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A ONE WAY DOPPLER TRACKING SYSTEM FOR SATELLITE TO SATELLITE TRACKING

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

An S-band tracking system employing one way Doppler applicable to the Tracking and Data Relay System (TDRS) has been evaluated. This evaluation is limited to the measurement system parameters. Orbit determination uncertainties, such as those associated with modeling of the geopotential function, solar pressure and atmospheric drag, are independent of the measurement system and therefore are not included in the evaluation.

The analysis assumes state-of-the-art components such as spacecraft oscillator long term stability of one part in 10^8 and ground station oscillator stability of one part in 10^{11}. Signal characteristics comparable with the present (1973) ranging systems are utilized in the analysis.

Predicated upon measurement system parameters, position uncertainties for the low orbiting spacecraft vary from 5 meters to 2 kilometers along track, 4 meters to 1 kilometer cross track, and 1 meter to 180 meters radially depending upon the tracking geometry and the high satellite position and velocity error assumptions.

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A ONE WAY DOPPLER TRACKING SYSTEM FOR SATELLITE TO SATELLITE TRACKING

1. INTRODUCTION

The Tracking and Data Relay Satellite (TDRS) System is envisioned as a system where by one or more synchronous satellites are employed in the generation and/or measurement of tracking data for lower orbiting (User) satellite orbit determination as well as the relaying of telemetry data from the lower orbiter to a ground facility. This paper is concerned with generation and/or measurement of tracking data and its application to orbit determination. The measurement data type, processing and the orbit dynamics of both satellites must be considered in the application of such a technique. The measurement of range or signal propagation delay between the synchronous satellite and the low orbiter defines a sphere centered at the former with a radius equal to the range. Another possibility is the determination of range rate between the two satellites or a combination of both range and range rate. The type of measurement obtained depends upon the number of independent measurements that can be performed and on the tracking data processing employed in the position or orbit determination program. The type and number of measurements is of course reduced if some a priori knowledge of the low orbiter trajectory or orbit is assumed and the data processing procedures include this a priori knowledge.

The purpose of this document is to evaluate an S-band tracking system for utilization in the TDRS System. The recommended system results in a very small impact upon the User satellite in that no formal ranging transponder is required. The only specific requirement is that the transmitted telemetry time ticks be traceable, time wise, to the received command time ticks. The analysis of the system is presented in Section 3 with the resulting measurement uncertainties of 12 meters bias and 24 meters noise in the range measurement and 0.2 cm/sec bias and 0.04 cm/sec noise in the range rate measurement when integrated over a ten second period.

The resulting orbital position uncertainties are analyzed in Section 4 and indicate from 30 to 110 meters uncertainty depending upon the tracking span and geometry.

2. SYSTEM DESCRIPTION

The tracking system being evaluated operates as follows. The command signals to an individual User are transmitted in the normal fashion (Fig. 1). That is,

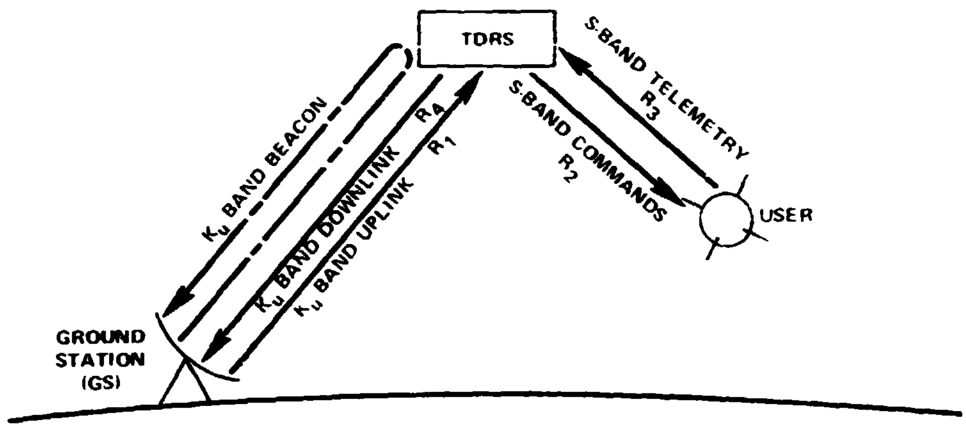


Figure 1. TDRS Tracking Configuration

the commands are transmitted on a K_u-band carrier from the ground station to the TDRS. The TDRS, utilizing its phase locked vco, frequency translates the carrier to S-band and transmits the command toward the User. The User receives, detects and decodes the commands. In this process time marks are either recognized or generated coherent, in the time domain, with the command code synchronization. This time mark is then employed in the User as a synchronization for the return link telemetry and the time mark is transmitted to the ground station over the reverse of the command link. Thus the equivalent of one event being transmitted thru the entire link has been simulated, the total delay time can be measured and the range sum $R_1 + R_2 + R_3 + R_4$ (Fig. 1) can be determined. The utilization of this measurement will be explained in Section 4.

In this system the range rate sum measurement is derived from the one way Doppler only. Thus no up or forward link or coherent carrier translation is required. One way range rate determinations may be made when ever the User is in line of site from the TDRS.

3. SYSTEM ANALYSIS

A detailed analysis of the VHF equivalent range and range rate determinations for this system is found in Reference 1. The system concepts include a Ground Station (GS) to TDRS turn around beacon. Measurements derived from this beacon are employed in the processing to compensate for the GS/TDRS range rate and the TDRS on board oscillator noise. With proper implementation, as described in Reference 1, some of the effects due to GS oscillator instabilities may be removed with the turn around beacon.

The GS/TDRS link is assumed to have a signal to noise spectral density (S/Φ) of at least 90 dB-Hz. Under this assumption, the effects of this link upon the ranging signals can be neglected. Depending upon the information bandwidth and the antenna gains this S/Φ should be realizable.

In establishing the uncertainties, several assumptions concerning the link and orbit dynamics are made. These are tabulated in Table 1.

Table 1

Parametric Assumptions

Orbital Dynamics $\dot{R}$ $\ddot{R}$	TDRS 1 m/s 1 mm/s^2	User 7 km/s 10 m/s^2
Link Power Budget S/Φ	TDRS-User 65 dB-Hz	User-TDRS 60 dB-Hz

3.1 Range System Analysis

The uncertainties in the range determination due to the instrument are 24 meters noise and 12 meters bias. An uncertainty in the velocity of light represents a range uncertainty of

$$\Delta R = \frac{\Delta C}{C} R \tag{1}$$

so that for a $\Delta C/C \approx 3 \times 10^{-7}$, $|\Delta R| \approx 12$m

The GS oscillator drift contributes a

$$\Delta R = \frac{\Delta f}{f} R \tag{2}$$

Assuming an oscillator long term stability of 10^{-10}, $|\Delta R| \approx 2.0$ centimeters. The equivalent propagation time delay will be determined utilizing a 2nd order tracking loop. For a 5 Hz loop and the dynamics of Table 1 the dynamic lag error is approximately 50 centimeters. The root sum square effect of these three bias type uncertainties is 12.0 meters.

The thermal noise error may be evaluated employing the clock frequency and

$$\sigma_R = \frac{C}{5.6\pi f_{CL}} (SNR)^{-1/2} \tag{3}$$

Since the system may be employed for a low data rate User a clock of 10 KHz is assumed and σ_R is approximately 24 meters for the link parameters given in Table 1 and a 5 Hertz tracking loop bandwidth.

The GS tracking loop voltage controlled oscillator (vco) contributes noise to the tracking uncertainties. This is due to the vco phase jitter. For a well designed crystal vco this jitter can be made as small as 3 nanoseconds, resulting in a 0.6 meter uncertainty in the range determination.

Quantization, assuming a 100 MHz clock in the GS, contributes another 0.6 meters to the system uncertainty and a 20 nanosecond timing jitter is responsible for 3.0 meters. The root sum squared range noise is 24.2 meters.

3.2 Range Rate System Analysis

The system employs a conventional Doppler cycle counting technique for range rate determination. The Doppler signal counted consists of a bias frequency (f_b) plus the Doppler offset on the one way carrier. The number (N) of Doppler plus bias cycles counted during a time (T) must satisfy the relationship

$$N = (f_b + f_D)T \tag{4}$$

There exists two implementations for Doppler cycle counting. The T-count implementation where by the counting time is held constant and the Doppler plus bias cycles counted in time T; and the N-count where by the number of Doppler plus bias cycles counted is held constant and the time to accumulate this number of cycles measured. The system may employ either the N or T count implementation. The uncertainties in both implementations are the same. However, in the T-count system some of the errors may be correlated, sample to sample, and this correlation may be employed to reduce the error in the data processing.

As in the Range determination, there is a bias in the range rate determination due to the uncertainty in the velocity of propagation. This error is a function of the range rate sum as follows:

$$\Delta \dot{R} = \frac{\Delta C}{C}(\dot{R}_3 + \dot{R}_4) \qquad \text{(Fig. 1)} \tag{5}$$

Once again for a $\Delta C/C \approx 3 \times 10^{-7}$ the range rate bias is 0.2 cm/sec maximum. Since this system determines range rate by Doppler measurements and the return link carrier only, the long term drift of the User spacecraft transmitter frequency (f_0) must be considered. Consider the spacecraft transmitter frequency to be ($f_0 + \Delta f_0$) instead of the nominal f_0.

