THE PREDICTION OF NONLINEAR THREE-DIMENSIONAL COMBUSTION INSTABILITY IN LIQUID ROCKETS WITH CONVENTIONAL NOZZLES

by

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FOREWORD

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ABSTRACT

An analytical technique is developed to solve nonlinear three-dimensional, transverse and axial combustion instability problems associated with liquid-propellant rocket motors. The Method of Weighted Residuals is used to determine the nonlinear stability characteristics of a cylindrical combustor with uniform injection of propellants at one end and a conventional DeLaval nozzle at the other end. Crocco's pressure sensitive time-lag model is used to describe the unsteady combustion process. The developed model predicts the transient behavior and nonlinear wave shapes as well as limit-cycle amplitudes and frequencies typical of unstable motor operation. The limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure waveforms exhibit sharp peaks and shallow minima, and the frequency of oscillation is within a few percent of the pure acoustic mode frequency. For axial instabilities, the theory predicts a steep-fronted wave moving back and forth along the combustor.
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SUMMARY

An approximate analytical technique has been developed for the solution of nonlinear three-dimensional, transverse and axial combustion instability problems that are frequently observed in liquid-propellant rocket motors. This theory is an extension and generalization of previous analyses, which could analyze either transverse or axial instabilities in liquid combustors with quasi-steady nozzles, to the practical situations of three-dimensional instabilities in combustors with conventional DeLaval nozzles. Unlike the quasi-steady nozzle, the presence of a conventional nozzle imposes restrictions upon the behavior of both the amplitudes and phases of the oscillations at the nozzle entrance plane. The Method of Weighted Residuals is used to determine the nonlinear stability characteristics of a cylindrical combustor with uniform injection of propellants at one end and a conventional nozzle at the other end. Crocco's pressure sensitive time-lag model is used to describe the unsteady combustion process. The developed model can predict the transient behavior and nonlinear wave shapes as well as limit-cycle amplitudes and frequencies typical of unstable motor operation. These results establish the relationship that exists between the resulting instability (i.e., waveform, final amplitude and final frequency), the combustion parameters (i.e., interaction index, n, and time-lag,  \( \bar{\tau} \)), and the chamber Mach number and length-to-diameter ratio. Results indicate that the limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure waveforms exhibit sharp peaks and shallow minima, and the frequency of oscillation is always within a few percent of the frequency of one of the chamber's acoustic modes. For axial instabilities, the theory predicts the presence of a steep-fronted wave moving back and forth along the combustor. In both cases calculations of pressure and velocity perturbations at the nozzle entrance plane show that the approximation to the nozzle boundary condition is very good. The theory described in this report represents the final stage in the development of a unified nonlinear theory for the solution of general three-dimensional, transverse and axial combustion instability problems.
INTRODUCTION

Observation of the behavior of unstable rocket motors indicates that combustion instability can be divided into two categories; that is, linear and nonlinear instabilities. Linear instabilities are spontaneous in nature, and they are usually an outgrowth of the random combustion and flow fluctuations present in the system. On the other hand, nonlinearly unstable motors require the introduction of a finite amplitude disturbance to produce (or trigger) combustion instability. In either case the instability, after a transient period, reaches a limiting maximum amplitude (i.e., limit-cycle amplitude) at which it oscillates with a given frequency that is usually close to the frequency of one of the chamber's acoustic modes. Pressure measurements taken during test firings of unstable motors indicate that the limit-cycle waveforms of transverse instabilities are non-sinusoidal; that is, they exhibit sharp peaks and flattened minima.\(^1\) On the other hand, experimental observations of axial instabilities indicate the presence of shock-like steep-fronted waves in the chamber.\(^2\) These results indicate that nonlinearities need to be considered in the theoretical treatment of combustion instability.

Any analytical treatment of combustion instability should be capable of solving nonlinear multi-dimensional combustion instability problems without exceeding memory core limitations of current computers and without requiring excessive computation time. To be of practical use, such a solution technique should be conceptually simple and easily adaptable for use by industry. This report describes the development and use of such a numerical solution technique.

Work on this problem has been in progress during the past several years, and due to its complexity, the problem had to be tackled in stages. In earlier investigations by these authors theories describing the nonlinear behavior of longitudinal\(^3,4\) and transverse\(^5,6\) instabilities in liquid combustors with quasi-steady nozzles were developed. These theories, which were based upon the application of the Method of Weighted Residuals (MWR), successfully
predicted the transient behavior, nonlinear waveforms, and limit-cycle ampli-
tudes of longitudinal and transverse instabilities in unstable liquid rockets. 
This report is concerned with the development of a generalized nonlinear 
theory that will be capable of analyzing three-dimensional, transverse and 
axial instabilities in the more practical situations where the combustors 
are attached to conventional nozzles. Obviously, this generalized theory 
will encompass the above-mentioned investigations as special cases. Contrary 
to the quasi-steady nozzle case, the presence of a conventional nozzle imposes 
both amplitude and phase boundary conditions that must be satisfied by the 
solutions of the problem at the nozzle entrance plane. The generalized the-
ory presented herein also provides a better description of the unsteady flow 
field in the vicinity of the nozzle entrance plane.

The application of the theory presented herein will be demonstrated 
by considering the nonlinear stability of a liquid-propellant rocket combus-
tor with uniform injection of propellants at one end and a conventional noz-
le at the other end. Crocco's pressure sensitive time lag model\textsuperscript{7} is used to 
describe the unsteady combustion process. In the sections to follow, the de-
velopment of the wave equation for the analysis of nonlinear combustion in-
stability in liquid rockets will be briefly described, the solution of this 
nonlinear wave equation will be outlined, and typical results will be present-
ed and discussed. User's Manuals and program listings for the computer pro-
grams used to solve these problems are included as appendices to this report.

**SYMBOLS**

- \( A_{\text{mn}}(t), B_{\text{mn}}(t) \) : time-dependent amplitudes in series given by Eq. (6)
- \( A_{\text{p}}(t) \) : time-dependent amplitudes in series given by Eq. (9)
- \( B(\delta) \) : boundary residual
- \( b_{\text{mn}} \) : complex axial acoustic eigenvalue
- \( c^* \) : velocity of sound, ft/sec
$C_0, C_1, C_2, C_3$ coefficients of linear terms in Eqs. (12)

$D_1, D_2, D_3, D_4$ coefficients of nonlinear terms in Eqs. (12)

$E(t)$ residual of Eq. (10)

$i$ imaginary unit, $\sqrt{-1}$

$J_m$ Bessel function of the first kind, order $m$

$l, m$ axial and tangential mode numbers, respectively

$n$ pressure interaction index

$p$ dimensionless pressure, $\gamma p^*/\rho_o c_o^2$

$r$ dimensionless radial coordinate, $r^*/R_c^*$

$R_c^*$ chamber radius, ft

$R_p(r)$ radial acoustic eigenfunction in Eq. (9)

$S_{mn}$ dimensionless transverse mode frequency

$t$ dimensionless time, $\frac{t^*}{(R_c^*/c_o^*)}$

$u$ dimensionless axial velocity, $u^*/c_o^*$

$V^\rightarrow$ dimensionless velocity vector, $V^*/c_o^*$

$W_m^\prime$ unsteady combustion mass source

$Y$ complex nozzle admittance
$z$  
  dimensionless axial coordinate, $z^*/R^*_c$

$Z_{lmn}(z), Z_p(z)$  
  axial acoustic eigenfunctions

$\gamma$  
  ratio of specific heats

$\varepsilon$  
  ordering parameter

$\Theta$  
  azimuthal coordinate

$\Theta_p(\Theta)$  
  tangential acoustic eigenfunction in Eq. (9)

$\rho$  
  dimensionless density, $\rho^*/\rho_o^*$

$\tau$  
  dimensionless pressure sensitive time lag, $\frac{\tau^*}{(R^*_c/c_o^*)}$

$\phi$  
  velocity potential

Subscripts:

$e$  
  evaluated at the nozzle entrance

$n$  
  radial mode number

$r, t, z, \theta$  
  partial differentiation with respect to $r, t, z,$ or $\theta$ respectively

$r, i$  
  real and imaginary parts of a complex quantity, respectively

$o$  
  stagnation quantity

Superscripts:

$p$  
  perturbation quantity, differentiation with respect to argument
Development of the Wave Equation

To keep the problem as simple as possible, yet still physically meaningful, the following assumptions are made. The gas phase in the combustor is assumed to consist of a single constituent which is thermally and calorically perfect. Transport phenomena, such as diffusion, viscosity, and heat conduction are neglected. The momentum interchange between the liquid and gas phases is neglected (see Appendix A for a discussion of this assumption), and the specific stagnation enthalpy of the unburned propellant is assumed constant throughout the chamber. The presence of burning propellant drops is represented by a distribution of unsteady mass sources and it is also assumed that the Mach number of the combustor's mean flow is small and that the waves have moderate amplitudes.

As a result of the last two assumptions, the governing conservation equations may be combined and the unsteady flow in the combustor can be described by a single nonlinear wave equation. The derivation of this equation appears in Refs. 8 and 9, where it was assumed that each perturbation quantity and the mean flow Mach number were of $O(\varepsilon)$, where $\varepsilon$ is an ordering parameter that is a measure of the wave amplitude. After neglecting all terms of $O(\epsilon^3)$ or higher and combining equations, one obtains the following nonlinear partial differential equation that describes the behavior of the velocity potential, $\phi$, of the combustor disturbance:

$$\nabla^2 \phi - \phi_{tt} = 2V \cdot \nabla \phi_t + \gamma (\nabla \phi) \phi_t + 2V \cdot \nabla \phi_t + (\gamma - 1) \phi_t \nabla^2 \phi + W' \quad (1)$$

Equation (1) is the desired wave equation, and it is similar to the inhomogeneous wave equation solved by Maslen and Moore in a related study on nonlinear acoustics. This equation accounts for the following effects: (1) the
effect of a steady state flow on the wave motion (viz., the first two terms on the right-hand side), (2) the coupling between the gas dynamical oscillations and the unsteady combustion process (viz., the last term on the right-hand side), and (3) the second order nonlinearities of the gas dynamical processes (viz., the third and fourth terms on the right-hand side).

In addition to satisfying Eq. (1), the desired solutions must satisfy rigid wall boundary conditions at the injector end of the chamber and at the chamber walls, while a nozzle admittance condition must be satisfied at the nozzle entrance. These boundary conditions are given (in a cylindrical coordinate system) by:

\[ \dot{\phi}_r = 0 \text{ at } r = 1 \]
\[ \dot{\phi}_z = 0 \text{ at } z = 0 \]
\[ B(\phi) = \dot{\phi}_z + \gamma Y \dot{\phi}_t = 0 \text{ at } z = z_e \]

(2)

The nozzle admittance, \( Y \), is a complex number defined by

\[ Y = Y_r + iY_i = (u'/p')_z = z_e \]

(3)

where \( u' \) is the dimensionless axial velocity perturbation and \( p' \) is the dimensionless pressure perturbation.

It should be pointed out that due to the absence of an appropriate nonlinear nozzle admittance boundary condition, the solutions of the problem are required to satisfy a linear nozzle admittance. Although inconsistent with the nonlinear wave equation, the linear nozzle admittance condition is used herein with the hope that the solution techniques developed herein will also be applicable when nonlinear nozzle admittance conditions become available. Also, the relative importance of nozzle nonlinearities is not known at the moment and it is quite possible that the linear nozzle boundary condition used herein adequately describes the flow conditions at the nozzle entrance.

The unsteady combustion process is represented by mass sources distributed throughout the volume of the chamber, and the response of the mass sources to pressure oscillations is assumed to be described by Crocco's pressure sensitive time-lag hypothesis.\(^7\) The mass source perturbation, \( W'_m \), is then given by:\(^5,8\)
where \( n \) is the pressure "interaction index" that describes the sensitivity of the combustion process to pressure oscillations, and \( \tau \), commonly referred to as the sensitive time-lag, is the part of the total combustion time-lag during which the combustion process is sensitive to pressure oscillations. The unsteady combustion response described by Eq. (4) is linear and the comments made above regarding the use of a linear nozzle admittance boundary condition are also applicable to this case.

Substituting Eq. (4) into Eq. (1) and expressing the resulting equation in a cylindrical coordinate system yields the following wave equation:

\[
W_m' = -\gamma n \frac{du}{dz} \left[ \phi_t(r, \theta, z, t) - \phi_t(r, \theta, z, t - \tau) \right]
\]

The combustor and nozzle geometries considered in this study, as well as the cylindrical coordinate system used in writing Eq. (5), are shown in Fig. 1.

**Method of Solution**

Since Eq. (5) has no known closed-form mathematical solution, it is necessary to resort to the use of either exact numerical solution techniques or approximate analytical techniques. For multi-dimensional problems, the exact numerical solution techniques generally exceed the computer storage capacities, therefore an approximate solution technique is used herein. The experience of previous investigators in the fields of structural stability and aeroelasticity indicates that an approximate solution technique known as the Method of Weighted Residuals may be effective in the solution of this nonlinear wave equation.

In order to employ the Method of Weighted Residuals in the solution on Eq. (5), it is first necessary to express the velocity potential, \( \phi \), as an
approximating series expansion, $\Phi$. The question naturally arises as to what form of series expansion should be used. Inasmuch as the experimentally observed pressure oscillations during combustion instability usually resemble the natural acoustic modes of the chamber, the velocity potential, $\Phi$, is expanded in terms of the natural acoustic modes of the chamber with unknown time-dependent amplitudes.

In previous analyses of related problems the approximate solutions were expressed in terms of the acoustic modes for a cylindrical chamber with solid wall boundary conditions at both the injector and the nozzle ends. Consequently, the approximation of the flow conditions at the nozzle entrance was poor. In the present analysis a better approximation to the flow at the nozzle entrance is obtained by expanding the velocity potential in terms of the acoustic eigenfunctions for a chamber with a solid wall boundary condition at the injector end and a nozzle admittance condition at the other end. This removes both the two-dimensionality and the quasi-steady nozzle restrictions imposed upon the previous investigations.

The velocity potential, $\Phi$, is therefore approximated by the following series expansion:

$$\Phi = \sum_{l} \sum_{m} \sum_{n} \left\{ A_{lmn}(t) \sin m\theta + B_{lmn}(t) \cos m\theta \right\} Z_{lmn}(z) J_{m}(S_{mn} r) \quad (6)$$

where the $A$'s and $B$'s are unknown complex functions of time, and the $Z$'s are the complex axial acoustic eigenfunctions. The complex form of the axial acoustic eigenfunctions is given by

$$Z_{lmn}(z) = \cosh(ib_{lmn} z) \quad (7)$$

where the $b_{lmn}$ are the axial acoustic eigenvalues which must satisfy the following transcendental equation:

$$b_{lmn}^2 \sin^2(b_{lmn} z_e) + \gamma^2 y^2 (S_{mn}^2 + b_{lmn}^2) \cos^2(b_{lmn} z_e) = 0 \quad (8)$$

Equations (7) and (8) are obtained by linearizing Eq. (5) and solving the resulting equation for the case of no mean flow or combustion (i.e., the acoustic case) subject to the boundary conditions specified in Eq. (2). Each term in the above expansion exactly satisfies the solid wall boundary conditions at the injector end (i.e., at $z = 0$) and at the chamber wall (i.e., at
r = 1); however, due to the unknown time dependence of Eq. (6) the nozzle
admittance condition imposed at $z = z_e$ is not exactly satisfied by the indi-
vidual terms. Including both the $\sin m\theta$ and $\cos m\theta$ terms in the expansion for
$\Phi$ allows for the possibility of either spinning or standing wave solutions.

In order to simplify the algebra involved in the application of the
Method of Weighted Residuals, the development of the associated computer
program, and the presentation of the results; the expansion of the velocity
potential is written as a single summation as follows:

$$\tilde{\Phi} = \sum_{p=1}^{N} A_p(t)Z_p(z)\Theta_p(\theta)R_p(r)$$

where the $A_p$'s are the unknown time-dependent amplitudes. In order to use
Eq. (9) a correspondence must be established between the index, $p$, in Eq. (9)
and the mode-numbers $\ell$, $m$, and $n$ in Eq. (6). Such a correspondence is given
in Table 1 for a three mode series consisting of the spinning first tangential
(1T) mode ($\ell = 0$, $m = 1$, $n = 1$), the spinning second tangential (2T) mode
($\ell = 0$, $m = 2$, $n = 1$), and the first radial (1R) mode ($\ell = 0$, $m = 0$, $n = 1$).

Table 1
Correspondence Between Eq. (6) and (9) for a Three-Mode Series

<table>
<thead>
<tr>
<th>$p$</th>
<th>Mode</th>
<th>$\ell(p)$</th>
<th>$m(p)$</th>
<th>$n(p)$</th>
<th>$A_p$</th>
<th>$\Theta_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1T</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$A_{011}(t)$</td>
<td>$\sin \theta$</td>
</tr>
<tr>
<td>2</td>
<td>1T</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$B_{011}(t)$</td>
<td>$\cos \theta$</td>
</tr>
<tr>
<td>3</td>
<td>2T</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>$A_{021}(t)$</td>
<td>$\sin 2\theta$</td>
</tr>
<tr>
<td>4</td>
<td>2T</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>$B_{021}(t)$</td>
<td>$\cos 2\theta$</td>
</tr>
<tr>
<td>5</td>
<td>1R</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$B_{001}(t)$</td>
<td>1</td>
</tr>
</tbody>
</table>

Before proceeding with the analysis, the wave equation (i.e., Eq. (1))
must be modified for use with the assumed complex solution given by Equation
(9). This modification is necessary because only the real part of the assum-
ed solution is physically meaningful. It can easily be shown that if $\Phi = \varphi + i\gamma$ is a solution to Eq. (1), the real part, $\varphi$, is not a solution to Eq. (1).
This failure of \( \varphi \) to satisfy Eq. (1) is due to the presence of the nonlinear terms in this equation. It can also be shown, however, that a modified wave equation can be constructed for which the real part of its solution satisfies the original wave equation (i.e., Eq. (1)). This modified wave equation is given by:

\[
E(\hat{\varphi}) = \nabla^2 \hat{\varphi} - \hat{\varphi}_{tt} - 2 \nabla \cdot \nabla \hat{\varphi} - \gamma (\nabla \cdot \nabla) \hat{\varphi}_t - W_m
\]

\[
- \frac{1 - i}{2} \left[ \nabla \cdot \nabla \hat{\varphi}_t + \nabla \cdot \nabla \hat{\varphi}^* \right] - \frac{1 + i}{2} \left[ \nabla \cdot \nabla \hat{\varphi}^* + \nabla \cdot \nabla \hat{\varphi}_t \right]
\]

\[
- \frac{\gamma - 1}{4} \left\{ (1 - i) \left[ \hat{\varphi}_t \nabla^2 \hat{\varphi} + \hat{\varphi}^*_t \nabla^2 \hat{\varphi}^* \right] \right\} = 0
\]

where \( \hat{\varphi}^* \) is the complex conjugate of \( \hat{\varphi} \). The derivation of this equation is discussed in Appendix B. Thus, the Method of Weighted Residuals will be used to obtain approximate solutions to Eq. (10) (i.e., \( \tilde{\varphi} = \hat{\varphi} + i\Psi \)) from which the real part, \( \tilde{\varphi} \), will be taken as the approximate solution of Eq. (1).

In order to obtain a solution, the unknown time-dependent mode-amplitudes (i.e., \( A_p(t) \)) are determined by the following mathematical procedure. The assumed series expansion, \( \tilde{\varphi} \), (i.e., Eq. (9)) is substituted into the wave equation (i.e., Eq. (10)) to form the equation residual, \( E(\tilde{\varphi}) \). Similarly, substituting the series expansion into the nozzle boundary condition (i.e., the last of Eq. (2)) yields the boundary residual, \( B(\tilde{\varphi}) \). In the event that these residuals are both identically zero, the solution is an exact solution. The residuals \( E(\tilde{\varphi}) \) and \( B(\tilde{\varphi}) \) represent the errors incurred by using the approximate solution, \( \tilde{\varphi} \).

According to the modified version of the Method of Weighted Residuals, developed by the authors in Refs. 5 and 8, the residuals \( E(\tilde{\varphi}) \) and \( B(\tilde{\varphi}) \) must satisfy the following orthogonality conditions:

\[
\int_0^{2\pi} \int_0^1 \int E(\tilde{\varphi}) Z_j^*(z)e_j(\theta) R_j(r) r dr d\theta dz = 0
\]

\[
- \int_0^{2\pi} \int_0^1 B(\tilde{\varphi}) Z_j^*(z_e)e_j(\theta) R_j(r) r dr d\theta = 0
\]

\[
j = 1, 2, \ldots N
\]
where in the present study the complex conjugate of the axial eigenfunction, \( Z_j^* \), is used in the weighting functions. The chosen weighting functions must correspond to the terms that appear in the assumed series solution; that is, Eq. (9).

Evaluating the spatial integrals in Eq. (11) yields the following system of \( N \) complex ordinary differential equations to be solved for the unknown complex amplitude functions, \( A_p(t) \):

\[
\sum_{j=1}^{N} \left\{ C_0(j,p) \frac{d^2A_p}{dt^2} + C_1(j,p)A_p(t) + \left[ C_2(j,p) - nC_3(j,p) \right] \frac{dA_p}{dt} \right\} + \sum_{j=1}^{N} \left\{ D_1(j,p,q) A_p \frac{dA_q}{dt} + D_2(j,p,q) A_p \frac{dA_q^*}{dt} \right\} = 0
\]

\( j = 1, 2, \ldots, N \)

The coefficients appearing in the above equations are determined by evaluating the various integrals of hyperbolic, trigonometric, and Bessel functions that arise from the spatial integrations indicated in Eq. (11). A user's manual for the computer program COEFFS3D used to calculate these coefficients is given in Appendix C.

The time-dependent behavior of an engine following the introduction of a disturbance is determined by specifying the form of the initial disturbance and then following the subsequent behavior of the individual modes by numerically integrating Eqs. (12). Once the time-dependence of the individual modes is known, the velocity potential, \( \tilde{\phi} \), is calculated from Eq. (9). The pressure perturbation at any location within the chamber is related to the real part of \( \tilde{\phi} \) (i.e., \( \phi \)) by the following second-order momentum equation (see Refs. 5 and 8):

\[
\tilde{p}' = -\gamma[\tilde{\phi}_t + \tilde{u}(z)\tilde{\phi}_z + \frac{1}{2}(\tilde{\phi}_r + \frac{1}{r^2}\tilde{\phi}_\theta + \tilde{\phi}_z) - \frac{1}{r^2}\tilde{\phi}_\theta] = 0
\]  (13)
A user's manual for the computer program, LCYC3D, which obtains numerical solutions of Eqs. (12) and (13) is given in Appendix D.

In summary, the theory presented in this section represents a two-stage simplification of the original problem. In the first stage the problem has been reduced to the solution of a single nonlinear, partial differential equation (i.e., Eq. (1)). In the second stage the solution was expanded in a series of acoustic modes with time-dependent coefficients and the Method of Weighted Residuals was used to replace the solution of the nonlinear partial differential equation with the solution of a system of nonlinear, ordinary differential equations (i.e., Eq. (12)). Typical numerical solutions of these equations will be presented and discussed in the following section.

RESULTS AND DISCUSSION

The generalized three-dimensional theory introduced in the previous section has been used to obtain both linear and nonlinear data for pure transverse modes and pure longitudinal modes for rocket motors with conventional nozzles. Nonlinear data for the first tangential (1T) mode and the first longitudinal (LL) mode has also been obtained for combustors with quasi-steady nozzles for comparison with the results of the previous two-dimensional theories.

Linear Solutions

Before proceeding with the nonlinear analysis, it was desired to obtain numerical solutions of the linearized equations (i.e., Eqs. (12) with $D_1 = D_2 = D_3 = D_4 = 0$) in order to determine how closely the approximate solutions satisfied the nozzle boundary condition. The linear solution is also needed for comparison with the corresponding nonlinear results. The linear solutions were obtained by assuming a one-mode series expansion consisting only of the mode under consideration. Due to the presence of the retarded variables (i.e., $d[A_{p}(t - \tau)]/dt$) in Eqs. (12), it is necessary to specify the initial amplitudes over the interval $-\tau \leq t \leq 0$. In this study the initial values were chosen such that the nozzle boundary condition was exactly satisfied during this initial time period. Solutions were obtained for values of $n$ and $\tau$ on the neutral stability limit (see Appendix E for the determination of neutral stability limits) for various conventional nozzle configurations. The nozzle admittance was expressed in the form, $Y = Ae^{i\varphi}$, where $A$ is the amplitude fac-
tor and φ is the phase shift. The pressure perturbation, \( p' \), and the axial velocity perturbation, \( u' \), at the nozzle entrance were calculated numerically for several values of the nozzle phase shift, \( φ \). These calculated values were then used to compute the ratios \( (u'/p')_{z=z_e} \), which were then compared with the specified nozzle admittance values. These results are shown in Tables (2) and (3) where \( A_n \) and \( \varphi_n \) are the computed values of the amplitude factor and phase shift, respectively. These results show that the approximation to the nozzle boundary condition is very good for both the 1T and 1L modes; that is, the maximum error in the amplitude ratio is about 5% and the maximum error in phase is approximately 0.5 degree. These results are in contrast with previous theoretical investigations where the representation of the unsteady flow conditions in the vicinity of the nozzle entrance was very poor.

Table 2. 1T Mode Linear Solutions (Numerical).

<table>
<thead>
<tr>
<th>( \varphi ) (Degrees)</th>
<th>( \tau )</th>
<th>( n )</th>
<th>( A_n - A )</th>
<th>( \varphi_n - \varphi ) (Degrees)</th>
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<tr>
<td>0</td>
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<td>0.66416</td>
<td>-0.029</td>
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</tr>
<tr>
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<td>0.003</td>
<td>0.4</td>
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<tr>
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<tr>
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<tr>
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<td>0.5</td>
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<tr>
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<td>0.3</td>
</tr>
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<tr>
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<tr>
<td>Degrees</td>
<td>A = 0.02</td>
<td>Error at Nozzle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>-----------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\varphi$</td>
<td>$\tau$</td>
<td>$n$</td>
<td>$\frac{A_n - A}{A}$</td>
</tr>
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</tr>
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<td>0.02</td>
<td>1.4</td>
<td>1.34523</td>
<td>0.049</td>
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</table>
Nonlinear Solutions

Nonlinear solutions have been computed for both the 1T mode and the 1L mode. For the 1T mode calculations a three mode series expansion consisting of the 1T, 2T (second tangential), and 1R (first radial) modes was used. These are the same modes that were included in the series expansion used in the previous two-dimensional transverse instability studies.\(^5,6\) In these studies it was shown that convergence was obtained with this three mode series; that is, the addition of higher transverse modes (i.e., 3T, 4T, etc.) to the basic series had little effect on the solution. The 1L mode computations were made using a series consisting of the first five longitudinal modes (i.e., 1L, 2L, 3L, 4L, and 5L). It has been shown by Lores and Zinn\(^3,4\) that convergence is obtained with this five-mode series.

Transverse Mode Solutions. Nonlinear solutions have been computed for rocket combustors with quasi-steady nozzles (i.e., real admittances) and also for nozzles with complex admittances. The quasi-steady nozzle solutions were generated for comparison with the results of the previous two-dimensional theory.\(^5\) For this case the nozzle admittance is given by:\(^13\)

\[
y_r = \frac{y - \frac{1}{2\pi} \frac{1}{u_e}}{2y} \quad Y_i = 0
\]

For nozzles with complex admittances the admittance was expressed in the form, \(Y = Ae^{i\phi}\). For both cases limit-cycle amplitudes and waveforms have been computed for both standing and spinning first tangential instability. This required three series terms to describe standing instability and five series terms to describe spinning instability. Typical computation times on a Univac 1108 computer to reach a limit-cycle were one minute for a standing wave and two minutes for a spinning wave.

Wall pressure waveforms \((r = 1)\) were computed at the injector face \((z = 0)\) and at the nozzle entrance \((z = z_e)\) for three azimuthal locations, \(\theta = 0^\circ, \theta = 45^\circ,\) and \(\theta = 90^\circ\). The initial conditions for standing waves were chosen such that a pressure anti-node occurred at \(\theta = 0^\circ\). Injector pressure waveforms for both standing and spinning instability are shown in Fig. 2 for combustors with quasi-steady nozzles. These waveforms exhibit sharp peaks.
Dimensionless Time, $t$

(a) Standing Wave: $n = 0.65$, $\tau = 1.7$, $\gamma = 1.2$, $\bar{u}_e = 0.2$, $z_e = 1.0$

(b) Spinning Wave: $n = 0.58$, $\tau = 1.7$, $\gamma = 1.2$, $\bar{u}_e = 0.2$, $z_e = 1.0$

Figure 2. Nonlinear Pressure Waveforms for the IT Mode.
and shallow minima; they are nearly identical in shape to those calculated using the previous two-dimensional theory.\textsuperscript{5,6} Comparison of injector and nozzle pressure waveforms ($\theta = 0^\circ$) shows that there is very little variation in pressure with axial position. These waveforms are in qualitative agreement with the results of pressure measurements taken during test firings of unstable rocket motors.\textsuperscript{1}

To check the accuracy of the approximation of the nozzle boundary condition, wall pressure and axial velocity waveforms were calculated at the nozzle entrance. The error at the nozzle boundary ($z = z_e$) is shown for nonlinear standing and spinning IT mode instabilities in Fig. 3. Here the axial velocity perturbation, $u'$, and the product of the quasi-steady nozzle admittance and the pressure perturbation, $Y_r p'$ are plotted as a function of time. The latter quantity is the axial velocity perturbation that would be obtained at the nozzle entrance if the nozzle boundary condition were exactly satisfied (i.e., the nozzle admittance condition requires that $u' = Y_r p'$ at $z = z_e$). Most of the discrepancy between the two curves is due to a slight phase shift between pressure and velocity and the second harmonic distortion of the pressure waveform resulting from the nonlinearities of the system. The nozzle boundary condition is satisfied in an average sense, however, for the ratio of the velocity amplitude (peak-to-peak) to pressure amplitude (peak-to-peak) is very close to the required value, $Y_r$.

In another study, limit-cycle amplitudes were calculated as a function of $n$ and $\bar{\tau}$ for standing IT mode instability. Values of $n$ in the linearly unstable region were chosen for below resonant ($\bar{\tau} = 1.9$), resonant ($\bar{\tau} = 1.706$), and above resonant ($\bar{\tau} = 1.5$) conditions. The resulting amplitudes are compared with those obtained with the two-dimensional theory in Fig. 4. This figure shows that the three-dimensional theory predicts a slightly higher limit-cycle amplitude than the two-dimensional theory for chambers with quasi-steady nozzles.

Figure 4 also shows that the three-dimensional theory, like the previous two-dimensional one, cannot predict triggering of IT mode instability by the introduction of finite amplitude disturbances. This result was expected since it was shown in Refs. 6 and 8 that the second order (i.e., $O(\epsilon^2)$) theory can predict triggering only for pure radial modes ($m = 0, n = 1, 2 \ldots$). Such triggering limits for the $1R$ mode are discussed in Ref. 9. It has also been
Figure 3. Nozzle Boundary Condition for Nonlinear 1T Mode Solutions for Quasi-Steady Nozzles.
Figure 4. Limit-Cycle Amplitudes for the 1T Mode.
shown, however, that triggering of IT mode instability can be described when
the $O(q^3)$ terms are retained in the analysis. The third order theory
given in Refs. 8 and 14 is limited to a single mode in the approximating
series expansions. A more general multi-mode third-order theory is now under
development and the results will be presented in a future publication. It
is also suspected that nonlinear unsteady combustion effects (not included in
the present analysis) may play an important role in the triggering phenomenon.

For nozzles with complex admittances a study was conducted to determine
the effect of the nozzle phase shift, $\varphi$, upon the limit-cycle amplitudes and
waveforms for both standing and spinning IT mode instability. The effect of
nozzle phase shift on the nonlinear pressure and velocity waveforms at the
nozzle entrance plane is shown in Fig. 5 for spinning waves. This figure
shows that, while $\varphi$ has little or no effect on the pressure waveforms, the
phase and shape of the velocity waveforms is strongly dependent on $\varphi$. The
effect of $\varphi$ on the limit-cycle amplitude for standing IT mode instability is
shown in Fig. 6. For a given value of $n$ and $\tilde{r}$ (in the linearly unstable re-
gion for the IT mode), Fig 6 shows a sinusoidal variation of limit-cycle am-
plitude with $\varphi$ having a maximum amplitude at about $\varphi = 200^\circ$ and a minimum am-
plitude at about $\varphi = 20^\circ$. In this connection, it should be pointed out that
according to linear results nozzle damping is a maximum at $\varphi = 0^\circ$ and a mini-
um at $\varphi = 180^\circ$; thus the observed shifts must be due to nonlinearities.

In order to determine how well the solutions approximate the nozzle
boundary condition, the amplitude ratio and phase shift between pressure and
velocity at the nozzle entrance have been calculated from the nonlinear solu-
tions and have been compared with the specified nozzle admittance condition.
Since the waveforms are non-sinusoidal, an approximate amplitude ratio, $A_c$, 
was calculated by taking the ratio of peak-to-peak velocity amplitude to
peak-to-peak pressure amplitude. The approximate phase shift, $\varphi_c$, was calcu-
lated from the following formula:

$$\varphi_c = \left[ \frac{t_p - t_u}{T} \right] \times 360 \quad (15)$$

where $t_p$ is the average of an ascending zero-crossing and the following de-
sceding zero-crossing for the pressure perturbation, $t_u$ is a similar average
Figure 5. Effect of Nozzle Phase Shift, $\varphi$, on Nozzle Waveforms for Spinning 1T Modes.
Figure 6. Effect of Nozzle Phase Shift on Limit-Cycle Amplitudes for the Standing 1T Mode.
for the velocity perturbation, and \( T \) is the period of oscillation. The results of this study are shown in Fig. 7 for both standing and spinning waves. For standing waves the calculated amplitude ratios are seen to be consistently higher than required by the nozzle admittance condition (dashed line), while for spinning waves the calculated amplitude ratios are lower than required. For both standing and spinning waves the calculated phase shifts are in excellent agreement with the imposed phase shifts. This study shows that the three-dimensional theory provides a good approximation to the nozzle boundary condition for the IT mode, considering that the nonlinear solutions are being forced to satisfy a linear boundary condition.

Longitudinal Mode Solutions. Letting \( m \) and \( n \) equal zero in Eq. (6) and using a series consisting of the first five longitudinal modes (i.e., \( \ell = 1, 2, \ldots, 5 \)), limit-cycle solutions were calculated for quasi-steady nozzles as well as for nozzles with complex admittances. The longitudinal mode solutions required somewhat longer computation times than the transverse mode solutions; the time required to reach a limit cycle was from three to four minutes on the Univac 1108 computer.

Longitudinal mode solutions for chambers with quasi-steady nozzles were compared with the solutions previously obtained by Lores and Zinn \(^3,4\) using a one-dimensional theory. Pressure waveforms at the injector face are compared for both resonant and off-resonant conditions in Fig. 8 which shows excellent agreement between the two theories. Pressure and velocity waveforms at the nozzle entrance as well as injector face pressure waveforms are shown in Fig. 9 for quasi-steady nozzles, while Fig. 10 shows waveforms at the nozzle entrance for nozzles with complex admittance (\( \phi = 45^\circ \) and \( \phi = 90^\circ \)). In each case the results indicate the presence of a steep-fronted pressure wave moving back and forth in the chamber. This behavior is in agreement with experimental observations of axial instabilities.\(^2\) The relation between pressure and velocity waveforms at the nozzle entrance is a fairly good approximation to the nozzle admittance condition (see Figs. 9 and 10) in spite of the highly nonlinear waveforms. The results of this investigation indicate that the three-dimensional nonlinear theory is applicable to longitudinal instabilities as well as transverse instabilities. The theory can also be used to investigate the nonlinear behavior of combined
Standing (n = 0.65)
Spinning (n = 0.58)

Figure 7. Nozzle Boundary Condition for Nonlinear 1T Mode Solutions.
Figure 8. Comparison of Nonlinear IL Mode Solutions for Quasi-Steady Nozzles.
Figure 9. Longitudinal Mode Waveforms for Quasi-Steady Nozzles.
Figure 10. Longitudinal Mode Waveforms for Nozzles with Complex Admittances.
longitudinal-transverse instabilities, although no results for instabilities of this type are presented in this report.

