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Unified Powered Flight Guidance

By

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The publication of this report does not constitute approval by the National Aeronautics and Space Administration of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.
PREFACE

This document reflects a complete revision of the orbiter powered flight guidance scheme. A unified approach to powered flight guidance has been taken to accommodate all phases of exo-atmospheric orbiter powered flight, from ascent through deorbit. The guidance scheme has been changed from the previous modified version of the Lambert Aim Point Maneuver Mode used in Apollo to one that employs Linear Tangent Guidance concepts. As such it supersedes the previous document, "Powered Flight Guidance", GN&C Equation Document No. 11, Rev. 2, April 1972. In addition, this document replaces the previous ascent phase equation document titled "Powered Ascent Guidance", GN&C Equation Document No. 1-71, January 1971.
FOREWORD


Gerald M. Levine
Division Leader, Guidance Analysis
NASA Programs Department
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NOMENCLATURE

\( a_L \) Specific force limit during SSME maneuvers

\( a_{T,i} \) Assumed thrust acceleration for \( i \)th phase

\( C_1, C_2 \) Constants used to define target (entry interface) velocity constraint, \( v_v = C_1 \cdot C_2 \cdot h \)

\( C_{i,v} \) Intermediate variable used to determine sensitivity of \( \ld \) to \( t_t \)

\( \text{SSME} \) Vacuum thrust of single SSME, OMS, or RCS engine, respectively

\( \text{OMS} \)

\( \text{RCS} \)

\( T_{i,i} \) Total assumed thrust for \( i \)th phase

\( z \) Gravity vector

\( H \) Thrust acceleration integral, \( \int_{t_0}^{t_o} (t_{T,i}(t)) \cdot t^2 \cdot dt \)

\( H_{i,i} \) \( i \)th element of \( H \), \( \int_{t_0}^{t_o} (t_{T,i}(t)) \cdot t^2 \cdot dt \)

\( \mathbf{I} \) Unit thrust vector

\( \mathbf{I}_d \) Unit vectors relative to desired trajectory plane:

\( \mathbf{I}_R \) is radial along \( \mathbf{I}_d \), \( \mathbf{I}_N \) is normal to desired trajectory plane, and \( \mathbf{I}_z \) is in the downrange direction. (\( \mathbf{I}_x = \mathbf{I}_y = \mathbf{I}_z \))

\( \mathbf{I}_N \) Unit vector in direction of the orbital angular momentum vector normal to the transfer plane (see Ref. 5)

\( \mathbf{I}_{rt} \) Unit vector in direction of \( \mathbf{I}_t \)
Element subscript variables

$J$  
Thrust acceleration integral, $\int_{t_{0,i}}^{t_{go,i}} \left( \frac{f_T}{m} \right) t \ dt$

$J_i$  
$i^{th}$ element of $J$, $\int_{t_{0,i-1}}^{t_{go,i}} \left( \frac{f_T}{m} \right) t \ dt$

$K_i$  
SSME throttle setting, $i^{th}$ phase ($1.0 = 100\%$)

$K_{\text{max}}$  
Maximum throttle setting of SSME

$k$  
Subscript variable referring to current phase number

$L$  
Thrust acceleration integral, $\int_{t_{0}}^{t_{go}} \left( \frac{f_T}{m} \right) \ dt$

$L_i$  
$i^{th}$ element of $L$, $\int_{t_{0,i-1}}^{t_{go,i}} \left( \frac{f_T}{m} \right) \ dt$

$m$  
Current simulated vehicle mass

$m_{0,i}$  
Mass at beginning of $i^{th}$ phase

$m_{\text{SSME}}$, $m_{\text{OMS}}$, $m_{\text{RCS}}$  
Mass flow rate of single SSME, OMS, or RCS engine, respectively

$m_i$  
Total mass flow rate, $i^{th}$ phase

$n$  
Number of thrust phases

$n_{\text{rev}}$  
Number of revolutions of coast (see Ref. 5)

$P$  
Thrust acceleration integral, $\int_{0}^{t_{go}} \int_{0}^{t} \left( \frac{f_T}{m} \right) s^2 ds \ dt$

$P_i$  
$i^{th}$ element of $P$, $\int_{t_{go,i-1}}^{t_{go,i}} \int_{t_{go,i-1}}^{t} \left( \frac{f_T}{m} \right) s^2 ds \ dt$

$Q$  
Thrust acceleration integral, $\int_{t_{0}}^{t_{go}} \int_{t_{0}}^{t} \left( \frac{f_T}{m} \right) s \ ds \ dt$

$Q_i$  
$i^{th}$ element of $Q$, $\int_{t_{go,i-1}}^{t_{go,i}} \int_{t_{go,i-1}}^{t} \left( \frac{f_T}{m} \right) s \ ds \ dt$
\( \mathbf{r} \)  
Vehicle position vector

\( \mathbf{r}_{\text{bias}} \)  
A position bias to account for effects of a rotating thrust vector  
\( (\mathbf{r}_{\text{bias}} = \mathbf{r}_{\text{go}} - \mathbf{r}_{\text{thrust}}) \)

\( \mathbf{r}_{\text{c1}} \)  
Vehicle position vector at beginning of gravity computation coast segment

\( \mathbf{r}_{\text{c2}} \)  
Vehicle position vector at end of gravity computation coast segment

\( \mathbf{r}_d \)  
Desired terminal (cutoff) position

\( r_d \)  
Desired radius magnitude at terminal (cutoff) position

\( \mathbf{r}_{\text{go}} \)  
Position-to-be-gained including bias (\( \mathbf{r}_{\text{thrust}} \) reflects true position-to-be-gained)

\( \mathbf{r}_{\text{goxy}} \)  
Projection of \( \mathbf{r}_{\text{go}} \) on plane defined by \( \mathbf{i}_x \) and \( \mathbf{i}_y \)

\( r_{\text{goz}} \)  
Component of \( \mathbf{r}_{\text{go}} \) along \( \mathbf{i}_z \) (downrange)

\( \mathbf{r}_{\text{grav}} \)  
Second integral of central force field gravitational acceleration over thrusting maneuver

\( \mathbf{r}_p \)  
Predicted terminal (cutoff) position

\( \mathbf{r}_{\text{ref}} \)  
Position on reference trajectory at time \( t_{\text{ref}} \)

\( \mathbf{r}_t \)  
Target position in inertial coordinates

\( \mathbf{e}_{\text{ref}} \)  
Entry interface target in Earth fixed coordinates

\( \mathbf{r}_{\text{thrust}} \)  
Second integral of thrust acceleration vector over thrusting maneuver
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>Thrust acceleration integral, ( \int_{0}^{t_{go}} \int_{0}^{t} \left( \frac{f_r}{m} \right) ds \ dt )</td>
</tr>
<tr>
<td>$s_{engoff}$</td>
<td>Switch indicating whether engine-off command has been issued, ( s_{engoff} = 0 ), command not issued, ( s_{engoff} = 1 ), command issued</td>
</tr>
<tr>
<td>$s_{guess}$</td>
<td>Switch set by Lambert routine indicating type of solution (see Ref. 5)</td>
</tr>
<tr>
<td>$S_i$</td>
<td>Ith element of $S$, ( \int_{t_{go,i-1}}^{t_{go,i}} \int_{t_{go,i-1}}^{t} \left( \frac{f_r}{m} \right) ds \ dt )</td>
</tr>
<tr>
<td>$s_{mode}$</td>
<td>Maneuver mode (see Ref. 5)</td>
</tr>
<tr>
<td>$s_{pass1}$</td>
<td>( s_{pass1} = 1 ), first active guidance call, ( s_{pass1} = 0 ), not first active guidance call</td>
</tr>
<tr>
<td>$s_{pert}$</td>
<td>Gravity perturbation switch (( &gt; 0 ) for non-Keplerian model)</td>
</tr>
<tr>
<td>$s_{phase,i}$</td>
<td>( s_{phase,i} = 0 ), constant thrust during phase $i$, ( s_{phase,i} = 1 ), constant acceleration during phase $i$</td>
</tr>
<tr>
<td>$s_{pre}$</td>
<td>Prethrust switch, ( s_{pre} = 1 ), prethrust, ( s_{pre} = 0 ), active guidance</td>
</tr>
<tr>
<td>$s_{proj}$</td>
<td>Switch indicating whether the initial and target position vectors are to be projected into the plane defined by ( \hat{\mathbf{i}}_X ) (see Ref. 5)</td>
</tr>
<tr>
<td>$s_{soln}$</td>
<td>Lambert solution type switch (see Ref. 5)</td>
</tr>
<tr>
<td>$S_{SSME,i}$</td>
<td>Number of SSME, OMS, RCS engines, respectively, assumed to be thrusting during $i$th phase</td>
</tr>
<tr>
<td>$S_{OMS,i}$</td>
<td>Number of SSME, OMS, RCS engines, respectively, assumed to be thrusting during $i$th phase</td>
</tr>
<tr>
<td>$S_{RCS,i}$</td>
<td>Number of SSME, OMS, RCS engines, respectively, assumed to be thrusting during $i$th phase</td>
</tr>
</tbody>
</table>
\( t \)  
Time associated with \( \mathbf{r}, \mathbf{v} \)

\( t_{b, i} \)  
Estimated burn time remaining in phase \( i \)

\( t_c \)  
Coast time between last and next-to-last phase for tank separation

\( t_{\text{ref}} \)  
Nominal cut-off time for reference trajectory mode

\( t_{\text{go}} \)  
Time-to-go until end of maneuver

\( t'_{\text{go}} \)  
\( t_{\text{go}} \) of previous call

\( t_{\text{go}, i} \)  
Time-to-go until end of \( i \)th phase

\( t_{\text{ig}} \)  
Ignition time of first phase

\( t_{\text{prev}} \)  
\( t \) of previous guidance cycle

\( t_t \)  
Time at target (point where terminal constraint defined)

\( u \)  
Iteration variable determined in Lambert Routine (see Ref. 5)

\( \mathbf{v} \)  
Vehicle velocity vector

\( \mathbf{v}_{\text{bias}} \)  
A velocity bias to account for effects of a rotating thrust vector \( \mathbf{v}_{\text{bias}} = \mathbf{v}_{\text{go}} - \mathbf{v}_{\text{thrust}} \)

\( \mathbf{v}_{\text{c1}} \)  
Vehicle velocity vector at beginning of gravity computation coast segment

\( \mathbf{v}_{\text{c2}} \)  
Vehicle velocity vector at end of gravity computation coast segment

\( \mathbf{v}_d \)  
Desired velocity vector at terminal (cutoff) position

\( \mathbf{v}_d' \)  
Desired velocity magnitude at terminal (cutoff) position

\( \mathbf{v}_d'' \)  
Value of \( \mathbf{v}_d \) resulting from perturbed time of flight
\( v_{\text{ex, i}} \)  
Effective exhaust gas velocity for phase \( i \)

\( v_{\text{go}} \)  
Velocity-to-be-gained including bias. \( v_{\text{thrust}} \) reflects true velocity-to-be-gained

\( v_{\text{goz}} \)  
Component of \( v_{\text{go}} \) along \( i_z \) (downrange)

\( v'_{\text{go}} \)  
Recomputed \( v_{\text{go}} \) to satisfy terminal constraints