Then

$$\Delta R^{\cdot} = \frac{C f_D}{f_o} - \frac{C f_D}{f_o + \Delta f_o} \approx R^{\cdot} \frac{\Delta f_o}{f_o} \tag{6}$$

the bias term associate with a spacecraft oscillator long term stability of 10^{-8} is then 0.07 mm/sec. The effects of the GS master oscillator drift is dependent upon the Doppler extractor implementation. The system will employ a "Frequency Independent Doppler Extractor" (Ref. 2). This technique takes advantage of the correlation between the bias frequency, the reference frequency (used to determine T in Equation 4) and the GS master oscillator and results in

$$\Delta R^{\cdot} = -R^{\cdot} \left(\frac{\Delta f_G}{f_G} \right) \tag{7}$$

Assuming an oscillator long term stability of 10^{-10} the bias due to the GS master oscillator is 0.0008 cm/sec and can be neglected.

The root sum square of the Doppler bias error is 0.2 cm/sec.

In either the N or T count system the quantization error is the same. In the N count, the start and stop pulses are treated as independent random variables. In the T count, the start pulse being derived from the reterence generator does not contribute a random error. However in the T count system two sucessive reference counts are required and the phase offsets of results in the same uncertainty as the N count and

$$\sigma_{R^{\cdot}} = \frac{C}{f_o} \left[\frac{f_b + f_D}{\sqrt{6}\, T\, f_r} \right] \tag{8}$$

The system should be specified such that $(f_b + f_D)/f_r \approx 5 \times 10^{-3}$. For this evaluation 1/T will be taken to be the reciprocal of the output data rate. Table 2 summarizes the qu' ıtization noise errors.

Table 2

T (sec)	$\sigma_{R^{\cdot}}$ (cm/sec)
10.0	0.003
1.0	0.03
0.5	0.06
0.25	0.12

The previous exploration does not include start and stop pulse jitter. It is felt that the time jitter and these pulses can be maintained to a few nanoseconds and can be neglected.

The effect of the User L.O. short term instability (phase jitter) may be determined from the expression

$$\sigma_{\dot{R}} = \frac{C}{2\pi f_o} \left(\frac{\Delta f_o}{f_o} \frac{2}{T} \right)^{1/2} \tag{9}$$

Employing a $\Delta f_0/f_0$ of 10^{-6} for the phase stability of the User L.O. frequency, Equation 9 reduces to

$$\sigma_{\dot{R}} \approx \frac{1}{3\sqrt{T} \times 10^2}$$

which is evaluated in Table 3.

Table 3

T (sec)	$\sigma_{\dot{R}}$(cm/sec)
10.0	0.001
1.0	0.0033
0.5	0.005
0.25	0.007

As the signal is propagated over the path R_3 R_4 of Figure 1 thermal noise is added which eventually shows up as phase noise in the Doppler extractor. The noise contribution of the TDRS/GS link (R_4 of Figure 1) may be neglected for the assumed signal to noise density conditions. The range rate uncertainty due to thermal noise is:

$$\sigma_{\dot{R}} = \frac{C}{2\pi f_o T} (SNR)^{-1/2} \tag{10}$$

where the signal to noise ratio (SNR) is determined in the loop bandwidth. Under the assumptions of Table 1 and considering a 3 dB carrier reduction due to modulation the range rate uncertainty for several sampling periods is given in Table 4.

The various noise and bias terms for the system are summarized in Table 5 for a 10 sec sampling time and a 5 Hertz tracking loop.

Table 4

T (sec)	$\sigma_{\dot{R}}$ (cm/sec)
10.0	0.0025
1.0	0.025
0.5	0.05
0.25	0.1

Table 5

	Noise	Bias
Range	24.2 meters	12 meters
Range Rate	0.004 cm/sec	0.2 cm/sec

3.3 Propagation Path Effects

The preprocessing of the data from this tracking system must include not only the uncertainties of Table 5 but also the uncertainties due to the troposphere, ionosphere and timing. The rms range fluctuations due to the troposphere have been analyzed by P. E. Schmid in Reference 3. The tropospheric range rate errors have been analyzed by C. A. Filippi in Reference 4. In our application the troposphere effects primarily the GS/TDRS and TDRS/GS links since it is assumed that the User is above the troposphere during the tracking interval. However, the Doppler extraction process is operating as if the entire link were at S-band and not K-band. Therefore, the effect is multiplied by the actual K-band/S-band ratio. The effect upon the range and range rate can be evaluated employing data from the GS/TDRS/GS beacon. It is assumed that this compensation will be part of the data preprocessing program.

Depending upon the altitude of the User and/or the line of sight between the TDRS and the User, the ionospheric effect may be negligible for a high altitude User directly beneath the TDRS to considerable for a low altitude User near the TDRS horizon. For the range measurement, the ionosphere is crossed twice by the S-band carrier and the effect must be included in the preprocessor. In Reference 4, the ionospheric effect is given as

$$\Delta R = \frac{40 \int N(r) dr}{f_0^2}$$

where f_0 is expressed in Hertz and the electron content is evaluated over the TDRS/User range limits and depends upon both the atmospheric condition and the angle associated with the propagation path. Values for $\int N(r)dr$ as cited in Reference 5 are 2×10^{16} to 4×10^{17} typically with a maximum of 1.5×10^{18} for vertical incidence. The error magnitudes associated with these are 3 to 54 meters typically and 200 meters maximum. This uncertainty must be modeled in the data preprocessor.

In evaluating the effect of the ionosphere upon range rate measurements the following criteria is established: the time delay fluctuations due to the ionosphere are short relative to the Doppler counting interval but long relative to the signal propagation times. Under these criteria the effect of the ionosphere may be written as

$$\Delta \dot{R} = \frac{40}{f_0^2} \frac{d}{dt} \int N(r)dr$$

for the TDRS/User link. The ionospheric effect on the GS/TDRS/GS link is negligible. The ionospheric effect may vary from 5 meters/sec at zero grazing angle (TDRS horizon) to 1.5 meters/sec at 70°. This uncertainty must be modeled and the model included in the data preprocessor.

4. TRAJECTORY DETERMINATION EVALUATION

The range and range rate uncertainties (noise and bias) were used to corrupt simulated tracking data. These corrupted data were then inputted to the Navigation Analysis Program (NAP) to ascertain their influence upon an orbit or trajectory determination.

Referring to Figure 2, the three vectors $\overline{X} = (x_1, x_2, x_3)$, $\overline{Y} = (y_1, y_2, y_3)$ and $\overline{S} = (s_1, s_2, s_3)$ represent the inertial geocentric coordinates of TDRS, User and the ground station respectively. Simulated observations for satellite to satellite two way range and one way range rate were synthesized from reference trajectories for the TDRS and User satellites. The reference trajectories were obtained by means of numerical integration of the equations of motion with given sets of initial conditions in the form of position and velocity state vectors at some epoch. These vectors are given in Table 6.

4.1 Range Simulation

The two way range observations were determined by measuring the time delay resulting from an event being propagated from the ground station via the TDRS to the User and returning over the same path. An event occurring at the ground station at time t, reaches the TDRS at some later time $t + T_3$. Thus, T_3 represents the propagation time over the GS/TDRS path. The event is then transmitted

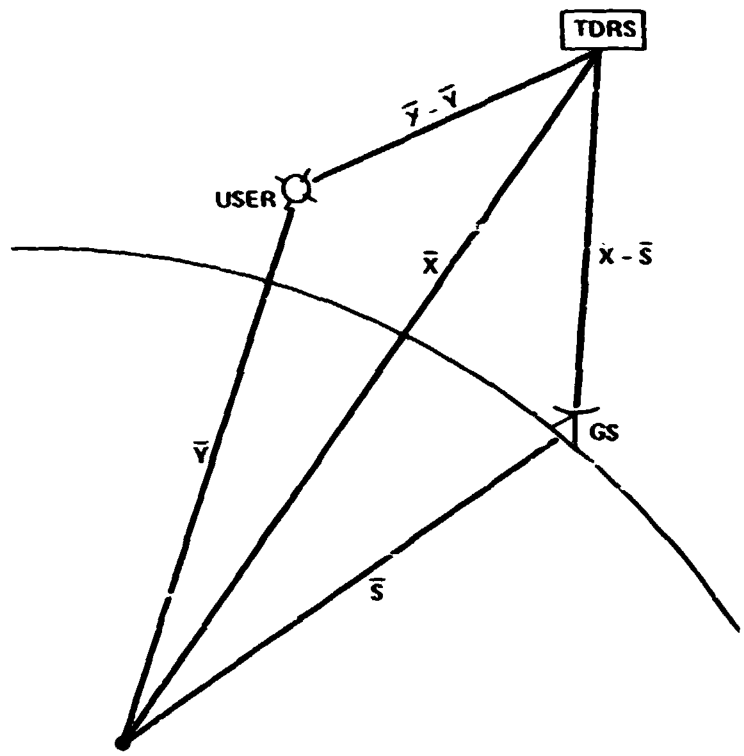


Figure 2. Simulation Tracking Geometry

from the TDRS to the User arriving at $t + T_3 + T_1$, (transponder delays are considered to be known and, therefore, may be equated to zero). The User returns a time derived signal to the TDRS reaching it at time $t + T_3 + T_1 + T_2$. The event is returned to the ground station and detected at time $t + T_3 + T_1 + T_2 + T_4$.