**CONCLUDING REMARKS**

A general three-dimensional second-order nonlinear theory has been developed for predicting the linear and nonlinear behavior of combustion instability in liquid-propellant rocket combustors. This theory contains previous analyses of transverse and longitudinal instabilities as special cases. Furthermore it extends the previous analyses which were applicable only to combustors with quasi-steady nozzles, to the more practical cases of combustors with conventional DeLaval nozzles. The present theory can be used to predict the stability characteristics of longitudinal, transverse and combined longitudinal-transverse modes for various liquid-propellant rocket motor designs.

Results obtained for combustors with quasi-steady nozzles are in excellent agreement with the predictions of previous theories for both transverse and longitudinal instabilities. For combustors with conventional nozzles the limit-cycle amplitude varies sinusoidally with nozzle phase shift, $\varphi$, having a maximum value at $\varphi = 200^\circ$ and a minimum value at $\varphi = 20^\circ$. The nozzle phase shift has a strong effect on the axial velocity waveforms at the nozzle entrance while having only a minor influence on the nonlinear pressure waveforms. In both cases, the nonlinear theory developed in this paper provides a good approximation to the unsteady flow conditions at the nozzle entrance plane. This is in contrast to the previous theories which provided a relatively poor approximation to the nozzle boundary condition.

The results presented in this report establish the relationship that exists between the resulting instability (i.e., waveform, final amplitude, and final frequency), the combustion parameters (i.e., interaction index, $n$, and time-lag $\tau$), and the chamber Mach number and length-to-diameter ratio. These results indicate that the limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure waveforms exhibit sharp peaks and shallow minima, and the frequency of oscillation is always within a few percent of the frequency of one of the chamber's acoustic modes. For axial instabilities, the theory predicts the presence of a steep-fronted wave moving back and forth along the combustor. In both cases the calculated pressure waveforms are in
good qualitative agreement with available experimental data.
APPENDIX A

MOMENTUM INTERCHANGE BETWEEN LIQUID AND GAS PHASES

The results presented in this report were obtained under the assumption that the momentum interchange between the liquid droplets and the burned gases is negligible. This assumption will now be relaxed for the special case of uniformly distributed combustion, and it will be shown that this momentum interchange is an important stabilizing effect.

Analysis

The momentum equation for two-phase flow was derived in Ref. 8 and is given by:

\[
\rho \left[ \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} \right] + \frac{1}{\gamma} \nabla p = -(\vec{V} - \vec{V}_L)(C + w_m)
\]  

(A-1)

where \( \vec{V} \) and \( \vec{V}_L \) are the gas and liquid velocity, respectively. The term on the right-hand-side of Eq. (A-1) represents a momentum source to the gas produced by the burning liquid drops. This momentum source consists of two parts: (1) the force necessary to accelerate the evolved gases from the droplet velocity to the gas velocity (i.e., the term \(-w_m(\vec{V} - \vec{V}_L)\)) and (2) the aerodynamic drag of the droplets (i.e., the term \(-C(\vec{V} - \vec{V}_L)\)).

In order to derive a wave equation for the velocity potential \( \phi \) it is necessary to make the following assumptions: (1) the drag term is negligible compared with the acceleration term, (2) liquid velocity fluctuations are negligible, and (3) the combustion is uniformly distributed throughout the chamber. Neglecting the drag term, perturbing, and neglecting third order quantities gives the following expression for the momentum source perturbation, \( M' \):

\[
M' = -(\vec{V}' - \vec{V}_L')w_m
\]  

(A-2)

This is simplified further by neglecting the liquid velocity perturbation,
introducing the velocity potential, and using the steady-state relation, 
\( \bar{W}_m = \frac{d\bar{u}}{dz} \), to obtain:

\[
M' = \frac{d\bar{u}}{dz} \nabla \phi
\]  
(A-3)

Finally, the assumption of uniformly distributed combustion gives \( \frac{d\bar{u}}{dz} = \text{constant} \) which yields:

\[
M' = -\nabla \left[ \frac{d\bar{u}}{dz} \phi \right]
\]  
(A-4)

Perturbing the left-hand-side of Eq. (A-1), introducing the velocity potential, and combining with Eq. (A-4) gives:

\[
\nabla \left[ \frac{\partial \phi}{\partial t} + \frac{1}{r} p' + \bar{u}_z \phi + \frac{d\bar{u}}{dz} \phi + \frac{1}{2} \nabla \phi \cdot \nabla \phi - \frac{1}{2} \bar{u}_t^2 \right] = 0
\]  
(A-5)

which can be integrated to obtain:

\[
p' = -\gamma \left[ \phi_t + \bar{u}_z \phi_z + \frac{d\bar{u}}{dz} \phi + \frac{1}{2} \nabla \phi \cdot \nabla \phi - \frac{1}{2} \bar{u}_t^2 \right]
\]  
(A-6)

Equation (A-6) is similar to Eq. (13), where the additional term \( (d\bar{u}/dz) \phi \) arises from the droplet momentum source. Following the procedure outlined in Ref. 8, the momentum equation given by Eq. (A-6) is combined with the continuity and energy equations to obtain the desired wave equation:

\[
\nabla^2 \phi - \phi_{tt} = 2\bar{u}_z \phi_{zt} + (\gamma + 1) \frac{d\bar{u}}{dz} \phi_t + 2\nabla \phi \cdot \nabla \phi_t + (\gamma - 1) \phi_t \nabla^2 \phi + W_m'
\]  
(A-7)

Comparing Eq. (A-7) with Eq. (1) shows that the droplet momentum source
appears only in the second term on the right-hand-side of this equation, where the factor $\gamma$ in Eq. (1) becomes $(\gamma + 1)$ in Eq. (A-7).

Applying the Method of Weighted Residuals to obtain approximate solutions to Eq. (A-7) yields a set of ordinary differential equations identical to Eqs. (12) where the coefficient $C_2(j,p)$ is now given by:

$$C_2(j,p) = \left\{ \int_0^{r_e} \bar{u}(z)Z_p^*Z_j^*dz + (\gamma + 1)\int_0^{r_e} \frac{e^{d_n}}{d_z}Z_p^*Z_j^*dz + \gamma Y_{p_e}(z_e)Z_j^*(z_e) \right\} X,$$

$$X \int_0^{2\pi} \int_0^{\infty} \int_0^{1} R_P R_J r dr d\theta$$

Equation (A-8) is readily obtained from Eq. (C-3) by replacing $\gamma$ in the second term by $\gamma + 1$.

**Linear Stability Limits**

Linear stability limits for the LL mode were calculated by the method described in Appendix E for the following two cases: (1) the droplet momentum source was included in the analysis and (2) the droplet momentum source was neglected. The results were compared with the linear stability limit calculated by Mitchell$^{15}$ on a plot of interaction index, $n$, versus stretched time-lag, $\mu$, where $\mu = \omega T/\pi$ (see Fig. A-1). This figure shows excellent agreement between the results of Mitchell (solid curve) and the present theory (circle symbols) when the droplet momentum source is included. Neglecting the droplet momentum source shifts the stability curve to much lower values of $n$ (dashed curve), which indicates that the droplet momentum source is an important stabilizing effect.

**Nonlinear Solutions**

In the second-order analysis presented in this report, the droplet momentum source affects the nonlinear solutions primarily by increasing the linear stability of the system. This is readily shown in Fig. (A-2) where the limit-cycle amplitude is plotted as a function of the displacement, $\delta n$, above the neutral stability limit. By plotting the limit-cycle amplitudes in this manner,
Figure A-1. Effect of Droplet Momentum Source on Linear Stability Limits for the IL Mode.
Figure A-2. Effect of Droplet Momentum Source on Limit-Cycle Amplitude.
the effect of the shift in the neutral stability curves is removed so that only the nonlinear effect of the momentum source is seen. Figure A-2 shows that, for equal displacements above the neutral stability limits, including the droplet momentum source results in a slightly smaller limit-cycle amplitude. This difference in limit-cycle amplitude is negligible for most practical purposes.

For combustors with uniformly distributed combustion it has been shown that the droplet momentum source is an important effect which is easily incorporated into the present analysis. Consequently the computer programs based on this theory include the droplet momentum source as an optional feature (see Appendices C, D, and E).

For chambers with non-uniform combustion distributions, Eqs. (A-6) and (A-7) are no longer applicable; however, the droplet momentum source can be taken into account in the following manner. Using the present theory with the droplet momentum source omitted, the neutral stability limit, \( n_1(\tau) \), is calculated and the limit-cycle amplitudes are determined as a function of \( \delta n \) as in Fig. A-2. In addition, the linear stability limit, \( n_2(\tau) \), is calculated using a linear theory which includes the droplet momentum source and is not restricted to uniformly distributed combustion (such as in Ref. (15)). Assuming that the nonlinear effect of the droplet momentum source is also small for non-uniformly distributed combustion and using the values of \( \delta n \) and \( n_2(\tau) \) calculated above, the desired plot of limit-cycle amplitude as a function of \( n \) is readily obtained.
APPENDIX B

USE OF COMPLEX VARIABLES IN THE SOLUTION OF NONLINEAR DIFFERENTIAL EQUATIONS

It is often convenient to use complex variables in the solution of the linear equations which arise in acoustics, combustion instability and related fields. In this case the solution is expressed in complex form, and the real part represents the physically meaningful solution. However, care must be used when applying this technique in the solution of nonlinear equations. The difficulties that are encountered in applying the complex variable technique to nonlinear problems will be illustrated by analyzing the following simplified example. Consider the nonlinear wave equation given by:

\[ \nabla^2 \psi - \psi_{tt} = \psi_t \tag{B-1} \]

A complex solution of Eq. (B-1) of the form \( \psi = \phi + i\psi \) would be useful only if its real part, \( \phi \), satisfies Eq. (B-1), which would be the case if the equation were linear. However, straightforward substitution of \( \psi = \phi + i\psi \) into Eq. (B-1) and separating its real and imaginary parts yields the following equation for \( \phi \):

\[ \nabla^2 \phi - \phi_{tt} = \phi\phi_t - \psi\psi_t \tag{B-2} \]

indicating that the real part, \( \phi \), does not satisfy Eq. (B-1) because of the extra term, \( -\psi\psi_t \), appearing on the right hand side. In order to eliminate this extra term, the form of the original differential equation (i.e., Eq. (B-1)) must be modified.

Since Eq. (B-1) supposedly describes some physical phenomenon, and since only the real part of the complex solution is physically meaningful, then the nonlinear term \( \phi\phi_t \) should really be expressed as the product \( \text{Re}(\psi) \times \text{Re}(\phi_t) \) which is equivalent to \( (\phi_t + \phi_t^* + \psi^* \phi_t + \phi^* \psi_t)/4 \). Substituting this expression into Eq. (B-1) yields:
\[ \nabla^2 \phi - \phi_{tt} = \frac{1}{\mu}[\phi_t^* \phi_t^* + \phi_t^* \phi_t^*] \]  
(B-3)

Substituting \( \psi = \varphi + i \psi \) into Eq. (B-3) and separating its real and imaginary parts yield:

\[ \nabla^2 \varphi - \varphi_{tt} = \varphi \psi_t \]  
(B-4)

\[ \nabla^2 \psi - \psi_{tt} = 0 \]

which shows that the real part of the solution of Eq. (B-3) satisfies the desired equation (i.e., Eq. (B-1)) and the imaginary part satisfies a homogeneous linear wave equation. This technique was applied to the solution of nonlinear combustion instability problems (i.e., to Eq. (I)), and the resulting modified wave equation was solved using the Method of Weighted Residuals. Due to the approximate nature of the Method of Weighted Residuals, however, the resulting solution contained an error term which grew without limit. Consequently, the above procedure had to be modified in order to obtain satisfactory solutions of Eq. (1) using the Method of Weighted Residuals.

An alternate technique is to modify Eq. (B-1) such that both the real and imaginary parts satisfy the original equation. This can be done by replacing terms of the form \( \phi \phi_t \) with \( \text{Re}(\phi) \text{Re}(\phi_t) + i \text{Im}(\phi) \text{Im}(\phi_t) \); using the relations:

\[ \text{Re}(\phi) \text{Re}(\phi_t) = \left( \frac{\phi + \phi^*}{2} \right) \left( \frac{\phi_t + \phi_t^*}{2} \right) = \frac{1}{\mu} \left[ \phi \phi_t^* + \phi^* \phi_t^* + \phi \phi_t^* + \phi^* \phi_t^* \right] \]  
(B-5)

\[ i \text{Im}(\phi) \text{Im}(\phi_t) = -i \left( \frac{\phi - \phi^*}{2} \right) \left( \frac{\phi_t - \phi_t^*}{2} \right) = \frac{1}{\mu} \left[ \phi \phi_t - \phi \phi_t^* - \phi^* \phi_t + \phi^* \phi_t^* \right] \]

in Eq. (B-1) gives:

\[ \nabla^2 \phi - \phi_{tt} = \frac{1}{\mu} \left[ (1 - i)(\phi \phi_t + \phi^* \phi_t^*) + (1 + i)(\phi \phi_t^* + \phi^* \phi_t) \right] \]  
(B-6)

Substituting \( \phi = \varphi + i \psi \) into Eq. (B-6) and separating into its real and imaginary parts yield:

\[ \nabla^2 \varphi - \varphi_{tt} = \varphi \psi_t \]  
(B-4)

\[ \nabla^2 \psi - \psi_{tt} = 0 \]
nary parts gives:

\[ \nabla^2 \varphi - \varphi_{tt} = \varphi \varphi_t \]  
(B-7)

\[ \nabla^2 \psi - \psi_{tt} = \psi \psi_t \]

which shows that both \( \varphi \) and \( \psi \) satisfy Eq. (B-1). Applying this method to the solution of Eq. (1) yields the modified wave equation (i.e., Eq. (10)) used in the present investigation.
Statement of the Problem

Program COEFFS3D calculates the coefficients of both the linear and non-linear terms which appear in Eqs. (12). These coefficients are required as input for Program LCYC3D (see Appendix D) which numerically integrates this system of equations. The coefficients that are required depend on the choice of terms to be included in the series solution for $\tilde{\phi}$ (see Eq. (9)), therefore this information must be provided as input to Program COEFFS3D. The output of Program COEFFS3D is either punched onto cards or stored on drum (FASTRAND) for input to Program LCYC3D.

The coefficients to be calculated are functions of various integrals of hyperbolic, trigonometric, and Bessel functions and are given by the following expressions:

\[
C_0(j,p) = \int_{0}^{\pi} \int_{\phi}^{\phi_0} \int_{0}^{\rho} \int_{0}^{1} R R R_r dr d\phi d\rho dz
\]

\[
C_1(j,p) = \left\{ \frac{2}{\pi} \left( \int_{0}^{\pi} \int_{\phi}^{\phi_0} \int_{0}^{1} R R R_r dr d\phi d\rho \int_{0}^{1} \frac{d}{dz} Z_p Z_j^* dz + \frac{d}{dz} Z_p Z_j^* dz + \frac{d}{dz} Z_p Z_j^* dz \right) \right\}
\]

\[
C_2(j,p) = \left\{ \int_{\phi}^{\phi_0} \int_{0}^{\rho} \int_{0}^{1} R R R_r dr d\phi d\rho \right\}
\]

\[
C_3(j,p) = \left\{ \gamma \int_{\phi}^{\phi_0} \int_{0}^{\rho} \int_{0}^{1} R R R_r dr d\phi d\rho \right\}
\]
\[ D_1(j,p,q) = \frac{1}{2}(1 - i) \left\{ T_1 \int_{0}^{z_e} Z^*_p Z^*_j dz + T_2 \left[ \int_{0}^{z_e} Z'_p Z'_j dz + \frac{\gamma - \frac{1}{2}}{2} \int_{0}^{z_e} Z''_p Z'_j dz \right] \right\} \] (C-5)

\[ D_2(j,p,q) = \frac{1}{2}(1 + i) \left\{ T_1 \int_{0}^{z_e} Z^*_p Z^*_j dz + T_2 \left[ \int_{0}^{z_e} (Z^*_p)' Z'_j dz + \frac{\gamma - \frac{1}{2}}{2} \int_{0}^{z_e} (Z^*_p)'' Z'_j dz \right] \right\} \] (C-6)

\[ D_3(j,p,q) = \frac{1}{2}(1 + i) \left\{ T_1 \int_{0}^{z_e} Z^*_p Z_j dz + T_2 \left[ \int_{0}^{z_e} (Z^*_p)' Z'_j dz + \frac{\gamma - \frac{1}{2}}{2} \int_{0}^{z_e} (Z^*_p)'' Z'_j dz \right] \right\} \] (C-7)

\[ D_4(j,p,q) = \frac{1}{2}(1 - i) \left\{ T_1 \int_{0}^{z_e} Z^*_p Z_j dz + T_2 \left[ \int_{0}^{z_e} (Z^*_p)'(Z^*_p)' Z'_j dz + \frac{\gamma - \frac{1}{2}}{2} \int_{0}^{z_e} (Z^*_p)'' Z'_j dz \right] \right\} \] (C-8)

where

\[ T_1 = \frac{2\pi}{2} \int_{0}^{2\pi} \frac{1}{r} \int_{0}^{2\pi} R'R' R r dr d\theta \int_{0}^{2\pi} R'R' R r dr - \frac{\gamma - \frac{1}{2}}{2} S_{mn}^2(p) \times \]

\[ T_2 = \frac{2\pi}{2} \int_{0}^{2\pi} \frac{1}{r} \int_{0}^{2\pi} R'R R r dr d\theta \int_{0}^{2\pi} R'R R r dr \]
In the equations on the prior page the notation of Eq. (9) is used; that is, a single index (i.e., j, p, or q) is used to identify a particular series term rather than the mode numbers used in Eq. (6). The index j identifies the equations in which a given coefficient appears which corresponds to the weighting function used in deriving that equation. For the coefficients of the linear terms (i.e., the C's) the index p identifies the amplitude function which the coefficient multiplies. For coefficients of the nonlinear terms, (i.e., the D's) p identifies the factor which is not differentiated with respect to time, (i.e., $A_p$ or $A_p^*$), while q identifies the differentiated factor (i.e. $dA_p/dt$ or $dA_p^*/dt$). Due to the complex nature of the axial eigenfunctions, the above coefficients are complex numbers.

Structure of the Numerical Calculations

A flow chart for Program COEFFS3D is shown in Figure (C-1). The program can be divided into five major sections: (1) input, (2) calculation of the complex linear coefficients, (3) calculation of the complex nonlinear coefficients, (4) obtaining coefficients of the equivalent uncoupled real system, and (5) output.

The inputs to the program include the various parameters describing the chamber geometry, the nozzle boundary condition, the modes included in the approximating series expansion, and various control numbers, as well as the roots of the Bessel functions.

In the second section the axial acoustic eigenvalues are calculated by means of Subroutines EIGVAL and FCNS, and the integrals of the products of two axial eigenfunctions are computed by means of Subroutines AXIAL1 and UBAR. The integrals involving radial and tangential eigenfunctions are evaluated by using the orthogonality properties of these functions. The complex linear coefficients are then calculated according to Eqs. (C-1) through (C-4) and are normalized by dividing by $C_0(j, j)$. In the third section the integrals of products of three Bessel functions are calculated using Subroutines RADIAL and JBES, while similar integrals involving azimuthal eigenfunctions and axial eigenfunctions are computed using Subroutines AZIMTL and AXIAL2 respectively. The normalized complex nonlinear coefficients are obtained from Eqs. (C-5) through (C-8) by dividing by $C_0(j, j)$.

In the fourth section the normalized complex coefficients are used to
Figure C-1. Flow Chart for Program COEFS3D
obtain the coefficients for the equivalent system of real differential equations obtained by separating the real and imaginary parts of the complex equations. Since the axial eigenfunctions are non-orthogonal, the resulting system of equations may be coupled in the second derivative terms. Therefore, a matrix inversion procedure is used to obtain the coefficients of an equivalent system which is not coupled in the second derivatives.

In the last section the computed values of the coefficients are either printed out, punched onto cards, or stored on drum (FASTRAND file) as desired.

Input Data

The input data consists of the chamber parameters (i.e., ratio of specific heats, steady state Mach number, and length-to-diameter ratio), the nozzle admittance ratio, various control numbers, and information indicating which modes are included in the approximate series expansion. Regarding the latter information, each term in the series is identified by the integer variable J. The nature of each term is specified by the four integers L(J), M(J), N(J), and NS(J), and each term is given a four character name NAME(J). In this manner the coefficients are identified by the integers J associated with the modes involved rather than the corresponding axial, azimuthal, and radial mode numbers.

The following comments pertain to the detailed description of the input. The location number refers to columns of the card. Three formats are used for input: "A" indicates alphanumeric characters, "I" indicates integers, and "F" indicates real numbers with a decimal point. For the "I" and "F" formats the values are placed in fields of five and ten locations, respectively, and the numbers must be placed in the rightmost locations of the allocated field.

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<th>Location</th>
<th>Type</th>
<th>Input Item</th>
<th>Comments</th>
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<td>A</td>
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<td>Title of Case</td>
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<td>F</td>
<td>UE</td>
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<td>F</td>
<td>ZCOMB</td>
<td>Length of combustion zone, $z_c/z_e$.</td>
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<td>NDROPS</td>
<td></td>
<td>If 0: droplet momentum source neglected. If 1: droplet momentum source included.</td>
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<td>If 0: quasi-steady nozzle. If 1: conventional nozzle.</td>
</tr>
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<td>I</td>
<td>NONLIN</td>
<td>If 0: linear terms only. If 1: linear and nonlinear terms.</td>
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<td>I</td>
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<td>If 0: Nonzero coefficients calculated. If 1: Small coefficients neglected.</td>
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<td>16-20</td>
<td>I</td>
<td>NOUT</td>
<td>If 0: printed output only. If 1: printed and written into FASTRAND file. If 2: FASTRAND only. If 3: card output only.</td>
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If NEGL = 1:

1 1-10 F SML Linear coefficients with absolute value less than SML neglected.

11-20 F SM2 Nonlinear coefficients with absolute value less than SM2 neglected.

End of input for NEGL = 1.

If NOZZLE = 1:

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<td>F</td>
<td>AMPL(J)</td>
<td>Amplitude factor of nozzle admittance, A.</td>
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<td>16-25</td>
<td>F</td>
<td>PHASE(J)</td>
<td>Phase of nozzle admittance, ( \phi ).</td>
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</tbody>
</table>

End of Input for NOZZLE = 1.

<table>
<thead>
<tr>
<th>NJMAX</th>
<th>1-5</th>
<th>I</th>
<th>J</th>
<th>Integer which identifies series term.</th>
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<tr>
<td></td>
<td>6-10</td>
<td>I</td>
<td>L(J)</td>
<td>Axial mode number, ( l ). (0 ≤ L(J) ≤ 10)</td>
</tr>
</tbody>
</table>
The first card gives a title (maximum 72 characters) used to identify the run. The second card gives the chamber parameters (i.e., \( \gamma, \bar{u}_e, L/D, z_c \)), determines whether the droplet momentum source is included in the analysis (see Appendix A), and specifies the type of nozzle (quasi-steady or conventional). If a quasi-steady nozzle is specified the nozzle admittance is calculated using Eqs. (14), and no further information concerning the nozzle is required. The control numbers are given on the third card. Due to computer storage limitations the series expansion is limited to ten terms, thus \( NJMAX \leq 10 \). The control number \( NEGL \) gives the option to neglect all coefficients with absolute value smaller than a given number, thus allowing a considerable saving in computation time when the equations are numerically integrated by Program LCYC3D. It has been found that neglecting coefficients with absolute value smaller than 0.1 (i.e., \( SML = SM2 = 0.1 \)) reduces the computation time by half and has a negligible effect on the resulting solutions. For conventional nozzles a series of \( NJMAX \) cards is read which gives the nozzle admittance (amplitude and phase) for each term in the series. This is followed by another series of \( NJMAX \) cards giving the mode numbers for each series term.

The proper input for program COEFFS3D will be illustrated with the following example. Suppose the velocity potential \( \phi \) is expressed in terms of the first tangential (1T), the second tangential (2T), and the first radial (1R) modes. It is also desired to investigate instability of the spinning type, therefore both \( \sin(m\theta) \) and \( \cos(m\theta) \) terms are included in the series. However, for the 1R mode (\( m=0 \)) there is no corresponding \( \sin(m\theta) \) term, therefore the resulting series will contain five terms. A nozzle admittance of \( A = 0.02 \) and \( \varphi = 45^\circ \) will be assumed for each term in the series, and coefficients smaller than 0.1 as well as the droplet momentum source will be neglected.
The output data will be punched on cards. A sample input for this case is given in Table (C-1) below.

Table C-1. Sample Input.

| S 6 7 T 2 4 | 5 1 3 1 | 0.1 | 1.0 0.1
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0.02 45.0</td>
<td>2 0.02 45.0</td>
<td>3 0.02 45.0</td>
<td>4 0.02 45.0</td>
</tr>
</tbody>
</table>
| 5 0.02 45.0 | 1 0.11 1 1 | 2 0.11 1 | 3 0.21 1
| 4 0.21 2 | 5 0.01 1 2 0.01 |

After the last card in the sequence described above is read, the program is executed and control returns to the input section. Thus, several cases can be executed on the same run. If no further cards are given the run is terminated.

In addition to the above card input, roots of the Bessel functions \( S_{mn} \) which give zero slope at \( r = 1 \) and the associated values \( J_m(S_{mn}) \) are needed for these calculations. These values were taken from Ref. (16) for \( m = 0, 1, \ldots 8 \) and \( n = 1, 2, \ldots 5 \); they are automatically put into the program by means of a DATA statement, which is an integral part of the program.
Complex Linear Coefficients

For NDROPS = 0 the complex linear coefficients are computed from Eqs. (C-1) through (C-4) and are stored in the complex array CC(KC,NJ,NP). For NDROPS = 1 the coefficients C2(j,p) are computed from Eq. (A-8).

In order to calculate these coefficients the following information is needed: (1) the axial acoustic eigenvalues, \( \beta_{mn} \), (2) the steady state Mach number distribution, \( \tilde{u}(z) \), (3) the orthogonality properties of the transverse eigenfunctions, and (4) the integrals of products of two axial eigenfunctions. The calculation of these quantities is described below.

Axial Acoustic Eigenvalues. The axial acoustic eigenvalues are determined by numerically solving the transcendental equation given by Eq. (8). This is done by first substituting \( \beta_{mn} = \epsilon_{mn} + iY_{mn} \) and \( Y = Y_r + iY_i \) into Eq. (8) and separating real and imaginary parts. This yields a pair of simultaneous equations of the form:

\[
\begin{align*}
f(\epsilon, \eta) &= 0 \quad (C-9) \\
g(\epsilon, \eta) &= 0
\end{align*}
\]

where

\[
f(\epsilon, \eta) = (\epsilon^2 - \eta^2)F(\epsilon, \eta) - 4\epsilon\eta H(\epsilon, \eta)
\]

\[
\begin{align*}
&+ \gamma^2\left\{(Y_r^2 - Y_i^2)(s_{mn}^2 + \epsilon^2 - \eta^2) - 4Y_rY_i\epsilon\eta\right\}G(\epsilon, \eta) \\
&+ 4\left[Y_rY_i(s_{mn}^2 + \epsilon^2 - \eta^2) + (Y_r^2 - Y_i^2)\epsilon\eta\right]H(\epsilon, \eta) \quad (C-10)
\end{align*}
\]
\[
g(\varepsilon, \eta) = (\varepsilon^2 - \eta^2)H(\varepsilon, \eta) + \varepsilon \eta F(\varepsilon, \eta) + \gamma^2 \left[ \left( \sum_{mn}^2 s^2_{mn} + \varepsilon^2 - \eta^2 \right) + (\eta^2 - \varepsilon^2) \varepsilon \eta \right] G(\varepsilon, \eta)
\]

\[
H(\varepsilon, \eta) = \sin(\varepsilon \eta) \cos(\varepsilon \eta) \sinh(\eta \varepsilon) \cosh(\eta \varepsilon)
\]

and

\[
F(\varepsilon, \eta) = \sin^2(\varepsilon \eta) \cosh^2(\eta \varepsilon) - \cos^2(\varepsilon \eta) \sinh^2(\eta \varepsilon)
\]

\[
G(\varepsilon, \eta) = \cos^2(\varepsilon \eta) \cosh^2(\eta \varepsilon) - \sin^2(\varepsilon \eta) \sinh^2(\eta \varepsilon)
\]

Equations (C-9) are solved by Subroutine EIGVAL using Newton’s Method for two unknowns.\(^{17}\) In this method successive approximations to the roots are generated by the recursion formulas:

\[
\varepsilon_{i+1} = \varepsilon_i - \left[ \frac{f g_i - g f_i}{J(f, g)} \right]_i
\]

\[
\eta_{i+1} = \eta_i - \left[ \frac{g f_i - f g_i}{J(f, g)} \right]_i
\]

where the Jacobian \(J(f, g)\) is given by:

\[
J(f, g) = f_i g_i \eta - g_i f_i \varepsilon
\]
and the subscripts indicate partial differentiation with respect to \( e \) and \( \eta \). The quantities \( f, g, f_e, f_\eta, g_e, g_\eta \) are calculated by the Subroutine FCNS. The iteration is started by assuming the following values for \( e \) and \( \eta \):

\[
\begin{align*}
\varepsilon_0 &= \varepsilon_m + a \cos(\beta) \\
\eta_0 &= a \sin(\beta)
\end{align*}
\]

where for \( \ell = 0 \):

\[
\begin{align*}
\varepsilon_m &= 0 \\
a &= 10A/z_e \\
\beta &= \Phi/2 + 45 \text{ (degrees)}
\end{align*}
\]

and for \( \ell \neq 0 \):

\[
\begin{align*}
\varepsilon_m &= \ell\pi/z_e \\
a &= A/z_e \\
\beta &= \Phi + 90 \text{ (degrees)}
\end{align*}
\]

The iteration is terminated when the errors \( \Delta e \) and \( \Delta \eta \) are smaller than \( 10^{-7} \).

If the iteration fails to converge after 40 iterations or the Jacobian vanishes a warning message is printed. FORTRAN listings of Subroutines EIGVAL and FCNS are given at the end of this appendix.

**Steady State Mach Number Distribution.** The steady state Mach number distribution is calculated by means of Subroutine UBAR which must be supplied by the user. This distribution must be of the form shown in Fig. (C-2) where the Mach number varies from zero at the injector face \( (z = 0) \) to its maximum value at the end of the combustion zone \( (z = z_c) \) and remains constant until the nozzle entrance \( (z = z_e) \) is reached. Thus the Mach number is given by

\[
\bar{u}(z) = U(z)\bar{u}_e \quad (0 \leq z \leq z_c)
\]
\[ \ddot{u}(z) = \ddot{u}_e \quad (z_c \leq z \leq z_e) \]

where \( U(0) = 0 \) and \( U(z_e) = 1 \). Although the function \( U(z) \) may be arbitrary, the results presented in this report were obtained using a linear Mach number distribution in the combustion zone (i.e., uniformly distributed combustion). Thus the function \( U(z) \) in the listing of UBAR provided herein is given by:

\[ U(z) = \frac{z}{z_c} \quad (C-18) \]

In addition to the Mach number distribution (\( NOPT = 1 \)), the first (\( NOPT = 2 \)) and second (\( NOPT = 3 \)) derivatives are also calculated.

Figure C-2. Steady-State Mach Number Distribution.
Orthogonality of Transverse Eigenfunctions. The tangential eigenfunctions have the following orthogonality properties:

\[ \int_0^{2\pi} \sin(m_p \theta) \sin(m_j \theta) d\theta = \int_0^{2\pi} \cos(m_p \theta) \cos(m_j \theta) d\theta = 0 \quad m_p \neq m_j \]

\[ = \pi \quad m_p = m_j \neq 0 \]  

\[ \int_0^{2\pi} \cos(m_p \theta) \cos(m_j \theta) d\theta = 2\pi \quad m_p = m_j = 0 \]

\[ \int_0^{2\pi} \sin(m_p \theta) \cos(m_j \theta) d\theta = 0 \quad \text{for all } m_p \text{ and } m_j \]  

For the special case of \( m_p = m_j = 0 \) the integral involving sines vanishes. The orthogonality property of the radial eigenfunctions is given by:

\[ \int_0^1 R_p R_j r dr = 0 \quad n_p \neq n_j \quad (m_p = m_j) \]

\[ \int_0^1 R_p R_j r dr = \frac{S_{mn}^2 - m^2}{2S_{mn}^2} \left[ J_m(s_{mn}) \right]^2 \quad n_p = n_j \quad (m_p = m_j) \]  

Since the tangential integrals vanish when \( m_p \neq m_j \), it is not necessary to calculate the radial integrals for \( m_p \neq m_j \). These orthogonality properties are used to calculate the integrals, \( \int_0^{2\pi} \theta \Theta d\theta \) and \( \int_0^1 R_p R_j r dr \), which appear in Eqs. (C-1) through (C-4). For a series containing pure transverse modes only (\( t = 0 \)), it is easily seen that all of the linear coefficients vanish except those corresponding to \( p = j \), yielding a system of equations which are not coupled in the linear terms.

Axial Integrals. The integrals of products of two axial eigenfunctions are calculated by Subroutine AXIALI. According to the value of the input parameter NOPT these integrals are calculated as follows:
The last two integrals, which involve the mean flow Mach number, are evaluated by means of Simpson's Rule. A FORTRAN listing of AXIAL1 is provided at the end of this appendix.

Complex Nonlinear Coefficients.

The complex nonlinear coefficients are calculated from Eqs. (C-5) through (C-8) and are stored in the complex arrays, CD1(NJ, NP, NQ), CD2(NJ, NP, NQ), CD3(NJ, NP, NQ), and CD4(NJ, NP, NQ).

In order to calculate these coefficients, the various integrals of axial, azimuthal, and radial eigenfunctions must be evaluated. Since many of the azimuthal integrals are zero they are evaluated first, and the remaining integrals are computed only if the corresponding azimuthal integral is nonzero. The subroutines used to calculate these integrals are described in the following paragraphs.

Azimuthal Integrals. The azimuthal integrals are calculated by Subroutine AZIMTL according to the value of NOPT as follows:

\[ \text{NOPT} = 1: \quad \int_0^{2\pi} \sum_{p,q} w_{p,q,j} d\theta \]
These integrals are easily evaluated analytically; for most values of $p$, $q$, and $j$ they are zero. The nonzero integrals are readily expressed in terms of the following integrals:

\[
\int_0^{2\pi} \cos(m_p \theta) \cos(m_q \theta) \cos(m_j \theta) d\theta = \frac{\pi}{2} \quad \text{for} \quad m_j = m_p + m_q,
\]

\[
m_p = m_j + m_q, \quad \text{or} \quad m_q = m_j + m_p \tag{C-23}
\]

\[
\int_0^{2\pi} \cos(m_p \theta) \sin(m_q \theta) \sin(m_j \theta) d\theta = \frac{\pi}{2} \quad \text{for} \quad m_q = m_p + m_j \quad \text{or} \quad m_j = m_p + m_q \tag{C-24}
\]

\[
\int_0^{2\pi} \cos(m_p \theta) \sin(m_q \theta) \sin(m_j \theta) d\theta = -\frac{\pi}{2} \quad \text{for} \quad m_p = m_q + m_j \tag{C-25}
\]

where $m_p$, $m_q$, and $m_j$ are nonzero. If any one of the tangential mode numbers is zero (corresponding to a radial mode) the following values are obtained:

\[
\int_0^{2\pi} \cos(m_p \theta) \cos(m_q \theta) \cos(m_j \theta) d\theta = 2\pi \quad \text{for} \quad m_p = m_q = m_j = 0
\]

\[
\quad = \pi \quad \text{for} \quad m_p = 0, \quad m_q = m_j;
\]

\[
\quad m_q = 0, \quad m_p = m_j;
\]

\[
\quad m_j = 0, \quad m_p = m_q \tag{C-26}
\]
Subroutine AZIMTL consists of two sections. In the first section the azimuthal integral is expressed as the product of a constant factor and one of the basic forms given in Eqs. (C-23) and (C-24). The second section is essentially a series of logical tests to determine if the mode numbers, \( m_p \), \( m_q \), and \( m_j \) satisfy any of the conditions for Eqs. (C-23) through (C-27). If any of these conditions is satisfied the appropriate value is multiplied by the corresponding factor determined in the first section and the product is assigned to the output variable (i.e., RESULT), otherwise the value zero is assigned.