\( v_{\text{grav}} \)  
First integral of central force field gravity acceleration over thrusting maneuver

\( v_{\text{ref}} \)  
Velocity on reference trajectory at time \( t_{\text{ref}} \)

\( v_{\text{t}} \)  
Velocity vector at target

\( v'_{\text{t}} \)  
Value of \( v_{\text{t}} \) resulting from perturbed time of flight

\( v_{\text{th}} \)  
Projection of \( v_{\text{t}} \) on horizontal plane

\( v'_{\text{th}} \)  
Projection of \( v'_{\text{t}} \) on horizontal plane

\( v_{\text{thrust}} \)  
First integral of thrust acceleration vector over thrusting maneuver

\( v_{\text{tv}} \)  
Projection of \( v_{\text{t}} \) on vertical

\( v'_{\text{tv}} \)  
Projection of \( v'_{\text{t}} \) on vertical

\( \gamma_{\text{d}} \)  
Desired inertial flight path angle at terminal (cutoff) position

\( \Delta r_c \)  
Position offset used in gravity computation

\( \Delta r_z \)  
Downrange component of terminal position error (used during reference trajectory mode)

\( \Delta t \)  
Guidance cycle time step

\( \Delta t_{\text{cutoff}} \)  
Value of \( t_{\text{go}} \) used to define time to issue engine cutoff command and terminate active steering computations
Δt_{\text{go}} \quad \text{Change in } t_{\text{go}} \text{ used in throttling computations}

Δt_{\text{tg}} \quad \text{Time interval before } t_{\text{tg}} \text{ to start active guidance calls}

Δv_{\text{c}} \quad \text{Impulse velocity increment used in gravity computations}

Δv_{\text{go}} \quad \text{Change in } v_{\text{go}}

Δv_{\text{sensed}} \quad \text{Total velocity change accumulated on accelerometers since last reading}

Δv_{\text{OMS}} \quad \text{Total characteristic velocity to be imparted during the last ascent phase by the OMS engines}

δt \quad \text{Perturbation in coast time used in deorbit required velocity computations to determine sensitivity to coast time}

c_{\text{cone}} \quad \text{Sine of half cone angle of ejection (see Ref. 5)}

ε_{\text{vgo}} \quad \text{Value of } |Δv_{\text{go}}| \text{ defining prethrust convergence limit}

\lambda \quad \text{Unit vector in direction of } v_{\text{go}}

\dot{\lambda} \quad \text{Time derivative of unit vector coincident with } \lambda \text{ but rotating with desired thrust vector turning rate, } \omega_{\ell}

ρ \quad \text{Pumping factor used in determining the change in } Δv_{\text{go}}

σ \quad \text{Scaling factor in required velocity computations}

τ_{\text{i}} \quad \text{Ratio of mass to mass flow rate for } \text{th phase}

θ \quad \text{Angle between } v_{\text{go}} \text{ and } \ell_f
\dot{\phi} \quad \text{Time rate of change of } \phi

\alpha_f \quad \text{Desired thrust vector turning rate}
1. INTRODUCTION

1.1 Objectives and Requirements

The primary objective of powered flight guidance is to issue proper steering and if necessary throttle commands during the thrusting portions of a mission such that the desired objectives of the maneuver are satisfied in a reasonably efficient manner. In addition, the navigated vehicle state vector must be maintained through the maneuver. These are obvious requirements of the powered flight guidance program. Additional objectives or requirements, which influence the overall design, are listed below:

1) The guidance program should be applicable to all powered maneuvers. This objective assumes that a single guidance routine is less expensive to design, code, and maintain in a flight computer than several smaller routines. This guidance scheme, referred to as Unified Powered Flight Guidance (UPFG), can be used for all exo-atmospheric orbiter thrusting maneuvers, from ascent through decrbit. Future versions of this scheme should probably include the atmospheric solid rocket boost phase for completeness and to minimize any difficulties in transition from one guidance phase to the next. A complete description of the many types of maneuvers possible with this UPFG routine is included in the following section.

2) The guidance program should be simple and flexible. Any guidance program which must handle the many types of maneuvers required of the space shuttle orbiter, from the relatively high accelerations during ascent to the very low accelerations experienced with a single Orbital Maneuvering System (OMS) engine, can probably not be classified as truly simple. However, by properly structuring the routine, the various requirements can be handled efficiently and the impact of new requirements can be minimized. This UPFG routine is structured such that communality of basic computations is maintained with specialized operations performed according to the mission phase and desired maneuver objectives. In addition, to minimize program size, maximum use is made of subroutines required for other GAN functions.
(3) **The premaneuver prediction of the maneuver \( \Delta v \) requirement should be reasonably accurate.** In order to assist the premaneuver targeting process and make mission critical decisions, the powered flight guidance scheme must accurately predict the maneuver \( \Delta v \), especially for long, low acceleration orbital maneuvers. This requirement can have a very significant effect upon the guidance algorithm design. Many guidance schemes are based upon approximations which become more accurate as the maneuver progresses. Although these schemes may satisfactorily calculate steering commands during the course of the maneuver, they may not adequately predict the maneuver \( \Delta v \). This unified guidance scheme is based upon modifications to the Linear Tangent Guidance (LTG) concept which improve overall accuracy for both premaneuver targeting and guidance. This improvement in the LTG concept was also required for one of the ascent guidance modes so that engine throttle changes, which are based upon an accurate prediction of the terminal (cutoff) state of the vehicle, could be properly calculated.

(4) **The guidance algorithm should satisfy primary maneuver objectives for nominal and perturbed conditions.** When practical, the guidance scheme should close the guidance loop around the true maneuver constraints, rather than a set of artificial constraints based upon nominal conditions. For example, during many on-orbit maneuvers the true constraints lie on the coasting trajectory subsequent to the maneuver. During the terminal phase rendezvous maneuver the objective is to intercept the target vehicle. During the deorbit maneuver the true maneuver constraints are at entry interface, where a pre-determined relationship between entry range, velocity, and flight path angle must be satisfied. It is possible to determine a set of artificial constraints, defined at thrust cutoff, which will satisfy the maneuver objectives. However, the process of accurately calculating these artificial constraints can significantly complicate the targeting process. In addition, the artificial constraints would necessarily have to be based upon a nominal coast trajectory after thrust cutoff. All perturbations during the maneuver
could result in a loss of performance (i.e. the true constraints are only partially satisfied) or an increase in total maneuver $\Delta v$, caused by trying to force the solution onto the nominal coast trajectory at thrust cutoff. Furthermore, if the guidance system satisfies true maneuver constraints, then much of the analysis necessary to determine guidance software performance under perturbed conditions can be eliminated.

5. The guidance algorithm should produce near fuel optimal maneuvers for nominal and perturbed conditions. The Linear Tangent Guidance (LTG) Equations, which form the basis for the UPFG scheme, are based upon classical optimization theory and appear to give excellent performance. However, it should be noted that even a truly optimal scheme will use excessive fuel if the ignition time is poorly chosen or the maneuver constraints result in an unnecessarily over-constrained maneuver which could be more efficiently performed by using more than one maneuver.

1.2 Types of Maneuvers

From a guidance viewpoint, the orbiter maneuvers can be conveniently separated into two classes. The first class, ascent, is characterized by a constraint on the vehicle altitude at thrust termination. The second class of maneuvers, on-orbit, does not require any constraint on the position at thrust termination. On-orbit maneuvers are typically intended to place the vehicle onto any coasting trajectory which satisfies a rendezvous intercept constraint or deorbit entry-interface condition. In these cases, any type of thrust cutoff position constraint would probably increase the maneuver $\Delta v$ unnecessarily.

Each of these classes of maneuvers can be further subdivided into individual maneuver modes with a particular set of objectives and a specified set of constraints which meet these objectives. These individual modes are listed below:

1. Standard Ascent Maneuver

This mode is intended for use during the ascent phase of most missions. The insertion conditions are specified preflight, and defined by a desired terminal (cutoff) altitude, velocity, flight path angle, and orbital plane. Thus all components of the terminal state are specified except the downrange component of position.
(1) **Ascent to Coast Reference Trajectory**

This mode is intended for use on Mission 3B, a time critical mission involving ascent, rendezvous, satellite retrieval, and deorbit with return to the launch site one revolution after liftoff. To satisfy the objectives of this mission in the presence of perturbations and small launch delays, the main engine throttles are used to improve control over the ascent and subsequent coasting trajectory. Introducing the throttle command as one additional degree of freedom in the guidance algorithm makes it possible to insert the orbiter onto a coast reference trajectory at cutoff. This coast reference trajectory can be determined preflight to provide proper closing velocity with the satellite for the rendezvous braking maneuvers.

(2) **Lambert Ascent Maneuver**

This mode can be used to insert the orbiter at a specified altitude onto a coasting trajectory which intercepts a specified position (target) at a specified time. It is similar to the coast reference trajectory mode, except that throttle commands for trajectory control are not used. Therefore, although the resulting coasting trajectory does intercept the target, it does not guarantee the proper closing velocity for successful braking. This mode can be used during the latter phases of ascent to a coast reference trajectory, when g-limiting considerations may override throttling. Since most perturbations occur well prior to g-limiting, the coast reference trajectory mode is designed to complete compensation for perturbations prior to g-limiting. During g-limiting, the Lambert ascent mode will maintain an intercept trajectory with only small deviations in terminal rendezvous closing velocity. It should be noted that no additional code is required to support this guidance mode, since it evolves as a natural result of two other modes, the on-orbit Lambert maneuver and the standard ascent.
(4) **Once-Around Abort**

This mode is intended for use during the latter portion of ascent in the event of an engine failure. The resulting coasting trajectory insures that proper entry interface conditions (range, velocity, and flight path angle) are achieved for successful reentry. As with other guidance modes, the insertion conditions necessary to satisfy true maneuver constraints, such as at entry interface, are recomputed every guidance cycle. Very little additional code is required to support this mode, since it evolves from a combination of the standard ascent and deorbit guidance modes.

(5) **Return-to-Launch-Site Abort**

(Studies are currently in process to determine desired thrust termination conditions).

(6) **External Delta-V Maneuver**

This maneuver is designed to guide the vehicle through a constant attitude maneuver which achieves a specified velocity change. This mode is used for small on-orbit maneuvers, such as rendezvous phasing maneuvers. It is similar to the Apollo External Delta-V maneuver mode.

(7) **Lambert On-orbit Maneuver**

This mode is designed to insert the vehicle onto a coasting trajectory which intercepts a specified position (target) at a specified time. This is typically a constant attitude maneuver and is intended primarily for rendezvous terminal phase and automatic braking. It is similar to the Lambert Ascent maneuver, except that no constraint is placed upon the vehicle altitude at thrust termination.
(8) **Deorbit Maneuver**

This mode is designed to place the vehicle onto a coasting trajectory which satisfies entry interface conditions. These entry interface conditions are assumed to be defined by a prescribed (possibly functional) relationship between entry range, velocity, and flight path angle. This mode is similar to the once-around abort maneuver, except that no constraint is placed upon the vehicle altitude at thrust termination.