Each of the delay times (t_i) defined above is expressable in terms of the inertial satellite and ground station coordinates. The relationships are:

$$CT_3 = \left(\sum_{k=1}^{3} (y_k(t + T_3) - s_k(t))^2 \right)^{1/2}$$

Let $t_1 = t + T_3$ (11)

$$CT_1 = \left(\sum_{k=1}^{3} (x_k(t_1 + T_1) - y_k(t_1))^2 \right)^{1/2}$$

Let $t_2 = t_1 + T_1$

$$CT_2 = \left(\sum_{k=1}^{3} (y_k(t_2 + T_2) - x_k(t_2))^2 \right)^{1/2}$$

(iii)

Let $t_3 = t_2 + T_2$

$$CT_4 = \left(\sum_{k=1}^{3} (y_k(t_3) - s_k(t_3 + T_4))^2 \right)^{1/2}$$

Table 6

Reference Trajectory

Reference Orbit Parameters		
Epoch- 00h-00m-00s		
Vector (true of date)		
	X	-7064.8802 KM
	Y	2508.6284
	Z	-1227.4734
	XD	-0.1974129 KM/SEC
	YD	2.2046003
	ZD	6.95402775
Vector (mean of 1950)		
	X	-7055.091084 KM
	Y	2543.221503
	Z	-1212.498994
	XD	-1717654498.0 KM/SEC
	YD	2.20572973
	ZD	6.95435038
Synch. Satellite vector (earth fixed coordinates)		
	X	11425.03 KM
	Y	-40565.529
	Z	380.053
	XD	0.0028836 KM/SEC
	YD	-0.003219
	ZD	0.079284

where C is the velocity of propagation for the signal.

These equations (11) were solved iteratively in the NAP-3 to yield the expression for the two way range simulated observation as:

$$R = CT_1 + CT_2 + CT_3 + CT_4 \qquad (12)$$

4.2 Range Rate Simulation

The Doppler data to be simulated includes the Doppler due to the TDRS/User relative motion plus the Doppler due to the TDRS/GS relative motion. The simulated range rate was determined based upon the return link carrier Doppler. The actual parameter which is measured is the time interval required to accumulate some fixed number (N) of Doppler plus bias frequency cycles as explained in 3.2. This time interval as given in Equation 4 is

$$T_D = \frac{N}{f_b + f_D} \qquad (4)$$

In terms of the link delays as defined in 4.1, the Doppler frequency is expressible as:

$$f_D = (f_r + f_s)\,\dot{T}_4 + f_s\,\dot{T}_3 + f_r\,\dot{T}_2 \qquad (13)$$

where* f_r = User/TDRS frequency
f_s = TDRS/GS frequency

and the time derivative $\dot{T}_k$ were evaluated from the set of nonlinear equations (11) with

$$\dot{T}_3 = \sum_{k=1}^{3} (\dot{y}_k(t + T_3) - \dot{s}_k(t))/c^2\dot{T}_3 \qquad (14)$$

$\dot{T}_2$ and $\dot{T}_4$ were similarly evaluated.

Simulated "one way range rate" observations were synthesized by means of the NAP-3 as follows. First the Doppler shift f_D was evaluated by setting Equation 13 equal to $(N/T_D - f_b)$. Then for the TDRS we assumed that $\dot{T}_3 = \dot{T}_4$ so that f_D may be written as

$$f_D = (\dot{T}_2 + \dot{T}_4)\,f_r + 2f_s\,\dot{T}_4 \qquad (15)$$

*Since the TDRS is phase locked to the GS, uplink Doppler is reflected in the downlink transmitted frequency and must be included.

We define the simulated "one way range rate" observation as:

$$\dot{R} = C(\dot{T}_4 + \dot{T}_2)$$
$$= \left(\frac{f_D}{f_r} - \frac{2f_s}{f_r}\dot{T}_4\right)C \qquad (16)$$

4.3 Orbit Discrepancies

The previously generated simulated "two way range" and "one way range rate" data provided inputs to the NAP-3 for numerous data reduction exercises. The effects of tracking geometry, random observation noise and unmodeled systematic errors and the resulting orbit estimates were investigated by performing numerous reductions under varying input conditions. Two general classes of NAP-3 reductions were considered.

a. Recovery of User state vector from simulated (noisey) observation data corrupted by systematic errors such as zero set and timing biases or TDRS and station location uncertainties.

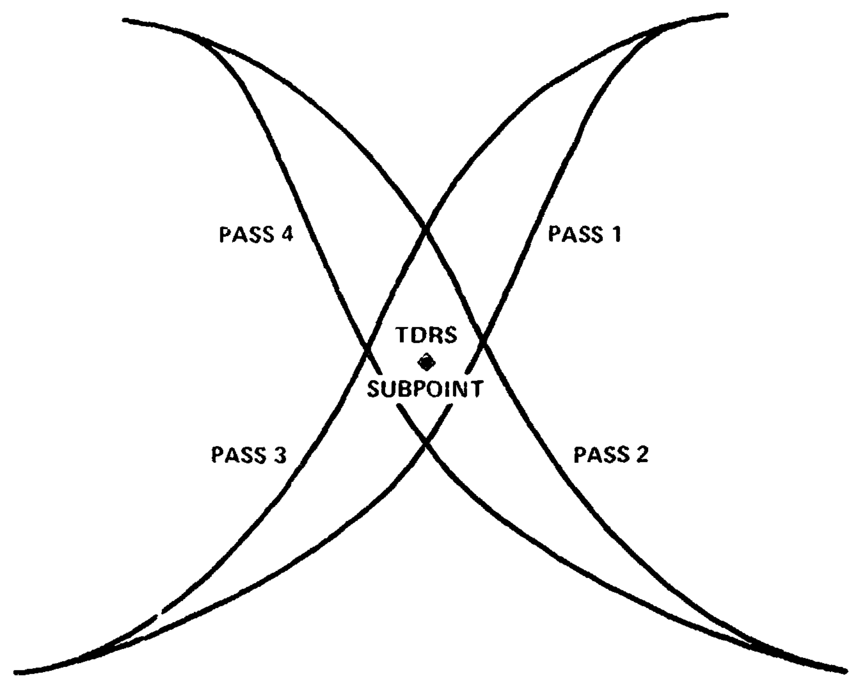
b. Recovery of User satellite state vector from noisey observation data under various configurations of tracking geometries.

The effects due to the various error sources under consideration were analyzed by examining the resulting orbital discrepancies. These discrepancies represent the differences between the reference orbit (orbit utilized in the generation of simulated data) and the orbit recovered, by means of the NAP, from the corrupted simulated data.

The systematic error sources listed in Table 7 were used in developing orbital uncertainties for various tracking geometries (Figs. 3 and 4).

Table 7

Ground Station Location	5 meters
TDRS Position	50 meters
TDRS Velocity	5 cm/sec
Range Sum Bias	12 meters
Range Sum Noise	24.2 meters
Range Rate Sum Bias	0.2 cm/sec
Range Rate Sum Noise	0.004 cm/sec

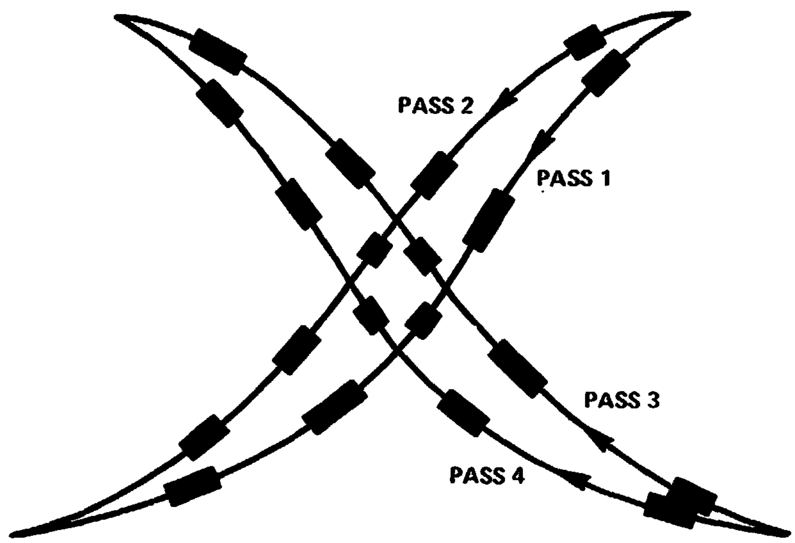


		Position	
a.	Passes # 1, 2, 3 and 4	Minimum	Maximum
	Along track uncertainty =	100 m	2000 m
	Cross track uncertainty =	40 m	1000 m
	Radial uncertainty =	20 m	180 m
b.	Passes # 1 and 2 only	Minimum	Maximum
	Along track uncertainty =	115 m	1700 m
	Cross track uncertainty =	80 m	300 m
	Radial uncertainty =	3 m	160 m

Figure 3. User Tracking Data Spans

CONCLUSIONS

The position uncertainty for several tracking geometries is shown in Figures 3 and 4. Comparison of the results from these figures might indicate that the use of more data (Fig. 3b) yields worse results (higher uncertainties) than the comparable case (Fig. 4c) which utilized less tracking data. While this may appear to be an inconsistency, it must be remembered that addition or more dense data improves the results only when considering random or noise like error sources. Bias type errors, on the other hand, tend to make the results worse as more biased data is introduced into the computations.



