUCTURE OF AIRLINE FARES

where $S = \text{capacity (seats) per aircraft}$,
$N = \text{mean flight demand}$,
$\sigma = \text{standard deviation of flight's demand}$

Expected schedule delay, $T$, is the sum of expected frequency delay and expected stochastic delay

(3) $T = T_f + T_s$

For a route with the distance and the aircraft type specified, we may compute the relationship between the cost per passenger, and the average load factor, as described in figure 1. The operating costs were estimated using a model developed by the C.A.B., which relates the cost per passenger to the ALF, and the performance and factor price parameters of the various aircraft types. For a specific level of mean daily demand (and its variance), we can then compute the expected schedule delay for any assumed level of capacity (or the ALF). On table 1 we indicate the levels of these delays that might be expected for a hypothetical route. As might be expected, as excess

Average Cost as Related to Average Load Factor

![Graph showing the relationship between cost per passenger and average load factor.](image)

**FIGURE 1**

Hypothetical Trip:
Distance = 600 miles
Aircraft = Three engine Turbo-Fan
EXPECTED DELAYS PER PASSENGER:

Hypothetical Route with
Distance = 600 miles
Avg. Passengers/Day = 800
Aircraft = Three Engine Turbo-Fan

\[
\begin{array}{cccc}
\text{ALF} & \text{Stochastic} & \text{Frequency} & \text{Schedule} & \text{Cost/Pax} \\
& \text{Delay} & \text{Delay} & \text{Delay} & \\
.40 & 6.90 & 23.86 & 30.76 & 43.84 \\
.44 & 9.07 & 24.92 & 33.99 & 40.99 \\
.48 & 11.87 & 25.93 & 37.80 & 38.61 \\
.52 & 15.54 & 26.90 & 42.44 & 36.59 \\
.56 & 20.40 & 27.82 & 48.22 & 34.85 \\
.60 & 26.97 & 28.71 & 55.68 & 33.34 \\
.64 & 36.05 & 29.57 & 65.62 & 32.01 \\
.68 & 48.96 & 30.40 & 79.36 & 30.84 \\
.72 & 68.03 & 31.21 & 99.24 & 29.79 \\
.76 & 97.60 & 31.99 & 129.59 & 28.85 \\
.80 & 146.63 & 32.74 & 179.37 & 28.00 \\
\end{array}
\]

Delays measured in minutes per passenger.
Cost is weighted average of coach and first class costs, inclusive of "fair" rate of return on capital.

TABLE 1

capacity is reduced, and approaches the mean demand (i.e., the ALF increases) the stochastic delay increases exponentially. On figure 2, we graph the relationship (in this market) between the average load factor and the expected level of schedule delay.

With the information contained in figures 1 and 2, we are now prepared to relate the costs per passenger with the level of expected schedule delay, or service quality. This "tradeoff" relationship is depicted in figure 3. This might be interpreted as the opportunity locus facing the regulators; if a high fare is chosen, the market equilibrium will generate a low ALF, and a high level of service quality; reduction of the fare implies an equilibrium with a higher ALF and a greater delay (or a lower level of service quality).\(^4\)

III. THE OPTIMAL REGULATED PRICE STRUCTURE

Having the information necessary to describe the technical tradeoff between price (cost) and service quality, the selection of an "optimal" price

\[^4\text{The tradeoff curve is drawn over a broad range, and without regard to demand elasticities. Since we assume that total revenues equal total costs, the range of feasible points of equilibrium would be constrained to be between some critical boundary prices. The feasible range, however, is rather wide in most markets.}\]
and implicit quality level may be investigated. It appears on first glance to be a straightforward maximization problem, in which one should choose that point where the technical tradeoff is consistent with that of the customers' preferences. This is a particularly difficult problem, however, if, as in this case, quality differentiation is constrained. The regulators must select a quality level for a population of customers whose preferences for quality may be diverse. The level chosen then, must compromise those aspects of service quality that are not separable among these customers.