The functional flow of the program will be described in the next section to aid in understanding the general method employed in this guidance scheme. Following this, the particular equations and flow charts relating to the various modes are presented in greater detail.
2. **FUNCTIONAL FLOW DIAGRAM**

JPG is designed to be called by a Servicer Routine at appropriate times during a powered maneuver. The first call can be made at any time prior to active guidance calls. This first, or prethrust, call is different from the active guidance calls in that certain required parameters are initialized and the compatibility of targeting information is checked and revised if necessary. The first call for active guidance is made at a specified time interval before ignition with following calls made at periodic intervals through-out the maneuver.

During the prethrust call, an iteration takes place which recycles the routine, without advancing the state vector, until convergence of the required velocity-to-be-gained takes place. During active guidance, however, a single pass through the routine is made each guidance cycle call, incorporating sensed velocity changes and updating accordingly.

This section will present the general functional flow of computations performed by UPFG. No attempt will be made here to define the actual equations used or to differentiate between the various modes of operation (other than to point out that several functional areas are not required by the External Delta-V mode). Detailed equations and flow charts will be covered in later sections.

The computational flow normally proceeds through nine distinct functional groupings or blocks of computations after each entry into UPFG. (see Figure 2-1) External Delta-V maneuvers, being less complicated, bypass two of these functional blocks. Each functional block of computations results in the determination of the values of specific variables or commands used by the following functional blocks. The equations employed within the blocks may vary depending upon the various maneuver modes but the resulting output list from each block is the same.

The first block of computations encountered is either initialization (Block 1) or update (Block 2) depending upon whether a prethrust or active guidance cycle call is being made. In Block 1 the initial values of required variables are set and the state is advanced on a coasting trajectory to the time when the first active guidance call will be made. In Block 2, after the accelerometers are read, the state is advanced based upon the sensed velocity change over the time elapsed since the last accelerometer reading. The sensed velocity change is also used to update certain other variables. Logic is included to exit the routine from this point if either (1) an active guidance call is being made prior to ignition or (2) an active guidance call is being made after an engine-off command has been issued. Prethrust flow does not use this exit path.
In the next block of computations (Block 3) the time-to-go remaining for the maneuver is computed. This computation involves solving for the phase elements of one of the thrust integrals. To avoid duplication these elements are saved to be summed in the next block.

Several scalar integrals relating to the thrust acceleration are computed in Block 4. These integrals are needed in following computations to predict the cut-off state and thrust direction.

Block 5 incorporates an input thrust turning rate or determines one for ascent and abort modes. The turning rate is utilized to determine a unit thrust direction according to 1.TG concepts. In addition, actual contributions to velocity and position due to incorporating the turning rate are determined and biases are computed which reflect the differences between actual contributions and the necessary velocity-to-be-gained and position-to-be-gained vectors. The biases are used in the routine to improve prediction.

The steering block (Block 6) remains to be determined. It must, however, set up steering commands based upon the unit thrust direction computed previously, taking into account autopilot requirements. This block of equations must differentiate between prethrust and active guidance calls when making these computations.

Block 7 and 8, which are skipped in the External Delta-V Mode, estimate the effects of gravity over the thrusting trajectory and incorporate these effects in the prediction of terminal conditions. Using these terminal conditions the velocity-to-be-gained is revised to force the predicted target conditions to satisfy the specified target conditions. This revision is necessary to prepare for the next pass through the routine.

Block 9, which determines and issues engine commands, is also to be determined at a later date when engine characteristics are better defined. It also must differentiate between prethrust and active guidance calls when formulating these commands.

This completes one pass, prethrust or active guidance, through the routine. From this point, if a prethrust call is being made and a change to velocity-to-be-gained greater than some limiting value has been computed, the flow returns to the beginning of Block 3. If the velocity-to-be-gained change has converged to less than the limiting value, or if an active guidance call is being made, the flow exits to the Servicer Routine to await the next call.
**Figure 2-1. Functional Flow Diagram**
3. INPUT AND OUTPUT VARIABLES

The Unified Powered Flight Guidance program is not designed to run independently throughout the maneuver. It must be called periodically by the Servicer Routine, first in prethrust and subsequently at the beginning of each active guidance cycle. Some variables require input values at each call and some variables are modified internally requiring only initial input values. Thus, the input list for UPFG can become complicated. Furthermore, certain variables optionally require input values (initially and/or subsequently) depending upon the maneuver mode desired. In an effort to alleviate some confusion the input will be listed below in two categories: input required on the prethrust call and input required each active guidance call. The first category will be further broken down into two groupings: those inputs required by all maneuver modes and those inputs optionally required by some modes and not by others. Variables that are modified internally only, and whose values affect future UPFG computations, will not be listed as input.

In addition to input which may be changed each call, a certain number of constants are required to be preset. These constants are not modified during program execution. They may be broken down into two categories: universal constants and program constants. Universal constants are those constants not related to UPFG such as gravitational constants, earth radius, expected thrust levels, etc. Program constants are physical parameters which are unique to UPFG such as convergence criterion, iteration limits, etc.

Sensors provide another source of input. The values of certain variables are updated by sensors on a continuous or semi-continuous basis. When a program uses the value of this variable it obtains the current value existing at the time the storage location for that variable is read. Sensors read and the variables they control are listed below.

Many variables are modified and assigned new values during the prethrust and active guidance calls. With common storage locations they could all be termed "output". The list below, however, contains only those variables, known to affect other routines, that have been assigned new values by UPFG. Those that only effect subsequent calls to UPFG have been omitted.

For on-orbit maneuvers (modes E, 7, and 8) the desired thrust vector turning rate $\omega_f$ is required as input to UPFG. It is anticipated that this quantity will be set to zero for External Delta-V and Lambert maneuvers, and computed in the de-orbit targeting program (Ref. 3) to minimize $\Delta v$ for the deorbit maneuver. If necessary, at a later date, an optimal turning rate for the Lambert maneuver could be determined.
### Input - Prethrust Call Only (All Modes)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
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<td>$s_{\text{pre}}$</td>
<td>Prethrust switch</td>
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<tr>
<td>$s_{\text{mode}}$</td>
<td>Maneuver mode</td>
</tr>
</tbody>
</table>

- $s_{\text{pre}}$:
  - 1: Prethrust
  - 0: Active guidance

- $s_{\text{mode}}$:
  - 1: Ascent, standard
  - 2: Ascent, reference trajectory
  - 3: Ascent, Lambert
  - 4: Ascent, once-around abort
  - 5: Ascent, return-to-launch-site abort
  - 6: On-orbit, external $\Delta v$
  - 7: On-orbit, Lambert
  - 8: On-orbit, deorbit

- $n$: Number of thrust phases
- $t_{\text{ig}}$: Ignition time, first phase
- $\mathbf{f}_i$: State vector
- $\mathbf{v}_i$: Estimated velocity-to-be-gained vector
- $z_{\text{SSME},i}$: Number of engines (SSME, OMS, RCS) for $i$th phase
- $z_{\text{OMS},i}$
- $z_{\text{RCS},i}$
- $s_{\text{phase},i}$: Phase switch, $i$th phase
  - 0: Constant thrust
  - 1: Constant acceleration
- $m_{0,i}$: Mass at beginning of phase $i$
## Input - Prethrust Call Only (Optional)

<table>
<thead>
<tr>
<th>Mode Required</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_c$</td>
<td>Coast time between last and next to last phase</td>
</tr>
<tr>
<td></td>
<td>$t_{b,i}$</td>
<td>Time remaining in $i^{th}$ phase</td>
</tr>
<tr>
<td></td>
<td>$\Delta v_{OMS}$</td>
<td>Velocity change to be imparted by OMS phase</td>
</tr>
<tr>
<td></td>
<td>$a_L$</td>
<td>Specific force limit during SSME maneuvers</td>
</tr>
<tr>
<td></td>
<td>$\hat{n}_y$</td>
<td>Unit vector normal to desired trajectory</td>
</tr>
<tr>
<td></td>
<td>$r_d$</td>
<td>Desired cutoff position</td>
</tr>
<tr>
<td></td>
<td>$r_d$</td>
<td>Desired radius magnitude of $r_d$</td>
</tr>
<tr>
<td></td>
<td>$v_d$</td>
<td>Desired velocity magnitude at cutoff</td>
</tr>
<tr>
<td></td>
<td>$\gamma_d$</td>
<td>Desired flight path angle at cutoff</td>
</tr>
<tr>
<td></td>
<td>$\hat{v}_d$</td>
<td>Desired velocity vector at cutoff</td>
</tr>
<tr>
<td></td>
<td>$t_{ref}$</td>
<td>Desired cutoff time</td>
</tr>
<tr>
<td></td>
<td>$K_1$</td>
<td>Desired throttle setting for SSME for $i^{th}$ phase</td>
</tr>
<tr>
<td></td>
<td>$\hat{N}$</td>
<td>Lambert unit normal to projection plane (see Ref. 5)</td>
</tr>
<tr>
<td></td>
<td>$n_{rev}$</td>
<td>Lambert number of revolutions (see Ref. 5)</td>
</tr>
<tr>
<td></td>
<td>$s_{soln}$</td>
<td>Lambert solution type switch (see Ref. 5)</td>
</tr>
<tr>
<td></td>
<td>$r_f$</td>
<td>Target position - inertial coordinates</td>
</tr>
<tr>
<td></td>
<td>$r_{tef}$</td>
<td>Target (entry interface) - Earth fixed coordinates</td>
</tr>
<tr>
<td></td>
<td>$t_t$</td>
<td>Time at target</td>
</tr>
<tr>
<td></td>
<td>$C_1$</td>
<td>Constants in target (entry interface) velocity constraint, $v_v \leq C_1 \cdot C_2 \cdot v_h$</td>
</tr>
<tr>
<td></td>
<td>$C_2$</td>
<td>$v_t$</td>
</tr>
</tbody>
</table>

Inputs for Mode 5 are to be determined.
### Input Required - Active Guidance Call (All Modes)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{SSME}^k$</td>
<td>No. of engines (SSME, OMS, RCS) to be considered on current phase ($k^{th}$ phase)</td>
</tr>
<tr>
<td>$s_{OMS, k}$</td>
<td>for current phase ($k^{th}$ phase)</td>
</tr>
</tbody>
</table>

### Program Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{cone}$</td>
<td>Lambert required sine of half cone angle of exclusion</td>
</tr>
<tr>
<td>$\epsilon_{vgo}$</td>
<td>Value of $</td>
</tr>
<tr>
<td>$\Delta t_{t0}$</td>
<td>Time interval before $t_{ig}$ to start active guidance calls</td>
</tr>
<tr>
<td>$\delta t$</td>
<td>Offset in coast time used in deorbit required velocity computations to determine sensitivity to coast time</td>
</tr>
<tr>
<td>$\Delta t_{cutoff}$</td>
<td>Value of $t_{go}$ used to define time to issue engine cutoff command and terminate active steering computations</td>
</tr>
<tr>
<td>$s_{pert}$</td>
<td>Gravity perturbation switch</td>
</tr>
</tbody>
</table>