		Minimum	Maximum
a.	Passes # 1, 2, 3 and 4	20 minute tracking during 24 hours	
	Along track uncertainty =	100 m	1900 m
	Cross track uncertainty =	50 m	1050 m
	Radial uncertainty =	25 m	150 m
b.	Same geometry as (a)	(TDRS velocity error = 0)	
	Along track uncertainty =	5 m	100 m
	Cross track uncertainty =	10 m	70 m
	Radial uncertainty =	2 m	6 m
c.	Passes # 1 and 2 only	10 minute tracking during 24 hours	
	Along track uncertainty =	125 m	1400 m
	Cross track uncertainty =	90 m	330 m
	Radial uncertainty =	4 m	150 m
d.	Same geometry as (c)	(TDRS velocity error = 0)	
	Along track uncertainty =	5 m	160 m
	Cross track uncertainty =	10 m	30 m
	Radial uncertainty =	1 m	8 m

Figure 4. User Tracking Data Spans

It should be noted from Figure 4 that the velocity uncertainty of the synchronous satellite is the dominant source of systematic error. The results listed in the figure demonstrate the marked improvement in the User orbit when this error source is eliminated.

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APPENDIX A

ERROR ANALYSIS

APPENDIX A

J. J. Lynn

ERROR ANALYSIS

User satellite orbital uncertainties as a function of systematic and random error sources in the observational data were investigated by means of error analysis techniques. This allowed for a considerable increase in the number of cases to be investigated because of the better utilization of computer time afforded by this technique as compared to the data simulation/reduction technique described in Section 4. When utilized in the error analysis mode of operation NAP-3 provides the user with the so called "consider mode" option. Using this option it is possible to obtain estimates of uncertainties associated with any recoverable parameter (such as User Satellite State Vector) as a function of non-recoverable (or consider) parameter uncertainties. For any given tracking geometry only one NAP-3 run is required to produce orbital uncertainties associated with any number and level of systematic and random error sources under consideration. A brief description of the underlying mathematical algorithms associated with this NAP-3 consider mode option follows.

Consider a linear observation model given by

$$Y = AX + BS + N \qquad \text{(A-1)}$$

where Y is an n x 1 vector of observations, X is a p x 1 vector of parameters to be estimated, S is a q x 1 vector of systematic error parameters and N is the vector associated with random observational noise. The matrices A (n x p) and B (n x q) are assumed to be known. Consider now a linear estimator denoted by L which operates on Y to provide an estimate $\hat{X}$ of X, namely $\hat{X} = LY$. Now making use of A-1 yields

$$\hat{X} = LAX + LBS + LN \qquad \text{(A-2)}$$

further restricting ourselves to those cases where L is an unbiased linear estimator so that LA = I, reduces the above expression to

$$\hat{X} = X + LBS + LN \qquad \text{(A-3)}$$

Now the vector S may be regarded as some constant but unknown vector of systematic errors. Our best estimate for this vector is $\hat{S} = 0$, and our confidence in this estimate is provided by some covariance matrix denoted by Σ_s. Mathematically this is equivalent to treating the vector S as a random vector with zero mean and covariance Σ_s.

Let us now evaluate the covariance associated with our estimate $\hat{X}$. From A-3 the expected value of $\hat{X}$, namely $E(\hat{X})$ is given by,

$$E(\hat{X}) = X + LBE(S) + LE(N) = X \qquad \text{(A-4)}$$

which is of course a consequence of the constraint of unbiasedness placed on the linear estimator L and the fact that $E(S) = E(N) = 0$. The covariance associated with this estimate is then given by

$$\Sigma_{\hat{X}} = E[(\hat{X} - X)(\hat{X} - X)^T] = LB\,\Sigma_S\,B^T L^T + L\,\Sigma_N\,L^T \qquad \text{(A-5)}$$

where $\Sigma_N = E(NN^T)$ and we have assumed that the random and systematic errors are independent e.g., $E(SN^T) = 0$.

Let L be selected to correspond to the Weighted Least Squares linear estimator given by,

$$L = (A^T WA)^{-1} A^T W \qquad \text{(A-6)}$$

with W representing the n x n weighting matrix. Substituting the expression for L given in A-6 into A-5 yields the following expression for the covariance of $\hat{X}$,

$$\begin{aligned} \Sigma_{\hat{X}} &= (A^T WA)^{-1} A^T W \Sigma_N WA (A^T WA)^{-1} + \\ &\quad (A^T WA)^{-1} A^T WB \Sigma_S B^T WA (A^T WA)^{-1} \end{aligned} \qquad \text{(A-7)}$$

Futhermore, if the weighting matrix W is chosen to be the inverse of the noise covariance matrix, e.g., $W = \Sigma_N^{-1}$, then the above expression reduces to

$$\Sigma_{\hat{X}} = (A^T WA)^{-1} + (A^T WA)^{-1} A^T WB \Sigma_S B^T WA (A^T WA)^{-1} \qquad \text{(A-8)}$$

The first term on the right-hand side of A-8 represents the contribution of the random observation noise to the uncertainty associated with our estimated vector $\hat{X}$, and the second term represents the contribution of the systematic error uncertainty Σ_S to our estimated vector uncertainty.

The positive definite matrix Σ_S may be expressed as a product of a matrix σ_S by its transpose σ_S^T. Let us now define a so called alias matrix AL as,

$$AL = (A^T WA)^{-1} A^T WB \sigma_S$$

Then A-8 may be rewritten as,

$$\Sigma_{\hat{X}} = (A^T WA)^{-1} + (AL)(AL)^T$$

For the most usual cases the matrix σ_S is a diagonal matrix whose elements represent the standard deviations associated with considered parameter errors. In such cases the concept of the alias matrix is very convenient because it gives the effect of an error in each considered parameter on the estimated parameters.

Consider now that the parameter vector X evaluated at any time t is related to its value at time t_0 and to the vector S at t_0 by the linear transformation given by

$$X(t) = V_1(t, t_0)\, X(t_0) + V_2(t, t_0)\, S(t_0) \tag{A-9}$$

Let $\hat{X}(t_0)$ represent the corresponding linear unbiased estimate for $X(t_0)$. Then the uncertainty associated with this estimate is given by means of A-5. Let it now be desired to evaluate corresponding uncertainty associated with the estimate of X at t or $\hat{X}(t)$. The relationship between $\hat{X}(t)$ and $\hat{X}(t_0)$ is given by

$$\hat{X}(t) = V_1(t, t_0)\, \hat{X}(t_0) + V_2(t, t_0)\, \hat{S}(t_0) \tag{A-10}$$

Then the covariance associated with $\hat{X}(t)$, and which will be denoted by $\Sigma_{\hat{X}_1}$ is given by

$$\begin{aligned} \Sigma_{\hat{X}_1} &= E\left\{(\hat{X}(t) - X(t))(\hat{X}(t) - X(t))^T\right\} \\ &= V_1\, \Sigma_{\hat{X}}\, V_1^T + V_2\, \Sigma_S\, V_2^T + V_1\, \Sigma_{\hat{X},S}\, V_2^T + V_2\, \Sigma_{S,\hat{X}}\, V_1^T \end{aligned} \tag{A-11}$$

where for the sake of brevity we have denoted the linear operators V_1 and V_2 without their agreements t and t_0. The cross covariance matrix $\Sigma_{\hat{X}S}$ is simply the expected value $E\left\{(\hat{X}(t_0) - X(t_0))\, S^T(t_0)\right\}$ which with the aid of A-3 may be expressed as,

$$\Sigma_{\hat{X},S} = -LB\, \Sigma_S \tag{A-12}$$

and similarly

$$\Sigma_{S,\hat{X}} = \Sigma_S\, B^T L^T \tag{A-13}$$

Finally, substituting A-5, A-12 and A-13 into A-11 yields the following expression for the covariance associated with the estimate $\hat{X}(t)$,

$$\begin{aligned} \Sigma_{\hat{X}_1} = {} & V_1\, LB\, \Sigma_S\, B^T L^T\, V_1^T + V_2\, \Sigma_S\, V_S^T - V_1 LB\, \Sigma_S\, V_2^T - V_2\, \Sigma_S\, B^T L^T V_1^T \\ & + V_1 L\, \Sigma_N\, L^T\, V_1^T \end{aligned} \tag{A-14}$$

where L is given by means of A-6. Alternately the above may also be expressed in terms of the alias matrix

$$\Sigma_{\hat{X}_1} = (V_1 \ (AL) - V_2 * \sigma_S)(V_1 \ (AL) - V_2 * \sigma_S)^T \qquad (A-15)$$

$$+ V_1 (A^T WA)^{-1} V_1^T$$

where the expression (V_1 $AL - V_2 * \sigma_S$) is defined as the propagated alias matrix.