Radial Integrals. Subroutine RADIAL calculates the radial integrals which appear in Eqs. (C-5) through (C-8) according to NOPT as follows:

\[
\begin{align*}
\text{NOPT} = 1 & : \int_0^1 R \, R \, R \, r dr \\
\text{NOPT} = 2 & : \int_0^1 R \, R \, R \, r dr \\
\text{NOPT} = 3 & : \int_0^1 R \, R \, R \, r dr
\end{align*}
\]

where the R's are the Bessel functions, \( J_{m_{mn}}(r) \). These integrals are computed numerically using Simpson's Rule with 100 subdivisions. In calculating the integrands the derivatives of the Bessel functions are given by:

\[
J_m'(S_{mn}r) = \frac{1}{2} \left[ J_{m-1}(S_{mn}r) - J_{m+1}(S_{mn}r) \right] \quad \text{for} \quad m = 1, 2, 3, \ldots
\]

\[
J_0'(S_{mn}r) = -J_1(S_{mn}r)
\]
The integrand of the second integral (NOPT = 2) is indeterminate at the lower limit of integration. However a limit exists, denoted by L, which vanishes with the following exceptions:

\[ L = S_{mn}(p)/2 \text{ for } m_p = 1, m_q = m_j = 0 \]

\[ L = S_{mn}(q)/2 \text{ for } m_q = 1, m_p = m_j = 0 \]

\[ L = S_{mn}(j)/2 \text{ for } m_j = 1, m_p = m_q = 0 \]

All of the calculations in Subroutine RADIAL are carried out in double precision arithmetic. The results are given as a single precision number.

Subroutine JBES computes the double precision Bessel functions which are needed for the above calculations. A description of this subroutine and a program listing are given in Chapter 23 of Ref. (18).

Axial Integrals. The integrals of the products of three axial eigenfunctions (see Eqs. (C-5) through (C-8)) are computed by Subroutine AXIAL2 according to the input parameters NOPT and NCONJ. The three basic forms are specified by NOPT as follows:

\[
\text{NOPT } = 1 : \quad \int_0^Z \frac{Z^e Z_p Z_q Z_j^*}{Z_p Z_q Z_j^*} \, dz
\]

\[
\text{NOPT } = 2 : \quad \int_0^Z \frac{Z^e Z_p Z_q Z_j^*}{Z_p Z_q Z_j^*} \, dz
\]

\[
\text{NOPT } = 3 : \quad \int_0^Z \frac{Z^e Z_p Z_q Z_j^*}{Z_p Z_q Z_j^*} \, dz
\]

When NCONJ = 1 these basic forms are calculated; these are the forms appearing in the expression for \( D_1(j, p, q) \) (see Eq. (C-5)). For NCONJ = 2 the second function in the integrand is replaced by its complex conjugate to obtain the
integrals appearing in the expression for $D_2(j,p,q)$. The integrals appearing in the expressions for $D_3(j,p,q)$ and $D_4(j,p,q)$ are obtained by setting $NCONJ = 3$ and $NCONJ = 4$ respectively.

The basic forms are calculated from the following analytical formulas:

\[
\int_0^{\frac{e}{p}} Z_{p}^{\ast} Z_{q}^{\ast} Z_{j}^{\ast} dz = \frac{1}{4} \left\{ \frac{\sinh \left[ i(b_p + b_q + b_j^*) z_e \right]}{i(b_p + b_q + b_j^*)} + \frac{\sinh \left[ i(b_p + b_q - b_j^*) z_e \right]}{i(b_p + b_q - b_j^*)} + \frac{\sinh \left[ i(b_p - b_q + b_j^*) z_e \right]}{i(b_p - b_q + b_j^*)} + \frac{\sinh \left[ i(b_p - b_q - b_j^*) z_e \right]}{i(b_p - b_q - b_j^*)} \right\} \quad (C-30)
\]

\[
\int_0^{\frac{e}{p}} Z_{p}^{\ast} Z_{q}^{\ast} Z_{j}^{\ast} dz = -\frac{1}{4} b_p b_q \left\{ \frac{\sinh \left[ i(b_p + b_q + b_j^*) z_e \right]}{i(b_p + b_q + b_j^*)} + \frac{\sinh \left[ i(b_p + b_q - b_j^*) z_e \right]}{i(b_p + b_q - b_j^*)} - \frac{\sinh \left[ i(b_p - b_q + b_j^*) z_e \right]}{i(b_p - b_q + b_j^*)} - \frac{\sinh \left[ i(b_p - b_q - b_j^*) z_e \right]}{i(b_p - b_q - b_j^*)} \right\} \quad (C-31)
\]

\[
\int_0^{\frac{e}{p}} Z_{p}^{\ast} Z_{q}^{\ast} Z_{j}^{\ast} dz = -b_p^2 \int_0^{\frac{e}{p}} Z_{p}^{\ast} Z_{q}^{\ast} Z_{j}^{\ast} dz \quad (C-32)
\]
The remaining forms are obtained from Eqs. (C-30) through (C-32) by replacing
the appropriate eigenvalues with their complex conjugates; thus, for NOPT = 2
b_q is replaced by b_q^* for NOPT = 3 b_p is replaced with b_p^*, and both b_p and
b_q are replaced by their conjugates for NOPT = 4.

FORTRAN listings for Subroutines AZIMTL, RADIAL, and AXIAL2 are given at
the end of this appendix.

Coefficients for Equivalent Real System.

Equations (12) are a system of complex differential equations to be
solved for the unknown complex amplitude functions, A_p(t). In order to solve
these equations numerically they must first be separated into their real and
imaginary parts. This is done by assuming that A_p(t) = F_p(t) + iG_p(t), sub-
stituting into Eqs. (12), and separating real and imaginary parts to obtain
an equivalent system of real differential equations that describe the behavior
of the F_p's and G_p's. Since these equations contain twice as many unknown
functions (i.e., F_p(t) and G_p(t)) as Eqs. (12), it is convenient to re-index
the unknown functions and their coefficients as follows:

\[
F_p(t) = B_{2p-1}(t)
\]

\[
G_p(t) = B_{2p}(t)
\]

(c-33)

Thus the B's with odd indices correspond to the real parts, F_p(t), and the
B's with even indices correspond to the imaginary parts, G_p(t). The corre-
responding set of differential equations is given by:

\[
\sum_{p=1}^{2N} \left[ c_0'(j,p) \frac{d^2B_p}{dt^2} + c_1'(j,p)B_p(t) + \left[ c_2'(j,p) - nc_3'(j,p) \right] \frac{dB_p}{dt} \right] +
\]

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The real coefficients in Eqs. (C-34) (i.e., \(C_0',C_1',C_2',C_3',\) and \(D'\)) are related to the complex coefficients in Eqs. (12) (i.e., \(C_0,...C_3, D_1,...D_4\)) as follows:

\[
\begin{align*}
C_k'(2j-1, 2p-1) &= \text{Re} \left[ C_k(j,p) \right] \\
C_k'(2j-1, 2p) &= -\text{Im} \left[ C_k(j,p) \right] \\
C_k'(2j, 2p-1) &= \text{Im} \left[ C_k(j,p) \right] \\
C_k'(2j, 2p) &= \text{Re} \left[ C_k(j,p) \right]
\end{align*}
\]  

for \(k = 0,1,2,3,\) \(j = 1,2,...N,\) \(p = 1,2,...N\) and:

\[
\begin{align*}
D'(2j-1,2p-1,2q-1) &= \text{Re} \left[ D_1(j,p,q) + D_2(j,p,q) + D_3(j,p,q) + D_4(j,p,q) \right] \\
D'(2j-1,2p-1,2q) &= \text{Im} \left[ -D_1(j,p,q) + D_2(j,p,q) - D_3(j,p,q) + D_4(j,p,q) \right] \\
D'(2j-1,2p,2q-1) &= \text{Im} \left[ -D_1(j,p,q) - D_2(j,p,q) + D_3(j,p,q) + D_4(j,p,q) \right] \\
D'(2j-1,2p,2q) &= \text{Re} \left[ -D_1(j,p,q) - D_2(j,p,q) + D_3(j,p,q) - D_4(j,p,q) \right]
\end{align*}
\]  

(C-36)
\( D'(2j,2p-1,2q-1) = \text{Im} \left[ D_1(j,p,q) + D_2(j,p,q) + D_3(j,p,q) + D_4(j,p,q) \right] \)
\( D'(2j,2p-1,2q) = \text{Re} \left[ D_1(j,p,q) - D_2(j,p,q) + D_3(j,p,q) - D_4(j,p,q) \right] \)
\( D'(2j,2p,2q-1) = \text{Re} \left[ D_1(j,p,q) + D_2(j,p,q) - D_3(j,p,q) - D_4(j,p,q) \right] \)
\( D'(2j,2p,2q) = \text{Im} \left[ -D_1(j,p,q) + D_2(j,p,q) + D_3(j,p,q) - D_4(j,p,q) \right] \)

for \( j = 1,2,...N, \ p = 1,2,...N, \ q = 1,2,...N \). The linear coefficients are stored in the arrays \( C_1(NJ, NP) \) for \( k = 0 \) and \( C(KC, NJ, NP) \) for \( k = 1,2,3 \). The nonlinear coefficients are stored in the array \( D(NJ, NP, NQ) \).

In general Eqs. (C-34) are coupled in the second derivatives; that is, they are of the form:

\[
\sum_{p=1}^{2N} \left\{ C'_0(j,p) \frac{d^2B}{dt^2} \right\} = g_j(B_1, B_2, ..., B_{2N}) \quad \text{(C-37)}
\]

where there are two or more \( C'_0 \) terms in each equation. This coupling results from the non-orthogonality of the axial eigenfunctions. In order to numerically integrate Eqs. (C-34), they must be decoupled by transforming to the form:

\[
\frac{d^2B_j}{dt^2} = f_j(B_1, B_2, ..., B_{2N}) \quad \text{(C-38)}
\]

in which only one second derivative appears in each equation. Using Eq. (C-38), it is seen that Eq. (C-37) can be expressed as

\[
C'_0 f = g \quad \text{(C-39)}
\]

where \( C'_0 \) is the \( 2N \times 2N \) matrix of coefficients of the coupled system, \( f \) is...
the column matrix corresponding to the right-hand-side of the decoupled
system, and \( g \) is the column matrix corresponding to the right-hand-side of
the coupled system. To decouple Eqs. (C-37), therefore, Eq. (C-39) is solved
for \( f \), thus:

\[
f = C_0^{-1} g
\]  

(C-40)

where \( C_0^{-1} \) is the inverse of the matrix \( C_0 \). Performing these operations and
equating the coefficients of like terms in \( f \) and \( C_0^{-1} g \) gives the following
relations:

\[
\begin{align*}
\tilde{c}_i(j,p) &= \sum_{k=1}^{2N} C_0^{-1}(j,k)c'_i(k,p) \quad i = 1,2,3 \\
\tilde{d}(j,p,q) &= \sum_{k=1}^{2N} C_0^{-1}(j,k)d'(k,p,q)
\end{align*}
\]  

(C-41)

where \( \tilde{c}_i \) and \( \tilde{d} \) are the corresponding coefficients of the decoupled system.
The matrix inverse, \( C_0^{-1} \), is computed by the subroutine GJR, which is a
standard Univac 1108 library program, and is stored in the array \( C_1(NJ,NP) \).
A listing of GJR and instructions for its use are given in Ref. (19).

The calculation of \( \tilde{c}_i(j,p) \) and \( \tilde{d}(j,p,q) \), which are the coefficients for
the equivalent set of real, decoupled equations, is the final step in the
computations performed by COEFFS3D. The coefficients are stored in the arrays
\( C(KC,NJ,NP) \) and \( D(NJ,NP,NQ) \), replacing those computed from Eqs. (C-35) and
(C-36). The output of these coefficients is described below.

Output

According to the value of the control number NOUT, the coefficients
calculated by Program COEFFS3D are printed, punched onto cards, or stored on
drum (FASTRAND). These three output modes will now be discussed individu-
ally.

Printed Output. Since the printed output cannot be used as input to
Program LCYC3D, the option "printed output only" (NOUT = O) is only used for checkout purposes. Printed output can also be obtained in conjunction with the drum storage mode (NOUT = 1). Since the printed output format can only accommodate five series terms (complex), it should only be used for NJMAX ≤ 5.

The first page of printed output gives a restatement of the input parameters. This page is headed by the title of the case (TITLE) which is followed by the ratio of specific heats (GAMMA), the steady state Mach number at the nozzle entrance (UE), the length-to-diameter ratio (L/D), and the length of the combustion zone as a fraction of the chamber length (ZCOMB). After statements concerning the presence or absence of the liquid droplet momentum source and the type of nozzle considered, a restatement of the input parameters J, L(J), M(J), N(J), NS(J), and NAME(J) which describe the terms in the series expansion of φ is given. This tabulation also includes additional parameters needed by Program LCYC3D: Smn, the dimensionless frequency of the mode (SMN); J_m(Smn), the associated value of the Bessel function (JM(SMN)); the real part (EPS) and the imaginary part (ETA) of the axial acoustic eigenvalue; and the real part (YR) and imaginary part (YI) of the nozzle admittance.

The next three pages give the decoupled linear coefficients, C1(j,p), C2(j,p), and C3(j,p). These coefficients are presented in the matrix format with the rows corresponding to the index j and the columns corresponding to the index p. The remaining pages give the decoupled nonlinear coefficients D(j,p,q) for each value of j. Here the rows correspond to the index p and the columns correspond to the index q.

A sample printed output for the five term series used in the sample input is given in Tables (C-2) through (C-4).

Drum Storage. When available drum storage, such as the FASTRAND system used with the Univac 1108, is the most convenient means of storing the output of Program COEFFS3D. In the absence of such a system, the program can be easily modified to store the coefficients on magnetic tape. In either case magnetic tape can be used as a back-up file or for permanent storage of the data. The control statements needed to execute these procedures depend upon the computer facilities being used and cannot be described in
Table C-2. Sample Printed Output, Page 1.

**IT,2T,IR SPINNING.**

**GAMMA = 1.20   UE = .20   L/D = .50000   ZCOMB = 1.00**

**DROPLET MOMENTUM SOURCE NEGLECTED**

<table>
<thead>
<tr>
<th>NAME</th>
<th>J</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>NS</th>
<th>SNN</th>
<th>JM(SNN)</th>
<th>EPS</th>
<th>ETA</th>
<th>YR</th>
<th>YI</th>
</tr>
</thead>
<tbody>
<tr>
<td>A011</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.84118</td>
<td>.58187</td>
<td>.08122</td>
<td>.19451</td>
<td>.01414</td>
<td>.01414</td>
</tr>
<tr>
<td>B011</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1.84118</td>
<td>.58187</td>
<td>.08122</td>
<td>.19451</td>
<td>.01414</td>
<td>.01414</td>
</tr>
<tr>
<td>A021</td>
<td>3</td>
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<td>2</td>
<td>1</td>
<td>1</td>
<td>3.05424</td>
<td>.48650</td>
<td>.10617</td>
<td>.25115</td>
<td>.01414</td>
<td>.01414</td>
</tr>
<tr>
<td>B021</td>
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<td>2</td>
<td>1</td>
<td>2</td>
<td>3.05424</td>
<td>.48650</td>
<td>.10617</td>
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<td>.01414</td>
<td>.01414</td>
</tr>
<tr>
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<td>0</td>
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<td>2</td>
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<td>.01414</td>
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Table C-3. Sample Printed Output, Page 2.

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<th>4</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
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<tr>
<td>2</td>
<td></td>
<td>-.000000</td>
<td>3.390599</td>
<td>.000000</td>
<td>.000000</td>
<td>.000000</td>
<td>.000000</td>
<td>.000000</td>
<td>.000000</td>
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<td>.000000</td>
<td>14.684905</td>
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</tbody>
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### Table C-4. Sample Printed Output, Page 5.

<table>
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<th>2</th>
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<th>4</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9.46170</td>
<td>-0.02679</td>
<td>4.46570</td>
<td>-0.15812</td>
<td>1.235038</td>
<td>-0.00450</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
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<tr>
<td>B</td>
<td>9.29412</td>
<td>1.235038</td>
<td>-0.00450</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
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<td>C</td>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
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<td>0.00000</td>
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<td>0.00000</td>
<td>0.00000</td>
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<td>0.00000</td>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
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<td>I</td>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
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<tr>
<td>J</td>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>
Card Output. When a drum or magnetic tape storage is not available, punched card output can be used (NOUT = 3). This method becomes unwieldy, however, when a large number of coefficients is involved since only one coefficient can be punched on a card. The format for both drum and card output is the same and is given below:

<table>
<thead>
<tr>
<th>Number of Cards</th>
<th>Location</th>
<th>Type</th>
<th>Output Item</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-10</td>
<td>F</td>
<td>GAMMA</td>
<td>Same as for input.</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>F</td>
<td>UE</td>
<td>Same as for input.</td>
</tr>
<tr>
<td></td>
<td>21-30</td>
<td>F</td>
<td>ZE</td>
<td>Dimensionless chamber length, ((2\ell/D)).</td>
</tr>
<tr>
<td></td>
<td>31-40</td>
<td>F</td>
<td>ZCOMB</td>
<td>Same as for input.</td>
</tr>
<tr>
<td>41-45</td>
<td>I</td>
<td></td>
<td>NDROPS</td>
<td>Same as for input.</td>
</tr>
<tr>
<td>46-50</td>
<td>I</td>
<td></td>
<td>NJMAX</td>
<td>Number of unknown functions, (B_p(t)) (see Eq. (C-34)).</td>
</tr>
</tbody>
</table>

\[ \text{NJMAX/2} \]

<table>
<thead>
<tr>
<th>Location</th>
<th>Type</th>
<th>Output Item</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>I</td>
<td>J</td>
<td>Same as input.</td>
</tr>
<tr>
<td>6-10</td>
<td>I</td>
<td>L(J)</td>
<td>&quot;</td>
</tr>
<tr>
<td>11-15</td>
<td>I</td>
<td>M(J)</td>
<td>&quot;</td>
</tr>
<tr>
<td>16-20</td>
<td>I</td>
<td>N(J)</td>
<td>&quot;</td>
</tr>
<tr>
<td>21-25</td>
<td>I</td>
<td>NS(J)</td>
<td>&quot;</td>
</tr>
<tr>
<td>26-35</td>
<td>F</td>
<td>S(J)</td>
<td>Root of Bessel function, (S_{mn}).</td>
</tr>
<tr>
<td>36-45</td>
<td>F</td>
<td>SJ(J)</td>
<td>Associated value of Bessel function, (J_m(S_{mn})).</td>
</tr>
<tr>
<td>46-50</td>
<td>A</td>
<td>NAME(J)</td>
<td>Same as input.</td>
</tr>
</tbody>
</table>

\[ \text{NJMAX/2} \]

<table>
<thead>
<tr>
<th>Location</th>
<th>Type</th>
<th>Output Item</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>I</td>
<td>J</td>
<td>Same as input.</td>
</tr>
<tr>
<td>6-15</td>
<td>F</td>
<td>YR</td>
<td>Real part of nozzle admittance, (Y_r).</td>
</tr>
<tr>
<td>16-25</td>
<td>F</td>
<td>YI</td>
<td>Imaginary part of nozzle admittance, (Y_i).</td>
</tr>
<tr>
<td>26-35</td>
<td>F</td>
<td>EPS</td>
<td>Real part of axial eigenvalue, (\varepsilon).</td>
</tr>
<tr>
<td>36-45</td>
<td>F</td>
<td>ETA</td>
<td>Imaginary part of axial eigenvalue, (\eta).</td>
</tr>
<tr>
<td>Number of Cards</td>
<td>Location</td>
<td>Type</td>
<td>Output Item</td>
</tr>
<tr>
<td>----------------</td>
<td>----------</td>
<td>------</td>
<td>-------------</td>
</tr>
<tr>
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<td>1-5</td>
<td>I</td>
<td>KMAX(1)</td>
</tr>
<tr>
<td>KMAX(1)</td>
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<td>I</td>
<td>NJ</td>
</tr>
<tr>
<td>6-10</td>
<td>I</td>
<td>NP</td>
<td></td>
</tr>
<tr>
<td>11-25</td>
<td>F</td>
<td>C(1,NJ,NP)</td>
<td>Linear coefficient, ( \tilde{C}_1(j,p) ).</td>
</tr>
<tr>
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<td>1-5</td>
<td>I</td>
<td>KMAX(2)</td>
</tr>
<tr>
<td>KMAX(2)</td>
<td>1-5</td>
<td>I</td>
<td>NJ</td>
</tr>
<tr>
<td>6-10</td>
<td>I</td>
<td>NP</td>
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<tr>
<td>11-25</td>
<td>F</td>
<td>C(2,NJ,NP)</td>
<td>Linear coefficient, ( \tilde{C}_2(j,p) ).</td>
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<tr>
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<td>KMAX(3)</td>
</tr>
<tr>
<td>KMAX(3)</td>
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<td>I</td>
<td>NJ</td>
</tr>
<tr>
<td>6-10</td>
<td>I</td>
<td>NP</td>
<td></td>
</tr>
<tr>
<td>11-25</td>
<td>F</td>
<td>C(3,NJ,NP)</td>
<td>Linear coefficient, ( \tilde{C}_3(j,p) ).</td>
</tr>
<tr>
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<td>1-5</td>
<td>I</td>
<td>KMAX(4)</td>
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<tr>
<td>KMAX(4)</td>
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<td>NJ</td>
</tr>
<tr>
<td>6-10</td>
<td>I</td>
<td>NP</td>
<td></td>
</tr>
<tr>
<td>11-15</td>
<td>I</td>
<td>NQ</td>
<td></td>
</tr>
<tr>
<td>16-30</td>
<td>F</td>
<td>D(NJ,NP,NQ)</td>
<td>Nonlinear coefficient, ( D(j,p,q) ).</td>
</tr>
</tbody>
</table>

The first card of output gives the chamber parameters \( \gamma, \bar{u}_e, L/D, \) and \( z_c/z_e \); the droplet momentum source control number, NDROPS; and the number of unknown real functions (i.e., \( B_p(t) \)), NJMAX. This is followed by NJMAX/2 cards (the number of unknown complex functions, \( A_p(t) \)) describing the terms included in the series expansion of \( \dot{\phi} \). The next NJMAX/2 cards gives the complex nozzle admittance (\( Y_r \) and \( Y_i \)) and the corresponding complex axial eigenvalue (\( \epsilon \) and \( \eta \)) for each complex series term. The linear coefficients are given in three sets of cards. The first card in the set gives the number of coefficients of the given type, while the remaining
cards give the indices $j$ and $p$ and the coefficient $\tilde{c}_i(j,p)$. The next card
gives the number of nonlinear coefficients and is followed by cards giving
the indices $j$, $p$, $q$ and the corresponding coefficient $D(j,p,q)$. Both linear
and nonlinear coefficients are given in a field of 15 spaces with six decimal
places. For $\text{NEGL} = 0$ only the nonzero coefficients (absolute value
greater than $10^{-5}$) are given, while for $\text{NEGL} = 1$ only linear coefficients
with absolute value greater than $\text{SM1}$ and nonlinear coefficients with abso-
lute value greater than $\text{SM2}$ are given.

A sample card output produced by the sample input of Table (C-1) is
given in Table (C-5) below.

**Table C-5. Sample Card Output.**

<p>| | | | | | | | | | |</p>
<table>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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69
FORTRAN Listing.

***************PROGRAM COEFFS3D***************

THIS PROGRAM COMPUTES THE COEFFICIENTS WHICH APPEAR
IN THE DIFFERENTIAL EQUATIONS WHICH GOVERN THE MODE-AMPLITUDE
FUNCTIONS. THESE COEFFICIENTS ARE STORED ON DRUM OR
PUNCHED ONTO CARDS FOR INPUT INTO PROGRAM LCYC3D.

THE FOLLOWING INPUTS ARE REQUIRED:
THE TITLE OF THE CASE.
GAMMA IS THE SPECIFIC HEAT RATIO.
UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.
RLD IS THE LENGTH-TO-DIAMETER RATIO.
ZCOMB IS THE LENGTH OF THE REGION OF UNIFORMLY DISTRIBUTED
COMBUSTION, EXPRESSED AS A FRACTION OF THE CHAMBER LENGTH.
NDROPS DETERMINES THE PRESENCE OF DROPLET MOMENTUM SOURCES:
NDROPS = 0 DROPLET MOMENTUM SOURCE NEGLECTED.
NDROPS = 1 DROPLET MOMENTUM SOURCE INCLUDED.
NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:
NOZZLE = 0 QUASI-STEADY.
NOZZLE = 1 CONVENTIONAL NOZZLE.
FOR CONVENTIONAL NOZZLE:
AMPL IS THE NOZZLE AMPLITUDE RATIO.
PHASE IS THE NOZZLE PHASE SHIFT.
NUMAX IS THE NUMBER OF MODE-AMPLITUDE FUNCTIONS IN THE ASSUMED
SERIES SOLUTION: NUMAX MUST NOT EXCEED 10.
THE COEFFICIENTS COMPUTED ARE DETERMINED BY NONLIN AS FOLLOWS:
NONLIN = 0 LINEAR COEFFICIENTS ONLY.
NONLIN = 1 BOTH LINEAR AND NONLINEAR COEFFICIENTS.
COEFFICIENTS TO BE NEGLECTED ARE DETERMINED BY NEGL
AS FOLLOWS:
NEGL = 0 TERMS SMALLER THAN 0.00001 ARE NEGLECTED.
NEGL = 1 LINEAR TERMS SMALLER THAN 0.001 AND NONLINEAR
TERMS SMALLER THAN 0.002 ARE NEGLECTED.
THE OUTPUT IS DETERMINED BY NOUT AS FOLLOWS:
NOUT = 0 PRINTED OUTPUT ONLY.
NOUT = 1 PRINTED AND STORED ON DRUM (FASTRAND FILE).
NOUT = 2 FASTRAND FILE ONLY.
NOUT = 3 CARD OUTPUT ONLY.
EACH MODE-AMPLITUDE IS ASSIGNED AN INTEGER J.
THE MODE IS SPECIFIED BY THE INDICES L(J), M(J), AND N(J).
L(J) IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED 10.
M(J) IS THE AZIMUTHAL MODE NUMBER AND MUST NOT EXCEED 8.
N(J) IS THE RADIAL MODE NUMBER AND MUST NOT EXCEED 5.
THE INTEGER NS(J) IS ASSIGNED AS FOLLOWS:
NS = 1 A-FUNCTION \sin(M*\theta) * \cosh(I*B*Z)
NS = 2 B-FUNCTION \cos(M*\theta) * \cosh(I*B*Z)
NAME(J) IS A FOUR-CHARACTER NAME.
DIMENSION L(10), N(10), NAME(10), S(10), SJ(10), TITLE(80),
RJR00T(10*,5), RJVAL(10*,5), CI(20*20), C(3*20*20),
D(20*20*20), AMPL(10), PHASE(10), AZI(2),
BES1(9*9*9), BES2(9*9*9), BES3(9*9*9),
V(2), JC(20), TSC(3*20), TSQ(20), KMAX(4),
COMPLEX CRSLT, CI, ZEJ, ZEF1, ZEF2, CZE, CAZ, CRAD,
G1, DCOEF, CGAM, CAX, B(10), BC(10), YNOZ(10),
CNORM(10), CSSQ(10), TANINT(2), RADINT(3),
AXINT(4*3), CC(4*10,10), CD(10,10,10),
CD2(10,10,10), AX(4), T1, T2, D1, D2, D3, D4,
CD3(10,10,10), CD4(10,10,10),
COMMON B /BLK2/ M(10), NS(10),

DATA INPUT.

PI = 3.1415927
SM1 = 0.00001
SM2 = 0.00001
CI = (0.0, 0.0)

INPUT ROOTS AND VALUES OF BESSEL FUNCTIONS.

DATA ((RJR00T(J,I)), J = 1,5), I = 1,9)/
1 3.83171, 7.01559, 10.17347, 13.32831, 16.47063,
2 1.84118, 5.33144, 8.56327, 11.70600, 14.86359,
3 3.05424, 6.70613, 9.69478, 13.17037, 16.34752,
4 4.02119, 8.01524, 11.34592, 14.78875, 18.03702,
5 5.31755, 9.28240, 12.68191, 16.09611, 20.14175,
6 6.41562, 10.51986, 13.98719, 17.31824, 20.75516,
7 7.50127, 11.73494, 15.26818, 18.37441, 21.93172,
8 8.57784, 12.93239, 16.52937, 19.94185, 23.26050,
9 9.64742, 14.15552, 17.77401, 21.29062, 24.57200,

DATA ((RJVAL(J,I)), J = 1,5), I = 1,9)/
1 -0.40276, 0.30012, 0.49707, 0.21363, -0.19647,
2 0.58187, -0.34613, 0.27330, 0.23330, 0.20701,
3 0.46500, -0.31353, 0.25474, 0.22088, 0.19794,
4 0.43439, -0.29116, 0.24074, 0.21097, 0.19042,
5 0.39953, -0.27438, 0.22959, 0.20276, 0.18403,
6 0.37409, -0.26109, 0.22039, 0.19580, 0.17849,
7 0.35414, -0.25017, 0.21261, 0.18978, 0.17363,
8 0.33793, -0.24096, 0.20588, 0.18449, 0.16929,
9 0.32438, -0.23303, 0.19998, -0.17979, 0.16539,

INPUT PARAMETERS.

READ (5,5000, END = 600) (TITLE(I), I = 1,72)
READ (5,5001) GAMMA, UE, RLD, ZCOMB, NDROPS, NOZZLE
IF (GAMMA .LE. 600), 600, 8
READ (5,5004) NMAX, NONLIN, NEGL, NOUT
IF (NEGL .EQ. 1) READ (5,5005) SM1, SM2
IF (NOZZLE EQ 1) GO TO 5

COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE.

Y = (GAMMA - 1.0) * UE/(2.0 * GAMMA)

DO 3 J = 1, NJMAX
  AMPL(J) = Y
  PHASE(J) = 0.0
3 CONTINUE

GO TO 7

5 DO 6 I = 1, NJMAX
  READ (5,5003) J, AMPL(J), PHASE(J)
6 CONTINUE

7 DO 10 I = 1, NJMAX
  READ (5,5002) J, L(J), M(J), N(J), NS(J), NAME(J)
10 CONTINUE

DO 12 J = 1, NJMAX
  THETA = PHASE(J) * PI/180.0
  YR = AMPL(J) * COS(THETA)
  YI = AMPL(J) * SIN(THETA)
  YNOZ(J) = CMPLX(YR,YI)
12 CONTINUE

ZE = 2.0 * RLD
CZE = CMPLX(ZE,0.0)
CGAM = CMPLX(GAMMA,0.0)
CAX = CGAM
IF (NDBOPS EQ 1) CAX = CGAM + (1.0,0.0)

***********************************************************************

ASSIGN ARRAYS FOR ROOTS OF BESSEL FUNCTIONS.

DO 20 J = 1, NJMAX
  IF ((M(J) EQ 0) AND (N(J) EQ 0)) GO TO 15
  MM = M(J) + 1
  NN = N(J)
  S(J) = RJROOT(MM,NN)
  SJ(J) = RJVAL(MM,NN)
  GO TO 25
15 S(J) = 0.0
25 SSQ = S(J) * S(J)
CSSQ(J) = CMPLX(SSQ,0.0)
20 CONTINUE

***********************************************************************

CALCULATE AXIAL ACOUSTIC EIGENVALUES.

FIND MAXIMUM VALUES OF L(J), M(J), AND N(J).

