EFFECT OF GAGE SIZE ON THE MEASUREMENT OF LOCAL HEAT FLUX

by Kenneth J. Baumeister and S. Stephen Papell

Lewis Research Center
Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • NOVEMBER 1973
Abstract

General formulas are derived for determining gage averaging errors of strip-type heat flux meters used in the measurement of one-dimensional heat flux distributions. In addition, a correction procedure is presented which allows a better estimate for the true value of the local heat flux. As an example of the technique, the formulas are applied to the cases of heat transfer to air slot jets impinging on flat and concave surfaces. For many practical problems, the use of very small heat flux gages is shown to be often unnecessary.
EFFECT OF GAGE SIZE ON THE MEASUREMENT OF LOCAL HEAT FLUX
by Kenneth J. Baumeister and S. Stephen Papell
Lewis Research Center

SUMMARY

General formulas are derived for determining gage averaging errors of strip-type heat flux meters used in the measurement of one-dimensional heat flux distributions. The local averaging error $e(x)$ is defined as the difference between the measured value of the heat flux and the local value which occurs at the center of the gage. The present report will develop some simple expressions for estimating $e(x)$ by the use of a truncated Taylor series approximation. In the analysis, the gage is assumed to operate under steady-state conditions and to be 'perfect'; that is, it is assumed to measure exactly the heat flux that falls on it. In actual practice, however, conduction and radiation losses must be accounted for by calculation or calibration. In addition, a correction procedure is presented which allows a better estimate for the true value of the local heat flux. As an example of the technique, the formulas are applied to the cases of heat transfer to air slot jets impinging on flat and concave surfaces, as might occur in the internal cooling channels of turbine blades. One-dimensional heat flux distributions occur when cooling is accomplished with slot jets or with round hole jets aligned in tightly packed rows. It has been found that for many practical problems, the use of very small heat flux gages is often unnecessary.

INTRODUCTION

The aviation and power-generation industries have been concerned with heat flux measurements (refs. 1 and 2). Prior to 1960, measurements of heat flux (ref. 3) were generally concerned with average values taken over large areas. However, because of interest in prediction of local temperature gradients and thermal stresses, such as in the thermal design of turbine blades, recent work has been concerned with local heat flux measurements.

In choosing a particular heat flux meter, such as a slug-type, Gardon, or ablation calorimeter, the experimenter is faced with the choice of gage size. The gage can be
assumed to measure the average heat flux across its face, rather than a local value at its center (see fig. 1). In principle, the smallest possible heat flux gage should be used for accurate local heat flux measurements. However, the smaller the size of the gage, the greater will be the fabrication, handling, and calibration problems, as well as cost. This is illustrated in figure 2. The averaging error \( e(x) \) (difference between center value and average measured value of the heat flux) will decrease for smaller gage sizes, as illustrated in figure 2. On the other hand, for very small gages, calibration errors may increase and costs will rise because of fabrication difficulties. Clearly, some optimum choice of gage will generally be required, depending on the heat flux distribution and fabrication and calibration difficulty. The role of the heat flux distribution will be discussed in detail in the main body of this report.

To estimate the optimum size heat flux gage, the experimenter would like to know the "averaging" error \( e(x) \). At the present time, however, the literature is void of any means of predicting \( e(x) \) for these gages. In the present report, some relatively simple expressions will be developed for estimating \( e(x) \) by the use of a truncated Taylor series approximation. With the resulting analytical expressions, the experimenter can conveniently estimate the maximum gage size permissible for a given acceptable experimental error, provided he knows the general shape of the heat flux distribution curve. In detail, the purpose of this report is as follows:

1. To present an expression for the error \( e(x) \) due to gage averaging for one-dimensional heat flux distributions as measured with a strip-type gage (The gage is assumed to operate under steady-state conditions and to be "perfect"; i.e., it is assumed to measure exactly the heat flux that falls on it. In actual practice, however, there are losses, such as thermal conduction leakage or thermal radiation losses. These must be accounted for by calculation or calibration.)
2. To present parametric plots of the error distribution \( e(x) \) for some common types of heat flux distributions that might occur in practice
3. To apply the analysis to some problems in jet impingement heat transfer
4. To present a means of correcting measured heat flux data to account for gage size

**SYMBOLS**

\[ a \quad \text{constant} \]
\[ B \quad \text{width of slot jet (ref. 4)} \]
\[ C, C_0, C_1, C_2 \quad \text{constants} \]
\[ D \quad \text{diameter of hole} \]
Some impinging jet experiments for rows of holes and slot jets will now be discussed to illustrate some of the problems involved with the measurement of the local heat flux. Also, as an illustrative example, the analytical techniques will be applied to some jet impingement data.