For these cases where the systematic error vector S does not contain dynamic parameters, the value of X at any time t does not depend on S but only on the value of X (t_0). In this case $V_2 \equiv 0$ and A-14 reduces to

$$\Sigma_{\hat{X}_1} = V_1 LB \Sigma_S B^T L^T V_1^T + V_1 L \Sigma_N L^T V_1^T \qquad (A-16)$$

The above simplification will for example be valid for those cases where the vector S represents systematic observation biases.

The above represents the fundamental error analysis consider option algorithm utilized in NAP. The actual observation model in NAP is however non-linear. In order to utilize the linear model the actual non-linear model is replaced by its linearized equivalent by expanding the observational equations in a Taylor series about some nominal or reference trajectory and retaining terms through first order only. The linear observation model given in A-1 still holds except that the vector Y is the vector of observational residuals, and the vector X represents the discrepancy vector defined as the difference between the parameter vector to be estimated and its a priori nominal value used in the linearization process. The matrices A and B represent the matrices of partial derivatives of the observation vector with respect to the parameter vectors X and S evaluated about the nominal a priori value for X. Thus if the non-linear observation model is given by

$$Y = F(X, S) + N \qquad (A-17)$$

Let X^o and S^o be the nominal values for X and S respectively. Then to a first order

$$Y - F(X^o, S^o) = F_X(X^o, S^o)(X - X_0) + F_S(X^o, S^o)(S - S_0^o) + N \qquad (A-18)$$

where F_X and F_S are partial derivatives of the vector valued function F with respect to X and S respectively. Equation A-18 is thus the linear equivalent of A-1 and our previous analysis for the uncertainty associated with our estimate $\hat{X}$ still holds true.

In our error analysis studies we considered the following systematic error sources

a. Two way range bias

b. One way range rate bias

c. Ground station coordinate offsets

d. TDRS position and velocity at epoch

e. Gravity parameter (GM) bias

A summary of the resulting error analysis exercizes is provided in the following tables. The entries under the column heading "Estimated Parameters" corresponds to our previously defined vector $\hat{X}$. The entry USER refers to the User satellite state vector and the entry labled TDRS refers to the TDRS state vector at epoch, and the entries labled STN, RB, and RDB refer to ground station coordinate offsets, range bias, and rate rate bias parameters respectively. Under the column heading "Systematic Error Sources" are listed the "consider" option parameters corresponding to our vector S. The numbers in parentheses appearing next to each such column entry represent the standard deviation associated with the corresponding element of the S vector. We have thus taken the covariance matrix Σ_S to be a diagonal matrix whose elements are the squares of these listed standard deviations. The resulting orbital uncertainties are listed in the last three columns of the tables. The first of these lists the User orbit position error at epoch time to which was taken at $O^h - O^m - O^s$. The last two columns give the corresponding minimum and maximum propagated errors (calculated according to A-14) during a 24 hour period starting with t_0. The actual tracking intervals are of course contained within this 24 hour period and are given below. The listed position uncertainties are calculated from the covariance matrix $\Sigma_{\hat{X}}$ or $\Sigma_{\hat{X}_1}$ which is of course associated with the estimated parameter vector X. This position uncertainty is simply the square root of the sum of the three diagonal elements of $\Sigma_{\hat{X}}$ which correspond to the three position coordinate of the User satellite state vector. For example let $X = (x_1\ x_2\ x_3\ x_4 \ldots\ x_p)$ and suppose x_1, x_2, x_3 represent the User satellite state vector position coordinates. Let $\Sigma_{\hat{X}}$, the associated covariance matrix be described by,

$$\Sigma_{\hat{X}} = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1 x_2} & \sigma_{x_1 x_3} & \cdots \\ \sigma_{x_2 x_1} & \sigma_{x_2}^2 & \cdots & \cdots \\ \vdots & \vdots & \sigma_{x_3}^2 & \\ \vdots & \vdots & & \ddots \quad \sigma_{x_p}^2 \end{bmatrix}$$

then, the listed orbit position uncertainty is evaluated as

$$(\sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2)^{1/2}.$$

Table A lists the results for the case where the tracking geometry is as depicted by Pass #1 in Figure A1. Table B lists the results for the case where the tracking geometry comprises the 4 data passes depicted in Figure A1. These passes cover the following times:

pass number	start time	stop time
1	2h - 17m	3h - 28m
2	4h - 08m	5h - 18m
3	14h - 25m	15h - 34m
4	16h - 16m	17h - 24m

Table C lists the corresponding results for the case where 8 data spans, each of 5 minutes in duration, are utilized. These are depicted in Figure A2.

The passes corresponding to Table C span the following times:

1. 2h - 17m to 2h - 22m
2. 3h - 23m to 3h - 28m
3. 4h - 08m to 4h - 13m
4. 5h - 13m to 5h - 18m
5. 14h - 25m to 14h - 30m
6. 15h - 29m to 15h - 34m
7. 16h - 16m to 16h - 21m
8. 17h - 19m to 17h - 24m

and the tracking data is taken at a rate of 6 data points per minute.

Table D lists the results for the cases where the tracking data comprise of 4 passes each of 5 minutes in duration as depicted in Figure A3. These

Table A

Case No.	Estimated Parameters	Systematic Error Sources	Resulting User Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
A.1	USER	None	19	5	101
A.2	USER, TDRS	None	1381	330	5686
A.3	USER, TDRS, STN	None	1381	333	5687
A.4	USER, TDRS, STN, RB, RDB	None	1382	333	5695
A.5	USER	RB(16.8m)	864	360	3920
A.6	USER	RDB(0.2cm/sec)	74	18	342
A.7	USER	RB(12m)	617	252	2800
A.8	USER	RB(12m), RDB(0.2cm/sec)	621	253	2820
A.9	USER	STN(5m)	258	105	1170
A.10	USER	STN(5m), RB(16.8), RDB(0.2cm/s)	904	369	4102
A.11	USER	Same as A.10+TDRS(50m, 5cm/s)	68330	27153	310052
A.12	USER	GM(1 part/10^6)	50m	—	—
A.13	USER	TDRS (1m)	155	63	709
A.14	USER	TDRS (10m)	1546	624	7015
A.15	USER	TDRS (0.1cm/sec)	1357	540	6161
A.16	USER	TDRS Z (100m)	33	13	127
A.17	USER, TDRS	RB(16.8), RDB(0.2cm/s)	1392	507	5728
A.18	USER, TDRS	RB(12), RDB(0.2cm/s)	1392	504	5751
A.19	USER, TDRS	RDB(0.2cm/s)	1391	505	5720

NOTE: Tracking interval for this case is from 2H - 17M to 3H - 28M corresponding to pass #1 in Figure A1. Data rate is 6 points/minute.
Range uncertainty = 3.3 m
Range Rate uncertainty = 0.12 cm/sec

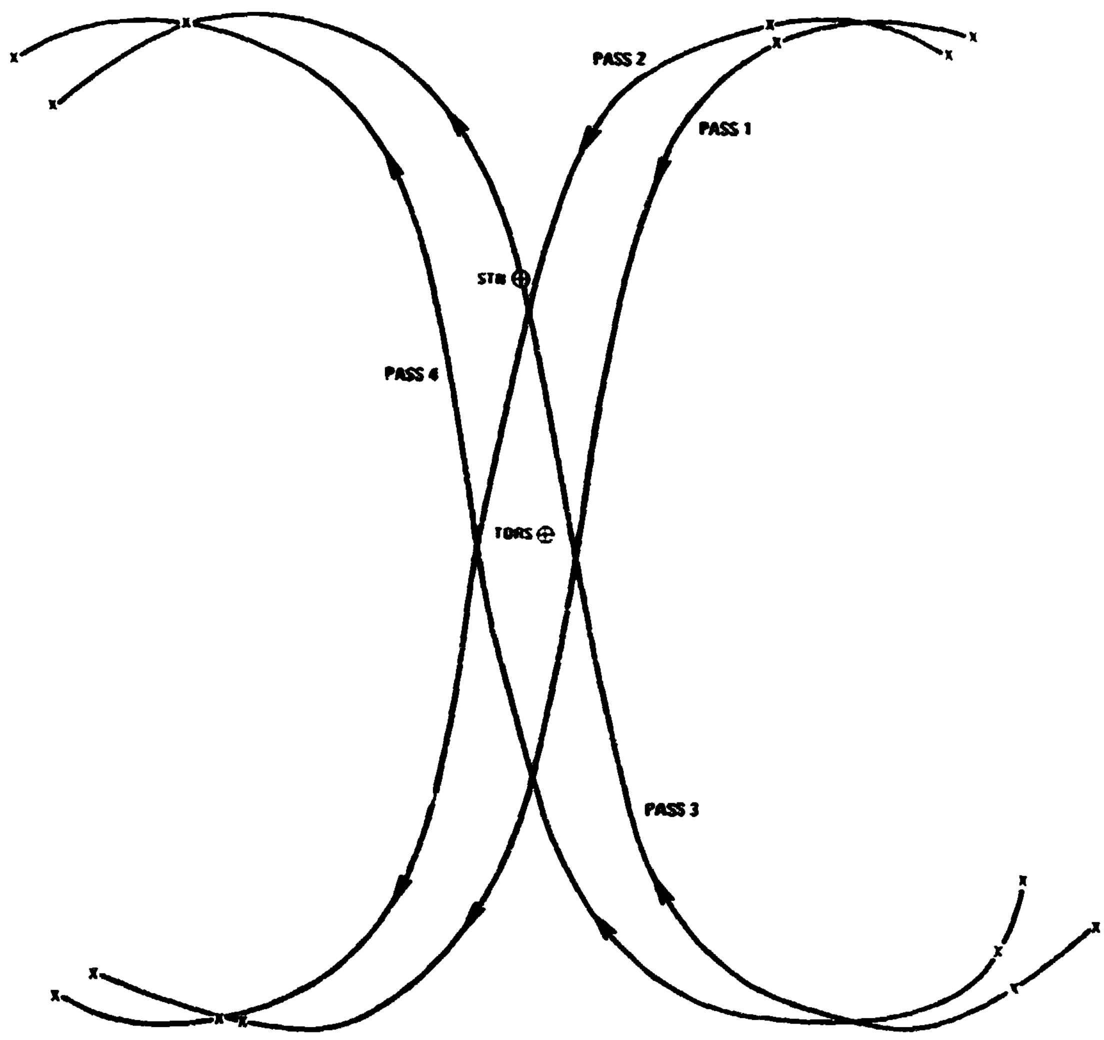

Figure A1. User Satellite Ground Track with Solid Lines Indicating the Data Tracking Intervals.