KN = 0
LMAX = 0
MMAX = 0
NMAX = 0
DO 30 J = 1, NJMAX
IF (L(J) GT LMAX) LMAX = L(J)
IF (M(J) GT MMAX) MMAX = M(J)
IF (N(J) GT NMAX) NMAX = N(J)
IF (N(J) NE N(1)) KN = 1
30 CONTINUE
LMAX = LMAX + 1
MMAX = MMAX + 1

C COMPUTE EIGENVALUES.
DO 40 J = 1, NJMAX
LL = L(J)
SM = S(J)
YAML = AMPL(J)
YPHASE = PHASE(J)
CALL EIGVAL(LL, SM, GAMMA, ZE, YAML, YPHASE, CRSLT)
B(J) = CRSLT
BC(J) = CONJG(CRSLT)
40 CONTINUE

C ************************************************************

C CALCULATE LINEAR COEFFICIENTS.
DO 100 NJ = 1, NJMAX
DO 100 NP = 1, NJMAX

C ZERO COEFFICIENT ARRAYS.
DO 105 KC = 1, 4
CC(KC, NJ, NP) = (0, 0, 0, 0)
105 CONTINUE

C ORTHOGONALITY PROPERTY OF TANGENTIAL EIGENFUNCTIONS.
IF (NS(NP) NE NS(NJ)) GO TO 100
IF (M(NP) NE M(NJ)) GO TO 100
IF (M(NJ) EQ 0) GO TO 112
AZ = PI
GO TO 120
112 IF (NS(NJ) EQ 1) GO TO 100
AZ = 2.0 * PI

C ORTHOGONALITY PROPERTY OF RADIAL EIGENFUNCTIONS.
120 IF (NP(NP) NE NP(NJ)) GO TO 100
IF (S(NP)) 125, 122, 125
125 SQ = M(NJ) * M(NJ)
SSQ = S(NP) * S(NP)
SJSQ = SJ(NJ) * SJ(NJ)
RAD = (SSQ - SQM) * SSQ/(2.0 * SSQ)
GO TO 127
122 RAD = 0.5
C
C CALCULATE AXIAL INTEGRALS.
127 DO 130 NOPT = 1, 4
CALL AXIAL1(NOPT, NP, NJ, UE, ZE, ZCOMB, CRSLT)
AX(NOPT) = CRSLT
130 CONTINUE
C
C EVALUATE FUNCTIONS AT NOZZLE END.
ZEJ = CCOSH(CI*BC(NJ)*CZE)
ZEP1 = CCOSH(CI*B(NP)*CZE)
ZEP2 = CI * B(NP) * CSINH(CI*B(NP)*CZE)
C
CAZ = CMPLX(AZ,0.0)
CRAD = CMPLX(RAD,0.0)
C
COEFFICIENT OF THE SECOND DERIVATIVE OF A(P).
CC(1,NJ,NP) = AX(1) * CAZ * CRAD
C
COEFFICIENT OF A(P).
CC(2,NJ,NP) = (CSSQ(NP)*AX(1) - AX(2) + ZEP2*ZEJ) * CAZ * CRAD
C
COEFFICIENT OF THE FIRST DERIVATIVE OF A(P).
CC(3,NJ,NP) = (CAX*AX(3) + (2.0,0.0)*AX(4)
        + CGAM*YN0Z(NP)*ZEP1*ZEJ) * CAZ * CRAD
C
COEFFICIENT OF THE RETARDED DERIVATIVE OF A(P).
CC(4,NJ,NP) = CGAM * AX(3) * CAZ * CRAD
C
100 CONTINUE
C
NORMALIZE LINEAR COEFFICIENTS.
DO 140 NJ = 1, NJMAX
CNORM(NJ) = CC(1,NJ,NJ)
DO 140 NP = 1, NJMAX
DO 140 KC = 1, 4
CC(KC,NJ,NP) = CC(KC,NJ,NP)/CNORM(NJ)
140 CONTINUE
C
*********************************************************************************************
C
COMPUTE NONLINEAR COEFFICIENTS.
C
IF (NONLIN EQ 0) GO TO 402
G1 = (CGAM - (1.0,0.0)) * (0.5,0.0)
C
COMPUTATIONS OF BESSEL INTEGRALS WHEN ALL SERIES TERMS HAVE THE
SAME RADIAL MODE NUMBER N(J).
IF (KN .EQ. 1) GO TO 170
DO 150 MP = 1, MMAX
DO 150 MQ = 1, MMAX
DO 150 MJ = 1, MMAX
BES1(MP, MQ, MJ) = 0.0
BES2(MP, MQ, MJ) = 0.0
BES3(MP, MQ, MJ) = 0.0
L1 = MP - 1
L2 = MQ - 1
L3 = MJ - 1
LM = L1 + L2
LN = L1 + L3
MN = L2 + L3
IF (((L3*EQ*LM) .OR. (L2*EQ*LN) .OR. (L1*EQ*MN)) ) GO TO 160
GO TO 150
160 IF (NMAX .EQ. 0) GO TO 165
A1 = RJROOT(MP, NMAX)
A2 = RJROOT(MQ, NMAX)
A3 = RJROOT(MJ, NMAX)
GO TO 167
165 A1 = 0.0
A2 = 0.0
A3 = 0.0
167 CALL RADIAL(1, L1, L2, L3, A1, A2, A3, RESULT)
BES1(MP, MQ, MJ) = RESULT
CALL RADIAL(2, L1, L2, L3, A1, A2, A3, RESULT)
BES2(MP, MQ, MJ) = RESULT
CALL RADIAL(3, L1, L2, L3, A1, A2, A3, RESULT)
BES3(MP, MQ, MJ) = RESULT
150 CONTINUE
C
170 DO 200 NJ = 1, NJMAX
DO 200 NP = 1, NJMAX
DO 200 NQ = 1, NJMAX
C
CD1(NJ, NP, NQ) = (0.0, 0.0)
CD2(NJ, NP, NQ) = (0.0, 0.0)
C
DO 210 J = 1, 2
CALL AZIKTL(J, NP, NQ, NJ, RESULT)
AZI(J) = RESULT
TANINT(J) = CMPLX(RESULT, 0.0)
210 CONTINUE
C
IF (AZI(1)) 220, 225, 220
225 IF (AZI(2)) 220, 200, 220
C
220 IF (KN .EQ. 0) GO TO 222
L1 = M(NP)
L2 = M(NQ)
L3 = M(NJ)
A1 = S(NP)
A2 = S(NQ)
A3 = S(NJ)
GO TO 244

222 MP = M(NP) + 1
MQ = M(NQ) + 1
MJ = M(NJ) + 1
RADINT(1) = CMPLX(BES1(MP, MQ, MJ), 0.0)
RADINT(2) = CMPLX(BES2(MP, MQ, MJ), 0.0)
RADINT(3) = CMPLX(BES3(MP, MQ, MJ), 0.0)

244 DO 240 J = 1, 3
IF (KN * EQ. 0) GO TO 242
CALL RADIAL (J, L1, L2, L3, A1, A2, A3, RESULT)
RADINT(J) = CMPLX(RESULT, 0.0)

242 DO 240 NC = 1, 4
CALL AXIAL2(J, NC, NP, NQ, NJ, ZE, CRSLT)
AXINT(NC, J) = CRSLT

240 CONTINUE

DO 250 J = 1, 4
T1 = G1 * CSSQ(NF) * AXINT(J, 1)
T2 = G1 * AXINT(J, 3)
D1 = AXINT(J, 1) * TANINT(1) * RADINT(3)
D2 = AXINT(J, 1) * TANINT(2) * RADINT(2)
D3 = AXINT(J, 2) * TANINT(1) * RADINT(1)
D4 = (T2 - T1) * TANINT(1) * RADINT(1)
DCOEF = (0.5 * 0.0) * (D1 + D2 + D3 + D4) / CNORM(NJ)
IF (J * EQ. 1) CD1(NJ, NP, NQ) = (1.0 - 1.0) * DCOEF
IF (J * EQ. 2) CD2(NJ, NP, NQ) = (1.0 - 1.0) * DCOEF
IF (J * EQ. 3) CD3(NJ, NP, NQ) = (1.0 - 1.0) * DCOEF
IF (J * EQ. 4) CD4(NJ, NP, NQ) = (1.0 - 1.0) * DCOEF

250 CONTINUE

200 CONTINUE

************************************************************
CALCULATE COEFFICIENTS FOR EQUIVALENT REAL SYSTEM.

402 DO 350 NJ = 1, NJMAX
NEWJ = (2 * NJ) - 1
NEWJ1 = NEWJ + 1
DO 350 NP = 1, NJMAX
NEWP = (2 * NP) - 1
NEWP1 = NEWP + 1

COEFFICIENTS OF LINEAR TERMS.
CCR = REAL(CC(1,NJ,JP))
CCI = AIMAG(CC(1,NJ,JP))
C1(NEWJ,NEWP) = CCR
C1(NEWJ,NEWP1) = -CCI
C1(NEWJ,NEWP) = CCI
C1(NEWJ,NEWP1) = CCR
DO 360 KC = 1,3
CCR = REAL(CC(KC+1,NJ,JP))
CCI = AIMAG(CC(KC+1,NJ,JP))
C(KC,NEWJ,NEWP) = CCR
C(KC,NEWJ,NEWP1) = -CCI
C(KC,NEWJ,NEWP) = CCI
C(KC,NEWJ,NEWP1) = CCR
360 CONTINUE
C
COEFFICIENTS OF NONLINEAR TERMS.
IF (NONLIN .EQ. 0) GO TO 350
DO 370 NQ = 1, NJMAX
NEWQ = (2 * NQ) - 1
NEWQ1 = NEWQ + 1
CD1R = REAL(CD1(NJ,JP,NQ))
CD1I = AIMAG(CD1(NJ,JP,NQ))
CD2R = REAL(CD2(NJ,JP,NQ))
CD2I = AIMAG(CD2(NJ,JP,NQ))
CD3R = REAL(CD3(NJ,JP,NQ))
CD3I = AIMAG(CD3(NJ,JP,NQ))
CD4R = REAL(CD4(NJ,JP,NQ))
CD4I = AIMAG(CD4(NJ,JP,NQ))
D(NEWJ,NEWP,NEWQ) = CD1R + CD2R + CD3R + CD4R
D(NEWJ,NEWP,NEWQ1) = -CD1I + CD2I - CD3I + CD4I
D(NEWJ,NEWP1,NEWQ) = -CD1I - CD2I + CD3I + CD4I
D(NEWJ,NEWP1,NEWQ1) = -CD1R + CD2R + CD3R - CD4R
D(NEWJ,NEWP,NEWQ) = CD1I + CD2I + CD3I + CD4I
D(NEWJ,NEWP1,NEWQ1) = CD1R - CD2R + CD3R - CD4R
D(NEWJ,NEWP1,NEWQ) = CD1I - CD2I + CD3I - CD4I
D(NEWJ,NEWP1,NEWQ1) = -CD1I + CD2I + CD3I - CD4I
370 CONTINUE
350 CONTINUE
C
*************************************************************************
COMPUTE COEFFICIENTS FOR THE EQUATIONS WHICH ARE DECOUPLED
IN THE SECOND DERIVATIVES.
C
DO 405 KC = 1, 4
KMAX(KC) = 0
405 CONTINUE
C
CALCULATE INVERSE OF THE MATRIX C1(I,J).
JMAX = NJMAX
C
NJMAX = 2 * NJMAX

V(1) = 1
CALL GJR(C1, 20, 20, NJMAX, 0, 500, JC, V)

USE INVERSE TO CALCULATE DECOUPLED COEFFICIENTS.

DO 410 NP = 1, NJMAX

LINEAR COEFFICIENTS.
DO 420 NJ = 1, NJMAX
DO 420 KC = 1, 3
TS(KC, NJ) = 0.0
DO 420 K = 1, NJMAX
CONTINUE
DO 430 NJ = 1, NJMAX
DO 430 KC = 1, 3
C(KC, NJ, NP) = TS(KC, NJ)
ABSVAL = ABS(C(KC, NJ, NP))
IF (ABSVAL * GE * SM1) KMAX(KC) = KMAX(KC) + 1
CONTINUE

NONLINEAR COEFFICIENTS.
IF (NONLIN = EQ 0) GO TO 410
DO 415 NQ = 1, NJMAX
DO 440 NJ = 1, NJMAX
TSQ(NJ) = 0.0
DO 440 K = 1, NJMAX
TSQ(NJ) = TSQ(NJ) + CI(NJ, K) * D(K, NP, NQ)
CONTINUE
DO 445 NJ = 1, NJMAX
D(NJ, NP, NQ) = TSQ(NJ)
ABSVAL = ABS(D(NJ, NP, NQ))
IF (ABSVAL * GE * SM2) KMAX(4) = KMAX(4) + 1
CONTINUE

410 CONTINUE

415 CONTINUE

OUTPUT.

IF (NOUT = EQ 2) GO TO 455

PRINTED OUTPUT.
WRITE (6, 6001) (TITLE(I), I = 1, 72)
WRITE (6, 6002) GAMMA, UE, RLD, ZCOMB
IF (NDROPS = EQ 0) WRITE (6, 6020)
IF (NDROPS *EQ* 1) WRITE (6,6021)
IF (NOZZLE *EQ* 0) WRITE (6,6012)
WRITE (6,6004)
DO 310 J = 1, JMAX
   WRITE (6,6003) NAME(J), J, L(J), M(J), N(J), NS(J),
   S(J), SJ(J), B(J), YNOZ(J)
1
310 CONTINUE
IF (NONLIN *EQ* 0) WRITE (6,6013)
C
C OUTPUT OF LINEAR COEFFICIENTS
C
DO 320 KC = 1, 3
IF (KC *EQ* 1) WRITE (6,6005)
IF (KC *EQ* 2) WRITE (6,6006)
IF (KC *EQ* 3) WRITE (6,6007)
WRITE (6,6008) (J, J = 1, NJMAX)
WRITE (6,6014)
DO 320 NJ = 1, NJMAX
WRITE (6,6009) NJ, (C(KC,NJ,NP), NP = 1, NJMAX)
320 CONTINUE

C
C OUTPUT OF NONLINEAR COEFFICIENTS
C
IF (NONLIN *EQ* 0) GO TO 452
DO 400 NJ = 1, NJMAX
WRITE (6,6010) NJ
WRITE (6,6011) (J, J = 1, NJMAX)
WRITE (6,6015)
DO 400 NP = 1, NJMAX
WRITE (6,6009) NP, (D(NJ,NP,NQ), NQ = 1, NJMAX)
400 CONTINUE

452 IF (NOUT *EQ* 0) GO TO 4
C
455 IF (NOUT *EQ* 3) GO TO 480
C
C WRITE COEFFICIENTS ON FASTRAND FILE
C
WRITE (9,7001) GAMMA, UE, ZE, ZCOMB, NDROPS, NJMAX
C
DO 450 J = 1, JMAX
   WRITE (9,7002) J, L(J), M(J), N(J), NS(J), S(J), SJ(J),
   NAME(J)
450 CONTINUE
C
DO 457 J = 1, JMAX
   WRITE (9,7006) J, YNOZ(J), B(J)
457 CONTINUE
C
DO 460 KC = 1, 3
   WRITE (9,7003) KMAX(KC)
DO 460 NJ = 1, NJMAX
DO 460 NP = 1, NJMAX
ABSVAL = ABS(C(KC,NJ, NP))
IF (ABSVAL o GEo SM1) WRITE (9,7004) NJ, NP, C(KC,NJ, NP)
460 CONTINUE

WRITE (9,7003) KMAX(4)
IF (NONLIN o EQ= 0) GO TO 4
DO 470 NJ = 1, NJMAX
DO 470 NP = 1, NJMAX
DO 470 NQ = 1, NJMAX
ABSVAL = ABS(D(NJ, NP, NQ))
IF (ABSVAL o GEo SM2) WRITE (9,7005) NJ, NP, NQ, D(NJ, NP, NQ)
470 CONTINUE
GO TO 4

PUNCHED CARD OUTPUT

PUNCH 7001 GAMMA, UE, ZE, ZCOMB, NDROPS, NJMAX

DO 482 J = 1, JMAX
PUNCH 7002 J, L(J), M(J), N(J), NS(J), S(J), SJ(J),
1 NAME(J)
482 CONTINUE

DO 484 J = 1, JMAX
PUNCH 7006 J, YNOZ(J), B(J)
484 CONTINUE

DO 486 KC = 1, 3
PUNCH 7003 KMAX(KC)
DO 486 NJ = 1, NJMAX
DO 486 NP = 1, NJMAX
ABSVAL = ABS(C(KC,NJ, NP))
IF (ABSVAL o GEo SM1) PUNCH 7004 NJ, NP, C(KC,NJ, NP)
486 CONTINUE

PUNCH 7003 KMAX(4)
IF (NONLIN o EQ= 0) GO TO 4
DO 488 NJ = 1, NJMAX
DO 488 NP = 1, NJMAX
DO 488 NQ = 1, NJMAX
ABSVAL = ABS(D(NJ, NP, NQ))
IF (ABSVAL o GEo SM2) PUNCH 7005 NJ, NP, NQ, D(NJ, NP, NQ)
488 CONTINUE
GO TO 4

ERROR EXIT

500 IF (JC(1)) 510, 510, 520
510 JC(1) = ABS(JC(1))
WRITE (6,6017) JC(1)
GO TO 4
**520 WRITE (6, 6018) JC(1)**
**GO TO 4**
**600 CONTINUE**

**C**

**C ***********************************************/

**C**

**C FORMAT SPECIFICATIONS**

**5000 FORMAT (72A1)**
**5001 FORMAT (4F10.0,2I5)**
**5002 FORMAT (5I5, 1X, A4)**
**5003 FORMAT (15, 2F10.0)**
**5004 FORMAT (4I5)**
**5005 FORMAT (2F10.0)**
**5006 FORMAT (1H1, 1X, 72A1)**
**5007 FORMAT (4F10.5, 2I5)**
**5008 FORMAT (5IL, 2F10.5, 2A4)**
**5009 FORMAT (15)**
**5010 FORMAT (215, F15.6)**
**5011 FORMAT (315, F15.6)**
**5012 FORMAT (15, 4FI0.5)**

**END**
SUBROUTINE EIGVAL(L, SMN, GAMMA, ZE, YAMPL, YPHASE, RESULT)

COMPLEX RESULT
COMMON /BLK1/ GSO, ABSQ, ALBET, SMNSQ

******************************************************************************

THIS SUBROUTINE COMPUTES THE COMPLEX AXIAL ACOUSTIC EIGENVALUES
FOR A CYLINDRICAL CHAMBER WITH A NOZZLE AND STORES THEM IN
RESULT.
THE EIGENVALUES ARE COMPUTED BY MEANS OF NEWTONS METHOD.

THE INPUT PARAMETERS ARE AS FOLLOWS:
L IS THE AXIAL MODE NUMBER.
SMN IS THE DIMENSIONLESS ACOUSTIC FREQUENCY.
GAMMA IS THE SPECIFIC HEAT RATIO.
ZE IS THE LENGTH-TO-RADIUS RATIO.
YAMPL IS THE NOZZLE AMPLITUDE FACTOR.
YPHASE IS THE NOZZLE PHASE SHIFT IN DEGREES.

******************************************************************************

PI = 3.1415927
ERR = 0.0000001

IF (YAMPL) .LE. 60. 5
CALCULATE CONSTANTS.
5 PHASE = YPHASE * PI/180.0
ALPHA = YAMPL * COS(PHASE)
BETA = YAMPL * SIN(PHASE)
GSO = GAMMA * GAMMA
ABSQ = (ALPHA * ALPHA) - (BETA * BETA)
ALBET = ALPHA * BETA
SMNSQ = SMN * SMN

ASSIGN INITIAL GUESS FOR EIGENVALUE.
IF (L .EQ. 0) GO TO 45
RL = L
PHI = PI/2.0 + PHASE
XM = RL * PI/ZE
A = YAMPL/ZE
X0 = XM + A*COS(PHI)
Y0 = A*SIN(PHI)
GO TO 47

45 PHI = PI/4.0 + 0.5*PHASE
A = YAMPL * 10.0/ZE
X0 = A * COS(PHI)
Y0 = A * SIN(PHI)

ITERATION USING NEWTONS METHOD FOR A SYSTEM OF TWO EQUATIONS
C IN TWO UNKNOWNS.

47 L1 = 0
X = X0
Y = Y0

40 CALL FCNS(X,Y,Z,E,F,G,FX,FY,GX,GY)
IF (L1 .EQ. 40) GO TO 50
RJFG = (FX * GY) - (GX * FY)
IF (RJFG) 20, 30, 20
20 DELTAX = (-F * GY + G * FY)/RJFG
DELTAY = (-G * FX + F * GX)/RJFG
L1 = L1 + 1
X = X + DELTAX
Y = Y + DELTAY
C TEST FOR CONVERGENCE.
IF (ABS(DELTAX) .GE. ERR .OR. ABS(DELTAY) .GE. ERR) GO TO 40
GO TO 10
C WARNING MESSAGES
30 WRITE (6,6005)
GO TO 10
50 WRITE (6,6006)
GO TO 10
C CASE OF HARD WALL (YAMPL = 0).
60 RL = L
X = RL * PI/ZE
Y = 0.0
C 10 RESULT = CMPLX(X,Y)
C FORMAT SPECIFICATIONS.
6005 FORMAT (2X/2X,16HJACOBIAN IS ZERO//)
6006 FORMAT (2X/2X,35HFAILED TO CONVERGE IN 40 ITERATIONS//)
RETURN
END
SUBROUTINE FCNS(X, Y, Z, E, F, G, FX, FY, GX, GY)

THIS SUBROUTINE COMPUTES THE FUNCTIONS F(X, Y) AND G(X, Y)
AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO X AND Y.

COMMON /BLK1/ GSQ, ABSQ, ALBET, SMNSQ

COMPUTE THE TRIGONOMETRIC FUNCTIONS, THE HYPERBOLIC FUNCTIONS
AND THEIR SQUARES.

I = 1
ARGX = ZE * X
ARGY = ZE * Y
10 SX = SIN(ARGX)
CX = COS(ARGX)
SHY = SINH(ARGY)
CHY = COSH(ARGY)
IF (I .EQ. 2) GO TO 20
SXSQ = SX * SX
CXSQ = CX * CX
SHYSQ = SHY * SHY
CHYSQ = CHY * CHY
ARGX = 2.0 * ARGX
ARGY = 2.0 * ARGY
I = 2
GO TO 10

COMPUTE TRANSCENDENTAL FUNCTIONS AND THEIR DERIVATIVES

20 FF = (SXSQ * CHYSQ) - (CXSQ * SHYSQ)
GG = (CXSQ * CHYSQ) - (SXSQ * SHYSQ)
HH = 0.25 * SX * SHY
FFX = ZE * SX * CHY
GGY = ZE * CX * SHY
FFY = -GGY
GGX = -FFX
HHX = 0.5 * GGY
HHY = 0.5 * FFX

COMPUTE FACTORS
XYSQ = (X * X) - (Y * Y)
XY = X * Y
SMNXY = SMNSQ + XYSQ
F1 = (ABSQ * SMNXY) - (4.0 * ALBET * XY)
F2 = (ALBET * SMNXY) + (ABSQ * XY)
G1 = (ABSQ * SMNXY) + (4.0 * ALBET * XY)
FX1 = (2.0 * X * ABSQ) - (4.0 * ALBET * Y)
FX2 = (2.0 * X * ALBET) + (ABSQ * Y)
FY1 = (-2.0 * Y * ABSQ) - (4.0 * ALBET * X)
FY2 = (-2.0 * Y * ALBET) + (ABSQ * X)
GX1 = (2.0 * X * ABSQ) + (4.0 * ALBET * Y)  
GY1 = (-2.0 * Y * ABSQ) + (4.0 * ALBET * X)

COMPUTE F(X, Y) AND G(X, Y)

F = (XYSQ * FF) - (4.0 * XY * HH)
1 + GSQ * ((F1 * GG) + (4.0 * F2 * HH))
G = (XYSQ * HH) + (XY * FF)
1 + GSQ * ((F2 * GG) - (G1 * HH))

COMPUTE THE PARTIAL DERIVATIVES OF F AND G

FX = (2.0 * X * FF) + (XYSQ * FFX)
1 -4.0 * ((Y * HH) + (XY * HHX))
2 + GSQ * ((FX1 * GG) + (F1 * GGX))
3 + (4.0 * FX2 * HH) + (4.0 * F2 * HHX))
FY = (-2.0 * Y * FF) + (XYSQ * FFY)
1 -4.0 * ((X * HH) + (XY * HHY))
2 + GSQ * ((FY1 * GG) + (F1 * GGY))
3 + (4.0 * FY2 * HH) + (4.0 * F2 * HHY))
GX = (2.0 * X * HH) + (XYSQ * HHX)
1 + (Y * FF) + (XY * FFX)
2 + GSQ * ((FX2 * GG) + (F2 * GGX))
3 -(GX1 * HH) - (G1 * HHX))
GY = (-2.0 * Y * HH) + (XYSQ * HHY)
1 + (X * FF) + (XY * FFY)
2 + GSQ * ((FY2 * GG) + (F2 * GGY))
3 -(GY1 * HH) - (G1 * HHY))

RETURN

END
SUBROUTINE AXIAL1 (NOPT, NP, NJ, UE, ZE, ZCOMB, RESULT)

THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL (0, ZE) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUE OF NOPT:

NOPT = 1 Z(NP) * ZC(NJ)
NOPT = 2 ZPP(NP) * ZC(NJ)
NOPT = 3 U * Z(NP) * ZC(NJ)
NOPT = 4 U * ZP(NP) * ZC(NJ)

IN THE ABOVE EQUATIONS:
Z(NP) IS THE AXIAL ACOUSTIC EIGENFUNCTION OF INDEX NP.
Z(NJ) IS THE AXIAL ACOUSTIC EIGENFUNCTION OF INDEX NJ.
ZC IS THE COMPLEX CONJUGATE OF THE AXIAL EIGENFUNCTION.
ZP AND ZPP ARE THE FIRST AND SECOND DERIVATIVES OF THE AXIAL EIGENFUNCTIONS RESPECTIVELY.
U IS THE STEADY STATE VELOCITY DISTRIBUTION AND UP IS ITS AXIAL DERIVATIVE.
THE VELOCITY DISTRIBUTION IS COMPUTED BY THE SUBROUTINE UBAR.

REAL MAG
COMPLEX CI, CZE, BP, BJ, T1, T2, CH, F1, F2, F3, CZ, ARG, S1, S2, S3, RESULT, FUNCT(500), B(10)

COMMON B

CI = (0.0-1.0)
CZE = CMPLX(ZE,0.0)
BP = B(NP)
BJ = CONJG(B(NJ))

IF (NOPT -GT. 2) GO TO 50
CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS FOR NOPT = 1 AND NOPT = 2:
ARG = (BP + BJ) * CI
MAG = CABS(ARG)
IF (MAG) 20, 25, 20
20 T1 = CSINH(ARG*CZE)/ARG
GO TO 30
25 T1 = CZE
30 ARG = (BP - BJ) * CI
MAG = CABS(ARG)
IF (MAG) 35, 40, 35
35 T2 = CSINH(ARG*CZE)/ARG
GO TO 45
40 T2 = CZE
45 RESULT = (T1 + T2) * (0.5, 0.0)
IF (NOPT -EQ 2) RESULT = -B(NP) * B(NP) * RESULT
GO TO 100
NUMERICAL EVALUATION OF INTEGRALS FOR NOPT = 3 AND NOPT = 4.

COMPUTE STEP SIZE FOR SIMPSON INTEGRATION.

50 N = 50
RN = N
RESULT = (0.0, 0.0)
IC = ZCOMB
IC = 2 - IC

DO 90 J = 1, IC
IF (J .EQ. 1) H = ZCOMB * ZE/RN
IF (J .EQ. 2) H = (1.0 - ZCOMB) * ZE/RN
IF (J .EQ. 1) Z0 = 0.0
IF (J .EQ. 2) Z0 = ZCOMB * ZE
NP1 = N + 1
CH = CMPLX(H, 0.0)

COMPUTE INTEGRANDS.

DO 60 I = 1, NP1
STEP = I - 1
Z = (STEP * H) + Z0
IF (I .EQ. 1) .AND. (J .EQ. 2) Z = Z + H/100.0
IF (NOPT .EQ. 3) CALL UBAR(2*UE*ZE*ZCOMB, Z, F)
IF (NOPT .EQ. 4) CALL UBAR(1*UE*ZE*ZCOMB, Z, F)
F1 = CMPLX(F, 0.0)
CZ = CMPLX(Z, 0.0)
ARG = CI * BP
IF (NOPT .EQ. 3) F2 = CCOSH(ARG*CZ)
IF (NOPT .EQ. 4) F2 = ARG * CSINH(ARG*CZ)
ARG = CI * BJ
F3 = CCOSH(ARG*CZ)
FUNCT(I) = F1 * F2 * F3
60 CONTINUE

PERFORM SIMPSON INTEGRATION.

NM1 = N - 1
S1 = FUNCT(1) + FUNCT(NP1)
S2 = (0.0, 0.0)
S3 = (0.0, 0.0)
DO 70 I = 2, N, 2
S2 = S2 + FUNCT(I)
70 CONTINUE
DO 80 I = 3, NM1, 2
S3 = S3 + FUNCT(I)
80 CONTINUE
RESULT = RESULT +
1 CH * (S1 + (4.0, 0.0) * S2 + (2.0, 0.0) * S3) / (3.0, 0.0)
90 CONTINUE

100 CONTINUE
RETURN
END
SUBROUTINE UBAR(NOPT, UE, ZE, ZCOMB, Z, RESULT)

THIS SUBROUTINE CALCULATES THE STEADY STATE VELOCITY DISTRIBUTION FOR UNIFORMLY DISTRIBUTED COMBUSTION COMPLETED AT Z = ZCOMB * ZE WHERE:
UE IS THE EXIT MACH NUMBER.
ZE IS THE DIMENSIONLESS LENGTH.
Z IS THE AXIAL COORDINATE.

IF NOPT = 1 THE DISTRIBUTION IS CALCULATED.
IF NOPT = 2 THE DERIVATIVE IS CALCULATED.
IF NOPT = 3 THE SECOND DERIVATIVE IS CALCULATED.

ECZ = ZCOMB * ZE
GO TO (10, 20, 30), NOPT
10 IF (Z .LE. ECZ) RESULT = UE * Z/ECZ
    IF (Z .GT. ECZ) RESULT = UE
    GO TO 40
20 IF (Z .LE. ECZ) RESULT = UE/ECZ
    IF (Z .GT. ECZ) RESULT = 0.0
    GO TO 40
30 RESULT = 0.0
40 CONTINUE
RETURN
END
SUBROUTINE AZIMTL(NOPT, NP, NQ, NJ, RESULT)

DIMENSION NFCN(3), SG(2)
COMMON /ELK2/ M(10), NS(10)

******************************************************************************

THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL
(0, 2*PI) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUE
OF NOPT:

NOPT = 1       TH(NP) * TH(NQ) * TH(NJ)
NOPT = 2       THP(NP) * THP(NQ) * TH(NJ)

IN THE ABOVE EQUATIONS:
TH(NP), TH(NQ), AND TH(NJ) ARE THE TANGENTIAL EIGENFUNCTIONS
AND NP, NQ, AND NJ ARE THEIR INDICES.
THP IS THE DERIVATIVE OF THE TANGENTIAL EIGENFUNCTIONS.

IF NS = 1       TH = SIN(M*THETA)
IF NS = 2       TH = COS(M*THETA)

******************************************************************************

RESULT = 0.0
FACTOR = 1.0
PI = 3.1415927

DISTINGUISH BETWEEN SINES AND COSINES.
DO 10 K1 = 1, 3
   NFCN(K1) = 1
   CONTINUE
   IF (NS(NJ) .EQ. 2) NFCN(3) = 2
   IF (NOPT .EQ. 2) GO TO 20
   IF (NS(NP) .EQ. 2) NFCN(1) = 2
   IF (NS(NQ) .EQ. 2) NFCN(2) = 2
   GO TO 30

20 IF (NS(NP) .EQ. 1) NFCN(1) = 2
   IF (NS(NQ) .EQ. 1) NFCN(2) = 2
   DO 40 K1 = 1, 2
       SG(K1) = 1.0
   IF (NFCN(K1) .EQ. 1) SG(K1) = -1.0
   CONTINUE
   FACTOR = SG(1) * SG(2) * M(NP) * M(NQ)

30 NSUM = 0
   DO 50 K1 = 1, 3
       NSUM = NSUM + NFCN(K1)
   CONTINUE

50 CONTINUE
IF ((NSUM .EQ. 3) .OR. (NSUM .EQ. 5)) GO TO 60
IF (NSUM .EQ. 4) GO TO 70
IF (NSUM .EQ. 6) GO TO 80
C
70 KOPT = 2
   IF (NFCN(1) .EQ. 2) GO TO 72
   GO TO 74
72 LL = M(NP)
   MM = M(NQ)
   NN = M(NJ)
   GO TO 90
74 IF (NFCN(2) .EQ. 2) GO TO 76
   GO TO 78
76 LL = M(NQ)
   MM = M(NP)
   NN = M(NJ)
   GO TO 90
78 LL = M(NJ)
   MM = M(NP)
   NN = M(NQ)
   GO TO 90
C
80 KOPT = 1
   LL = M(NP)
   MM = M(NQ)
   NN = M(NJ)
C
C   COMPUTE VALUES OF THE INTEGRALS.
C
90 IF ((LL.NE.0) .AND. (MM.NE.0) .AND. (NN.NE.0)) GO TO 101
   GO TO 103
101 LM = LL + MM
   LN = LL + NN
   MN = MM + NN
   IF ((NN.EQ.LM) .OR. (MM.EQ.LN)) RESULT = PI/2.0
   IF (LL .EQ. MN) GO TO 102
   GO TO 104
102 IF (KOPT .EQ. 1) RESULT = PI/2.0
   IF (KOPT .EQ. 2) RESULT = -PI/2.0
   GO TO 104
103 IF ((LL.EQ.0) .AND. (MM.EQ.0) .AND. (NN.EQ.0)) GO TO 105
   IF ((KOPT.EQ.1) .AND. (NN.EQ.0) .AND. (LL.EQ.MM)) RESULT = PI
   IF ((KOPT.EQ.1) .AND. (MM.EQ.0) .AND. (LL.EQ.NN)) RESULT = PI
   IF ((LL.EQ.0) .AND. (MM .EQ. NN)) RESULT = PI
   GO TO 104
105 IF (KOPT .EQ. 1) RESULT = 2.0 * PI
104 CONTINUE
   RESULT = FACTOR * RESULT
60 CONTINUE
   RETURN
END

91
SUBROUTINE RADIAL(NOPT,L,M,N,A,B,C,RESULT)

THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL
(0,1) OF THE FOLLOWING PRODUCTS OF THREE BESSEL FUNCTIONS:

NOPT = 1  JL(A*R) * JM(B*R) * JN(C*R) * R

NOPT = 2  JL(A*R) * JM(B*R) * JN(C*R)/R

NOPT = 3  JPL(A*R) * JPM(B*R) * JN(C*R) * R

JL IS THE BESSEL FUNCTION OF FIRST KIND OF ORDER L
JPL IS THE DERIVATIVE OF JL WITH RESPECT TO R
L, M, N ARE NON-NEGATIVE INTEGERS
A, B, C ARE REAL NUMBERS

DIMENSION FUNCT(200)
DOUBLE PRECISION DN, DH, DSTEP, DR, ARG1, ARG2, ARG3,
1  BES1, BES2, BES3, BESH, BESL, PROD,
2  FUNCT, BESLIM, S1, S2, S3

NN = 100
DN = NN
DH = 1.0/DN
NP1 = NN + 1

DO 10 I = 1, NP1
DSTEP = I - 1
DR = DH * DSTEP
ARG1 = A * DR
ARG2 = B * DR
ARG3 = C * DR

CALL JBES(N, ARG3, BES3, S500)
IF (NOPT .EQ. 3) GO TO 101
CALL JBES(L, ARG1, BES1, S500)
CALL JBES(M, ARG2, BES2, S500)
GO TO 102

101 IF (L .EQ. 0) GO TO 103
CALL JBES(L+1, ARG1, BESH, S500)
CALL JBES(L-1, ARG1, BESL, S500)
BES1 = A * (BESL - BESH)/2.0
GO TO 104

103 CALL JBES(1, ARG1, BES1, S500)
BES1 = -BES1 * A

104 IF (M .EQ. 0) GO TO 105
CALL JBES(M+1, ARG2, BESH, S500)
CALL JBES(M-1, ARG2, BESL, S500)
BES2 = B * (BESL - BESH)/2.0
GO TO 102
105 CALL JBES(1, ARG2, BES2, $.500)
BES2 = -BES2 * B
102 PROD = BES1 * BES2 * BES3
C
   IF (NOPT .EQ. 2) GO TO 110
   FUNCT(I) = PROD * DR
   GO TO 10
110 IF (I .EQ. 1) GO TO 111
   FUNCT(I) = PROD/DR
   GO TO 10
111 BESLIM = 0.0
   IF ((L .EQ. 1) .AND. (M .EQ. 0) .AND. (N .EQ. 0)) BESLIM = A/2.0
   IF ((L .EQ. 0) .AND. (M .EQ. 1) .AND. (N .EQ. 0)) BESLIM = B/2.0
   IF ((L .EQ. 0) .AND. (M .EQ. 0) .AND. (N .EQ. 1)) BESLIM = C/2.0
   FUNCT(I) = BESLIM
10 CONTINUE
C
   NM1 = NN - 1
   S1 = FUNCT(I) + FUNCT(NP1)
   S2 = 0.0
   S3 = 0.0
   DO 20 I = 2, NN, 2
      S2 = S2 + FUNCT(I)
   20 CONTINUE
      DO 30 I = 3, NM1, 2
      S3 = S3 + FUNCT(I)
   30 CONTINUE
   RESULT = DH * (S1 + 4.0*S2 + 2.0*S3)/3.0
   GO TO 501
500 WRITE (6, 6000)
6000 FORMAT (1H1, 'OHERROR JBES')
501 CONTINUE
      RETURN
END
SUBROUTINE AXIAL2( NOPT, NCONJ, NP, NQ, NJ, ZE, RESULT)

THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL (0, ZE) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUES OF NOPT AND NCONJ:

FOR NCONJ = 1 AND:
NOPT = 1  Z(NP) * Z(NQ) * ZC(NJ)
NOPT = 2  ZP(NP) * ZP(NQ) * ZC(NJ)
NOPT = 3  ZPP(NP) * Z(NQ) * ZC(NJ)

FOR NCONJ = 2 AND:
NOPT = 1  Z(NP) * ZC(NQ) * ZC(NJ)
NOPT = 2  ZP(NP) * ZPC(NQ) * ZC(NJ)
NOPT = 3  ZPPC(NP) * ZC(NQ) * ZC(NJ)

FOR NCONJ = 3 AND:
NOPT = 1  ZC(NP) * Z(NQ) * ZC(NJ)
NOPT = 2  ZPC(NP) * ZP(NQ) * ZC(NJ)
NOPT = 3  ZPPC(NP) * ZC(NQ) * ZC(NJ)

FOR NCONJ = 4 AND:
NOPT = 1  ZC(NP) * ZC(NQ) * ZC(NJ)
NOPT = 2  ZPC(NP) * ZPC(NQ) * ZC(NJ)
NOPT = 3  ZPPC(NP) * ZC(NQ) * ZC(NJ)

IN THE ABOVE EQUATIONS:
Z(NP), Z(NQ), AND Z(NJ) ARE THE AXIAL ACOUSTIC EIGENFUNCTIONS AND NP, NQ, AND NJ ARE THEIR INDICES.
ZP IS THE FIRST DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.
ZPP IS THE SECOND DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.
ZC AND ZPC ARE COMPLEX CONJUGATES OF Z AND ZP RESPECTIVELY.