Gardon and Akfirat (ref. 4) measured the local heat transfer coefficients under impinging axisymmetrical and slotted jets. Figure 3 shows some of their typical experimental results. They used a small 0.9-millimeter-diameter Gardon-type heat flux meter. Because of the small size of their meter, we would naturally expect very good local measurements. On the other hand, Tabakoff and Clevenger (ref. 5) used relatively large (3.8-cm wide) heat flux measurement devices (electrical heaters) to measure their heat transfer coefficient distribution. The validity of their local measurements is certainly subject to question. Figure 4 shows a typical experimental result from reference 5.
for rows of holes impinging on a concave surface. The question arises, how much
detailed local information can we deduce from the measured heat transfer coefficient dis-
tribution? Of course, one obvious problem with such gages, as used in reference 5, is
the possibility that finely structured peaks might occur and be undetected. Obviously,
such measurements would lead to greatly underestimated temperature and thermal stress
distributions if peaks occurred.

The manner in which the gage is used is also extremely important in sizing the gage.
If the gage can be positioned at all values of $x$, as was done by Gardon and Akfirat in
reference 4 by moving their plate and heat flux gage relative to a fixed jet, the local
peaks have a much better chance of being observed. On the other hand, if for a particu-
lar experiment the gages must be placed in a fixed position, as was done in reference 5
for the data shown in figure 4 herein, then it will not be possible to determine if any
local peaks occur within one gage size. Also, for very small fixed gages, local peaks
between the gages will also go undetected. A local peak, for example, might occur if the
boundary layer changes from laminar to turbulent flow, that is, if transition occurs.

In addition to sizing gages for new experiments, there is also a need to judge past
work. For example, how accurate was Gardon and Akfirat's measurement of the peak
heat transfer coefficient (fig. 3(a)), or how much higher is the local heat transfer coeffi-
cient at the stagnation point than the average value reported by Tabakoff and Clevenger
(fig. 4)?

Averaging Technique

Consider the one-dimensional heat flux distribution $q(x)$, as shown in figure 5,
which is being measured by a heat flux meter of size $\Delta x$. For the purpose of this re-
port, we assume that the gage measures exactly the total heat flux that falls on it; that
is, we assume that we can account for all losses from the heat flux meter, such as tem-
perature gradients along the wall, and correct for them. In this section, we are con-
cerned only with the problem of a large probe diameter washing out local effects.

The average heat flux measured by the gage can be expressed as

$$
\overline{q}_{\text{meas}}(\eta) = \frac{1}{\Delta \eta} \int_{\eta - \Delta \eta/2}^{\eta + \Delta \eta/2} q(\eta) d\eta
$$

(1)

where $\eta$ is a dimensionless distance chosen as

$$
\eta = \frac{x}{L}; \quad \Delta \eta = \frac{\Delta x}{L}
$$

(2)
Thus, the error in the measurement of the local heat flux can be expressed as

\[
e(\eta) = \frac{\bar{q}_{\text{meas}}(\eta) - q(\eta)}{q(\eta)} = \int \frac{q(\eta) d\eta}{\Delta \eta q(\eta)} - 1 \tag{3}
\]

By using equation (3), we can evaluate the errors associated with some common \( q(\eta) \) distributions for various dimensionless probe sizes \( \Delta \eta \).

**General Case**

Before we begin investigating specific heat flux distributions, let us consider properties of \( q(\eta) \) which could lead to large measurement errors. The function \( q(\eta) \) can be expanded in a Taylor series about any point "a" as follows:

\[
q(\eta) = q(a) + (\eta - a)q'(a) + \frac{(\eta - a)^2}{2!} q''(a) + \frac{(\eta - a)^3}{3!} q'''(a) + \frac{(\eta - a)^4}{4!} q^{IV}(a) + \ldots \tag{4}
\]

where \((I, II, \ldots)\) represents derivatives with respect to \( \eta \).