### Universal Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>$f_{SSME}$</td>
<td>Full thrust of single SSME, OMS, RCS engine</td>
</tr>
<tr>
<td>$f_{OMS}$</td>
<td>Full thrust of single SSME, OMS, RCS engine</td>
</tr>
<tr>
<td>$f_{RCS}$</td>
<td>Full thrust of single SSME, OMS, RCS engine</td>
</tr>
<tr>
<td>$K_{max}$</td>
<td>Maximum throttle setting of SSME</td>
</tr>
<tr>
<td>$m_{SSME}$</td>
<td>Mass flow rate of single SSME, OMS, RCS engine at full thrust</td>
</tr>
<tr>
<td>$m_{OMS}$</td>
<td>Mass flow rate of single SSME, OMS, RCS engine at full thrust</td>
</tr>
<tr>
<td>$m_{RCS}$</td>
<td>Mass flow rate of single SSME, OMS, RCS engine at full thrust</td>
</tr>
</tbody>
</table>
Sensors Required

<table>
<thead>
<tr>
<th>Sensed Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta v$-sensed</td>
<td>Total velocity change accumulated on accelerometers since last reading</td>
</tr>
<tr>
<td>$t$</td>
<td>Actual time when accelerometers are read (will be associated with state vector)</td>
</tr>
</tbody>
</table>

Output

Output from this program will be in the form of attitude and engine commands and navigation state. These include:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-$</td>
<td>Steering command (TBD)</td>
</tr>
<tr>
<td>$-$</td>
<td>Ignition (engine-on) command (TBD)</td>
</tr>
<tr>
<td>$k_e$</td>
<td>Throttle command</td>
</tr>
<tr>
<td>$-$</td>
<td>Cutoff (engine-off) command (TBD)</td>
</tr>
<tr>
<td>$t$</td>
<td>State vector</td>
</tr>
<tr>
<td>$r$</td>
<td>State vector</td>
</tr>
<tr>
<td>$v$</td>
<td>State vector</td>
</tr>
</tbody>
</table>
4. DESCRIPTION OF EQUATIONS

The guidance scheme presented in this document evolved from a rather extensive modification of the Linear Tangent Guidance (LTG) concept described in Ref. 1. The original LTG concept was designed for an orbiter ascent maneuver in which the thrust cutoff altitude, orbital plane, velocity, and flight path angle were constrained to specified values determined prior to the flight. The current unified scheme retains that basic guidance mode, the standard ascent mode as described in the introduction. However, to adapt the LTG concept to the various shuttle maneuvers, from ascent through deorbit, several important changes were made.

First, the program was restructured to efficiently accommodate the various maneuver modes and to permit the calculation of steering commands early in the guidance cycle, thus minimizing the computational lag. Second, the addition of a required velocity calculation toward the end of each guidance cycle was made. The basic standard ascent mode does not constrain either the downrange component of position or the time of thrust cutoff, and therefore the subsequent coasting trajectory can vary considerably depending upon ignition delays, atmospheric effects during boost, engine perturbations or failures, and any other factors which might affect the trajectory. By including a required velocity calculation every guidance cycle, based upon the predicted cutoff position, the true maneuver objectives can be satisfied and the effects of perturbations minimized. The third important change to the LTG concept involved the elimination of any cutoff position constraint for on-orbit maneuvers. An alternate equation, which uses the input vehicle turning rate, simplifies premaneuver targeting and minimizes maneuver \( \Delta v \). The fourth change involved the design of a new scheme for the prediction of the effects of gravity on the powered trajectory. The original LTG scheme was not suitable for long, low thrust orbital maneuvers such as deorbit. A new scheme, based upon a conic coasting trajectory, appears to give good performance for all maneuvers. Finally, to support the ascent to reference trajectory guidance mode, equations were added to account for a rotating thrust vector. These additional equations improve the prediction of both the maneuver time and cutoff position. They are required for proper calculation of throttling commands during ascent, and they provide increased premaneuver prediction accuracy in all modes.

As described in Section 2, the Unified Powered Flight Guidance (UPFG) routine consists of nine blocks of in-line computations connected by the logic necessary to perform either the prethrust function or the active guidance function for the eight possible maneuver modes. A single entry and exit point are maintained for these computations. This section will describe the computations required for one pass, entry to exit, through UPFG.
Figure 5-1 shows the sequencing of the major computational block. Immediately after entry into UPFG a test is made on the value of the switch, \( s_{\text{pre}} \). If \( s_{\text{pre}} = 1 \), a prethrust call is being made; \( s_{\text{pre}} = 0 \) implies an active guidance call. For prethrust calls, initialization in Block 1 is performed. For active guidance calls, Block 1 is bypassed and an update in Block 2 is performed.

4.1 Initialization (Block 1)

This block of computations provides the necessary one-time initialization of certain variables required to set-up the prethrust calculations. These variables include \( k \), the maneuver phase counter. It should be noted that the first phase of each maneuver is denoted "1". Several switches are also set and the values of variables which may be defined by input, depending upon maneuver mode, are initialized. An initial value of \( \bar{r}_{\text{grav}} \) is required, since the calculation of \( \bar{r}_{\text{go}} \) in Block 5 occurs prior to the calculation of gravity effects in Block 7. To estimate \( \bar{r}_{\text{grav}} \), it is assumed that the acceleration of gravity will be equal to its present value over the entire maneuver. Then, if a one second maneuver is assumed for simplicity,

\[
\bar{r}_{\text{grav}} = \frac{1}{2} \frac{\mu r}{r^3}
\]

Later, in Block 5, after the maneuver time-to-go is computed, \( \bar{r}_{\text{grav}} \) is adjusted for the estimated maneuver time as follows,

\[
\bar{r}_{\text{grav}}(\text{new}) = \frac{\bar{r}_{\text{grav}}(\text{new})}{\bar{r}_{\text{grav}}(\text{previous})} \]

where \( \bar{r}_{\text{go}}(\text{previous}) \) has been set to 1. On subsequent passes through Block 5 a previous \( \bar{r}_{\text{go}} \) will have been set in the previous pass and the revision to \( \bar{r}_{\text{grav}} \) will proceed normally.

Finally, the effective engine parameters for the array of engines to be used during each phase, \( i = 1 \) to \( n \), are determined. These engine parameters include the total thrust \( T_{\text{r},i} \), mass-flow-rate \( \dot{m}_{\text{r},i} \), the exhaust velocity \( v_{\text{ex},i} \), the initial acceleration \( a_{T,i} \), and the ratio of mass to mass-flow-rate \( \tau_{\text{r},i} \) for each phase of the maneuver.

The last operation in Block 1 advances the state \( \tau_{\text{r},i} \) from the input values to the time when the first active guidance call will be made at \( t = t_{\text{ig}} - \Delta t_{10} \). This is accomplished by calling the Precision State Extrapolate - Routine (Ref. 7). It should be noted that this sets up the prethrust operations as if thrusting is expected to commence at \( t_{\text{ig}} - \Delta t_{10} \) instead of \( t_{12} \). If \( \Delta t_{10} \) is small this is not expected to significantly affect the results and it simplifies the program. An al-
ternative would be to extrapolate the state to $t_{ig}$ in the prethrust call and each active guidance call prior to $t_{ig}$. It should be noted that this method of initialization, although satisfactory for on-orbit maneuvers, is not applicable to ascent. In future revisions of this document a scheme which is satisfactory for both ascent and on-orbit maneuvers will be included.

The computations now proceed to Block 3 since the update (Block 2) is skipped during the prethrust call.

4.2 **Update (Block 2)**

Block 2, which is skipped in the prethrust call, is the first block encountered during each active guidance call. This block acquires the velocity change sensed by the accelerometers and accounts for that velocity change by updating affected variables to the time when the accelerometers were read. The routine which reads the accelerometers and clock has not been defined but it is assumed that it will require some sort of initialization on the first call such that velocity changes on succeeding calls can be obtained by differencing from a previous reading. Thus, on the first call the velocity change is assumed to be zero.

To allow for a variable guidance cycle time step, the time step is computed each cycle as follows,

$$\Delta t = t - t_{prev}$$

where $t$ is the current clock reading at the time the accelerometers are read and $t_{prev}$ is the previous time the accelerometers were read. On the first active guidance call $\Delta t$ will be nearly zero since $t_{prev}$ was set in prethrust to $t_{ig} - \Delta t_0$, the expected time of the first active guidance call. The variable $t_{prev}$ is set to $t$ for use later.

The velocity-to-be-gained is updated using the velocity change sensed by the accelerometers since the last guidance call as follows.

$$v_{go} \text{ (new)} = v_{go} \text{ (previous)} - \Delta v_{sensed}$$

This gives an excellent estimate of the velocity-to-be-gained for subsequent guidance calculations. Near the end of the guidance routine this value of $v_{go}$ is adjusted slightly to account for any changes in desired cutoff velocity and to prepare for the next guidance cycle.

It should be noted that on the first active guidance call (denoted by $s_{pass} = 1$) $\Delta v_{sensed}$ is assumed to be zero. Therefore, updating $v_{go}$ is bypassed under these circumstances and

$$\Delta v_{sensed} = 0$$
$$s_{pass} = 0$$
Next, the state, \( r, y \), is advanced to the time of the latest accelerometer reading by calling the Powered Flight Navigation Routine (Ref. 6).

The remainder of the update block, which is bypassed when \( t \leq t_{ig} \), results in decrementing the \( t_{go} \) values for each phase, \( i \), by \( \Delta t \)

\[
\begin{align*}
t_{go,i}^{\text{new}} & = t_{go,i}^{\text{previous}} - \Delta t \\
t_{b,k}^{\text{new}} & = t_{b,k}^{\text{previous}} - \Delta t 
\end{align*}
\]
during thrusting. Also decremented are the current mass, \( m \), and current phase burn time \( t_b \)

\[
\begin{align*}
m^{\text{new}} & = m^{\text{previous}} - \dot{m}_k \Delta t \\
t_{b,k}^{\text{new}} & = t_{b,k}^{\text{previous}} - \Delta t 
\end{align*}
\]

If \( t_{b,k} \) becomes less than or equal to zero the end of that phase has been reached. Under these circumstances a change of phase is performed by setting the \( t_{go} \) for the current phase to zero and then incrementing the phase number by one.

\[
\begin{align*}
t_{go,k} & = 0 \\
k^{\text{new}} & = k^{\text{previous}} + 1 
\end{align*}
\]

It should be noted at this point that mass, \( m \), has not been adjusted to reflect step changes in this variable that may occur during a phase change due to such things as tank jettison or staging. It is expected that some routine external to UPFG will maintain current mass following phase changes for use by both Guidance and Control.

During the coast period between the last and next-to-last phases of an ascent maneuver the coast time, \( t_c \), is decremented by \( \Delta t \).

\[
t_{c}^{\text{new}} = t_{c}^{\text{previous}} - \Delta t 
\]

No changes to mass which would account for external tank jettison have been made here as this function is assumed to be performed by a routine external to UPFG. Burn times remaining are also assumed to be constant during the coast time.

Block 3 normally follows Block 2 during active guidance calls unless \( t < t_{ig} \) or unless the thrust-off command has been issued as indicated by \( s_{\text{engoff}} = 1 \).