REAL  MAG
COMPLEX  CI, CF, CZE, BP, BQ, BJ, SUM, RESULT,
I  ARG(4), FUNCT(4), B(10)
COMMON  B

CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS.
CI = (0.0, 1.0)
CF = (0.25, 0.0)
CZE = COMPLX(ZE, 0.0)
BP = B(NP)
BQ = B(NQ)
BJ = CONJG(B(NJ))
IF (NCONJ .EQ. 2) .OR. (NCONJ .EQ. 4) THEN
   BQ = CONJG(BQ)
ELSE
   BP = CONJG(BP)
ENDIF
ARG(1) = (BP + BQ + BJ) * CI
ARG(2) = (BP + BQ - BJ) * CI
ARG(3) = (BP - BQ + BJ) * CI
ARG(4) = (BP - BQ - BJ) * CI
DO 10 J = 1, 4
MAG = CABS(ARG(J))
IF (MAG) 12, 15, 12
12 FUNCT(J) = CSINH(ARG(J)*CZE)/ARG(J)
GO TO 10
15 FUNCT(J) = CZE
10 CONTINUE
IF (NOPT .EQ. 2) GO TO 30
SUM = FUNCT(1) + FUNCT(2) + FUNCT(3) + FUNCT(4)
RESULT = CF * SUM
IF (NOPT .EQ. 3) RESULT = -BP * BP * RESULT
GO TO 50
30 SUM = FUNCT(1) + FUNCT(2) - FUNCT(3) - FUNCT(4)
RESULT = -CF * BP * BQ * SUM
50 CONTINUE
RETURN
END
APPENDIX D

PROGRAM LCYC3D: A USER'S MANUAL

General Description

Using the three-dimensional second-order theory described in this report, Program LCYC3D calculates the nonlinear stability characteristics of a cylindrical combustion chamber with distributed combustion and a conventional nozzle. The response of the burning rate to pressure oscillations is described by Crocco's time-lag model. For given values of the operating parameters (i.e., $n$, $\bar{T}$, $\gamma$, $\bar{u}_e$, and L/D), a given series expansion, and a given initial disturbance, Program LCYC3D integrates Eqs. (C-38) to obtain the time behavior of the unknown mode-amplitude functions (i.e., $B_j(t)$). From this information, a time history of the pressure oscillation is determined. The program determines the final amplitude of the pressure oscillation attained in a linearly unstable engine (i.e., limit-cycle amplitude). Since the second-order analysis does not predict "triggering", however, the threshold amplitude above which a finite amplitude disturbance can trigger instability in a linearly stable engine (i.e., triggering limit) is not calculated by Program LCYC3D. For either transient or limit-cycle conditions, the program prints out time histories of both pressure and axial velocity perturbations from which the amplitude, frequency, and wave shapes can be determined. The option to produce plotted output using a CALCOMP plotter is also provided.

Program Structure

A flow chart for Program LCYC3D is given in Fig. (D-1). This program performs the following operations: (1) reads the input data, (2) calculates the initial conditions, (3) numerically integrates the differential equations, (4) tests for limit cycles (optional), and (5) prints and plots the resulting solutions.

The inputs to the program include the data generated by Program COEFFS3D, the combustion parameters $n$ and $\bar{T}$, various control numbers, and a description of the initial disturbance. The data from COEFFS3D is read first and then printed out. Next the space dependent coefficients appearing in the series
Input from
COEFFS3D

Printout of Coefficients
(Optional)

Coefficients
for $\xi_t, \xi_z$

Printout of
$\gamma$-Coefficients

Read
Input
Data

Read
Initial
Amplitudes
of $B_{21}(t)$

Calculate
Initial Amplitudes
of $B_{21}(t)$

Printout of
Initial
Amplitudes

Linear
Coefficients
$C_nC, nC_3$

Step-Size

Amplitude
Functions
for $-Tt \leq 0$

Printout of
Initial Values
for $-Tt \leq 0$

$\text{t=0}$

$I=NDIV+1$

Amplitude
Functions at $t+\Delta t$

Pressure and
Velocity at $t+\Delta t$

Maximum and
Minimum Values

$\text{Does More Data Follow?}$

Yes

Stop

No

$\text{NTES=0}$

Assign
Final Values
to Initial Values

Yes

$\text{Limit Cycle}$

$\text{No}$

$\text{0}$

$\text{1}$

$\text{NTES}$

Yes

$\text{is } t < 250\Delta t$

No

$\text{MAXNO}$

$\text{<500}$

Printout of
Pressure Max.
and Min.

$\text{<500}$

$\text{NTES}$

$\text{1}$

$\text{0}$

$\text{< TSTART}$

$\text{TIME}$

$\text{No}$

Increase $t$
by $\Delta t$

$\text{TIME} \geq \text{TQUIT}$

Printout of
Pressure and
Velocity at $t+\Delta t$

$\text{TIME} \geq \text{TSTART}$

Printout of
Pressure and
Velocity at $t+\Delta t$

Assign Data
to Plot Arrays
for $t+\Delta t$

$\text{PLOTS?}$

Yes

No

$\text{Plot Array Pull}$

$\text{Is Plot Array Pull}$

Yes

Plot Data in Arrays

No

$\text{Stop}$

$\text{Figure D-1. Flow Chart for Program LCYC3D.}$
expansions for $\hat{\phi}_t$, $\hat{\phi}_\theta$, and $\hat{\phi}_z$ are computed and printed out. These coefficients are calculated by Subroutine PHICFS for use in the computation of the pressure and axial velocity perturbations. The remaining input data is then read, and following program execution, control is returned to this point (see Fig. D-1) so that several cases (i.e., different values of $n$ and $\tau$) may be run for a given set of coefficients generated by COEFFS3D.

After input of the initial amplitudes of the real parts (i.e., $B_{2j-1}(t)$) of the complex amplitude functions, the initial amplitudes of the imaginary parts (i.e., $B_{2j}(t)$) are calculated such that the nozzle admittance condition is satisfied for $-\tau \leq t \leq 0$. These amplitudes are then printed out. Next the integration step-size, $\Delta t$, is calculated such that the interval $-\tau \leq t \leq 0$ is divided into $NDIV$ equal increments. Assuming a sinusoidal initial disturbance, the initial amplitudes of $B_{2j-1}(t)$ and $B_{2j}(t)$ are used to calculate these functions and their derivatives at each of the $NDIV + 1$ discrete points in $-\tau \leq t \leq 0$. These values are needed in order to start the numerical solution of the differential equations (i.e., Eqs. (C-38)).

After input of the initial amplitudes of the real parts (i.e., $B_{2j-1}(t)$) of the complex amplitude functions, the initial amplitudes of the imaginary parts (i.e., $B_{2j}(t)$) are calculated such that the nozzle admittance condition is satisfied for $-\tau \leq t \leq 0$. These amplitudes are then printed out. Next the integration step-size, $\Delta t$, is calculated such that the interval $-\tau \leq t \leq 0$ is divided into $NDIV$ equal increments. Assuming a sinusoidal initial disturbance, the initial amplitudes of $B_{2j-1}(t)$ and $B_{2j}(t)$ are used to calculate these functions and their derivatives at each of the $NDIV + 1$ discrete points in $-\tau \leq t \leq 0$. These values are needed in order to start the numerical solution of the differential equations (i.e., Eqs. (C-38)).

The initial values of the amplitude functions are stored in the array $U(I,J)$ where the index $I$ varies from $1 (t = -\tau)$ to $NDIV + 1 (t = 0)$ and the index $J$ identifies the function. The corresponding initial values of the pressure and velocity perturbations are then printed out. This section also calculates the coefficients $\tilde{C}_2(j,p) - n\tilde{C}_3(j,p)$ and $n\tilde{C}_3(j,p)$ which are the coefficients of $dB_p/dt$ and $d[B_p(t - \tau)]/dt$ in Eqs. (C-38).

After the starting values are calculated, Eqs. (C-38) are solved using a modified form of the fourth order Runge Kutta method. Starting at $t = 0$ ($I = NDIV+1$), the amplitude functions at $t + \Delta t$ are calculated, using the Subroutine RHS to evaluate the functions $f_j(B_1,B_2,\ldots,B_{2N})$ on the right hand sides of Eqs. (C-38). The amplitude functions and the coefficients from PHICFS are then used to compute the pressure and axial velocity perturbations by Subroutine PRSVEL. The values of the amplitude functions at $t + \Delta t$ are stored in $U(I+1,J)$, while the pressure and axial velocity perturbations are stored in the arrays PRESS(npres) and AXVEL(npres) where NPRES specifies the locations in the chamber where the data is calculated. Pressure data at one location (specified by NLOC) is also stored in the array PRESS(I+1). After checking for maximum and minimum values of $U(I,J)$ and PRESS(I), the data may
be printed out (if NTEST = 0 and TSTART ≤ t ≤ TQUIT) or stored in plot arrays as desired. The time is then increased by Δt (i.e., I is increased by 1) and the calculations are repeated. This process continues until 250 integration steps have been computed (t = 250Δt), after which transfer is made to the limit-cycle section.

In the limit-cycle section a test for a limit-cycle is made if NTEST = 1. If the test is satisfied, NTEST is set to zero so that no further tests will be made and the results can be printed or plotted. In either case the final values (for 250-NDIV ≤ I ≤ 250) replace the initial values (for 1 ≤ I ≤ NDIV+1) in the arrays U(I,J) and PRS(I), I is again assigned the value NDIV+1, and another 250 integration steps are calculated. This process continues until one of the following conditions is satisfied: (1) NTEST = 0 and t > TQUIT, (2) a limit-cycle is reached and t > TQUIT, and (3) more than 250 cycles of the pressure oscillation have been computed (MAXNO > 500). At this point the numerical calculations are terminated and the time history of the pressure amplitude (maxima and minima) are printed out and/or plotted as desired.

As can be seen from Fig. D-l the output is not confined to a single section of the program but is produced in several different sections. Thus data is printed out or plotted shortly after it is calculated, which greatly reduces the amount of core storage required. All plots are generated by Subroutine GRAPHS which uses standard Univac 1108 plot routines.

FORTRAN listings of Program LCYC3D and Subroutines PHICFS, PRSVEL, RHS, and GRAPHS are provided at the end of this appendix.

Input Data

A precise definition of the input data required to run the computer program is given below. This input data consists of three parts: (1) the control number NOUTCF, (2) the parameters and coefficients generated by Program COEFFS3D and (3) the data describing the cases to be run (see Fig. D-l). For each input case the following information must be provided: (1) the combustion parameters n and T; (2) a series of control numbers; and (3) information describing the initial disturbance.

The control number NOUTCF determines whether the coefficients from COEFFS3D will be printed, and it appears on the first card of input. This
card is followed by the coefficient deck generated by COEFFS3D and the data describing the cases to be run. Since the coefficient data has already been described in Appendix C, it will be omitted from the following detailed description of the input. As in Appendix C the location number refers to the columns of the card. Again three formats are used for input: "A" indicates alphanumeric characters, "I" indicates integers, and "F" indicates real numbers with a decimal point. For the "I" formats the values are placed in fields of five locations, while a field of ten locations is used with the "F" formats. In either case the numbers must be placed in the rightmost locations of the allocated field.

<table>
<thead>
<tr>
<th>No. of Cards</th>
<th>Location</th>
<th>Type</th>
<th>Input Item</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-5</td>
<td>I</td>
<td>NOUTCF</td>
<td>If 0: coefficients are not printed out.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>If 1: linear coefficients only are printed out.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>If 2: all coefficients are printed out.</td>
</tr>
<tr>
<td>1</td>
<td>1-72</td>
<td>A</td>
<td>TITLE</td>
<td>Title used to label plots.</td>
</tr>
<tr>
<td>1</td>
<td>1-10</td>
<td>F</td>
<td>EN</td>
<td>Interaction index, n.</td>
</tr>
<tr>
<td>11-20</td>
<td>F</td>
<td>TAU</td>
<td>Time-lag, ( \tau ).</td>
<td></td>
</tr>
<tr>
<td>21-30</td>
<td>F</td>
<td>H</td>
<td>Time-increment for numerical integration, ( \Delta t ). *</td>
<td></td>
</tr>
<tr>
<td>31-40</td>
<td>F</td>
<td>TSTART</td>
<td>Time at which output of solutions begins.</td>
<td></td>
</tr>
<tr>
<td>41-50</td>
<td>F</td>
<td>TQUIT</td>
<td>Time at which output of solutions ends.</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1-5</td>
<td>I</td>
<td>NTEST</td>
<td>If 0: compute transient behavior.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>If 1: compute limit-cycle behavior.</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td>I</td>
<td>JMODE</td>
<td>Identifies the amplitude function used to test for limit-cycles.</td>
</tr>
</tbody>
</table>

* This value is adjusted slightly by the program to divide the interval \(-\tau \leq t \leq 0\) into NDIV equal parts.
<table>
<thead>
<tr>
<th>No. of Cards</th>
<th>Location</th>
<th>Type</th>
<th>Input Item</th>
<th>Comments</th>
</tr>
</thead>
</table>
| 11-15       | I        | NLOC |            | Determines location for wall pressure maxima and minima.  
|             |          |      |            | If 1: $z = 0, \theta = 0^\circ$  
|             |          |      |            | If 2: $z = 0, \theta = 45^\circ$  
|             |          |      |            | If 3: $z = 0, \theta = 90^\circ$  |
| 16-20       | I        | NTERMS |          | Number of amplitude functions given initial values.  
|             |          |      |            | If 0: all instability zones retained.  
|             |          |      |            | If 1: secondary zones eliminated.  |
| 21-25       | I        | NPZ  |            | Determines how secondary instability zones are handled.  
|             |          |      |            | If 0: all instability zones retained.  
|             |          |      |            | If 1: secondary zones eliminated.  |
| 26-30       | I        | NOUT |            | Determines output.  
|             |          |      |            | If 0: printed output only.  
|             |          |      |            | If $1 \leq \text{NOUT} \leq 6$: both printed and plotted output, NOUT gives number of last plot produced.  |

If $1 \leq \text{NOUT} \leq 6$ the following two cards are read:

<table>
<thead>
<tr>
<th>1</th>
<th>1-10</th>
<th>F</th>
<th>YHI(1)</th>
<th>Maximum ordinate for pressure plots.</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-20</td>
<td>F</td>
<td>YHI(5)</td>
<td></td>
<td>Maximum ordinate for velocity plots.</td>
</tr>
<tr>
<td>21-30</td>
<td>F</td>
<td>YLAB(1)</td>
<td></td>
<td>Interval for ordinate labeling of pressure plots.</td>
</tr>
<tr>
<td>31-40</td>
<td>F</td>
<td>YLAB(5)</td>
<td></td>
<td>Interval for ordinate labeling of velocity plots.</td>
</tr>
<tr>
<td>1</td>
<td>1-5</td>
<td>I</td>
<td>ITICY(1)</td>
<td>Number of ordinate tic marks for pressure plots.</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td>I</td>
<td>ITICY(5)</td>
<td>Number of ordinate tic marks for velocity plots.</td>
</tr>
<tr>
<td></td>
<td>11-15</td>
<td>I</td>
<td>NFIRST</td>
<td>Gives the number of the first plot produced.</td>
</tr>
</tbody>
</table>
|             | 16-20    | I    | NOMIT     | If 0: amplitude plot produced.  
|             |          |      |           | If 1: amplitude plot omitted.  |
The input data describing the cases to be run is given on a series of three or more cards. These cards are preceded by a title card which gives a title (TITLE) to be used to identify any plots produced by the run. This title appears before the first plot generated and does not appear on the printed output. The title card is included only for the first case of the run; on all subsequent cases it is omitted.

The first card of the series gives the interaction index, n, and the time-lag, $\tau$, for the motor under consideration (EN and TAU); the time-increment, $\Delta t$, used in the numerical integrations (H); and the times (TSTART and TQUIT) at which output begins and ends. For all cases considered in this report a time-increment (dimensionless) of $H = 0.050$ was used, which gives about 70 steps per cycle for the $1T$ mode. For $\tau = 1.7$ this input value was adjusted by the program to obtain $H = 0.04857$ which divides $-\tau <= t <= 0$ into 35 equal parts. For transient cases (NTEST = 0) printed output is given for TSTART <= t <= TQUIT. When the limit-cycle behavior is calculated (NTEST = 1), TSTART and TQUIT are measured from the time at which the limit-cycle is reached, $t_{LC}$. Thus the limit-cycle solutions are printed out for $(t_{LC} + TSTART) <= t <= (t_{LC} + TQUIT)$. Two or three cycles of limit-cycle data for the $1T$ mode are obtained with TSTART = 0 and TQUIT = 10. For plotted output, the time axis is always 10 units long, therefore (TQUIT - TSTART) > 10 to obtain plots.

The second card of the series gives the control numbers, NTEST, JMODE, NLOC, NTERMS, NPZ, and NOUT. The task to be performed by Program LCYC3D is specified by NTEST. If NTEST = 0 the transient behavior (growth or decay) of the pressure oscillation is determined, while for NTEST = 1 the program
searches for a limit-cycle amplitude. JMODE identifies the "principal" series term, the amplitude function used in the limit-cycle test. This is usually the lowest frequency mode (i.e., 1T or 1L) in the approximating series expansion. NLOC gives the location at which the amplitude-time history (maxima and minima) of the wall pressure perturbation is calculated. The number of complex series terms $A_i(t)$ receiving initial values is specified by NTERMS, while all other series terms are initially zero. The parameter NPZ determines how the secondary instability zones (phantom zones) are handled by Program LCYC3D. For NPZ = 1 the phantom zones are eliminated by dropping the combustion terms for a given mode when $\tau > \tau_{\text{cut}}$ where:

$$\tau_{\text{cut}} = \frac{2\pi}{\omega} = \frac{2\pi}{\omega} \left[ \frac{g_{mn}^2}{\sigma_e^2} + \frac{L}{\sigma_e^2} \right]^{\frac{1}{2}}$$  \hspace{1cm} (D-1)

A similar procedure was used in the axial instability studies by Lores and Zinn. The transverse instability data presented herein was obtained with NPZ = 0, while NPZ = 1 was used in the axial instability studies to facilitate comparison with the results of Ref. (3). The last control number NOUT determines which plots, if any, are produced. For NOUT = 0 no plots are produced. For $1 \leq \text{NOUT} \leq 6$, NOUT gives the number of the last plot produced, where the plots are numbered as given in Table D-1 below:

<table>
<thead>
<tr>
<th>No. of Plot (NPL)</th>
<th>Quantity Plotted</th>
<th>Axial Location</th>
<th>Azimuthal Coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pressure</td>
<td>Injector</td>
<td>0°</td>
</tr>
<tr>
<td>2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>45°</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>90°</td>
</tr>
<tr>
<td>4</td>
<td>&quot;</td>
<td>Nozzle</td>
<td>0°</td>
</tr>
<tr>
<td>5</td>
<td>Axial Velocity</td>
<td>&quot;</td>
<td>0°</td>
</tr>
<tr>
<td>6</td>
<td>Nozzle Boundary</td>
<td>&quot;</td>
<td>0°</td>
</tr>
<tr>
<td></td>
<td>Term</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The nozzle boundary term given on the last plot is discussed later in this appendix.

If plots are produced, two additional cards are needed to give the maximum and minimum values of the variables to be plotted, \( YHI(NPLOT) \) and \( YLO(NPLOT) \); the intervals for ordinate labeling (\( YLAB(NPLOT) \)); and the number of ordinate tic marks, \( ITICY(NPLOT) \). All of the plots are symmetric about the time-axis so that \( YLO(NPLOT) = -YHI(NPLOT) \), and \( ITICY(NPLOT) \) must be negative to obtain the centerline. Since the ordinate scales and labeling are the same for all pressure plots (\( NPLOT = 1,2,3,4 \)) this data is read for \( NPLOT = 1 \) only; likewise the data for the last two plots is read for \( NPLOT = 5 \) only. In addition \( NFIRST \) gives the number of the first plot produced, giving additional control over the number of plots produced. \( NOMIT \) determines whether a plot of pressure amplitude versus time (location specified by \( NLOC \)) is produced.

The remaining cards give the initial amplitudes of the complex series terms, \( A_j(t) \), needed to start the numerical integration. Only the amplitudes of the real parts, \( B_{2j-1}(t) \), are given on these cards, while the amplitudes of the imaginary parts, \( B_{2j}(t) \), are determined from the nozzle admittance condition. For each value of \( J \) the amplitudes \( AST \) and \( ACT \) are assigned to the arrays \( AS(NP) \) and \( AC(NP) \) where \( NP = 2J - 1 \). The computation of the amplitudes of the imaginary parts, \( AS(NP + 1) \) and \( AC(NP + 1) \), is discussed later. The initial values of the series terms are then calculated from the formula:

\[
B_p(t) = AS(NP) \sin(\omega_p t) + AC(NP) \cos(\omega_p t) \quad (-\pi \leq t \leq 0) \quad (D-2)
\]

where \( \omega_p \) is the acoustic frequency. The derivatives, \( dB_p/dt \), are also required for starting the numerical integration; they are obtained simply by differentiating Eq. (D-2).

The proper input for pure standing and pure spinning single-mode initial disturbances is given as follows. For a standing mode, only the \( \cos(m\theta) \) terms are retained in the series and \( NTERMS = 1 \). A single card is read giving the amplitude of the initial disturbance. For a spinning mode, both \( \sin(m\theta) \) and
cos(mω) terms are included in the series expansion. It is convenient to pair these terms such that the index J corresponds to a sin(mω) term and J + 1 corresponds to a cos(mω) term. For an initial disturbance of amplitude A spinning in the counterclockwise direction (θ increasing), NTERMS = 2 and two cards are read giving the following data:

\[
\begin{align*}
J & : \text{AST} = A \text{ and } \text{ACT} = 0 \\
J + 1 & : \text{AST} = 0 \text{ and } \text{ACT} = A
\end{align*}
\]

(D-3)

In both cases above initial amplitudes are required only for the mode initially present, and the initial amplitudes of all other modes included in the series expansion are zero.

The proper input for Program LCYC3D will be illustrated with the following example. Assuming that the velocity potential ψ is expressed in terms of the 1R, 1T, and 2T modes, it is desired to determine the limit-cycle behavior of a linearly unstable engine (n = 0.57486, \( \bar{\tau} = 1.7 \), \( \bar{u}_e = 0.2 \), L/D = 0.5) with a nozzle admittance of A = 0.02 and \( \varphi = 45^\circ \). Sample input is given for the case of a spinning 1T mode disturbance of amplitude 0.3. The principal series term is the cos(mω) term for the 1T mode (i.e., B_{011}(t)), thus JMODE = 2. Plots are desired for the pressure, axial velocity, and nozzle boundary condition at the nozzle entrance, thus NOUT = 6 and NFIRST = 4.

To run the case described above the data deck must be assembled as follows. The card specifying NOUTCF is followed by the coefficient deck produced by Program COEFFS3D; in this example it contains the information given in the sample output for COEFFS3D shown in Appendix C. The coefficient deck is followed by the data for the case to be run as shown in the sample input below:

---

* This is the same case used to illustrate Program COEFFS3D.
Coefficients in Series for $\dot{\phi}_t$, $\dot{\phi}_\theta$, and $\dot{\phi}_z$.

As seen from Eq. (13) the real parts of the time and space derivatives of the velocity potential (i.e., $\phi_t$, $\phi_r$, $\phi_\theta$, $\phi_z$) are needed in order to compute the pressure perturbation. Differentiating the complex series expansion given by Eq. (9) and evaluating at the chamber wall ($r = 1$) gives the following expansions:

$$\dot{\phi}_t = \sum_{p=1}^{N} \frac{dA_p}{dt} Z_p(z) \dot{\phi}_p(\theta) R_p(1) = \sum_{p=1}^{N} C_{t}(p,z,\theta) \frac{dA_p}{dt}$$

$$\dot{\phi}_\theta = \sum_{p=1}^{N} A_p(t) Z_p(z) \dot{\theta}_p(\theta) R_p(1) = \sum_{p=1}^{N} C_{\theta}(p,z,\theta) A_p(t)$$

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\[ \phi_z = \sum_{p=1}^{N} A_p(t) z_p(z) \Theta_p(\theta) R_p(l) = \sum_{p=1}^{N} C_z(p, z, \theta) A_p(t) \] (D-6)

where the complex coefficients \( C_t, C_\theta, \) and \( C_z \) are functions of \( z \) and \( \theta \). The quantity, \( \phi_r \), is not needed since \( \phi_r = 0 \) at the chamber wall. The complex coefficients \( C_t, C_\theta, \) and \( C_z \) are calculated by Subroutine PHICFS and are assigned to the variables, \( C1, C2, \) and \( C3 \) respectively. The coefficients in the series expansions for the corresponding real parts (i.e., \( \phi_t, \phi_\theta, \phi_z \)) are related to the complex coefficients by:

\[ C_t^{(2p-1, z, \theta)} = \text{Re}[C_t(p, z, \theta)] \]  
\[ C_t^{(2p, z, \theta)} = -\text{Im}[C_t(p, z, \theta)] \] (D-7)

where similar relations hold for \( C_\theta \) and \( C_z \). The real coefficients are stored in the arrays \( \text{CFT(NPRES, NP)}, \text{CFTH(NPRES, NP)}, \) and \( \text{CFZ(NPRES, NP)} \) where \( \text{NPRES} \) determines the location in the chamber as given in Table D-3 below:

<table>
<thead>
<tr>
<th>NPRES</th>
<th>Axial Location ((z))</th>
<th>Azimuthal Location ((\theta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0°</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>45°</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>90°</td>
</tr>
<tr>
<td>4</td>
<td>( z_e )</td>
<td>0°</td>
</tr>
<tr>
<td>5</td>
<td>( z_e )</td>
<td>45°</td>
</tr>
<tr>
<td>6</td>
<td>( z_e )</td>
<td>90°</td>
</tr>
</tbody>
</table>

Initial Amplitudes

The initial amplitudes of the real parts of the complex series terms (i.e., \( B_{2j-1}(t) \)) are specified in the input to the program. The initial
amplitudes of the imaginary parts (i.e., $B_{2j}(t)$), however, are calculated such that the nozzle admittance condition is satisfied for $-\tau \leq t \leq 0$. This is done by introducing the linear expressions for $u'$ and $p'$ into the nozzle admittance relation and assuming periodic solutions. This yields a set of linear algebraic equations relating the amplitudes of the real and imaginary parts of the complex series terms. For given values of the amplitudes of the real parts, $AS(NP)$ and $AC(NP)$, these equations are solved to obtain the amplitudes of the imaginary parts, $AS(NP + 1)$ and $AC(NP + 1)$. The following formulas are used in this calculation.

\[ AS(NJ + 1) = -(r_2a_1 - r_1a_2) / (a_1^2 + a_2^2) \]  
\[ AC(NJ + 1) = (r_1a_1 + r_2a_2) / (a_1^2 + a_2^2) \]  

where

\[ r_1 = a_3 \left[ AC(NJ) \right] - a_4 \left[ AS(NJ) \right] \]  
\[ r_2 = a_3 \left[ AC(NJ) \right] - a_4 \left[ AS(NJ) \right] \]

and

\[ a_1 = (1 + \gamma Y_{r\tilde{u}_e})CFZ(NPRES, NJ+1) - \gamma Y_{i\omega_j}CFT(NPRES, NJ+1) \]  
\[ a_2 = \gamma Y_{r\omega_j}CFT(NPRES, NJ+1) + \gamma Y_{i\tilde{u}_e}CFZ(NPRES, NJ+1) \]  
\[ a_3 = -(1 + \gamma Y_{r\tilde{u}_e})CFZ(NPRES, NJ) + \gamma Y_{i\omega_j}CFT(NPRES, NJ) \]  
\[ a_4 = \gamma Y_{r\omega_j}CFT(NPRES, NJ) + \gamma Y_{i\tilde{u}_e}CFZ(NPRES, NJ) \]

In Eqs. (D-8) through (D-10) $\omega_j$ is the acoustic frequency and CFT and CFZ are
the coefficients in the series for $\varphi_t$ and $\varphi_z$ computed previously. The above conditions are applied at a pressure anti-node for each series term, therefore $NPRES = 4 (z = z_e, \theta = 0^\circ)$ for a $\cos(m\theta)$ term and $NPRES = 6 (z = z_e, \theta = 90^\circ)$ for a $\sin(m\theta)$ term.

For nozzles with phase shifts of $\varphi = 90^\circ$ and $\varphi = 270^\circ$ the quantity $a_1^2 + a_2^2$ vanishes and Eqs. (D-8) become indeterminate. In these cases the amplitudes of the imaginary parts are given by:

$$AS(NJ + 1) = AC(NJ)$$

$$AC(NJ + 1) = AS(NJ)$$

which provides a good approximation to the nozzle admittance condition.

**Integration of the Differential Equations**

For purposes of numerical integration Eqs. (C-38) are written as an equivalent system of first order differential equations as follows:

$$\frac{dB_j}{dt} = B'_j$$

$$\frac{dB'_j}{dt} = f_j(B_p, B'_p)$$

where the dependent variables are now $B_j$ and $B'_j$. These equations are solved numerically using the fourth order Runge-Kutta method. Due to the presence of retarded variables in Eqs. (D-12) and (D-13) the formulas (see Ref. 21) used in the Runge-Kutta method must be slightly modified.

The appropriate formulas for applying the Runge-Kutta method to problems involving a time-delay are readily obtained by considering a single equation of the following form:

$$\frac{dx}{dt} = f(x,t) + g[x(t - \tau)]$$

(D-14)
Noting that at any step of the integration the value of $x(t - \tau)$ has already been determined from previous steps, the function $g$ can be considered to be a known function of time $g(t)$.

Since $x(t)$ is computed only at discrete points $x_n(t_n)$ it is desired that the retarded variable $x(t_n - \tau)$ will coincide with such previously computed points. This can be accomplished by choosing the step-size $\Delta t$ such that it divides the time-lag $\tau$ into $k$ equal increments. Thus $\tau = k\Delta t$ and the Runge-Kutta formulas which apply to Eq. (D-14) can now be written as:

\[
x_{n+1} = x_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)
\]

\[
k_1 = \left\{ f(x_n, t_n) + g(x_{n-k}) \right\} \Delta t
\]

\[
k_2 = \left\{ f(x_n + \frac{k_1}{2}, t_n + \frac{\Delta t}{2}) + g(x_{n-k+\frac{1}{2}}) \right\} \Delta t \quad (D-15)
\]

\[
k_3 = \left\{ f(x_n + \frac{k_2}{2}, t_n + \frac{\Delta t}{2}) + g(x_{n-k+\frac{1}{2}}) \right\} \Delta t
\]

\[
k_4 = \left\{ f(x_n + k_3, t_n + \Delta t) + g(x_{n-k+1}) \right\} \Delta t
\]

Equations (D-15) are readily extended to handle the system of equations given by Eqs. (D-12) and (D-13). It is seen from Eqs. (D-15) that $k$ values of the dependent variables prior to the initial values are needed to start the integration.

Although the initial wave shape can be an arbitrary function of time, it is assumed that initially the mode-amplitudes are sinusoidal functions of time oscillating with the natural frequency $\omega_j$. Thus each mode-amplitude function is expressed in the following form:

\[
B_j(t) = A_0(J)\sin(\omega_j t) + A_0(J)\cos(\omega_j t)
\]

(D-16)
\[ B_j(t) = w_j \left[ A_S(j) \cos(w_j t) - A_C(j) \sin(w_j t) \right] \]

where \(-\tau \leq t \leq 0\).

In Program LCYC3D both the functions \(B_j(t)\) and the derivatives \(B_j'(t)\) are stored in the same array \(U(I,J)\). The \(B_j(t)\) (N functions) are stored in the first half of the array \((1 \leq J \leq N)\), while the remaining space \((N + 1 \leq J \leq 2N)\) is used to store the values of \(B_j'(t)\). Thus for a given value of \(j\) \((1 \leq j \leq N)\), \(B_j(t)\) is stored in \(U(I,J)\) and \(B_j'(t)\) is stored in \(U(I,J+N)\). In addition the retarded variables \(B_j'(t-\tau)\) are stored in the array \(RV(J,K)\) as follows:

\[ RV(J,1) = B_j'(t-\tau) \]
\[ RV(J,2) = RV(J,3) = B_j'(t-\tau + \Delta t/2) \]  
\[ RV(J,4) = B_j'(t-\tau + \Delta t) \]

The values of \(B_j'(t-\tau + \Delta t/2)\) are computed from \(B_j'(t-\tau)\), \(B_j'(t-\tau + \Delta t)\), and \(B_j'(t-\tau + 2\Delta t)\) using a three-point interpolation.

**Pressure and Axial Velocity Perturbations**

From the calculated time dependence of the series terms Program LCYC3D computes the dimensionless pressure perturbation, \(p'\), with the aid of Eqs. (D-4) through (D-6) and either Eq. (13) for NDROPS = 0 or Eq. (A-6) for NDROPS = 1. The pressure is calculated at the injector face \((z = 0)\) and the nozzle entrance plane \((z = z_e)\) for three angular positions along the periphery of the chamber \((i.e., r = 1; \theta = 0^\circ, 45^\circ, 90^\circ)\). The results are stored in the array \(PRESS(NPRES)\) where \(NPRES\) gives the location according to Table D-3. The axial velocity perturbation at the nozzle entrance, \(u'_e\), is calculated for \(\theta = 0^\circ, 45^\circ, 90^\circ\) using the relation \(u' = \varphi_z\) and Eq. (D-6), and the results are stored in \(AXVEL(K)\), where \(K = NPRES-3\). In addition the quantity, \(Re[-\gamma X\ddot{\varphi}_t]\), is calculated at the nozzle entrance for \(\theta = 0^\circ\) and assigned to the variable \(YPHI\). From Eq. (2) it is seen that \(YPHI\) is the axial velocity.
at the nozzle entrance (i.e., \( u'_e \)) if the nozzle admittance condition is exactly satisfied. Since the solutions generated by Program LCYC3D are approximate, the difference between \( u'_e \) and \( YPHI \) is a measure of the accuracy of this approximation at the nozzle boundary.

**Maximum and Minimum Values**

In order to determine the transient behavior and limit-cycle amplitudes it is necessary to follow the growth or decay of the amplitudes of the series terms and the pressure perturbation. The maxima and minima of the principal series term (specified by JMODE) are assigned to the array \( UMAX(MAXNO) \) where \( MAXNO \) is a counter variable. For the pressure perturbation, maximum and minimum values at the location specified by NLOC are stored in \( PMAX(MAXP) \), and the corresponding times of maximum and minimum are stored in \( TIMAX(MAXP) \). Since the solutions are calculated only at discrete points, the maximum and minimum values are computed using a three-point interpolation scheme.