The simplest approach to this problem is to attempt to discover the tradeoff preferred by the typical traveler, or the implicit value the traveler places on time he is delayed. By assigning such a price, we can determine an "optimal" level of price and quality, which minimizes total trip cost for

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6 This approach, while used persuasively in valuing some delays in transportation, such as congestion delays, should be approached cautiously here. The time lost through congestion is irretrievably lost, whereas schedule delays may have alternative uses. Ideally, we would like to discover the tradeoff of demand

\[ \frac{\partial T}{\partial P} \]

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that "typical" traveler, inclusive of the value of delay times. In figure 3 we indicate that optimal level where the slope of the technical tradeoff between cost and delays equals the assumed value of time. Alternatively, we may represent the minimization problem with a marginal analysis, such as contained in figure 4. Here we indicate with the curve labeled "C6," the reduction in cost per passenger (fare) of a 2% increase in the average load factor, as a function of the load factor. We also indicate with the curves labeled MDC, the implicit value of the additional delay caused by a 2% increase in the average load factor, with time valued at $5.00 and $10.00 per hour. Cost minimization occurs at that ALF where the fare reduction caused by the increase of the ALF by 2% just equals the marginal delay cost (MDC); in this market between .59 and .66.

As pointed out above, the technical tradeoff between price and service quality varies with changes in the distance, size and dispersion of demand. This has the effect, then, of changing the optimal ALF chosen for markets with different characteristics. We should expect, for example, that the optimal load factor should be greater, ceteris paribus, for a long flight than a short one. The delay for either route is related to the average load factor of the system, or the relative number of empty seats flown, on the average. Thus,
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Least Trip Cost Analysis

![Graph with curves C6, MDC, MDC* and labels](image)

- **Curve C6** = Cost reduction of 2% increase in average load factor
- **Curve MDC** = Marginal delay cost from 2% increase in average load factor; time valued at $10.00/hr.
- **Curve MDC*** = Marginal delay cost; time valued at $5.00/hr.

**FIGURE 4**

While the delay associated with any given load factor is equal for both routes, *ceteris paribus*, the cost reduction (in dollars) per passenger, of a slight increase in the average load factor is much greater for the long route than the short one. In figure 5 we demonstrate this effect graphically. The curve C22 represents the cost reductions for a trip of 2200 miles, from an increase in the ALF of 2%. As can be seen, the least trip cost occurs at an ALF of .59 for the 600 mile trip, and at approximately .68 for the 2200 mile trip. On figure 6, we portray the range of "optimal" ALF's for a market of a given size, as the distance is increased.

We should also expect that the market size should affect the optimal average load factor. The stochastic delays are derived by first computing the probabilities of being delayed one, two, three or more flight intervals; and then multiplying each by the average interval between flights. In comparing a large and small market, with all other characteristics being identical, we find that the probabilities of being delayed are similar for operations at a given average load factor in either market. However, the expected delays are less in the larger market, as the flight frequencies would be greater, and the average interval between flights would be shorter, for any given ALF. Hence, we would expect that the optimal average load factor in the larger market would be greater than that in the smaller market. On figure 7, we describe the analysis graphically. In this case, the marginal cost reduction...
least Cost Average Load Factor Analysis as Distance is Varied

Curve C6 = Cost reduction of 2% increase in ALF for trip length of 600 miles
C22 = Cost reduction of 2% increase in ALF for trip length of 2200 miles
MDC = Marginal delay cost

FIGURE 5

curve, C6, is identical for both markets. The marginal delay costs associated with a market of mean demand of 3200 (labelled MDC32) lie below those associated with a mean demand of 800 (labelled MDC8). Hence, we find that the optimal ALF for the smaller market is approximately .60, while that of the larger market is approximately .64. Figure 8 describes the optimal average load factors continuously against market size, as measured by mean daily demand.

The delay model by which the relationship between the cost and the level of service delays were estimated contains a number of assumptions and approximations from limited data of the characteristics of the stochastic demand distributions. Hence, the relationship should be considered tentative in the quantitative sense. However, the model, when tested indirectly by comparing the forecast distributions of average load factors in specific markets with those observed, was found to be reasonably accurate. In any case the qualitative assumptions of the model (i.e., the signs of the partial derivatives) are reliable, and we are thus prepared to defend the qualitative conclusions; i.e., that load factors on long hauls should be higher than on short hauls, ceteris paribus, and higher in dense markets than in thin markets. The measure of the delay, re-
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Range of Optimal Average Load Factors as Related to Distance; mean daily demand = 800.

Curve H represents optimal load factors consistent with time valued at $5.00/hr. Curve L represents optimal load factors with time valued at $10.00/hr.

