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ADHESIVE-BONDED SCARF AND STEPPED-LAP JOINTS

TECHNICAL REPORT

by

L. J. HART-SMITH

Prepared under Contract NAS1-11234
Douglas Aircraft Company
McDonnell Douglas Corporation
3855 Lakewood Blvd
Long Beach, California 90846

January 1973

for

Langley Research Center
Hampton, Virginia 23366

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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ABSTRACT

Continuum mechanics solutions are derived for the static load-carrying capacity of scarf and stepped-lap adhesive-bonded joints. The analyses account for adhesive plasticity and adherend stiffness imbalance and thermal mismatch. The scarf joint solutions include a simple algebraic formula which serves as a close lower bound, within a small fraction of a per cent of the true answer for most practical geometries and materials. The scarf joint solutions are believed to be the first such results ever obtained for dissimilar adherends. Digital computer programs have been developed and, for the stepped-lap joints, the critical adherend and adhesive stresses are computed for each step. The scarf joint solutions exhibit grossly different behavior from that for double-lap joints for long overlaps inasmuch as that the potential bond shear strength continues to increase with indefinitely long overlaps on the scarf joints. The stepped-lap joint solutions exhibit some characteristics of both the scarf and double-lap joints. The stepped-lap computer program handles arbitrary (different) step lengths and thicknesses and the solutions obtained have clarified potentially weak design details and the remedies. Indeed, the program has been used effectively to optimize the joint proportions.

KEYWORD DESCRIPTORS

Bonded Joints
Adhesive Stresses and Strains
Adherend Stiffness Imbalance
Adherend Thermal Mismatch
Computer Analysis Programs
Scarf Joints
Stepped-Lap Joints
Static Strength
Elastic-Plastic Formulation
Advanced Composite Joints

FOREWORD

This report was prepared by the Douglas Aircraft Company, McDonnell Douglas Corporation, Long Beach, California under the terms of Contract NAS1-11234. One summary report (NASA CR 2218) and four technical reports (NASA CR 112235, -6, -7, and -8) cover the work, which was performed between November 1971 and January 1973. The program was sponsored by the National Aeronautics and Space Administration's Langley Research Center, Hampton, Virginia. Dr. M. F. Card and Mr. H. G. Bush were the Contracting Agency's Technical Monitors.

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ILLUSTRATIONS

SYMBOLS

$A_0,..A_n$	=	Coefficients of power series for shear stress distribution in adhesive layer
a,c	=	Extents of plastic stress state in adhesive at ends of bonded joint (in.)
b	=	Extent of elastic trough in adhesive (in.)
C,D	=	Integration constants
CTHERM	=	Non-dimensionalized adherend thermal mismatch coefficient
d	=	Length of elastic zone in adhesive bond (in.)
E	=	Young's modulus (longitudinal) for adherend (psi)
ETR	=	Adherend extensional stiffness ratio
$F_{..}$	=	Adherend allowable (or ultimate) stress (psi)
G	=	Adhesive shear modulus for elastic-plastic representation (psi)
ℓ	=	Overlap (length of bond) (in.)
P	=	Applied direct load on entire joint (lb in. / in.)
SGNLD	=	Distinguisher between tensile and compressive shear loads
T	=	Direct stress resultants in adherends (lb / in.)
ΔT	=	Temperature change ($T_{operating}$ - T_{cure})
t	=	Thickness of adherend (in.)
x	=	Axial (longitudinal) coordinate parallel to direction of load
α	=	Coefficient of thermal expansion (/°F)
γ	=	Adhesive shear strain
γ_e	=	Elastic adhesive shear strain
γ_p	=	Plastic adhesive shear strain
δ	=	Axial (longitudinal) displacement of adherend (in.)
ζ,ξ,χ	=	Non-dimensionalized axial coordinates (different origin and/or sense from x)

η = Thickness of adhesive layer (in.)

θ = Scarf angle (small) (°)

λ = Exponent of elastic shear stress distribution (in.$^{-1}$)

ν = Poisson's ratio for adherend(s)

τ = Adhesive shear stress (psi)

τ_{av} = Average adhesive shear stress (psi)

τ_p = Plastic (maximum) adhesive shear stress (psi)

ϕ = x/ℓ = Non-dimensionalized coordinate

SUBSCRIPTS

a,c = Adhesive (cement)

e,p = Elastic and plastic values

i,o = Inner and outer adherends of symmetric bonded joint

1,2 = Different adherends at each end of joint

1,2,..n = Power series counter

SUMMARY

It has long been known that bonded scarf joints have a higher efficiency than uniform lap joints and that the latter are limited in strength and unsuitable for joining thicker sections. What has not been well understood until recently is that, in the bonding together of dissimilar adherends in a scarf joint, any adherend stiffness imbalance or thermal mismatch imposes a limitation on the joint efficiency. As a consequence the adhesive layer is not (essentially) uniformly stressed along its length as it is for a scarf joint between identical adherends. One objective of this report is to analyze and quantify these limitations on efficiency of unbalanced scarf joints. In doing so, adhesive plasticity is accounted for by the Douglas elastic-plastic model which has been demonstrated to be effective for uniform lap joints. One dominant characteristic deduced for scarf joints is that for long overlaps, regardless of any adhesive ductility and/or adherend thermal mismatch, the ratio of the average adhesive shear stress to the peak adhesive shear stress is equal to the lower ratio (<1) of the adherend extensional stiffnesses. The governing differential equations do not possess an explicit solution in terms of standard functions, so a series solution was employed. Even so, an algebraic expression was deduced for a lower bound which proved to be so close to the more precise solutions that it could be employed directly for practically all realistic joint proportions. Severe adverse effects of adherend thermal mismatch are confined to a specific overlap range. The effects decrease asymptotically to zero for very short or very long overlaps.

Stepped-lap joints represent a cross between scarf joints and uniform lap joints. The stepped-lap joint overcomes the upper limit on joint strength of uniform lap joints but retains the severe adhesive strain concentration at the end of each step. One advantage of stepped-lap joints over scarf joints is that the alignment and fit is far less critical when there are joints on more than a single interface. Another is that it is more suitable for boron-epoxy laminates than is a scarf joint because of the thick brittle filaments. This is particularly important for the titanium edge members frequently used in conjunction with boron-epoxy panels. Because the graphite fibers are so much thinner and more flexible than boron filaments, the former can take advantage of the higher efficiency of the scarf joint.

Digital computer FORTRAN IV programs are included for the iterative solutions necessary for these problems. The scarf joint solutions are in terms of non-dimensionalized parameters. The stepped-lap joint program is dimensional and permits each step to be varied independently so as to be able to identify and improve the most critical detail(s) of the joint. One key factor in the design of stepped-lap joints is that the bond load transfer is concentrated at the end of the joint from which the softer (less stiff) adherend extends. Consequently, it is necessary to restrict the length of the end step of the stiffer adherend to prevent it from being overloaded. Another characteristic of stepped-lap joints identified by the analyses is that the end three steps of the more critical end dominate the internal load distribution and effectively determine the load capacity. The steps at the less critical end are found to have practically no effect on the load capacity.

1. INTRODUCTION

It is generally recognized that, in the bonding together of thick sections, the use of either scarf or stepped-lap joints is mandatory if an acceptable structural efficiency is to be realized. References (1) and (2) explain how, for uniform lap joints, the maximum possible joint efficiency decreases with increasing thickness (extensional stiffness) of the members being bonded together. The objective of this report is to apply the elastic-plastic adhesive analysis techniques developed in References (1) and (2) to the scarf and stepped-lap joints. The approach used remains that of continuum mechanics rather than finite elements. The governing differential equations were relatively straightforward to set up but, in most cases, specific closed-form solutions could not be derived. Severe numerical accuracy problems had to be overcome in developing the FORTRAN IV digital computer programs employed and this phase of the work represented by far the bulk of the investigation. The computer programs are listed in the Appendices and representative non-dimensionalized solutions are illustrated to show the effect of the governing scarf joint parameters. Specific solutions are presented for stepped-lap joints.

This scarf joint analysis is concerned with the non-uniform adhesive shear stresses necessarily associated with the bonding together of dissimilar adherends. It is well-known that the stresses are uniform if the adherends are identical. It has only recently begun to be appreciated that the adhesive shear stresses are markedly non-uniform if the adherends are dissimilar. Indeed, the literature contains very few references to this problem. The mechanism whereby these non-uniform stresses are developed is illustrated in Figure 1 for the case of thermal mismatch between stiffness-balanced adherends. The first publication on scarf joints between dissimilar adherends appears to be that of Lubkin [Reference (3)] who, in 1957 sought the particular scarf angle associated with uniform adhesive stress for a particular ratio of adherend elastic moduli. He omitted consideration of any adherend thermal dissimilarity. Unfortunately the predictions of his equation [10] are such as to indicate the appropriate scarf angle θ is so great (typically in excess of 45 degrees) as to be of no practical interest for bonding aerospace materials together. For realistic adhesives and adherend materials, the scarf angle should be restricted to less than 4 degrees in order for the potential bond

strength to exceed the adherend strength(s). Working independently, in 1971, the present author [Reference (4)] and Erdogan and Ratwani [Reference (5)] demonstrated by calculation the non-uniform adhesive shear stress associated with scarf joints between dissimilar adherends. The former work was based on a perfectly-plastic adhesive analysis, while the latter derived from a linearly-elastic formulation. Consequently neither afforded a complete solution but both demonstrated clearly that the adhesive load transfer is concentrated at that end of the joint from which the softer adherend extends. The present solution utilizes an elastic-plastic adhesive model with linearly elastic adherends and accounts for adherend stiffness and thermal imbalances. Eccentricities in the load path are excluded and, in keeping with common design practice, the scarf angle is considered to be so small that adhesive tension (or compression) stresses may be neglected in comparison with the shear stresses.

In 1968, an elastic finite-element analysis of scarf joints was performed by Richards [see Reference (6)]. Boron/epoxy-to-boron/epoxy and boron/epoxy-to-aluminum joints were analyzed. Thermal effects were neglected. In the former case, relatively small (<4%) stress concentrations were identified in the vicinity of the ends of the scarf. Their existence had not been demonstrated prior to that investigation. In the latter case a markedly non-uniform stress distribution was deduced, with significantly more load being transferred to and from the 0° plies in the laminate than occurred with the ± 45° plies. This is to be expected in view of the much lower modulus of the cross plies.

While the mathematical complexity of equations governing the scarf joint has restricted the number of solutions obtained, a number of investigations of the stepped-lap adhesive-bonded joint have been performed. Finite-element elastic solutions are reported in References (5) to (9) but none of these include any thermal mismatch effects. Reference (10) included adhesive and adherend non-linear behavior in the analysis but, for the stepped-lap joint, encountered convergence difficulties at high load levels. Grimes, Calcote, Wah, et al [Reference (10)] also performed non-linear iterative theoretical analyses of double-lap, single-lap and stepped-lap joints which they compared with their discrete element analyses, showing good agreement for the first two. They also formulated the scarf joint equations (see their Appendix A) in greater detail than is done here, but were unable to solve them. Corvelli and Saleme

[Reference (11)] developed analysis techniques for bonded joints which included analytical solutions for stepped-lap joints, but in a less comprehensive form than presented here.

Past attempts to include non-linear adhesive behavior in the analytical solutions have centered around the Ramberg-Osgood representation which has a smooth continuous characteristic. This has precluded the derivation of any explicit closed-form solutions. The present author had earlier derived such solutions for double- and single-lap adhesive-bonded joints using an elastic-plastic adhesive formulation [see References (12) and (13)]. These showed that the adhesive shear strain energy per unit bond area was the necessary and sufficient adhesive characteristic governing the potential bond shear strength. The precise shape of the stress-strain curve appeared to be unimportant. This belief was further reinforced in Reference (1) by the derivation of precisely the same potential bond shear-strength for any arbitrary bi-elastic adhesive characteristic having the same strain energy and failure stress and strain. In addition, the author's elastic-plastic solution was in good agreement with the discrete element solutions by Teodosiadis [Reference (14)], who represented the adhesive and interlaminar shear characteristics by six straight segments. The success of this elastic-plastic adhesive approach in these simpler problems led to the decision to apply the same techniques to the scarf and stepped-lap joints in this report.

The adhesive-bonded stepped-lap joint is of practical interest principally because of extensive use in the bonding of boron-epoxy to titanium edge members. The boron filaments are too thick (0.005 inch), and too hard to machine, to be as suitable for the more efficient scarf joints as the very thin graphite fibers are. In practice the stepped-lap joint contains a large number of small steps and closely approximates the behavior of the equivalent scarf joint. The only difference is marked for very brittle (high-temperature) adhesives and is the adhesive shear stress (and strain) concentrations at the ends of each step, particularly at the outermost steps. It transpired that peel stresses imposed more severe limitations for thick double- and single-lap joints than did the adhesive shear stresses [see References (1) and (2)]. In actual design practice for scarf and stepped-lap joints, the slope is small and the end step is

invariably thin so there is no way for severe peel stresses to develop. For any unusual stepped-lap joint, with a thick outer end step, the analysis in Reference (1) can be employed to assess any potential peel problem.

This report considers in turn elastic and elastic-plastic analyses of scarf and stepped-lap joints and discusses parametric effects and design procedures. The digital computer programs prepared from the analyses are recorded in the Appendices, along with brief instructions for their use.

2. ELASTIC ANALYSIS OF SCARF JOINTS

Figure 2 depicts the geometry and nomenclature for the analysis of a non-eccentric bonded scarf joint. The diagram serves for both the elastic and elastic-plastic solutions. In the former case, the plastic adhesive zones should be considered removed. That is, set $a = c \equiv 0$ and $b = \ell$. The scarf angle θ is considered so small that $\cos\theta \simeq 1$ and $\theta \simeq 0$. In other words, the effect of adhesive peel stresses is omitted from consideration. This is quite legitimate for the small scarf angles associated with practical aerospace materials.

The conditions of horizontal equilibrium for a differential element dx within the joint are

$$\frac{dT_1}{dx} + \tau = 0 \quad , \qquad \frac{dT_2}{dx} - \tau = 0 \quad . \tag{1}$$

The stress-strain relations for the adherend materials, accounting for thermo-elastic effects, yield

$$\frac{d\delta_1}{dx} = \frac{T_1}{(Et)_1} + \alpha_1 \Delta T \quad , \qquad \frac{d\delta_2}{dx} = \frac{T_2}{(Et)_2} + \alpha_2 \Delta T \quad , \tag{2}$$

in which the adherend thicknesses, as a function of the axial coordinate x are

$$(Et)_1 = E_1 t_1 \left(1 - \frac{x}{\ell}\right) \quad , \qquad (Et)_2 = E_2 t_2 \left(\frac{x}{\ell}\right) \quad . \tag{3}$$

The adhesive shear strain is taken to be uniform across the thickness of the bond. That is

$$\gamma = (\delta_2 - \delta_1)/\eta \quad . \tag{4}$$

The elastic adhesive shear stress follows as

$$\tau = G\gamma = G(\delta_2 - \delta_1)/\eta \quad . \tag{5}$$

In solving these equations it is desirable to non-dimensionalize the solution with respect to the peak adhesive shear stress τ_p and the bond overlap. Thus, introducing the non-dimensionalized axial co-ordinate

$$\phi = x/\ell \quad , \tag{6}$$

a series solution is sought, having the form

$$\frac{\tau}{\tau_p} = \sum_1^{\infty} A_n \phi^{n-1} \quad . \tag{7}$$

We define the adherend 1 end of the joint as critical so that

$$A_1 \equiv 1 \quad , \tag{8}$$

if necessary by interchange of the identifying subscripts 1 and 2. While a single non-linear differential equation has been derived from the equations above, it cannot be solved directly. This is why a series solution is employed here and, in this case, it is more straightforward to work in terms of the equations above than the derivative governing equation.

The solution proceeds from equation (7). Substitution into equation (1) yields, for the adherend forces per unit width,

$$T_1 = \tau_{av}\ell - \tau_p \ell \sum_1^{\infty} \frac{A_n}{n} \phi^n \quad , \qquad T_2 = \tau_p \ell \sum_1^{\infty} \frac{A_n}{n} \phi^n \quad . \tag{9}$$

Now equation (5) is differentiated.

$$\frac{d(\tau / \tau_p)}{d\phi} = \frac{G}{\tau_p \eta} \left[\frac{d\delta_2}{d\phi} - \frac{d\delta_1}{d\phi} \right] \quad . \tag{10}$$

Substitution of the series (7) and (9), with the aid of equations (2), leads to the solution

$$\sum_1^{\infty} (n-1) A_n \phi^{(n-2)} = \frac{G\ell}{\tau_p \eta} \left\{ (\alpha_2 - \alpha_1)\Delta T + \frac{\tau_p \ell}{E_2 t_2} \sum_1^{\infty} \frac{A_n}{n} \phi^{(n-1)} \right.$$
$$\left. - \frac{\tau_{av}\ell}{E_1 t_1} \frac{1}{(1 - \phi)} + \frac{\tau_p \ell}{E_1 t_1 (1 - \phi)} \sum_1^{\infty} \frac{A_n}{n} \phi^n \right\} \quad . \tag{11}$$

Multiplication throughout by $(1 - \phi)$ converts the equation directly into a form suitable for solution by recurrence relations.

$$(1-\phi)\sum_{1}^{\infty}(n-1)A_n\phi^{(n-2)} = \frac{G\ell}{\tau_p\eta}(\alpha_2-\alpha_1)\Delta T(1-\phi) - \frac{G\ell^2\,\tau_{av}}{\eta E_1 t_1 \tau_p} + \frac{G\ell^2}{\eta}\left[\frac{(1-\phi)}{E_2t_2}\sum_{1}^{\infty}\frac{A_n}{n}\phi^{(n-1)} + \frac{1}{E_1t_1}\sum_{1}^{\infty}\frac{A_n}{n}\phi^{n}\right] \quad . \qquad (12)$$

In order to give the solution the greatest coverage with the minimum number of independent variables, certain non-dimensional parameters are introduced. The non-dimensionalized overlap is given by the square root of

$$(\lambda\ell)^2 = \frac{G\ell^2}{\eta}\left[\frac{1}{E_1t_1}+\frac{1}{E_2t_2}\right] = \frac{\tau_p\ell^2}{\eta\gamma_e}\left[\frac{1}{E_1t_1}+\frac{1}{E_2t_2}\right] \quad , \qquad (13)$$

the non-dimensionalized thermal mismatch term is

$$\mathrm{CTHERM}(1) = \frac{\lambda(\alpha_2-\alpha_1)\Delta T}{\tau_p\left(\frac{1}{E_1t_1}+\frac{1}{E_2t_2}\right)} \quad , \qquad \mathrm{CTHERM}(2) = -\,\mathrm{CTHERM}(1) \quad , \qquad (14)$$

and the adherend stiffness ratio is

$$\mathrm{ETR}(1) = E_1t_1\,/\,E_2t_2 \quad , \qquad \mathrm{ETR}(2) = E_2t_2\,/\,E_1t_1 \quad . \qquad (15)$$

It is interesting to note that precisely the same variables govern the double-lap joint [see Reference (1)]. Equation (12) then becomes

$$\sum_{1}^{\infty}[(n-1)A_n - (n-2)A_{n-1}]\phi^{(n-2)} = (\lambda\ell)\times\mathrm{CTHERM}(1)\times(1-\phi) - \frac{(\lambda\ell)^2}{[1+\mathrm{ETR}(1)]}\times\frac{\tau_{av}}{\tau_p} + \frac{(\lambda\ell)^2\mathrm{ETR}(1)}{[1+\mathrm{ETR}(1)]}\sum_{1}^{\infty}\frac{A_n}{n}\phi^{(n-1)} + \frac{(\lambda\ell)^2[1-\mathrm{ETR}(1)]}{[1+\mathrm{ETR}(1)]}\sum_{1}^{\infty}\frac{A_n}{n}\phi^{n} \quad . \qquad (16)$$

By rearranging the limits of the series it follows that

$$\sum_{1}^{\infty}[(n+1)A_{n+2} - nA_{n+1}]\phi^{n} = (\lambda\ell)\times \mathrm{CTHERM}(1)\times(1-\phi) - \frac{(\lambda\ell)^2}{[1+\mathrm{ETR}(1)]}\times\frac{\tau_{av}}{\tau_p}$$
$$+ \frac{(\lambda\ell)^2\mathrm{ETR}(1)}{[1+\mathrm{ETR}(1)]}\sum_{1}^{\infty}\frac{A_{n+1}}{n-1}\phi^n + \frac{(\lambda\ell)^2[1-\mathrm{ETR}(1)]}{[1+\mathrm{ETR}(1)]}\sum_{1}^{\infty}\frac{A_n}{n}\phi^n \quad . \tag{17}$$

For large values of n, on setting to zero the coefficient of the term ϕ^{n-2}, the recurrence relation is deduced as

$$A_n = \left\{(n-2)A_{n-1} + (\lambda\ell)^2\left[\left(\frac{\mathrm{ETR}(1)}{1+\mathrm{ETR}(1)}\right)\frac{A_{n-1}}{n-1} + \left(\frac{1-\mathrm{ETR}(1)}{1+\mathrm{ETR}(1)}\right)\frac{A_{n-2}}{n-2}\right]\right\} / (n-1) \, . \tag{18}$$

It remains now to establish the initial conditions by examining the coefficients of the ϕ^0 and ϕ^1 terms. From the coefficient of ϕ^0,

$$A_2 = (\lambda\ell)\ \mathrm{CTHERM}(1) - (\lambda\ell)^2\left[\frac{1}{1+\mathrm{ETR}(1)}\right]\frac{\tau_{av}}{\tau_p} + (\lambda\ell)^2\left[\frac{\mathrm{ETR}(1)}{1+\mathrm{ETR}(1)}\right]A_1 \tag{19}$$

while, from the coefficient of ϕ^1,

$$2A_3 - A_2 = -(\lambda\ell)\ \mathrm{CTHERM}(1) + \frac{(\lambda\ell)^2}{[1+\mathrm{ETR}(1)]}\left\{[1-\mathrm{ETR}(1)]A_1 + \mathrm{ETR}(1)\frac{A_2}{2}\right\} . \tag{20}$$

It follows from equation (19) that, quite generally, for long overlaps (large values of $\lambda\ell$),

$$\frac{\tau_{av}}{\tau_p} \to \mathrm{ETR}(1) \leq 1 \ . \qquad \text{(Interchange 1 and 2 if necessary.)} \tag{21}$$

This surprisingly simple result proves to dominate the entire behavior of bonded scarf joints, even for elastic-plastic adhesives. This equation demonstrates conclusively the importance of maintaining adherend stiffness balance whenever possible. When this is maintained, in the absence of any thermal mismatch, the adhesive is essentially uniformly stressed throughout the entire overlap of any length. The only minor exception is the local end effect identified by Richards in Reference (6).

Returning now to the solution, in terms of equations (18) to (20), it follows by integrating equation (7) that

$$\frac{\tau_{av}}{\tau_p} = \sum_{1}^{\infty} \frac{A_n}{n} \quad . \tag{22}$$

In using this series it is necessary to employ two arbitrary constants to satisfy the boundary conditions. The first two are chosen. That is

$$\frac{\tau_{av}}{\tau_p} = A_1 \times \mathrm{SIG}(3) + A_2 \times \mathrm{SIG}(4) \tag{23}$$

and, because of equation (8),

$$\frac{\tau_{av}}{\tau_p} = \mathrm{SIG}(3) + A_2 \times \mathrm{SIG}(4) \quad . \tag{24}$$

The summations SIG(3) and SIG(4) are the quantities formed by evaluating the coefficients in equation (22) by means of equations (20) and (18) after setting, in turn,

$$A_1 = 1 \;, \qquad A_2 = 0 \qquad \text{for } \mathrm{SIG}(3) \tag{25}$$

and

$$A_1 = 0 \;, \qquad A_2 = 1 \qquad \text{for } \mathrm{SIG}(4) \quad . \tag{26}$$

The solution procedure employed in the FORTRAN IV digital computer program listed in Appendix A1 is as follows. The coefficient A_3 for each set of initial values (25) and (26) is evaluated in terms of equation (20). Then a number of higher order coefficients are evaluated in turn through the recurrence relation (18), the same number being evaluated for SIG(3) as for SIG(4). The results of these summations are then substituted into equation (19) which takes on the form

$$A_2 \left\{ 1 + \frac{(\lambda \ell)^2}{[1 + \mathrm{ETR}(1)]} \mathrm{SIG}(4) \right\} = (\lambda \ell)\ \mathrm{CTHERM}(1) + \frac{(\lambda \ell)^2 \mathrm{ETR}(1)}{[1 + \mathrm{ETR}(1)]} - \frac{(\lambda \ell)^2}{[1 + \mathrm{ETR}(1)]} \mathrm{SIG}(3) \quad . \tag{27}$$

The unknown A_2 is then to be evaluated and substituted into equation (19) re-arranged in the form

$$\frac{\tau_{av}}{\tau_p} = \mathrm{ETR}(1) + \frac{[1 + \mathrm{ETR}(1)]\ \mathrm{CTHERM}(1)}{(\lambda\ell)} - \frac{[1+\mathrm{ETR}(1)]}{(\lambda\ell)^2} A_2 \quad . \tag{28}$$

This equation establishes the potential bond shear strength.

The detailed discussion of parametric effects is presented in Section 5 but certain features of the mathematics of the numerical solution merit elaboration at this stage. The most important feature is the decision to evaluate the terms A_n/n of the average stress series (22) directly rather than the quantities A_n of the series (7). To do so, equation (18) is re-organized to the form

$$\left(\frac{A_n}{n}\right) = \frac{(n-1)(n-2)\left(\frac{A_{n-1}}{n-1}\right) + (\lambda\ell)^2\left[\frac{\mathrm{ETR}(1)}{1+\mathrm{ETR}(1)}\left(\frac{A_{n-1}}{n-1}\right) + \frac{1-\mathrm{ETR}(1)}{1+\mathrm{ETR}(1)}\left(\frac{A_{n-2}}{n-2}\right)\right]}{[n(n-1)]} \; . \tag{29}$$

The reason for this is the factor $(\lambda\ell)^2$ in equations (29) and (18). Because of this, for long overlaps, a much higher value of n is needed to reach negligible values of A_n from equation (18) than to reach negligible values of A_n/n from equation (29). Indeed, even with the use of equation (29) rather than equation (18) it remained impossible to compute reliable internal stress distributions for long overlap joints, even with as many as 50 terms of the shear stress series because of overflow in the computer. Such a computation is of little importance, however, since the critical location must be at one end or other of the joint. In spite of this problem, however, equation (27) converges rapidly, usually within the first five successive evaluations (for progressively increasing n) of SIG(3) and SIG(4). The program in Appendix A1 used 20 terms. In addition to this, because A_2 is divided by $(\lambda\ell)^2$ in equation (28), an extremely reliable value of τ_{av}/τ_p can be computed readily. The program identifies the more critical end by the simple expedient of estimating the strength starting from each end of the joint and selecting the lower value. It is obvious that a computation of $\tau_{av}/\tau_p > 1$ signifies simply that condition (8) was violated. A negative value of τ_{av}/τ_p indicates such severe thermal mismatch between adherends that the joint will break apart prior to application of any mechanical loads.

The computation of joint strength proceeding from the other end of the joint is effected by simply interchanging the subscripts 1 and 2 on all affected quantities. With regard to adherend stiffness imbalance alone, it is always possible to identify from equation (28) that the more critical end (1) is that for which ETR(1) ≤ 1. The possible ambiguity arises as the result of the thermal mismatch terms. Since CTHERM(1) may be either negative or positive independently of whether ETR(1) is less than or greater than unity, severe thermal mismatch may nullify or even overpower any stiffness imbalance effects. This possibility is evidently greatest for short overlaps because of the factor $(\lambda \ell)$ in the denominator of the thermal term in equation (28). It follows that the critical end of the joint between given adherends may well change as the overlap changes and, indeed, such behavior was predicted by the computer program output.

Equations (1) and (2) have been set up for applied tensile loads in the adherends. In the event that the applied load is compressive, it can be seen with reference to Figure 2 that all quantities except the thermal strain terms will change sign. This implies that, in the absence of any thermal mismatch effects, the same end of the joint is critical for both tensile and compressive adherend loads and that the joint strength is the same. Rather than change the sign of all quantities with the exception of the thermal terms, the program merely changes the signs of CTHERM(1) and CTHERM(2) to account for compressive loading rather than tensile loading. It should be noted that, as a consequence, the opposite end of the same joint may be critical for a reversed load and that the strength may not be the same if there is also stiffness imbalance between the adherends. Likewise, just as for double-lap joints, if the thermal mismatch terms nullify any stiffness imbalance effects for one load direction, they must aggravate the stress concentrations for a load in the reverse direction. By analogy with the double-lap joint analyses in Reference (1), the case of in-plane shear loading is covered by the analysis above replacing E_1 and E_2 in equation (2) and those equations based on it by the shear moduli G_1 and G_2 and neglecting the thermal affects which induce bond stresses at right angles to those of concern for mechanical in-plane shear loads except at the sides of the joint. The direct adherend forces T_1 and T_2 are replaced by shear forces S_1 and S_2 per unit length. A more precise representation of thermal effects for in-plane shear loading would necessarily require a two-dimensional

analysis rather than the one-dimensional solution above and the justification for doing so is minimized by the small amount of adhesive plasticity that even the real brittle adhesives exhibit.

3. ELASTIC-PLASTIC ANALYSIS OF SCARF JOINTS

The preceding elastic analysis covers essentially the most difficult formulative portions of the elastic-plastic scarf joint analysis. New numerical difficulties of major proportions were encountered in the generation of specific answers by the computer program, but the plastic part of the analysis is straightforward. The necessary additional geometry and nomenclature are identified in Figure 2. Equations (1), (2) and (4) continue to apply, with the substitutions

$$dx = \ell \, d\xi = \ell \, d\chi = \ell \, d\zeta \tag{30}$$

as appropriate. Equation (5) is supplemented by the relation

$$\tau = \tau_p \qquad \text{for} \quad 0 \le \xi \le a \quad \text{and} \quad 0 \le \zeta \le c \; . \tag{31}$$

The relations (4) for the adherend stiffnesses are replaced by

$$(Et)_1 = E_1 t_1 (1 - \xi) = E_1 t_1 (1 - \frac{a}{\ell} - \chi) = E_1 t_1 (1 - \frac{a}{\ell} - \frac{b}{\ell} - \zeta) \tag{32}$$

and

$$(Et)_2 = E_2 t_2 \xi = E_2 t_2 \left(\frac{a}{\ell} + \chi\right) = E_2 t_2 \left(\frac{a}{\ell} + \frac{b}{\ell} + \zeta\right) \; . \tag{33}$$

In the elastic zone, the location of which has yet to be determined, the same power series solution is sought:

$$\frac{\tau}{\tau_p} = \sum_1^{\infty} A_n \chi^{(n-1)} \qquad \text{or} \qquad \frac{\gamma}{\gamma_e} = \sum_1^{\infty} A_n \chi^{(n-1)} \; , \tag{34}$$

again with $A_1 = 1$ by definition of adherend 1 as the more highly loaded end of the joint.

In the left adhesive plastic zone of the joint illustrated in Figure 2, the adherend forces per unit width follow from equations (1) and (31) as

$$T_1 = \tau_{av} \ell - \tau_p \ell \xi \qquad \text{and} \qquad T_2 = \tau_p \ell \xi \; . \tag{35}$$

Substitution into equation (4), making use of equations (2), yields

$$\gamma = \frac{\delta_2 - \delta_1}{\eta} = \frac{1}{\eta}\left[(\alpha_2 - \alpha_1)\Delta T\ell\xi + \int_0^\xi \frac{T_2\ell d\xi}{(Et)_2} - \int_0^\xi \frac{T_1\ell d\xi}{(Et)_1}\right] , \tag{36}$$

$$\gamma = \frac{1}{\eta}\left[(\alpha_2 - \alpha_1)\Delta T\ell\xi + \frac{\tau_p\ell^2}{E_2t_2}\xi + \int_0^\xi \frac{(\tau_p - \tau_{av})\ell^2}{(E_1t_1)(1-\xi)}d\xi - \frac{\tau_p\ell^2}{E_1t_1}\xi\right] + C , \tag{37}$$

$$\gamma = \frac{1}{\eta}(\alpha_2 - \alpha_1)\Delta T\ell\xi + \tau_p\ell^2\left[\frac{1}{E_2t_2} - \frac{1}{E_1t_1}\right]\xi - \frac{(\tau_p - \tau_{av})\ell^2}{E_1t_1}\ell n(1-\xi) + C . \tag{38}$$

The appropriate boundary conditions are that

$$\gamma = \gamma_e + \gamma_p \qquad \text{at } \xi = 0 \tag{39}$$

and

$$\gamma = \gamma_e \qquad \text{at } \xi = a/\ell . \tag{40}$$

Consequently, from equations (39) and (38),

$$C = (\gamma_e + \gamma_p) \tag{41}$$

so that, from equations (40) and (38),

$$\gamma_p = -\frac{1}{\eta}\left[(\alpha_2 - \alpha_1)\Delta T\ell\left(\frac{a}{\ell}\right) + \tau_p\ell^2\left(\frac{1}{E_2t_2} - \frac{1}{E_1t_1}\right)\left(\frac{a}{\ell}\right) - \frac{(\tau_p - \tau_{av})\ell^2}{E_1t_1}\ell n\left(1 - \frac{a}{\ell}\right)\right] . \tag{42}$$

Equation (42) may be non-dimensionalized by use of the quantities in equations (13) to (15). It then adopts the form

$$\left(\frac{\gamma_p}{\gamma_e}\right) = -(\lambda\ell)\ \mathrm{CTHERM}(1)\left(\frac{a}{\ell}\right) + (\lambda\ell)^2\left(\frac{a}{\ell}\right)\frac{1 - \mathrm{ETR}(1)}{1 + \mathrm{ETR}(1)} + (\lambda\ell)^2\frac{[1 - (\tau_{av}/\tau_p)]}{[1 + \mathrm{ETR}(1)]}\ell n\left(1 - \frac{a}{\ell}\right) . \tag{43}$$

In solving for the joint strength it is necessary to maintain continuity at the transition ($\xi = a/\ell$) from plastic to elastic adhesive behavior. The continuity of adherend stresses requires that there be no change in $d\gamma/dx$.
From equations (4) and (2)

$$\frac{d\gamma}{dx} = \frac{1}{\eta}\left[(\alpha_2 - \alpha_1)\Delta T + \frac{T_2}{(Et)_2} - \frac{T_1}{(Et)_1}\right] \tag{44}$$

or, in non-dimensionalized form, for the plastic side of the transition

$$\left.\frac{d(\gamma/\gamma_e)}{d\xi}\right|_{\xi = a/\ell} = (\lambda\ell)\ \mathrm{CTHERM}(1) - (\lambda\ell)^2 \frac{1 - \mathrm{ETR}(1)}{1 + \mathrm{ETR}(1)} + \frac{(\lambda\ell)^2[1 - (\tau_{av}/\tau_p)]}{[1 + \mathrm{ETR}(1)][1 - (a/\ell)]} \quad . \tag{45}$$

For the elastic side, equation (34) requires that

$$\left.\frac{d(\gamma/\gamma_e)}{d\chi}\right|_{\chi = 0} = A_2 \quad . \tag{46}$$

Since $A_1 \equiv 1$, the elastic stress distribution can now be evaluated by a recurrence formula, just as in Section 2.