**Calculation of Limit-Cycle Amplitude**

A limit-cycle amplitude is calculated by specifying an initial disturbance and continuing the step-by-step integration of Eqs. (D-12) and (D-13) until a periodic solution is obtained; that is, the amplitude of the oscillation remains essentially constant. The test for convergence to a limit cycle is performed upon a single series term, usually the most important term in the series, in the following manner. After the first 500 integration steps, usually about 10 cycles for the IT mode, the amplitude of the principal series term \( A_1 \) is compared with its amplitude after 250 integration steps \( A_0 \). If the change in amplitude \( |A_1 - A_0| \) is greater than the maximum permissible change \( \epsilon \), the calculations are continued and the change in amplitude during the next 250 integration steps is calculated. The process is repeated until \( |A_k - A_{k-1}| < \epsilon \) at which point the computation is terminated. The amplitudes used in the above calculations are determined by averaging the absolute values of \( UMAX(MAXNO) \) over the last two complete cycles for each 250 integration steps. A value of \( \epsilon = 0.001 \) is used in Program LCYC3D which gives sufficient accuracy for most cases.
Output

Printed Output. The printed output produced by Program LCYC3D consists of the five sections discussed below.

Section 1 is a restatement of the input from Program COEFF3D. It includes the following information: (a) the ratio of specific heats (GAMMA), the mean flow Mach number at the nozzle entrance (UE), the dimensionless chamber length (ZE), the length of the combustion zone as a fraction of the chamber length (ZCOMB), and the number of series terms (real) NJMAX; (b) a statement regarding the presence or absence of the droplet momentum source; (c) the parameters which describe and identify each term in the series expansion; (d) the nozzle admittance (YR and YI) and the axial acoustic eigenvalue (EPS and ETA) for each series term; (e) the nonzero linear coefficients, C(KC, NJ, NP); and (f) the nonzero nonlinear coefficients, D(NJ, NP, NQ). The nonlinear coefficients are omitted from the output for NOUTCF = 1, and no coefficients are printed out for NOUTCF = 0.

Section 2 gives the coefficients needed for computation of the wall pressure waveforms; that is, the coefficients in the series for $\varphi_t$, $\varphi_\theta$, and $\varphi_z$. These are given for each of the NJMAX series terms at each of the six locations specified by NPRES (see Table D-3).

Section 3 gives the initial amplitudes (AS(J) and AC(J)) of all series terms included in the assumed initial disturbance. This section also states whether the limit-cycle behavior is calculated and whether plots are produced.

Section 4 gives the time-dependent solutions for the following quantities: (a) the injector pressure perturbation at $\theta = 0^\circ$, $45^\circ$, $90^\circ$; (b) the nozzle pressure perturbation at $\theta = 0^\circ$, $45^\circ$, $90^\circ$; (c) the nozzle axial velocity perturbation at $\theta = 0^\circ$, $45^\circ$, $90^\circ$; and (d) the nozzle boundary term, $Re[-\gamma Y_t]$, at $\theta = 0^\circ$. This output is given in two parts: (1) the initial values for $-\tau \leq t \leq 0$ and (2) the solutions for $t_1 \leq t \leq t_f$, where $t_1$ and $t_f$ are determined by the input parameters TSTART and TQUIT (see discussion on Input). On the first page of each part a heading gives the interaction index, n, and the time-lag, $\tau$, and the chamber parameters, $\gamma$, $\bar{u}_e$, and L/D.

Section 5 gives the time history of the pressure amplitude (maximum and minimum values) for the chamber location specified by NLOC. This information
is printed as an array of number pairs giving the value of the pressure maximum or minimum (upper number) and the corresponding time of maximum or minimum (lower number). This information is useful in determining the growth (or decay) rate of the transient solutions, and it provides a check on the convergence of the solution to a limit-cycle.

**Plotted Output.** According to the values of NOUT and NFIRST the pressure and axial velocity waveforms given in Section 4 of the printed output may be plotted using a Calcomp plotter. The data over the dimensionless time interval for printed output, \( t_i \leq t \leq t_f \), is plotted in sections of 10 units in length beginning at \( t = t_i \). Thus for each quantity plotted, \( N \) plots are produced where \( N \) is the largest multiple of 10 contained in the interval \( t_i \leq t \leq t_f \). The data left over (i.e., for \( t_i + 10N \leq t \leq t_f \)) is not plotted. All quantities to be plotted for a given time interval are plotted before proceeding to the next time interval.

The data given in Section 5 of the printed output (pressure maxima only) is also plotted if NOUT > 0 and NOMIT = 0. The abscissa and ordinate ranges for this plot are not specified in the input, but are calculated such that all of the data falls within these ranges. This plot is always the last plot produced.

All of the above plots are scaled to fit on standard 8\( \frac{1}{2} \)" x 11" paper and scissor-lines are plotted for trimming plots to this size. The data is plotted as individual points using a small circle symbol, and all of the values computed during the given time interval are plotted. Before the first plot is produced the identifying title (see Input) is printed.

**Sample Output.** The following sample output illustrates the printed and plotted output produced by Program LCYC3D for the sample input given in Table D-2.
Table D-4. Sample Output, Section I.

**GAMMA = 1.200, UE = 0.200, ZE = 1,00000, ZCOMB = 1.00, NJMAX = 10**

**DROPLET MOMENTUM SOURCE IS NEGLECTED**

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<thead>
<tr>
<th>NAME</th>
<th>J</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>NS</th>
<th>SMN</th>
<th>JM(SMN)</th>
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<td>.58187</td>
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<td>2</td>
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<td>2</td>
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</tbody>
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<table>
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<th>YI</th>
<th>EPS</th>
<th>ETA</th>
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<tbody>
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<td>.01414</td>
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<td>.19451</td>
</tr>
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<tr>
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<td>.01414</td>
<td>.01414</td>
<td>.10617</td>
<td>.25115</td>
</tr>
<tr>
<td>5</td>
<td>.01414</td>
<td>.01414</td>
<td>.11993</td>
<td>.28170</td>
</tr>
</tbody>
</table>

**NUMBER OF COEFFICIENTS C(1,NJ, NP) IS 10**

\[
\begin{align*}
C(1, 1, 1) &= 3.39060 \\
C(1, 2, 2) &= 3.39060 \\
C(1, 3, 3) &= 3.39060 \\
C(1, 4, 4) &= 3.39060 \\
C(1, 5, 5) &= 9.33021 \\
C(1, 6, 6) &= 9.33021 \\
C(1, 7, 7) &= 9.33021 \\
C(1, 8, 8) &= 9.33021 \\
C(1, 9, 9) &= 14.68491 \\
C(1, 10, 10) &= 14.68491 \\
\end{align*}
\]

**NUMBER OF COEFFICIENTS C(2,NJ, NP) IS 10**

\[
\begin{align*}
C(2, 1, 1) &= .26153 \\
C(2, 2, 2) &= .26153 \\
C(2, 3, 3) &= .26153 \\
C(2, 4, 4) &= .26153 \\
C(2, 5, 5) &= .26457 \\
C(2, 6, 6) &= .26457 \\
C(2, 7, 7) &= .26457 \\
C(2, 8, 8) &= .26457 \\
C(2, 9, 9) &= .26654 \\
C(2, 10, 10) &= .26654 \\
\end{align*}
\]

**NUMBER OF COEFFICIENTS C(3,NJ, NP) IS 10**

\[
\begin{align*}
C(3, 1, 1) &= .24000 \\
C(3, 2, 2) &= .24000 \\
C(3, 3, 3) &= .24000 \\
C(3, 4, 4) &= .24000 \\
C(3, 5, 5) &= .24000 \\
C(3, 6, 6) &= .24000 \\
C(3, 7, 7) &= .24000 \\
\end{align*}
\]
Table D-4. (Continued)

\[
\begin{array}{l}
\text{C(3, 8, 8) = .24000} \\
\text{C(3, 9, 9) = .24000} \\
\text{C(3,10,10) = .24000}
\end{array}
\]

**NUMBER OF COEFFICIENTS D(NJ, NP, NQ) IS 50**

| \( \text{D(1, 1, 7)} \) | \(-1.73504\) |
| \( \text{D(1, 1, 9)} \) | \(-2.33866\) |
| \( \text{D(1, 3, 5)} \) | \(1.73504\) |
| \( \text{D(1, 5, 3)} \) | \(1.49783\) |
| \( \text{D(1, 7, 1)} \) | \(-1.49783\) |
| \( \text{D(1, 9, 1)} \) | \(-1.96281\) |
| \( \text{D(2, 2, 8)} \) | \(-1.73505\) |
| \( \text{D(2, 2,10)} \) | \(-2.33867\) |
| \( \text{D(2, 4, 6)} \) | \(1.73505\) |
| \( \text{D(2, 6, 4)} \) | \(1.49784\) |
| \( \text{D(2, 8, 2)} \) | \(-1.49784\) |
| \( \text{D(2,10, 2)} \) | \(-1.96282\) |
| \( \text{D(3, 1, 5)} \) | \(1.73504\) |
| \( \text{D(3, 3, 7)} \) | \(1.73504\) |
| \( \text{D(3, 3, 9)} \) | \(-2.33866\) |
| \( \text{D(3, 5, 1)} \) | \(1.49783\) |
| \( \text{D(3, 7, 3)} \) | \(1.49783\) |
| \( \text{D(3, 9, 3)} \) | \(-1.96281\) |
| \( \text{D(4, 2, 6)} \) | \(1.73505\) |
| \( \text{D(4, 4, 6)} \) | \(1.73505\) |
| \( \text{D(4, 4,10)} \) | \(-2.33867\) |
| \( \text{D(4, 6, 2)} \) | \(1.49784\) |
| \( \text{D(4, 8, 4)} \) | \(1.49784\) |
| \( \text{D(4,10, 4)} \) | \(-1.96282\) |
| \( \text{D(5, 1, 3)} \) | \(-1.13133\) |
| \( \text{D(5, 3, 1)} \) | \(-1.13133\) |
| \( \text{D(5, 5, 9)} \) | \(-3.07465\) |
| \( \text{D(5, 9, 5)} \) | \(-2.81865\) |
| \( \text{D(6, 2, 4)} \) | \(-1.13132\) |
| \( \text{D(6, 4, 2)} \) | \(-1.13132\) |
| \( \text{D(6, 6,10)} \) | \(-3.07469\) |
| \( \text{D(6,10, 6)} \) | \(-2.81868\) |
| \( \text{D(7, 1, 1)} \) | \(1.13133\) |
| \( \text{D(7, 3, 3)} \) | \(-1.13133\) |
| \( \text{D(7, 7, 9)} \) | \(-3.07465\) |
| \( \text{D(7, 9, 7)} \) | \(-2.81865\) |
| \( \text{D(8, 2, 2)} \) | \(1.13132\) |
| \( \text{D(8, 4, 4)} \) | \(-1.13132\) |
| \( \text{D(8, 6,10)} \) | \(-3.07469\) |
| \( \text{D(8,10, 6)} \) | \(-2.81866\) |
| \( \text{D(9, 1, 1)} \) | \(1.04087\) |
| \( \text{D(9, 3, 3)} \) | \(1.04087\) |
| \( \text{D(9, 5, 5)} \) | \(-2.1090\) |
| \( \text{D(9, 7, 7)} \) | \(-2.1090\) |
| \( \text{D(9, 9, 9)} \) | \(4.18784\) |
| \( \text{D(10, 2, 2)} \) | \(1.04087\) |
| \( \text{D(10, 4, 4)} \) | \(1.04087\) |
| \( \text{D(10, 6, 6)} \) | \(-2.1091\) |
| \( \text{D(10, 8, 8)} \) | \(-2.1091\) |
| \( \text{D(10,10,10)} \) | \(4.18793\) |
Table D-5. Sample Output, Section 2.

COEFFICIENTS FOR COMPUTATION OF WALL PRESSURE WAVEFORMS

<table>
<thead>
<tr>
<th>J</th>
<th>Z (DEGREES)</th>
<th>COEFFICIENTS IN SERIES FOR:</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>THETA TIME</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DERIVATIVE</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
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<td>0.0</td>
</tr>
<tr>
<td>5</td>
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</tr>
<tr>
<td>6</td>
<td>0.000</td>
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</tr>
<tr>
<td>7</td>
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<td>0.0</td>
</tr>
<tr>
<td>8</td>
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</tr>
<tr>
<td>9</td>
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</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>0.0</td>
</tr>
</tbody>
</table>

| 1 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 2 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 3 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 4 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 5 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 6 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 7 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 8 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
| 9 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |
|10 | 0.000       | 45.0       | 411442         | 411442         | 0.0000000       |

| 1 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 2 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 3 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 4 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 5 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 6 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 7 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 8 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
| 9 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |
|10 | 0.000       | 90.0       | 5818700        | 0.0000000       | 0.0000000       |

| 1 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 2 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 3 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 4 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 5 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 6 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 7 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 8 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
| 9 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |
|10 | 1.000       | 0.0        | 0.0000000       | 0.0000000       | 0.0000000       |

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Table D-5. (Continued)

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Table D-6. Sample Output, Section 3.

INITIAL CONDITIONS ARE OF THE FORM:

\[ U(i,j) = AC(j) \cdot \cos(FREQ \cdot T) + AS(j) \cdot \sin(FREQ \cdot T)) \cdot \exp(DAMP \cdot T) \]

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THE LIMIT-CYCLE BEHAVIOR IS CALCULATED.

THIS RUN PRODUCES PLOTTED OUTPUT.
Table D-7. Sample Output, Section 4.

**Combustion Parameters:** Interaction Index = 0.57486

**Motor Parameters:**
- Gamma = 1.20000
- Exit Mach Number = 0.20000
- Length/Diameter = 0.50000

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Figure D-2. Sample Pressure Plot.
Figure D-3. Sample Amplitude Plot.
*************** PROGRAM LCYC3D ***************

THIS PROGRAM CALCULATES THE NONLINEAR BEHAVIOR OF
TRANSVERSE, AXIAL, OR COMBINED LONGITUDINAL-TRANSVERSE
INSTABILITIES IN A CYLINDRICAL COMBUSTION CHAMBER WITH
UNIFORM PROPELLANT INJECTION, DISTRIBUTED COMBUSTION
PROCESS, AND A CONVENTIONAL NOZZLE. THE COMBUSTION PROCESS
IS DESCRIBED BY CROCCO'S TIME-LAG MODEL. BOTH TRANSIENT
AND LIMIT-CYCLE SOLUTIONS ARE CALCULATED.

THE FOLLOWING INPUTS ARE REQUIRED:

1. THE CONTROL NUMBER, NOUTCF.
2. THE COEFFICIENTS FROM PROGRAM COEFFS3D.
3. THE DATA DECK.

NOUTCF DETERMINES PRINTOUT OF COEFFICIENTS:
   IF NOUTCF = 0 COEFFICIENTS ARE NOT PRINTED OUT.
   IF NOUTCF = 1 LINEAR COEFFICIENTS ONLY ARE PRINTED OUT.
   IF NOUTCF = 2 ALL COEFFICIENTS ARE PRINTED OUT.

THE DATA DECK CONSISTS OF THE FOLLOWING CARDS:

FIRST CARD:

EN IS THE INTERACTION INDEX.
TAU IS THE TIME LAG.
H IS THE INTEGRATION STEP SIZE.
TSTART IS THE TIME AT WHICH OUTPUT STARTS.
TQUIT IS THE TIME AT WHICH COMPUTATIONS ARE TERMINATED.

SECOND CARD:

NTEST IS TASK CONTROL NUMBER:
   IF NTEST = 0 COMPUTE TRANSIENT BEHAVIOR.
   IF NTEST = 1 COMPUTE THE LIMIT-CYCLE BEHAVIOR.

JMODE IS THE MODE-AMPLITUDE USED TO TEST FOR LIMIT-CYCLES.

NLOC DETERMINES THE LOCATION OF THE WALL PRESSURE MAXIMA
AND MINIMA:
   IF NLOC = 1 LOCATION IS Z = 0, THETA = 0 DEGREES.
   IF NLOC = 2 LOCATION IS Z = 0, THETA = 45 DEGREES.
   IF NLOC = 3 LOCATION IS Z = 0, THETA = 90 DEGREES.

NTERMS DETERMINES THE NUMBER OF TERMS GIVEN INITIAL VALUES.

NPZ DETERMINES HOW SECONDARY STABILITY ZONES (PHANTOM
ZONES) ARE HANDLED.
   IF NPZ = 0 PHANTOM ZONES ARE RETAINED.
   IF NPZ = 1 PHANTOM ZONES ARE ELIMINATED.

NOUT IS THE OUTPUT CONTROL NUMBER.
   IF NOUT = 0 PRINTED OUTPUT ONLY.
   IF NOUT > 0 BOTH PRINTED AND FLOTTED OUTPUT, NOUT
DETERMINES THE NUMBER OF THE LAST PLOT PRODUCED.

DATA FOR SETTING UP PLOTS (THIRD AND FOURTH CARDS):

YHI(1) IS THE MAXIMUM ORDIminate FOR PRESSURE PLOTS.
YHI(5) IS THE MAXIMUM ORDIminate FOR VELOCITY PLOTS.
NOTE: THE ORDImate SCALES FOR PRESSURE AND VELOCITY PLOTS ARE SYMMETRIC ABOUT ZERO.
YLAB IS THE INTERVAL FOR ORDImate LABELING FOR ABOVE PLOTS.
ITY CY IS THE NUMBER OF ORDImate TIC MARKS FOR ABOVE PLOTS.
NOTE: ITICY SHOULD BE NEGATIVE FOR PRESSURE AND VELOCITY PLOTS TO OBTAIN CENTERLINE.
NFIRST IS THE NUMBER OF THE FIRST PLOT PRODUCED.
NOMIT DETERMINES WHETHER AMPLITUDE PLOT IS PRODUCED:
IF NOMIT = 0 AMPLITUDE PLOT IS PRODUCED.
IF NOMIT = 1 AMPLITUDE PLOT IS OMITTED.

INITIAL AMPLITUDES OF F-FUNCTIONS (REMAINING CARDS):

AS(J) IS THE AMPLITUDE OF THE SINE TERM.
AC(J) IS THE AMPLITUDE OF THE COSINE TERM.

COMPLEX YNOZ(10), B(10), C1, C2, C3, CPHT(10), CSUM, A
DIMENSION L(10), N(10), S(10), NAME(10), AS(20), AC(20),
1 UC(20,40), AAC(4), Y(40), FZ(4,40), YP(40), UZ(40),
2 CP(3,20,20), FR01(20), DMP1(20), UMAX(500), UAVEG(100),
3 Z(6), ANGLE(6), THETA(6), CFT(6,20), YI(20),
4 CFTH(6,20), CFZ(6,20), PRESS(6), AXVEL(3), YR(20),
5 TPL0T(500), YPL0T(6,500), DUMMYT(500), DUMMYY(500),
6 IBUF(3000), ITT(4), IITY1(7), IITY2(7), IITY3(7),
7 IITY4(7), IITY5(6), TAUCUT(20), IITY6(6),
8 ITP(3), TITLE(12), PRS(500), Tl(500), FMAX(500),
9 TIMAX(500), YL0(6), YHI(6), YLAB(6), ITICY(6),

COMMON RIV(20,4), C(3,20,20), D(20,400),
1 KFMAX(3,20), IC(3,20,20), KPME(20),
2 IDP(20,400), IDQ(20,400)
COMMON /BLK2/ M(10), N5(10), SJ(10), B
COMMON /BLK3/ NJMAX, NLMAX, GAMMA, COEF(3,20)

DATA
1 IYT1/INJECTOR PRESSURE PERTURBATION, THETA = 0'/
2 IYT2/INJECTOR PRESSURE PERTURBATION, THETA = 45'/
3 IYT3/INJECTOR PRESSURE PERTURBATION, THETA = 90'/
4 IYT4/NOZZLE PRESSURE PERTURBATION, THETA = 0'/
5 IYT5/NOZZLE AXIAL VELOCITY, THETA = 0'/
6 IYT6/NOZZLE B.C. (RE(-GAMMA*Y*PHIT)) AT THETA = 0'
LAST = 250
ERR = 0.001
TDEL = 10.0
NPT = 0
AA(1) = 0.0
AA(2) = 0.5
AA(3) = 0.5
AA(4) = 1.0
PI = 3.1415927
READ (5,5003) NOUTCF

*********** COEFFICIENT INPUT SECTION ****************************

THIS VERSION OF LCYC3D READS THE COEFFICIENT DATA FROM
A FASTRAND FILE GENERATED BY PROGRAM COEFF3D. TO READ
THIS DATA FROM CARDS, USE READ (5, XXX) INSTEAD OF
READ (9, XXX) IN THIS SECTION.

INPUT OF MOTOR PARAMETERS AND NUMBER OF TERMS.
READ (9,5001) GAMMA, UE, ZE, ZCOMB, NDROPS, NJMAX
WRITE (6,6001) GAMMA, UE, ZE, ZCOMB, NDROPS, NJMAX
IF (NDROPS .EQ. 0) WRITE (6,6030)
IF (NDROPS .EQ. 1) WRITE (6,6031)
NU = 2 * NJMAX
JMX = NJMAX/2
RLD = 0.5 * ZE

WRITE (6,6002)

INPUT OF DESCRIPTION OF SERIES EXPANSION.
DO 10 K = 1, JMX
READ (9,5002) NJ, L(NJ), M(NJ), N(NJ), NS(NJ), S(NJ), SJ(NJ),
1 NAME(NJ)
WRITE (6,6003) NAME(NJ), NJ, L(NJ), M(NJ), N(NJ), NS(NJ),
1 S(NJ), SJ(NJ)
10 CONTINUE

WRITE (6,6010)
DO 15 K = 1, JMX
READ (9,5010) J, YNOZ(J), B(J)
WRITE (6,6015) J, YNOZ(J), B(J)
NJ = (2 * J) - 1
YR(NJ) = REAL(YNOZ(J))
YI(NJ) = AIMAG(YNOZ(J))
YR(NJ+1) = YR(NJ)
YI(NJ+1) = YI(NJ)
15 CONTINUE
ZERO LINEAR COEFFICIENT ARRAYS.
DO 20 KC = 1, 3
DO 20 NJ = 1, 20
DO 20 NP = 1, 20
C(KC,NJ,NP) = 0.0
CP(KC,NJ,NP) = 0.0
20 CONTINUE

ZERO NONLINEAR COEFFICIENT ARRAY.
DO 30 NJ = 1, 20
DO 30 NPQ = 1, 400
D(NJ,NPQ) = 0.0
30 CONTINUE

INPUT OF LINEAR COEFFICIENTS.
DO 40 KC = 1, 3
READ (9,5003) KMAX
IF (NOUTCF .GT. 0) WRITE (6,6004) KC, KMAX
IF (KMAX .LE. 0) GO TO 40
DO 45 K = 1, KMAX
READ (9,5004) NJ, NP, CP(KC,NJ,NP)
IF (NOUTCF .GT. 0) WRITE (6,6005) KC, NJ, NP, CP(KC,NJ,NP)
45 CONTINUE
40 CONTINUE

INPUT OF NONLINEAR COEFFICIENTS.
READ (9,5003) NLMAX
IF (NOUTCF .EQ. 2) WRITE (6,6006) NLMAX
IF (NLMAX .EQ. 0) GO TO 50
DO 52 NJ = 1, 20
KPQMAX(NJ) = 0
52 CONTINUE
DO 55 K = 1, NLMAX
READ (9,5005) NJ, NP, NQ, DT
IF (NOUTCF .EQ. 2) WRITE (6,6007) NJ, NP, NQ, DT
KPQMAX(NJ) = KPQMAX(NJ) + 1
KPO = KPQMAX(NJ)
IDP(NJ,KPO) = NP
IDQ(NJ,KPO) = NQ
D(NJ,KPO) = DT
55 CONTINUE
50 CONTINUE

*************** PRESSURE COEFFICIENT SECTION ***********************

CALCULATE SPATIAL COORDINATES FOR PRESSURE COMPUTATION.
DO 51 NPRES = 1, 3
Z(NPRES) = 0.0
RTHETA = NPRES - 1
**DATA INPUT SECTION**

**ZERO INITIAL VALUE AND FREQUENCY ARRAYS**

```fortran
5 DO 57 K = 1, NJMAX
   AS(K) = 0.0
   AC(K) = 0.0
   FREQ(K) = 0.0
57 CONTINUE
```

**READ CO-PRODUCTION AND CONTROL PARAMETERS**

```fortran
READ (5,5006, END = 300) EN, TAU, H, TSTART, TQUIT
```

**READ CONTROL NUMBERS**

```fortran
READ (5,5008) NTEST, JMODE, NLOC, NTERMS, NPZ, NOUT
```

**JMODE = (2 * JMODE) - 1**
JPMODE = JMODE + NJMAX
IF (NOUT GT 0) NPT = 1

IF (NOUT EQ 0) GO TO 9
READ DATA FOR SETTING UP PLOTS.
READ (5,5009) YHI(1), YHI(5), YLAB(1), YLAB(5)
READ (5,5008) ITICY(1), ITICY(5), NFIRST, NOMIT

********** INITIAL AMPLITUDES SECTION **********

9 DO 58 K = 1, NTERMS

INPUT INITIAL AMPLITUDES FOR F-FUNCTIONS.
READ (5,5007) J, AST, ACT
NJ = (2 * J) - 1
AS(NJ) = AST
AC(NJ) = ACT

CALCULATE FREQUENCY AND DAMPING.
RL = L(J)
AX = RL * PI/ZE
AXSQ = AX * AX
SSQ = S(J) * S(J)
FRQ1(NJ) = SQRT(SSQ + AXSQ)
DMP1(NJ) = 0.0
FRQ1(NJ+1) = FRQ1(NJ)
DMP1(NJ+1) = DMP1(NJ)

CALCULATE INITIAL AMPLITUDES FOR G-FUNCTIONS.

IF (FRQ1(NJ)) 58, 58, 581
581 GYRU = GAMMA*YR(NJ)*UE
GYIF = GAMMA*YI(NJ)*FRQ1(NJ)
GYRF = GAMMA*YR(NJ)*FRQ1(NJ)
GYIU = GAMMA*YI(NJ)*UE

NPRES = 4
IF (NS(J) EQ 1) NPRES = 6

A1 = (1.0 + GYRU)*CFZ(NPRES,NJ+1)
1 - GYIF*CFT(NPRES,NJ+1)
A2 = GYRF*CFT(NPRES,NJ+1) + GYIU*CFZ(NPRES,NJ+1)
A3 = -(1.0 + GYRU)*CFZ(NPRES,NJ) + GYIF*CFT(NPRES,NJ)
A4 = GYRF*CFT(NPRES,NJ) + GYIU*CFZ(NPRES,NJ)

DET = A1*A1 + A2*A2
IF (DET LT 0.0000301) GO TO 583
R1 = A3*AC(NJ) - A4*AS(NJ)
R2 = -A4*AC(NJ) - A3*AS(NJ)
AC(NJ+1) = (R1*A1 + R2*A2)/DET
AS(NJ+1) = -(R2*A1 - R1*A2)/DET
GO TO 58
583 AC(NJ+1) = -AS(NJ)
    AS(NJ+1) = AC(NJ)

58 CONTINUE

OUTPUT OF INITIAL AMPLITUDES.
WRITE (6,6016)
DO 590 J = 1, NJMAX
     IF (AS(J)) 591, 592, 591
591 WRITE (6,6017) J, IMPI(J), FRQI(J), AC(J), AS(J)
592 CONTINUE
     IF (NTEST .EQ. 0) WRITE (6,6025)
     IF (NTEST .EQ. 1) WRITE (6,6026)
     IF (NPZ .EQ. 1) WRITE (6,6028)
     IF (NOUT .GE. 1) WRITE (6,6027)

************* LINEAR COEFFICIENTS SECTION ***********************

DO 59 KC = 1, 3
DO 59 NJ = 1, 10
KMAX(KC,NJ) = 0
59 CONTINUE

IF (NPZ .EQ. 0) GO TO 605
DO 602 J = 1, JMOM
NJ = (2 * J) - 1
RL = L(J)
AX = RL * PI/ZE
AXS = AX * AX
SSQ = S(J) * S(J)
OMEGA = SQRT(SSQ + AXS)
TAUCUT(NJ) = 2.0 * PI/OMEGA
TAUCUT(NJ+1) = TAUCUT(NJ)
602 CONTINUE

DO 604 NJ = 1, NJMAX
DO 604 NP = 1, NJMAX
IF (TAU .GT. TAUCUT(NP)) CP(3,NJ,NP) = 0.0
604 CONTINUE

COMPUTE LINEAR COEFFICIENTS FOR GIVEN VALUES OF EN AND TAU.
605 DO 60 NJ = 1, NJMAX
DO 60 NP = 1, NJMAX
CT = CP(1,NJ,NP)
IF (CT) 61, 62, 61
61 KPMAX(1,NJ) = KPMAX(1,NJ) + 1
KP = KPMAX(1,NJ)
IC(1,NJ,KP) = NP
C(1,NJ,KP) = CT
62 CT = CP(2,NJ,NP) - EN*CP(3,NJ,NP)
IF (CT) 63, 64, 63
63 KPMAX(2,NJ) = KPMAX(2,NJ) + 1
KP = KPMAX(2,NJ)
IC(2,NJ,KP) = NP
C(2,NJ,KP) = CT
64 CT = EN * CP(3,NJ,NP)
IF (CT) 65, 66, 65
65 KPMAX(3,NJ) = KPMAX(3,NJ) + 1
KP = KPMAX(3,NJ)
IC(3,NJ,KP) = NP
C(3,NJ,KP) = CT
66 CONTINUE

C ************* STEP-SIZE COMPUTATION ****************************

NDIV = 1.0 + TAU/H
RN = NDIV
H = TAU/RN
H6 = H/6.0

C ************** INITIAL VALUES SECTION ***********************

WRITE (6,6008) EN, TAU, GAMMA, UE, RLD
WRITE (6,6009)
WRITE (6,6022) (ANGLE(J), J = 1,6), (ANGLE(J), J = 1,3)
WRITE (6,6012)
NP1 = NDIV + 1
DO 70 I = 1, NP1
NSTEP = I - NP1
RSTEP = NSTEP
TIME = RSTEP * H
TI(I) = TIME
DO 75 J = 1, NJMAX
JP = J + NJMAX
IF (AC(J)) 751, 753, 751
753 IF (AS(J)) 751, 752, 751
752 UI(J) = 0.0
UI(JP) = 0.0
GO TO 75
751 ARG = FRQ(J) * TIME
FSIN = SIN(ARG)
FCOS = COS(ARG)
FEXP = EXP(DMP(J)*TIME)
UI(J) = (AS(J)*FSIN + AC(J)*FCOS) * FEXP
U(I,JP) = ((AS(J) * FCOS) - (AC(J) * FSIN)) * FRQ1(J) * FEXP
1 + DMPI(J) * U(I,J)

75 CONTINUE
C CALCULATE INITIAL VALUES OF PRESSURE AND VELOCITY.
DO 704 NPRES = 1, 6
DO 702 J = 1, NJMAX
COEF(I,J) = CFT(NPRES,J)
COEF(2,J) = CFTH(NPRES,J)
COEF(3,J) = CFZ(NPRES,J)
702 CONTINUE
DO 703 J = 1, NU
Y(J) = U(I,J)
703 CONTINUE
UBAR = 0.0
IF (NPRES > 3) UBAR = UE
UMS = 0.0
IF ((NDROPS.EQ.1) .AND. (NPRES.LT.4)) UMS = UE/(ZE*ZCOMB)
CALL PRSVEL(UBAR, UMS, Y, P, VTH, VZ)
PRESS(NPRES) = P
IF (NPRES > 3) AXVEL(NPRES - 3) = VZ
704 CONTINUE
PRS(I) = PRESS(NLOC)
C
C CALCULATE INITIAL VALUES OF NOZZLE B.C.
CSUM = (0.0,0.0)
DO 710 J = 1, JMX
JP = NJMAX + (2 * J) - 1
FT = Y(JP)
GT = Y(JP+1)
A = CMPLX(FT,GT)
CSUM = CSUM + YNOZ(J) * CPHIT(J) * A
710 CONTINUE
SUM = REAL(CSUM)
YPHI = -GAMMA * SUM
WRITE (6,6011) NSTEP, TIME, (PRESS(J), J = 1,6),
1 (AXVEL(J), J = 1,3), YPHI
70 CONTINUE
C
C WRITE (6,6008) EN, TAU, GAMMA, UE, RLD
WRITE (6,6022) (ANGLE(J), J = 1,6), (ANGLE(J), J = 1,3)
C
C *************** INITIALIZE CONTROL NUMBERS ********************
LINE = 8
K = 0
MAXNO = 0
MAXP = 0
IF (NOUT .EQ. 0) GO TO 100
JPLLOT = 0
TMIN = TSTART

132
TPAX = TSTART + TDEL
YLO(1) = -YHI(1)
DO 90 J = 2,4
YHI(J) = YHI(1)
YLO(J) = YLO(1)
YLAB(J) = YLAB(1)
ITICY(J) = ITICY(1)
90 CONTINUE
YLO(5) = -YHI(5)
YHI(6) = YHI(5)
YLO(6) = YLO(5)
YLAB(6) = YLAB(5)
ITICY(6) = ITICY(5)

C

C ******************** NUMERICAL CALCULATIONS SECTION ********************
C

100 I = NP1
C
C  RUNGE-KUTTA INTEGRATION SCHEME
C
105 NSTEP = (I - NP1 + (LAST - NP1) * K)
STEP = NSTEP
TIME = RSTEP * H
TI(I) = TIME
DO 110 J = 1, NJMAX
JP = J + NJMAX
RV(J) = U(J, I)
RV(J, 2) = 0.375 * RV(J, 1) + 0.75 * RV(J, 4) - 0.125 * U(J, I)
110 CONTINUE
DO 120 J = 1, NU
Y(J) = U(I, J)
120 CONTINUE
CALL RHS(NU, Y, YP)
DO 130 J = 1, NU
FZ(I, J) = YP(J)
130 CONTINUE
DO 140 II = 2, 4
DO 144 J = 1, NU
UZ(J) = Y(J) + AA(II) * H * FZ(II, J)
144 CONTINUE
CALL RHS(NU, II, UZ, YP)
DO 148 J = 1, NU
FZ(II, J) = YP(J)
148 CONTINUE
DO 150 J = 1, NU
U(I+1, J) = Y(J) + (FZ(1, J) + 2.0 * (FZ(2, J) + FZ(3, J)) + FZ(4, J)) * H6
150 CONTINUE
C CALCULATE PRESSURE TIME HISTORIES
DO 154 NPRES = 1, 6
DO 152 J = 1, NJMAX
COEF(I,J) = CFT(NPRES,J)
COEF(2,J) = CFTH(NPRES,J)
COEF(3,J) = CFZ(NPRES,J)
152 CONTINUE
UBAR = 0.0
IF (NPRES *GT* 3) UBAR = UE
UMS = 0.0
IF ((NDROPS*EQ*1) *AND* (NPRES*LT*4)) UMS = UE/(Z*ZCOMB)
CALL PRSVEL(UBAR,UMS,Y,F,VTH,VZ)
PRESS(NPRES) = F
IF (NPRES *GT* 3) AXVEL(NPRES - 3) = VZ
154 CONTINUE
PRS(I) = PRESS(NLOC)

C CALCULATE VALUES OF NOZZLE B.C.
CSUM = (0.0,0.0)
DO 650 J = 1, JMX
JP = NJMAX + (2 * J) - 1
FT = Y(JP)
GT = Y(JP+1)
A = CMFLX(FT,GT)
CSUM = CSUM + YNOZ(J) * CPHIT(J) * A
650 CONTINUE
SUM = REAL(CSUM)
YPHI = -GAMMA * SUM

C DETERMINE MAXIMA AND MINIMA OF PRINCIPAL MODE-AMPLITUDE
C FUNCTION FOR USE IN DETERMINING LIMIT-CYCLE BEHAVIOR.
C IF (UI*JFMODE) * U(I+1,JFMODE)) 170, 170, 160
170 FDEN = U(I,JFMODE) - U(I+1,JFMODE)
IF (PDEN) 171, 160, 171
171 PP = U(I,JFMODE)/PDEN
FA = (PP - 1.0) * PP * 0.5
PB = 1.0 - (PP * PP)
FC = (PP + 1.0) * PP * 0.5
MAXNO = MAXNO + 1
UMAX(MAXNO) = FA*U(I-1,JMODE) + PB*U(I,JMODE) + FC*U(I+1,JMODE)
IF (MAXNO *GE* 500) GO TO 250
160 CONTINUE