Substituting equation (4) into equation (3) and performing the integration gives

\[
e(\eta) = \frac{q(a)}{q(\eta)} - 1 + \frac{q'(a)}{2q(\eta)} (\eta - a) + \frac{q''(a)}{2q(\eta)} \left( (\eta - a)^2 + \frac{\Delta \eta^2}{12} \right) + \frac{q'''(a)}{24q(\eta)} \left[ 4(\eta - a)^3 + \Delta \eta^2 (\eta - a) \right] + \frac{q^{IV}(a)}{24} \left[ (\eta - a)^4 + \frac{\Delta \eta^2}{2} (\eta - a)^2 + \frac{\Delta \eta^4}{80} \right] + \ldots \tag{5}
\]

The location of the parameter "a" could be at some fixed point in the coordinate system, such as \( \eta = 0 \), or at the center of the heat flux gage. The simplest expression results when we expand about the center of the gage \( (a = \eta) \). In this case, equation (5) becomes

\[
e(\eta) = \frac{q''(\eta)}{q(\eta)} \frac{\Delta \eta^2}{24} + \frac{q^{IV}(\eta)}{q(\eta)} \frac{\Delta \eta^4}{1920} + \ldots \tag{6}
\]

or

\[
e(\eta) = \frac{q''(\eta)}{q(\eta)} \frac{\Delta \eta^2}{24} \left[ 1 + \frac{q^{IV}(\eta)}{q''(\eta)} \frac{\Delta \eta^2}{80} + \ldots \right] \tag{7}
\]
For many practical problems, the contribution of the fourth and higher order terms in equation (5) may be neglected. That is,

$$\left| \frac{q^{IV}(\eta)}{q^{II}(\eta)} \frac{\Delta \eta^2}{2^0} \right| \ll 1 \quad (8)$$

Thus, the error in the heat flux meter can be expressed as

$$e(\eta) \approx \frac{q^{II}(\eta)}{q(\eta)} \frac{\Delta \eta^2}{2^4} \quad (9)$$

The error, therefore, is proportional to the square of the size of the heat flux meter and to the second spatial derivative of heat flux.

Equation (9) can now be solved to determine the size of the heat flux gage for a given error:

$$\Delta \eta = \sqrt{\frac{2^4 e(\eta)q(\eta)}{q^{II}(\eta)}} \quad (10)$$

For calculational purposes, the heat flux $q(\eta)$ is often expressed in terms of a heat transfer coefficient $h(\eta)$ as follows:

$$q(\eta) = h(\eta)\Delta T(\eta) \quad (11)$$

Substituting equation (11) into equation (9) gives the following for the error:

$$e(\eta) = \left( \frac{1}{h} \frac{\partial^2 h}{\partial \eta^2} + 2 \frac{\partial \ln h}{\partial \eta} \frac{\partial \ln \Delta T}{\partial \eta} + \frac{1}{\Delta T} \frac{\partial^2 \Delta T}{\partial \eta^2} \right) \frac{\Delta \eta^2}{2^4} \quad (12)$$

The spatial temperature derivatives will, of course, depend on the particular flow situation being measured.

Peaks

The expression for the error just derived is inconvenient to use when any part of the heat flux gage crosses a peak, such as might occur at a stagnation point, because of the large number of terms that would be needed to express a continuous function for the peak. In the previous analysis, it was assumed that the odd derivatives were continuous across the gage center, at $\eta = 0$, and thus did not contribute in the integration of equation (3). However, at a flux peak, symmetry is now assumed to exist for all terms in the Taylor
expansion. Hence, equation (3) becomes

$$e(0) = \frac{2}{\Delta \eta} \int_0^{\Delta \eta/2} q(\eta) d\eta - 1$$  \hspace{1cm} (13)$$

For this special case, the parameter "a" in equation (4) takes on the value of zero, and the Taylor series reduces to a Maclaurin series. Substituting the Maclaurin series into equation (13) and performing the specified integration gives

$$e(0) = \frac{q'(0)}{4} \frac{\Delta \eta}{q(0)} + \frac{q''(0)}{24} \frac{\Delta \eta^2}{q(0)} + \frac{q'''(0)}{192} \frac{\Delta \eta^3}{q(0)} + \frac{q''''(0)}{1920} \frac{\Delta \eta^4}{q(0)} + \ldots$$ \hspace{1cm} (14)$$
or

$$e(0) = \left[ \frac{q'(0)}{4} \frac{\Delta \eta}{q(0)} + \frac{q''(0)}{24} \frac{\Delta \eta^2}{q(0)} + \frac{q'''(0)}{192} \frac{\Delta \eta^3}{q(0)} + \frac{q''''(0)}{1920} \frac{\Delta \eta^4}{q(0)} + \ldots \right]$$ \hspace{1cm} (15)$$

As can be seen from equation (15), both the odd and even derivatives contributed to the error, in contrast to equation (7), in which only the even order terms contributed to the error. Consequently, the estimate to order $\Delta \eta^2$ is

$$e(0) \approx \frac{q'(0)}{4} \frac{\Delta \eta}{q(0)} + \frac{q''(0)}{24} \frac{\Delta \eta^2}{q(0)}$$ \hspace{1cm} (16)$$

Note, if the heat flux is expanded in terms of even functions such that the first derivatives are zero at the origin, then equations (9) and (16) are identical.

