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SHAPED CASSEGRAIN REFLECTOR ANTENNA

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ABSTRACT

Design equations are developed to compute the reflector surfaces required to produce an uniform illumination on the main reflector of a cassegrain system when the feed pattern is specified. The final equations are somewhat simple and straightforward to solve (using a computer) compared to the ones which exist already in the literature. Step by step procedure for solving the design equations is discussed in detail.

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INTRODUCTION

In recent years efforts have been made to improve the efficiency of a dual reflector system by mechanically shaping both subreflector and main reflector (approximately paraboloid).[1,2,3] The general technique is to specify the feed pattern and its phase center and compute the surfaces required to produce a specified illumination on the main reflector. One hundred percent illumination efficiency can be obtained by specifying uniform illumination on the main reflector and very low spillover can be achieved by properly choosing the separation between the feed phase center and the subreflector.

The purpose of this communication is to present a set of design equations for a Shaped Cassegrain Reflector Antenna when the feed pattern is given either as an analytical expression or as an experimentally measured pattern. The approach used is analogous to that of Galindo[1] except that the independent variable in the present case is taken to be the feed pattern angular variable. This is a logical choice because the feed pattern is the one which is usually specified. Also, the resultant equations are somewhat simpler and are not coupled like those of Williams.[2]

DESIGN EQUATIONS

The cassegrain reflector system to be discussed is shown in Figure 1. The feed power pattern is assumed to be $F(\theta_1)$ and its phase center is located as shown in Figure 1. The design equations will be developed for providing uniform illumination across the main reflector. The optical principles used in developing the required equations are

1. conservation of energy flow along the ray trajectories
2. the path length $r_1 + r_2 + r_3$ remains constant: which results in planar phase front on the main reflector
3. application of Snell's law on the subreflector.

In addition to these three constraints, another constraint between x_1, y_1 and θ_1 will be obtained directly from the geometry of the reflector system. Taking θ_1 as the independent variable, there are four more variables, namely x_1, y_1, x_2 and y_2, to be solved from the four constraints discussed above.

Applying the principle of conservation of energy flow along the ray trajectories, the following relation is obtained[2] between x_2 and the assumed independent variable θ_1

$$x_2^2 = x_{2max}^2 \frac{\int_0^{\theta_1} F(\theta_1) \sin\theta_1 \, d\theta_1}{\int_0^{\theta_{1max}} F(\theta_1) \sin\theta_1 \, d\theta_1} \tag{1}$$

Equation (1) is one constraint from which x_2 can be determined as a function of θ_1. The equal path length condition gives the second constraint, i.e.,

$$r_1 + r_2 + r_3 = C_p \text{ a constant} \tag{2}$$

where $r_1 = (\beta - y_1) \sec \theta_1$,

$r_2 = \sqrt{(x_2 - x_1)^2 + (a + \beta + y_2 - y_1)^2}$,

$r_3 = y_2$, and C_p can be determined using boundary conditions as will be explained later on.

Equation (2) can be rewritten as

$$r_1 + r_2 + y_2 = C_p. \tag{3}$$

The relation given by equation (3) will be used later on. Substituting for r_2 in equation (3) and solving for y_2, it is not difficult to show that[1]

$$y_2 = \frac{(C_p - r_1)^2 - (a + \beta - y_1)^2 - (x_2 - x_1)^2}{2(a + \beta - y_1 + C_p - r_1)}. \tag{4}$$

Equation (4) is the same as equation (32) of Galindo. However, note that there is an error in his equation. The sign before C_p should be +.