Under certain combinations of stiffness and thermal mismatch between adherends there will be no second plastic adhesive shear stress zone at the far end of the joint while under others there will be. In the former case, the evaluation of the elastic adhesive shear stress at $\chi = 1 - (a/\ell)$ by means of the series (34) will lead to a result $\tau_{end}/\tau_p \leq 1$. A value of this ratio greater than unity indicates a need for evaluating the affects of the presence of a second plastic adhesive zone, at the far end of the joint. Referring again to Figure 2, the adherend forces per unit width are evaluated through equations (1) and (31) as

$$T_1 = \tau_p \ell(\frac{c}{\ell} - \zeta) \quad , \qquad T_2 = \tau_{av}\ell - \tau_p \ell(\frac{c}{\ell} - \zeta) \quad . \tag{47}$$

Substitution of equations (47) and (33) into equation (44) leads to the expression

$$\frac{d(\gamma/\gamma_e)}{d\zeta} = \frac{G}{\tau_p n}(\alpha_2 - \alpha_1)\Delta T\ell - \frac{\tau_p \ell^2}{E_1 t_1} + \frac{\tau_p \ell^2}{E_2 t_2} - \frac{(\tau_p - \tau_{av})\ell^2}{(E_2 t_2)[1 - \frac{c}{\ell} + \zeta]} \quad . \tag{48}$$

The transition relation at $\zeta = 0$ follows as

$$\left.\frac{d(\gamma/\gamma_e)}{d\gamma}\right|_{\zeta = 0} = (\lambda\ell)\ \mathrm{CTHERM}(1) - (\lambda\ell)^2\left[\frac{1 - \mathrm{ETR}(1)}{1 + \mathrm{ETR}(1)}\right] - (\lambda\ell)^2\frac{[1 - (\tau_{av}/\tau_p)]\mathrm{ETR}(1)}{[1 + \mathrm{ETR}(1)](1 - \frac{c}{\ell})} \quad . \tag{49}$$

Equation (48) may be integrated once, yielding

$$\left(\frac{\gamma}{\gamma_e}\right) = (\lambda\ell)\ \mathrm{CTHERM}(1)\ \zeta - (\lambda\ell)^2 \frac{1-\mathrm{ETR}(1)}{1+\mathrm{ETR}(1)}\ \zeta -$$

$$- (\lambda\ell)^2 \frac{[1-(\tau_{av}/\tau_p)]\mathrm{ETR}(1)}{[1+\mathrm{ETR}(1)]}\ \ell n\left(1-\frac{c}{\ell}+\zeta\right) + C \quad , \tag{50}$$

in which, since

$$\gamma = \gamma_e \qquad \text{at} \quad \zeta = 0 \quad , \tag{51}$$

$$1 = -(\lambda\ell)^2 \frac{[1-(\tau_{av}/\tau_p)]\mathrm{ETR}(1)}{[1+\mathrm{ETR}(1)]}\ \ell n\left(1-\frac{c}{\ell}\right) + C \quad . \tag{52}$$

Signifying by γ_{max} the peak adhesive shear strain at the less critical (by definition) right hand end of the joint,

$$\left(\frac{\gamma_p}{\gamma_e}\right) > \frac{\gamma_{max}}{\gamma_e} = (\lambda\ell)\ \mathrm{CTHERM}(1)\left(\frac{c}{\ell}\right) - (\lambda\ell)^2\left[\frac{1-\mathrm{ETR}(1)}{1+\mathrm{ETR}(1)}\right]\left(\frac{c}{\ell}\right)$$

$$+ (\lambda\ell)^2 \frac{[1-(\tau_{av}/\tau_p)]\mathrm{ETR}(1)}{[1+\mathrm{ETR}(1)]}\ \ell n\left(1-\frac{c}{\ell}\right) \quad . \tag{53}$$

A comparison of equations (43) and (53) shows complete consistency upon interchanging subscripts 1 and 2. While equation (53) could be employed to identify whether the left or right hand end of the joint in Figure 2 is more critical once the extent of the second plastic zone (c/ℓ) had been established, there is an inherent numerical difficulty in the step by step computation of the strength by the procedure outlined above. It was explained in Section 2 that, for the perfectly-elastic adhesive, only the average adhesive shear stress could be computed and not the stress distribution as a function of position along the joint. In the computer program in Appendix A3, the only reason why it proved possible to evaluate the extent of the elastic trough, for long overlaps, was the factor $[1-(a/\ell)-(c/\ell)]^{n-1} < 1$ in equation (34). At high values of n, this very small term was able to overpower the influence of the $(\lambda\ell)^2$ factor in the numerator of the recurrence formula (18). This numerical accuracy problem prevented the reliable evaluation of $d(\gamma/\gamma_e)/d\chi$ at $\chi = (b/\ell)$ to match boundary conditions at the transition of the second plastic adhesive zone. Consequently an iterative solution had to be employed to evaluate the maximum possible

extent of the elastic trough.

Referring to equations (45) and (46), it can be seen that, in the iterative solution process, the second term of the elastic adhesive shear stress series A_2 depends on the preceeding estimates of both (a/ℓ) and (τ_{av}/τ_p). In the early development of the digital computer program for elastic-plastic scarf joints insurmountable convergence difficulties were encountered if the initial estimates for (τ_{av}/τ_p) and (a/ℓ) were not sufficiently close to the true values. This difficulty was eventually overcome by the following technique. Equation (43) was re-arranged to read

$$\left(\frac{\tau_{av}}{\tau_p}\right) = 1 - \frac{\left\{\dfrac{[1+\mathrm{ETR}(1)]}{(\lambda\ell)^2}\left(\dfrac{\gamma_p}{\gamma_e}\right) + \left[\dfrac{[1+\mathrm{ETR}(1)]\mathrm{CTHERM}(1)}{(\lambda\ell)} - [1-\mathrm{ETR}(1)]\right]\left(\dfrac{a}{\ell}\right)\right\}}{\ell n[1-(a/\ell)]} \,. \quad (54)$$

This can be differentiated with respect to (a/ℓ) so that

$d(\tau_{av}/\tau_p)\ /\ d(a/\ell) = 0$ when

$$\left(1-\frac{a}{\ell}\right)\ell n\left(1-\frac{a}{\ell}\right) = \frac{\left\{\dfrac{[1+\mathrm{ETR}(1)]}{(\lambda\ell)^2}\left(\dfrac{\gamma_p}{\gamma_e}\right) + \left[\dfrac{[1+\mathrm{ETR}(1)]\mathrm{CTHERM}(1)}{(\lambda\ell)} - [1-\mathrm{ETR}(1)]\right]\left(\dfrac{a}{\ell}\right)\right\}}{[1-\mathrm{ETR}(1)] - \dfrac{[1+\mathrm{ETR}(1)]\mathrm{CTHERM}(1)}{(\lambda\ell)}} \,. \quad (55)$$

Substitution of equation (55) into equation (54) yields, for the minimum (stationary) value of (τ_{av}/τ_p)

$$\frac{\tau_{av}}{\tau_p} = \mathrm{ETR}(1) + \frac{[1+\mathrm{ETR}(1)]\mathrm{CTHERM}(1)}{(\lambda\ell)} + \frac{a}{\ell}\left\{[1-\mathrm{ETR}(1)] - \frac{[1+\mathrm{ETR}(1)]\mathrm{CTHERM}(1)}{(\lambda\ell)}\right\} \,. \quad (56)$$

This is evidently consistent with the elastic solution $(a/\ell) = 0$ for large overlaps and, upon subsequent comparison with the more precisely estimated joint strengths, proved to be an extremely close lower bound for all cases of practical interest. It is significantly conservative only for very short overlaps [small values of $(\lambda\ell)$] or very brittle adhesives [very small values of (γ_p/γ_e)]. The adhesive shear strain capacity γ_p is involved in equation (56) implicitly through the extent (a/ℓ) of the plastic zone. Equation (55)

is solved by iteration to evaluate (a/ℓ) and the result substituted into equation (56) or (54). Appendix A2 contains a listing of the FORTRAN IV digital computer program employed to solve equations (55) and (54), together with sample outputs and brief user instructions. The iteration technique eventually adopted proved to be quite convergent, after other re-arrangements of equation (55) demonstrated strongly divergent characteristics.

This program in Appendix A2 served to provide the initial estimates of (a/ℓ) and (τ_{av}/τ_p) in the more precise solution listed in Appendix A3. The sequence of variables used in the solution is (a/ℓ), (τ_{av}/τ_p) and (c/ℓ) after which (τ_{av}/τ_p) is recomputed and the estimate of (a/ℓ) adjusted until convergence is attained. In those cases in which the critical end is not evident by inspection, the potential bond shear strength is computed from each end of the joint and the lower value adopted. Brief user instructions and sample outputs are included in Appendix A3.

The analyses above for scarf joints pertain to adhesive shear stresses and it is demonstrated that a small enough scarf angle can always be found to transfer the full adherend strength through the bond with an adequate margin. There is, of course, a potential problem with the adherend strength(s) if the scarf angle is too small. Specifically, one adherend will fail if the scarf angle θ is so small that

$$\theta < \tau_p/F_u \;, \tag{57}$$

(where F_u is the ultimate adherend stress in tension, compression, or shear, as appropriate) at the more critical end of the joint (identified by the adhesive shear stress analysis). Should this situation arise, the solution is to decrease the adherend stiffness imbalance across the joint by local reinforcement of the softer adherend. It is evident from equation (17) that this potential problem of breaking off the tip of (usually) the stiffer adherend is more likely to arise with the brittle adhesives (higher values of peak adhesive shear stress τ_p) than with ductile adhesives. This is one important reason for preferring to effect the load transfer with a shorter overlap of ductile adhesive than with a longer overlap of brittle adhesive. The extreme case of making the overlap so extremely long that the peak adhesive shear

stress actually developed is restricted to a small fraction of its capacity when adherend failure occurs outside the joint has theoretical appeal only, frequently being quite impractical.

4. DISCUSSION OF PARAMETRIC EFFECTS

Representative solutions from Sections 2 and 3 for unbalanced bonded scarf joints are illustrated in Figures 3 through 7. Figures 3 and 4 show the separate effects of adherend stiffness and thermal mismatch, respectively, on the elastic joint strength. The deviations from unity in the (τ_{av}/τ_p) ratio, for a given overlap $(\lambda \ell)$, are proportional to the individual imbalances. The effect of stiffness imbalance is a smooth decrease from a fully-efficient bond $(\tau_{av} = \tau_p)$ to a less efficient bond $(\tau_{av} < \tau_p)$ asymptoting towards the solution given in equation (21). This diagram, more than any other, characterizes the dominant feature of the scarf joint behavior. This is that the potential bond strength continues to increase indefinitely with increasing overlap. This is in marked contrast to the behavior of uniform lap joints [References (1) and (2)], which develop maximum strengths which remain effectively constant beyond intermediate overlaps. The effect of this characteristic on the potential bond strength of scarf joints is that, by making the scarf angle sufficiently small, one can always design a joint in which the potential bond strength exceeds the adherend strength by any specified factor. This is amply demonstrated by curve D in Figure 4. While adherend stiffness and thermal mismatch combine to decrease the bond efficiency below the unit value of curve A, the bond strength for long overlaps ends up being proportional to the overlap. As a consequence of this characteristic, the elastic adhesive shear stresses play a far more important role in the strength of scarf joints than they do in the case of uniform lap joints. Nevertheless, it would be erroneous to conclude that one could always design an unbalanced scarf joint within the capabilities of an elastic adhesive. The limiting problem is that, as the scarf angle becomes very small, there is a strong probability of breaking off the tip of the stiffer adherend. While not as acute a design detail problem as its counterpart for stepped-lap joints, this feature restricts the scarf angle to exceed the value

$$\theta = \mathrm{ARCTAN}(\tau_p/F_u) \tag{58}$$

in which F_u is the adherend ultimate strength (in tension, compression, or shear, as appropriate for the applied load).

The effect of adherend thermal mismatch on the potential bond strength of scarf joints is shown in Figure 4. It is clear that the effects are insignificant for very short and very long overlaps, being significant only for those

overlaps of practical interest. The effects are maximum at $(\lambda \ell) = 2$ for all values of the thermal mismatch coefficient CTHERM.

Figure 5 shows the interaction between adherend stiffness and thermal mismatch. Curves B, D and E represent one set of solutions, with curve B showing the effect of stiffness imbalance alone. Curve D adds the influence of compounding thermal mismatch as well. Curve E demonstrates the behavior of self-cancelling adherend imbalances at $(\lambda \ell) = 3$. For values of $(\lambda \ell)$ less than 3, the thermal mismatch effects dominate over those arising from stiffness mismatch and the more critical end of the joint is reversed. Curves A, C and F form another set showing how, for severe adherend thermal mismatch, there is a range of overlaps for which the residual thermal stresses are so severe that the joint will split apart without the application of any mechanical loads. Quite unlike the behavior of uniform lap joints [References (1) and (2)], this problem can be eliminated completely by sufficient extension of the overlap.

Just as is the case for uniform lap joints adhesive plasticity can increase the potential bond shear strength. The extent of this strength increase is shown in Figures 6 and 7 for stiffness and thermal mismatch, respectively. For each amount of adhesive plastic shear strain, there is an associated overlap below which the bond can be uniformly stressed. For indefinitely large overlaps the asymptotic solution (21) again holds, masking completely the influence of any adhesive plasticity. In the overlaps of practical interest, the actual amount of adhesive plasticity available from real structural adhesives can improve the potential joint strength greatly. One benefit of using a ductile adhesive of moderately high peak shear stress rather than a brittle adhesive of very high peak shear stress is that the joint is better able to withstand the variation in joint load which inevitably occurs as the result of manufacturing imperfections and non-uniform load distribution. Another benefit is that the problem of breaking off the tip of the adherend at the more critically loaded end [see equation (58)] is greatly alleviated. If the tip of the stronger adherend were allowed to be broken off, this would impose an effective net area loss on the cross-section of the weaker adherend.

5. ELASTIC ANALYSIS OF STEPPED-LAP JOINTS

The analysis for the strength of stepped-lap adhesive-bonded joints contains features of both the uniform lap joints [References (1) and (2)] and the scarf joint above. Peel stress problems are ignored on the grounds that the outermost end steps are invariably thin enough (in good design practice) not to induce significant peel stresses in the adhesive. Likewise, the small eccentricity in the load path has been ignored in the interests of obtaining a useful uncomplicated design tool.

A representative idealized stepped-lap joint is shown in Figure 8, along with the sign convention and nomenclature necessary for the analysis. Just as for the scarf joint analysis, the same diagram serves also for the elastic-plastic analysis, so it contains information not necessary for the elastic analysis. This begins with the equilibrium equations for a differential element of one of the steps.

$$\frac{dT_o}{dx} + 2\tau = 0 \quad , \qquad \frac{dT_i}{dx} - 2\tau = 0 \quad . \tag{59}$$

Here the subscripts o and i refer to the "outer" and "inner" adherends, respectively, and the factors 2 in equations (59) account for the two bond surfaces surrounding the inner adherend. Consequently the adherend thicknesses t_o and t_i refer to the total cross-section and the forces T_o and T_i do likewise. The nature of the solution is such that it is, on occasions, necessary to interchange the subscripts o and i mathematically. The thermo-elastic relations for the adherends are

$$\frac{d\delta_o}{dx} = \frac{T_o}{E_o t_o} + \alpha_o \Delta T \quad , \qquad \frac{d\delta_i}{dx} = \frac{T_i}{E_i t_i} + \alpha_i \Delta T \quad . \tag{60}$$

The adhesive shear strain, for tensile lap shear loading, is

$$\gamma = (\delta_i - \delta_o) / \eta \quad . \tag{61}$$

while the elastic adhesive shear stress is related to the shear strain by the relation

$$\tau = G\gamma = G(\delta_i - \delta_o)/\eta \quad . \tag{62}$$

The solution proceeds just as in Reference (1).

$$\frac{d\tau}{dx} = \frac{G}{\eta}\left[\frac{d\delta_i}{dx} - \frac{d\delta_o}{dx}\right] = \frac{G}{\eta}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right] \quad . \tag{63}$$

$$\frac{d^2\tau}{dx^2} = \frac{G}{\eta}\left[\frac{2}{E_i t_i} + \frac{2}{E_o t_o}\right]\tau = \lambda^2\tau \quad . \tag{64}$$

The solution of equation (64) is

$$\tau = A\cosh(\lambda x) + B\sinh(\lambda x) \tag{65}$$

where the integration constants A and B are to be determined by boundary conditions for each step. Substitution of equation into equation (59) yields

$$T_o = T_{o_{ref}} - 2\frac{A}{\lambda}\sinh(\lambda x) - 2\frac{B}{\lambda}[\cosh(\lambda x) - 1] \tag{66}$$

and

$$T_i = T_{i_{ref}} + 2\frac{A}{\lambda}\sinh(\lambda x) + 2\frac{B}{\lambda}[\cosh(\lambda x) - 1] \quad . \tag{67}$$

The values of $T_{o_{ref}}$ and $T_{i_{ref}}$ depend upon the origin of x adopted. In the solution it proves convenient to adopt the start of each step as the origin for that step. Integrating again, by means of equations (69),

$$\delta_o = \delta_{o_{ref}} + \alpha_o\Delta Tx + \frac{1}{E_o t_o}\left[T_{o_{ref}}x - 2\frac{A}{\lambda^2}\cosh(\lambda x) - 2\frac{B}{\lambda^2}[\sinh(\lambda x) - (\lambda x)]\right] \tag{68}$$

and

$$\delta_i = \delta_{i_{ref}} + \alpha_i\Delta Tx + \frac{1}{E_i t_i}\left[T_{i_{ref}}x + 2\frac{A}{\lambda^2}\cosh(\lambda x) + 2\frac{B}{\lambda^2}[\sinh(\lambda x) - (\lambda x)]\right] \quad . \tag{69}$$

In the FORTRAN IV digital computer program, listed in Appendix A4, used to solve the equations above for the elastic stepped-lap joint, the technique of solution is as follows. The solution proceeds, one joint step at a time starting with assumed values of the load and initial adhesive shear strain (or stress). The latter is set at the maximum adhesive allowable and remains so unless it is computed that the peak adhesive shear strain is greater elsewhere (most probably at the other end of the joint) in which case the initial strain is reduced as much as necessary to avoid exceeding the allowable. The key

equation in the solution is equation (65). The integration constant A is evaluated as the specified (or subsequently computed) adhesive shear stress at the start of the step under consideration.

$$A = \tau_{x=0} \quad . \tag{70}$$

The other constant B derives from equation (63), also evaluated at the start of that step. That is

$$\frac{d\tau}{dx} = A\lambda \sinh(\lambda x) + B\lambda \cosh(\lambda x) = \frac{G}{\eta}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right] \tag{71}$$

so that at $x = 0$

$$B = \frac{G}{\eta\lambda}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right]_{x=0} \quad . \tag{72}$$

The values of τ, T_o, T_i, δ_o and δ_i at the end of that step then follow from equations (65), (66), (67), (68), (69) and (62), respectively. If, after one complete set of computations, the load computed to be transferred out of the far end of the joint does not match that assumed to act at the near (starting) end, the initial estimate is adjusted until the two quantities do match. At that stage, a check is made throughout the joint, step by step, to identify the most critical adhesive and adherend locations. If any negative margins are identified, the load and peak adhesive shear stress are reduced as much as is necessary to eliminate them.

While the formulation of the equations and analysis scheme above is quite straightforward, the actual numerical solution of the problem proved to be quite difficult. Even with double precision it was almost invariably impossible to compute values for all steps of the joint in a single pass, even if the initial conditions (load and peak adhesive shear stress) were precisely correct to 16 significant figures. A change of 1 in the 16th significant digit of an initial condition would frequently effect a change by a factor of up to $\pm 10^7$ in a quantity computed in the fourth or fifth step. This was not the result of a poorly conditioned mathematical formulation. It follows directly from strong physical characteristics of stepped-lap joints. It is the nature of stepped-lap joints, be they bonded or bolted, that any non-uniformities in the load transfer are dominated by the geometry and materials of the end three

steps. What happens in between has only negligible effect on the critical loads which almost invariably occur at one end or other of the joint. Likewise, in a uniform lap joint, practically all the load is transferred through the end three (rows of) bolts or through a narrow effective end zone of adhesive. Because of this characteristic the initial coding of the equations led to a highly accurate estimate of the load (assuming that the adhesive was critical at one end of the joint) but was unable to compute the internal loads and check on the adherend strength margin. The technique finally employed for dealing with this problem took advantage of the seemingly undesirable characteristics and is summarized as follows. By printing out intermediate computations it became clear that, if the initial load estimate on a given step was too high (even if only minutely), on the step just before computations for a subsequent step caused overflows and underflows in the computer the computations would diverge in a characteristic way, precisely the opposite of that for an initial underestimate of that load. Therefore upper and lower bounds were placed on the load estimate and the trial load was taken as the average of these. If the trial load was found to be too high, it served as the new upper bound and, were it too low, it was used to raise the lower bound. This technique was found to bring the upper and lower bounds into precise agreement rapidly. Once this had occurred the computations for the start of that step were frozen and the solution proceeded to perturb each successive step in turn, using the same convergence check above, until the load transferred out of the far end of the joint precisely equalled that input at the near end. Then a check is made, at the ends of each step, on the adhesive and adherend stresses to ensure that neither exceeds the allowable. Due allowance is made for the sign of the quantities involved. In the absence of any thermal mismatch this last operation of checking on the allowables can be performed by simple linear scaling. However, if there is any adherend thermal mismatch present, this adjustment must be performed by iteration since, as is evident from equation (62), the thermal stress terms do not scale in proportion to the adhesive and adherend stresses. A necessary check on the accuracy of the numerical processes has been accomplished by checking that precisely the same solution is obtained regardless of whether the computations commence at the more critically loaded end of the joint or at the other end.

In view of the numerical problems encountered with this analytical solution, it stands to reason that they will have their counterpart in any finite-element solution. Very fine grids would be needed in the high stress gradient areas.

6. ELASTIC-PLASTIC ANALYSIS OF STEPPED-LAP JOINTS

In addition to the equations of Section 5 for the perfectly elastic analysis of stepped-lap joints, the elastic-plastic analysis requires, instead of equation (62), that

$$\tau = \tau_p \qquad \text{for } \gamma \geq \gamma_e \ , \tag{73}$$

and

$$\tau = G\gamma \qquad \text{for } \gamma \leq \gamma_e \ . \tag{74}$$

The elastic-plastic solution is best carried out in terms of the adhesive shear strains rather than the shear stresses. In the plastic adhesive zones, from equations (61) and (60),

$$\frac{d\gamma}{dx} = \frac{1}{\eta}\left[\frac{d\delta_i}{dx} - \frac{d\delta_o}{dx}\right] = \frac{1}{\eta}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right] \tag{75}$$

whence, from equations (59)

$$\frac{d^2\gamma}{dx^2} = \frac{2}{\eta}\left[\frac{1}{E_i t_i} + \frac{1}{E_o t_o}\right]\tau_p = \frac{\lambda^2}{G}\tau_p = \text{constant} \ . \tag{76}$$

Therefore, in the plastic zone,

$$\gamma = \frac{\lambda^2}{2G}\tau_p x^2 + Cx + D \tag{77}$$

and

$$T_o = T_{o_{ref}} - 2\tau_p x \ , \qquad T_i = T_{i_{ref}} + 2\tau_p x \tag{78}$$

while

$$\left.\begin{aligned} \delta_o &= \delta_{o_{ref}} + \alpha_o \Delta T x + \frac{1}{E_o t_o}\left[T_{o_{ref}} x - \tau_p x^2\right] \\ \text{and} \qquad & \\ \delta_i &= \delta_{i_{ref}} + \alpha_i \Delta T x + \frac{1}{E_i t_i}\left[T_{i_{ref}} x + \tau_p x^2\right] \end{aligned}\right\} . \tag{79}$$

In equation (77), D is set equal to γ at the start of any step, since a new zero for x is chosen at that location for each step. The other constant C follows from equations (75) and (77). Thus

$$C = \left.\frac{d\gamma}{dx}\right|_{x=0} = \frac{1}{\eta}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right]_{x=0} \quad . \tag{80}$$

Very few individual steps of stepped-lap joints have fully-plastic adhesive throughout the entire joint. Any adhesive plasticity is frequently confined to the end(s) of the step(s). Therefore, in performing an elastic-plastic analysis of a stepped-lap joint, it is necessary to be able to compute the extent of the plastic zones. Therefore, beginning at the left hand end of the step element shown in Figure 8 and assuming a sufficiently high load intensity for the adhesive to be in the plastic state, the first computation is that of the maximum possible extent of the plastic zone. This is then compared with the actual extent of the step. If necessary, a second computation is performed of the maximum possible extent of the elastic trough in that same step. Starting from equation (77) with $\gamma = \gamma_{ref}$ at $x = 0$,

$$\gamma = \frac{\lambda^2}{2G}\tau_p x^2 + Cx + \gamma_{ref} \tag{81}$$

where the constant C is given by equation (80). It is necessary to find the lesser value of x for which $\gamma = \gamma_e$. Equation (81) is re-arranged to read

$$\frac{\lambda^2\tau_p}{2G} x_p{}^2 + Cx_p + (\gamma_{ref} - \gamma_e) = 0 \tag{82}$$

so that the maximum extent of plastic adhesive zone is given by

$$x_p = -C \pm \sqrt{C^2 - 2\lambda^2\gamma_e(\gamma_{ref} - \gamma_e)} \quad . \tag{83}$$

Now, since $C = d\gamma/dx < 0$ at $x = 0$ the minus sign in front of the radical holds. Once x_p has been computed, it is compared with the step length ℓ_{step}. If $x_p > \ell_{step}$, that particular step is fully-plastic throughout and the values of the various quantities at the far end of the step are evaluated from equations (73) to (80). Should x_p be less than ℓ_{step}, the difference is examined elastically, to see whether it remains elastic throughout or becomes plastic again at the far end. For $x_p < \ell_{step}$, the values of the various stresses, strains,

displacements and forces are evaluated in terms of equations (73) to (79) and the subscripts pe serve to identify the plastic-to-elastic transition. Likewise ep identifies the possible elastic-to-plastic transition at the far end of the joint. It is necessary that $d\gamma/dx$ be maintained at these transitions, as is evident from equation (75). The maximum possible extent of elastic trough must be deduced from equation (65). In doing so, it is mathematically far simpler to shift the x origin to the middle of the elastic trough (of extent $2x_e$) so that

$$\tau = \tau_p \cosh(\lambda x) \,/\, \cosh(\lambda x_e) \tag{84}$$

At the pe transition ($x = -x_e$) equation (62) requires that

$$\frac{d\tau}{dx} = \frac{G}{\eta}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right]_{pe} = -\tau_p \lambda \tanh(\lambda x_e) \tag{85}$$

so that the elastic trough could extend, if ℓ_{step} were great enough, a distance

$$2\lambda x_e = \tanh^{-1}\left\{-\frac{1}{\lambda\eta\gamma_e}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right]_{pe}\right\} . \tag{86}$$

By use of known formulas for hyperbolic functions in terms of exporentials and the interrelation between exponential and logarithmic functions, the solution (85) is more conveniently expressed as

$$2x_e = \frac{1}{\lambda}\,\ell n\left\{\frac{1 - \dfrac{1}{\lambda\eta\gamma_e}\left[\dfrac{T_i}{E_i t_i} - \dfrac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right]_{pe}}{1 + \dfrac{1}{\lambda\eta\gamma_e}\left[\dfrac{T_i}{E_i t_i} - \dfrac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right]_{pe}}\right\} . \tag{87}$$

In the event that x_e does not extend beyond the far (right hand) end of the step being analyzed, it is necessary to compute the load transferred between the adherends throughout the elastic trough. In doing so, it is quite simple to take the value of $2x_e$ from equation (87) and substitute it back into equations (65) to (72) for the standard elastic analysis of the preceding section. Should the elastic trough not extend to the far end of the step under analysis, equations (73) to (80) are employed for the plastic zone to the end of the step.

Equation (77) now becomes

$$\gamma = \frac{\lambda^2 \tau_p}{2G} x^2 + Cx + \gamma_{ep} \tag{88}$$

with

$$\tau = \tau_p \qquad \text{for} \quad x > x_{ep} \quad . \tag{89}$$

The constant C in equation (88) is evaluated in terms of equation (75)

$$C = \left.\frac{d\gamma}{dx}\right|_{ep} = \frac{1}{\eta}\left[\frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o)\Delta T\right]_{ep} \quad . \tag{90}$$

In the last steps of the joint at the far end, the adhesive may be fully plastic throughout in which case, in equation (87), γ_{ep} should be replaced by γ_{ref}. Likewise, in those steps, near the middle of the joint, in which the adhesive shear strains are so small as not to reach the plastic state at either end of the step, the step will be elastic throughout and equations (65) to (72) are employed in the analysis. Towards the far end of the joint there may be a step which starts elastically and becomes plastic. In this case the actual extent of elastic behavior is determined by iteration, using equations (65) to (72) with a cut off (either positive or negative) on the shear stress.

If it should transpire that, at the end of the step, γ exceeds $(\gamma_e + \gamma_p)$ or T_i or T_o exceed their respective allowables, this does not cause any analytical difficulty. An iterative procedure is employed in the analysis to reduce the external load and initial adhesive strain whenever necessary. While this does not represent any analytical difficulty, one should recognize that exceeding the allowables on an inner step can occur only as the result of poor detail design. The improvement of such details can increase the potential joint strength.

No new numerical difficulties were encountered in the program listed in Appendix A5 for the elastic-plastic analysis of stepped-lap joints which did not have a direct counterpart in the perfectly elastic analysis. However, the logic associated with keeping track of the locations of the transitions between elastic and plastic adhesive behavior, and vice versa, as they moved with each successive iteration posed a formidable problem. One small computational

problem was that, if the load estimate at some early stage in the iteration sequence was too far removed from the correct value, the computer would predict physically unrealizable large negative shear strains in the adhesive. A special set of instructions was prepared for this quirk.

The computer program, as basically written, checks simultaneously for the allowable adherend and adhesive strengths at the most critical locations in each step. Since stepped-lap joints are frequently more critical in the adherend than in the adhesive, a special feature has been added to increase greatly all adherend strengths artificially in order to print out also the potential adhesive bond strength and confirm that it exceeds the adherend strength by an adequate margin.

The analysis above is presented for the case of tensile lap shear loads being positive and the sign convention is in accordance. The computer programs have been so coded that, by a single input for the variable SGNLD, the respective solutions for tensile shear loading (SGNLD = +1) and compressive shear loading (SGNLD = -1) can be printed out. In the event that there are simultaneous stiffness and thermal mismatches between the adherends, the joint strength will not be the same for each load sense. Such a situation is common in the bonding of titanium edge members to boron-epoxy panels.

7. DISCUSSION OF DESIGN OF STEPPED-LAP JOINTS

The digital computer programs developed above to analyze stepped-lap joints can serve also as a useful design tool. Three clear dominant joint characteristics have been confirmed by studies with this program. The first is that the joint load capacity is defined by the end three steps at the more critical end of the joint. If other steps have a significant influence it will be adverse and be due to poor design detailing. The second is that, once the joint is essentially well-designed, quite major changes can be made to other than the critical end three steps without any significant impact on the joint strength. Third is that, in a well-designed joint, it is the very end step that is likely to precipitate joint failure unless its length is restricted in the design process. The necessary restriction is that the product of maximum adhesive shear stress and total bond area on the end step must not exceed the product of adherend material allowable and, cross section of the end step. Consequently, a ductile adhesive with higher strain energy provides stronger joints than a brittle adhesive with higher peak stress but less strain energy. It should be noted also that minimizing adherend stiffness imbalance increases the potential bond shear strength.

Mathematically speaking, the stepped-lap family of joints represent perturbations about the scarf joint solution. These perturbations become progressively greater as the number of steps decreases until the stepped-lap solution reduces to a single-lap joint for one step. Stepped-lap joints with only two or three steps are usually confined to thin adherends for which the potential bond shear strength is far in excess of the adherend(s) strength. In such cases the added strain concentrations in the bond due to the step discontinuities are not very important. Most applications of stepped-lap joints contain a large number of steps and, with a ductile adhesive softening the most severe of the adhesive stress spikes, the behavior very closely approaches that of the scarf joint. For this reason, preliminary design of practical stepped-lap joints by means of the scarf joint solution appears to be quite realistic. In doing so, however, one should exercise caution with regard to the critical end step of the adherend. The stepped-lap joint analysis, and practical experience, have identified the end step of the stiffer adherend as a prime candidate for the most critical design detail. If the extensional modulus of a composite adherend is

significantly less than that of a metal adherend to which it is bonded, most of the shear load transfer will be concentrated at the composite end of the joint with the probable result that tip fracture of the stiffer adherend will occur. One simple remedy to this potential difficulty is to be found in the concept of the dual-slope scarf joint illustrated in the upper part of Figure 9. In this joint, in order to protect the tip of the adherend, the scarf angle θ_1 is set to exceed

$$\theta_{1_{min}} = \tau_p \;/\; F_u \tag{91}$$

in which τ_p is the peak adhesive shear stress and F_u is the appropriate adherend allowable stress in tension, compression, or shear as dictated by the nature of the applied load. The next step in the preliminary design process is to estimate the total scarf length necessary to effect the transfer of the entire load P. A reasonable approximation to this is given by the approximation

$$P = \left(\frac{\tau_{av}}{\tau_p}\right)\tau_p \ell \simeq \left[\frac{E_1 t_1}{E_2 t_2}\right]\tau_p \ell \tag{92}$$

for the asymptotic scarf joint solution for very long overlaps, whence

$$\ell \simeq \frac{P}{\tau_p}\left[\frac{E_2 t_2}{E_1 t_1}\right] \quad . \tag{93}$$

The optimum location of the transition from scarf angle θ_1 to θ_2 can then be determined by trial and error using the stepped-lap joint computer program developed in Section 6. As a preliminary guide, it is suggested that one third of the total thickness be tried. The conversion of this conceptual scarf joint design into a practical stepped-lap joint is illustrated schematically in the lower part of Figure 9. It should be noted that the steps are thinner in the more critical load transfer region, and at the extreme opposite end for a single step to minimize potential peel stress problems. Normally peel stresses will not be a problem with stepped-lap joints for practical design configurations but the double-lap joint analysis can serve as a check if appropriate. The larger step sizes in the lightly loaded area effect an economy of fabrication which offsets the greater expense of proper detailing in the more critical areas.