FIGURE 6

relationship could be refined with more extensive data on the demands for individual flights over a wide variety of city pairs.

IV. CHARACTERISTICS OF THE EXISTING STRUCTURE OF AVERAGE LOAD FACTORS

It is interesting to compare the pattern of average load factors that has developed in the industry, with the pattern we have suggested. In one instance, the relationship of fares and the average load factor to length of haul (distance), the industry's pattern has been mildly perverse.

One well known characteristic of airline costs is that the average cost of capacity per mile declines significantly with increases in distance. On figure 9 we describe the average cost per passenger mile at various distances, assuming that load factors are held constant. The source of this nonlinearity is the rather substantial fixed or "terminal" cost per flight, which does not vary with distance. The C.A.B. has, from time to time, investigated the cost and fare "taper," to see if they were in close correspondence. The Domestic Air Fare Study of 1967, confused the issue, however, by principally computing the cost "taper" with load factors that varied with distance. Although actual load factor relationships with distance were not exhibited in this study,

7 The principal analyses and discussions centered on a cost taper derived with load factors varying from .58 at 250 miles to .48 at 2,500 miles. See Domestic Air Fares: A Study, Civil Aeronautics Board, Jan. 1968.
Least Cost Average Load Factor Analysis as Market Size is Varied

one can only assume that the varying load factors chosen were typical of the existing pattern. The determination of the study was that the fare (actually the weighted "yield") taper was not as steep as the cost taper; if this were so it would explain why the load factors were lower for long hauls. Following that study, a number of fare adjustments have been made to increase the fare taper, presumably to be consistent with a cost taper with constant load factors.

The only data currently available to the public concerning the ALF's in the various markets, is that generated by the current General Fare Investigation. From this, we have data on capacity and traffic on each of 353 non-stop routes, by all certificated carriers during selected months of 1969. We are thus able to analyze the relationship of average load factors to the market's characteristics with cross section regression analysis. This analysis indicates that the average load factor is most strongly influenced by the level of competition, e.g., the number of carriers serving the market. The load factors tend to be higher in large markets than in small markets, but even after adjusting for those effects, there yet remained (in 1969) an inverse relation between the average load factor and distance. The results of these regressions are summarized in table 2.
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Range of Optimal Average Load Factors as Market Size is Varied

Distance = 600 miles
Curve labelled H represents optimal ALF's with time valued at $5.00/hr;
Curve labelled L represents optimal ALF's with time valued at $10.00/hr.

FIGURE 8

V. CONCLUSIONS

We have demonstrated that the price level and structures set by the C.A.B. tends to determine the average load factor of the air transport system. Moreover, the level of service quality and the average costs of the system are closely related to the average load factor. By qualitative analysis with simple assumptions concerning the relationship, one can conclude that average load factors should be higher in long haul markets than in short haul markets, and higher in dense markets than in thin markets. The actual specification of desirable load factor standards depends on the quantitative description of the technical tradeoff between price (cost) and service quality, and a measure of the traveler's preference (tradeoff) between price and service quality. With the limited data currently available, delay models were constructed to approximate these tradeoffs, and from these a range of "optimal" average load factors were computed.

APPENDIX

THE ESTIMATION OF SCHEDULE DELAYS

Schedule delay arises from two sources:
(a) That a traveler's desired departure time does not coincide with a scheduled flight ("frequency delay"), and
(b) That the desired flight is filled, and the traveler must take another flight (stochastic delay).

Frequency delay (type "(a)") was estimated by simulation. The daily pattern of demand (Figure 2) of a typical route was transformed into a discrete frequency distribution. A procedure was used to schedule "F" flights during the day, such that each flight faced demand of equal size. The difference between each traveler's desired departure time and the nearest scheduled flight was computed, and their absolute values summed for all travelers. The mean, or average delay for each traveler was computed. The procedure was repeated for F+ 1, F+ 2, etc., thus generating the average or "expected" value of frequency delays as a function of the daily flight frequency. These observations were fitted to the function

\[ T_f = 92F - 456 \]

where \( T_f \) is the expected frequency delay, per passenger (measured in minutes) and \( F \) is the daily flight frequency.