If either of these two conditions exists, all further computations are bypassed and UPFG is exited.
4.3 Time-To-Go

The UPFC program is designed to provide guidance for all exo-atmospheric maneuvers, including ascent-to-orbit. The complexity of the time-to-go computations is dependent upon the number of distinct thrust phases in the maneuver. All orbital maneuvers nominally have only one distinct thrust phase (OMS constant thrust), while an ascent maneuver will have at least three distinct thrust phases (i.e. (1) SSME constant thrust prior to g-limiting, (2) SSME constant acceleration during g-limiting and (3) OMS constant thrust after tank separation). The equations that are described in this section are the equations required to compute the time-to-go until maneuver completion for a multi-phase ascent maneuver. The equations required for any single-phase maneuver are merely a subset of these equations; in particular, they are the same equations that will be solved during an ascent maneuver when actually in the final thrust phase of the maneuver (OMS constant thrust phase).

To compute the time-to-go it is first necessary to compute estimates of the current values of thrust magnitude \( f_{T,k} \), mass flow rate \( \dot{m}_k \), thrust acceleration \( a_{T,k} \), effective exhaust velocity \( v_{ex,k} \), and mass to mass flow rate ratio, \( \tau_k \), where \( k \) is an index referring to the current thrust phase. Estimates of \( f_{T,k} \) and \( \dot{m}_k \) are given by

\[
\begin{align*}
    f_{T,k} &= f_{SSME,k} f_{OMS,k} f_{RCS,k} \\
    \dot{m}_k &= \dot{m}_{SSME,k} + \dot{m}_{OMS,k} + \dot{m}_{RCS,k}
\end{align*}
\]

If \( \dot{m}_k \neq 0 \), \( a_{T,k}, v_{ex,k} \) and \( \tau_k \) are computed as follows:

\[
\begin{align*}
    a_{T,k} &= \frac{f_{T,k}}{m} \\
    v_{ex,k} &= \frac{f_{T,k}}{\dot{m}_k} m_k \\
    \tau_k &= \frac{v_{ex,k}}{a_{T,k}}
\end{align*}
\]

where \( m \) is the current estimated mass of the vehicle. If \( \dot{m}_k = 0 \), the last three equations are bypassed and the previous estimates of \( a_{T,k}, v_{ex,k} \) and \( \tau_k \) are used.

If there are two or more thrust phases remaining (including the current phase), the assumed burn times, \( t_{b,i} \), of each phase are used to compute the velocity change due to thrust, \( L_i \), that will be applied during each phase \( i \) from the current phase to the \( n-2 \) phase (\( n \) denotes the number of phases in the maneuver). If phase \( i \) has constant thrust,

\[
L_i = -v_{ex,i} \ln \left( \frac{t_i - t_{b,i}}{t_i} \right)
\]
and if phase $i$ has constant acceleration

$$L_i = a_i \cdot t_{b,i}$$

where $a_i$ is the SSME acceleration limit.

Since the velocity change to be applied in the $n^{th}$ phase, $L_n$, (after tank staging) is predefined, the velocity change to be applied in the $n-1$ phase can be determined by

$$L_j = \left| v_{\text{go}} \right| - \sum_{i=k}^{n-2} L_i - L_n$$

where $j = n - 1$ and $v_{\text{go}}$ is the velocity-to-be-gained vector computed in Block 2. If there are only two phases remaining ($k = n - 1$), then

$$L_j = \left| v_{\text{go}} \right| - L_n$$

where again $j = n - 1$. During the final phase of the maneuver $L_j$ is given by

$$L_j = \left| v_{\text{go}} \right|$$

where, in this case, $j = n$.

Having determined $L_j$, the burn time remaining in the $j^{th}$ phase can be computed. If phase $j$ has constant thrust

$$t_{b,j} = \tau_j \left( 1 - e^{-L_j/v_{\text{ev},j}} \right)$$

or if phase $j$ has constant acceleration

$$t_{b,j} = \frac{L_j}{a_i}$$

The time-to-go until the end of phase $i$, $t_{\text{go},i}$, for $i = k, \ldots, n$ is required in Block 4 in order to evaluate the thrust integrals $T$, $J$, $S$, $Q$, $H$, and $P$. These times are given by

$$t_{\text{go},i} = t_{\text{go},i-1} - t_{b,i} \quad \text{for } i < n$$

and where

$$t_{\text{go},i-1} = 0 \quad \text{for } i > k.$$
where \( t_c \) is the coast time during external task separation between phase \( n-1 \) and phase \( n \). Although \( t_c \) is not actually part of the maneuver time, it is added to the time-to-go until the end of phase \( n \) in order to maintain continuity of the steering commands.

4.4 Integrals of Thrust

The LTG guidance concept requires the evaluation of several thrust integrals. These integrals are defined as follows

\[
\begin{align*}
L &= \int_0^{t_{go}} (f/m) \, dt & S &= \int_0^{t_{go}} \left[ \int_0^t (t/f/m) \, ds \right] \, dt \\
J &= \int_0^{t_{go}} (f/m) \, t \, dt & Q &= \int_0^{t_{go}} \left[ \int_0^t (t/f/m) \, s \, ds \right] \, dt \\
H &= \int_0^{t_{go}} (f/m) \, t^2 \, dt & P &= \int_0^{t_{go}} \left[ \int_0^t (t/f/m) \, s^2 \, ds \right] \, dt
\end{align*}
\]

where \((f/m)\) is the thrust acceleration, \( t_{go} \) is the time-to-go until the end of the maneuver, and \( t \) and \( s \) are variables of integration. For the space shuttle vehicle, it is assumed that either \((f/m)\) is constant or \( f \) (thrust) and \( m \) (mass flow rate) are constant, therefore, these integrals can be integrated in closed form.

The UPPG program is designed to accommodate multi-thrust-phase maneuvers as well as single-thrust-phase maneuvers. Therefore, the above integrals must be evaluated piece by piece since each thrust phase has a distinct thrust profile. This is accomplished by evaluating the thrust integrals separately for each phase and then summing them up. The thrust integrals for each phase \( i \) are defined as follows

\[
\begin{align*}
L_i &= \int_{t_{go,i-1}}^{t_{go,i}} (f_{T,i}/m_i) \, dt & S_i &= \int_{t_{go,i-1}}^{t_{go,i}} \left[ \int_{t_{go,i-1}}^t (f_{T,i}/m_i) \, ds \right] \, dt \\
J_i &= \int_{t_{go,i-1}}^{t_{go,i}} (f_{T,i}/m_i) \, t \, dt & Q_i &= \int_{t_{go,i-1}}^{t_{go,i}} \left[ \int_{t_{go,i-1}}^t (f_{T,i}/m_i) \, s \, ds \right] \, dt \\
H_i &= \int_{t_{go,i-1}}^{t_{go,i}} (f_{T,i}/m_i)^2 \, t^2 \, dt & P_i &= \int_{t_{go,i-1}}^{t_{go,i}} \left[ \int_{t_{go,i-1}}^t (f_{T,i}/m_i)^2 \, s^2 \, ds \right] \, dt
\end{align*}
\]
where \( \{T_i, \frac{v_i}{r_i}\} \) defines the thrust profile for phase \( i \), and \( t_{go,i} \) is the time-to-go until the end of phase \( i \) (note: \( t_{go,i-1} = 0 \) for \( i < k \)).

The computation of the thrust integrals for each phase is performed in two steps. This is done in order to minimize computer memory requirements and computation time. In the first step, the equations vary depending upon the type of thrust phase, while for the second step, the equations are identical for both types of phases. Also, it should be noted that \( H_i \) is not explicitly computed. It can be shown using integration by parts that \( H_i \), which is the sum of the \( H_i \)'s, can be computed directly as a function of the time-to-go and the integrals \( J \) and \( Q \). The evaluation of \( L_i \) has already been described in Section 4.3 because it is required in the computation of time-to-go. However, for the sake of completeness, it will be described again in this section.

Step 1 in evaluating the thrust integrals is to compute \( L_i \) and then to compute part of \( J_i, S_i, Q_i, \) and \( P_i \). If phase \( i \) has constant thrust then

\[
L_i = -v_{ex,i} \ln \left( \frac{\tau_i - b_{i}}{\tau_i} \right)
\]

\[
J_i = L_i \tau_i - v_{ex,i} t_{b,i}
\]

\[
S_i = -J_i + t_{b,i} L_i
\]

\[
Q_i = S_i (\tau_i + t_{go,i-1}) - \frac{1}{2} v_{ex,i} t_{b,i}^2
\]

\[
P_i = Q_i (\tau_i + t_{go,i-1}) - \frac{1}{2} v_{ex,i} t_{b,i}^2 \left( \frac{1}{3} t_{b,i} + t_{go,i-1} \right)
\]

and if phase \( i \) has constant acceleration then

\[
L_i = a_L t_{b,i}
\]

\[
J_i = \frac{1}{2} L_i t_{b,i}
\]

\[
S_i = J_i
\]

\[
Q_i = \frac{1}{3} t_{b,i} + t_{go,i-1}
\]

\[
P_i = \frac{1}{6} S_i (t_{go,i}^2 + 2 t_{go,i} t_{go,i-1} + 3 t_{go,i-1}^2)
\]
In step 2, the remainder of $J_i$, $S_i$, $Q_i$ and $P_i$ are computed as follows

\[ J_1 = J_i + L_i t_{g0,i-1} \]
\[ S_1 = S_i + L t_{b,i} \]
\[ Q_1 = Q_i + J t_{b,i} \]
\[ P_1 = P_i + H t_{b,i} \]

where $L$, $J$, and $H$ are the total thrust integrals from the current phase to the $i$-th phase. If $i = n$, the effects of the coast time, $t_c$, between phases $n-1$ and $n$ are then added in as follows

\[ S_i = S_i + L t_c \]
\[ Q_i = Q_i + J t_c \]
\[ P_i = P_i + H t_c \]

Having evaluated the thrust integrals for each phase, the total thrust integrals are given by

\[ L = \sum_{i=k}^{n} L_i \]
\[ J = \sum_{i=k}^{n} J_i \]

\[ S = \sum_{i=k}^{n} S_i \]
\[ Q = \sum_{i=k}^{n} Q_i \]

\[ P = \sum_{i=k}^{n} P_i \]
\[ H = t_{g0,i} J - Q \]

It should be pointed out that $H$ must be evaluated as the total thrust integral from the current phase to the $i$-th phase for each phase because it is required in the computation of each $P_i$. 
4.5 Turning Rate

The main results of this block of computations are the desired unit thrust direction, \( \mathbf{1}_f \), and a vector, \( \mathbf{\lambda} \), which is associated with the thrust turning rate.

The first operation in this block is to define \( \mathbf{\lambda} \), a vector in the direction of \( \mathbf{v}_g \), the velocity-to-be-gained. The unit vector, \( \hat{\mathbf{\lambda}} \), is the vector about which an expansion is later made to determine \( \mathbf{1}_f \). From this point one of four different methods will be employed depending upon \( s_{mode} \).

The first operation in this block is to define \( \mathbf{\lambda} \), a vector in the direction of \( \mathbf{v}_g \), the velocity-to-be-gained. The unit vector, \( \mathbf{1}_f \), is the vector about which an expansion is later made to determine \( \mathbf{1}_f \). From this point one of four different methods will be employed depending upon \( s_{mode} \).