For reasons evident from the discussion above, the dual-slope scarf joint has merits in its own right as well as for a model for approximate stepped-lap joint analysis. The steepening of the scarf angle at one end is particularly important for the brittle adhesives for which τ_p is much higher than for the ductile adhesives. This greater importance follows from equation (91).

One characteristic of the internal stress distribution within stepped-lap bonded joints is directly traceable to double-lap joint phenomena and has no counterpart in scarf joint behavior. This characteristic is that, once each or any step is sufficiently long to contain a fully-developed elastic trough in the adhesive shear-stress distribution, an increase in that step length does not alter the joint shear strength. Indeed, as confirmed by application of the computer programs A4EF and A4EG, the internal adherend and adhesive stresses at the ends of each and every step are invariant with respect to such step length increases, whether one, some, or all of the step lengths are increased. That this should be so follows directly from the governing equations for each step of the joint. These are precisely the same as for an unbalanced double-lap joint, the shear strength of which is independent of overlap beyond some value. The impact of this phenomenon on the design of stepped-lap bonded joints is that, if analysis indicates inadequate bond strength and the overlap is already reasonably great, no further increase in step lengths can accomplish an improvement in joint strength. It is necessary to increase the number of steps and decrease the incremental step thickness.

The technique of refining the preliminary analysis developed by the rules above is as follows. An analysis is performed, and the limiting (critical) detail identified. If this is the strength of the end step of the stiffer adherend, the appropriate procedure is to decrease this length and increase the length of the other steps. A halving of the step thickness increment and doubling of the number of steps at the more critical end of the joint will also help. This situation can be identified by a solution in which the maximum adhesive shear strain developed is less than the allowable. In rare instances it may not be the very end but one or two steps inside which are critical. The procedure for improving the joint strength is the same. Reduce the length of the critical steps and increase the others. In doing so, it should be remembered that any fully-elastic step will not transfer much more load even if its length is

increased. Furthermore, if the adhesive shear stresses at each end of the step are less than their plastic value, increasing the step length indefinitely will not introduce a plastic zone. If the adhesive shear strain is predicted to be the limiting feature rather than the adherend strength, the joint strength may be improved by increasing the number of joint steps. In doing so, steps at one end of the joint will tend to become critical and length increases in the remaining (elastic) steps will continue to increase the joint strength, but at a decreasing rate. The behavior of bonded scarf joints (Figure 6) serves to explain this approach. Since the average bond stress on a scarf joint approaches a fixed fraction of the maximum bond shear stress, an overlap sufficiently long can always be found to develop a potential strength 50 percent in excess of the adherend strength. The only inherent difficulty in this approach is the care needed not to exceed the adherend allowables near the more critical end of the joint. One may look upon an optimally designed stepped-lap joint as an approximation to a dual slope scarf joint with a small angle at the less critical end to build up the total load transferred and a steeper angle (still small) at the more critical end to prevent breaking off the tip of the adherend.

In the presence of adherend thermal mismatch (advanced composite-to-metal for example), a reversal of load direction can reverse the more critical and less critical ends of the joint. Therefore it is necessary in such cases to design for both the maximum tensile shear and compressive shear loads to be applied.

Figures 10 to 12 illustrate solutions obtained to stepped-lap bonded joint analyses using the computer programs above. The joint is drawn to scale in Figures 10 and 11 and the material properties can be found in the sample printout included in the Appendix. The brittle and ductile adhesives referred to are, respectively, Narmco Metlbond 329 and Hysol EA951 which have the shear characteristics illustrated in Figure 13. The elastic solutions in Figure 10 show dramatically the sharp spikes in the shear stress distribution at the ends of each step. These spikes, separated by relatively lightly-loaded troughs, represent the influence of the uniform thickness steps. It is evident also from Figure 10 that the ductile adhesive, with its lower modulus and higher elastic shear strain carries slightly more load elastically than does the brittle adhesive. Figures 10 to 12 omit the influence of thermal mismatch between adherends and, had this been included, the elastic strength disparity in

Figure 10 would have been very pronounced in favor of the ductile adhesive for a tensile shear loading. Figure 11 shows the computed bond shear stress distributions, corresponding with Figure 10, when the adhesive properties are modified to account for plasticity. As is to be expected from the adhesive characteristics in Figure 13, this modification does not increase the joint strength of the brittle adhesive sufficiently for the bond to be stronger than the weaker adherend. The ductile adhesive, on the other hand, is computed to have a potential bond strength nearly as great as the strength of the titanium outside the joint. Actually, by the time the adhesive has used up only 15 percent of its total shear strain capacity, the load level is so high as to cause the end (thin) titanium step to yield, as shown in the middle illustration of Figure 11. The ductile adhesive has a considerable strength margin over the composite adherend. Figure 12 demonstrates how the theory identifies the end metal step as being prone to fatigue failure, even though the end step had been shortened to alleviate the problem. In the static load case the theory predicts that, once the titanium has yielded locally, as shown in the second illustration of Figure 12, the load level will increase until failure occurs in the composite at the end of the joint, as shown in the fourth illustration. Figures 11 and 12 depict only the most critical conditions within each step because the computer program does not normally output a continuous solution. The adhesive shear stress distribution throughout the lightly loaded regions is not crucial to the design/analysis cycle. For illustrative purposes one can easily artificially divide each step into a number of short segments in order to avoid adding another computation sequence to the programs. This has been demonstrated to be free from convergence problems (as confirmed by Figure 10) but, naturally, takes more computer time.

The following table enumerates a number of solutions obtained with the stepped-lap joint computer programs above. The effects of thermal stresses are included, as also is the influence of the direction of the applied load. Of interest is the way in which the adherend thermal and stiffness imbalances compound to decrease the joint strengths for tensile loading while they counteract each other for the compressive loading. The failure modes predicted are identified by the comment codes 1 through 5 which are explained at the foot of the table. All cases except those for optimized step lengths have the joint geometry shown in Figure 10. In optimizing the joint proportions, the computer program

STRENGTHS OF STEPPED-LAP ADHESIVE-BONDED JOINTS

JOINTS OF TITANIUM TO ISOTROPIC HTS GRAPHITE-EPOXY

TITANIUM 0.25 IN. THICK GRAPHITE-EPOXY 0.264 IN. THICK

FAILURE LOADS (LBS/INCH)

ΔT / ADHESIVE	0° TENSION & COMPRESSION	$-280^{\circ}F$ TENSION	$-280^{\circ}F$ COMPRESSION	OPTIMIZED STEP LENGTHS $-280^{\circ}F$ TENSION	OPTIMIZED STEP LENGTHS $-280^{\circ}F$ COMPRESSION	$-400^{\circ}F$ TENSION	$-400^{\circ}F$ COMPRESSION
EPON 951							
PURELY-ELASTIC	7829	4927	10730	4367	9990	3683	9203
ELASTIC-PLASTIC	14430	11866	16997	18180	18182	10769	17821
POTENTIAL BOND STRENGTH	28362	26099	30569	23257	27299	25123	45000
COMMENTS	1, 2	1, 2	1, 4	5	5	1, 2	1, 2
METLBOND 329							
PURELY-ELASTIC	6764	3812	8552			2547	6152
ELASTIC-PLASTIC	13505	10555	16457			9290	17720
COMMENTS	3	3	3			3	3

COMMENT LEGEND :
1. TITANIUM YIELDS ON END (THIN) STEP
2. FAILURE IN COMPOSITE OUTSIDE JOINT AT 18216 LB/IN.
3. FAILURE IN ADHESIVE AT COMPOSITE END OF JOINT
4. FAILURE IN COMPOSITE ONE STEP IN FROM TITANIUM END OF JOINT
5. FAILURE IN COMPOSITE ONE STEP IN FROM COMPOSITE END

ΔT = OPERATING TEMPERATURE - CURE TEMPERATURE OF ADHESIVE

was used to identify the most critical location and the step lengths were modified by hand for re-analysis until the minimum tensile and compressive joint strengths matched the composite adherend strength. This took only two iterations to achieve the results shown and this feature is one of the more beneficial merits of the complete internal joint analysis.

Figure 14 illustrates the bond shear stress distributions for both ductile and brittle adhesives. A comparison is effected between a joint of uniform step lengths, at left, and that with optimized lengths, at right. A small loss in elastic joint strength is incurred by shortening the end steps (and some of this could be recovered by increasing the lengths of the other steps to compensate) but the problem of yielding the end titanium step has been eliminated for the ductile adhesive. It is interesting to observe that the brittle adhesive had insufficient strain energy in shear for the problem to arise. Another important phenomenon revealed is that the ductile adhesive uses up only about a third of its ultimate shear strain capacity in breaking the composite adherend just outside the joint. This leaves a generous margin for dealing with

stress concentration due to irregularities in load intensity or bond thickness across the width of the joint. Because of these ever-present considerations, the brittle adhesive should not be expected to develop the full predicted joint strength over each inch of a wide panel. Failure would be initiated by a local effect and then be propagated rapidly.

Figure 14 omits consideration of thermal effects in order not to complicate the comparisons made. Figure 15 includes these effects for both tensile and compressive shear loading with the ductile adhesive. This figure compares the performance of the preliminary design (Figures 10 and 11) with the optimized design. Improvements in ultimate compressive strength and tensile fatigue load capacity are demonstrated.

8. CONCLUSION

This report presents elastic and elastic-plastic analysis methods for adhesive-bonded scarf and stepped-lap joints. The solutions obtained are analytic in form and the necessary digital computer FORTRAN IV programs are listed in the Appendices. These solutions are believed to be the first for such joints which account for adhesive plasticity. They include also the effects of adherend stiffness- and thermal-mismatch. While the precise solutions require iterative numerical solutions, explicit algebraic formulas are derived for a close lower-bound on the strength of scarf joints. The dominant characteristic of scarf joints is that, for long overlaps, the average bond stress asymptotes towards a fixed fraction of the peak bond stress, that fraction equalling the lesser ratio of adherend extensional stiffnesses. Unlike uniform lap joints, which reach a definite strength limit which cannot be exceeded by using longer overlaps, the potential bond strength of scarf joints increases indefinitely with overlap so that a design can always be devised in which the failure is forced to occur outside the joint. In using this approach, however, it is necessary also to check on the adherend stresses at the tip of the stiffer adherend to ensure that the scarf angle is not too small. Stepped-lap joints exhibit some characteristics of both the scarf joint and uniform double-lap joints. Those steps near the middle of a stepped-lap joint carry significantly more load than that transferred in the corresponding area of a uniform lap joint but the load so transferred is usually not a major contribution. Most of the load transfer is effected through the end three steps at one or both ends of the joint, depending on the nature of the adherend imbalances and the direction of the load. Within each step, since the governing equations are precisely the same as for an unbalanced double-lap joint, it is found that no further load can be transferred once the overlap has exceeded a determinable value. In other words, unlike scarf joints, the potential shear strength of stepped-lap joints cannot be increased indefinitely by increasing the overlap(s). The appropriate procedure is to employ more steps of finer thickness increments in order to augment the load capacity. Because scarf and stepped-lap joints can efficiently transfer load between thicker adherends than is possible with uniform double-lap joints, the latter are restricted to thinner sections in practical applications.

The inclusion of adhesive plasticity in the analysis has a marked effect on the

strength predictions. On the other hand, the elastic adhesive stresses play a far more important role in the behavior of scarf and stepped-lap joints than they do for uniform lap joints. The inclusion in the analyses of thermal mismatch effects permits their application to the bonding of titanium to the advanced composite laminates and explains how the joint strength changes with the load direction in such a situation.

The elastic-plastic analysis of the internal stresses within stepped-lap bonded joints provides sufficient information for the joint proportions to be optimized. Analyses should be performed for each load direction and at the extremes of the environmental temperature range, taking due account of material property changes with temperature, in the optimization sequence.

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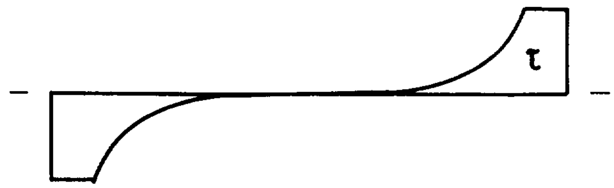
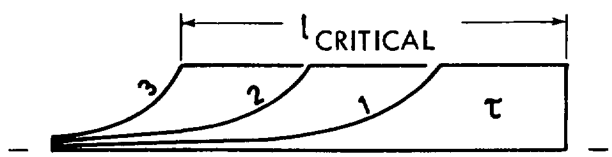
SCARF JOINT BETWEEN DISSIMILAR MATERIALS ($\alpha_1 > \alpha_2$, $E_1t_1 = E_2t_2$)

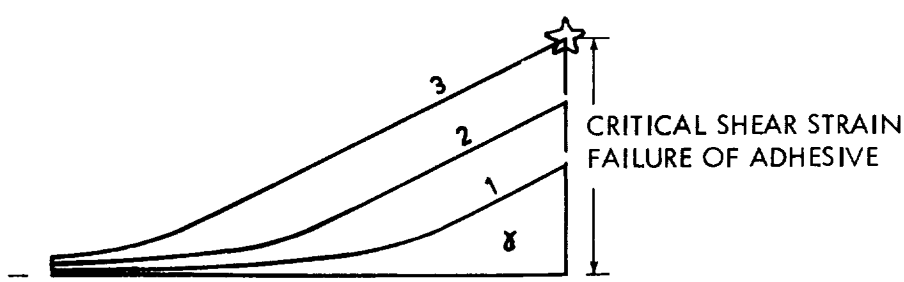


RESIDUAL ADHESIVE SHEAR STRESS

RESIDUAL ADHESIVE SHEAR STRAIN



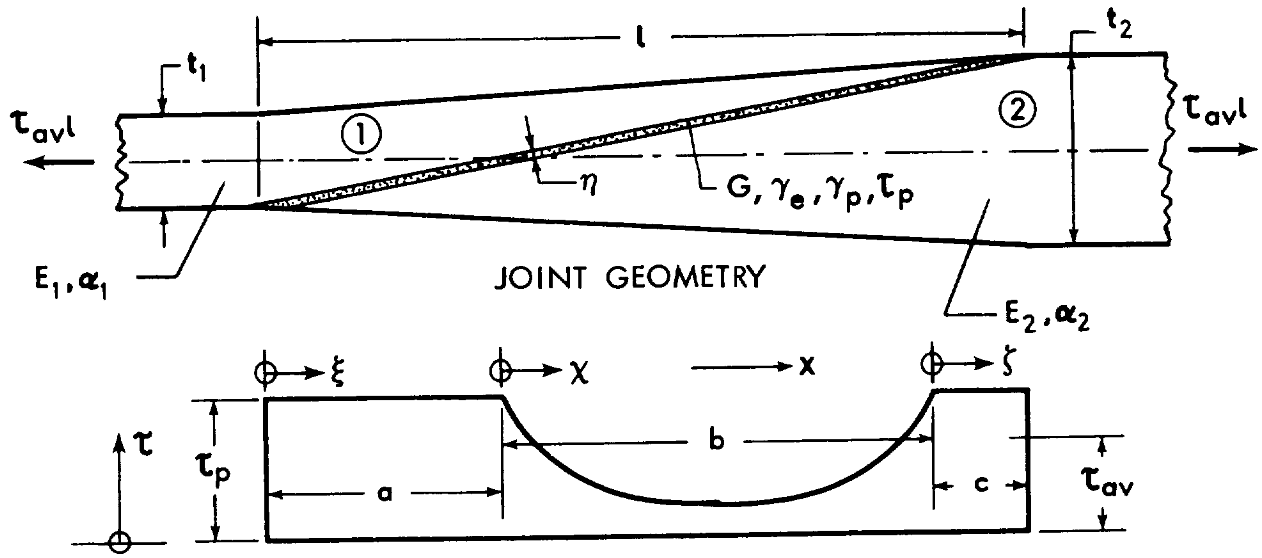
ADHESIVE SHEAR STRESSES
UNDER PROGRESSIVELY INCREASING TENSILE LOADS



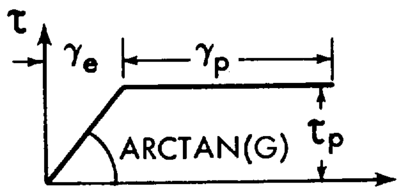
CORRESPONDING ADHESIVE SHEAR STRAINS

(NOTE THAT THE STRESSES AND STRAINS FOR PARTIAL LOADING, ABOVE, WOULD ALSO INDICATE THE BEHAVIOR UNDER STIFFNESS IMBALANCE IF $\alpha_1 = \alpha_2$ AND $E_1t_1 > E_2t_2$ BUT, WHEREAS THE CRITICAL END REVERSES WITH LOAD DIRECTION FOR THERMAL IMBALANCE, IT REMAINS THE SAME FOR STIFFNESS IMBALANCE.)

FIGURE 1. EXPLANATION OF NON-UNIFORM ADHESIVE SHEAR STRESSES IN BONDED SCARF JOINTS BETWEEN DISSIMILAR ADHERENDS



ADHESIVE BEHAVIOUR



ADHESIVE SHEAR CHARACTERISTIC

$$(\lambda l)^2 = \frac{G}{\eta}\left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right) l^2$$

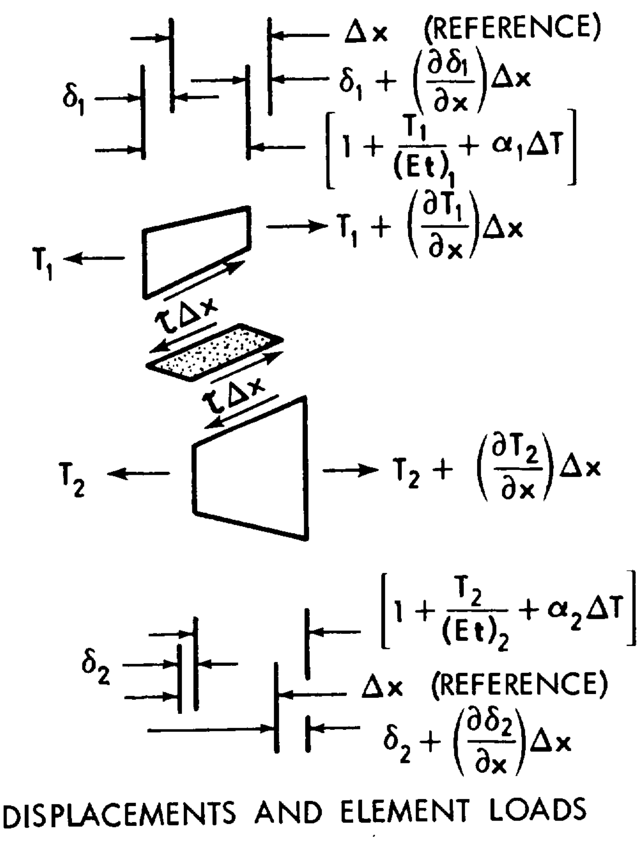
$$\mathrm{ETR(1)} = E_1 t_1 / E_2 t_2$$

$$\mathrm{ETR(2)} = E_2 t_2 / E_1 t_1$$

$$\mathrm{CTHERM(1)} = \frac{(\alpha_2 - \alpha_1)\Delta T \lambda}{\tau_p\left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right)}$$

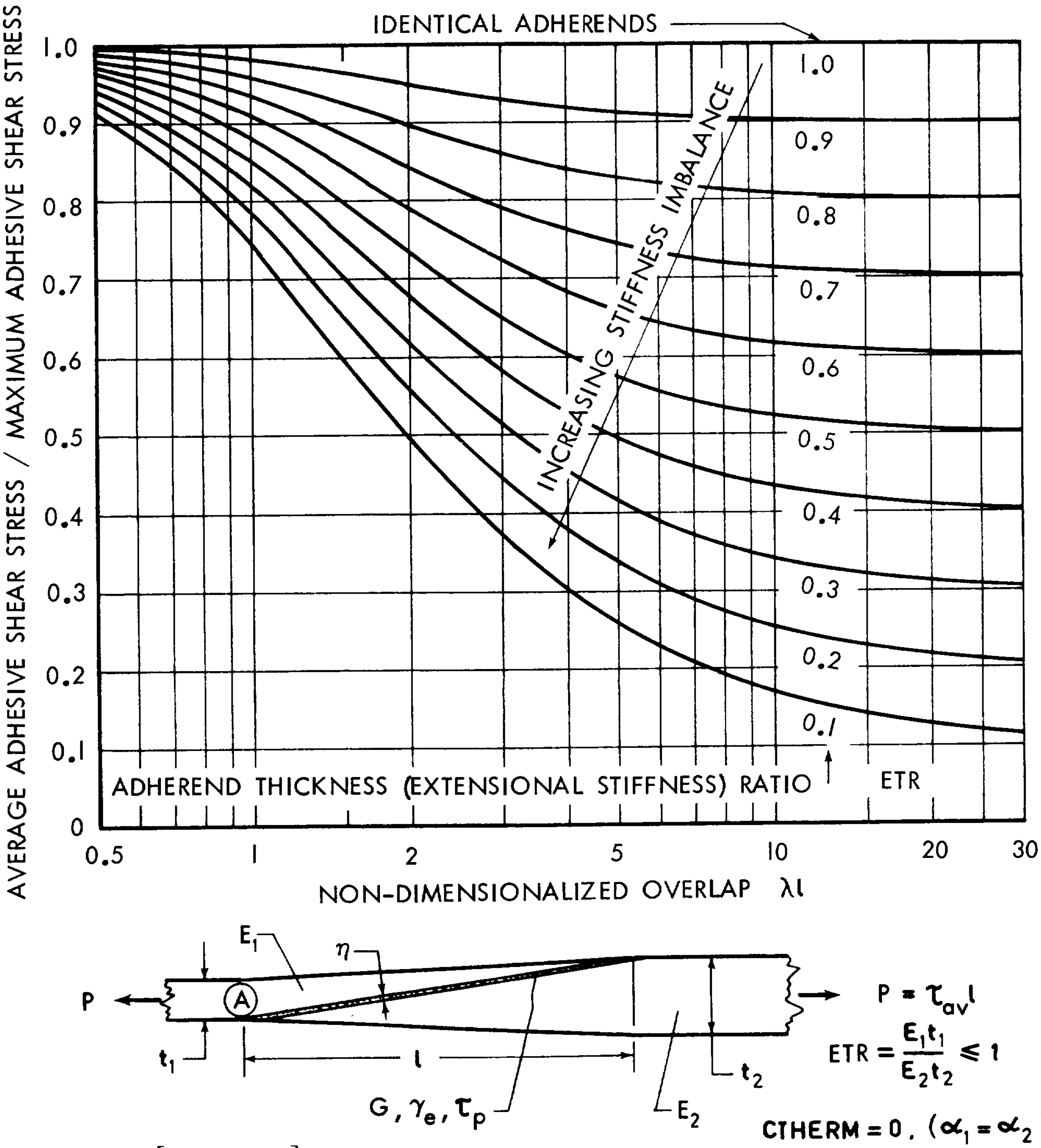
$$\mathrm{CTHERM(2)} = \frac{(\alpha_1 - \alpha_2)\Delta T \lambda}{\tau_p\left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right)}$$

NON-DIMENSIONALIZED
JOINT PARAMETERS



DISPLACEMENTS AND ELEMENT LOADS

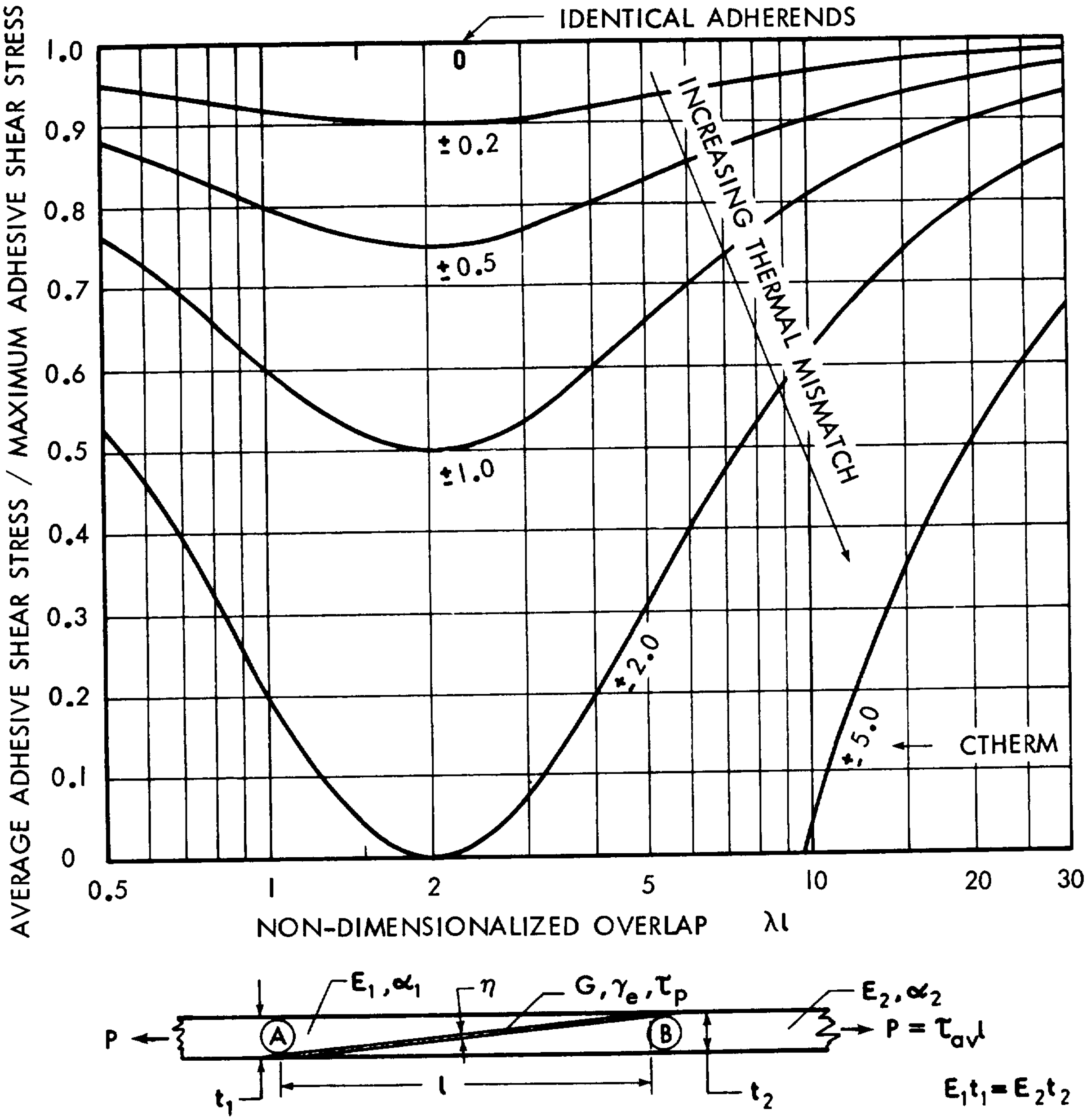
FIGURE 2. NOTATION AND GEOMETRY FOR ADHESIVE-BONDED SCARF JOINT ANALYSIS



$$\lambda^2 = \frac{G}{\eta}\left[\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right]$$

LOCATION A IS CRITICAL FOR BOTH POSITIVE (TENSILE LAP-SHEAR) AND NEGATIVE (COMPRESSIVE LAP-SHEAR) VALUES OF LOAD P

FIGURE 3. EFFECT OF ADHEREND STIFFNESS IMBALANCE ON ELASTIC STRENGTH OF BONDED SCARF JOINTS



$$\lambda^2 = \frac{G}{\eta}\left[\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right] \qquad \text{CTHERM} = \frac{(\alpha_2 - \alpha_1)\,\Delta T\,\lambda}{\tau_p\left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right)}, \qquad \Delta T = T_{\text{operating}} - T_{\text{stress free}}$$

LOCATION A CRITICAL FOR CTHERM < 0 AND P > 0
LOCATION A CRITICAL FOR CTHERM > 0 AND P < 0
LOCATION B CRITICAL FOR CTHERM < 0 AND P < 0
LOCATION B CRITICAL FOR CTHERM > 0 AND P > 0