Table B

Case No.	Estimated Parameters	Systematic Error Sources	Resulting User Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
B. 1	USER	None	0. 24	0. 12	0. 27
B. 2	USER, TDRS	None	6. 5	1. 8	7. 0
B. 3	USER, TDRS, STN	None	42. 0	11. 0	43. 0
B. 4	USER, TDRS, STN, RB, RDB	None	110. 0	30. 0	115. 0
B. 5	USER	RB(16. 8m)	3. 6	0. 6	5. 0
B. 6	USER	RDB(0. 2cm/sec)	3. 0	1. 5	3. 0
B. 7	USER	RB(12m)	2. 6	0. 5	4. 0
B. 8	USER	RB(12m), RDB(0. 2cm/sec)	4. 0	2. 0	4. 0
B. 9	USER	STN(5m)	1. 0	1. 0	2. 0
B. 10	USER	TDRS(pos. 1m)	2. 2	1. 0	3. 0
B. 11	USER	TDRS velocity (0. 1cm/sec)	36. 0	11. 0	57. 0
B. 12	USER	TDRS Z coord (100m)	11. 0	7. 0	17. 0
B. 13	USER, TDRS	RB(16. 8), RDB(0. 2cm/sec)	145. 0	42. 0	150. 0
B. 14	USER, TDRS	RB(12m), RDB(0. 2cm/sec)	104. 0	31. 0	110. 0
B. 15	USER, TDRS	RDB(0. 2cm/sec)	16. 0	8. 0	18. 0

NOTE: The assumed values for random observational noise are 3. 3 meters for range and 0. 12cm/sec for range rate. Data rate is at 6 points/minute. The tracking data includes a total of 1672 points for range and 1672 points for range rate. The tracking geometry is depicted in Figure 1.

Table C

Case No.	Estimated Parameters	Systematic Error Sources	Resulting Orbit Position Uncertainties during 24 hr Period		
			At Epoch	Min.	Max.
C.1	USER	None	0.8	0.5	1.0
C.2	USER, TDRS	None	16.0	5.0	17.0
C.3	USER, TDRS, STN	None	44.0	14.0	47.0
C.4	USER, TDRS, STN, RB, RDB	None	114.0	31.0	117.0
C.5	USER	RB(16.8m)	4.3	4.0	20.0
C.6	USER	RBD(0.2cm/sec)	3.6	2.5	3.6
C.7	USER	RB(12m)	3.0	3.0	14.0
C.8	USER	RB(12m), RDB(0.2cm/sec)	4.7	4.0	14.0
C.9	USER	STN(5m)	1.5	1.0	6.0
C.10	USER	TDRS(pos. 1m)	2.6	1.0	5.0
C.11	USER	TDRS velocity (0.1cm/sec)	43.0	15.0	87.0
C.12	USER	TDRS Z coord (100m)	12.0	5.0	13.0
C.13	USER, TDRS	RB(16.8), RDB(0.2cm/sec)	150.0	42.0	155.0
C.14	USER, TDRS	RB(12m), RDB(0.2cm/sec)	111.0	30.0	114.0
C.15	USER, TDRS	RDB(0.2cm/sec)	42.0	7.0	44.0

NOTE: The assumed values for random observational noise are 3.3m and 0.12cm/sec for range and range rate respectively. Data rate is at 6 points/minute. The tracking data includes a total of 248 points for range and 248 points for range rate.

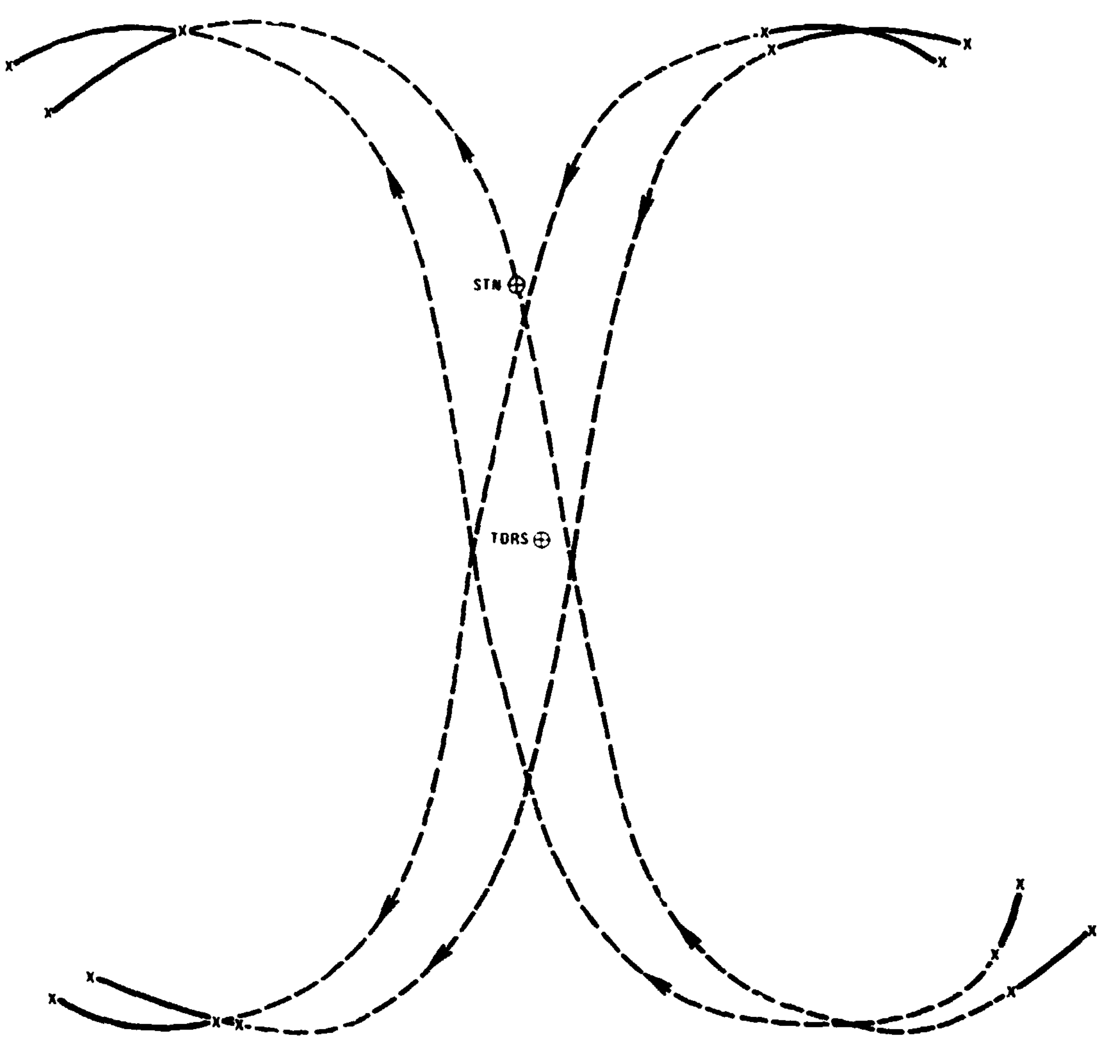

Figure A2. User Satellite Ground Track with Solid Lines Indicating the Data Tracking Intervals.

Table D

Case No.	Estimated Parameters	Systematic Error Sources	Resulting Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
D.1	USER	None	5	2	5
D.2	USER, TDRS	None	186	130	198
D.3	USER, TDRS, STN	None	186	130	198
D.4	USER, TDRS, STN, RB, RDB	None	187	130	199
D.5	USER	RB(16.8m)	104	31	114
D.6					
D.7	USER	RB(12m)	75	22	82
D.8					
D.9	USER	STN(5m)	31	9	33
D.10	USER	TDRS(pos. 1m)	22	6	24
D.11	USER	TDRS velocity (0.1cm/sec)	281	53	290
D.12	USER	TDRS Z coord (100m)	25	15	42
D.13					
D.14					
D.15					

NOTE: The above results are based on range data only. The assumed value for random observational noise is 3.3 m. Data rate is at 6/minute. A total of 248 range data points are included in the tracking data.