Applying Snell's law at the subreflector leads to the third constraint, which is given below

$$\frac{dy_1}{dx_1} = \tan\left(\frac{\theta_1 - \theta_2}{2}\right) \tag{5}$$

where

$$\theta_2 = \arctan\left[\frac{x_2 - x_1}{a + \beta + y_2 - y_1}\right]. \tag{6}$$

The fourth constraint can be obtained directly from Figure 1 and is given by

$$x_1 = (\beta - y_1) \tan \theta_1. \tag{7}$$

From the four constraints given by equations (1), (4), (5) and (7) it is noted that the solution of x_2 could be obtained directly from equation (1), and in addition if the solution for y_1 is known, equations (7) and (4) could be used to solve for x_1 and y_2 respectively. Therefore, the problem now is to solve for y_1 from the constraint given by equation (5). It is possible to eliminate the unknowns x_1 and y_2 from equation (5) by using the equations (7) and (4) and then solve for y_1 in terms of the known variables θ_1 and x_2. However, this is a complicated procedure and results in a cumbersome equation for y_1. The following procedure results in a simpler equation for y_1. Since θ_1 is assumed to be an independent variable, $dy_1/d\theta_1$ is obtained from the following relation

$$\frac{dy_1}{d\theta_1} = \frac{dy_1}{dx_1}\frac{dx_1}{d\theta_1}. \tag{8}$$

Substituting dy_1/dx_1 from equation (5) and finding $dx_1/d\theta_1$ from equation (7), it can be shown that

$$\frac{dy_1}{d\theta_1} = \frac{(\beta - y_1)\sec^2\theta_1 \tan\left(\frac{\theta_1 - \theta_2}{2}\right)}{1 + \tan\theta_1 \tan\left(\frac{\theta_1 - \theta_2}{2}\right)}. \tag{9}$$

Since $\tan(\theta_2/2)$ is needed in equation (9), the following trigonometric identity is used

$$\tan\left(\frac{\theta_2}{2}\right) = \frac{\sin\theta_2}{1 + \cos\theta_2}. \tag{10}$$

Using equation (6) to find $\sin \theta_2$ and $\cos \theta_2$ and making use of the relation (3) to eliminate y_2 and equation (7) to eliminate x_1, it can be shown that

$$\tan\left(\frac{\theta_2}{2}\right) = \frac{x_2 - (\beta - y_1)\tan\theta_1}{a + \beta - y_1 + C_p - (\beta - y_1)\sec\theta_1} . \tag{11}$$

Equation (9), together with equation (11), can be solved for y_1 using one of the several numerical methods applicable for solving differential equations.[4] Even after the above simplification, it is noted that it takes more time on the computer to solve for y_1 than any other variable, so any simplification in solving for y_1 is welcome.

Boundary conditions are applied to determine the constants β, C_p and θ_{2max} by choosing x_{2max}, x_{1max}, θ_{1max} and a. x_{2max} is determined from the required gain considerations. x_{1max} is usually assumed to be around $0.1x_{2max}$. θ_{1max} is determined from the feed pattern and the amount of spillover allowed. The constant a can be determined by assuming an equivalent parabola with specified F/x_{2max} ratio. That means the feed phase center will be placed at the same position as that of an equivalent parabola hyperbola system. The other constants can be found from Figure 1 and are given below.

$$\beta = \frac{x_{1max}}{\tan\theta_{1max}}, \quad a = \left(\frac{F}{x_{2max}} - \frac{x_{2max}}{4F}\right)(x_{2max} - x_{1max}) - \beta$$

$$\theta_{2max} = \arc\tan\left(\frac{x_{2max} - x_{1max}}{a+\beta}\right) \tag{12}$$

and

$$C_p = \frac{\beta}{\cos\theta_{1max}} + \frac{a+\beta}{\cos\theta_{2max}} .$$

Finally, the step by step procedure to design a shaped cassegrain reflector is to first determine the required constants given in (12), then equation (1) is used to solve for x_2 as a function of θ_1. Once x_2 is known, equation (9) along with (11) are used to solve for y_1. Knowing y_1, equation (7) is used to solve for x_1. Knowing x_2, y_1, x_1, equation (4) is used to find the last unknown y_2.