FIGURE 4. EFFECT OF ADHEREND THERMAL MISMATCH ON ELASTIC STRENGTH OF BONDED SCARF JOINTS

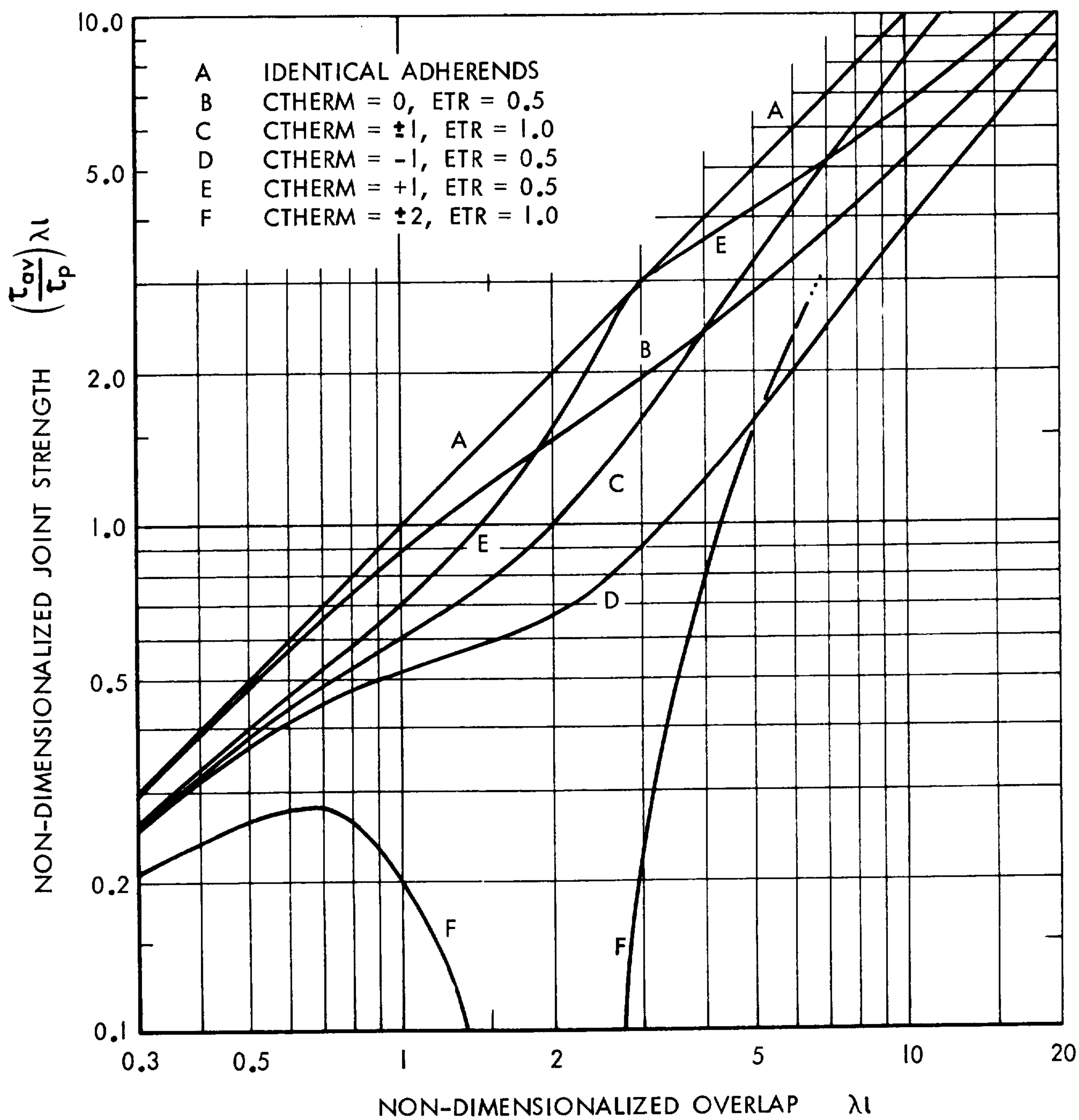


FIGURE 5. INTERACTION OF ADHERAND STIFFNESS AND THERMAL IMBALANCES FOR ELASTIC BONDED SCARF JOINTS

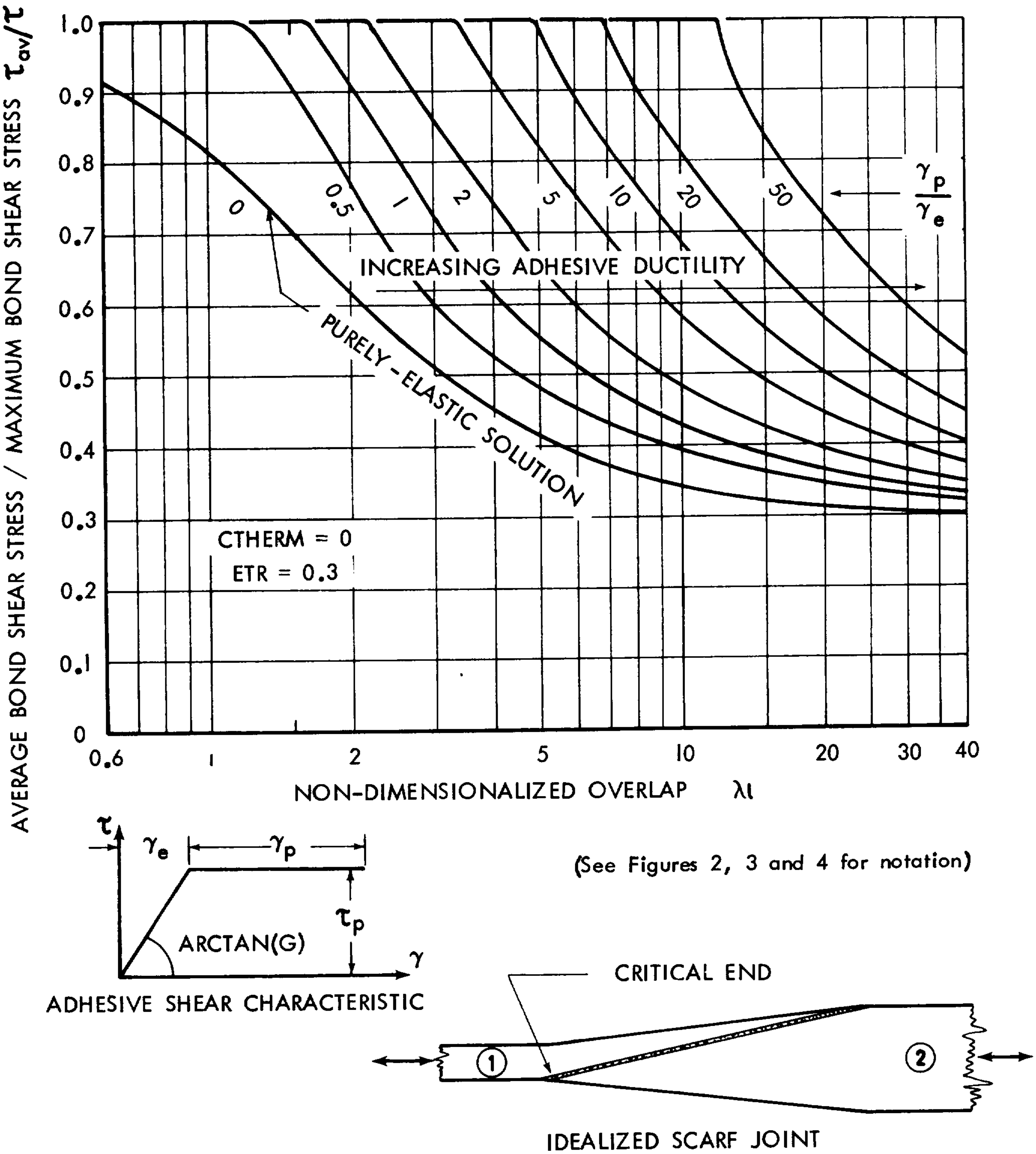
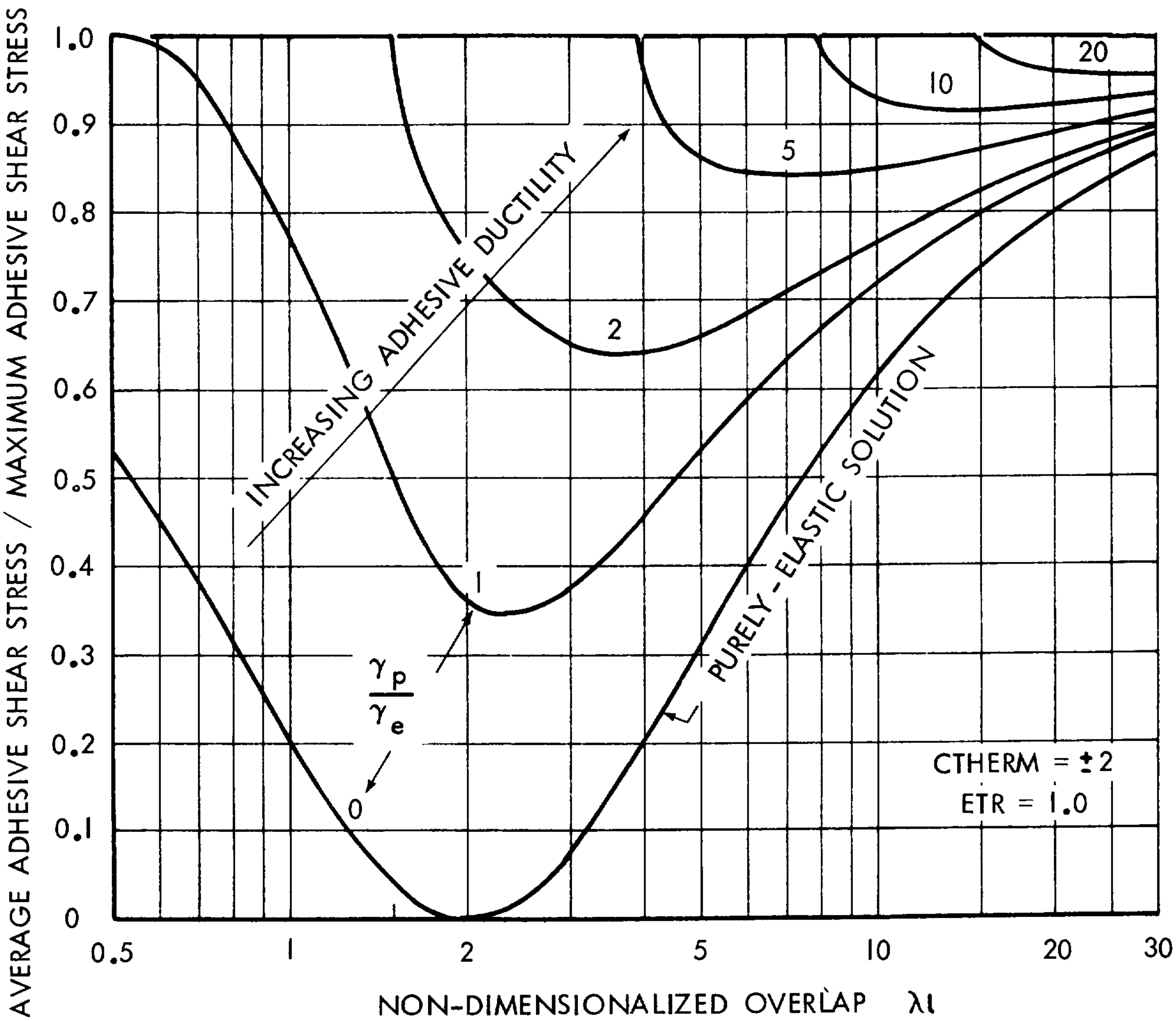


FIGURE 6. EFFECT OF ADHESIVE PLASTICITY IN REDUCING STRENGTH LOSS DUE TO ADHEREND STIFFNESS IMBALANCE FOR BONDED SCARF JOINTS



(See Figures 2, 3 and 4 for notation)

FIGURE 7. EFFECT OF ADHESIVE PLASTICITY IN REDUCING STRENGTH LOSS DUE TO ADHEREND THERMAL MISMATCH FOR BONDED SCARF JOINTS

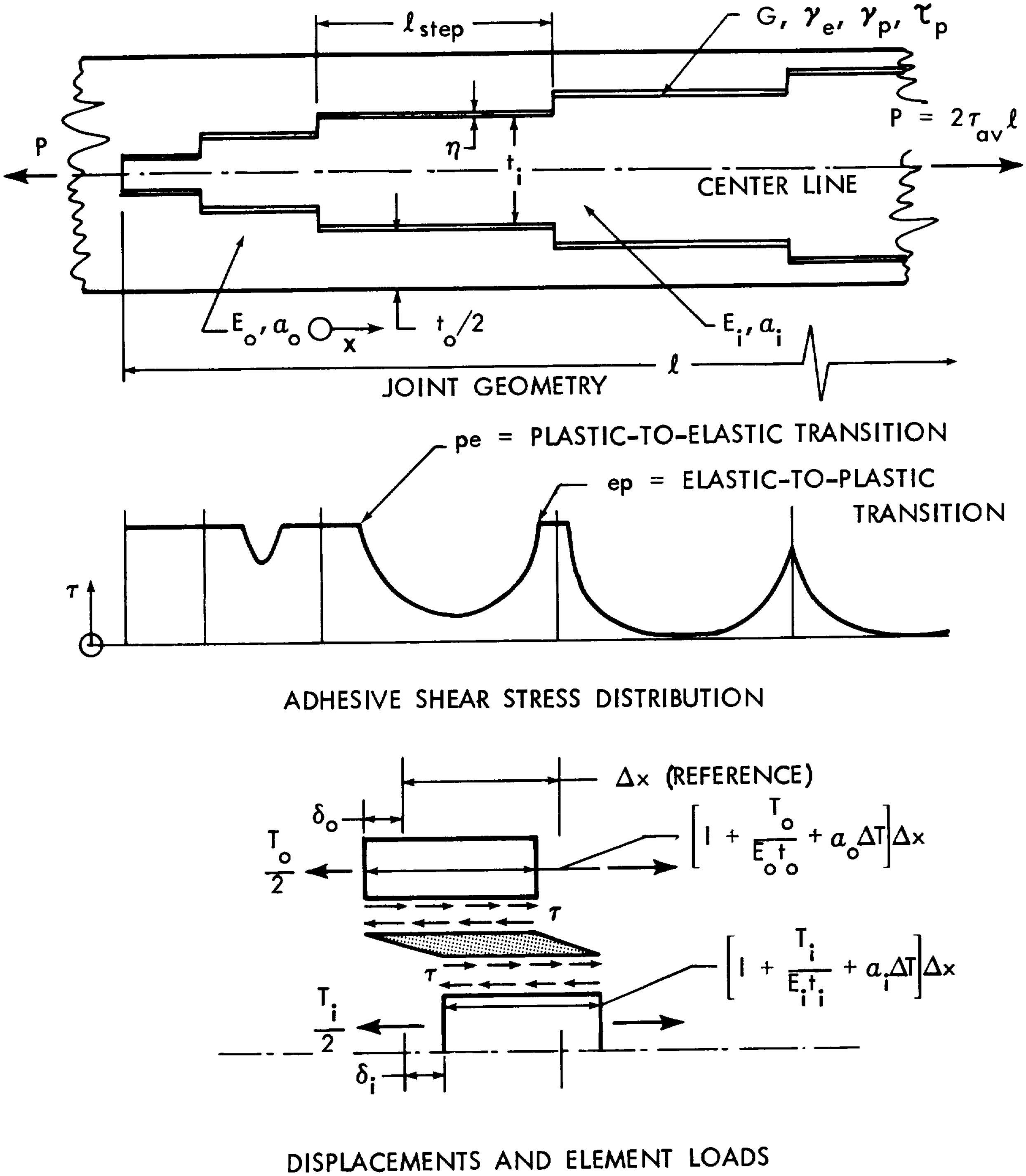
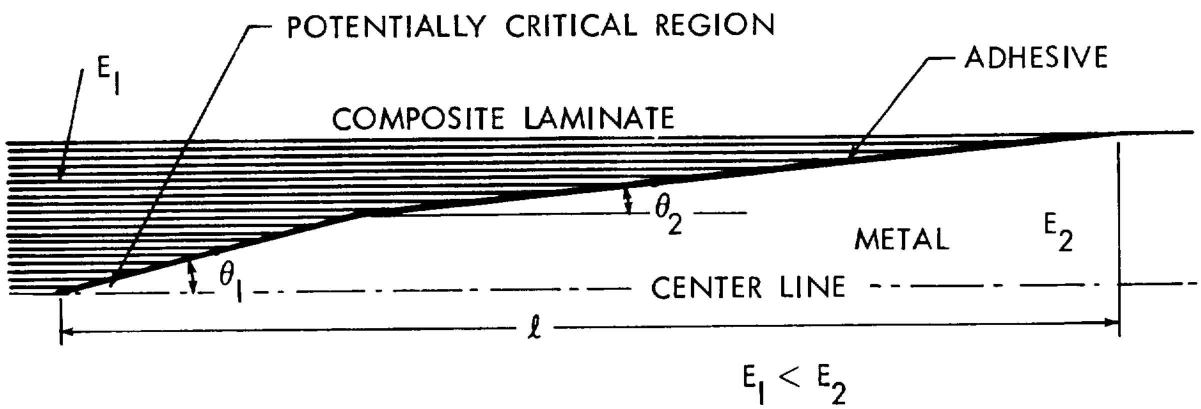
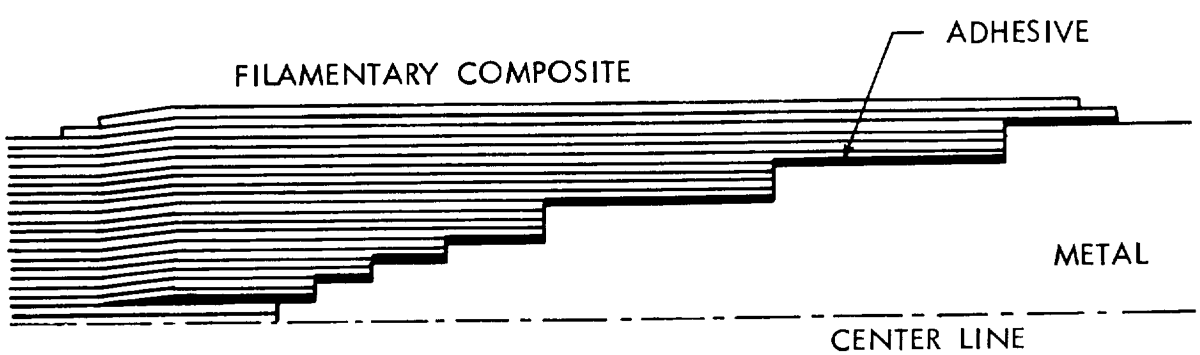


FIGURE 8. NOTATION AND GEOMETRY FOR ADHESIVE-BONDED STEPPED-LAP JOINT ANALYSIS



(A) . IDEALIZED DUAL-SLOPE SCARF JOINT TO PROTECT TIP OF STIFFER ADHEREND



(B) . REPRESENTATION OF OPTIMIZED STEP PROPORTIONS FOR STEPPED-LAP JOINT WITH MODULUS OF COMPOSITE LAMINATE LESS THAN THAT OF METAL ADHEREND

FIGURE 9. PRACTICAL PROPORTIONING OF STEPPED-LAP JOINTS TO PROTECT AGAINST FATIGUE FAILURES AT TIP OF METAL ADHEREND

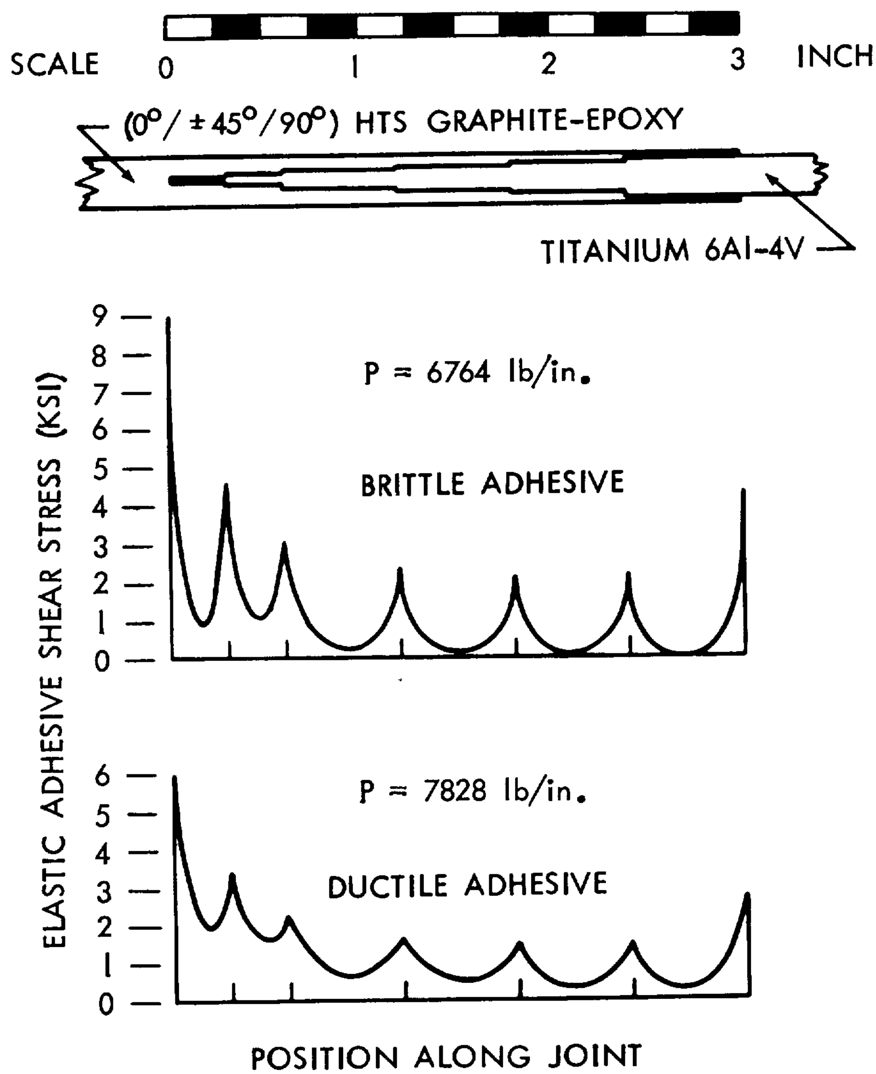


FIGURE 10. ELASTIC SHEAR STRESS DISTRIBUTIONS FOR BRITTLE AND DUCTILE ADHESIVES IN BONDED STEPPED-LAP JOINTS

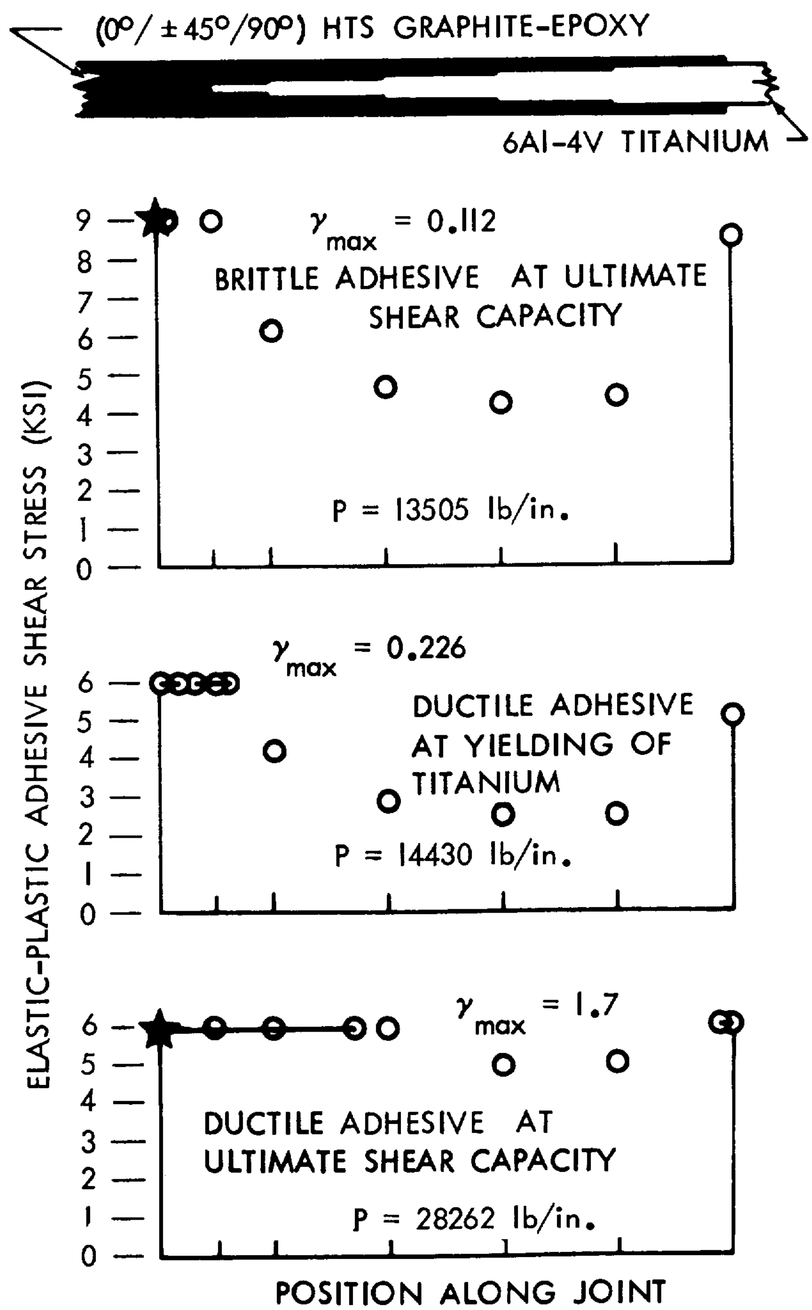


FIGURE 11. ELASTIC-PLASTIC SHEAR STRESS DISTRIBUTIONS FOR BRITTLE AND DUCTILE ADHESIVES IN BONDED STEPPED-LAP JOINTS

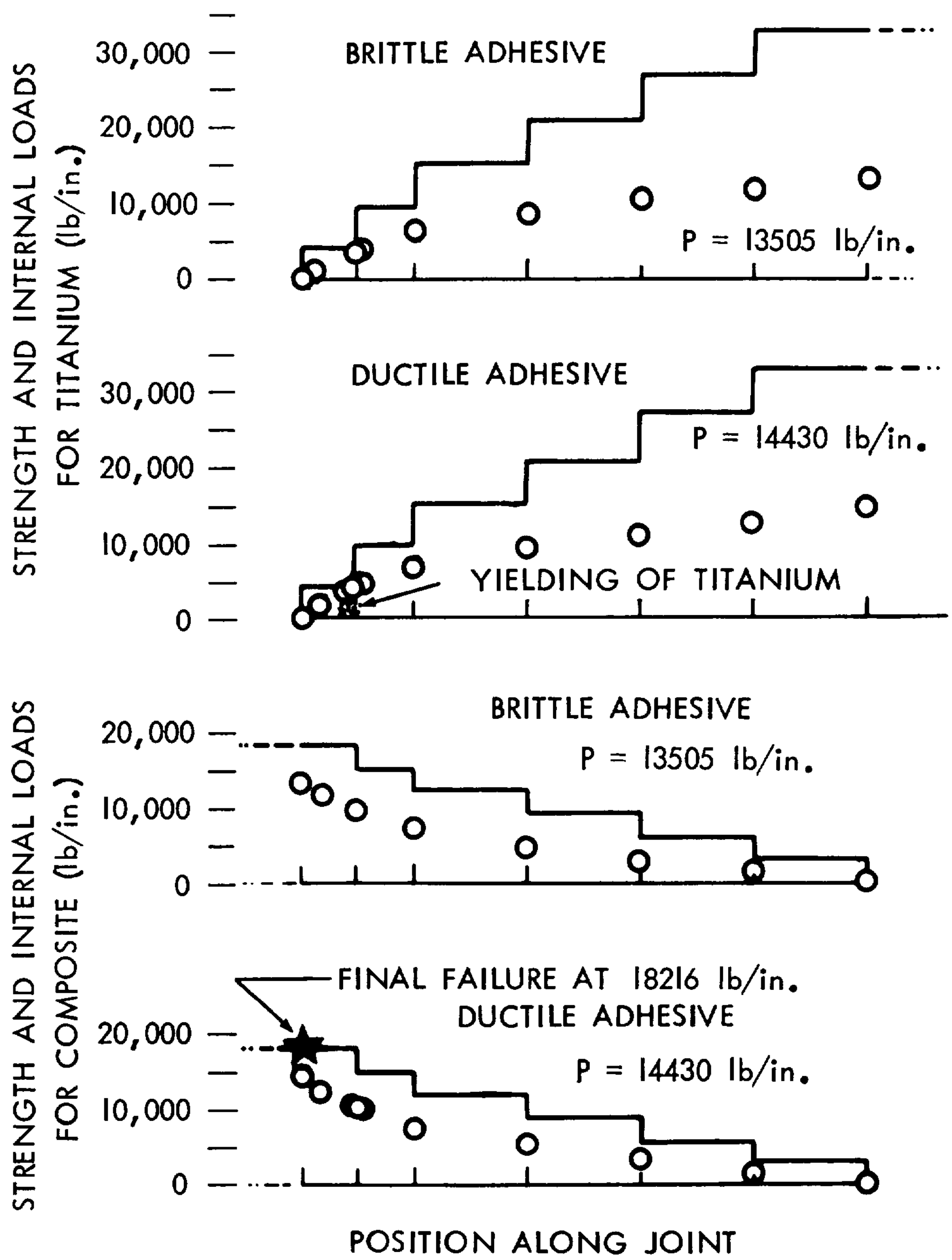


FIGURE 12. ADHEREND STRENGTHS AND INTERNAL LOADS FOR BONDED STEPPED-LAP JOINTS

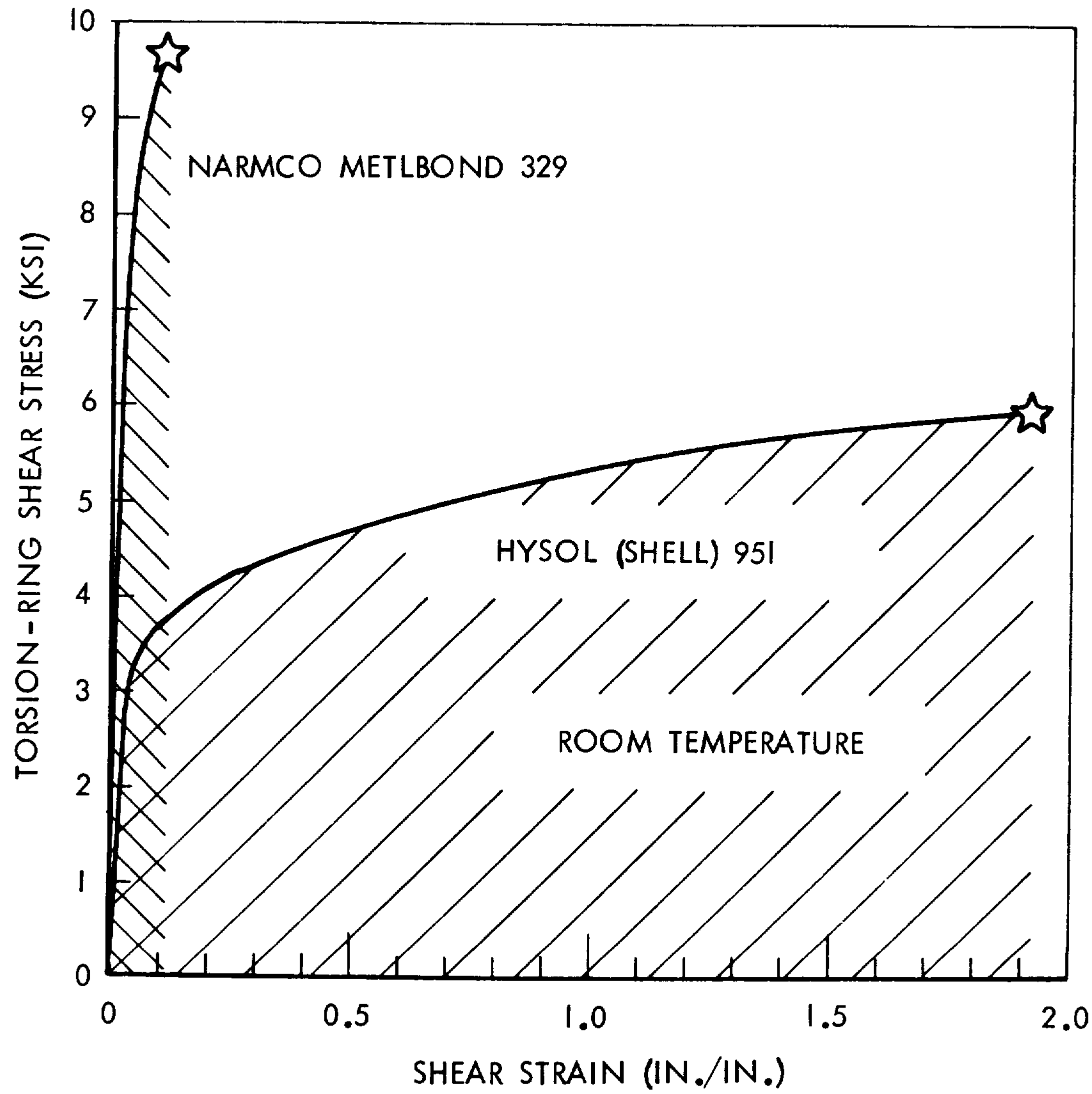


FIGURE 13. COMPARISON OF SHEAR STRESS-STRAIN CHARACTERISTICS FOR BRITTLE AND DUCTILE ADHESIVES

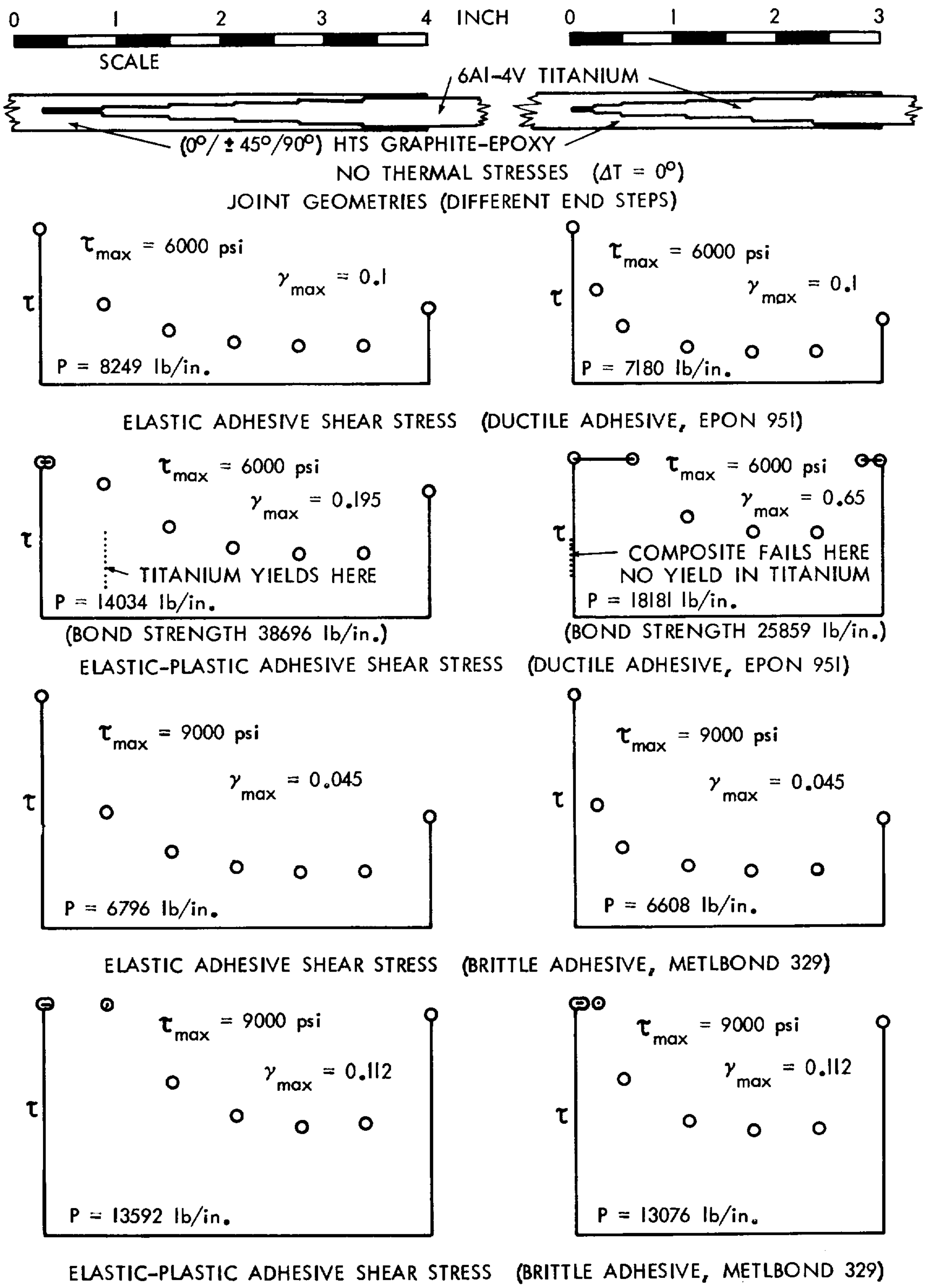
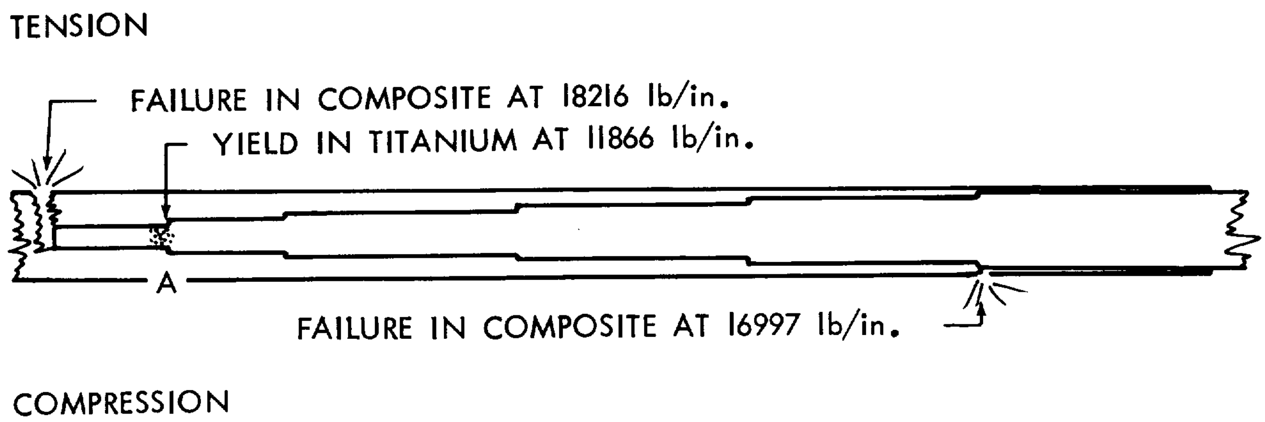
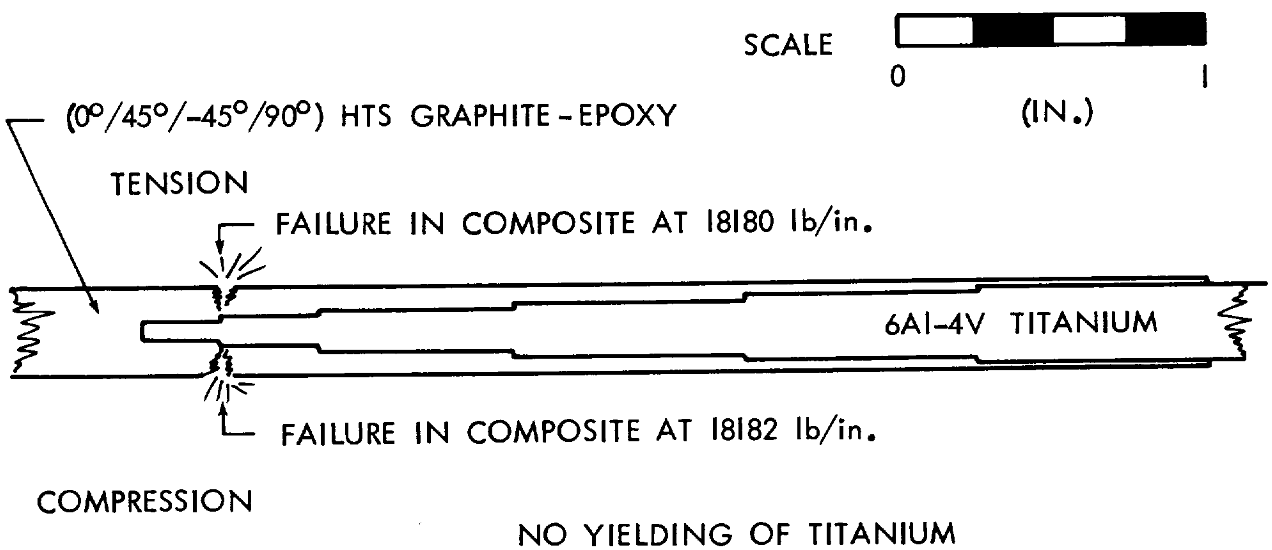


FIGURE 14. COMPARISON BETWEEN STEPPED-LAP JOINTS WITH UNIFORM STEP LENGTHS AND WITH OPTIMIZED STEP LENGTHS



NOTE THAT TITANIUM END STEPS WERE ALREADY SHORTENED DURING PRELIMINARY DESIGN. WITH UNIFORM STEPS 0.75 IN. LONG THROUGHOUT, PREMATURE FATIGUE FAILURE WOULD OCCUR AT A, FOLLOWED BY FAILURE OF COMPOSITE AT THE SAME (REDUCED) SECTION.