C DETERMINE MAXIMUM AND MINIMUM PRESSURE AT LOCATION SPECIFIED
C BY NLOC.
DPL = PRS(I) - PRS(I-1)
DPS = PRS(I-1) - PRS(I-2)
IF (DPL*DP5) 173, 173, 175
173 FNUM = PRS(I-2) - PRS(I)
PDEN = 2.0 * (PRS(I-2) + PRS(I) - 2.0 *PRS(I-1))
IF (PDEN) 174, 175, 174
174 PP = PNUM/PDEN
FA = (PF - 1.0) * PP * 0.5
PB = 1.0 - (PP * PP)
PC = (PP + 1.0) * PP * 0.5
MAXF = MAXF + 1
PMAX(MAXF) = FA*PRS(I-2) + PB*PRS(I-1) + PC*PRS(I)
TIMAX(MAXF) = TI(I-1) + FF*HH
IF (MAXF .GE. 500) GO TO 250
175 CONTINUE
C
IF (NTEST .EQ. 1) GO TO 155
IF (TIME .LT. TSTART) GO TO 155
IF ((NOUT .EQ. 0) .OR. (NOUT .GT. 6)) GO TO 156
C
*************** TIME HISTORY PLOTTING SECTION ***************
C
IF (TMAX .GT. TQUIT) GO TO 156
IF ((TIME .GT. TMAX) .OR. (JFLOT .GE. 500)) GO TO 1000
C
JFLOT = JFLOT + 1
C
FILL TIME ARRAY FOR PLOTTING.
TLOT(JFLOT) = TIME
C
FILL INJECTOR PRESSURE ARRAYS FOR PLOTTING (THETA = 0, 45, 90)
DO 1001 J = 1, 3
YFLOT(J,JFLOT) = PRESS(J)
1001 CONTINUE
C
FILL NOZZLE PRESSURE ARRAY FOR PLOTTING (THETA = 0)
YFLOT(4,JFLOT) = PHRES(4)
C
FILL NOZZLE AXIAL VELOCITY ARRAY FOR PLOTTING (THETA = 0)
YFLOT(5,JFLOT) = AXVEL(1)
C
FILL NOZZLE B.C. ARRAY FOR PLOTTING (THETA = 0).
YFLOT(6,JFLOT) = YPHI
C
GO TO 156
C
1000 NUM = JFLOT
C
PLOT TIME HISTORIES.
C
DO 1020 NLOT = NFIRST, NOUT
C
JFLOT = 0
C
ASSIGN PLOTTING PARAMETERS.
YMIN = YL0(NFLOT)
YMAX = YHI(NFLOT)
NTICY = ITICY(NFLOT)
DELY = YLAB(NFLOT)

ELIMINATE POINTS THAT ARE OUT OF THE ORIGINATE RANGE.
DO 1010 J = 1, NUM
IF ((YFLOT(NPLOT*J) .LT. YMIN) .OR. (YFLOT(NPLOT*J) .GT. YMAX))
  1010 CONTINUE

IF (JFLOT +EQ+ 0) GO TO 1020
GO TO (1011, 1012, 1013, 1014, 1015, 1016), NFLOT

PLOT INJECTOR PRESSURE AT THETA = 0 DEGREES.
1011 CALL GRAPHS(1BUF, 3000, 4, JFLOT, 1, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY1, 21, 41, DUMMYT, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020

PLOT INJECTOR PRESSURE AT THETA = 45 DEGREES.
1012 IF (M(JMODE) .EQ. 0) GO TO 1020
CALL GRAPHS(1BUF, 3000, 4, JFLOT, 1, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY2, 21, 42, DUMMYT, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020

PLOT INJECTOR PRESSURE AT THETA = 90 DEGREES.
1013 IF (M(JMODE) .EQ. 0) GO TO 1020
CALL GRAPHS(1BUF, 3000, 4, JFLOT, 1, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY3, 21, 43, DUMMYT, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020

PLOT NOZZLE PRESSURE AT THETA = 0 DEGREES.
1014 CALL GRAPHS(1BUF, 3000, 4, JFLOT, 1, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY4, 21, 39, DUMMYT, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020

PLOT NOZZLE AXIAL VELOCITY AT THETA = 0 DEGREES.
1015 CALL GRAPHS(1BUF, 3000, 4, JFLOT, 1, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY5, 21, 32, DUMMYT, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020

PLOT NOZZLE B.C. AT THETA = 0 DEGREES.
1016 CALL GRAPHS(1BUF, 3000, 4, JFLOT, 1, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY6, 21, 44, DUMMYT, DUMMYY, 2.0, DELY, TITLE)

1020 CONTINUE
C REASSIGN PLOTTING PARAMETERS FOR NEXT SET OF PLOTS.
JPLOT = 0
TMIN = TMAX
TMAX = TMAX + TDEL
C
*************** TIME HISTORY PRINTED OUTPUT SECTION ***************

156 WRITE (6,6011) NSTEP, TIME, (FRESS(J), J = 1,6),
        (AXVEL(J), J = 1,3), YPHI
LINE = LINE + 1
157 IF (TIME .GT. TQUIT) GO TO 250
IF (LINE .LT. 52) GO TO 155
WRITE (6,6013)
WRITE (6,6022) (ANGLE(J), J = 1,6), (ANGLE(J), J = 1,3)
LINE = 4

155 I = I + 1
IF (I .LT. LAST) GO TO 105
C
*************** LIMIT-CYCLE SECTION ***************

C TEST FOR LIMIT CYCLE.
K = K + 1
IF (((NTEST .EQ. 0) .OR. (MAXNO .LT. 80)) GO TO 190
UTOT = 0.0
DO 180 J = 0, 3
JMAX = MAXNO - J
UTOT = UTOT + ABS(UMAX(JMAX))
180 CONTINUE
UAVG(K) = UTOT/4.0
IF (K .EQ. 1) GO TO 190
CHANGE = UAVG(K) - UAVG(K-1)
ABSCHG = ABS(CHANGE/UAVG(K))
IF (ABSCHG .GT. ERR) GO TO 190
TM = TIME/2.0
ITM = TM
ITM = 2*ITM + 2
TM = ITM
TSTART = TM + TSTART
TQUIT = TM + TQUIT
TMIN = TSTART
TMAX = TSTART + TDEL
NTEST = 0
C
C RE-ASSIGN ARRAYS.
190 DO 200 I = 1, NP1
ILAST = LAST - NP1 + I
PRS(I) = PHS(ILAST)
TI(I) = TI(ILAST)
DO 200 J = 1, NU
U(I,J) = U(ILAST,J)
200 CONTINUE
GO TO 100

C
C
C *************** PRESSURE MAXIMA AND MINIMA PRINTOUT ***************
C
250 WRITE (6,6023) Z(NLOC), ANGLE(NLOC), MAXP
LINE = 4
DO 255 JST = 1, MAXP, 8
JSTART = JST
JSTOP = JST + 7
IF (JSTOP .GT. MAXP) JSTOP = MAXP
WRITE (6,6024) (PMAX(J), JST, JSTOP)
WRITE (6,6024) (TIMAX(J), JST, JSTOP)
WRITE (6,6014)
LINE = LINE + 3
IF (LINE .LT. 52) GO TO 255
LINE = 0
WRITE (6,6013)
255 CONTINUE
IF ((NOUT .EQ. 0) .OR. (NOMIT .EQ. 1)) GO TO 5
C
C *************** PRESSURE MAXIMA PLOTTING SECTION ***************
C
C DETERMINE LARGEST VALUE OF PMAX.
AMPMAX = 0.0
DO 260 J = 1, MAXP
IF (PMAX(J) .LT. AMPMAX) GO TO 260
AMPMAX = PMAX(J)
260 CONTINUE
C
C RANGE OF PLOT AND COORDINATE LABELING.
ITM = AMPMAX + 1.0
AMPMAX = ITM
ITM = 1.0 + TIMAX(MAXP)/50.0
TMAX = ITM * 50
DELX = TMAX/10.0
DELY = AMPMAX/10.0
C
C ELIMINATE NEGATIVE VALUES.
JFLOT = 0
DO 262 J = 1, MAXP
IF (PMAX(J)) 262, 262, 264
264 JFLOT = JFLOT + 1
DUMMY(JFLOT) = TIMAX(J)
DUMMY(JFLOT) = PMAX(J)
262 CONTINUE
C PLOT VALUES.
CALL GRAPHS(1BUF, 3000, 4, JFLOT, 101, 101, TMAX, AMPMAX, 0, 0, 0, 0,
1 ITT, ITP, 21, 14, DUMMY, DUMMY, DELX, DELY, TITLE)
C GO TO 5
C TURN OFF PLOTTING ROUTINE.
300 IF (NPT .EQ. 1) CALL SHPARG
C
*************** READ FORMAT SPECIFICATIONS ***************
C
5000 FORMAT (12A6)
5001 FORMAT (4F10.0, 2I5)
5002 FORMAT (5I5, 2F10.5, 1X, A4)
5003 FORMAT (15)
5004 FORMAT (2I5, F15.6)
5005 FORMAT (3I5, F15.6)
5006 FORMAT (5F10.0)
5007 FORMAT (15, 2F10.0)
5008 FORMAT (7I5)
5009 FORMAT (7F10.0)
5010 FORMAT (15, 4F10.5)
C
*************** WRITE FORMAT SPECIFICATIONS ***************
C
6001 FORMAT (1H1, 9H GAMMA = , F5.3, 5X, SHUE = , F5.3,
1 5X, SHZE = , F8.5, 5X, 6HZCOMB = , F5.2,
2 5X, 6HNJMAX = , I2//)
6002 FORMAT (2X, 29HNAME J L M N NS, 7X, 3HSMN, 3X,
1 7HJM(SMN)/)
6003 FORMAT (2X, 4A4, 5I5, 2F10.5)
6004 FORMAT (1H0, 26H NUMBER OF COEFFICIENTS C(11, 10H, NJ, NP) IS 15//)
6005 FORMAT (2X, 2HC(11, 1H, 12, 1H, 12, 4H) = , F10.5)
6006 FORMAT (1H0, 38H NUMBER OF COEFFICIENTS D(NJ, NP, NQ) IS 15//)
6007 FORMAT (2X, 2HD(12, 1H, 12, 1H, 12, 4H) = , F10.5)
6008 FORMAT (1H1, 45H COMBUSTION PARAMETERS: INTERACTION INDEX = , F7.5,
1 12H, 11H TIME-LAG = , F7.5/2X, 17H MOTOR PARAMETERS: 19X,
2 8HGAMMA = , F7.5, 23H EXIT MACH NUMBER = , F7.5,
3 22H LENGTH/DIAMETER = , F7.5//)
6009 FORMAT (2X, 18H INITIAL CONDITIONS//)
6010 FORMAT (1H0, 5X, 1HJ, 8X, 2HYR, 8X, 2HYI, 7X, 3HEPS, 7X, 3HETA//)
6011 FORMAT (2X, 15, F12.5, 10F10.5)
6012 FORMAT (1H0)
6013 FORMAT (1H1)
6014 FORMAT (1H )
6015 FORMAT (2X, 15, 4F10.5)
6016 FORMAT (1H1, 36H INITIAL CONDITIONS ARE OF THE FORM://
1 2X, 49HUC(J, J) = AC(J) * COS(FREQ*T) + AS(J) * SIN(FREQ*T)),
2 14H * EXP(DAMP*T),//6X, 1HJ, 8X, 7HDAMPING,
3 6X, 9HFREQUENCY, 10X, SHAC(J), 10X, SHAS(J)///

139
6017 FORMAT (2X,15,F15.8)
6020 FORMAT (1H1,'46H COEFFICIENTS FOR COMPUTATION OF WALL PRESSURE,
1 10H WAVEFORMS///43X,27HCOEFFICIENTS IN SERIES FOR://
2 22X,5HTHETA, 10X, 4HTIME, 10X, 5HTHETA, 10X, 5HAXIAL/
3 6X, 1HJ, 9X, 1HZ, 3X, 9H(DEGREES), 5X, 10HDERIVATIVE,
4 SX, 10HDERIVATIVE, SX, 10HDERIVATIVE///)
6021 FORMAT (2X,15,F10.3,F12.1,3F15.7)
6022 FORMAT (26X,17HINJECTOR PRESSURE,14X,15HNOZZLE PRESSURE,
1 12X,21HNOZZLE AXIAL VELOCITY/3X,4HSTEP,8X,4HTIME,
2 F5,0,5H DEG•F5,0,5H DEG•F5,0,5H DEG•F5,0,5H DEG•
3 F5,0,5H DEG•F5,0,5H DEG•F5,0,5H DEG•F5,0,5H DEG•
4 F5,0,5H DEG•F5,0,5H DEG•F5,0,5H DEG•,6X,4HYPHI///)
6023 FORMAT (1H1,'38H PRESSURE MAXIMA AND MINIMA AT: Z = ,F5.2,
1 11H THETA = ,F4.1/19H VALUES COMPUTED: ,13///)
6024 FORMAT (1H,7X,8F13.6)
6025 FORMAT (2X,'2X,37H THE TRANSIENT BEHAVIOR IS CALCULATED•
6026 FORMAT (2X,'2X,39H THE LIMIT-CYCLE BEHAVIOR IS CALCULATED•
6027 FORMAT (2X,'2X,33H THIS RUN PRODUCES PLOTTED OUTPUT•
6028 FORMAT (2X,'2X,'THE PHANTOM ZONES ARE ELIMINATED•
6030 FORMAT (2X,'DROPLET MOMENTUM SOURCE IS NEGLECTED•
6031 FORMAT (2X,'DROPLET MOMENTUM SOURCE IS INCLUDED•
END
SUBROUTINE PHICFS(NF,Z, THETA, CT, CTH, CZ)

   THIS SUBROUTINE COMPUTES THE COEFFICIENTS NEEDED TO
   CALCULATE THE WALL PRESSURE PERTURBATION.

   NP IS THE INDEX OF THE COMPLEX SERIES TERM.
   Z IS THE AXIAL LOCATION.
   THETA IS THE AZIMUTHAL LOCATION.
   CT IS THE COEFFICIENT IN THE SERIES FOR THE TIME DERIVATIVE OF
   THE VELOCITY POTENTIAL.
   CTH IS THE COEFFICIENT IN THE SERIES FOR THE THETA DERIVATIVE
   OF THE VELOCITY POTENTIAL.
   CZ IS THE COEFFICIENT IN THE SERIES FOR THE AXIAL DERIVATIVE
   OF THE VELOCITY POTENTIAL.

   COMPLEX CI, CZ, CAXI, CAXIZ, CRAD, CAZI, CAZITH,
   COMMON /BLK2/ M(10), NS(10), SJ(10), B

   CI = (0.0, 1.0)
   CZ = CMPLX(Z,0.0)
   CAXI = CCOSH(CI * B(NP) * CZ)
   CAXIZ = CI * B(NP) * CSINH(CI * B(NP) * CZ)
   CRAD = CMPLX(SJ(NP),0.0)
   EM = M(NF)
   ARG = EM * THETA
   FSIN = SIN(ARG)
   FCOS = COS(ARG)
   AZI = FCOS
   IF (NS(NP) .EQ. 1) AZI = FSIN
   AZITH = EM * FCOS
   IF (NS(NP) .EQ. 2) AZITH = -EM * FSIN
   CAZI = CMPLX(AZI,0.0)
   CAZITH = CMPLX(AZITH,0.0)

   CT = CAZI * CAXI * CRAD
   CTH = CAZITH * CAXI * CRAD
   CZ = CAZI * CAXIZ * CRAD

RETURN
END
SUBROUTINE PRSVEL(UBAR, UMS, Y, P, VTH, VZ)

THIS SUBROUTINE COMPUTES THE WALL PRESSURE AND VELOCITY.

UBAR IS THE LOCAL AXIAL STEADY STATE MACH NUMBER.
UMS IS THE DERIVATIVE OF THE MACH NUMBER FOR THE CASE
WHEN DROPLET MOMENTUM SOURCES ARE INCLUDED.
Y IS THE ARRAY CONTAINING VALUES OF THE MODE-AMPLITUDE
FUNCTIONS AND THEIR DERIVATIVES.
P IS THE VALUE OF THE WALL PRESSURE PERTURBATION.
VTH IS THE TANGENTIAL COMPONENT OF VELOCITY AT THE WALL.
VZ IS THE AXIAL COMPONENT OF VELOCITY AT THE WALL.

DIMENSION Y(40), SUM(4), SUMSQ(3)
COMMON /BLK3/ NJMAX, NLMAX, GAMMA, COEF(3, 20)

DO 10 I = 1, 4
SUM(I) = 0.0
10 CONTINUE

DO 20 I = 1, NJMAX
DO 20 J = 1, NJMAX
JY = J
IF (I .EQ. 1) JY = J + NJMAX
II = I
IF (I .EQ. 4) II = 1
SUM(I) = SUM(I) + Y(JY) * COEF(II, J)
20 CONTINUE

PLIN = SUM(1) + UBAR*SUM(3) + UMS*SUM(4)
PNL = 0.0
IF (NLMAX .EQ. 0) GO TO 40
DO 30 I = 1, 3
SUMSQ(I) = SUM(I) * SUM(I)
30 CONTINUE
PNL = 0.5 * (SUMSQ(2) + SUMSQ(3) - SUMSQ(1))

40 P = -GAMMA * (PLIN + PNL)
VTH = SUM(2)
VZ = SUM(3)

RETURN
END
SUBROUTINE RHS(NU,II,U,UP)

DIMENSION U(NU), UP(NU)
COMMON RV(20,4), C(3,20,20), D(20,400),
1 KPMAX(3,20), IC(3,20,20), KFQMAX(20),
2 IDF(20,400), IDQ(20,400)
 COMMON /BLK3/ NJMAX, NLMAX, GAMMA, COEF(3,20)

DO 10 NJ = 1, NJMAX
NJP = NJ + NJMAX
UP(NJP) = U(NJP)
SL1 = 0.0
SL2 = 0.0
SL3 = 0.0
SNL = 0.0
MAX = KPMAX(1,NJ)
IF (MAX .EQ. 0) GO TO 25
DO 20 KP = 1, MAX
NP = IC(NJ,KP)
SL1 = SL1 + (C(1,NJ,KP) * U(NP))
20 CONTINUE
25 MAX = KPMAX(2,NJ)
IF (MAX .EQ. 0) GO TO 35
DO 30 KP = 1, MAX
NPP = IC(2,NJ,KP) + NJMAX
SL2 = SL2 + (C(2,NJ,KP) * U(NPP))
30 CONTINUE
35 MAX = KPMAX(3,NJ)
IF (MAX .EQ. 0) GO TO 45
DO 40 KP = 1, MAX
NP = IC(3,NJ,KP)
SL3 = SL3 + (C(3,NJ,KP) * RV(NP,II))
40 CONTINUE
45 IF (NLMAX .EQ. 0) GO TO 55
MAX = KPMAX(NJ)
IF (MAX .EQ. 0) GO TO 55
DO 50 KPQ = 1, MAX
NP = IDP(NJ,KPQ)
NQP = IDQ(NJ,KPQ) + NJMAX
SNL = SNL + (D(NJ,KPQ) * U(NP) * U(NQP))
50 CONTINUE
55 UP(NJP) = -(SL1 + SL2 + SL3 + SNL)
10 CONTINUE
RETURN
END
COMPILER (FLD=ABS)
SUBROUTINE GRAPHS(IBUF,NLOC,LDEV,NTOT,NTICX,NTICY,
1 XMAX,YMAX,XMIN,YMIN,ITITLX,ITITLY,LTILTX,LTITLY,XARRAY,
2 YARRAY,DELX,DELY,TITLE)

C- IDENTIFIER MEANING TYPE
C
C IBUF: ADDRESS OF BUFFER AREA FOR PLOT OUTPUT INTEGER
C NLOC: NUMBER OF LOCATIONS IN BUFFER AREA (>=2000) INTEGER
C LDEV: LOGICAL DEVICE NUMBER FOR PLOT INTEGER
C NTOT: NUMBER OF POINTS TO BE PLOTTED INTEGER
C NTICX: NUMBER OF TIC MARKS ON ABSCISSA (>=2) INTEGER
C NTICY: NUMBER OF TIC MARKS ON ORDINATE (>=2) INTEGER
C XMAX: UPPER LIMIT OF ABSCISSA DOMAIN REAL
C YMAX: UPPER LIMIT OF ORDINATE RANGE REAL
C XMIN: LOWER LIMIT OF ABSCISSA DOMAIN REAL
C YMIN: LOWER LIMIT OF ORDINATE RANGE REAL
C ITITLX: ABSCISSA LABEL FIELDATA ARRAY
C ITITLY: ORDINATE LABEL FIELDATA ARRAY
C
C LTITLX: NUMBER OF CHARACTERS IN ITITLX INTEGER
C LTITLY: NUMBER OF CHARACTERS IN ITITLY INTEGER
C XARRAY: ABSCISSA POINTS IN TERMS OF XMIN-XMAX COORD'S REAL ARRAY
C YARRAY: ORDINATE POINTS IN TERMS OF YMIN-YMAX COORD'S REAL ARRAY
C DELX1 INTERVALS OF ABSCISSA TIC MARK LABELING REAL
C IN TERMS OF XMIN-XMAX COORDINATES REAL
C DELY1 INTERVALS OF ORDINATE TIC MARK LABELING REAL
C IN TERMS OF YMIN-YMAX COORDINATES REAL
C TITLE: LABEL FOR THE WHOLE RUN FIELDATA ARRAY
C
C-DIMENSION IBUF(NLOC),XARRAY(NTOT),YARRAY(NTOT),ITITLX(1),
1 ITITLY(1),YTITL(100)
DIMENSION TITLE(1)
C- FIXED BASIC PARAMETERS
C
C LOGICAL ZERO
DEFINE ZER0=NDEC*LT.0.AND.-ABS(FPN)*LT.5
1 OR=NDEC.6T.0.AND.-ABS(FPN)*LT.5.*10.*((-NDEC-1)
DEFINE DNDEC=NDEC-FLD(0,36,ZERO)*NDEC-FLD(0,36,ZERO)
DEFINE IFIX(FARG)=INT(FARG+.5)
DATA J/1/
DATA HEIGHT/*.105/
DATA INTEG/1/
DATA APSCIS/8/
DATA ORDNA/6/
DATA ICODE/-1/
DATA TOPMAR/1.5/
DATA BOTMAR/1.5/
REAL LEFMAR
DATA LEFMAR/1.9/
DATA RYTMAR/1.1/
DATA FACT/1.0/
DATA MAXIS/1.0/
DATA MLINE/1.0/
DATA HTLAB/.105/

19 INITIAL COMPUTATION OF DERIVED PARAMETERS
AND INITIAL PLOTS CALL
20 SKIPS PRELIMINARIES FOR 2ND AND SUBSEQUENT CALLS

GO TO (19,20), J

YDIT(1) = 3./19.
TICKLE = HEIGHT/2.
ROTFAC = -3./14. * HEIGHT + 4./7. * HEIGHT
STARTL = 6. * HEIGHT + ROTFAC + TICKLE
SEPLAB = STARTL + 1.5 * HEIGHT
SYMBLH = 0.070
REAL LABSEP
LABSEP = 4. * HEIGHT
ASTART = 2. * HEIGHT
DO 1 I = 1, 100
   YDIT(I) = YDIT(I - 1) + (2 * MOD(I, 2) + 1)/19.
   YDIT(100) = YDIT(100) + .5
   CALL PLOTS(IBUF, NLOC, LLEV)
   CALL FACTOR(I°)
   J = 2
   CALL SYMBOL (HEIGHT, 36 * HEIGHT + 5.5, HEIGHT, TITLE, 270.0, 72)
   CALL PLOT(1., -5., -3)
DO 2 I = 2, 100
   CALL PLOT(0., YDIT(I), 3 - MOD(I, 2))
   DO 33 I = 1, 100
33 YDIT(I) = YDIT(I) - ABSCLS - RYTMAR

RESER ORIGIN

XPAGE = BOTMAR + ORDINA
GO TO 2019
20 XPAGE = BOTMAR + ORDINA + TOPMAR
2019 CALL WHERE(REXPAGE, RYPAGE, FACT)
YPAGE = RYPAGE - LEFMAR
CALL PLOT(XPAGE, YPAGE, -3)
CALL FACTOR(FACT)
DRAW AXES AND LABELING MAXIS TIMES

DO 100 I = 1, IMAX
100 CALL MYAXIS

DRAW POINTS, OPTIONAL CENTERLINE, AND PAGE SCISSORLINE MLINE TIMES

DO 200 I = 1, MLINE
200 CALL MYLINE
RETURN

ENTRY POINT SHPARG
TERMATE PLOTTING SEQUENCE

ENTRY SHPARG
CALL WHERE(RXPAGE, RYPAGE, I)
CALL PLOT(RXPAGE, RYPAGE, 999)
RETURN

SUBROUTINE MYAXIS (INTERNAL)

SUBROUTINE MYAXIS
STARTL = 6 * HEIGHT + ROTFAC + TICKLE
IMAX = IFIX((YMAX - YMIN)/DELY)
TICSEP = ORDINA/(ABS(NTICY) - 1)
CALL DENDEC(YMAX*DELY*NDEC)
K = 1
N = (ABS(NTICY)/IMAX) - 1 + MOD(ABS(NTICY), 2)
DO 9 I = 0, IMAX
GO TO (11, 12), K
11 IF(2 * I LT IMAX)GO TO 12
CALL AXLAB(0., ITITLY, LTITLY, HTLAB)
K = 2
12 FPN = YMAX - I * DELY
IF(ZERO)FPN = 0.
TMID = 1.
XPAGE = - I * ORDINA/IMAX - .5 * HEIGHT
IF(FPN)113, 122, 118
113 IF(NDEC - 2)115, 114, 112
114 YPAGE = STARTL
813 815
GO TO 112

115 IF(NDEC = 1)117,116,112
116 YPAGE = STARTL - HEIGHT
GO TO 112
117 IF(ABS(FPN) = 100.)119,116,116
119 IF(ABS(FPN) = 10.)120,121,121
120 YPAGE = STARTL - 3 * HEIGHT
GO TO 112
121 YPAGE = STARTL - 2 * HEIGHT
GO TO 112
122 YPAGE = STARTL - 4 * HEIGHT
GO TO 112
118 IF(NDEC = 2)123,116,112
123 IF(NDEC = 1)125,124,112
124 IF(FPN = 10.)121,116,116
125 IF(FPN = 10.)122,120,126
126 IF(FPN = 100.)120,121,127
127 IF(FPN = 1000.)121,116,128
128 IF(FPN = 10000.)116,114,114
112 NNDEC = DNDEC
CALL NUMBER(XPAGE,YPAGE,HEIGHT,FPN,270.,NNDEC)
XPAGE = - 1 * (ORDINA/IMAX)
DO 10 JJ = 1,N
YPAGE = TICKLE * TMID
CALL PLOT(XPAGE,YPAGE,3)
YPAGE = YPAGE * (- 1 + I/IMAX * .5)
CALL PLOT(XPAGE,YPAGE,2)
IF(I/IMAX) 110, 110, 9
110 YPAGE = 0
CALL PLOT(XPAGE,YPAGE,3)
XPAGE = XPAGE - TICSEP
CALL PLOT(XPAGE, YPAGE,2)
TMID = .5
10 CONTINUE
9 CONTINUE
K = 1
IMAX = IFIX((XMAX - XMIN)/DELX)
TICSEP = ABS CIS/(NTICX - 1)
XPAGE = - ASTART - ORDINA
CALL DENDEC(XMAX,DELX,NDEC)
DO 28 I = 0,IMAX
STARTL = - I * ABSCIS/IMAX
GO TO (24,25)*K
24 IF(2 * I.LT.IMAX)GO TO 25
CALL AXLAB(270.,ITITLX,LTILX,HTLAB)
K = 2
XPAGE = - ASTART - ORDINA
25 FPN = XMIN + I * DELX
IF(ZERO)FPN = 0.
IF(FPN)813,822,818
813 IF(NDEC = 2)815,817,23
814 YPAGE = STARTL + 16./7. * HEIGHT
GO TO 23
815 IF(NDEC = 1)817,816,23
YPAGE = STARTL + 25/14 * HEIGHT
GO TO 23

IF(ABS(FPN) - 100.)819,816,816

YPAGE = STARTL + 11/14 * HEIGHT
GO TO 23

YPAGE = STARTL + 9/7 * HEIGHT
GO TO 23

YPAGE = STARTL + 2/7 * HEIGHT
GO TO 23

IF(NDEC - 2)623,816,23

IF(NDEC - 1)824,824J23
IF(FPN - 100*)821a816,816
IF(FPN - 1000-)821.616.828
IF(FPN - 10000.)8161614J'814
23 NNDEC = NNDEC

CALL NUMBER(XPAGE, YPAGE, HEIGHT, FPN, 270o s, NNDEC)
N = (NTICKX/IMAX) - 1 + MOD(NTICKX, 2)
DO 26 I = IMAX, 0, - 1
TMID = I.
YPAGE = - I * ABSCIS/IMAX
DO 27 JJ = 1, N
XPAGE = - ORDINA - TICKLE * TMID
CALL PLOT(XPAGE, YPAGE, 3)
XPAGE = XPAGE + (TICKLE + FLDO.36, I•NE.0) * TICKLE) * TMID
CALL PLOT(XPAGE, YPAGE, 2)
IF(I)11,26,111
111 XPAGE = - ORDINA
CALL PLOT(XPAGE, YPAGE, 3)
YPAGE = YPAGE + TICSEP
CALL PLOT(XPAGE, YPAGE, 2)
TMID = +5
27 CONTINUE
26 CONTINUE
RETURN

C ---------------------------------------------------
C
C SUBROUTINE MYLINE (INTERNAL)
C ---------------------------------------------------

SUBROUTINE MYLINE
ITOP = IFIX((ABSCIS + RYMAR + .5)/11. * 99.)
IBOT = IFIX(RYMAR/11. * 99.)
DO 17 I = 1, NTICX
XPAGE = (YARRAY(I) - YMAX)/(YMAX - YMIN) * ORDINA
YPAGE = (XMIN - XARRAY(I))/(XMAX - XMIN) * ABSCIS
CALL SYMBOL(XPAGE, YPAGE, SYMREL, INTEQ, 270o s, ICODE)
IF(NTICY. GE.0)GO TO 22
XPAGE = - ORDINA/2.
YPAGE = - ABSCIS
CALL PLOT(XPAGE, YPAGE, 3)
DO 18 I = IBOT, ITOP
18 CALL PLOT(XPAGE,YDIT(1),3 - MOD(I,2))

22 XPAGE = TOPMAR
YPAGE = - ABSCIS - RTMAR -.5
CALL PLOT(XPAGE,YPAGE,3)
DO 21 I = 1,100

21 CALL PLOT(XPAGE,YDIT(1),3 - MOD(I,2))
RETURN

C SUBROUTINE AXLAB (INTERNAL)

C SUBROUTINE AXLAB(ANGLE,IBCD,NCHAR,HEIGHT)
DIMENSION IBCD(7)
LOGICAL S
INTEGER OSQ,' S'/
K = 2
NCHAR = NCHARX
S = .FALSE.*
IF(ABS(ANGLE) > GT.1) GO TO 30
XPAGE = - ORDNA/2* - NCHAR * HEIGHT/2
YPAGE = SEPLAB
GO TO 31

30 XPAGE = - ORDNA - LABSEP
YPAGE = - ABSCIS/2* + NCHAR * HEIGHT/2

31 LSTART = 6 * MOD(NCHAR,6) - 12
IF(LSTART.EQ. - 12) LSTART = 24
LOOK = NCHAR/6 + 1
IF(LLOOK.EQ. 6) GO TO 13
IF(FLD(0,12,'S') .NE. FLD(LOOK,12,IBCD(LOOK))) GO TO 15
GO TO 14

13 IF(FLD(0,6,'S') .NE. FLD(30,6,IBCD(LOOK - 1))) GO TO 14
IF(FLD(0,6,'S') .NE. FLD(30,6,IBCD(LOOK))) GO TO 14

15 NCHAR = NCHAR - 1
S = .TRUE.*

14 CALL SYMBOL(XPAGE,YPAGE,HEIGHT,IBCD,ANGLE,NCHAR)
IF(S) CALL SYMBOL(999,999,2 * HEIGHT/3,OSQ,ANGLE,2)
RETURN

C SUBROUTINE DENDEC (INTERNAL)

C SUBROUTINE DENDEC(QMAX,DELQ,NDEC)
IF(INTABS(QMAX) * GE.10) GO TO 5
IF(AMOD(ABS(QMAX - DELQ),1) * GE.01) GO TO 7
NDEC = 1
RETURN

5 NDEC = - 1
RETURN

7 NDEC = 2
RETURN
END
APPENDIX E

USER'S MANUAL FOR THE LINEAR STABILITY
PROGRAMS: LINSOL AND LSTB3D

General Description

Two auxiliary programs, LINSOL and LSTB3D, calculate the linear stability characteristics of a cylindrical combustion chamber with distributed combustion and a conventional nozzle. For given values of the operating parameters (i.e., n, τ, γ, \( \bar{u}_e \), and L/D) and a given nozzle admittance (i.e., A and φ), Program LINSOL calculates the growth rate, \( \Lambda \), and the frequency, \( \omega \), of a given acoustic mode. For given values of \( \tau \) Program LSTB3D calculates the corresponding values of n and \( \omega \) for neutral stability (\( \Lambda = 0 \)). These programs are based on an analytical solution of the linearized version of Eqs. (12).

After a discussion of the linear analysis, Programs LINSOL and LSTB3D will be described.

Linear Analysis

For a single acoustic mode, dropping the nonlinear terms in Eqs. (12) yields the following linear equation:

\[
\frac{d^2 A}{dt^2} + C_1 A + (C_2 - nC_3) \frac{dA}{dt} + nC_3 \frac{d[A(t - \tau)]}{dt} = 0 \tag{E-1}
\]

where \( A(t) \) is the unknown complex amplitude function for the mode under consideration and the coefficients are obtained from Eqs. (C-1) through (C-4) by dividing by \( C_0 \). Thus the coefficients are complex numbers given by:

\[
C_1 = S^2_{mn} + \frac{Z'(z_e)Z^*(z_e) - \int_0^e Z''Z^*dz}{\int_0^e ZZ^*dz} \tag{E-2}
\]
The linear solutions are determined by substituting a solution of the form:

$$A(t) = ae^{(\Lambda + iw)t}$$  \hspace{1cm} (E-5)

into Eq. (E-1) and separating real and imaginary parts to obtain:

$$\omega^2 = C_1 + \Lambda^2 + (C_2 - nC_3)\Lambda - C_2i\omega + C_3ne^{-\Lambda^2}((\Lambda \cos \omega^n + \omega \sin \omega^n)$$  \hspace{1cm} (E-6)

$$\Lambda = -\left\{ \frac{C_1i + (C_2 - nC_3)\omega + nC_3e^{-\Lambda^2}\omega \cos \omega^n}{2\omega + C_2i - nC_3e^{-\Lambda^2}\sin \omega^n} \right\}$$  \hspace{1cm} (E-7)

where $C_1 = C_{1r} + iC_{1i}$, $C_2 = C_{2r} + iC_{2i}$, and $C_3$ is always real. The above equations are solved numerically by Program LINSOL to obtain the growth rate, $\Lambda$, and the frequency, $\omega$, for given values of $n$ and $\tau$.

The equations describing the neutral stability limits are obtained by substituting $\Lambda = 0$ into Eqs. (E-6) and (E-7). Solving the resulting equations
for \( n \) and \( \omega^2 \) gives:

\[
n = \frac{C_2r + \frac{C_1r}{\omega}}{C_3(1 - \cos \omega \tau)} \quad (E-8)
\]

\[
\omega^2 = C_1r + \omega(nC_3 \sin \omega \tau - C_2r) \quad (E-9)
\]

which are solved numerically by Program LSTB3D.