Modes 1 through 5, ascent and abort, follow closely the LTG method of determining the rate \( \dot{\mathbf{\lambda}} \), by first using a desired burn-out position, \( \mathbf{r}_d \), to estimate \( \mathbf{r}_{go} \)

\[
\mathbf{r}_{go} = \mathbf{r}_d - (\mathbf{r} \cdot \mathbf{v}_g) \mathbf{v}_g + \mathbf{r}_{grav}
\]

The effect of gravity on this \( \mathbf{r}_{go} \) is given by \( \mathbf{r}_{grav} \), which is estimated using \( \mathbf{r}_{grav} \) from the previous call as follows,

\[
\mathbf{r}_{grav(new)} = \left( \frac{t_{g0(new)}}{t_{g0(previous)}} \right)^2 \mathbf{r}_{grav(previous)}
\]

The projection of \( \mathbf{r}_{go} \) on the plane normal to the downrange direction, \( \mathbf{r}_{goxy} \), is given by

\[
\mathbf{r}_{goxy} = \mathbf{r}_{go} - (\mathbf{z} \cdot \mathbf{r}_{go}) \mathbf{z}
\]

Using the integral, \( S \), computed previously, the downrange component of \( \mathbf{r}_{go} \) can be modified by the LTG relationship

\[
\mathbf{r}_{goz} = \frac{(S - \mathbf{\lambda} \cdot \mathbf{r}_{goxy})}{\mathbf{\lambda} \cdot \mathbf{z}}
\]

and a new \( \mathbf{r}_{go} \) is thus found to be

\[
\mathbf{r}_{go} = \mathbf{r}_{goxy} + \mathbf{r}_{goz} \mathbf{z} + \mathbf{r}_{bias}
\]

In this equation the effects of a rotating thrust vector are included by the term, \( \mathbf{r}_{bias} \), which was computed on the previous guidance cycle in Block 5. The rate, \( \dot{\mathbf{\lambda}} \), which corresponds to the velocity of the tip of a unit vector coincident with \( \mathbf{\lambda} \) but rotating with the desired unit thrust vector rotation rate is now obtained using the integrals, \( L \), \( J \), \( S \), and \( Q \):

\[
\dot{\mathbf{\lambda}} = \frac{(\mathbf{r}_{go} - S \mathbf{\lambda})}{(Q - S J / L)}
\]
Note that:

\[ \dot{\lambda} \neq \frac{d}{dt}(\lambda) \]

For modes 6-8, a rotation rate, \( \omega_f \), is input and \( \dot{\lambda} \) is determined by

\[ \dot{\lambda} = \omega_f \text{unit}\left(\frac{\lambda \times r}{x} \frac{\lambda}{\lambda}\right) \]

The predicted unit thrust direction at time, \( t \), is given by

\[ \text{unit}\left(\frac{\lambda - (J/L) \dot{\lambda}}{\lambda}\right) \]

It is recognized that the results of integrating a rotating thrust vector can be significantly different from \( r_{go} \) and \( v_{go} \) if the rotation angle is large.

The angle \( \phi \) between \( \lambda \), a unit vector in the direction of \( v_{go} \), and \( \frac{\lambda}{\lambda} \), the unit thrust direction, is given by

\[ \phi = \cos^{-1}\left(\frac{i_{\lambda}}{\lambda}\right) \]

Since the linear tangent guidance equations are designed to align \( \lambda \) and the thrust direction at the time \( J/L \) (\( J/L \) is approximately the midpoint of the maneuver), then

\[ \dot{\phi} = -\phi \frac{L}{J} \]

Based upon \( \phi \) and \( \dot{\phi} \), the first and second integrals of the thrust acceleration are given by

\[ x_{\text{thrust}} = \int_0^{t_{go}} \frac{r_{go}}{m} \left[ \lambda \cos(\phi + \dot{\phi}t) + \frac{\dot{\lambda}}{|\lambda|} \sin(\phi + \dot{\phi}t) \right] dt \]

\[ r_{\text{thrust}} = \int_0^{t_{go}} \int_0^t \frac{r_{go}}{m} \left[ \lambda \cos(\phi + \dot{\phi}t) + \frac{\dot{\lambda}}{|\lambda|} \int_{\phi_{in}}^{\phi} \phi + \dot{\phi}t \right] dt \]

This can be simplified by assuming that

\[ \sin(\phi + \dot{\phi}t) = \phi + \dot{\phi}t \]

and

\[ \cos(\phi + \dot{\phi}t) = 1 - (\phi + \dot{\phi}t)^2/2 \]

Using the thrust integrals computed in Block 4, the actual (considering rotation) change in position and velocity due to thrust is computed as follows,

\[ v_{\text{thrust}} = \left( L - \frac{1}{2} \frac{L}{2} \phi^2 - J \phi \frac{\dot{\phi}}{2} H \frac{\phi^2}{2} \right) \lambda \]

\[ -(L \phi + J \dot{\phi}) \text{unit}\left(\frac{\lambda}{\lambda}\right) \]

\[ r_{\text{thrust}} = \left( S - \frac{1}{2} \frac{S}{2} \phi^2 - Q \phi \frac{\dot{\phi}}{2} P \frac{\phi^2}{2} \right) \lambda \]

\[ -(S \phi + Q \dot{\phi}) \text{unit}\left(\frac{\lambda}{\lambda}\right) \]
It may be noted that \( \mathbf{v}_{\text{thrust}} \) and \( \mathbf{r}_{\text{thrust}} \) are resolved into components parallel and normal to \( \hat{\Lambda} \) (parallel to \( \Lambda \)).

Biases to thrust cutoff velocity and position are computed by

\[
\begin{align*}
\mathbf{v}_{\text{bias}} &= \mathbf{v}_{\text{go}} - \mathbf{v}_{\text{thrust}} \\
\mathbf{r}_{\text{bias}} &= \mathbf{r}_{\text{go}} - \mathbf{r}_{\text{thrust}}
\end{align*}
\]

4.6 **Steering Commands**

This block of computations is to be determined at a later date. It will receive the vector \( \hat{\Lambda} \) and \( \dot{\hat{\Lambda}} \) and determine steering commands based upon the unit thrust direction as determined by an equation of the form

\[
\mathbf{i}_f = \text{unit} \left[ \hat{\Lambda} - (J/L) \dot{\hat{\Lambda}} \right]
\]

In order to issue steering commands to the autopilot, lead terms may be added as required.

At the end of the prethrust call steering commands will take the form of a command to maneuver to the ignition attitude but during active guidance calls a turning rate may be implied. Logic will be included to incorporate this and to activate and deactivate steering at the proper times.
4.7 Prediction of Gravity Effects (Block 4)

The solution of the LTG equations requires a prediction of both the first and second integrals of gravity over the thrusting maneuver. The technique originally devised for ascent, and described in Ref. 1, was not appropriate for the long, low-thrust on-orbit maneuvers such as deorbit. Therefore a new technique has been devised which is applicable to all maneuvers. This technique is based upon a coasting trajectory which is constructed such that it remains 'close' to the powered trajectory throughout the maneuver. The effects of gravity on the powered trajectory are then assumed to approximate the effects of gravity on the coasting trajectory. Thus the Kepler (Conic State Extrapolation) Routine can be used to determine the required integrals of gravity. Figure 4-1 illustrates this concept.

Figure 4-1. Prediction of Gravity Effects
To construct this special coasting trajectory, assume for the moment that the maneuver takes place in field-free space. The initial conditions \( t = 0 \) for the powered trajectory are defined by \( r \) and \( v \). The first and second integrals of the thrust acceleration over the powered maneuver, \( \dot{x}_{\text{thrust}} \) and \( x_{\text{thrust}} \), are described in Section 4.5. Therefore, at thrust cutoff the position and velocity on the powered trajectory are given by

\[
\begin{align*}
\mathbf{r}_{\text{cutoff}} &= \mathbf{r} + \mathbf{v} \, t_{\text{go}} + x_{\text{thrust}} \\
\mathbf{v}_{\text{cutoff}} &= \mathbf{v} + \mathbf{v}_{\text{thrust}}
\end{align*}
\]

where \( t_{\text{go}} \) is the maneuver time. The resulting trajectories in field-free space are illustrated in Figure 4-2, where coasting trajectories simply appear as straight lines.

\[\text{Figure 4-2. Prediction of Gravity Effects - Field Free Space}\]
The initial state \( (t = 0, \mathbf{r}, \mathbf{v}) \) and cutoff state \( (t = t_{\text{go}}, \mathbf{r}_{\text{cutoff}}, \mathbf{v}_{\text{cutoff}}) \) in field free space are completely defined. A cubic equation can be used to model the state vector on the powered trajectory as a function of time,

\[
\begin{align*}
\mathbf{r}_p(t) &= A + B t + C t^2 + D t^3 \\
\mathbf{v}_p(t) &= B + 2 C t + 3 D t^2
\end{align*}
\]

where \( \mathbf{r}_p(t) \) is the position. The velocity \( \mathbf{v}_p(t) \) is equal to \( \frac{d \mathbf{r}_p(t)}{dt} \). The four vector coefficients, \( A, B, C, \) and \( D \) can be determined such that the boundary conditions on position and velocity at the initial and final times are satisfied. It should be noted that in actual practice it is not necessary to actually solve for these coefficients. They are merely used to aid in describing the concept.

A coasting trajectory can now be constructed which remains 'close' to the powered trajectory. The position on this coasting trajectory \( \mathbf{r}_c(t) \) is defined by the linear equation

\[
\mathbf{r}_c(t) = A' + B' t
\]

The velocity \( \mathbf{v}_c(t) \) is constant and equal to \( B' \).

To determine \( A' \) and \( B' \), the following integrals must be satisfied.

\[
\begin{align*}
\int_0^{t_{\text{go}}} [\mathbf{r}_c(t) - \mathbf{r}_p(t)] \, dt &= 0 \\
\int_0^{t_{\text{go}}} [\mathbf{r}_c(t) - \mathbf{r}_p(t)] [t_{\text{go}} - t] \, dt &= 0
\end{align*}
\]

Since gravity is strictly a function of position, the first integral insures that the average position difference (or error) is zero. In addition, since errors in the initial position (and gravity) have more time to propagate and thus have more influence on the total position error, the second integral weights the error as a function of time. Using these integrals, the initial position \( A' \) and the initial velocity \( B' \) on the coasting trajectory can be easily computed. Using \( A' \) and \( B' \), the initial conditions for the coasting trajectory, \( \mathbf{r}_{c1} \) and \( \mathbf{v}_{c1} \), reduce to the following simple form:

\[
\begin{align*}
\mathbf{r}_{c1} &= \mathbf{r} - \frac{1}{10} \mathbf{r}_{\text{thrust}} - \frac{1}{30} \mathbf{v}_{\text{thrust}} t_{\text{go}} \\
\mathbf{v}_{c1} &= \mathbf{v} + \frac{6}{3} \mathbf{r}_{\text{thrust}} - \frac{1}{10} \mathbf{v}_{\text{thrust}} t_{\text{go}}
\end{align*}
\]

4-15
The Kepler Routine is used to extrapolate these initial conditions through the time $t_{go}$, thus obtaining $r_{c2}$ and $v_{c2}$. Then the effects of gravity on the coasting trajectory, which approximate the effects on the powered trajectory, are given by

$$
\begin{align*}
\mathbf{r}_{\text{grav}} &= r_{c2} - r_{c1} \\
\mathbf{v}_{\text{grav}} &= v_{c2} - v_{c1} - v_{c1} t_{go}
\end{align*}
$$

4.8 Velocity-to-be-Gained

A more accurate prediction of the cut-off position is now obtained as follows.

$$
\mathbf{r}_p = \mathbf{r} + v t_{go} + \mathbf{r}_{\text{grav}} + \mathbf{r}_{\text{thrust}}
$$

For on-orbit Lambert and deorbit maneuvers ($s_{\text{mode}} = 7, 8$) a desired cutoff position is given by

$$
\mathbf{r}_d = \mathbf{r}_p
$$

For aborts and ascents (other than reference trajectory) the thrust cutoff altitude is constrained, therefore

$$
\mathbf{r}_d = \mathbf{r}_d \text{ unit (r}_p^\perp)
$$

After determination of $\mathbf{r}_d$, this block splits into three branches according to mode: standard ascent, ascent to reference trajectory and the remaining modes.