(A) PRELIMINARY DESIGN



(B) OPTIMIZED DESIGN

DUCTILE ADHESIVE CURED AT 350°F.
STRENGTHS CALCULATED AT ROOM TEMPERATURE.
STRENGTH OF COMPOSITE ADHEREND OUTSIDE JOINT = 18216 lb/in.
POTENTIAL BOND SHEAR STRENGTH WOULD EXCEED 23257 lb/in. IN EVERY CASE SHOWN IF ADHERENDS WERE SUFFICIENTLY STRONG.

FIGURE 15. OPTIMIZATION OF DETAILS IN STEPPED-LAP BONDED JOINTS

APPENDICES

A.1 Computer Program A4EC For Elastic Strength of Bonded Scarf Joints

The FORTRAN IV digital computer program associated with the analysis in Section 2 is listed below. This program has been checked out thoroughly and sample solutions are illustrated in Section 4. Only shear stresses are considered, with the peel stresses neglected in accordance with the very small scarf angles used in practice. As discussed in Section 2, there are severe convergence problems associated with the series solution to this problem. While the average shear stresses computed are considered very reliable, no computation sequence for the stress distribution was found which was considered sufficiently accurate over the far end of the joint ($x/\ell \simeq 1$). The peak shear stress is located correctly by program A4EC at one end or other of the overlap. The only real need for a shear stress distribution is as an intermediate step in the computation of the internal adherend stresses. Since the convergence of the series was enhanced greatly by prior integration into the contributions to the average shear stress, it is recommended that any attempt to pursue the adherend stress distribution should proceed along similar lines. The adhesive shear stress distribution series can be integrated mathematically so that a more tractable series solution is obtained for the adherend stresses. The first two terms follow from the average shear stress solution and the subsequent ones would derive from recurrence formulae. The condition under which a need for such information could arise is the possible breaking off of the thin tip of the stiffer adherend for a very small scarf angle. Such a situation is unlikely for perfectly elastic adhesives because the shear stress drops off very rapidly away from the ends. A simpler procedure is available for the elastic-plastic adhesive.

The format of the input data necessary to operate the A4EC computer program is as follows:

CARD 1:

FORMAT (4I5)

IMAX = Number of thermal mismatch coefficients. IMAX .LE. 20.

JMAX = Number of non-dimensionalized overlaps. JMAX .LE. 40. (Note that this is one more than the number of overlaps to be read in. The limiting case of OL(1)=0 is set by the program.)

KMAX = Number of adherend stiffness imbalances. KMAX .LE. 10. (Note that this controls the number of columns of answers printed across the page and cannot be increased indefinitely.)

NMAX = Number of terms in power series. 10 .LE. NMAX .LE. 50. (Note NMAX = 20 is recommended.)

CARDS 2, 2A, 2B, etc.:

FORMAT (12F6.2)

OL(J)= Non-dimensionalized overlaps. Number restricted to 40 by dimension statement. (Note that OL(J) must be read in in ascending order and that OL(2), which is the first entry on card 2, must not exceed 0.5 because of internal computations. OL(1) = 0 is set by program as limiting case.) Values of OL(J) exceeding 50 are impractically large.

CARDS 3, 3A, 3B, etc.:

FORMAT (10F5.2)

ETR(K)=Adherend stiffness ratios $(E_1t_1)/(E_2t_2)$. Number restricted to 10 by dimension statement. (Subscripts 1 and 2 must be identified so that 0 .LT. ETR .LE. 1. Array should be read in in ascending or descending order.)

CARDS 4, 4A, 4B, etc.:

FORMAT (10F7.3)

CTHERM(I) = Adherend thermal mismatch coefficients in non-dimensionalized form. Number restricted to 20 by dimension statement. (Note that equal and opposite values must be read in consecutively to account for the difference between tensile and compressive application of the shear load. Values of up to ±5 are sufficient for the available range of adhesives. Greater values are usually associated with failure of the joint under residual thermal stresses alone.)

The complete listing follows, along with sample output pages. The output tables come in pairs with the ratio of the average to maximum adhesive shear stress (τ_{av}/τ_p) and the non-dimensionalized joint strength $(\tau_{av}/\tau_p)(\lambda\ell)$ as functions of the adherend extensional stiffness ratio $\mathrm{ETR} = E_1t_1/E_2t_2 \leq 1$ horizontally and the non-dimensionalized joint overlap $\lambda\ell = \sqrt{\frac{G}{\eta}\left(\frac{1}{E_1t_1} + \frac{1}{E_2t_2}\right)\ell^2}$ vertically. Each table is prepared for a single value of thermal mismatch coefficient $\mathrm{CTHERM} = \dfrac{(\alpha_2 - \alpha_1)\Delta T\lambda}{\tau_p\left(\frac{1}{E_1t_1} + \frac{1}{E_2t_2}\right)}$ and equal and opposite values are treated in turn to cover both tensile and compressive shear loadings.

```
CDECK A4EC
C ELASTIC ANALYSIS OF UNBALANCED SCARF JOINTS
C NON-DIMENSIONALIZED FORMULATION
C AVERAGE STRESSES
C STIFFNESS AND THERMAL IMBALANCES ACCOUNTED FOR
C DATA PRESENTATION FOR TENSILE SHEAR LOADING
C CHANGE SIGN OF CTHERM TO USE FOR COMPRESSIVE SHEAR LOADS
C DIMENSION  OL(J), ETR(K), CTHERM(I), A(N,2), TRATIO(N,2),
C     1   TAUAVG(J,K), ICRTND(J,K), STRGTH(J,K), SIG(N,2),
      DIMENSION OL(40), ETR(10), CTHERM(20), A(50,2),          SIG(50,2),
     1     TRATIO(50,2), TAUAVG(40,10), ICRTND(40,10), STRGTH(40,10)
C READ IN ARRAY SIZES
      READ (5,10) IMAX, JMAX, KMAX, NMAX
   10 FORMAT (4I5)
C IMAX .LE. 20, JMAX .LE. 40, KMAX .LE. 10, NMAX .LE. 50 .AND. .GE. 10
C READ IN NON-DIMENSIONALIZED OVERLAP ARRAY
      OL(1) = 0.
C OL(2) MUST BE .LE. 0.5 BECAUSE OF A SUBSEQUENT TREND CHECK
C OL(J) MUST BE IN ASCENDING ORDER
      READ (5,20) (OL(J), J = 2, JMAX)
   20 FORMAT (12F6.2)
C READ IN STIFFNESS IMBALANCE ARRAY
C IDENTIFY ADHERENDS 1 AND 2 SUCH THAT ETR(K) = (ET)1/(ET)2 .LE. 1.
      READ (5,30) (ETR(K), K = 1, KMAX)
   30 FORMAT (10F5.2)
C READ IN NON-DIMENSIONALIZED THERMAL MISMATCH COEFFICIENTS
      READ (5,40) (CTHERM(I), I = 1, IMAX)
   40 FORMAT (10F7.3)
C PRINT OUT INPUT DATA
      WRITE (6,50) IMAX, JMAX, KMAX, NMAX
   50 FORMAT (1H1, 9H  IMAX = , I2, 9H  JMAX = , I2, 9H  KMAX = , I2,
     1   9H  NMAX = , I2)
      WRITE (6,60) (OL(J), J = 1, JMAX)
   60 FORMAT (10H  OVERLAPS/, 12F6.2)
      WRITE (6,70) (ETR(K), K = 1, KMAX)
   70 FORMAT (22H  STIFFNESS IMBALANCES/, 10F5.2)
      WRITE (6,80) (CTHERM(I), I = 1, IMAX)
   80 FORMAT (20H  THERMAL MISMATCHES/, 10F7.3)
C SET UNIFORM STRESS FOR ZERO OVERLAP
      DO 90 K = 1, KMAX
      TAUAVG(1,K) = 1.
   90 STRGTH(1,K) = 0.
C START OF COMPUTATION DO LOOPS
      DO 310 I = 1, IMAX
      DO 180 K = 1, KMAX
      DO 180 J = 2, JMAX
C ESTABLISH ADHEREND 1 END OF JOINT AS REFERENCE
C SUBSEQUENTLY CHECK WHETHER ADHEREND 1 END OR ADHEREND 2 END IS CRITICA
      NCRTND = 1
      THERMC = CTHERM(I)
      VR = ETR(K)
      IF ((VR .NE. 1.) .OR. (THERMC .NE. 0.)) GO TO 100
C SET UNIFORM STRESS FOR BALANCED JOINTS
      TAUAVG(J,K) = 1.
      STRGTH(J,K) = OL(J)
      ICRTND(J,K) = 0
      GO TO 180
  100 V1 = 1. + VR
      V2 = VR / V1
      V3 = (1. - VR) / V1
      OLAP = OL(J)
      OLAP2 = OLAP * OLAP
C COMPUTE INITIAL TERMS OF SERIES, ASSUMING A(1,1)=A(2,2)=1. & A(1,2)=A(
      A(1,1) = 1.
      A(2,1) = 0.
      A(3,1) = (OLAP/6.) * (-THERMC + V3*OLAP)
      A(1,2) = 0.
      A(2,2) = 0.5
      A(3,2) = (OLAP2 / 6.) * ( V2/2. ) + 1./6.
C COMPUTE NMAX TERMS OF AVERAGE STRESS POWER SERIES
      DO 110 N = 4, NMAX
      DO 110 M = 1, 2
  110 A(N,M) = ( ((N-2)*(N-1) + OLAP2 * V2) * A(N-1,M)
     1            + V3 * OLAP2 * A(N-2,M) ) / (N*(N-1))
C COMPUTE A2 THROUGH RAPID CONVERGENCE OF AVERAGE STRESS
C NOTE THAT INDIVIDUAL TERMS IN DISTRIBUTION DO NOT CONVERGE AS RAPIDLY
      SIG(3,1) = 1. + A(3,1)
      SIG(3,2) = 0.5 + A(3,2)
      DO 120 N = 4, NMAX
      SIG(N,1) = SIG(N-1,1) + A(N,1)
  120 SIG(N,2) = SIG(N-1,2) + A(N,2)
C COMPUTE A2(NMAX)
      A2SAVE = (THERMC/OLAP + V2 - (SIG(NMAX,1)/V1) )   /
     1     ( (SIG(NMAX,2)/V1) + (1./OLAP2) )
C COMPUTE AVERAGE SHEAR STRESS IN BOND
      TRATIO(J,NCRTND) = V1 * (V2 + THERMC/OLAP - A2SAVE/OLAP2)
C CHECK WHICH END OF JOINT IS CRITICAL
      IF (NCRTND .EQ. 2) GO TO 130
```

```
      IF (THERMC .LT. 0.) GO TO 130
C IF ADHEREND 2 END IS CRITICAL INTERCHANGE 1 AND 2 AND RECOMPUTE
      NCRTND = 2
      VR = 1. / VR
      THERMC = - THERMC
      GO TO 100
  130 CONTINUE
C IDENTIFY AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS & CRITICAL END
C NOTE THAT INITIAL SELECTION CRITERIA ASSUME ONE END OF JOINT OR
C     1 OTHER IS CRITICAL AND PRECLUDE POSSIBILITY OF MAXIMUM STRESS IN
C     2  MIDDLE. SUBSEQUENT SEPARATE CHECK ON THIS CONDITION.
      IF (CTHERM(I) .GT. 0.) GO TO 140
C BOTH IMBALANCES WILL INEVITABLY COMPOUND FOR CTHERM(I) .LT. 0.
C     1  SINCE ETR(K) .LE. 1..  HENCE NCRTND = 1
      ICRTND(J,K) = 1
      IF (TRATIO(J,1) .GE. 0.) GO TO 160
      IF (TRATIO(J,1) .LT. 0.) GO TO 150
  140 CONTINUE
C IDENTIFY MORE POWERFUL IMBALANCE FOR NULLIFYING BEHAVIOUR (NCRTND =  )
C     1  CTHERM(I) .GT. 0. .AND. ETR(K) .LE. 1.
C COVER SITUATION WHERE THERMAL IMBALANCE DOMINATES OVER STIFFNESS
C     1  IMBALANCE.  NOTE NEED OL(2) .LE. 0.5 FOR THIS CHECK.
      ICRTND(J,K) = 2
      IF ((TRATIO(2,1).GT.1.).AND.(TRATIO(J,2).LT.0.))GO TO 150
      IF ((TRATIO(2,1).GT.1.).AND.(TRATIO(J,2).LE.1.))GO TO 170
C CHECK IF TWO IMBALANCES PRECISELY CANCEL
      ICRTND(J,K) = 0
      IF ((TRATIO(J,1) .EQ. 1.) .AND. (TRATIO(J,2) .EQ. 1.)) GO TO 160
C CHECK IF STIFFNESS IMBALANCE DOMINATES
      ICRTND(J,K) = 1
      IF ((TRATIO(J,1) .LE. 1.) .AND. (TRATIO(J,1) .GE. 0.) .AND.
     1   (TRATIO(J,2) .GT. 1.)) GO TO 160
C ALL POSSIBILITIES FOR EITHER END CRITICAL CHECKED OUT
C ONLY POSSIBILITY REMAINING IS THAT TAUMAX IS IN MIDDLE OF JOINT
C NOTE THAT THIS PHENOMENON ARISES ONLY FOR JOINTS BROKEN WITHOUT LOAD
C     1  WHEN THE LOAD IN THE OPPOSITE SENSE IS EXAMINED
C COMBINATION OF SEVERE THERMAL MISMATCH AND EXCESSIVE LENGTH IS NECESSA
C IDENTIFY FAILURE CASES BY ASTERISKS
      TAUAVG(J,K) = 100.
      STRGTH(J,K) = 1000.
      ICRTND (J,K) = 10
      GO TO 180
C ZERO STRENGTH ATTAINED
  150 TAUAVG(J,K) = 0.
      STRGTH(J,K) = 0.
      GO TO 180
C ADHEREND 1 END OF JOINT CRITICAL
  160 TAUAVG(J,K) = TRATIO(J,1)
      STRGTH(J,K) = TAUAVG(J,K) * OL(J)
      GO TO 180
C ADHEREND 2 END OF JOINT CRITICAL
  170 TAUAVG(J,K) = TRATIO(J,2)
      STRGTH(J,K) = TAUAVG(J,K) * OL(J)
  180 CONTINUE
C IDENTIFY CRITICAL END OF JOINT FOR ZERO OVERLAP
      DO 190 K = 1, KMAX
  190 ICRTND(1,K) = ICRTND(2,K)
C HENCE NEED OL(2) .LE. 0.2
      IF (CTHERM(I) .NE. 0.) GO TO 210
C PRINT OUT SPECIAL HEADING FOR ZERO THERMAL MISMATCH BETWEEN ADHERENDS
      WRITE (6,200) (ETR(K), K = 1, KMAX)
  200 FORMAT (1H1,10(/),31X, 48HADHESIVE-BONDED SCARF JOINTS  (ELASTIC A
     1NALYSIS)/, 39X, 31HNON-DIMENSIONALIZED FORMULATION//,
     2  38X, 33HZERO THERMAL MISMATCH COEFFICIENT//,
     3  68X, 28H0 = BOTH ENDS EQUALLY LOADED/, 20X, 72HAVERAGE SHEAR STR
     4ESS / MAXIMUM SHEAR STRESS    ,  1 = SOFT ET END CRITICAL/,
     5  68X, 25H2 = STIFF ET END CRITICAL//,
     6 8H  SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
     7  7H    L/T/, 7H  RATIO/, 1H+, 4X, 10F10.1//)
      GO TO 230
  210 THERMC = - CTHERM(I)
C PRINT OUT HEADING
      WRITE (6,220) CTHERM(I), THERMC, (ETR(K), K = 1, KMAX)
  220 FORMAT (1H1,10(/),31X, 48HADHESIVE-BONDED SCARF JOINTS  (ELASTIC A
     1NALYSIS)/, 39X, 31HNON-DIMENSIONALIZED FORMULATION//, 17X, 31HTHER
     2MAL MISMATCH COEFFICIENT = , F6.3, 17H FOR TENSION,  = ,F6.3, 16H
     3FOR COMPRESSION//,68X, 28H0 = BOTH ENDS EQUALLY LOADED/,
     4            20X, 72HAVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS   ,
     51 = SOFT ET END CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL//,
     6  9H  SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
     7  7H    L/T/, 7H  RATIO/, 1H+, 4X, 10F10.1/)
  230 CONTINUE
C PRINT OUT TABULATIONS OF AVERAGE BOND STRESSES
      DO 250 J = 1, JMAX
      WRITE (6,240) OL(J), ((TAUAVG(J,K),ICRTND(J,K)), K = 1, KMAX)
  240 FORMAT (1H , F6.2, 2X, 10(F7.5, 1X, I1, 1X))
  250 CONTINUE
      IF (CTHERM(I) .NE. 0.) GO TO 270
```

```
C PRINT OUT SPECIAL HEADING FOR ZERO THERMAL MISMATCH BETWEEN ADHERENDS
      WRITE (6,260) (ETR(K), K = 1, KMAX)
  260 FORMAT (1H1,10(/),31X, 48HADHESIVE-BONDED SCARF JOINTS  (ELASTIC A
     1NALYSIS)/, 39X, 31HNON-DIMENSIONALIZED FORMULATION//,
     2  38X, 33HZERO THERMAL MISMATCH COEFFICIENT//,
     3  68X, 28HO = BOTH ENDS EQUALLY LOADED/, 20X, 72HNON-DIMENSIONALIZ
     4ED STRENGTH   ,                          1 = SOFT ET END CRITICAL/,
     5  68X, 25H2 = STIFF ET END CRITICAL//,
     6  8H  SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
     7  7H     L/T/, 7H  RATIO/, 1H+, 4X, 10F10.1//)
      GO TO 290
C PRINT OUT HEADINGS
  270 WRITE (6,280) CTHERM(I), THERMC, (ETR(K), K = 1, KMAX)
  280 FORMAT (1H1,10(/),31X, 48HADHESIVE-BONDED SCARF JOINTS  (ELASTIC A
     1NALYSIS)/, 39X, 31HNON-DIMENSIONALIZED FORMULATION//, 17X, 31HTHER
     2MAL MISMATCH COEFFICIENT = , F6.3, 17H FOR TENSION,   = ,F6.3, 16H
     3FOR COMPRESSION//,68X, 28HO = BOTH ENDS EQUALLY LOADED/,
     4             20X, 72HNON-DIMENSIONALIZED STRENGTH   ,
     51 = SOFT ET END CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL//,
     6  8H  SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
     7  7H     L/T/, 7H  RATIO/, 1H+, 4X, 10F10.1/)
  290 CONTINUE
C PRINT OUT TABULATIONS OF NON-DIMENSIONALIZED STRENGTHS
      DO 310 J = 1, JMAX
      WRITE (6,300) OL(J), ((STRGTH(J,K), ICRTND(J,K)), K = 1, KMAX)
  300 FORMAT (1H , F6.2, 2X, 10(F7.4, 1X, I1, 1X))
  310 CONTINUE
      WRITE (6,320)
  320 FORMAT (1H1, 18H PROGRAM COMPLETED/)
      STOP
      END
```

ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

NON-DIMENSIONALIZED STRENGTH ,
0 = BOTH ENDS EQUALLY LOADED
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2
0.20	0.1832 2	0.1826 2	0.1822 2	0.1818 2	0.1814 2	0.1811 2	0.1808 2	0.1806 2	0.1804 2	0.1802 2
0.50	0.4245 2	0.4161 2	0.4093 2	0.4035 2	0.3987 2	0.3945 2	0.3909 2	0.3877 2	0.3849 2	0.3823 2
1.00	0.9012 2	0.8301 2	0.7765 2	0.7347 2	0.7012 2	0.6736 2	0.6507 2	0.6312 2	0.6145 2	0.6000 2
1.20	1.1853 2	1.0527 2	0.9576 2	0.8861 2	0.8304 2	0.7957 2	0.7491 2	0.7186 2	0.6927 2	0.6706 2
1.50	1.3881 1	1.5000 2	1.3016 2	1.1615 2	1.0571 2	0.9765 2	0.9123 2	0.8601 2	0.8166 2	0.7800 2
1.70	1.4858 1	1.6219 1	1.5915 2	1.3857 2	1.2372 2	1.1251 2	1.0375 2	0.9671 2	0.9093 2	0.8611 2
2.00	1.6072 1	1.7784 1	1.9453 1	1.7899 2	1.5561 2	1.3850 2	1.2545 2	1.1516 2	1.0685 2	1.0000 2
2.50	1.7635 1	1.9898 1	2.2144 1	2.4369 1	2.2136 2	1.9151 2	1.6943 2	1.5244 2	1.3898 2	1.2805 2
3.00	1.8839 1	2.1618 1	2.4410 1	2.7205 1	3.0000 2	2.5462 2	2.2171 2	1.9676 2	1.7723 2	1.6154 2
4.00	2.0664 1	2.4416 1	2.8242 1	3.2124 1	3.6046 1	4.0000 2	3.4262 2	2.9971 2	2.6647 2	2.4000 2
5.00	2.2084 1	2.6771 1	3.1597 1	3.6525 1	4.1531 1	4.6596 1	4.7517 2	4.1334 2	3.6556 2	3.2759 2
6.00	2.3299 1	2.8909 1	3.4722 1	4.0681 1	4.6748 1	5.2897 1	5.9108 1	5.3214 2	4.6966 2	4.2000 2
8.00	2.5422 1	3.2882 1	4.0670 1	4.8679 1	5.6841 1	6.5112 1	7.3463 1	7.7637 2	6.8473 2	6.1176 2
10.00	2.7354 1	3.6639 1	4.6469 1	5.6528 1	6.6766 1	7.7127 1	8.7574 1	9.8094 1	9.0370 2	8.0769 2
12.00	2.9203 1	4.0443 1	5.2233 1	6.4342 1	7.6647 1	8.9080 1	10.1598 1	11.4178 1	11.2424 2	10.0541 2
15.00	3.1914 1	4.6063 1	6.0890 1	7.6074 1	9.1463 1	10.6979 1	12.2578 1	13.8232 1	14.5630 2	13.0349 2
17.00	3.3708 1	4.9923 1	6.6683 1	8.3915 1	10.1353 1	11.8913 1	13.6551 1	15.4243 1	16.7807 2	15.0273 2
20.00	3.6401 1	5.5495 1	7.5412 1	9.5710 1	11.6208 1	13.6822 1	15.7508 1	17.8243 1	19.9010 1	13.0198 2
25.00	4.0920 1	6.5035 1	9.0051 1	11.5442 1	14.1019 1	16.6699 1	19.2443 1	21.8227 1	24.4040 1	23.0127 2
30.00	4.5493 1	7.4667 1	10.4775 1	13.5241 1	16.5877 1	19.6606 1	22.7399 1	25.8208 1	28.9051 1	28.0088 2
35.00	5.0119 1	8.4371 1	11.9560 1	15.5087 1	19.0769 1	22.6533 1	26.2346 1	29.8190 1	33.4055 1	33.0065 2
40.00	5.4792 1	9.4129 1	13.4390 1	17.4966 1	21.5684 1	25.6475 1	29.7310 1	33.8173 1	37.9055 1	38.0050 2
45.00	5.9507 1	10.3930 1	14.9252 1	19.4869 1	24.0615 1	28.6428 1	33.2281 1	37.8159 1	42.4054 1	43.0039 2
50.00	6.4257 1	11.3763 1	16.4138 1	21.4739 1	26.5559 1	31.6390 1	36.7257 1	41.8147 1	46.9052 1	48.0032 2

ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,
0 = BOTH ENDS EQUALLY LOADED
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
0.20	0.91603 2	0.91323 2	0.91096 2	0.90883 2	0.90707 2	0.90554 2	0.90420 2	0.90301 2	0.90194 2	0.90099 2
0.50	0.84893 2	0.83225 2	0.81855 2	0.80708 2	0.79735 2	0.78898 2	0.78171 2	0.77534 2	0.76971 2	0.76470 2
1.00	0.90116 2	0.93009 2	0.77653 2	0.73471 2	0.70116 2	0.67365 2	0.65067 2	0.63120 2	0.61449 2	0.60000 2
1.20	0.98775 2	0.87725 2	0.79801 2	0.73842 2	0.69196 2	0.65475 2	0.62426 2	0.59883 2	0.57729 2	0.55882 2
1.50	0.92538 1	1.00000 2	0.86776 2	0.77430 2	0.73477 2	0.65101 2	0.60823 2	0.57337 2	0.54442 2	0.52000 2
1.70	0.87400 1	0.95406 1	0.93616 2	0.81511 2	0.72779 2	0.66184 2	0.61028 2	0.56898 2	0.53491 2	0.50653 2
2.00	0.80360 1	0.88920 1	0.97265 1	0.89497 2	0.77806 2	0.69251 2	0.62723 2	0.57581 2	0.53426 2	0.50000 2
2.50	0.70542 1	0.79593 1	0.88575 1	0.97474 1	0.88544 2	0.76604 2	0.67771 2	0.60976 2	0.55590 2	0.51219 2
3.00	0.62796 1	0.72061 1	0.81366 1	0.90685 1	1.00000 2	0.84875 2	0.73902 2	0.65588 2	0.59078 2	0.53846 2
4.00	0.51660 1	0.61040 1	0.70606 1	0.80309 1	0.90116 1	1.00000 2	0.85654 2	0.74928 2	0.66619 2	0.60000 2
5.00	0.44169 1	0.53543 1	0.63194 1	0.73050 1	0.83061 1	0.93192 1	0.95034 2	0.82669 2	0.73113 2	0.65517 2
6.00	0.38831 1	0.48181 1	0.57870 1	0.67802 1	0.77914 1	0.88161 1	0.98514 1	0.88690 2	0.78277 2	0.70000 2
8.00	0.31778 1	0.41102 1	0.50838 1	0.60849 1	0.71052 1	0.81390 1	0.91828 1	0.97046 2	0.85591 2	0.76471 2
10.00	0.27354 1	0.36689 1	0.46469 1	0.56528 1	0.66766 1	0.77127 1	0.87574 1	0.98084 1	0.90370 2	0.80769 2
12.00	0.24336 1	0.33702 1	0.43528 1	0.53619 1	0.63873 1	0.74233 1	0.84665 1	0.95149 1	0.93686 2	0.83784 2
15.00	0.21276 1	0.30709 1	0.40593 1	0.50716 1	0.60976 1	0.71319 1	0.81718 1	0.92155 1	0.97086 2	0.86900 2
17.00	0.19828 1	0.29308 1	0.39226 1	0.49362 1	0.59619 1	0.69949 1	0.80324 1	0.90731 1	0.98710 2	0.88396 2
20.00	0.18200 1	0.27748 1	0.37706 1	0.47855 1	0.53104 1	0.68411 1	0.78754 1	0.89121 1	0.99505 1	0.90099 2
25.00	0.16368 1	0.26014 1	0.35020 1	0.46177 1	0.56408 1	0.66680 1	0.76977 1	0.87291 1	0.97616 1	0.92051 2
30.00	0.15164 1	0.24889 1	0.34925 1	0.45030 1	0.55292 1	0.65535 1	0.75796 1	0.86069 1	0.96350 1	0.93363 2
35.00	0.14320 1	0.24106 1	0.34160 1	0.44311 1	0.54505 1	0.64724 1	0.74956 1	0.85197 1	0.95444 1	0.94304 2
40.00	0.13698 1	0.23532 1	0.33597 1	0.43742 1	0.53921 1	0.64119 1	0.74328 1	0.84543 1	0.94764 1	0.95012 2
45.00	0.13224 1	0.23095 1	0.33167 1	0.43304 1	0.53470 1	0.63651 1	0.73840 1	0.84035 1	0.94234 1	0.95564 2
50.00	0.12851 1	0.22753 1	0.32828 1	0.42958 1	0.53112 1	0.63278 1	0.73451 1	0.83629 1	0.93810 1	0.96006 2

ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

NON-DIMENSIONALIZED STRENGTH ,

0 = BOTH ENDS EQUALLY LOADED
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

EXTENSIONAL STIFFNESS (THICKNESS) RATIO

SCALED L/T RATIO	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1
0.20	0.1773 1	0.1778 1	0.1783 1	0.1786 1	0.1790 1	0.1793 1	0.1796 1	0.1799 1	0.1800 1	0.1802 1
0.50	0.3464 1	0.3527 1	0.3581 1	0.3629 1	0.3671 1	0.3708 1	0.3741 1	0.3771 1	0.3799 1	0.3823 1
1.00	0.4249 1	0.4516 1	0.4759 1	0.4981 1	0.5185 1	0.5374 1	0.5548 1	0.5710 1	0.5860 1	0.6000 1
1.20	0.4206 1	0.4567 1	0.4901 1	0.5213 1	0.5503 1	0.5775 1	0.6030 1	0.6269 1	0.6494 1	0.6706 1
1.50	0.4031 1	0.4535 1	0.5015 1	0.5472 1	0.5907 1	0.6321 1	0.6717 1	0.7094 1	0.7455 1	0.7800 1
1.70	0.3899 1	0.4504 1	0.5086 1	0.5647 1	0.6187 1	0.6708 1	0.7210 1	0.7694 1	0.8161 1	0.8611 1
2.00	0.3727 1	0.4498 1	0.5234 1	0.5964 1	0.6678 1	0.7375 1	0.8055 1	0.8719 1	0.9367 1	1.0000 1
2.50	0.3547 1	0.4593 1	0.5642 1	0.6688 1	0.7729 1	0.8763 1	0.9788 1	1.0804 1	1.1810 1	1.2805 1
3.00	0.3492 1	0.4955 1	0.6241 1	0.7644 1	0.9056 1	1.0475 1	1.1896 1	1.3317 1	1.4737 1	1.6154 1
4.00	0.3648 1	0.5715 1	0.7862 1	1.0070 1	1.2324 1	1.4614 1	1.6932 1	1.9271 1	2.1629 1	2.4000 1
5.00	0.4022 1	0.6861 1	0.9848 1	1.2943 1	1.6123 1	1.9366 1	2.2661 1	2.5996 1	2.9364 1	3.2759 1
6.00	0.4521 1	0.8181 1	1.2058 1	1.6093 1	2.0245 1	2.4485 1	2.8793 1	3.3155 1	3.7560 1	4.2000 1
8.00	0.5738 1	1.1135 1	1.6890 1	2.2886 1	2.9051 1	3.5336 1	4.1710 1	4.8151 1	5.4643 1	6.1176 1
10.00	0.7123 1	1.4343 1	2.2052 1	3.0066 1	3.8280 1	4.6631 1	5.5077 1	6.3593 1	7.2161 1	8.0769 1
12.00	0.8610 1	1.7712 1	2.7417 1	3.7473 1	4.7745 1	5.8159 1	6.8669 1	7.9248 1	8.9876 1	10.0541 1
15.00	1.0963 1	2.2966 1	3.5706 1	4.8338 1	6.2195 1	7.5691 1	8.9279 1	10.2929 1	11.6623 1	13.0349 1
17.00	1.2593 1	2.6563 1	4.1342 1	5.6525 1	7.1932 1	9.7474 1	10.3102 1	11.8789 1	13.4516 1	15.0273 1
20.00	1.5104 1	3.2062 1	4.9906 1	6.8162 1	8.6635 1	10.5233 1	12.3911 1	14.2640 1	16.1406 1	18.0198 1
25.00	1.9426 1	4.1423 1	6.4380 1	8.7739 1	11.1297 1	13.4966 1	15.8703 1	18.2484 1	20.6294 1	23.0127 1
30.00	2.3874 1	5.0948 1	7.9009 1	10.7451 1	13.6073 1	16.4792 1	19.3570 1	22.2386 1	25.1228 1	28.0088 1
35.00	2.8417 1	6.0583 1	9.3735 1	12.7243 1	16.0913 1	19.4670 1	22.8479 1	26.2321 1	29.6185 1	33.0065 1
40.00	3.3031 1	7.0295 1	10.8525 1	14.7086 1	18.5795 1	22.4581 1	26.3413 1	30.2274 1	34.1155 1	38.0050 1
45.00	3.7703 1	8.0053 1	12.3360 1	16.6964 1	21.0703 1	25.4512 1	29.8362 1	34.2239 1	38.6133 1	43.0039 1
50.00	4.2420 1	8.9872 1	13.8226 1	18.6866 1	23.5630 1	28.4458 1	33.3323 1	38.7212 1	43.1116 1	48.0032 1

ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,

0 = BOTH ENDS EQUALLY LOADED
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

EXTENSIONAL STIFFNESS (THICKNESS) RATIO

SCALED L/T RATIO	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
0.20	0.88632 1	0.88990 1	0.89129 1	0.89325 1	0.89496 1	0.89646 1	0.89778 1	0.89897 1	0.90003 1	0.90099 1
0.50	0.69288 1	0.70537 1	0.71621 1	0.72571 1	0.73410 1	0.74157 1	0.74826 1	0.75428 1	0.75973 1	0.76470 1
1.00	0.42491 1	0.45156 1	0.47586 1	0.49810 1	0.51854 1	0.53737 1	0.55480 1	0.57096 1	0.58598 1	0.60000 1
1.20	0.35053 1	0.38056 1	0.40944 1	0.43440 1	0.45862 1	0.48126 1	0.50249 1	0.52241 1	0.54116 1	0.55882 1
1.50	0.26871 1	0.30236 1	0.33435 1	0.36479 1	0.39378 1	0.42141 1	0.44777 1	0.47294 1	0.49699 1	0.52000 1
1.70	0.22938 1	0.26492 1	0.29917 1	0.33217 1	0.36396 1	0.39459 1	0.42411 1	0.45257 1	0.48003 1	0.50653 1
2.00	0.18637 1	0.22442 1	0.26171 1	0.29821 1	0.33388 1	0.36873 1	0.40275 1	0.43596 1	0.46837 1	0.50000 1
2.50	0.14187 1	0.18374 1	0.22567 1	0.26752 1	0.30916 1	0.35051 1	0.39153 1	0.43216 1	0.47239 1	0.51219 1
3.00	0.11640 1	0.15182 1	0.20804 1	0.25479 1	0.30187 1	0.34915 1	0.39652 1	0.44390 1	0.49123 1	0.53846 1
4.00	0.09121 1	0.14287 1	0.19655 1	0.25175 1	0.30810 1	0.36534 1	0.42329 1	0.48179 1	0.54072 1	0.60000 1
5.00	0.08043 1	0.13722 1	0.19695 1	0.25887 1	0.32245 1	0.38733 1	0.45322 1	0.51992 1	0.58728 1	0.65517 1
6.00	0.07536 1	0.13635 1	0.20097 1	0.26822 1	0.33741 1	0.40808 1	0.47998 1	0.55258 1	0.62600 1	0.70000 1
8.00	0.07173 1	0.13019 1	0.21112 1	0.28608 1	0.36313 1	0.44170 1	0.52137 1	0.60189 1	0.68304 1	0.76471 1
10.00	0.07123 1	0.14343 1	0.22052 1	0.30066 1	0.38280 1	0.46631 1	0.55077 1	0.63593 1	0.72161 1	0.80769 1
12.00	0.07175 1	0.14760 1	0.22848 1	0.31227 1	0.39788 1	0.48466 1	0.57224 1	0.66040 1	0.74896 1	0.83784 1
15.00	0.07309 1	0.15310 1	0.23904 1	0.32559 1	0.41463 1	0.50461 1	0.59519 1	0.68619 1	0.77748 1	0.86900 1
17.00	0.07407 1	0.15625 1	0.24319 1	0.33250 1	0.42313 1	0.51455 1	0.60648 1	0.69876 1	0.79127 1	0.88396 1
20.00	0.07552 1	0.16031 1	0.24953 1	0.34081 1	0.43317 1	0.52617 1	0.61955 1	0.71320 1	0.80703 1	0.90099 1
25.00	0.07770 1	0.16569 1	0.25752 1	0.35096 1	0.44519 1	0.53986 1	0.63481 1	0.72993 1	0.82518 1	0.92051 1
30.00	0.07958 1	0.16983 1	0.26336 1	0.35817 1	0.45358 1	0.54931 1	0.64523 1	0.74129 1	0.83743 1	0.93363 1
35.00	0.08119 1	0.17310 1	0.26781 1	0.36355 1	0.45975 1	0.55620 1	0.65280 1	0.74949 1	0.84624 1	0.94304 1
40.00	0.08258 1	0.17574 1	0.27131 1	0.36772 1	0.46449 1	0.56145 1	0.65853 1	0.75569 1	0.85289 1	0.95012 1
45.00	0.08378 1	0.17792 1	0.27413 1	0.37103 1	0.46823 1	0.56558 1	0.66303 1	0.76053 1	0.85807 1	0.95564 1
50.00	0.08484 1	0.17974 1	0.27645 1	0.37373 1	0.47126 1	0.56892 1	0.66665 1	0.76442 1	0.86223 1	0.96006 1

A.2 Computer Program A4ED For Lower Bound Elastic-Plastic Strength of Bonded Scarf Joints

This FORTRAN IV digital computer program covers a simple efficient approximate solution for the elastic-plastic strength of most bonded scarf joints of practical proportions and materials. Its development was needed as a sufficiently close starting point for convergence to proceed in the more precise program A4EE. It transpired, on examination of the equivalent results computed by A4EE that the quicker computations of A4ED were satisfactory as final answers provided that (1) and adhesive non-linear behavior was not negligible, i.e., that $\gamma_p/\gamma_e > 0.5$, (2) the thermal mismatch coefficient is not too high, i.e., that CTHERM < 2, and (3) that the stiffness mismatch between adherends be not too great, i.e., that $0.2 \leq \text{ETR} \leq 1$.