Now, we will apply the general theory to two simple example profiles. Let us assume that the heat flux distribution can be approximated by either a second-order polynomial or an exponential distribution. These two distributions were chosen because they can be used to approximate many practical heat flux distributions. Also, we can illustrate a case where the truncated equations cannot be used, and we must rely on an exact integration of equation (3) or (13) to determine $e(x)$.

**Polynomial**

An expression for the heat flux can be written in terms of a polynomial of the form

$$q(\eta) = C_0 + C_1 \eta + C_2 \eta^2$$ \hspace{1cm} (17)$$
The preceding second-order polynomial should be sufficient for describing many measured heat flux distributions. On the other hand, if more terms are needed in the polynomial, the extension is straightforward.

Since the third and higher order derivatives of equation (17) are zero, the truncated series expressions for the error are exact. Thus, the errors are

\[
e(\eta) = \frac{C_2 \Delta \eta^2 / 12}{C_0 + C_1 \eta + C_2 \eta^2} \quad \text{for} \quad \eta > \frac{\Delta \eta}{2}
\]

and

\[
e(0) = \frac{C_1 \Delta \eta}{C_0} + \frac{C_2 \Delta \eta^2}{12 C_0} \quad \text{for} \quad \eta = 0
\]

**Exponential**

The exponential distribution for the heat flux can be expressed as

\[
q(\eta) = Ce^{-\beta \eta}
\]

Substituting equation (20) into equations (3) and (13) and solving for the error distributions gives

\[
e(\eta) = \frac{2}{\beta \Delta \eta} \sinh \frac{\beta \Delta \eta}{2} - 1 \quad \text{for} \quad \eta > \frac{\Delta \eta}{2}
\]

\[
e(0) = \frac{2}{\beta \Delta \eta} \left(1 - e^{-\beta \Delta \eta/2}\right) - 1 \quad \text{for} \quad \eta = 0
\]

If we use the truncated series expressions, equations (9) and (16), the error distribution becomes

\[
e(\eta) = \frac{\Delta \eta^2 \beta^2}{24} \quad \text{for} \quad \eta > \frac{\Delta \eta}{2}
\]

\[
e(0) = \frac{-\beta \Delta \eta + \beta^2 \Delta \eta^2}{4} \quad \text{for} \quad \eta = 0
\]
In equation (24), we have kept the first two terms in the series, since this will make both equations (23) and (24) accurate to at least order $\Delta \eta$ squared.

The choice of the exponential representation is convenient, since the error is independent of position $\eta$, which was not the case for the second-order polynomial. Thus, a single plot will suffice for a complete mapping of the errors associated with a heat flux distribution, which can be approximated by an exponential.

Parametric plots for the errors involved with the exponential approximation are displayed in figures 6 and 7. Figure 6 considers measurements for $\eta > \Delta \eta/2$, while figure 7 considers measurements at a point of symmetry. As seen in these figures, for $\beta$ values less than 2, the truncated series expression for the error can be used without significant error. For $\beta$ values greater than 2, an exact integration is required. However, for most practical problems the value of $\beta$ is much less than 2.

Next, we shall use the results of this section to estimate the error involved in the measurements of Garden and Akfirat (ref. 4) and of Tabakoff and Clevenger (ref. 5).

**IMPINGING JET EXAMPLES**

The results of the last section will be applied to the problem of heat transfer from impinging jets. In particular, we will examine the data of Garden and Akfirat (ref. 4) for a two-dimensional jet, and the data of Tabakoff and Clevenger (ref. 5) for a row of hole jets impinging on a curved surface. In both experiments, the heat flux is essentially one-dimensional. The data of Tabakoff and Clevenger, shown in figure 3, are of particular interest, since we wish to examine the effect of very large heat flux meters on local measurements.

Tabakoff and Clevenger used strip-type heat flux meters; consequently, the analysis just presented is directly applicable to their data. On the other hand, Garden and Akfirat used small cylindrical meters. As a first approximation, we assume the present analysis can be applied to such data.