4.8.1 Standard Ascent, $s_{\text{mode}} = 1$

The required velocity at burnout, $v_d$, to satisfy the terminal constraints for this mode is a function of the inputs: $i_y, r_d, v_d, \gamma_d$. Since $r_d$ was already used to determine $\mathbf{r}_d$, the remaining three variables (1 vector and 2 scalars) are utilized in the following manner

$$
\begin{align*}
\mathbf{i}_x &= \text{unit (r}_d^\perp) \\
\mathbf{i}_z &= \mathbf{i}_x \times \mathbf{i}_y \\
\mathbf{v}_d &= v_d \begin{bmatrix}
\mathbf{i}_x^T \\
\mathbf{i}_y \\
\mathbf{i}_z
\end{bmatrix}
\begin{bmatrix}
\sin \gamma_d \\
0 \\
\cos \gamma_d
\end{bmatrix}
\end{align*}
$$

Note that $\mathbf{i}_x$ and $\mathbf{i}_z$ define the radial and downrange directions with respect to the cutoff state. The original LTG equations, described in Ref. 1, used the current vehicle position to define these directions. This resulted in continuous rotation of the desired terminal velocity. The input normal to the transfer plane, $\mathbf{i}_y$, is directed in the opposite sense to the current orbital angular velocity vector.
4.8.2 Ascent to Reference Trajectory ($s_{mode} = 2$)

The objective of this mode is to intercept a coast reference trajectory for the single orbit rendezvous on Mission 3B. Throttling of the Space Shuttle Main Engines is used to increase the number of guidance control variables (i.e., degrees of freedom) by one, and thereby enable the guidance algorithm to control the downrange component of position at insertion.

The following process is used to accomplish this. First, the Conic State Extrapolation Routine (Ref. 4) is used to extrapolate the reference trajectory state from the input values ($r_{ref}^p, v_{ref}^p$) to the predicted thrust cutoff time $t + t_{go}$. (It should be noted that for proper operation of the guidance equations during the initial pass through the program, it is assumed that the input reference state ($r_{ref}, v_{ref}$) corresponds to the nominal time of thrust cutoff determined from preflight simulation). The extrapolated state ($r_d^p, v_d^p$) can be used to determine the error in the downrange component of position at cutoff [$i_z \cdot (r_d - r_p^p)$]. The other components of the extrapolated state ($r_d^p, v_d^p$) are used to update $v_{go}$ and $r_{go}$ on the subsequent guidance cycle.

Using this error in downrange position, it is necessary to determine a change in the time-to-go which drives this error to zero. Based upon the simplifying assumption of a flat earth and constant acceleration, it can be shown that the change in relative position $\Delta r_x$ resulting from changes in $t_{go}$ is given by

$$\frac{d}{d t_{go}} \Delta r_x = -\frac{i_z \cdot v_{go}}{2}$$

Then the change in $t_{go}$ necessary to drive the position error to zero can be determined from

$$\Delta t_{go} = -2 \frac{i_z \cdot (r_d - r_p^p)}{i_z \cdot v_{go}}$$

The new SSME throttle setting necessary to achieve this change in time-to-go is based upon the assumption that the maneuver velocity-to-be-gained is fairly insensitive to changes in throttle setting. Thus

$$K_k = \frac{K_k \cdot \frac{t_{b,k}}{t_{b,k} + \Delta t_{go}}}{K_{max}}$$

where $t_{b,k}$ is the burn time remaining in the current maneuver phase. The throttle setting $K_k$ is limited to a maximum value $K_{max}$.

4-17
On the last guidance cycle prior to the constant acceleration (g-limited) phase, the maneuver is converted to the Lambert ascent mode \( s_{\text{mode}} = 3 \) and ascent is completed in that mode. This is necessary since throttling for trajectory control and g-limiting may be incompatible. However, use of an artificial g-limit slightly less than the true limit, or a slight relaxation of the g-limit constraint could make it possible (with minor changes) to maintain the reference trajectory throttling mode throughout the second maneuver phase prior to tank separation. The Lambert ascent mode, however, will insure intercept with the target satellite with only minor variation in closing velocity.

4.8.3 Modes Requiring Lambert Solutions, \( s_{\text{mode}} = 3, 4, 7, 8 \)

The remaining modes (with the exception of \( s_{\text{mode}} = 6 \), External Delta-V, which does not enter this block) require calls to the Conic Required Velocity Determination Routine (Ref. 5). This routine solves a Lambert problem to determine the initial velocity required to satisfy certain terminal constraints.

The straight Lambert modes, ascent and on-orbit \( (s_{\text{mode}} = 3, 7) \) require one call to the Conic Required Velocity Routine to determine, \( \mathbf{v}_d \), the desired velocity to coast from the end of the maneuver to the target. \( \mathbf{r}_t \), in the given time interval, \( t - (t + t_{\text{go}}) \).

Modes 4, 5, 8, abort and deorbit require atmospheric reentry with the entry point at a constant location relative to the rotating earth. It is assumed that a constraint of the form mentioned earlier is required of the vertical, \( \mathbf{v}_{\text{tv}} \), and horizontal, \( \mathbf{v}_{\text{th}} \), components of entry interface velocity \( \mathbf{v}_t \).

\[
\mathbf{v}_{\text{tv}} = C_1 \mathbf{v}_{\text{tv}} + C_2 \mathbf{v}_{\text{th}}
\]

For small entry angles (-1.0 to -1.7 degrees) and entry velocities around 26,000 ft/sec this constraint is nearly equivalent to expressing the entry angle as a linear function of the entry velocity.

The only remaining variable that can be adjusted to cause the entry point to satisfy this terminal constraint is the coast time between cutoff and entry interface, \( t - (t + t_{\text{go}}) \). In other words, since \( t \) and \( t_{\text{go}} \) are fixed by previous computations, \( t \), the time at entry interface, must be adjusted. Using the currently assumed \( t \) and entry interface point relative to earth, \( \mathbf{r}_{\text{ref}} \), an inertial entry interface position, \( \mathbf{r}_t \), is determined by a call to the Earth Fixed to Inertial Routine. Then \( \mathbf{r}_d \), \( \mathbf{r}_t \), and the coast time \( t - (t + t_{\text{go}}) \) are input to the Conic Required Velocity Routine to obtain the desired velocity at cutoff, \( \mathbf{v}_d \), and velocity at entry interface, \( \mathbf{v}_t \). The entry interface velocity, \( \mathbf{v}_t \), will not in general lie on the constraint defined above but at some other point as shown in Figure 4-3.
we find that, in general $C_{1C} \neq C_1$ and therefore, $v_t$ does not lie on the constraint. Thus we must vary $t_t$ by some value such that $C_{1C}$ goes to $C_1$. In order to determine the sensitivity of $C_{1C}$ to changes in $t_t$ we repeat the call to the Conic Required Velocity Routine with $t_t$ perturbed by a small $\delta t$. In a manner similar to before a new point $(v'_{th}, v'_{tv})$ is found on Figure 4-3. The factor

$$\sigma = \frac{C_1 - C_{1C}}{(v'_{tv} - v_{tv}) - C_2 (v'_{th} - v_{th})}$$
represents the required change in $C_{1c}$ (necessary to extrapolate $v_{tv}$ and $v_{th}$ to the constraint line) divided by the change in $C_{1c}$ experienced by perturbing the entry interface time by $\delta t$. It is used to extrapolate values for the entry interface time, $t_e$ and desired cutoff velocity, $v_{d}$, which should result in near satisfaction of the constraint.

$$t_{(\text{new})} = t_{(\text{previous})} + \sigma \delta t$$

$$v_{d_{(\text{new})}} = v_{d_{(\text{previous})}} + \sigma (v'_{d} - v'_{d_{(\text{previous})}})$$

The accuracy of this technique depends upon how much the region over which the extrapolation takes place varies from the linearized assumptions. Each guidance cycle, however, should bring the extrapolated point closer to the constraint.

4.8.4 Revising $v_{go}$

Changes in desired terminal velocity, $v_{d}$, affect the predicted terminal position (and desired terminal velocity) on the subsequent guidance cycle. If this effect is ignored, the desired terminal velocity is overcorrected and a small oscillation in desired terminal velocity is induced. To eliminate this overcorrection, a damping factor, $\rho$, is introduced for modes 4, 5, 8 as follows:

$$\Delta v_{go} = \rho (v'_{go} - v_{go})$$

where $v'_{go}$ is the new velocity-to-be-gained assuming no damping and is given by

$$v'_{go} = v_{d} - v_{grav} + v_{bias}$$

The damping factor is computed by making use of the approximate partial derivative of desired velocity with respect to terminal position

$$\frac{\partial v_{d}}{\partial r_{d}} = \begin{bmatrix} \frac{1}{t'_{c} - t' - t'_{go}} & 0 & 0 \\ 0 & \frac{1}{t'_{c} - t' - t'_{go}} & 0 \\ 0 & 0 & \frac{1}{t'_{c} - t' - t'_{go}} \end{bmatrix}$$

where $t'_{c} - t' - t'_{go}$ is the coast time from cutoff to entry interface. It can be shown that

$$\rho = \frac{1}{1 + \frac{\varepsilon_{go}}{2 (t'_{c} - t' - t'_{go})}}$$

4-20
Note that initially $0 < \rho < 1$ and $\rho$ goes to one at the end of the burn.

Velocity-to-be-gained for the next pass through the routine is now revised by

$$\dot{v}_{go} = \dot{v}_{go} + \Delta\dot{v}_{go}$$

### 4.9 Throttle Commands

The throttle setting, $K_k$, for the Space Shuttle Main Engine and the engine-on/off commands are issued by the last and as yet to be determined block of equations. The throttle setting will be assumed to be input each cycle for the acceleration limited phases or will be computed in the required velocity block of the previous cycle for Mode 2 (Reference Trajectory). The array of switches, $s_{SSME,i}$, $s_{OMS,i}$ and $s_{RCS,i}$, will be utilized to determine which engines will be operating in each phase.

During the prethrust call, on the last iteration to convergence engine ignition commands for $t_{ig}$ will be issued. The appropriate time to issue these commands will be determined by the conditions: $s_{pre} = 0$, $\Delta t = 0$. 