The input data for program A4ED is precisely the same as for program A4EE with the exception that γ_p/γ_e for the adhesive cannot be equal to zero for A4EE. In other words, perfectly elastic adhesive behavior must be excluded from A4EE. On the other hand, the values computed by A4ED for zero adhesive plasticity are unduly conservative.

A listing of the program and sample outputs follow.

```
CDECK A4ED
C ELASTIC-PLASTIC ANALYSIS OF UNBALANCED SCARF JOINTS
C LOWER BOUND ANALYSIS PROVIDED WHICH IS ACCURATE FOR DESIGN
C NON-DIMENSIONALIZED AVERAGE SHEAR STRESSES COMPUTED
C NON-DIMENSIONALIZED JOINT STRENGTHS COMPUTED
C RANGE OF ADHESIVE DUCTILITIES INCLUDED
C RANGES OF ADHEREND STIFFNESS AND THERMAL IMBALANCES ACCOUNTED FOR
C DATA PRESENTATION FOR TENSILE SHEAR LOADING
C CHANGE SIGN OF CTHERM TO USE FOR COMPRESSIVE SHEAR LOADS
C SET CTHERM .EQ. 0. AND REPLACE ADHEREND ET'S WITH GT'S FOR IN-PLANE
C     1  (EDGEWISE) SHEAR LOADING
C
C DIMENSION OL(J), ETR(K), CTHERM(I), GPOVGE(L), A(N), TRATIO(J,NCRTND),
C     1  TAUAVG(J,K), ICRTND(J,K), STRGTH(J,K), THERMC(NCRTND),
C     2  VR(NCRTND), VU(NCRTND), VL(NCRTND), OLTRNT(NCRTND),
C     3  OLTRNC(NCRTND), TRANSL(K)
      DIMENSION OL(40), ETR(10), CTHERM(20), GPOVGE(20), A(50),
     1  TRATIO(40,2), TAUAVG(40,10), STRGTH(40,10), ICRTND(40,10),
     2  THERMC(2), VR(2), VU(2), VL(2), OLTRNT(2), OLTRNC(2), TRANSL(10)
C
C READ IN INPUT DATA
C READ IN ARRAY SIZES
      READ (5,10) IMAX, JMAX, KMAX, LMAX, NMAX
   10 FORMAT (5I5)
C IMAX .LE. 20, JMAX .LE. 40, KMAX .LE. 10, LMAX .LE. 20,
C     1  NMAX .LE. 50 .AND.  .GE. 10.
C READ IN NON-DIMENSIONALIZED OVERLAP ARRAY
      OL(1) = 0.
C OL(J) MUST BE IN ASCENDING ORDER
C OL(2) MUST BE LESS THAN 0.2 FOR IDENTIFICATION OF CRITICAL END
C     1  OF JOINT OF ZERO OVERLAP (LIMITING CASE)
C OL(J) .LT. 100. FOR COMPATIBILITY WITH FORMAT STATEMENTS 470 & 590
      READ (5,20) (OL(J), J = 2, JMAX)
C NOTE JMAX ONE MORE THAN INPUT VALUES ON CARD(S)
   20 FORMAT (12F6.2)
C READ IN STIFFNESS IMBALANCE ARRAY
C IDENTIFY ADHERENDS SUCH THAT ETR(K) = (ET)1/(ET)2 .LE. 1.
C STIFFNESS RATIOS SHOULD BE IN ASCENDING OR DESCENDING ORDER
C ETR(K) SHOULD INCLUDE VALUE 1. BUT MUST EXCLUDE VALUE 0.
      READ (5,30) (ETR(K), K = 1, KMAX)
   30 FORMAT (10F5.2)
C READ IN NON-DIMENSIONALIZED THERMAL MISMATCH COEFFICIENTS
C CTHERM .PROPNL. (ALPHA(2)-ALPHA(1))*(OPERATING TEMP. - CURE TEMP.)
C NEED CTHERM(I) ARRAY TO CONTAIN BOTH POSITIVE AND NEGATIVE VALUES
C     1  TO COVER BOTH TENSILE AND COMPRESSIVE LOADS
      READ (5,40) (CTHERM(I), I = 1, IMAX)
   40 FORMAT (10F7.3)
C READ IN PLASTIC-TO-ELASTIC STRAIN RATIO ARRAY
C GPOVGE(L) MUST BE .GT. 0. FOR ELASTIC-PLASTIC ANALYSIS
C PURELY ELASTIC SOLUTION OBTAINED FROM SEPARATE PROCEDURE
      READ (5,50) (GPOVGE(L), L = 1, LMAX)
   50 FORMAT (14F5.2)
C
C PRINT OUT INPUT DATA
      WRITE (6,60) IMAX, JMAX, KMAX, LMAX, NMAX
   60 FORMAT (1H1, 9H  IMAX = ,I2, 9H  JMAX = ,I2, 9H  KMAX = ,I2,
     1  9H  LMAX = ,I2, 9H  NMAX = ,I2)
      WRITE (6,70)
   70 FORMAT (10H  OVERLAPS)
      WRITE (6,80) (OL(J), J = 1, JMAX)
   80 FORMAT (12F6.2)
      WRITE (6,90)
   90 FORMAT (22H  STIFFNESS IMBALANCES)
      WRITE (6,100) (ETR(K), K = 1, KMAX)
  100 FORMAT (14F5.2)
      WRITE (6,110)
  110 FORMAT (20H  THERMAL MISMATCHES)
      WRITE (6,120) (CTHERM(I), I = 1, IMAX)
  120 FORMAT (10F7.3)
      WRITE (6,130)
  130 FORMAT (34H  PLASTIC TO ELASTIC STRAIN RATIOS)
      WRITE (6,140) (GPOVGE(L), L = 1, LMAX)
  140 FORMAT (14F5.1)
C
C START COMPUTATIONAL DO LOOPS
      DO 620 L = 1, LMAX
      GAMMAR = GPOVGE(L)
C ENSURE EXCLUSION OF NEGATIVE PLASTICITY IN ADHESIVE (ERROR IN DATA)
      IF (GAMMAR .LT. 0.) GO TO 620
      DO 620 I = 1, IMAX
      THERMC(1) = CTHERM(I)
      THERMC(2) = - THERMC(1)
      DO 350 K = 1, KMAX
      VR(1) = ETR(K)
      VR(2) = 1. / VR(1)
      VU(1) = 1. - VR(1)
      VL(1) = 1. + VR(1)
      VU(2) = 1. - VR(2)
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      VL(2) = 1. + VR(2)
C
C SPECIAL PROCEDURE FOR PURELY ELASTIC ADHESIVE JOINT
      IF (GAMMAR .GT. 0.) GO TO 160
C SET ZERO TRANSITIONAL LENGTH FOR PURELY-ELASTIC JOINTS
      TRANSL(K) = 0.
      DO 150 J = 2, JMAX
      OLAP = OL(J)
      DO 150 NCRTND = 1, 2
  150 TRATIO(J,NCRTND) =1. - VU(NCRTND) + VL(NCRTND)*THERMC(NCRTND)/OLAP
      GO TO 270
C
C ESTABLISH TRANSITIONAL OVERLAPS FROM FULLY-PLASTIC TO ELASTIC-PLASTIC
C     1  BEHAVIOUR AS REFERENCE LENGTH FOR START OF ITERATIONS
C SPECIAL PROCEDURE FOR LESS THAN COMPLETELY UNBALANCED JOINTS
  160 IF ( (THERMC(1) .EQ. 0.) .AND. (VR(1) .EQ. 1.) ) GO TO 170
      IF (THERMC(1) .EQ. 0.) GO TO 180
      IF (VR(1) .EQ. 1.) GO TO 190
C IF NONE OF THESE, JOINT CONTAINS BOTH IMBALANCES
      GO TO 200
C SET INFINITE TRANSITIONAL OVERLAP FOR IDENTICAL ADHERENDS
  170 OLTRNT(1) = 1000000.
      OLTRNT(2) = 1000000.
C     OLTRNC(1) = 1000000.
C     OLTRNC(2) = 1000000.
      GO TO 210
C SET TRANSITIONAL OVERLAPS FOR STIFFNESS IMBALANCE ONLY
C IN THE ABSENCE OF THERMAL MISMATCH, SAME END IS CRITICAL FOR BOTH
C     1  TENSILE SHEAR AND COMPRESSIVE SHEAR LOADING
  180 IF (VU(1) .GT. 0.) OLTRNT(1) = SQRT(GAMMAR*VL(1)/VU(1))
      IF (VU(1) .LE. 0.) OLTRNT(1) = 1000000.
      IF (VU(2) .GT. 0.) OLTRNT(2) = SQRT(GAMMAR*VL(2)/VU(2))
      IF (VU(2) .LE. 0.) OLTRNT(2) = 1000000.
C     IF (VU(1) .GT. 0.) OLTRNC(1) = SQRT(GAMMAR*VL(1)/VU(1))
C     IF (VU(1) .LE. 0.) OLTRNC(1) = 1000000.
C     IF (VU(2) .GT. 0.) OLTRNC(2) = SQRT(GAMMAR*VL(2)/VU(2))
C     IF (VU(2) .LE. 0.) OLTRNC(2) = 1000000.
      GO TO 210
C SET TRANSITIONAL OVERLAPS FOR THERMAL MISMATCH ONLY
  190 IF (THERMC(1) .LT. 0.) OLTRNT(1) = -GAMMAR/THERMC(1)
      IF (THERMC(1) .GE. 0.) OLTRNT(1) = 1000000.
      IF (THERMC(2) .LT. 0.) OLTRNT(2) = -GAMMAR/THERMC(2)
      IF (THERMC(2) .GE. 0.) OLTRNT(2) = 1000000.
C     IF (THERMC(1) .GE. 0.) OLTRNC(1) = GAMMAR/THERMC(1)
C     IF (THERMC(1) .LT. 0.) OLTRNC(1) = 1000000.
C     IF (THERMC(2) .GE. 0.) OLTRNC(2) = GAMMAR/THERMC(2)
C     IF (THERMC(2) .LT. 0.) OLTRNC(2) = 1000000.
      GO TO 210
  200 CONTINUE
C STANDARD PROCEDURE FOR COMPLETELY UNBALANCED JOINTS
      V1 = THERMC(1) * VL(1) / (2. * VU(1))
      V2 = THERMC(2) * VL(2) / (2. * VU(2))
      V3 = V1*V1 + GAMMAR*VL(1)/VU(1)
      V4 = V2*V2 + GAMMAR*VL(2)/VU(2)
C ESTABLISH TRANSITIONAL OVERLAPS BELOW WHICH JOINT IS FULLY PLASTIC
C NEXT FOUR STATEMENTS APPLY FOR TENSILE SHEAR LOADING
      IF (V3 .GE. 0.) OLTRNT(1) = V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
      IF ( (V3 .LT. 0.) .OR. (OLTRNT(1) .LE. 0.) ) OLTRNT(1) = 1000000.
      IF (V4 .GE. 0.) OLTRNT(2) = V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
      IF ( (V4 .LT. 0.) .OR. (OLTRNT(2) .LE. 0.) ) OLTRNT(2) = 1000000.
C IF ICRTND .EQ. 2 FOR SHORT OVERLAPS, OLTRNT(1) WILL BE COMPUTED VERY
C     1  LARGE, AND VICE VERSA
C THIS IS PHYSICALLY REALISTIC AND DOES NOT LEAD TO IMPOSSIBLE COMPUTING
C IF BOTH V3 AND V4 ARE POSITIVE, EITHER OLTRNT(1) OR OLTRNT(2) WILL BE
C     1  COMPUTED NEGATIVE.  NEED TO PREVENT COMPUTATIONS BASED ON THIS
C     2  UNREAL SITUATION.  HENCE CHECKS ABOVE AND BELOW
C NEXT FOUR STATEMENTS WOULD APPLY FOR COMPRESSIVE SHEAR LOADING
C     IF (V3 .GE. 0.) OLTRNC(1) = -V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
C     IF ( (V3 .LT. 0.) .OR. (OLTRNC(1) .LE. 0.) ) OLTRNC(1) = 1000000.
C     IF (V4 .GE. 0.) OLTRNC(2) = -V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
C     IF ( (V4 .LT. 0.) .OR. (OLTRNC(2) .LE. 0.) ) OLTRNC(2) = 1000000.
C
  210 DO 260 NCRTND = 1, 2
C SET UNIFORM STRESS FOR SHORT OVERLAPS
      DO 220 J = 2, JMAX
      JSAVE = J
```