**Program LINSOL**

**Program Structure.** A flow chart for Program LINSOL is given in Fig. (E-1). This program consists of the following major sections: (1) input, (2) calculation of the coefficients \( C_1, C_2, \) and \( C_3 \), (3) iterative solution for \( \Lambda \) and \( \omega \), and (4) output.

**Input.** The input data required by Program LINSOL includes: (1) a title for the run, (2) the chamber parameters \( \gamma, u_e, L/D, \) and \( z_e/z_e \), (3) several control numbers, (4) the nozzle admittance, (5) the mode under consideration, and (6) the values of \( n \) and \( \tau \) for the cases to be run. This data is described in the following table where the location number refers to the columns of the card and the following three formats are used: alphanumeric characters (A), integers (I), and numbers with a decimal point (F). For the "I" formats the values are placed in fields of five locations, while a field of ten locations is used with the "F" formats. In either case the numbers must be placed in the rightmost locations of the allocated field.

<table>
<thead>
<tr>
<th>No. of Cards</th>
<th>Location</th>
<th>Type</th>
<th>Input Item</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-72</td>
<td>A</td>
<td>TITLE</td>
<td>Title of run.</td>
</tr>
<tr>
<td>1</td>
<td>1-10</td>
<td>F</td>
<td>GAMMA</td>
<td>Specific heat ratio, ( \gamma ).</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>F</td>
<td>UE</td>
<td>Steady state Mach number at nozzle entrance, ( u_e ).</td>
</tr>
<tr>
<td></td>
<td>21-30</td>
<td>F</td>
<td>RLD</td>
<td>Length-to-diameter ratio, ( L/D = z_e/2 ).</td>
</tr>
</tbody>
</table>

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Figure E-1. Flow Chart for Program LINSOL.
No. of Cards | Location | Type | Input Item | Comments
---|---|---|---|---
31-40 | F | ZCOMB | Length of combustion zone, $z_c/z_e$.
41-45 | I | NDROPS | If 0: droplet momentum source neglected.
 |  | | If 1: droplet momentum source included.
46-50 | I | NOZZLE | If 0: quasi-steady nozzle.
 |  | | If 1: conventional nozzle.
51-55 | I | NOPT | If 1: all coefficients included.
 |  | | If 2: imaginary parts neglected.

If NOZZLE = 1:

1

1-10 | F | YAMPL | Amplitude factor of nozzle admittance, $A$.
11-20 | F | YPHASE | Phase of nozzle admittance, $\varphi$.

End of input for NOZZLE = 1.

1

1-5 | I | L | Axial mode number, $\ell$ ($0 \leq \ell \leq 10$).
6-10 | I | M | Tangential mode number, $m$ ($0 \leq m \leq 8$).
11-15 | I | N | Radial mode number, $n$ ($0 \leq n \leq 5$).
16-20 | I | NCASES | Number of cases to be run (NCASES ≤ 100).

NCASES 1-10 | F | TAU | Time-lag, $\tau$.
11-20 | F | EN | Interaction Index, $n$.

The title on the first card should identify the mode under consideration. On the second card of input all quantities are the same as those given in the input to COEFTS3D (see Appendix C) except NOPT. NOPT gives the option to neglect the imaginary parts of the coefficients $C_1$ and $C_2$ which are an order of magnitude smaller than the corresponding real parts. Neglecting these...
imaginary parts (NOPT = 2) yields linear solutions consistent with the non-
linear solutions obtained when the small coefficients are neglected (NEGL = 1
in input to COEFFS3D). The values of n and \( \tilde{r} \) for the cases to be run are
given on a series of NCASES cards. These cards are all read and the values
of \( \tilde{r} \) and n are stored in the arrays TAU(J) and EN(J) before any computations
are made.

In addition to the above card input, the acoustic frequencies \( s_{mn} \) are
also needed for these calculations. As in Program COEFFS3D these values are
given in a DATA statement, which is an integral part of the program.

Calculation of \( C_1, C_2, \) and \( C_3 \). In this section the coefficients \( C_1, C_2, \)
and \( C_3 \) appearing in Eqs. (E-6) and (E-7) are calculated using Eqs. (E-2)
through (E-4). As in Program COEFFS3D the axial acoustic eigenvalues neces-
slary for these computations are calculated by Subroutines EIGVAL and FCNS, and
the integrals of the products of two axial eigenfunctions appearing in Eqs.
(E-2) through (E-4) are computed by Subroutines AXIAL1 and UBAR. Listings of
these subroutines are given in Appendix C.

Iterative Solution for \( A \) and \( \omega \). Equations (E-6) and (E-7) are of the
form:

\[
\omega^2 = C_{lr} + f(A,\omega)
\]

\[
A = g(A,\omega)
\]

(E-10)

where the quantity \( f(A,\omega) \) is small compared to \( C_{lr} \) and \( A \) is small in most
cases. Starting with an initial guess of

\[
\omega_1 = \sqrt{s_{mn}^2 + \frac{4\pi^2}{z_e^2}}
\]

(E-11)

\[
A_1 = 0
\]
Eqs. (E-10) are solved iteratively using the following recursion formulas:

\[ \omega_{k+1}^2 = C_{lr} + f(\Lambda_k, \omega_k) \]  
\[ \Lambda_{k+1} = g(\Lambda_k, \omega_k) \]  

(E-12)

At each step of the iteration the quantities \( \Delta \Lambda \) and \( \Delta \omega \) are calculated, where

\[ \Delta \Lambda = | \Lambda_{k+1} - \Lambda_k | \]  
\[ \Delta \omega = | \omega_{k+1} - \omega_k | \]  

(E-13)

and the computations are terminated when \( k = 40 \) or when \( \Delta \Lambda \) and \( \Delta \omega \) are less than \( \epsilon = 10^{-6} \). The process usually converges in less than 15 iterations.

Output. The output generated by Program LINSOL consists of a restatement of the input data followed by the calculated results in tabular form. For each case the tabulated results give the values of \( \tau \) and \( n \) (TAU and EN), the corresponding values of the growth rate \( \Lambda \) and the frequency \( \omega \) (LAMBDa and OMEGA), and the number of iterations (ITER). When ITER is 40 the last values of \( \Lambda \) and \( \omega \) are given followed by the warning message "FAILED TO CONVERGE."

Sample Input and Output. A sample input for the LT mode is given in Table E-1 followed by the resulting output in Table E-2.

Program LSTB3D

Program Structure. A flow chart for Program LSTB3D is given in Figure (E-2). This program consists of the following major sections: (1) input, (2) calculation of the coefficients \( C_1, C_2, \) and \( C_3 \), (3) iterative solution for \( \alpha \) and \( \omega \) for neutral stability, and (4) output.
Table E-1. Sample Input for LINSOL.

<table>
<thead>
<tr>
<th>IT MODE</th>
<th>DROPLET MOMENTUM SOURCE NEGLECTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAMMA</td>
<td>1.20</td>
</tr>
<tr>
<td>UE</td>
<td>0.20</td>
</tr>
<tr>
<td>L/D</td>
<td>0.50</td>
</tr>
<tr>
<td>ZCOMB</td>
<td>1.00</td>
</tr>
<tr>
<td>AMPL</td>
<td>0.02</td>
</tr>
<tr>
<td>PHASE</td>
<td>45.0</td>
</tr>
</tbody>
</table>

Table E-2. Sample Output for LINSOL.

<table>
<thead>
<tr>
<th>Tau</th>
<th>En</th>
<th>Lambda</th>
<th>Omega</th>
<th>Iter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.40</td>
<td>5.0000</td>
<td>0.1789</td>
<td>1.86593</td>
<td>7</td>
</tr>
<tr>
<td>1.40</td>
<td>5.8396</td>
<td>0.0000</td>
<td>1.87005</td>
<td>7</td>
</tr>
<tr>
<td>1.40</td>
<td>6.0000</td>
<td>0.0039</td>
<td>1.87078</td>
<td>7</td>
</tr>
<tr>
<td>1.70</td>
<td>5.0000</td>
<td>0.0075</td>
<td>1.83602</td>
<td>7</td>
</tr>
<tr>
<td>1.70</td>
<td>5.4490</td>
<td>0.0000</td>
<td>1.83612</td>
<td>6</td>
</tr>
<tr>
<td>1.70</td>
<td>6.0000</td>
<td>0.01176</td>
<td>1.83618</td>
<td>7</td>
</tr>
<tr>
<td>2.00</td>
<td>5.0000</td>
<td>0.0137</td>
<td>1.80691</td>
<td>8</td>
</tr>
<tr>
<td>2.00</td>
<td>5.7562</td>
<td>0.0000</td>
<td>1.80410</td>
<td>8</td>
</tr>
<tr>
<td>2.00</td>
<td>6.0000</td>
<td>0.00487</td>
<td>1.80322</td>
<td>8</td>
</tr>
</tbody>
</table>
Acoustic Frequencies

Input of Chamber Parameters

Input of Nozzle Admittance

Input of $\tau_{\text{min}}, \tau_{\text{max}}, \Delta\tau$

Subroutine EIGVAL

Subroutine AXIAL1

Calculate Coefficients $C_1, C_2, C_3$

$\omega_1 = \sqrt{C_{lr}}$

Output of Chamber Parameters

$\tau = \tau_{\text{min}}$

STOP

No

Does More Data Follow?

Yes

Is $\tau > \tau_{\text{max}}$?

Yes

Increase $\tau$ by $\Delta\tau$

No

Output of $\tau, n, \omega, k$

Print "FAILED TO CONVERGE"

Yes

Is $K=0$?

No

Compute $n_k, \omega_{k+1}$

Increase $K$ by 1

Yes

Is $K=1$?

No

Compute $\Delta n, \Delta \omega$

No

$\Delta n \leq \epsilon$ and $\Delta \omega \leq \epsilon$?

Yes

Figure E-2. Flow Chart for Program LSTB3D.
Input. The input data required by Program LSTB3D is basically the same as required by Program LINSOL. The first two cards, which give the title of the case, the chamber parameters, and the control numbers, are identical in content and format to those required by LINSOL. The third card gives the mode numbers \( l \), \( m \), and \( n \) and is followed by a card giving the nozzle admittance if a conventional nozzle is specified. The last card gives the values of \( \bar{\tau} \) for the cases to be run. A detailed description of this input is given below.

<table>
<thead>
<tr>
<th>No. of Cards</th>
<th>Location</th>
<th>Type</th>
<th>Input Item</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-72</td>
<td>A</td>
<td>TITLE</td>
<td>See input for LINSOL.</td>
</tr>
<tr>
<td>1</td>
<td>1-40</td>
<td>F</td>
<td>GAMMA, UE, RLD, ZCOMB</td>
<td>See input for LINSOL.</td>
</tr>
<tr>
<td></td>
<td>41-55</td>
<td>I</td>
<td>NDROPS, NOZZLE, NOPT</td>
<td>See input for LINSOL.</td>
</tr>
<tr>
<td>1</td>
<td>1-15</td>
<td>I</td>
<td>L, M, N</td>
<td>See input for LINSOL.</td>
</tr>
</tbody>
</table>

If NOZZLE = 1:

| 1            | 1-20     | F    | YAMPL, YPHASE  | See input for LINSOL.          |

End of input for NOZZLE = 1.

| 1            | 1-10     | F    | TAUMIN         | Smallest value of \( \bar{\tau} \). |
| 11-20        | F        |      | TAUMAX         | Largest value of \( \bar{\tau} \). |
| 21-30        | F        |      | DELTAU         | Increment in \( \bar{\tau} \).    |

The last card gives the values of \( \bar{\tau} \) which are used in the computation of the neutral stability limit. Thus computations are begun for \( \bar{\tau} = TAUMIN \), \( \bar{\tau} \) is increased by increments of DELTAU, and computations are terminated when \( \bar{\tau} \geq TAUMAX \).

After completion of the computations program control returns to the read statement for the nozzle admittance, thus neutral stability curves can be calculated for several different nozzles for the same set of chamber and mode parameters.

Calculation of \( C_1 \), \( C_2 \), and \( C_3 \). The calculation of the coefficients \( C_1 \), \( C_2 \), and \( C_3 \) appearing in Eqs. (E-8) and (E-9) is performed in the same manner as
Iterative Solution for $n$ and $\omega$. The values of $n$ and $\omega$ for neutral stability are calculated for each value of $\tau$ by solving Eqs. (E-8) and (E-9) using the following iteration scheme:

$$n_k = \frac{C_2 + C_{11}/\omega_k}{C_3(1 - \cos \omega_k \tau)}$$

$$\omega_{k+1}^2 = C_{1r} + \omega_k(n_k C_3 \sin \omega_k \tau - C_{21})$$

(E-14)

The iteration is started by using $\omega_1 = \sqrt{C_{1r}}$ and is stopped when $k = 40$ or $\Delta n$ and $\Delta \omega$ are less than $\varepsilon = 10^{-6}$. Convergence is usually obtained in less than 20 iterations.

Output. The output generated by Program LSTB3D consists of a restatement of the input data followed by the calculated results in tabular form. For each value of $\tau$ in the range $\text{TAUMIN} \leq \tau \leq \text{TAUMAX}$, the tabulated results give the value of $\tau$ (TAU), the corresponding values of $n$ and $\omega$ for neutral stability ($\chi_\nu$ and $\chi$), and the number of iterations (ITER). If ITER is 40 the last values of $n$ and $\omega$ computed are given followed by the warning message "FAILED TO CONVERGE."

Sample Input and Output. A sample input for the 1T mode is given in Table E-3 and is followed by the resulting output in Table E-4.

Table E-3. Sample Input for LSTB3D.
Table E-4. Sample Output for LSTB3D.

**IT MODE.**

**DROPLET MOMENTUM SOURCE NEGLECTED**

\[
\begin{array}{cccc}
\text{GAMMA} & 1.20 & \text{UE} & -0.20 \\
\text{RLD} & 0.50000 & \text{ZCOMB} & 1.00 \\
\text{AMPL} & 0.02000 & \text{PHASE} & 45.00 \\
\end{array}
\]

<table>
<thead>
<tr>
<th>TAU</th>
<th>EN</th>
<th>OMEGA</th>
<th>ITER</th>
</tr>
</thead>
<tbody>
<tr>
<td>6E0000</td>
<td>66353</td>
<td>2.03102</td>
<td>6</td>
</tr>
<tr>
<td>7E0000</td>
<td>31671</td>
<td>1.99646</td>
<td>6</td>
</tr>
<tr>
<td>8E0000</td>
<td>08482</td>
<td>1.96911</td>
<td>6</td>
</tr>
<tr>
<td>9E0000</td>
<td>92333</td>
<td>1.94663</td>
<td>6</td>
</tr>
<tr>
<td>1.0E0000</td>
<td>80765</td>
<td>1.92753</td>
<td>6</td>
</tr>
<tr>
<td>1.1E0000</td>
<td>72330</td>
<td>1.91089</td>
<td>6</td>
</tr>
<tr>
<td>1.2E0000</td>
<td>66137</td>
<td>1.89605</td>
<td>6</td>
</tr>
<tr>
<td>1.3E0000</td>
<td>61616</td>
<td>1.88255</td>
<td>6</td>
</tr>
<tr>
<td>1.4E0000</td>
<td>58396</td>
<td>1.87005</td>
<td>6</td>
</tr>
<tr>
<td>1.5E0000</td>
<td>56230</td>
<td>1.85827</td>
<td>6</td>
</tr>
<tr>
<td>1.6E0000</td>
<td>54961</td>
<td>1.84702</td>
<td>5</td>
</tr>
<tr>
<td>1.7E0000</td>
<td>54490</td>
<td>1.83612</td>
<td>5</td>
</tr>
<tr>
<td>1.8E0000</td>
<td>54769</td>
<td>1.82542</td>
<td>6</td>
</tr>
<tr>
<td>1.9E0000</td>
<td>55785</td>
<td>1.81479</td>
<td>7</td>
</tr>
<tr>
<td>2.0E0000</td>
<td>57562</td>
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161
*************** PROGRAM LINSOL ***************

THIS PROGRAM COMPUTES THE DAMPING (LAMBDA) AND FREQUENCY (OMEGA) FOR GIVEN VALUES OF THE INTERACTION INDEX (EN) AND THE TIME-LAG (TAU). THIS PROGRAM IS BASED ON AN ANALYTICAL SOLUTION OF THE COMPLEX DIFFERENTIAL EQUATION.

THE FOLLOWING INPUTS ARE REQUIRED:

FIRST CARD:
THE TITLE OF THE CASE.

SECOND CARD:
GAMMA IS THE SPECIFIC HEAT RATIO.
UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.
RLD IS THE LENGTH-TO-DIAMETER RATIO.
ZCOMB IS THE LENGTH OF THE COMBUSTION ZONE, EXPRESSED AS A FRACTION OF THE CHAMBER LENGTH.
NDROPS DETERMINES THE PRESENCE OF DROPLET MOMENTUM SOURCES:
   NDROPS = 0 DROPLET MOMENTUM SOURCE NEGLECTED.
   NDROPS = 1 DROPLET MOMENTUM SOURCE INCLUDED.
NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:
   NOZZLE = 0 QUASI-STEADY
   NOZZLE = 1 CONVENTIONAL NOZZLE
NOPT SPECIFIES THE SOLUTIONS DESIRED:
   NOPT = 1 COUPLING COEFFICIENTS INCLUDED.
   NOPT = 2 COUPLING COEFFICIENTS NEGLECTED.

THIRD CARD (FOR CONVENTIONAL NOZZLE ONLY):
   YAMPL IS THE AMPLITUDE OF THE NOZZLE ADMITTANCE.
   YPHASE IS THE PHASE OF THE NOZZLE ADMITTANCE.

FOURTH CARD:
The mode is specified by the indices L, M, and N.
L IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED 10.
M IS THE AZIMUTHAL MODE NUMBER AND MUST NOT EXCEED 8.
N IS THE RADIAL MODE NUMBER AND MUST NOT EXCEED 5.
NCASES IS THE NUMBER OF CASES TO BE RUN.

REMAINING CARDS:
TAU IS THE TIME LAG.
EN IS THE INTERACTION INDEX.

******************************************************************************

COMPLEX YNOZ, RESULT, B(10), BC, AX(4), CI, CZE
DIMENSION TITLE(72), 
1 RJROOT(10,5), 
2 DC(5), OMEGA(100), 
3 EN(100), TAU(100)
REAL LAMBDA(100)
COMMON B
*************** DATA INPUT SECTION ********************

ERR = 0.000001
PI = 3.1415927
CI = (0.0, 1.0)

INPUT ROOTS AND VALUES OF BESSEL FUNCTIONS:
DATA ((RJR00T(I,J), J = 1,5), I = 1,9)/
1 3.83171, 7.01559, 10.17347, 13.32369, 16.47063,
2 1.84118, 5.33144, 8.53632, 11.70600, 14.86359,
3 3.05424, 6.70613, 9.96947, 13.17037, 16.34752,
4 4.20119, 8.01524, 11.34592, 14.58585, 17.78875,
5 5.31755, 9.28240, 12.68191, 15.96411, 19.19603,
6 6.41562, 10.51966, 13.98719, 17.31284, 20.57551,
7 7.50127, 11.73494, 15.26818, 18.63744, 21.93172,
8 8.57784, 12.93239, 16.52937, 19.94185, 23.26805,

INPUT PARAMETERS:
READ (5,5000) (TITLE(I), I = 1,72)
READ (5,5001) GAMMA, UE, RLD, ZCOMB, NDROPS, NOZZLE, NOPT
IF (NOZZLE EQ 1) GO TO 5

COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE:
YMPL = (GAMMA - 1.0) * UE/(2.0 * GAMMA)
YPHASE = 0.0
GO TO 7

READ (5,5002) YMPL, YPHASE
READ (5,5003) L, M, N, NCASES

THETA = YPHASE * PI/180.0
YR = YMPL * COS(THETA)
YI = YMPL * SIN(THETA)
YNOZ = CMPLX(YR,YI)

ZE = 2.0 * RLD
CZE = CMPLX(ZE,0.0)
CGAM = CMPLX(GAMMA,0.0)
CAX = CGAM
IF (NDROPS EQ 1) CAX = CGAM + (1.0,0.0)

DO 10 J = 1, NCASES
READ (5,5002) TAU(J), EN(J)
10 CONTINUE

*************** PRELIMINARY CALCULATIONS ********************

ASSIGN ARRAYS FOR ROOTS OF BESSEL FUNCTIONS:
IF ((M EQ 0) .AND. (N EQ 0)) GO TO 15
MM = M + 1
NN = N
SMN = RJROOT(MM,NN)
GO TO 20

15 SMN = 0.0

20 SSQ = SMN * SMN
CSSQ = CMPLX(SSQ, 0.0)

CALCULATE AXIAL ACOUSTIC EIGENVALUES:
CALL EIGVAL(L, SMN, GAMMA, ZE, YAMPL, YPHASE, RESULT)
B(1) = RESULT
BC = CONJG(RESULT)

********** CALCULATE AXIAL INTEGRALS **********

DO 100 NT = 1, 4
CALL AXIAL1(NT, 1, UE, ZE, ZCOMB, RESULT)
AX(NT) = RESULT
100 CONTINUE

********** CALCULATE VALUES AT NOZZLE ENTRANCE **********

ZEJ = CCOSH(CI*BC*CZE)
ZEP1 = CCOSH(CI*B(1)*CZE)
ZEP2 = CI * B(1) * CSINH(CI*B(1)*CZE)

********** CALCULATE COEFFICIENTS **********

CC = (CSSQ*AX(1) - AX(2) + ZEP2*ZEJ)/AX(1)
CD = (CAX*AX(3) + (2.0, 0.0)*AX(4)
1 + Cgam*YNOZ*ZEP1*ZEJ)/AX(1)
CE = Cgam*AX(3)/AX(1)

D(1) = REAL(CC)
D(3) = REAL(CD)
D(5) = REAL(CE)
IF (NOPT .EQ. 2) GO TO 50
D(2) = AIMAG(CC)
D(4) = AIMAG(CD)
GO TO 55
50 D(2) = 0.0
D(4) = 0.0

****** CALCULATION OF DAMPING AND FREQUENCY ******

55 WRITE (6,6001) (TITLE(I), I = 1, 72)
IF (NDROPS .EQ. 0) WRITE (6,6020)
IF (NDROPS .EQ. 1) WRITE (6,6021)
IF (NOPT .EQ. 2) WRITE (6,6015)
WRITE (6,6002) GAMMA, UE, KLD, ZCOMB
IF (NOZLLE .EQ. 0) WRITE (6,6012)
WRITE (6,6005) YAMPL, YPHASE
WRITE (6,6011) LINE = 14
C CALCULATE INITIAL GUESSES FOR FREQUENCY.
RL = L
AXI = RL * PI/ZE
AXSQ = AXI * AXI
SSQ = SMN * SMN
FRQ = SQRT(SSQ + AXSQ)

C

DO 200 J = 1, NCASES
C2R = D(3) - EN(J) * D(5)
C3 = EN(J) * D(5)
C
LAMBDA(1) = 0.0
OMEGA(1) = FRQ
C
K = 1
210 X = LAMBDA(K)
Y = OMEGA(K)
XT = X * TAU(J)
YT = Y * TAU(J)
EX = EXP(-XT)
SN = SIN(YT)
CS = COS(YT)
XSQ = X * X
WSQ = D(1) + XSQ + C2R*X - D(4)*Y
1 + C3*EX*(X*CS + Y*SN)
A = D(2) + C2R*Y + C3*EX*Y*CS
BB = 2.0*Y + D(4) - C3*EX*SN
C
OMEGA(K+1) = SQRT(WSQ)
LAMBDA(K+1) = -A/BB
C
IF (K .EQ. 40) GO TO 216
DX = ABS(LAMBDA(K+1) - LAMBDA(K))
DY = ABS(OMEGA(K+1) - OMEGA(K))
K = K + 1
IF ((DX .LT. ERR) .AND. (DY .LT. ERR)) GO TO 217
GO TO 210
C
216 WRITE (6,6009) TAU(J), EN(J), LAMBDA(K), OMEGA(K), K
GO TO 220
C
217 WRITE (6,6008) TAU(J), EN(J), LAMBDA(K), OMEGA(K), K
C
220 LINE = LINE + 2
IF (LINE .LT. 54) GO TO 200
WRITE (6,6007)
WRITE (6,6011)
LINE = 4
C
200 CONTINUE
************ FORMAT SPECIFICATIONS ***********

READ FORMATS
5000  FORMAT (72A1)
5001  FORMAT (4F10.0,31S)
5002  FORMAT (2F10.0)
5003  FORMAT (4I5)

WRITE FORMATS
6001  FORMAT (1H1,1X,72A1/)
6002  FORMAT (2X,8HGAMMA = ,F5.2,5X,SHUE = ,F5.2,5X,6HL/D = ,F8.5,
        1     5X,8HZCOMB = ,F5.2/)
6005  FORMAT (2X,7HAMEL = ,F8.5,5X,8PHASE = ,F6.1/)
6007  FORMAT (1H )
6008  FORMAT (2X,F5.3,F8.5,2F10.5,16/)
6009  FORMAT (2X,F5.3,F8.5,2F10.5,16,5X,18HFAILED TO CONVERGE/)
6011  FORMAT (2X,//4X,3HTAU,6X,2HIN,4X,6HAMBDA,5X,5HOMECA,
        1     2X,4HITER/)
6012  FORMAT (2X,19HQUASI-STEADY NOZZLE/)
6015  FORMAT (2X,24HCPLING TERMS NEGLECTED/)
6020  FORMAT (2X,'DROPLET MOMENTUM SOURCE NEGLECTED'/)
6021  FORMAT (2X,'DROPLET MOMENTUM SOURCE INCLUDED'/)
END
*************** PROGRAM LSTB3D ***************

THIS PROGRAM COMPUTES THE LINEAR STABILITY LIMITS CONSISTENT WITH THE THREE-DIMENSIONAL SECOND-ORDER THEORY.

THE FOLLOWING INPUTS ARE REQUIRED:

FIRST CARD:
THE TITLE OF THE CASE

SECOND CARD:
GAMMA IS THE SPECIFIC HEAT RATIO.
UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.
RLD IS THE LENGTH-TO-DIAMETER RATIO.
ZCOMB IS THE LENGTH OF THE COMBUSTION ZONE, EXPRESSED AS A FRACTION OF THE CHAMBER LENGTH.
NDROPS DETERMINES THE PRESENCE OF DROPLET MOMENTUM SOURCES:
NDROPS = 0 DROPLET MOMENTUM SOURCE NEGLECTED.
NDROPS = 1 DROPLET MOMENTUM SOURCE INCLUDED.
NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:
NOZZLE = 0 QUASI-STEADY
NOZZLE = 1 CONVENTIONAL NOZZLE
NOPT SPECIFIES WHICH SOLUTION WILL BE COMPUTED:
NOPT = 1 COUPLING COEFFICIENTS INCLUDED.
NOPT = 2 COUPLING COEFFICIENTS NEGLECTED.

THIRD CARD:
The mode is specified by the indices L, M, and N.
L IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED 10.
M IS THE AZIMUTHAL MODE NUMBER AND MUST NOT EXCEED 8.
N IS THE RADIAL MODE NUMBER AND MUST NOT EXCEED 5.

FOURTH CARD (IF CONVENTIONAL NOZZLE):
YAMPL IS THE AMPLITUDE OF THE NOZZLE ADMITTANCE.
YPHASE IS THE PHASE OF THE NOZZLE ADMITTANCE.

REMAINING CARDS:
TAMIN IS THE MINIMUM VALUE OF THE TIME-LAG.
TAMAX IS THE MAXIMUM VALUE OF THE TIME-LAG.
DELTAU IS THE INCREMENT IN TIME-LAG.

******************************************

COMPLEX  YNOZ, RESULT, B(10), BC, AX(4), CI, CZE
DIMENSION TITLE(72),
1 RJROOT(10,5),
2 OMEGA(100), EN(100)
COMMON B
************** DATA INPUT SECTION ****************************

ERR = 0.000001
PI = 3.1415927
CI = (0.0, 1.0)

INPUT ROOTS AND VALUES OF BESSEL FUNCTIONS:
DATA ((RJROOT(I,J), J = 1,5), I = 1,9):
  1  3.83171,  7.01559, 10.17347, 13.32369, 16.47063,
  2  1.84118,  5.33144,  8.53632, 11.70600, 14.86359,
  3  3.05424,  6.70613,  9.96947, 13.17037, 16.34752,
  4  4.20119,  8.01524, 11.34592, 14.58585, 17.88757,
  5  5.31755,  9.28240, 12.68191, 15.96411, 19.19603,
  6  6.41562, 10.51986, 13.98719, 17.31284, 20.57551,
  7  7.50127, 11.73494, 15.26816, 18.63744, 21.93172,
  8  8.57784, 12.93239, 16.52937, 19.94185, 23.26805,

INPUT PARAMETERS:
READ (5,5000) (TITLE(I), I = 1, 72)
READ (5,5001) GAMMA, UE, RLD, ZCOMB, NDROPS, NOZZLE, NOPT
READ (5,5002) L, M, N
8 IF (NOZZLE == 1) GO TO 5
C COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE:
YAMPL = (GAMMA - 1.0) * UE/(2.0 * GAMMA)
YPHASE = 0.0
GO TO 7
5 READ (5,5003, END = 300) YAMPL, YPHASE
7 READ (5,5003, END = 300) TAUMIN, TAUMAX, DELTAU

THETA = YPHASE * PI/180.0
YR = YAMPL * COS(THETA)
YI = YAMPL * SIN(THETA)
YNOZ = CMPLX(YR,YI)

ZE = 2.0 * RLD
CZE = CMPLX(ZE,0.0)
CGAM = CMPLX(GAMMA,0.0)
CAX = CGAM
IF (NDROPS == 1) CAX = CGAM + (1.0,0.0)

************** PRELIMINARY CALCULATIONS ****************************

ASSIGN ARRAYS FOR ROOTS OF BESSEL FUNCTIONS:
IF ((M EQ. 0) AND (N EQ. 0)) GO TO 15
MM = M + 1
NN = N
SMN = RJROOT(MM,NN)
GO TO 20
15 SMN = 0.0
20 SSQ = SMN * SMN
CSSQ = CMPLX(SSQ,0.0)
C CALCULATE AXIAL ACOUSTIC EIGENVALUES.
CALL EIGVAL(L, SMN, GAMMA, ZE, YAMPL, YPHASE, RESULT)
B(1) = RESULT
BC = CONJG(RESULT)

C ************* CALCULATE AXIAL INTEGRALS ***********************
DO 100 NT = 1, 4
CALL AXIAL1(NT, 1, 1, UE, ZE, ZCOMB, RESULT)
AX(NT) = RESULT
100 CONTINUE

C ************* CALCULATE VALUES AT NOZZLE ENTRANCE *************
ZEJ = CCOSH(CI*BC*CZE)
ZEP1 = CCOSH(CI*B(1)*CZE)
ZEP2 = CI * B(1) * CSINH(CI*B(1)*CZE)

C ************* CALCULATE COEFFICIENTS ***********************
CC = (CSSQ*AX(1) - AX(2) + ZEP2*ZEJ)/AX(1)
CD = (CAX*AX(3) + (2.0, 0.0)*AX(4)
     + CGAM*YNOZ*ZEPI*ZEJ)/AX(1)
CE = CGAM*AX(3)/AX(1)

C CI = REAL(CC)
DI = REAL(CD)
E = REAL(CE)
IF (NOPT .EQ. 2) GO TO 50
C2 = AIMAG(CC)
D2 = AIMAG(CD)
50 GOTO 55
55 OMEGA(1) = SQRT(C1)

WRITE (6, 6001) (TITLE(J), J = 1, 72)
IF (NDROPS .EQ. 0) WRITE (6, 6025)
IF (NDROPS .EQ. 1) WRITE (6, 6026)
IF (NOPT .EQ. 2) WRITE (6, 6022)
WRITE (6, 6002) GAMMA, UE, RLD, ZCOMB
IF (NOZZLE .EQ. 0) WRITE (6, 6012)
WRITE (6, 6005) YAMPL, YPHASE
WRITE (6, 6010)
LINE = 12

C TAU = TAUMIN
370 IF (TAU .GT. TAUMAX) GO TO 8
C

K = 1
310 WT = OMEGA(K) * TAU
    BB = (D1 + C2/OMEGA(K))/E
    EN(K) = BB/(1 + COS(WT))
    G = (E*EN(K)*SIN(WT) - D2) * OMEGA(K)
    OMEGA(K+1) = SQRT(C1 + G)
    IF (K .EQ. 40) GO TO 316
    IF (K .EQ. 1) GO TO 311
    DN = ABS(EN(K) - EN(K-1))
    DW = ABS(OMEGA(K+1) - OMEGA(K))
    IF ((DN .LT. ERR) .AND. (DW .LT. ERR)) GO TO 317
311 K = K + 1
    GO TO 310
C
316 WRITE (6,6013) TAU, EN(K), OMEGA(K), K
    GO TO 318
317 WRITE (6,6014) TAU, EN(K), OMEGA(K), K
C
318 LINE = LINE + 2
    TAU = TAU + DELTAU
    IF ((LINE .LT. 60) .OR. (TAU .GT. TAU MAX)) GO TO 370
    WRITE (6,6015)
    WRITE (6,6010)
    LINE = 6
    GO TO 370
C
300 CONTINUE
C
*************** FORMAT SPECIFICATIONS ****************************
C
C
READ FORMATS
5000 FORMAT (72A1)
5001 FORMAT (4F10.0,315)
5002 FORMAT (315)
5003 FORMAT (3F10.0)
C
WRITE FORMATS
6001 FORMAT (1H1,1X,72A1/)
6002 FORMAT (2X,6HGAMMA = ,F5.2,5X,5HUE = ,F5.2,5X,6HRLD = ,F8.5,1,
             5X,8HZCOMB = ,F5.2/)
6003 FORMAT (2X,A4,515,4F10.5/)
6005 FORMAT (2X,7HAMPL = ,F8.5,5X,8HPhase = ,F7.2/)
6007 FORMAT (1H )
6008 FORMAT (1H0)
6010 FORMAT (2X//8X,3HTAU,8X,2HEN,5X,5HOMECA,6X,4HTER/)
6012 FORMAT (2X,19QUASI-STEADY NOZZLE/)
6013 FORMAT (2X,3F10.5,110,5X,19H FAILED TO CONVERGE/)
6014 FORMAT (2X,3F10.5,110/)
6015 FORMAT (1H1)
6022 FORMAT (2X,24COUPLING TERMS NEGLECTED/)
6025 FORMAT (2X,'DROPLT MOMENTUM SOURCE NEGLECTED'/)
6026 FORMAT (2X,'DROPLT MOMENTUM SOURCE INCLUDED'/)
END
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**Title and Subtitle**

THE PREDICTION OF NONLINEAR THREE-DIMENSIONAL COMBUSTION INSTABILITY IN LIQUID ROCKETS WITH CONVENTIONAL NOZZLES

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**Abstract**

An analytical technique is developed to solve nonlinear three-dimensional, transverse and axial combustion instability problems associated with liquid-propellant rocket motors. The Method of Weighted Residuals is used to determine the nonlinear stability characteristics of a cylindrical combustor with uniform injection of propellants at one end and a conventional DeLaval nozzle at the other end. Crocco’s pressure sensitive time-lag model is used to describe the unsteady combustion process. The developed model predicts the transient behavior and nonlinear wave shapes as well as limit-cycle amplitudes and frequencies typical of unstable motor operation. The limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure wave-forms exhibit sharp peaks and shallow minima, and the frequency of oscillation is within a few percent of the pure acoustic mode frequency. For axial instabilities, the theory predicts a steep-fronted wave moving back and forth along the combustor.

**Key Words (Suggested by Author(s))**

Combustion instability
Liquid rockets
Method of Weighted Residuals

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