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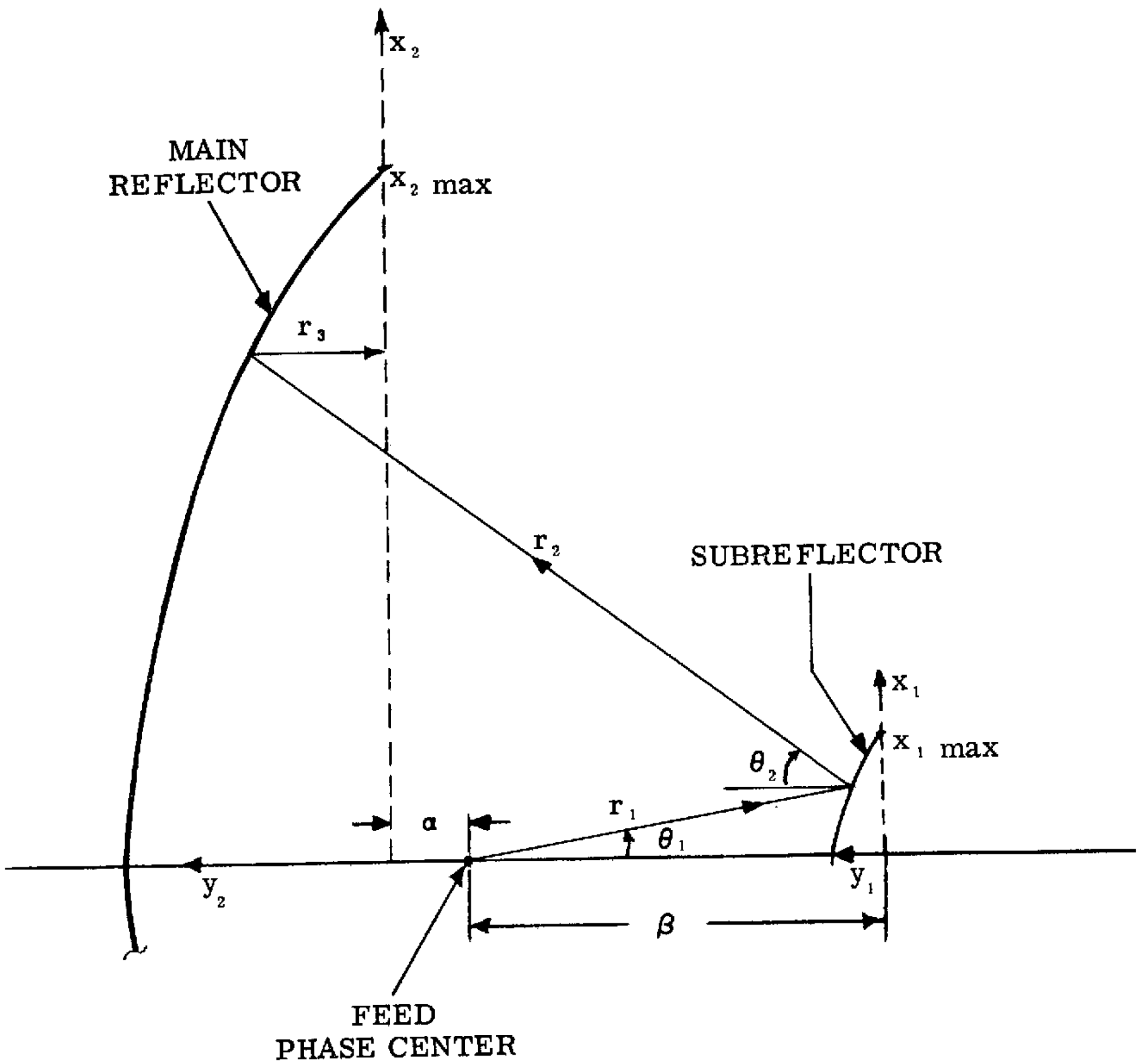

Figure 1. Shaped Cassegrain Reflector System.