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      IF (OL(J) .GT. OLTRNT(NCRTND)) GO TO 230
C IF NOT, JOINT IS FULLY PLASTIC
  220 TRATIO(J,NCRTND) = 1.
      IF (JSAVE .EQ. JMAX) GO TO 260
C
C COMPUTE JOINT STRENGTH FOR ELASTIC-PLASTIC ADHESIVE BEHAVIOUR
  230 DO 250 J = JSAVE, JMAX
      OLAP = OL(J)
      OLAP2 = OLAP * OLAP
C COMPUTE AOVERL FOR MINIMUM VALUE OF TAVGTP BY ITERATION
C SET INITIAL ESTIMATE OF EXTENT OF PLASTIC ZONE FROM TRANSITIONAL OLAP
      AOVERL = OLTRNT(NCRTND) / OLAP
      DO 240 N = 1, NMAX
      ARMDR = 1. - AOVERL
      AOVERL = -ARMDR*ALOG(ARMDR) + (GAMMAR / ((VU(NCRTND)/VL(NCRTND))*
     1 OLAP2 - THERMC(NCRTND)*OLAP))
      IF (AOVERL .GT. 0.999) AOVERL = 0.999
      IF (AOVERL .LT. 0.001) AOVERL = 0.001
  240 CONTINUE
      TRATIO(J,NCRTND) = 1.
      IF (AOVERL .GT. 0.9999) GO TO 250
C COMPUTE CORRESPONDING AVERAGE SHEAR STRESS
      TRATIO(J,NCRTND) = 1. - (VL(NCRTND)*GAMMAR/OLAP2 + (VL(NCRTND)*
     1 THERMC(NCRTND)/OLAP - VU(NCRTND)) * AOVERL) / ALOG(ARMDR)
  250 CONTINUE
  260 CONTINUE
C
C VALUES COMPUTED ARE NOW STORED IN TRATIO(J,NCRTND)
  270 DO 340 J = 2, JMAX
      OLAP = OL(J)
      TAU1 = TRATIO(J,1)
      TAU2 = TRATIO(J,2)
      IF ( (TAU1 .LT. 1.) .OR. (TAU2 .LT. 1.) ) GO TO 280
C IF SO, JOINT IS NOT FULLY PLASTIC
C IF NOT, IDENTIFY CRITICAL END OF JOINT FROM SHEAR STRAIN GRADIENT
      GRADNT = THERMC(1) - OLAP*VU(1)/VL(1)
      IF (GRADNT .LT. 0.) ICRTND(J,K) = 1
      IF (GRADNT .EQ. 0.) ICRTND(J,K) = 0
      IF (GRADNT .GT. 0.) ICRTND(J,K) = 2
      TAUAVG(J,K) = 1.
      STRGTH(J,K) = OLAP
C TRANSITIONAL OVERLAPS ALREADY COMPUTED FOR ELASTIC ADHESIVE
C BYPASS RECOMPUTATION.  THIS APPLIES TO ELASTIC-PLASTIC ADHESIVES
      IF (GAMMAR .EQ. 0.) GO TO 340
      MCRTND = ICRTND(J,K)
      IF (MCRTND .EQ. 0) MCRTND = 1
      TRANSL(K) = OLTRNT(MCRTND)
      GO TO 340
  280 DIFFNC = TAU1 - TAU2
C IF DIFFNC .LT. 0., NCRTND .EQ. 1
C IF DIFFNC .EQ. 0., NCRTND .EQ. 0
C IF DIFFNC .GT. 0., NCRTND .EQ. 2
      IF (DIFFNC)290,300,310
C ADHEREND (1) END OF JOINT CRITICAL
  290 TAUAVG(J,K) = TAU1
      STRGTH(J,K) = TAU1 * OLAP
      ICRTND(J,K) = 1
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
      IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)
      GO TO 320
C BOTH ENDS OF JOINT EQUALLY CRITICAL FROM NULLIFYING (OR ZERO)
C     1 ADHEREND IMBALANCES
  300 TAUAVG(J,K) = TAU1
      STRGTH(J,K) = TAU1 * OLAP
      ICRTND(J,K) = 0
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
      IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)
      GO TO 320
C ADHEREND (2) END OF JOINT CRITICAL
  310 TAUAVG(J,K) = TAU2
      STRGTH(J,K) = TAU2 * OLAP
      ICRTND(J,K) = 2
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
      IF (J .EQ. 2) TRANSL(K) = OLTRNT(2)
C COVER CASES OF ZERO OR NEGATIVE ESTIMATED STRENGTHS
  320 IF (TAUAVG(J,K) .GT. 0.) GO TO 330
C IF NOT, JOINT HAS BROKEN DUE TO THERMAL STRESSES WITHOUT EXTERNAL LOAD
      TAUAVG(J,K) = 0.
      STRGTH(J,K) = 0.
      GO TO 340
  330 IF (TAUAVG(J,K) .LE. 1.) GO TO 340
C IF NOT, THERE HAS BEEN A COMPUTATIONAL MISTAKE
C PRINT ASTERISKS TO IDENTIFY ERROR
C RERUN WITH GREATER VALUE OF NMAX
      TAUAVG(J,K) = 100.
      STRGTH(J,K) = 1000.
  340 CONTINUE
  350 CONTINUE
```

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C                                                                       A4ED2650
C SET UNIFORM STRESS FOR ZERO OVERLAP                                   A4ED2660
      DO 360 K = 1, KMAX                                                A4ED2670
      TAUAVG(1,K) = 1.                                                  A4ED2680
      STRGTH(1,K) = 0.                                                  A4ED2690
  360 ICRTND(1,K) = ICRTND(2,K)                                         A4ED2700
C HENCE NEED FOR OL(2) TO BE SMALL ENOUGH TO BE LESS THAN THAT AT WHICH A4ED2710
C    1  NCRTND CHANGES                                                  A4ED2720
C                                                                       A4ED2730
C END OF COMPUTATIONS. START PRINTING OUT OF TABULATED RESULTS          A4ED2740
C                                                                       A4ED2750
C PRINT OUT AVERAGE STRESS HEADING                                      A4ED2760
      WRITE (6,370)                                                     A4ED2770
  370 FORMAT (1H1/, 5(1H0/), 27X, 56HADHESIVE-BONDED SCARF JOINTS  (ELASA4ED2780
     1TIC-PLASTIC ANALYSIS)/,                                           A4ED2790
     2  39X, 31HNON-DIMENSIONALIZED FORMULATION/)                       A4ED2800
      IF (GAMMAR .NE. 0.) GO TO 390                                     A4ED2810
      WRITE (6,380)                                                     A4ED2820
  380 FORMAT (1H0, 42X, 23HPURELY ELASTIC ADHESIVE)                     A4ED2830
      GO TO 410                                                         A4ED2840
  390 WRITE (6,400) GAMMAR                                              A4ED2850
  400 FORMAT(1H0, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIOA4ED2860
     1 = , F5.2)                                                        A4ED2870
  410 IF (CTHERM(I) .NE. 0.) GO TO 430                                  A4ED2880
      WRITE (6,420)                                                     A4ED2890
  420 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT)           A4ED2900
      GO TO 450                                                         A4ED2910
  430 WRITE (6,440) THERMC(1), THERMC(2)                                A4ED2920
  440 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3,       A4ED2930
     1  17H FOR TENSION,   = , F6.3, 16H FOR COMPRESSION)               A4ED2940
  450 WRITE (6,460) (ETR(K), K = 1, KMAX)                               A4ED2950
  460 FORMAT(  1H0,  67X, 30H0 = BOTH ENDS EQUALLY CRITICAL/, 20X,      A4ED2960
     1  72HAVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS   ,  1 = SOFT ET EA4ED2970
     2ND CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL/,                 A4ED2980
     3  8H0 SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,   A4ED2990
     4  7H     L/T/, 7H  RATIO, F7.1, 9F10.1/, 1H )                     A4ED3000
C WRITE OUT TABULATIONS OF AVERAGE BOND STRESSES                        A4ED3010
      DO 480 J = 1, JMAX                                                A4ED3020
      WRITE (6,470) OL(J), ((TAUAVG(J,K), ICRTND(J,K)), K = 1, KMAX)    A4ED3030
  470 FORMAT (1H , F6.2, 2X, 10(F7.5, 1X, I1, 1X))                      A4ED3040
  480 CONTINUE                                                          A4ED3050
C                                                                       A4ED3060
C PRINT OUT JOINT STRENGTH HEADING                                      A4ED3070
      WRITE (6,490)                                                     A4ED3080
  490 FORMAT (1H1/, 5(1H0/), 27X, 56HADHESIVE-BONDED SCARF JOINTS  (ELASA4ED3090
     1TIC-PLASTIC ANALYSIS)/,                                           A4ED3100
     2  39X, 31HNON-DIMENSIONALIZED FORMULATION/)                       A4ED3110
      IF (GAMMAR .NE. 0.) GO TO 510                                     A4ED3120
      WRITE (6,500)                                                     A4ED3130
  500 FORMAT (1H0, 42X, 23HPURELY ELASTIC ADHESIVE)                     A4ED3140
      GO TO 530                                                         A4ED3150
  510 WRITE (6,520) GAMMAR                                              A4ED3160
  520 FORMAT(1H0, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIOA4ED3170
     1 = , F5.2)                                                        A4ED3180
  530 IF (CTHERM(I) .NE. 0.) GO TO 550                                  A4ED3190
      WRITE (6,540)                                                     A4ED3200
  540 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT)           A4ED3210
      GO TO 570                                                         A4ED3220
  550 WRITE (6,560) THERMC(1), THERMC(2)                                A4ED3230
  560 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3,       A4ED3240
     1  17H FOR TENSION,   = , F6.3, 16H FOR COMPRESSION)               A4ED3250
  570 WRITE (6,580) (ETR(K), K = 1, KMAX)                               A4ED3260
  580 FORMAT(  1H0,  67X, 30H0 = BOTH ENDS EQUALLY CRITICAL/, 20X,      A4ED3270
     1  72HNON-DIMENSIONALIZED JOINT STRENGTH   ,          1 = SOFT ET EA4ED3280
     2ND CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL/,                 A4ED3290
     3  8H0 SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,   A4ED3300
     4  7H     L/T/, 7H  RATIO, F7.1, 9F10.1/, 1H )                     A4ED3310
C WRITE OUT TABULATIONS OF JOINT STRENGTHS                              A4ED3320
      DO 600 J = 1, JMAX                                                A4ED3330
      WRITE (6,590) OL(J), ((STRGTH(J,K), ICRTND(J,K)), K = 1, KMAX)    A4ED3340
  590 FORMAT (1H , F6.2, 2X, 10(F7.4, 1X, I1, 1X))                      A4ED3350
  600 CONTINUE                                                          A4ED3360
C WRITE OUT TRANSITIONAL JOINT STRENGTHS                                A4ED3370
      WRITE (6,610) (TRANSL(K), K = 1, KMAX)                            A4ED3380
  610 FORMAT (8H0 TRANSL, 1X, 10(F7.4, 3X))                             A4ED3390
  620 CONTINUE                                                          A4ED3400
C                                                                       A4ED3410
      WRITE (6,630)                                                     A4ED3420
  630 FORMAT (1H1, 18H PROGRAM COMPLETED)                               A4ED3430
      STOP                                                              A4ED3440
      END                                                               A4ED3450
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ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH ,

0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2	0.0 2
0.20	0.2000 2	0.2000 2	0.2000 2	0.2000 2	0.2000 2	0.2000 2	0.2000 2	0.2000 2	0.2000 2	0.2000 2
0.50	0.5000 2	0.5000 2	0.5000 2	0.5000 2	0.5000 2	0.5000 2	0.5000 2	0.5000 2	0.5000 2	0.5000 2
1.00	1.0000 2	1.0000 2	1.0000 2	1.0000 2	1.0000 2	1.0000 2	1.0000 2	1.0000 2	1.0000 2	1.0000 2
1.20	1.2000 2	1.2000 2	1.2000 2	1.2000 2	1.2000 2	1.2000 2	1.2000 2	1.2000 2	1.2000 2	1.2000 2
1.50	1.5000 1	1.5000 0	1.5000 2	1.5000 2	1.5000 2	1.5000 2	1.5000 2	1.5000 2	1.5000 2	1.5000 2
1.70	1.7000 1	1.7000 1	1.7000 2	1.7000 2	1.7000 2	1.7000 2	1.7000 2	1.7000 2	1.7000 2	1.7000 2
2.00	2.0000 1	2.0000 1	2.0000 1	2.0000 2	2.0000 2	2.0000 2	2.0000 2	2.0000 2	2.0000 2	2.0000 2
2.50	2.5000 1	2.5000 1	2.5000 1	2.5000 1	2.5000 2	2.5000 2	2.5000 2	2.5000 2	2.5000 2	2.5000 2
3.00	3.0000 1	3.0000 1	3.0000 1	3.0000 1	3.0000 0	3.0000 2	3.0000 2	3.0000 2	3.0000 2	3.0000 2
4.00	3.6044 1	3.9980 1	4.0000 1	4.0000 1	4.0000 1	4.0000 0	4.0000 2	4.0000 2	4.0000 2	4.0000 2
5.00	3.9361 1	4.3495 1	4.7374 1	4.9984 1	5.0000 1	5.0000 1	5.0000 2	5.0000 2	5.0000 2	5.0000 2
6.00	4.1817 1	4.7235 1	5.2105 1	5.6279 1	5.9985 1	6.0000 1	6.0000 1	6.0000 2	6.0000 2	5.9980 2
8.00	4.5565 1	5.3297 1	6.0557 1	6.7256 1	7.3222 1	7.9959 1	8.0000 1	8.0000 2	8.0000 2	7.9824 2
10.00	4.8582 1	5.8486 1	6.7955 1	7.6917 1	8.5245 1	9.2690 1	9.9987 1	10.0000 1	10.0000 2	9.6266 2
12.00	5.1248 1	6.3263 1	7.4962 1	8.5985 1	9.6526 1	10.6237 1	11.4915 1	12.0000 1	12.0000 2	11.5188 2
15.00	5.4906 1	7.0028 1	8.4752 1	9.9027 1	11.2764 1	12.5802 1	13.7802 1	14.7760 1	15.0000 2	14.3908 2
17.00	5.7213 1	7.4394 1	9.1166 1	10.7510 1	12.3334 1	13.8491 1	15.2683 1	16.5094 1	17.0000 2	16.3217 2
20.00	6.0556 1	8.0782 1	10.0626 1	12.0045 1	13.9765 1	15.7251 1	17.4641 1	19.0473 1	20.0000 1	19.2352 2
25.00	6.5942 1	9.1229 1	11.6144 1	14.0647 1	16.4670 1	18.8093 1	21.0690 1	23.1924 1	24.7854 1	24.1230 2
30.00	7.1195 1	10.1524 1	13.1486 1	16.1044 1	19.0133 1	21.9645 1	24.6371 1	27.2839 1	29.6205 1	29.0372 2
35.00	7.6368 1	11.1731 1	14.6728 1	18.1325 1	21.5462 1	24.9034 1	28.1843 1	31.3471 1	34.2362 1	33.9688 2
40.00	8.1490 1	12.1881 1	16.1907 1	20.1535 1	24.0707 1	27.9324 1	31.7201 1	35.3935 1	38.8159 1	38.9126 2
45.00	8.6574 1	13.1993 1	17.7044 1	22.1696 1	26.5895 1	30.9548 1	35.2474 1	39.4290 1	43.3747 1	43.3654 2
50.00	9.1630 1	14.2075 1	19.2150 1	24.1825 1	29.1048 1	33.9727 1	38.7691 1	43.4570 1	47.9200 1	48.8249 2
TRANSL	3.1576	3.5895	4.1142	4.7761	5.6533	6.8990	8.8633	12.5777	23.1106	5.0000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,

0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
0.20	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
0.50	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
1.00	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
1.20	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
1.50	1.00000 1	1.00000 0	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
1.70	1.00000 1	1.00000 1	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
2.00	1.00000 1	1.00000 1	1.00000 1	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
2.50	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
3.00	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 0	1.00000 2	1.00000 2	1.00000 2	1.00000 2	1.00000 2
4.00	0.90110 1	0.99950 1	1.00000 1	1.00000 1	1.00000 1	1.00000 0	1.00000 2	1.00000 2	1.00000 2	1.00000 2
5.00	0.78722 1	0.86990 1	0.94749 1	0.99968 1	1.00000 1	1.00000 1	1.00000 2	1.00000 2	1.00000 2	1.00000 2
6.00	0.69695 1	0.78725 1	0.86842 1	0.93798 1	0.99975 1	1.00000 1	1.00000 1	1.00000 2	1.00000 2	0.99957 2
8.00	0.56956 1	0.66571 1	0.75696 1	0.84070 1	0.91527 1	0.99824 1	1.00000 1	1.00000 2	1.00000 2	0.99780 2
10.00	0.48592 1	0.58486 1	0.67955 1	0.76917 1	0.85245 1	0.92690 1	0.99987 1	1.00000 1	1.00000 2	0.96266 2
12.00	0.42707 1	0.52719 1	0.62395 1	0.71654 1	0.80439 1	0.88572 1	0.95580 1	1.00000 1	1.00000 2	0.95990 2
15.00	0.36604 1	0.46685 1	0.56502 1	0.66018 1	0.75176 1	0.83868 1	0.91863 1	0.98507 1	1.00000 2	0.95939 2
17.00	0.33655 1	0.43755 1	0.53627 1	0.63241 1	0.72549 1	0.81465 1	0.89914 1	0.97114 1	1.00000 2	0.96010 2
20.00	0.30278 1	0.40391 1	0.50313 1	0.60023 1	0.69483 1	0.78625 1	0.87321 1	0.95237 1	1.00000 1	0.96176 2
25.00	0.26377 1	0.36491 1	0.46458 1	0.56259 1	0.65368 1	0.75237 1	0.84275 1	0.92770 1	0.99142 1	0.96492 2
30.00	0.23732 1	0.33841 1	0.43829 1	0.53681 1	0.63378 1	0.72892 1	0.82124 1	0.90946 1	0.98735 1	0.96791 2
35.00	0.21820 1	0.31923 1	0.41922 1	0.51807 1	0.61561 1	0.71153 1	0.80528 1	0.89563 1	0.97818 1	0.97054 2
40.00	0.20372 1	0.30470 1	0.40477 1	0.50384 1	0.60177 1	0.69831 1	0.79300 1	0.88494 1	0.97040 1	0.97282 2
45.00	0.19239 1	0.29332 1	0.39343 1	0.49266 1	0.59088 1	0.68788 1	0.78328 1	0.87620 1	0.96388 1	0.97479 2
50.00	0.18326 1	0.28415 1	0.38430 1	0.48365 1	0.58210 1	0.67945 1	0.77538 1	0.86914 1	0.95840 1	0.97650 2

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH ,
0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1
0.20	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1
0.50	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1
1.00	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1
1.20	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1
1.50	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1
1.70	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1
2.00	1.9971 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1
2.50	2.4966 1	2.4959 1	2.4969 1	2.4971 1	2.5000 1	2.5000 1	2.5000 1	2.5000 1	2.5000 1	2.5000 1
3.00	2.2502 1	2.4413 1	2.9956 1	2.9968 1	2.9970 1	2.9972 1	3.0000 1	3.0000 1	3.0000 1	3.0000 1
4.00	2.3992 1	2.6877 1	2.9539 1	3.1978 1	3.4186 1	3.9719 1	3.9971 1	3.9974 1	4.0000 1	4.0000 1
5.00	2.5200 1	2.9045 1	3.2655 1	3.6028 1	3.9155 1	4.2022 1	4.4605 1	4.9972 1	4.9976 1	5.0000 1
6.00	2.6306 1	3.1116 1	3.5672 1	3.9992 1	4.4044 1	4.7817 1	5.1294 1	5.4400 1	5.9975 1	5.9990 1
8.00	2.8403 1	3.5158 1	4.1650 1	4.7871 1	5.3806 1	5.9430 1	6.4707 1	6.9574 1	7.3933 1	7.9824 1
10.00	3.0443 1	3.9160 1	4.7601 1	5.5757 1	6.3609 1	7.1127 1	7.8262 1	8.4939 1	9.1026 1	9.6266 1
12.00	3.2452 1	4.3151 1	5.3556 1	6.3665 1	7.3455 1	8.7893 1	9.1921 1	10.0450 1	10.8318 1	11.5189 1
15.00	3.5475 1	4.9134 1	6.2499 1	7.5559 1	8.8282 1	10.0632 1	11.2543 1	12.3903 1	13.4511 1	14.3908 1
17.00	3.7478 1	5.3123 1	6.8469 1	9.3502 1	9.8194 1	11.2501 1	12.6351 1	13.9624 1	15.2093 1	16.3217 1
20.00	4.0480 1	5.9108 1	7.7433 1	9.5436 1	11.3090 1	13.0345 1	14.7123 1	16.3292 1	17.9589 1	19.2352 1
25.00	4.5473 1	6.9086 1	9.2386 1	11.5356 1	13.7964 1	16.0153 1	19.1851 1	20.2891 1	22.2970 1	24.1230 1
30.00	5.0446 1	7.9057 1	10.7347 1	13.5298 1	16.2976 1	19.0027 1	21.5660 1	24.2608 1	26.7525 1	29.0372 1
35.00	5.5387 1	8.9012 1	12.2304 1	15.5247 1	18.7808 1	21.9930 1	25.1519 1	28.2398 1	31.2190 1	33.9589 1
40.00	6.0292 1	9.9940 1	13.7248 1	17.5195 1	21.2750 1	24.9854 1	28.6410 1	32.2236 1	35.6929 1	39.9126 1
45.00	6.5159 1	10.9839 1	15.2171 1	19.5134 1	23.7692 1	27.9788 1	32.1322 1	36.2106 1	40.1718 1	43.8654 1
50.00	6.9990 1	11.8708 1	16.7071 1	21.5057 1	26.2629 1	30.9727 1	35.6249 1	40.2001 1	44.6546 1	48.8249 1
TRANSL	1.9354	2.0895	2.2570	2.4427	2.6533	2.8990	3.1967	3.5777	4.1107	5.0000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,
0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
0.20	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
0.50	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.00	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.20	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.50	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.70	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
2.00	0.99855 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
2.50	0.99866 1	0.99872 1	0.99378 1	0.99884 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
3.00	0.75007 1	0.81377 1	0.99887 1	0.99893 1	0.99700 1	0.99907 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
4.00	0.59979 1	0.67192 1	0.73847 1	0.79944 1	0.85466 1	0.99297 1	0.99928 1	0.99935 1	1.00000 1	1.00000 1
5.00	0.50400 1	0.58091 1	0.65311 1	0.72056 1	0.79309 1	0.84043 1	0.89209 1	0.99944 1	0.99952 1	1.00000 1
6.00	0.43843 1	0.51860 1	0.59465 1	0.66653 1	0.73406 1	0.79695 1	0.85473 1	0.90655 1	0.99958 1	0.99967 1
8.00	0.35504 1	0.43948 1	0.52062 1	0.59838 1	0.67257 1	0.74288 1	0.80893 1	0.86969 1	0.92416 1	0.99780 1
10.00	0.30443 1	0.39160 1	0.47601 1	0.55757 1	0.63509 1	0.71127 1	0.78262 1	0.84937 1	0.91026 1	0.96266 1
12.00	0.27051 1	0.35959 1	0.44630 1	0.53054 1	0.61213 1	0.69077 1	0.76601 1	0.83704 1	0.90265 1	0.95990 1
15.00	0.23650 1	0.32756 1	0.41666 1	0.50372 1	0.58855 1	0.67038 1	0.75028 1	0.82602 1	0.89674 1	0.95939 1
17.00	0.22046 1	0.31249 1	0.40276 1	0.49119 1	0.57761 1	0.66177 1	0.74324 1	0.82132 1	0.89467 1	0.96010 1
20.00	0.20240 1	0.29554 1	0.38716 1	0.47718 1	0.56545 1	0.65172 1	0.73562 1	0.81646 1	0.89295 1	0.96176 1
25.00	0.18189 1	0.27634 1	0.36954 1	0.46143 1	0.55186 1	0.64053 1	0.72740 1	0.81156 1	0.89188 1	0.96492 1
30.00	0.16815 1	0.26352 1	0.35782 1	0.45099 1	0.54292 1	0.63343 1	0.72220 1	0.80869 1	0.89175 1	0.96791 1
35.00	0.15825 1	0.25432 1	0.34944 1	0.44356 1	0.53659 1	0.62837 1	0.71863 1	0.80695 1	0.89197 1	0.97054 1
40.00	0.15073 1	0.24735 1	0.34312 1	0.43799 1	0.53187 1	0.62463 1	0.71602 1	0.80559 1	0.89232 1	0.97282 1
45.00	0.14480 1	0.24187 1	0.33816 1	0.43363 1	0.52820 1	0.62175 1	0.71405 1	0.80468 1	0.89271 1	0.97479 1
50.00	0.13998 1	0.23742 1	0.33414 1	0.43011 1	0.52526 1	0.61945 1	0.71250 1	0.80400 1	0.89309 1	0.97650 1

A.3 Computer Program A4EE For Elastic-Plastic Strength of Bonded Scarf Joints

This FORTRAN IV digital computer program provides for the precise series solution for the average shear stress on bonded scarf joints with small scarf angles. It accounts for adherend stiffness and thermal imbalance as well as adhesive plasticity. The governing analysis is presented in Sections 3 and 4. This program A4EE will not handle perfectly elastic adhesives for which the program A4EC was developed. Severe convergence difficulties were encountered in the development of the numerical program. This contributed to the omission of a solution for the adherend and adhesive shear stress distributions. Whether or not the adherend allowable stresses are exceeded can be determined simply by evaluating the ratio of the adhesive peak shear stress to the adherend allowable direct stress. If this ratio exceeds the tangent of the scarf angle, the scarf angle is too small and the tip will either break off or be yielded depending on the nature of the adherend material.

The input data required to operate program A4EE is as follows.

CARD 1:

FORMAT (5I5)

IMAX = Number of thermal mismatch coefficients. IMAX .LE. 20.

JMAX = Number of non-dimensionalized overlaps. JMAX .LE. 40.
(Note that this is one more than the number of overlaps to be read in. The limiting case of OL(1)=0 is set by the program.)

KMAX = Number of adherend stiffness imbalances. KMAX .LE. 10.
(Note that this controls the number of answers printed across the page and cannot be increased indefinitely for a single pass through the program.)

LMAX = Number of plastic-to-elastic adhesive shear strain ratios.
LMAX .LE. 20.

NMAX = Number of terms in power series. 10 .LE. NMAX .LE. 50.
(Note NMAX = 20 is recommended.)

CARDS 2, 2A, 2B, etc.:

FORMAT (12F6.2)

OL(J) = Non-dimensionalized overlaps. Number restricted to 40 by dimension statement. (Note that OL(J) must be read in in ascending order and that OL(2), which is the first entry on card 2, must not exceed 0.5 because of internal computations. OL(1) = 0 is set by the program as a limiting case.) Values of OL(J) exceeding 50 are impractically large.

CARDS 3, 3A, 3B, etc.:

FORMAT (10F5.2)

ETR(K) = Adherend stiffness ratios $(E_1t_1)/(E_2t_2)$. Number of values restricted to 10 by dimension statement. (Subscripts 1 and 2 must be identified such that $0 < \mathrm{ETR(K)} \leq 1$. Array should be read in in ascending or descending order.)

CARDS 4, 4A, 4B, etc.:

FORMAT (10F7.3)

CTHERM(I) = Adherend thermal mismatch coefficients in non-dimensionalized form. Number restricted to 20 by dimension statement. (Note that equal and opposite values must be read in consecutively to account for the difference between tensile and compressive application of the shear load. Values up to ±5 are sufficient for the available range of adhesives and adherends. Greater values are usually associated with failure of the joint under residual thermal stresses alone.)

CARDS 5, 5A, 5B, etc.:

FORMAT (14F5.2)

GPOVGE(L) = Ratio of adhesive plastic-to-elastic strain ratios. Number of entries restricted to 20 by dimension statement. (Value of zero, for elastic case, is rejected by program A4EE to prevent breakdown of the computational sequence, but accepted by A4ED.)

A complete listing and sample outputs follow. The output tables come in pairs with the ratio of the average to maximum adhesive shear stress (τ_{av}/τ_p) and the non-dimensionalized joint strength $(\tau_{av}/\tau_p)(\lambda\ell)$ as functions of the adherend extensional stiffness ratio ETR $= E_1t_1/E_2t_2 \leq 1$ horizontally and the non-dimensionalized joint overlap $\lambda\ell = \sqrt{\frac{G}{\eta}\left(\frac{1}{E_1t_1} + \frac{1}{E_2t_2}\right)\ell^2}$ vertically. Each table is prepared for a single value of thermal mismatch coefficient

$$\text{CTHERM} = \frac{(\alpha_2 - \alpha_1)\Delta T\lambda}{\tau_p\left(\frac{1}{E_1t_1} + \frac{1}{E_2t_2}\right)}$$

and equal and opposite values are treated in turn to cover both tensile and compressive shear loadings. Each table is prepared for a single value of the plastic-to-elastic adhesive shear strain ratio γ_p/γ_e. The quantity TRANSL listed at the foot of each column of the non-dimensionalized strength table defines the transitional overlap at which the adhesive behavior changes from fully-plastic to elastic-plastic.

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CDECK A4EE
C ELASTIC-PLASTIC ANALYSIS OF UNBALANCED SCARF JOINTS
C PRECISE SOLUTION, NOT LOWER BOUND
C NON-DIMENSIONALIZED AVERAGE SHEAR STRESSES COMPUTED
C NON-DIMENSIONALIZED JOINT STRENGTHS COMPUTED
C RANGE OF ADHESIVE DUCTILITIES INCLUDED
C RANGES OF ADHEREND STIFFNESS AND THERMAL IMBALANCES ACCOUNTED FOR
C DATA PRESENTATION FOR TENSILE SHEAR LOADING
C CHANGE SIGN OF CTHERM TO USE FOR COMPRESSIVE SHEAR LOADS
C SET CTHERM .EQ. 0. AND REPLACE ADHEREND ET'S WITH GT'S FOR IN-PLANE
C     1   (EDGEWISE) SHEAR LOADING
C
C DIMENSION OL(J), ETR(K), CTHERM(I), GPOVGE(L), TRATIO(J,NCRTND),
C     1   A(N), TAUAVG(J,K), STRGTH(J,K), ICRTND(J,K), THERMC(NCRTND),
C     2   VR(NCRTND), VU(NCRTND), VL(NCRTND), OLTRNT(NCRTND),
C     3   OLTRNC(NCRTND), TRANSL(K), TAUEND(N1), SAVOTP(M), TAVOTP(M),
C     4   BOVERL(N1)
      DIMENSION OL(40), ETR(10), CTHERM(20), GPOVGE(20), A(50),
     1   TRATIO(40,2), TAUAVG(40,10), STRGTH(40,10), ICRTND(40,10),
     2  THERMC(2), VR(2), VU(2), VL(2), OLTRNT(2), OLTRNC(2), TRANSL(10),
     3   TAUEND(50), SAVOTP(50), TAVOTP(50), BOVERL(50)
C
C READ IN INPUT DATA
C READ IN ARRAY SIZES
      READ (5,10) IMAX, JMAX, KMAX, LMAX, NMAX
   10 FORMAT (5I5)
C IMAX .LE. 20, JMAX .LE. 40, KMAX .LE. 10, LMAX .LE. 20,
C     1   NMAX .LE. 50 .AND.  .GE. 10.
C READ IN NON-DIMENSIONALIZED OVERLAP ARRAY
      OL(1) = 0.
C OL(J) MUST BE IN ASCENDING ORDER
C OL(2) MUST BE LESS THAN 0.2 FOR IDENTIFICATION OF CRITICAL END
C     1   OF JOINT OF ZERO OVERLAP (LIMITING CASE)
C OL(J) .LT. 100. FOR COMPATIBILITY WITH FORMAT STATEMENTS 550 & 640
      READ (5,20) (OL(J), J = 2, JMAX)
C NOTE JMAX ONE MORE THAN INPUT VALUES ON CARD(S)
   20 FORMAT (12F6.2)
C READ IN STIFFNESS IMBALANCE ARRAY
C IDENTIFY ADHERENDS SUCH THAT ETR(K) = (ET)1/(ET)2 .LE. 1.
C STIFFNESS RATIOS SHOULD BE IN ASCENDING OR DESCENDING ORDER
C ETR(K) SHOULD INCLUDE VALUE 1. BUT MUST EXCLUDE VALUE 0.
      READ (5,30) (ETR(K), K = 1, KMAX)
   30 FORMAT (10F5.2)
C READ IN NON-DIMENSIONALIZED THERMAL MISMATCH COEFFICIENTS
C CTHERM .PROPNL. (ALPHA(2)-ALPHA(1))*(OPERATING TEMP. - CURE TEMP.)
C NEED CTHERM(I) ARRAY TO CONTAIN BOTH POSITIVE AND NEGATIVE VALUES
C     1   TO COVER BOTH TENSILE AND COMPRESSIVE LOADS
      READ (5,40) (CTHERM(I), I = 1, IMAX)
   40 FORMAT (10F7.3)
C READ IN PLASTIC-TO-ELASTIC STRAIN RATIO ARRAY
C GPOVGE(L) MUST BE .GT. 0. FOR ELASTIC-PLASTIC ANALYSIS
C PURELY ELASTIC SOLUTION OBTAINED FROM SEPARATE PROCEDURE
      READ (5,50) (GPOVGE(L), L = 1, LMAX)
   50 FORMAT (14F5.2)
C
C PRINT OUT INPUT DATA
      WRITE (6,60) IMAX, JMAX, KMAX, LMAX, NMAX
   60 FORMAT (1H1, 9H  IMAX = ,I2, 9H  JMAX = ,I2, 9H  KMAX = ,I2,
     1   9H  LMAX = ,I2, 9H  NMAX = ,I2)
      WRITE (6,70)
   70 FORMAT (10H  OVERLAPS)
      WRITE (6,80) (OL(J), J = 1, JMAX)
   80 FORMAT (12F6.2)
      WRITE (6,90)
   90 FORMAT (22H  STIFFNESS IMBALANCES)
      WRITE (6,100) (ETR(K), K = 1, KMAX)
  100 FORMAT (14F5.2)
      WRITE (6,110)
  110 FORMAT (20H  THERMAL MISMATCHES)
      WRITE (6,120) (CTHERM(I), I = 1, IMAX)
  120 FORMAT (10F7.3)
      WRITE (6,130)
  130 FORMAT (34H  PLASTIC TO ELASTIC STRAIN RATIOS)
      WRITE (6,140) (GPOVGE(L), L = 1, LMAX)
  140 FORMAT (14F5.1)
C
C STORE CONSTANTS
      ANMAX = NMAX
      A(1) = 1.
      TAUEND(1) = 1.
C
C START COMPUTATIONAL DO LOOPS
      DO 670 L = 1, LMAX
      GAMMAR = GPOVGE(L)
C ENSURE EXCLUSION OF NEGATIVE PLASTICITY IN ADHESIVE (ERROR IN DATA)
C EXCLUDE PURELY-ELASTIC ADHESIVE. SEPARATE PROGRAM NEEDED
      IF (GAMMAR .LE. 0.) GO TO 670
      DO 670 I = 1, IMAX
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      THERMC(1) = CTHERM(I)
      THERMC(2) = - THERMC(1)
      DO 460 K = 1, KMAX
      VR(1) = ETR(K)
      VR(2) = 1. / VR(1)
      VU(1) = 1. - VR(1)
      VL(1) = 1. + VR(1)
      VU(2) = 1. - VR(2)
      VL(2) = 1. + VR(2)
C
C ESTABLISH TRANSITIONAL OVERLAPS FROM FULLY-PLASTIC TO ELASTIC-PLASTIC
C    1  BEHAVIOUR AS REFERENCE LENGTH FOR START OF ITERATIONS
C SPECIAL PROCEDURE FOR LESS THAN COMPLETELY UNBALANCED JOINTS
      IF ( (THERMC(1) .EQ. 0.) .AND. (VR(1) .EQ. 1.) ) GO TO 150
      IF (THERMC(1) .EQ. 0.) GO TO 160
      IF (VR(1) .EQ. 1.) GO TO 170
C IF NONE OF THESE, JOINT CONTAINS BOTH IMBALANCES
      GO TO 180
C SET INFINITE TRANSITIONAL OVERLAP FOR IDENTICAL ADHERENDS
  150 OLTRNT(1) = 1000000.
      OLTRNT(2) = 1000000.
C     OLTRNC(1) = 1000000.
C     OLTRNC(2) = 1000000.
      GO TO 190
C SET TRANSITIONAL OVERLAPS FOR STIFFNESS IMBALANCE ONLY
C IN THE ABSENCE OF THERMAL MISMATCH, SAME END IS CRITICAL FOR BOTH
C    1  TENSILE SHEAR AND COMPRESSIVE SHEAR LOADING
  160 IF (VU(1) .GT. 0.) OLTRNT(1) = SQRT(GAMMAR*VL(1)/VU(1))
      IF (VU(1) .LE. 0.) OLTRNT(1) = 1000000.
      IF (VU(2) .GT. 0.) OLTRNT(2) = SQRT(GAMMAR*VL(2)/VU(2))
      IF (VU(2) .LE. 0.) OLTRNT(2) = 1000000.
C     IF (VU(1) .GT. 0.) OLTRNC(1) = SQRT(GAMMAR*VL(1)/VU(1))
C     IF (VU(1) .LE. 0.) OLTRNC(1) = 1000000.
C     IF (VU(2) .GT. 0.) OLTRNC(2) = SQRT(GAMMAR*VL(2)/VU(2))
C     IF (VU(2) .LE. 0.) OLTRNC(2) = 1000000.
      GO TO 190
C SET TRANSITIONAL OVERLAPS FOR THERMAL MISMATCH ONLY
  170 IF (THERMC(1) .LT. 0.) OLTRNT(1) = -GAMMAR/THERMC(1)
      IF (THERMC(1) .GE. 0.) OLTRNT(1) = 1000000.
      IF (THERMC(2) .LT. 0.) OLTRNT(2) = -GAMMAR/THERMC(2)
      IF (THERMC(2) .GE. 0.) OLTRNT(2) = 1000000.
C     IF (THERMC(1) .GE. 0.) OLTRNC(1) = GAMMAR/THERMC(1)
C     IF (THERMC(1) .LT. 0.) OLTRNC(1) = 1000000.
C     IF (THERMC(2) .GE. 0.) OLTRNC(2) = GAMMAR/THERMC(2)
C     IF (THERMC(2) .LT. 0.) OLTRNC(2) = 1000000.
      GO TO 190
  180 CONTINUE
C STANDARD PROCEDURE FOR COMPLETELY UNBALANCED JOINTS
      V1 = THERMC(1) * VL(1) / (2. * VU(1))
      V2 = THERMC(2) * VL(2) / (2. * VU(2))
      V3 = V1*V1 + GAMMAR*VL(1)/VU(1)
      V4 = V2*V2 + GAMMAR*VL(2)/VU(2)
C ESTABLISH TRANSITIONAL OVERLAPS BELOW WHICH JOINT IS FULLY PLASTIC
C NEXT FOUR STATEMENTS APPLY FOR TENSILE SHEAR LOADING
      IF (V3 .GE. 0.) OLTRNT(1) = V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
      IF ( (V3 .LT. 0.) .OR. (OLTRNT(1) .LE. 0.) ) OLTRNT(1) = 1000000.
      IF (V4 .GE. 0.) OLTRNT(2) = V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
      IF ( (V4 .LT. 0.) .OR. (OLTRNT(2) .LE. 0.) ) OLTRNT(2) = 1000000.
C IF ICRTND .EQ. 2 FOR SHORT OVERLAPS, OLTRNT(1) WILL BE COMPUTED VERY
C    1  LARGE, AND VICE VERSA
C THIS IS PHYSICALLY REALISTIC AND DOES NOT LEAD TO IMPOSSIBLE COMPUTING
C IF BOTH V3 AND V4 ARE POSITIVE, EITHER OLTRNT(1) OR OLTRNT(2) WILL BE
C    1  COMPUTED NEGATIVE.  NEED TO PREVENT COMPUTATIONS BASED ON THIS
C    2  UNREAL SITUATION.  HENCE CHECKS ABOVE AND BELOW
C NEXT FOUR STATEMENTS WOULD APPLY FOR COMPRESSIVE SHEAR LOADING
C     IF (V3 .GE. 0.) OLTRNC(1) = -V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
C     IF ( (V3 .LT. 0.) .OR. (OLTRNC(1) .LE. 0.) ) OLTRNC(1) = 1000000.
C     IF (V4 .GE. 0.) OLTRNC(2) = -V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
C     IF ( (V4 .LT. 0.) .OR. (OLTRNC(2) .LE. 0.) ) OLTRNC(2) = 1000000.
C
  190 DO 380 NCRTND = 1, 2
      THERM = THERMC(NCRTND)
      VRREF = VR(NCRTND)
      VUREF = VU(NCRTND)
      VLREF = VL(NCRTND)
C SET UNIFORM STRESS FOR SHORT OVERLAPS
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      DO 200 J = 2, JMAX
      JSAVE = J
      IF (OL(J) .GT. OLTRNT(NCRTND)) GO TO 210
C IF NOT, JOINT IS FULLY PLASTIC
  200 TRATIO(J,NCRTND) = 1.
      IF (JSAVE .EQ. JMAX) GO TO 380
C
C COMPUTE JOINT STRENGTH FOR ELASTIC-PLASTIC ADHESIVE BEHAVIOUR
  210 DO 380 J = JSAVE, JMAX
      OLAP = OL(J)
      OLAP2 = OLAP * OLAP
C COMPUTE AOVERL FOR MINIMUM VALUE OF TAVOTP BY ITERATION
C SET INITIAL ESTIMATE OF EXTENT OF PLASTIC ZONE FROM TRANSITIONAL OLAP
      AOVERL = OLTRNT(NCRTND) / OLAP
      N2MAX = 2 * NMAX
      DO 220 N = 1, N2MAX
      ARMDR = 1. - AOVERL
      AOVERL = -ARMDR*ALOG(ARMDR) + (GAMMAR / ((VUREF/VLREF)*OLAP2
     1 - THERM * OLAP))
      IF (AOVERL .GT. 0.9999) AOVERL = 0.9999
      IF (AOVERL .LT. 0.001) AOVERL = 0.001
  220 CONTINUE
      TRATIO(J,NCRTND) = 1.
      IF (AOVERL .EQ. 0.9999) GO TO 230
C COMPUTE CORRESPONDING AVERAGE SHEAR STRESS
      TAUREF = 1. - (VLREF*GAMMAR/OLAP2 + (VLREF*THERM/OLAP - VUREF) *
     1 AOVERL) / ALOG(1.-AOVERL)
      TRATIO(J,NCRTND) = TAUREF
  230 CONTINUE
      AREF = AOVERL * 0.999
C THE FACTOR IS TO PREVENT DIVERGENCE IN THE SERIES COEFFICIENTS
C SET MINIMUM POSSIBLE VALUE OF AVOERL, AT WHICH TAVOTP .EQ. 1.
      AMIN = GAMMAR / ((VUREF/VLREF)*OLAP2 - THERM*OLAP)
C TRUE EXTENT OF FIRST PLASTIC ZONE BOUNDED WITHIN AMIN AND AREF
      ADEL = (AREF - AMIN) / (ANMAX - 1.)
C MINIMUM VALUES OF TAVOTP ARE NOW COMPUTED.  THESE APPROXIMATE THE TRUE
C     1  SOLUTIONS FOR ALL BUT SHORT OVERLAPS OR THICK ADHERENDS
C     2  IN CONJUNCTION WITH SEVERE ADHEREND MISMATCH AND/OR BRITTLE
C     3  ADHESIVES.  REFINE ANSWER BY PRECISE SOLUTION IN POWER SERIES
C COMPUTE JOINT STRENGTH FOR ELASTIC-PLASTIC ADHESIVE BEHAVIOUR
      DO 360 M = 1, NMAX
      AM = M
      AOVERL = AREF - (AM - 1.)*ADEL
      ARMDR = 1. - AOVERL
C COMPUTE ASSOCIATED AVERAGE BOND STRESS
      TAVOTP(M) = 1. - (VLREF*GAMMAR/OLAP2 + (VLREF*THERM/OLAP - VUREF)
     1 * AOVERL) / ALOG(ARMDR)
C
C START COMPUTING ELASTIC STRESS SERIES
C     A(1) = 1.
C ESTABLISH A(2) AT START OF ELASTIC ZONE FROM CONTINUITY OF SHEAR
C     1  STRAINS IN ADHESIVE AT TRANSITION.  THIS ENSURES ADHEREND STRESS
C     2  CONTINUITY
      A(2) = THERM*OLAP - OLAP2*(VUREF - (1.-TAVOTP(M))/ARMDR) / VLREF
C A(2) SHOULD BE .LT. 0. FOR AOVERL .LT. AREF
C A(2) SHOULD BE .EQ. 0. FOR AOVERL .EQ. AREF
C A(2) SHOULD BE .GT. 0. FOR AOVERL .GT. AREF
      A(3) = (A(2) - THERM*OLAP + OLAP2*VUREF/VLREF) / (2.*ARMDR)
C CONVERT STRESS TERMS INTO AVERAGE STRESS TERMS BY DIVIDING BY N
      A(2) = A(2) / 2.
      A(3) = A(3) / 3.
C COMPUTE SUBSEQUENT TERMS FROM RECURRENCE FORMULA
      DO 240 N = 4, NMAX
      NSAVE = N
      AN = N
      A(N) = ((2.*AOVERL - 1.)*(AN-2.)*(AN-1.)*A(N-1) +
     1  (AN-3.)*(AN-2.)*A(N-2) + (OLAP2/VLREF)*
     2  ((AOVERL*VUREF + VRREF)*A(N-2) + VUREF*A(N-3))) /
     3  (AOVERL*ARMDR*(AN-1.)*AN)
      IF (ABS(A(N)) .LT. 1.E50) GO TO 240
C IF NOT, OVERFLOW IS IMMINENT, SO CUT DOWN ON NMAX
      GO TO 250
  240 CONTINUE
      GO TO 270
  250 DO 260 N = NSAVE, NMAX
  260 A(N) = 0.
C ESTIMATE ELASTIC ADHESIVE STRESS AT OTHER END OF JOINT, OR IDENTIFY
C     1  EXISTENCE OF SECOND PLASTIC ADHESIVE ZONE, AS APPROPRIATE
C START BY ASSUMING NO SECOND PLASTIC ADHESIVE ZONE
  270 COVERL = 0.
C     TAUEND(1) = 1.
      TAUEND(NMAX) = 1.
      DO 280 N = 2, NMAX
      AN = N
  280 TAUEND(NMAX) = TAUEND(NMAX) + A(N)*(ARMDR**(N-1))*AN
      IF (TAUEND(NMAX) .LE. 1.) GO TO 310
C IF SO, ONLY THE ONE PLASTIC ZONE, AT THE NCRTND REFERENCE END
C IF NOT, HAVE IDENTIFIED EXISTENCE OF SECOND PLASTIC ZONE, AT OTHER END
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C
C PROCEDURE FOR SECOND PLASTIC ZONE
C USE LINEAR INTERPOLATION TO ESTIMATE COVERL (EXTENT OF SECOND PLASTIC
C    1  ZONE)
      DELBOL = (1. - AOVERL) / (ANMAX - 1.)
      BOVERL(1) = 1. - AOVERL
      DO 300 N1 = 2, NMAX
      BOVERL(N1) = BOVERL(N1-1) + DELBOL
      TAUEND(N1) = 1.
      DO 290 N = 2, NMAX
      AN = N
      V = BOVERL(N1)**(N-1)
  290 TAUEND(N1) = TAUEND(N1) + A(N)*V*AN
C CHECK FOR CONVERGENCE OF COVERL
      IF ((1.0001.GT.TAUEND(N1)) .AND. (0.9999.LT.TAUEND(N1))) GO TO 310
C IF NOT, ITERATE ON COVERL
C COMPUTE VALUE OF COVERL NEEDED TO RESTRICT STRESSES TO ELASTIC LEVEL
      IF (TAUEND(N1) .LT. 1.) GO TO 300
C IF SO, ESTIMATE OF BOVERL IS INSUFFICIENT AND THAT OF COVERL EXCESSIVE
C IF NOT, CORRECT TRANSITION LIES BETWEEN N1 AND N1-1 LOCATIONS
      COVERL = 1. - AOVERL - BOVERL(N1-1) -
     1  ((1. - TAUEND(N1-1)) / (TAUEND(N1) - TAUEND(N1-1)))
      GO TO 310
  300 CONTINUE
C DEFER CHECK ON WHETHER COVERL IS SO LARGE THAT CRITICAL END OF JOINT
C    1   IS AT OTHER END UNTIL AFTER CONVERGENCE OF AOVERL IS ESTABLISHED
C
  310 SAVOTP(M) = 1.
      BOVL = 1. - AOVERL - COVERL
C EVALUATE AVERAGE STRESS IN TERMS OF SERIES COEFFICIENTS
      DO 320 N = 2, NMAX
      AN = N
  320 SAVOTP(M) = SAVOTP(M) + A(N)*(BOVL**(N))
C
C CHECK ON CONVERGENCE OF AOVERL
      IF (SAVOTP(M) .GT. 1.) GO TO 360
C IF SO, CANNOT HAVE CONVERGED YET
      IF ( (SAVOTP(M) .LT. TAVOTP(M)) .AND. (M .EQ. 1) ) GO TO 330
C IF SO, SOLUTION IS NUMERICALLY INDISTINGUISHIBLE FROM THE LOWER BOUND
C NEED BOTH M .EQ. 1 VALUES AND M .EQ. 2 VALUES FOR FIRST CHECK
      IF (M .EQ. 1) GO TO 360
C PROTECT AGAINST DIVISION BY ZERO
      IF ( (TAVOTP(M) .EQ. 0.) .AND. (SAVOTP(M) .EQ. 0.) ) GO TO 340
C IF SO, CONVERGENCE ESTABLISHED
      IF ( (SAVOTP(M) .LT. 0.00001) .AND. (SAVOTP(M) .GT. -0.00001) )
     1  RATIO = 1. + TAVOTP(M)
      IF ( (TAVOTP(M) .LT. 0.00001) .AND. (TAVOTP(M) .GT. -0.00001) )
     1  RATIO = 1. + SAVOTP(M)
C IF NONE OF THE ABOVE, NO FURTHER FAILURE CASES LEFT TO CHECK FOR
      RATIO = SAVOTP(M) / TAVOTP(M)
C CHECK ON CONVERGENCE OF JOINT STRENGTH PREDICTIONS
      IF ( (1.0001 .GT. RATIO) .AND. (0.9999 .LT. RATIO) ) GO TO 350
C IF SO, CONVERGENCE IS ESTABLISHED
C IF NOT, NEED TO RE-ESTIMATE AOVERL
C USE LINEAR INTERPOLATION TO ESTIMATE AOVERL (EXTENT OF FIRST PLASTIC
C    1  ZONE)
      IF (SAVOTP(M) .GT. TAVOTP(M)) GO TO 360
C IF SO, CONVERGENCE OF AOVERL NOT YET ESTABLISHED
C IF NOT, CORRECT VALUE OF AOVERL LIES BETWEEN M AND M-1 LOCATIONS
      TRATIO(J,NCRTND) = TAVOTP(M-1) + (TAVOTP(M) - TAVOTP(M-1)) *
     1  (1. - TAVOTP(M-1) / SAVOTP(M-1)) /
     2  (1. - (SAVOTP(M) - TAVOTP(M) + TAVOTP(M-1)) / SAVOTP(M-1))
      GO TO 370
  330 TRATIO(J,NCRTND) = TAUREF
      GO TO 370
  340 TRATIO(J,NCRTND) = 0.
      GO TO 370
  350 TRATIO(J,NCRTND) = TAVOTP(M)
      GO TO 370
  360 CONTINUE
C IF REFINEMENT HAS NOT CONVERGED, USE LOWER BOUND ESTIMATE
C TRATIO(J,NCRTND) = TAVOTP(1), AS SET EARLIER
C PROTECT AGAINST ACCUMULATED NUMERICAL ERRORS
C USE LOWER BOUND SOLUTION IF REFINEMENT RESULTS IN STILL LOWER VALUES
  370 IF (TRATIO(J,NCRTND) .LT. TAVOTP(1)) TRATIO(J,NCRTND) = TAVOTP(1)
  380 CONTINUE
C
C CONVERGENCE OF AOVERL ESTABLISHED.  RECORD AVERAGE SHEAR STRESS
C VALUES COMPUTED ARE NOW STORED IN TRATIO(J,NCRTND)
C NEED TO SELECT LOWER VALUE TO IDENTIFY CRITICAL END OF JOINT
      DO 450 J = 2, JMAX
      OLAP = OL(J)
      TAU1 = TRATIO(J,1)
      TAU2 = TRATIO(J,2)
      IF ( (TAU1 .LT. 1.) .OR. (TAU2 .LT. 1.) ) GO TO 390
C IF SO, JOINT IS NOT FULLY PLASTIC
C IF NOT, IDENTIFY CRITICAL END OF JOINT FROM SHEAR STRAIN GRADIENT
      GRADNT = THERMC(1) - OLAP*VU(1)/VL(1)
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      IF (GRADNT .LT. 0.) ICRTND(J,K) = 1                               A4EE3530
      IF (GRADNT .EQ. 0.) ICRTND(J,K) = 0                               A4EE3540
      IF (GRADNT .GT. 0.) ICRTND(J,K) = 2                               A4EE3550
      TAUAVG(J,K) = 1.                                                  A4EE3560
      STRGTH(J,K) = OLAP                                                A4EE3570
      MCRTND = ICRTND(J,K)                                              A4EE3580
      IF (MCRTND .EQ. 0) MCRTND = 1                                     A4EE3590
      TRANSL(K) = OLTRNT(MCRTND)                                        A4EE3600
      GO TO 450                                                         A4EE3610
  390 DIFFNC = TAU1 - TAU2                                              A4EE3620
C IF DIFFNC .LT. 0., NCRTND .EQ. 1                                      A4EE3630
C IF DIFFNC .EQ. 0., NCRTND .EQ. 0                                      A4EE3640
C IF DIFFNC .GT. 0., NCRTND .EQ. 2                                      A4EE3650
      IF (DIFFNC)400,410,420                                            A4EE3660
C ADHEREND (1) END OF JOINT CRITICAL                                    A4EE3670
  400 TAUAVG(J,K) = TAU1                                                A4EE3680
      STRGTH(J,K) = TAU1 * OLAP                                         A4EE3690
      ICRTND(J,K) = 1                                                   A4EE3700
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)          A4EE3710
      IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)                               A4EE3720
      GO TO 430                                                         A4EE3730
C BOTH ENDS OF JOINT EQUALLY CRITICAL FROM NULLIFYING (OR ZERO)         A4EE3740
C    1  ADHEREND IMBALANCES                                             A4EE3750
  410 TAUAVG(J,K) = TAU1                                                A4EE3760
      STRGTH(J,K) = TAU1 * OLAP                                         A4EE3770
      ICRTND(J,K) = 0                                                   A4EE3780
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)          A4EE3790
      IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)                               A4EE3800
      GO TO 430                                                         A4EE3810
C ADHEREND (2) END OF JOINT CRITICAL                                    A4EE3820
  420 TAUAVG(J,K) = TAU2                                                A4EE3830
      STRGTH(J,K) = TAU2 * OLAP                                         A4EE3840
      ICRTND(J,K) = 2                                                   A4EE3850
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)          A4EE3860
      IF (J .EQ. 2) TRANSL(K) = OLTRNT(2)                               A4EE3870
C COVER CASES OF ZERO OR NEGATIVE ESTIMATED STRENGTHS                   A4EE3880
  430 IF (TAUAVG(J,K) .GT. 0.) GO TO 440                                A4EE3890
C IF NOT, JOINT HAS BROKEN DUE TO THERMAL STRESSES WITHOUT EXTERNAL LOADA4EE3900
      TAUAVG(J,K) = 0.                                                  A4EE3910
      STRGTH(J,K) = 0.                                                  A4EE3920
      GO TO 450                                                         A4EE3930
  440 IF (TAUAVG(J,K) .LE. 1.) GO TO 450                                A4EE3940
C IF NOT, THERE HAS BEEN A COMPUTATIONAL MISTAKE                        A4EE3950
C PRINT ASTERISKS TO IDENTIFY ERROR                                     A4EE3960
C RERUN WITH GREATER VALUE OF NMAX                                      A4EE3970
      TAUAVG(J,K) = 100.                                                A4EE3980
      STRGTH(J,K) = 1000.                                               A4EE3990
  450 CONTINUE                                                          A4EE4000
  460 CONTINUE                                                          A4EE4010
C                                                                       A4EE4020
C SET UNIFORM STRESS FOR ZERO OVERLAP                                   A4EE4030
      DO 470 K = 1, KMAX                                                A4EE4040
      TAUAVG(1,K) = 1.                                                  A4EE4050
      STRGTH(1,K) = 0.                                                  A4EE4060
  470 ICRTND(1,K) = ICRTND(2,K)                                         A4EE4070
C HENCE NEED FOR OL(2) TO BE SMALL ENOUGH TO BE LESS THAN THAT AT WHICH A4EE4080
C    1  NCRTND CHANGES                                                  A4EE4090
C                                                                       A4EE4100
C END OF COMPUTATIONS. START PRINTING OUT OF TABULATED RESULTS          A4EE4110
C                                                                       A4EE4120
C PRINT OUT AVERAGE STRESS HEADING                                      A4EE4130
      WRITE (6,480)                                                     A4EE4140
  480 FORMAT (1H1/, 5(1H0/), 27X, 56HADHESIVE-BONDED SCARF JOINTS  (ELASA4EE4150
     1TIC-PLASTIC ANALYSIS)/,                                           A4EE4160
     2  39X, 31HNON-DIMENSIONALIZED FORMULATION/)                       A4EE4170
      WRITE (6,490) GAMMAR                                              A4EE4180
  490 FORMAT(1H0, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIOA4EE4190
     1 = , F5.2)                                                        A4EE4200
      IF (CTHERM(I) .NE. 0.) GO TO 510                                  A4EE4210
      WRITE (6,500)                                                     A4EE4220
  500 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT)           A4EE4230
      GO TO 530                                                         A4EE4240
  510 WRITE (6,520) THERMC(1), THERMC(2)                                A4EE4250
  520 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3,       A4EE4260
     1  17H FOR TENSION,   = , F6.3, 16H FOR COMPRESSION)               A4EE4270
  530 WRITE (6,540) (ETR(K), K = 1, KMAX)                               A4EE4280
  540 FORMAT(  1H0,  67X, 30H0 = BOTH ENDS EQUALLY CRITICAL/, 20X,      A4EE4290
     1  72HAVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS   ,  1 = SOFT ET EA4EE4300
     2ND CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL/,                 A4EE4310
     3  8H0 SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,   A4EE4320
     4  7H    L/T/, 7H  RATIO, F7.1, 9F10.1/, 1H )                      A4EE4330
C WRITE OUT TABULATIONS OF AVERAGE BOND STRESSES                        A4EE4340
      DO 560 J = 1, JMAX                                                A4EE4350
      WRITE (6,550) OL(J), ((TAUAVG(J,K), ICRTND(J,K)), K = 1, KMAX)    A4EE4360
  550 FORMAT (1H , F6.2, 2X, 10(F7.5, 1X, I1, 1X))                      A4EE4370
  560 CONTINUE                                                          A4EE4380
C                                                                       A4EE4390
C PRINT OUT JOINT STRENGTH HEADING                                      A4EE4400
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      WRITE (6,570)
  570 FORMAT (1H1/, 5(1HO/), 27X, 56HADHESIVE-BONDED SCARF JOINTS  (ELAS
     1TIC-PLASTIC ANALYSIS)/,
     2  39X, 31HNON-DIMENSIONALIZED FORMULATION/)
      WRITE (6,580) GAMMAR
  580 FORMAT(1HO, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO
     1 = , F5.2)
      IF (CTHERM(I) .NE. 0.) GO TO 600
      WRITE (6,590)
  590 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT)
      GO TO 620
  600 WRITE (6,610) THERMC(1), THERMC(2)
  610 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3,
     1  17H FOR TENSION,  = , F6.3, 16H FOR COMPRESSION)
  620 WRITE (6,630) (ETR(K), K = 1, KMAX)
  630 FORMAT(  1HO,  67X, 30HO = BOTH ENDS EQUALLY CRITICAL/, 20X,
     1  72HNON-DIMENSIONALIZED JOINT STRENGTH   ,          1 = SOFT ET E
     2ND CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL/,
     3  8HO SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
     4  7H    L/T/, 7H  RATIO, F7.1, 9F10.1/, 1H )
C WRITE OUT TABULATIONS OF JOINT STRENGTHS
      DO 650 J = 1, JMAX
      WRITE (6,640) OL(J), ((STRGTH(J,K), ICRTND(J,K)), K = 1, KMAX)
  640 FORMAT (1H , F6.2, 2X, 10(F7.4, 1X, I1, 1X))
  650 CONTINUE
C WRITE OUT TRANSITIONAL JOINT STRENGTHS
      WRITE (6,660) (TRANSL(K), K = 1, KMAX)
  660 FORMAT (8HO TRANSL, 1X, 10(F7.4, 3X))
  670 CONTINUE
C
      WRITE (6,680)
  680 FORMAT (1H1, 18H PROGRAM COMPLETED)
      STOP
      END
```