To estimate stochastic delay, we characterized the problem as a queuing phenomenon, and described it as a Markov process. To do this, we assumed that each flight faces a random demand with mean \( N_f \) and standard deviation \( \sigma_f \). We describe the state of the system by a variable \( Q \), defined as the number of passengers desiring space on a given flight. Assuming that the distribution of demand is normal, we can then assign probabilities to a one step transition matrix. An example of such a one step transition matrix is
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CROSS-SECTION ANALYSIS BY MARKET OF AVERAGE LOAD FACTORS

"t"-Statistics in Parentheses

All Markets:
1. ALF = .588 - 2 × 10^-4 × DISTANCE + .8 × 10^-6 × PAX - .07 NO CARRIERS
   (1.4) (6.5) R^2 = .213
2. ALF = .244 - .018 LOG DIST + .073 LOG PAX + .146 LOG C
   (1.8) (1.1) (5.5) R^2 = .144

One Carrier Markets:
3. ALF = .494 - 3 × 10^-4 DISTANCE + 1.4 × 10^-6 PAX
   (1.6) (4.1) R^2 = .128
4. ALF = .303 - .016 LOG DIST + .059 LOG PAX
   (1.2) (6.4) R^2 = .238

Two Carrier Markets:
5. ALF = .349 - 3 × 10^-4 DISTANCE + 1.9 × 10^-6 PAX
   (0.1) (16.10) R^2 = .572
6. ALF = .153 - .019 LOG DIST + .121 LOG PAX
   (0.8) (4.5) R^2 = .145

Three Carrier Markets:
7. ALF = .495 - 2 × 10^-4 DISTANCE + 1.1 × 10^-6 PAX
   (0.8) (0.8) R^2 = .024
8. ALF = .371 - .017 LOG DIST + .031 LOG PAX
   (1.4) (2.2) R^2 = .105

Four Carrier Markets:
9. ALF = .464 + .5 × 10^-4 DISTANCE + 1 × 10^-6 PAX
   (1.0) (2.8) R^2 = .62
10. ALF = .107 + .013 LOG DIST + .045 LOG PAX
    (0.5) (2.2) R^2 = .495

TABLE 2

given in Table A1. The row and column headings identify the state of the system, or the number of travelers desiring a seat on the flight. The row headings indicate the possible states of the system at any time T_0, while the column headings indicate the possible states of the system at time T_0 + 1. The entries in the matrix are the conditional probabilities. For example, if the state (number of passengers) at time T_0 were .4 of the mean demand, the probability that at time T_0 + 1 there would be a demand of .4N_f is .1; that there would be a demand of 1.2N_f is .187, etc. If at time T_0 the demand exceeded the capacity, then of course the demand at time T_0 + 1 must reflect this "overflow." Hence, the conditional probabilities would change, as indicated in the matrix. These probabilities are defined with respect to a given capacity, measured in units of "X" where

(2) X = \frac{S - N_f}{\sigma_f}
where \( S = \) aircraft capacity.

The "steady state" of the Markov process defines the probabilities that \( Q \) is of any given size. Comparing these probabilities with the aircraft capacity, we can estimate the probability of being delayed by one, two, three or more flights. By multiplying these probabilities by the average headway interval, we can estimate the expected delay associated with any relative capacity, "X." By computing many values of delays, as \( X \) is changed, we then fitted the function:

\[
T_s = 0.455\left(\frac{N}{\sigma}\right)^{-0.645} \left(\frac{S-N}{\sigma}\right)^{-1.790} \times \text{ (headway interval)}.
\]

**One Step Transition Matrix**

\( X = 0.575 \)

<table>
<thead>
<tr>
<th>State (queue length) at ( T_0 + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.049</td>
</tr>
<tr>
<td>0.40N</td>
</tr>
<tr>
<td>0.87N</td>
</tr>
<tr>
<td>1.22N</td>
</tr>
<tr>
<td>1.47N</td>
</tr>
<tr>
<td>1.73N</td>
</tr>
<tr>
<td>2.00N</td>
</tr>
<tr>
<td>2.27N</td>
</tr>
<tr>
<td>2.53N</td>
</tr>
<tr>
<td>3.07N</td>
</tr>
</tbody>
</table>

Note: Matrix condensed for expository purposes; computations were made using 33 x 33 matrix.

N represents the mean demand per flight period.

**TABLE A1**