Gardon and Akfirat's data for the slot jet ($Z_\eta/B = 8$, see fig. 3(a)) near the stagnation point will be examined first. In the vicinity of the stagnation point, the heat transfer coefficient can be fitted by the equation

$$h(\eta) = 85 - 13\eta^2 \quad \text{for} \quad \eta \leq 1$$

(25)

where

$$\eta = \frac{x}{B}$$

(26)

We will assume that the wall-to-bulk temperature gradients are negligible; thus, the
heat transfer coefficient and the heat flux will have a one-to-one correspondence in the expressions for the error, and they may be used interchangeably. Thus, if equation (25) is combined with equations (17) and (19), the expression for the error becomes

\[ e(\eta) = -0.0127 \Delta \eta^2 \]  
(27)

The 0.9-millimeter-diameter gage used by Gardon and Akfirat had a \( \Delta \eta \) value of 0.282. If the value of 0.282 is substituted into equation (27), the error in the absolute value of the heat transfer coefficient is calculated to be -0.1 percent, which is quite acceptable. At all other points in the flow field, the error will be negligible. Had the gage diameter been increased by one order of magnitude, from 0.9 to 9.0 millimeter, the error would be about 10 percent.

As shown in figure 8, the data (fig. 4) of Tabakoff and Clevenger (ref. 5) can be fitted by a polynomial of the form

\[ h \propto 2.65 - 3.77 + 1.8 \eta^2 \]  
(28)

where

\[ \eta = \frac{x}{2 \times \text{heater width}} \]  
(29)

Since only three measured values are presented to the left of the stagnation point, these values are fitted exactly by a second-order polynomial. The error at the stagnation point in this case becomes

\[ e(\eta) = -0.35 \Delta \eta + 0.057 \Delta \eta^2 \]  
(30)

In this particular case, \( \Delta \eta \) equals 0.5, and the error in the absolute value at the stagnation point is -16 percent.

The assumption was made here that the heat transfer coefficient is a monotonically decreasing function for \( \eta < 1/2 \). Had any singularities or peaks existed in this range, they would have gone undetected.

The error at the second station in figure 4 can be evaluated from equation (18) to be 3 percent. If the heat flux is monotonically decreasing, we see that the error in the measurement for this relatively large gage is quite acceptable. Also, if the gage were movable instead of being fixed, some peaks could even be detected by this gage.
CORRECTION PROCEDURE

The application of corrections to raw data is commonplace in engineering. The temperature distributions on the insides of tubes are often determined from measurements taken on the outsides of tubes by appropriate consideration of the thermal conductivities of the tubes and other operating parameters. In a similar manner, the heat flux measurements can also be corrected to account for the error due to probe averaging.

A better estimate for the "true" heat flux can be determined from the measured heat flux by solving the simple integral equation

\[
\frac{1}{\Delta \eta} \int_{\eta-\Delta \eta/2}^{\eta+\Delta \eta/2} q(\eta) d\eta = \bar{q}_{\text{meas}}(\eta)
\] (31)

For the exponential and polynomial cases, this integral reduces to

\[
q(\eta) = \frac{\bar{q}_{\text{meas}}(\eta)}{e(\eta) + 1}
\] (32)

where the various expressions for \( e(\eta) \) have been given earlier in this report.

This correction technique is simple to use. Thus, it may be possible to use relatively large heat flux meters in obtaining acceptable local data.

For example, for the Tabakoff and Clevenger (ref. 5) data, using equation (32) in conjunction with equation (30) gives a better estimate of the true value of the actual heat transfer coefficient at the stagnation point. The calculated value at the stagnation point is labeled "calculated peak" in figure 8.

CONCLUDING REMARKS

General formulas are derived for determining gage averaging errors of strip-type heat flux meters used in the measurement of one-dimensional heat flux distribution. In addition, a correction procedure is presented which allows a better estimate of the true value of the local heat flux. As an example of the technique, the formulas are applied to the cases of heat transfer to air slot jets impinging on flat and concave surfaces. For many practical problems, it is possible to use a relatively large gage to obtain acceptable local heat flux measurements, provided that the gage is small enough to detect any peaks which might occur in the heat flux distribution.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 3, 1973,
501-04.
REFERENCES


Figure 1. - Relation between local and "average" heat flux for nonlinear heat flux.

Calibration and fabrication error
Averaging error, e(Δx)

Figure 2. - Effect of gage size on measured error.

Figure 3. - Lateral variation of local heat transfer coefficient between a plate and an impinging two-dimensional air jet. Width of slot jet, B, 0.3175 centimeter (1/8 in.). (Data from ref. 4.)

(a) Impinging jet.
(b) Wall jet.

Figure 4. - Heat transfer for impinging hole jets on a concave surface. Hole diameter, D, 0.635 centimeter (1/4 in.); spacing-to-diameter ratio, H/D, 1; height-to-diameter ratio, Zn/D, 8.8. (Data from ref. 5.)
Figure 5. - Heat flux distribution and meter geometry.

Figure 6. - Error as a function of dimensionless width of probe $\Delta \eta$ and constant $\beta$ for exponential heat flux.
"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546