4-21
5. **DETAILED FLOW DIAGRAMS**

This section contains detailed flow diagrams of the computations and logic used for Unified Powered Flight Guidance. The overall flow is illustrated on the first diagram (Figure 5-1) which shows the nine blocks of computations connected by the necessary logic to form a single routine, UPFG. One entry and one exit point exist for UPFG with internal branching, as indicated, to perform prethrust or active guidance for the various maneuver modes.

The following diagrams (Figures 5-2 through 5-10) further detail each of the nine blocks of in-line computations of UPFG.
Figure 5-1a. Main Routine
Figure 5-1b. Main Routine (Cont.)
Figure 5-2. Block 1 - Initialization
ENTER Block 2

Read Accelerometers and Clock (TBD)
Output: $\Delta \dot{y}_{\text{sensed}}$, t

$\Delta t = t - t_{\text{prev}}$
$t_{\text{prev}} = t$

$s_{\text{pass1}} = 1$

Yes

No

$\dot{v}_{\text{go}} = \dot{v}_{\text{go}} - \Delta \dot{y}_{\text{sensed}}$

$\Delta \dot{y}_{\text{sensed}} = 0$
$s_{\text{pass1}} = 0$

Call Powered Flight Navigation Routine (Ref. 6)
Input: $\mathbf{r}[\mathbf{r}(t), \mathbf{v}(t), \Delta t, \Delta \dot{y}_{\text{sensed}}, \mathbf{g}[\mathbf{g}(t)]$,
$s_{\text{pert}}$
Output: $\mathbf{r}[\mathbf{r}(t + \Delta t), \mathbf{v}(t + \Delta t), \mathbf{g}[\mathbf{g}(t + \Delta t)]$

$k = n$

$t > t_{\text{ig}}$

Yes

No

$t_{\text{c}} > 0$

Yes

No

Figure 5-3a. Block 2 - Update
Figure 5-3b. Block 2 - Update (Cont.)
Figure 5-4a. Block 3 - Time-To-Go
Figure 5-4b. Block 3 - Time-To-Go (Cont.)
Figure 5-5. Block 4 - Integrals of Thrust
Figure 5-6. Block 5 - Turning Rate

Note: if $\lambda = 0$, unit ($\dot{\lambda}$) may cause computational difficulties.
Determine unit thrust direction based upon:
\[ \hat{t}_f = \text{unit} \left[ \hat{u} - \left( \frac{u}{L} \right) \hat{\omega} \right] \]

\[ s_{pre} = 0 \]

No (Prethrust)

Yes (Active Guidance or Last Prethrust Cycle)

(Last Prethrust Cycle)

Yes \( \Delta t = 0 \)

No (Active Guidance)

Determine and issue initial attitude orientation command based upon unit thrust direction, \( \hat{t}_f \) (TBD)

Determine and issue steering commands for active guidance based upon unit thrust direction, \( \hat{t}_f \) (TBD)

EXIT Block 6

Figure 5-7. Block 6 - Steering Command
Figure 5-8. Block 7 - Gravity Effects

\[ \Delta r_c = -\frac{1}{10} r_{\text{thrust}} \quad \Delta v_c = \frac{6}{5} \frac{r_{\text{thrust}}}{t_{\text{go}}} - \frac{1}{10} v_{\text{thrust}} \]

\[ r_{c1} = r + \Delta r_c \quad v_{c1} = v + \Delta v_c \]

Call Conic State Extrapolation Routine (Ref. 4)

Input: \( r_{c1} \), \( v_{c1} \), \( t_{\text{go}} \), \( \Delta t \)
Output: \( r_{c2} \), \( v_{c2} \)

\[ v_{\text{grav}} = v_{c2} - v_{c1} \]

\[ r_{\text{grav}} = r_{c2} - r_{c1} - v_{c1} t_{\text{go}} \]

ENTER Block 7

EXIT Block 7
\[ \mathbf{r}_p = \mathbf{r} + \mathbf{v}_{tg} + \mathbf{v}_{grav} + \mathbf{v}_{thrust} \]

(On-orbit, Lambert, Deorbit) 7, 8

1, 3, 4, 5

\[ \mathbf{r}_d = \mathbf{r}_p \]

\[ \mathbf{r}_d = \mathbf{r}_d \text{ unit}(\mathbf{r}_p) \]

\[ t_{b,k} < \Delta t \]

Yes

No

Call Conic State Extrapolation Routine (Ref. 4)

Input: \[ \mathbf{r}_{ref}, \mathbf{v}_{ref}, \mathbf{v}_{tg}, (t + t_{tg} - t_{ref})/\Delta t \]

Output: \[ \mathbf{r}_d(r) \]

\[ \Delta r_z = \mathbf{i}_z \cdot (\mathbf{r}_d - \mathbf{r}_p) \]

\[ v_{goz} = \mathbf{i}_z \cdot \mathbf{v}_{go} \]

\[ \Delta t_{go} = -\Delta r_z / v_{goz} \]

\[ K_k = \frac{K_k}{t_{b,k} + \Delta t_{go}} \]

\[ i_x = \text{unit}(\mathbf{r}_d) \]

\[ i_z = i_x \times i_y \]

\[ \mathbf{r}_d = \mathbf{v}_d \begin{bmatrix} i_x \sin \gamma_d \\ 0 \\ i_z \cos \gamma_d \end{bmatrix} \]

Figure 5-9a, Block 8 - Velocity-To-Be-Gained

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Figure 5-9b. Block 8 - Velocity-To-E. Gained (Cont.)
Figure 5-9c. Block 8 - Velocity-To-Be-Gained (Cont.)
Determine engine-off time as function of \((t + t_{go})\) and engine characteristics (TBD)

\[ s_{engoff} = 1 \]

\[ s_{pre} = 0 \] (Prethrust)

\[ \Delta t = 0 \] (Active Guidance)

\[ \text{Issue engine-on command, } t_{ig} \]

\[ \text{EXIT Block 9} \]

**Figure 5-10. Block 9 - Throttle Commands**
6. **SUPPLEMENTARY INFORMATION**

Several details of the UPFG program require further study or definition. They are listed below, not necessarily in order of importance.

6.1 **Targeting Assistance**

Since the deorbit maneuver may be a long, low acceleration burn with a single OMS engine, traditional targeting techniques based upon an impulsive maneuver are not adequate. The powered flight guidance routine must assist the targeting to determine the $\Delta v$ required for the finite thrust maneuver and compute the optimal ignition time. Thus an additional logical path through UPFG program, very similar to the prethrust path, should be included in future revisions. This path could be used iteratively by the deorbit targeting program to search for the optimal ignition time resulting in minimum $\Delta v$.

6.2 **Return-to-Launch-Site Abort**

Considerable effort will be required to define the thrust cutoff condition for successful RTLS abort maneuvers. Requirements on dynamic pressure at external tank separation, orbiter glide back capability, maximum loads, maximum vehicle turning rates, fuel depletion requirements, and other factors enter into the development of RTLS powered maneuver constraints.

6.3 **Ascent Initialization**

The problems of efficiently initializing UPFG during an ascent maneuver have not been addressed in sufficient detail. It is probably desirable to include the boost phase guidance in the overall UPFG scheme. This should simplify somewhat the initialization and transition from the relatively simple atmospheric phase guidance to the explicit LTG concept used in UPFG. The problems associated with changing maneuver objectives during ascent due to engine failures have not been explored. The best method of switching from a standard ascent maneuver to some abort mode must be determined. The reader should note that the prethrust process described in this report is primarily tailored for the on-orbit maneuvers.

6.4 **External Tank Disposal**

The implications and requirements of the external tank disposal require further study. In this document, it is assumed that tank separation will take place during a short coast phase after the ascent velocity-to-be-gained has been driven to a prespecified value ($< 150$ fps). Then the OMS engines complete the ascent maneuver. If this technique does not insure a sufficiently accurate external tank impact, then further refinement will have to be considered.
If the OMS maneuver following tank separation is sufficiently large (≈ 300 fps) such that the length of the OMS maneuver is comparable in length to the phase from SRM separation to tank separation, then the guidance equations must be modified. The multi-phase LTG equations, which result in a linear vehicle pitch rate, are not optimal for an acceleration profile with a high initial acceleration and a very low final acceleration. The final OMS maneuver could probably be treated as a constant attitude phase without unduly complicating the equations.

6.5 Compensation for Non-Keplerian Gravity Effects

No attempt has been made to include a technique for compensation of non-Keplerian gravity effects in this initial version of UPFG. Further work is required to determine whether this compensation should be accomplished during premaneuver targeting or during the maneuver. A combination of premaneuver compensation with small adjustments during the maneuver will probably produce good accuracy with minimal code.

6.6 Ascent Maneuver Phase Changes

The ascent maneuver is divided into several maneuver phases, which may include a constant thrust phase, a constant acceleration phase, a final OMS phase, and possibly a phase with an assumed SSME failure. Transition from one phase to the next may be a function of acceleration, time, or velocity-to-be-gained. To solve for several integrals of the thrust acceleration over the total maneuver time-to-go, the times at which phase changes occur must be prespecified or calculated. Since mission perturbations may alter these times, it may be desirable to modify them during the maneuver based upon sensed acceleration.

6.7 Steering Commands

The development of steering equations to generate commands for the control system is incomplete. The steering equations will combine desired vehicle attitude, desired vehicle rate, and sensed acceleration to produce a steering command.

6.8 Throttle Commands

An algorithm to estimate current acceleration and calculate an engine throttle setting for the constant acceleration (g-limited) phase of ascent has not been developed. The frequency with which these commands must be issued, to prevent a sawtooth profile, and the engine response to commands should be evaluated to develop this algorithm.

6.9 Vehicle Mass Estimate

The maintenance of the estimated vehicle mass should be accomplished external to the UPFG program since this information is also required by the control system software. In addition, step changes in the vehicle mass due to events such as tank disposal or satellite deployment will obviously have to be handled elsewhere. However, for completeness, an equation has been included in this document (Block 2) to decrement vehicle mass for simulation purposes.
6.10 **Engine Failure**

The failure of an engine during any mission phase could possibly be detected through changes in sensed acceleration, however it is assumed that more reliable information will be available from the performance monitoring system. Therefore, the UPFG equations have been written with the assumption that an external routine will notify the UPFG routine of changes in engine status through the input switches.

6.11 **Throttle Lag**

The ascent to reference trajectory guidance mode requires use of the SSME throttles. To make the proper adjustments in engine throttle settings, the currently commanded throttle settings are used in combination with nominal engine performance data. To compensate for engine performance perturbation and throttle non-linearity, it may be desirable to use sensed acceleration information. However, this will introduce problems due to both guidance computational delays and engine throttle response. Thus this problem requires further study.

6.12 **Alternate Input Schemes**

The input list for each maneuver mode, described in Section 3, has been designed with the intent to minimize the size of the flight program. A certain amount of ground calculation is required for the ascent guidance modes, however on-orbit input is consistent with current CSDL targeting concepts. Future revisions could include alternate input schemes for ascent maneuvers which may be easier to use but place additional calculation burdens upon the guidance computer.
REFERENCES