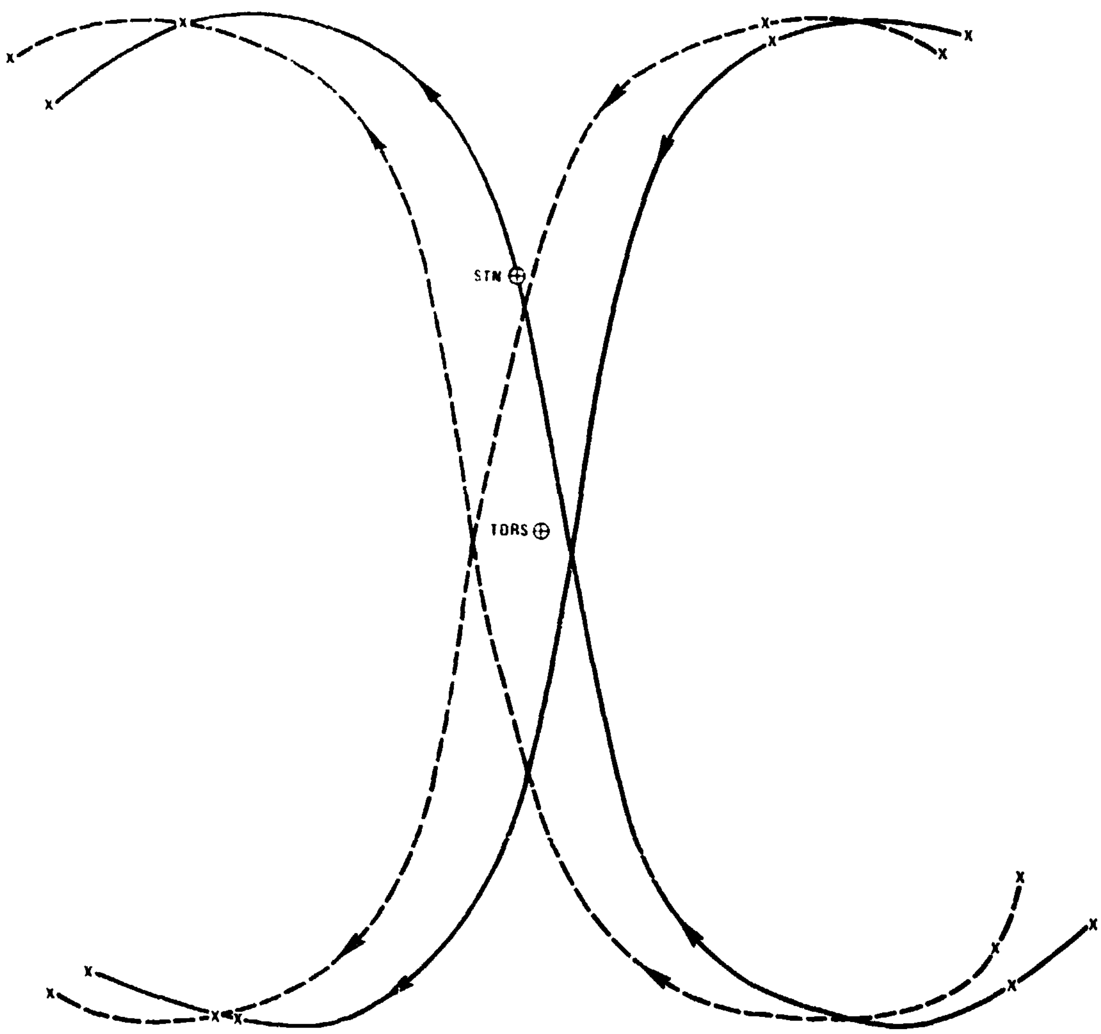

Figure A3. User Satellite Ground Track with Solid Lines Indicating the Data Tracking Intervals.

passes span the following times:

1. 2h - 17m to 2h - 22m
2. 3h - 23m to 3h - 28m
3. 14h - 25m to 14h - 30m
4. 15h - 29m to 15h - 34m

Tables E through H lists the results for various cases corresponding to the tracking geometry depicted in Figure A4, where a total of 20 data spans each of 1 minute in duration and with 5 such 1 minute spans per revolution are considered.

The times spanned by these passes are:

1. 2h - 30m to 2h - 31m
2. 2h - 40m to 2h - 41m
3. 2h - 50m to 2h - 51m
4. 3h - 0m to 3h - 01m
5. 3h - 10m to 3h - 11m
6. 4h - 20m to 4h - 21m
7. 4h - 30m to 4h - 31m
8. 4h - 40m to 4h - 41m
9. 4h - 50m to 4h - 51m
10. 5h - 0m to 5h - 01m
11. 14h - 40m to 14h - 41m
12. 14h - 50m to 14h - 51m
13. 15h - 0m to 15h - 01m

Table E

Case No.	Estimated Parameters	Systematic Error Sources	Resulting Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
E.1	USER	None	1.5	1	2.5
E.2	USER, TDRS	None	68	23	73
E.3	USER, TDRS, STN	None	77	26	82
E.4	USER, TDRS, STN, RB, RDB	None	123	50	130
E.5	USER	RB(16.8m)	59	30	138
E.6	USER	RDB(0.2cm/sec)	4	2	4
E.7	USER	RB(12m)	42	26	98
E.8	USER	RB(12m), RDB(0.2cm/sec)	42	26	98
E.9	USER	STN(5m)	17	12	41
E.10	USER	TDRS(pos. 1m)	14	6	30
E.11	USER	TDRS velocity (0.1cm/sec)	199	96	395
E.12	USER	TDRS Z coord (100m)	20	4	20
E.13	USER, TDRS	RB(16.8), RDB(0.2cm/sec)	147	60	158
E.14	USER, TDRS	RB(12m), RDB(0.2cm/sec)	119	55	130
E.15	USER, TDRS	RDB(0.2cm/sec)	80	41	96

NOTE: The assumed values for random observational noise are 3.3m and 0.12cm/sec for range and range rate respectively. Data rate is at 6 points/minute. The tracking data includes a total of 124 points for range and 124 points for range rate.

Table F

Case No.	Estimated Parameters	Systematic Error Sources	Resulting Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
F.1	USER	None	0.85	0.5	1.0
F.2	USER, TDRS	None	26.0	7.0	28.0
F.3	USER, TDRS, STN	None	48.0	13.0	50.0
F.4	USER, TDRS, STN, RB, RDB	None	115.0	38.0	120.0
F.5	USER	RB(16.8m)	4.8	1.0	6.0
F.6	USER	RDB(0.2cm/sec)	2.8	2.0	3.0
F.7	USER	RB(12m)	3.4	1.0	4.0
F.8	USER	RB(12m), RDB(0.2cm/sec)	4.3	2.0	5.0
F.9	USER	STN(5m)	1.6	0.6	2.0
F.10	USER	STN(5m), RB(16.8), RDB(0.2cm/s)	5.7	3.0	6.0
F.11	USER	Same as F.10+TDRS(50m, 5cm/s)	1723.0	650.0	2357.0
F.12	USER	GM(1 part/10^7)	3.5	—	—
F.13	USER	TDRS (1m)	2.5	1.0	3.0
F.14	USER	TDRS (10m)	24.0	8.0	27.0
F.15	USER	TDRS (0.1cm/sec)	34.0	13.0	47.0
F.16	USER	TDRS Z (100m)	9.5	7.0	16.0
F.17	USER, TDRS	RB(16.8), RDB(0.2cm/s)	143.0	40.0	150.0
F.18	USER, TDRS	RB(12), RDB(0.2cm/s)	104.0	29.0	110.0
F.19	USER, TDRS	RDB(0.2cm/s)	26.0	10.0	27.0
F.20	USER, TDRS	GM(1 part/10^7)	72.0	—	—
F.21	USER, TDRS	GM, RB(16.8), RDB(0.2), STN(5)	164.0	—	—

NOTE: 5 Minutes of tracking on each rev. for a total of 4 revs. (Figure 4). Each tracking interval duration is 1 min maximum. Data rate is 6/min. A total of 14 OR, R, data points.
Range noise 3.3 m
Range rate noise 0.0012m/sec.

Table G

Case No.	Estimated Parameters	Systematic Error Sources	Resulting Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
G.1	USER	None	2.9	2.0	3.0
G.2	USER, TDRS	None	87.0	25.0	90.0
G.3	USER, TDRS, STN	None	93.0	26.0	96.0
G.4	USER, TDRS, STN, RB, RDB	None	127.0	36.0	130.0
G.5	USER	RB(16.8m)	3.0	2.0	3.5
G.6	USER	RDB(0.2cm/sec)	4.1	3.0	4.3
G.7	USER	RB(12m)	3.0	2.0	3.3
G.8	USER	RB(12m), RDB(0.2cm/sec)	4.2	3.0	4.5
G.9	USER	STN(5m)	2.9	2.0	3.3
G.10	USER	STN(5m), RB(16.8), RDB(0.2cm/s)	4.3	3.3	4.4
G.11	USER	Same as G.10+TDRS(50m, 5cm/s)	1428.0	245.0	2130.0
G.12	USER	GM(1 part/10^7)	4.5	—	—
G.13	USER	TDRS (1m)	3.3	2.0	4.0
G.14	USER	TDRS (10m)	17.4	5.0	23.0
G.15	USER	TDRS (0.1cm/sec)	28.6	7.0	43.0
G.16	USER	TDRS Z (100m)	10.0	8.0	15.0
G.17	USER, TDRS	RB(16.8), RDB(0.2cm/s)	141.0	40.0	146.0
G.18	USER, TDRS	RB(12), RDB(0.2cm/s)	118.0	35.0	122.0
G.19	USER, TDRS	RDB(0.2cm/s)	87.0	26.0	90.0
G.20	USER, TDRS	GM(1 part/10^7)	100.0	—	—
G.21	USER, TDRS	GM, RB(16.8), RDB(0.2), STN(5)	152.00	—	—

NOTE: 5 Minutes of total tracking per revolution for a total of 4 revolutions (Figure 4). The duration of each continuous tracking interval is 1 minute and the data rate is 6/minute.
Range noise - 24.2 meter
Range rate noise - 0.39cm/sec.

Table H

Case No.	Estimated Parameters	Systematic Error Sources	Resulting Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
H.1	USER	None	1.5	1.0	1.8
H.2	USER, TDRS	None	44.0	12.0	47.0
H.3	USER, TDRS, STN	None	60.0	17.0	62.0
H.4	USER, TDRS, STN, RB, RDB	None	118.0	38.0	122.0
H.5	USER	RB(16.8m)	9.5	2.0	10.0
H.6	USER	RDB(0.2cm/sec)	2.7	2.0	3.0
H.7	USER	RB(12m)	6.9	1.5	8.0
H.8	USER	RB(12m), RDB(0.2cm/sec)	7.0	2.0	8.0
H.9	USER	STN(5m)	3.0	1.0	3.0
H.10	USER	STN(5m), RB(16.8), RDB(0.2cm/s)	10.0	3.0	12.0
H.11	USER	Same as H.10+TDRS(50m, 5cm/s)	2258.0	1000.0	2800.0
H.12	USER	GM(1 part/10^7)	4.0	—	—
H.13	USER	TDRS (1m)	3.6	2.0	4.0
H.14	USER	TDRS (10m)	33.0	12.0	36.0
H.15	USER	TDRS (0.1cm/sec)	45.0	20.0	50.0
H.16	USER	TDRS Z (100m)	9.0	8.0	16.0
H.17	USER, TDRS	RB(16.8), RDB(0.2cm/s)	142.0	40.0	150.0
H.18	USER, TDRS	RB(12), RDB(0.2cm/s)	106.0	30.0	110.0
H.19	USER, TDRS	RDB(0.2cm/s)	45.0	14.0	47.0
H.20	USER, TDRS	GM(1 part/10^7)	82.0	—	—
H.21	USER, TDRS	GM, RB(16.8), RDB(0.2), STN(5)	163.0	—	—

Range noise - 3.9 meters
Range rate noise - 0.22cm/sec

See Figure 4 for tracking geometry

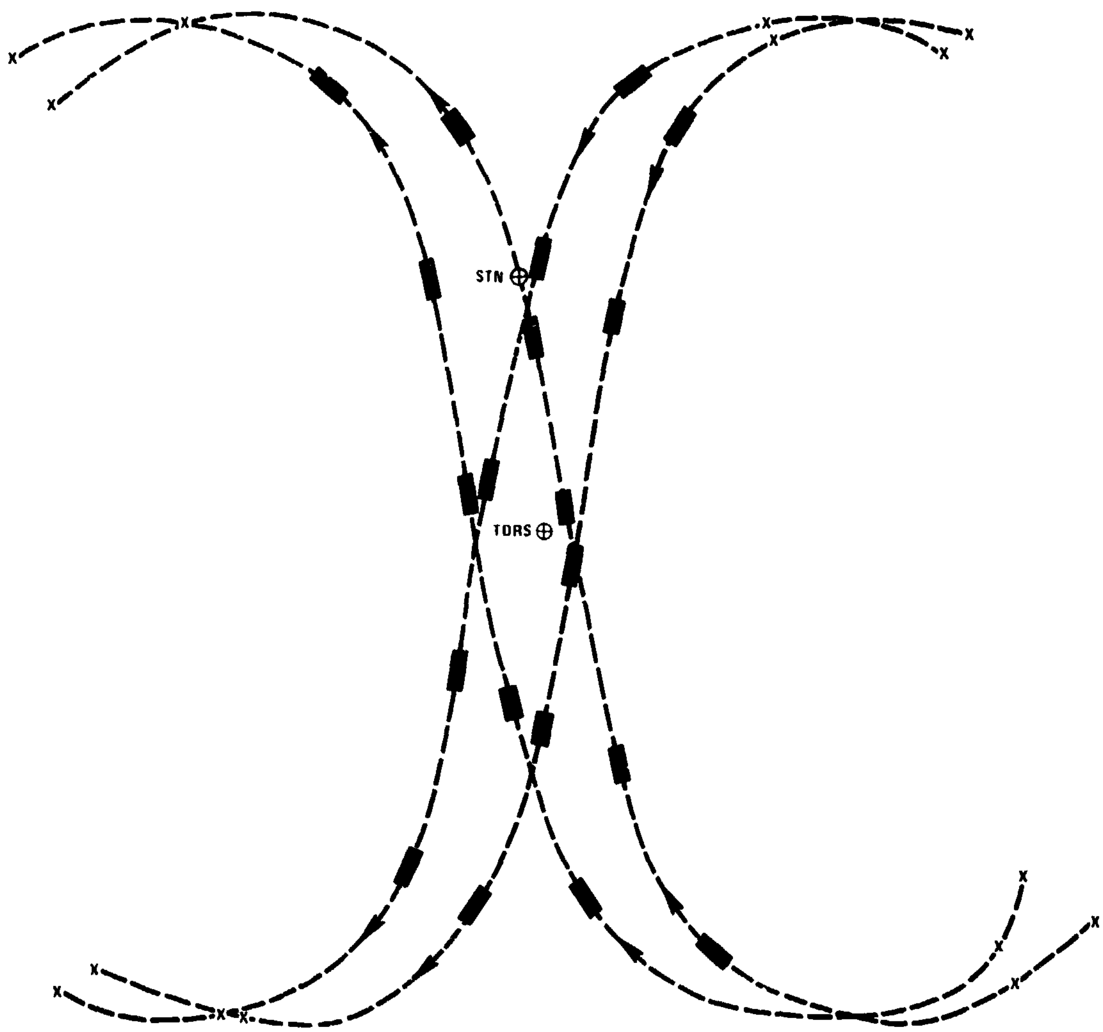

Figure A4. User Satellite Ground Track with Solid Lines Indicating the Data Intervals

14. 15 h - 10 m to 15 h - 11 m

15. 15 h - 20 m to 15 h - 21 m

16. 16 h - 35 m to 16 h - 36 m

17. 16 h - 45 m to 16 h - 46 m

18. 16 h - 55 m to 16 h - 56 m

19. 17 h - 05 m to 17 h - 06 m

20. 17 h - 15 m to 17 h - 16 m

Table I

Case No.	Estimated Parameters	Systematic Error Sources	Resulting Orbit Position Uncertainties During 24 hr Period		
			At Epoch	Min.	Max.
I.1	USER	None	2.0	1.0	2.5
I.2	USER, TDRS	None	61.0	17.0	64.0
I.3	USER, TDRS, STN	None	72.0	20.0	74.0
I.4	USER, TDRS, STN, RB, RDB	None	121.0	33.0	125.0
I.5	USER	RB(16.8m)	3.0	1.5	3.0
I.6	USER	RDB(0.2cm/sec)	3.5	2.5	4.0
I.7	USER	RB(12m)	2.5	1.5	3.5
I.8	USER	RB(12m), RDB(0.2cm/sec)	3.8	2.5	4.0
I.9	USER	STN(5m)	2.1	1.4	2.5
I.10	USER	STN(5m), RB(16.8), RDB(0.2cm/s)	4.1	3.0	4.2
I.11	USER	Same as I.10+TDRS(50m, 5cm/s)	1487.0	411.0	2106.6
I.12	USER	GM(1 part/10^7)	4.0	—	—
I.13	USER	TDRS (1m)	2.8	1.8	3.0
I.14	USER	TDRS (10m)	19.0	5.0	24.0
I.15	USER	TDRS (0.1cm/sec)	30.0	7.0	10.0
I.16	USER	TDRS Z (100m)	10.0	8.0	18.0
I.17	USER, TDRS	RB(16.8), RDB(0.2cm/s)	142.0	40.0	145.0
I.18	USER, TDRS	RB(12), RDB(0.2cm/s)	110.0	31.0	114.0
I.19	USER, TDRS	RDB(0.2cm/s)	61.0	21.0	64.0
I.20	USER, TDRS	GM(1 part/10^7)	85.0	—	—
I.21	USER, TDRS	GM, RB(16.8), RDB(0.2), STN(5)	157.0	—	—

Range noise - 12.4m
Range rate noise - 0.28cm/s

See Figure A4 for tracking geometry