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH ,

0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0	
0.0	0.0	2	0.0	2	0.0	2	0.0	2	0.0	2	0.0	2	0.0	2	0.0	2	0.0	2	0.0	2
0.20	0.2000	2	0.2000	2	0.2000	2	0.2000	2	0.2000	2	0.2000	2	0.2000	2	0.2000	2	0.2000	2	0.2000	2
0.50	0.5000	2	0.5000	2	0.5000	2	0.5000	2	0.5000	2	0.5000	2	0.5000	2	0.5000	2	0.5000	2	0.5000	2
1.00	1.0000	2	1.0000	2	1.0000	2	1.0000	2	1.0000	2	1.0000	2	1.0000	2	1.0000	2	1.0000	2	1.0000	2
1.20	1.2000	2	1.2000	2	1.2000	2	1.2000	2	1.2000	2	1.2000	2	1.2000	2	1.2000	2	1.2000	2	1.2000	2
1.50	1.5000	1	1.5000	0	1.5000	2	1.5000	2	1.5000	2	1.5000	2	1.5000	2	1.5000	2	1.5000	2	1.5000	2
1.70	1.7000	1	1.7000	1	1.7000	2	1.7000	2	1.7000	2	1.7000	2	1.7000	2	1.7000	2	1.7000	2	1.7000	2
2.00	2.0000	1	2.0000	1	2.0000	1	2.0000	2	2.0000	2	2.0000	2	2.0000	2	2.0000	2	2.0000	2	2.0000	2
2.50	2.5000	1	2.5000	1	2.5000	1	2.5000	1	2.5000	2	2.5000	2	2.5000	2	2.5000	2	2.5000	2	2.5000	2
3.00	3.0000	1	3.0000	1	3.0000	1	3.0000	1	3.0000	0	3.0000	2	3.0000	2	3.0000	2	3.0000	2	3.0000	2
4.00	3.6484	1	3.9998	1	4.0000	1	4.0000	1	4.0000	1	4.0000	0	4.0000	2	4.0000	2	4.0000	2	4.0000	2
5.00	4.0026	1	4.3962	1	4.7267	1	5.0000	1	5.0000	1	5.0000	1	5.0000	2	5.0000	2	5.0000	2	5.0000	2
6.00	4.2598	1	4.7826	1	5.2518	1	5.6524	1	5.9999	1	6.0000	1	6.0000	1	6.0000	2	6.0000	2	5.9999	2
8.00	4.5976	1	5.3763	1	6.1055	1	6.7624	1	7.3451	1	7.9402	1	8.0000	1	8.0000	2	8.0000	2	7.9253	2
10.00	4.8582	1	5.8512	1	6.8081	1	7.7193	1	8.5517	1	9.2849	1	9.9997	1	10.0000	1	10.0000	2	9.6345	2
12.00	5.1248	1	6.3263	1	7.4862	1	8.6009	1	9.6666	1	10.6464	1	11.4904	1	12.0000	1	12.0000	2	11.5248	2
15.00	5.4906	1	7.0028	1	8.4752	1	9.9027	1	11.2764	1	12.5839	1	13.7902	1	14.7790	1	15.0000	2	14.3952	2
17.00	5.7213	1	7.4384	1	9.1166	1	10.7510	1	12.3334	1	13.8491	1	15.2743	1	16.5133	1	17.0000	2	16.3252	2
20.00	6.0557	1	8.0782	1	10.0626	1	12.0046	1	13.8965	1	15.7251	1	17.4641	1	19.0517	1	20.0000	1	19.2379	2
25.00	6.5943	1	9.1229	1	11.6144	1	14.0647	1	16.4670	1	18.8093	1	21.0690	1	23.1927	1	24.9678	1	24.1246	2
30.00	7.1200	1	10.1525	1	13.1486	1	16.1044	1	19.0134	1	21.8645	1	24.6371	1	27.2939	1	29.6214	1	29.0380	2
35.00	7.6382	1	11.1736	1	14.6729	1	18.1325	1	21.5462	1	24.9034	1	28.1948	1	31.3471	1	34.2370	1	33.9691	2
40.00	8.1518	1	12.1893	1	16.1911	1	20.1535	1	24.0706	1	27.9324	1	31.7201	1	35.3935	1	38.8160	1	38.9126	2
45.00	8.6624	1	13.2016	1	17.7052	1	22.1698	1	26.5896	1	30.9547	1	35.2474	1	39.4290	1	43.3747	1	43.8654	2
50.00	9.1709	1	14.2113	1	19.2164	1	24.1828	1	29.1047	1	33.9726	1	38.7690	1	43.4570	1	47.9200	1	48.8249	2
TRANSL	3.1576		3.5895		4.1142		4.7761		5.6533		6.8990		8.8633		12.5777		23.1106		5.0000	

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,

0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0	
0.0	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
0.20	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
0.50	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
1.00	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
1.20	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
1.50	1.00000	1	1.00000	0	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
1.70	1.00000	1	1.00000	1	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
2.00	1.00000	1	1.00000	1	1.00000	1	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
2.50	1.00000	1	1.00000	1	1.00000	1	1.00000	1	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
3.00	1.00000	1	1.00000	1	1.00000	1	1.00000	1	1.00000	0	1.00000	2	1.00000	2	1.00000	2	1.00000	2	1.00000	2
4.00	0.91211	1	0.99994	1	1.00000	1	1.00000	1	1.00000	1	1.00000	0	1.00000	2	1.00000	2	1.00000	2	1.00000	2
5.00	0.80052	1	0.87925	1	0.94534	1	0.99999	1	1.00000	1	1.00000	1	1.00000	2	1.00000	2	1.00000	2	1.00000	2
6.00	0.70996	1	0.79709	1	0.87531	1	0.94206	1	0.99999	1	1.00000	1	1.00000	1	1.00000	2	1.00000	2	0.99998	2
8.00	0.57345	1	0.67203	1	0.76319	1	0.84530	1	0.91814	1	0.99253	1	1.00000	1	1.00000	2	1.00000	2	0.99066	2
10.00	0.48582	1	0.58512	1	0.68081	1	0.77193	1	0.85517	1	0.92849	1	0.99997	1	1.00000	1	1.00000	2	0.96345	2
12.00	0.42707	1	0.52719	1	0.62385	1	0.71675	1	0.80555	1	0.88720	1	0.95753	1	1.00000	1	1.00000	2	0.96040	2
15.00	0.36604	1	0.46685	1	0.56502	1	0.66018	1	0.75176	1	0.83893	1	0.91935	1	0.98527	1	1.00000	2	0.95968	2
17.00	0.33655	1	0.43755	1	0.53627	1	0.63241	1	0.72549	1	0.81465	1	0.89849	1	0.97137	1	1.00000	2	0.96031	2
20.00	0.30278	1	0.40391	1	0.50313	1	0.60023	1	0.69483	1	0.78625	1	0.87321	1	0.95259	1	1.00000	1	0.96189	2
25.00	0.26377	1	0.36492	1	0.46458	1	0.56259	1	0.65868	1	0.75237	1	0.84276	1	0.92771	1	0.99871	1	0.96498	2
30.00	0.23733	1	0.33842	1	0.43829	1	0.53681	1	0.63378	1	0.72882	1	0.82124	1	0.90946	1	0.99738	1	0.96793	2
35.00	0.21823	1	0.31925	1	0.41923	1	0.51907	1	0.61561	1	0.71153	1	0.80528	1	0.89563	1	0.97820	1	0.97055	2
40.00	0.20380	1	0.30473	1	0.40478	1	0.50384	1	0.60177	1	0.69831	1	0.79300	1	0.88484	1	0.97040	1	0.97282	2
45.00	0.19250	1	0.29337	1	0.39345	1	0.49266	1	0.59088	1	0.68789	1	0.78328	1	0.87620	1	0.96388	1	0.97479	2
50.00	0.18342	1	0.28423	1	0.38433	1	0.48366	1	0.58209	1	0.67945	1	0.77538	1	0.86914	1	0.95840	1	0.97650	2

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH ,
0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1	0.0 1
0.20	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1	0.2000 1
0.50	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1	0.5000 1
1.00	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1	1.0000 1
1.20	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1	1.2000 1
1.50	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1	1.5000 1
1.70	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1	1.7000 1
2.00	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1	2.0000 1
2.50	2.4992 1	2.4996 1	2.4999 1	2.5000 1	2.5000 1	2.5000 1	2.5000 1	2.5000 1	2.5000 1	2.5000 1
3.00	2.3553 1	2.5329 1	2.9991 1	2.9995 1	2.9999 1	3.0000 1	3.0000 1	3.0000 1	3.0000 1	3.0000 1
4.00	2.5137 1	2.7894 1	3.0434 1	3.2754 1	3.4845 1	3.8805 1	3.9996 1	3.9999 1	4.0000 1	4.0000 1
5.00	2.6328 1	3.0062 1	3.3544 1	3.6794 1	3.9806 1	4.2563 1	4.5041 1	4.9992 1	4.9998 1	5.0000 1
6.00	2.6965 1	3.1912 1	3.6503 1	4.0727 1	4.4663 1	4.8326 1	5.1691 1	5.4712 1	5.9993 1	5.9999 1
8.00	2.8421 1	3.5216 1	4.1808 1	4.8180 1	5.4241 1	5.9866 1	6.5054 1	6.9831 1	7.4109 1	7.9253 1
10.00	3.0443 1	3.9160 1	4.7601 1	5.5762 1	6.3663 1	7.1293 1	7.8520 1	8.5154 1	9.1169 1	9.6345 1
12.00	3.2462 1	4.3151 1	5.3556 1	6.3665 1	7.3455 1	8.2899 1	9.1993 1	10.0606 1	10.8436 1	11.5248 1
15.00	3.5475 1	4.9134 1	6.2500 1	7.5558 1	8.8282 1	10.0633 1	11.2543 1	12.3923 1	13.4594 1	14.3952 1
17.00	3.7478 1	5.3123 1	6.8469 1	8.3502 1	9.8194 1	11.2501 1	12.6351 1	13.9624 1	15.2136 1	16.3252 1
20.00	4.0481 1	5.9108 1	7.7433 1	9.5437 1	11.3090 1	13.0345 1	14.7123 1	16.3292 1	17.8594 1	19.2379 1
25.00	4.5482 1	6.9090 1	9.2397 1	11.5357 1	13.7965 1	16.0158 1	18.1851 1	20.2891 1	22.2970 1	24.1246 1
30.00	5.0482 1	7.9076 1	10.7355 1	13.5301 1	16.2877 1	19.0028 1	21.6660 1	24.2608 1	26.7525 1	29.0380 1
35.00	5.5482 1	8.9065 1	12.2331 1	15.5259 1	18.7812 1	21.9932 1	25.1519 1	28.2398 1	31.2190 1	33.9691 1
40.00	6.0484 1	9.9058 1	13.7313 1	17.5227 1	21.2763 1	24.9858 1	28.6411 1	32.2236 1	35.6929 1	38.9126 1
45.00	6.5487 1	10.9053 1	15.2299 1	19.5203 1	23.7723 1	27.9799 1	32.1325 1	36.2107 1	40.1718 1	43.8654 1
50.00	7.0494 1	11.9051 1	16.7289 1	21.5183 1	26.2692 1	30.9752 1	35.6255 1	40.2001 1	44.6546 1	48.8249 1
TRANSL	1.9354	2.0895	2.2570	2.4427	2.6533	2.8990	3.1967	3.5777	4.1107	5.0000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,
0 = BOTH ENDS EQUALLY CRITICAL
1 = SOFT ET END CRITICAL
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
0.20	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
0.50	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.00	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.20	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.50	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
1.70	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
2.00	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
2.50	0.99967 1	0.99984 1	0.99996 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
3.00	0.78511 1	0.94430 1	0.99968 1	0.99984 1	0.99995 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1	1.00000 1
4.00	0.62842 1	0.69735 1	0.76086 1	0.81986 1	0.87112 1	0.97012 1	0.99990 1	0.99998 1	1.00000 1	1.00000 1
5.00	0.52655 1	0.60124 1	0.67088 1	0.73587 1	0.79611 1	0.85127 1	0.90083 1	0.99985 1	0.99996 1	1.00000 1
6.00	0.44942 1	0.53186 1	0.60838 1	0.67878 1	0.74438 1	0.80544 1	0.86152 1	0.91187 1	0.99988 1	0.99998 1
8.00	0.35526 1	0.44020 1	0.52260 1	0.60225 1	0.67901 1	0.74833 1	0.81317 1	0.87289 1	0.92636 1	0.99066 1
10.00	0.30443 1	0.39160 1	0.47601 1	0.55762 1	0.63663 1	0.71293 1	0.78521 1	0.85154 1	0.91169 1	0.96345 1
12.00	0.27051 1	0.35959 1	0.44630 1	0.53054 1	0.61213 1	0.69083 1	0.76661 1	0.83838 1	0.90363 1	0.96040 1
15.00	0.23650 1	0.32756 1	0.41666 1	0.50372 1	0.59855 1	0.67088 1	0.75028 1	0.82615 1	0.89729 1	0.95968 1
17.00	0.22046 1	0.31249 1	0.40276 1	0.49119 1	0.57761 1	0.66177 1	0.74324 1	0.82132 1	0.89492 1	0.96031 1
20.00	0.20241 1	0.29554 1	0.38716 1	0.47718 1	0.56545 1	0.65172 1	0.73562 1	0.81646 1	0.89297 1	0.96189 1
25.00	0.18193 1	0.27636 1	0.36955 1	0.46143 1	0.55186 1	0.64063 1	0.72740 1	0.81156 1	0.89188 1	0.96498 1
30.00	0.16827 1	0.26359 1	0.35785 1	0.45100 1	0.54292 1	0.63343 1	0.72220 1	0.80869 1	0.89175 1	0.96793 1
35.00	0.15852 1	0.25447 1	0.34952 1	0.44360 1	0.53661 1	0.62838 1	0.71863 1	0.80685 1	0.89197 1	0.97055 1
40.00	0.15121 1	0.24764 1	0.34328 1	0.43807 1	0.53191 1	0.62464 1	0.71603 1	0.80559 1	0.89232 1	0.97282 1
45.00	0.14553 1	0.24234 1	0.33844 1	0.43378 1	0.52827 1	0.62178 1	0.71405 1	0.80468 1	0.89271 1	0.97479 1
50.00	0.14099 1	0.23810 1	0.33458 1	0.43037 1	0.52538 1	0.61950 1	0.71251 1	0.80400 1	0.89309 1	0.97650 1

A.4 Computer Program A4EF For Elastic Strength of Stepped-Lap Bonded Joints

The analysis in Section 5 has been prepared as the FORTRAN IV digital computer program A4EF. The program computes the elastic joint strength of any stepped-lap bonded joint and prints out the most critical adherend and adhesive stresses for each step of the joint. In order to obtain a more complete internal stress distribution, each step can be subdivided and a series of shorter steps input instead. The input data is printed out to supplement the solution output. Eccentricities are excluded from the joint and a symmetric two-sided bonded joint is analyzed in which the thicknesses of the two outer adherends are lumped together in evaluating the joint strengths. The reason for this is the greater utilization of the back-to-back stepped-lap joint than of the single-sided joint. A single-sided joint can be analyzed with this program in one of two ways. One can add a mirror image of the actual joint and halve the strength predicted for this joint of twice the actual thickness and twice the bond area or one can change certain factors of 2, identified in the listing, to 1 for single-sided joints. The program accounts for arbitrary combinations of adherend stiffness and thermal imbalances as well as non-uniform step thickness increments and step lengths. It has been used successfully in optimizing the joint proportions in order to maximize the joint strength.

A complete listing of the program A4EF follows after the input and output have been described.

CARD 1:

FORMAT (I2)

M = Number of configurations (each requiring a complete set of data) to be solved.

CARDS 2, 2A:

FORMAT (8F10.3)

TAUMAX = τ_p = Peak adhesive shear stress.

G = Elastic adhesive shear modulus.

GAMMAX = $\gamma_e + \gamma_p$ = Maximum adhesive shear strain. (This may be set less than γ_e to cover partial loads.)

GAMMAE = γ_e = Elastic adhesive shear strain.

ETA = η = Bond line thickness.

ALPHAO = α_o = Coefficient of thermal expansion of outer adherend.

ALPHAI = α_i = Coefficient of thermal expansion of inner adherend.

DELTMP = $\Delta T = T_{operating} - T_{stress\text{-}free} \approx T_{operating} - T_{cure}$
= Temperature differential.

SGNLD = +1 for tensile shear load, and
= -1 for compressive shear load.

ANSTEP = Number of steps in the joint. This serves to control the number of adherend property cards read in.

CARDS 3, 3A, 3B, ..etc.., 3(N = ANSTEP + 1)

FORMAT (7F10.3)

THICKO(N) = Sum of thicknesses of outer adherends for nth step.

THICKI(N) = Thickness of nth step of inner adherend.

STEPL(N) = Length of nth step.

ETOTR(N) = Net extensional stiffness of outer adherends at nth step.

ETINR(N) = Extensional stiffness of inner adherend at nth step.

STROTR(N) = Net strength of outer adherends at nth step.

STRINR(N) = Strength of inner adherend at nth step.

The output is in tabular form with one row devoted to each step or step portion. Those entries not defined in the input description above are: TAU the adhesive shear stress, GAMMA the adhesive shear strain, DELTAO the displacement of the outer adherends, DELTAI the displacement of the inner adherend, with TOUTER and TINNER being the loads (σt) in the outer and inner adherends, respectively.

The more accurate solution is obtained by starting the iterative solution from the more critically loaded end. Therefore, in those cases in which the a priori identification of the more critical end is not possible, the program outputs solutions from each end, and the second one is to be preferred. Such cases have been run and the computational procedure in double precision has been shown to be sufficiently accurate from either end. The need for this higher precision on IBM computers arises from the precision loss throughout the nested do loops in the iteration sequence. The greater number of significant digits employed by CDC machines has been found to obviate the need for this and the program can be modified to single-precision operation on CDC machines in a straightforward manner.

```
CDECK A4EF
C STEPPED-LAP ADHESIVE-BONDED-JOINTS
C PERFECTLY-ELASTIC SOLUTIONS
C JOINT ANALYSIS PROGRAM
C SOLUTION EXAMINES ADHESIVE SHEAR STRESS AND ADHEREND NORMAL (AXIAL)
C     1  STRESS BUT OMITS CONSIDERATION OF ADHESIVE PEEL STRESS ON THE
C     2  GROUND THAT OUTER END STEP IS USUALLY SUFFICIENTLY THIN FOR
C     3  PEEL STRESS PROBLEMS NOT TO ARISE
C NOTE THAT CONVERGENCE PROBLEM IS ACUTE FOR STEPPED-LAP JOINTS, EVEN
C     1  WITH DOUBLE-PRECISION.  STEPS TAKEN HERE TO CONSTRAIN TENDENCY
C     2  TO DIVERGE (BY FREEZING SOLUTION ONE STEP AT A TIME) HAVE BEEN
C     3  ADOPTED AFTER TRYING BOTH MORE AND LESS STRINGENT TECHNIQUES
C NOTE ALSO THAT CONVERGENCE DIFFICULTIES ARE PROBLEM DEPENDENT, BEING
C     1  MORE SEVERE FOR BRITTLE (HIGH MODULUS) ADHESIVES.  LOW MODULUS
C     2  ADHESIVES PROVED AMENABLE TO A CONVERGENT SOLUTION IN ONLY A
C     3  SINGLE PASS STRAIGHT THROUGH THE JOINT FROM END TO END, IN
C     4  SINGLE-PRECISION, WITH ONLY A SMALL LOSS OF ACCURACY IN LATER
C     5  STEPS.
C THE UNDERLYING DIFFICULTY IS ONE OF NUMERICAL ACCURACY LOSS IN THE
C     1  PRESENCE OF EXTREMELY HIGH ADHESIVE SHEAR STRESS GRADIENTS AT
C     2  BOTH ENDS OF EACH OF THE OUTER STEPS.
C NOTE THAT PROGRAM CANNOT HANDLE PROPERLY A JOINT WITH SUCH HIGH
C     1  RESIDUAL THERMAL STRESSES THAT IT BREAKS APART PRIOR TO
C     2  APPLICATION OF MECHANICAL LOADS.  ANSWER FROM ONE END OF JOINT
C     4  WILL BE ZERO, BUT FROM OTHER END WILL BE LARGE AND POSITIVE.
C     5  ACTUALLY, THE LATTER ANSWER IS FOR A LOAD OF REVERSED SIGN, WITH
C     6  THE THERMAL STRESSES HELPING RATHER THAN HINDERING.  SUCH A
C     7  SITUATION CAN BE SPOTTED IF THE SHEAR STRESS IN THE FORMER
C     8  SOLUTION IS NEGATIVE AT THE START OF THE JOINT
      DIMENSION TOUTER(50), TINNER(50), GAMMA(50), TAU(50), DELTAO(50),
     1  DELTAI(50), STEPL(50), THICKO(50), THICKI(50), ETOTR(50),
     2  ETINR(50), STROTR(50), STRINR(50), STEP(50), THCKNO(50),
     3  THCKNI(50), ETOUTR(50), ETINNR(50), STRGTR(50), STRGNR(50)
      DOUBLE PRECISION TOUTER, TINNER, GAMMA, TAU, DELTAO, DELTAI,
     1  TMAX, TMIN, TLOAD, A, B, C, D, E, F, ALAMDA, DELT, DELOLT,
     2  C1, C2, C3, C4, C5, V, STEP, THCKNO, THCKNI, ETOUTR, ETINNR,
     3  STRGTR, STRGNR, TCHECK, STEPL, THICKO, THICKI, ETOTR, ETINR,
     4  STROTR, STRINR, TAUUPR, TAULWR
      READ (5,10) M
   10 FORMAT (I2)
C M .EQ. NUMBER OF JOINT CONFIGURATIONS TO BE SOLVED
C READ IN MATERIAL PROPERTIES
      DO 390 MCOUNT = 1, M
      NRVRS = 0
      JFLAG = 1
C JFLAG IDENTIFIES END OF JOINT FROM WHICH ANALYSIS COMMENCES
      READ (5,20) TAUMAX, G, GAMMAX, GAMMAE, ETA, ALPHAO, ALPHAI,
     1   DELTMP, SGNLD, ANSTEP
   20 FORMAT (8F10.3)
      NSTEPS = ANSTEP
      MSTEPS = NSTEPS + 1
C READ IN JOINT GEOMETRY
      READ (5,30) ((THICKO(N), THICKI(N), STEPL(N), ETOTR(N), ETINR(N),
     1    STROTR(N), STRINR(N)), N = 1, MSTEPS)
   30 FORMAT (7F10.3)
C CHECK ON CONSISTENCY OF ADHESIVE DATA
      VCHECK = G * GAMMAE
      R = TAUMAX / VCHECK
      IF ((1.001 .LT. R) .OR. (0.999 .GT. R)) GO TO 390
      IF (GAMMAX .GE. GAMMAE) GO TO 40
C IF NOT, REDUCE PEAK SHEAR STRESS TO LESS THAN MAXIMUM ELASTIC VALUE
      TAUMAX = G * GAMMAX
C SET UP RECURRING CONSTANTS
   40 C1 = G / ETA
      C2 = 2. * C1
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
   50 C3 = ALPHAO * DELTMP
      C4 = ALPHAI * DELTMP
      C5 = C4 - C3
C
C PRINT OUT INPUT DATA
      WRITE (6,60) ALPHAO, ALPHAI, DELTMP
   60 FORMAT (1H1/, 5(1H0/), 11H INPUT DATA//, 10H ALPHAO = , F10.7,
     1  13H (PER DEG. F), 3X, 9HALPHAI = , F10.7, 13H (PER DEG. F),
     2  3X, 9HDELTMP = , F6.1, 9H (DEG. F)//,
     3  1H , 5X, 1HN, 2X, 5HSTEPL, 1X, 6HTHICKO, 1X, 6HTHICKI, 4X,
     4  6HSTROTR, 5X, 6HSTRINR, 5X, 5HETOTR, 5X, 5HETINR//)
      DO 80 N = 1, MSTEPS
      WRITE (6,70) N, STEPL(N), THICKO(N), THICKI(N), STROTR(N),
     1 STRINR(N), ETOTR(N), ETINR(N)
   70 FORMAT (1H , 4X, I2, 3(1X, F6.4), 2(1X, F10.1), 2(1X, F10.1))
   80 CONTINUE
C
C ESTIMATE MAXIMUM POSSIBLE BOND CAPACITY FOR FULLY-PLASTIC ADHESIVE
      PBOND = TAUMAX * OLAP * 2.
C NOTE FACTOR 2. INCLUDED FOR DOUBLE-SIDED JOINT
C REDUCE TO 1. IF JOINT HAS ONLY ONE SIDE BONDED
```

```
C PERFECTLY-ELASTIC BOND CAPACITY WILL BE CLOSER TO ASYMPTOTE OF SCARF
C     1  JOINT SOLUTION AND IS SIGNIFICANTLY LOWER THAN PLASTIC ESTIMATE
C ACTUAL LOAD CAPACITY MAY BE SIGNIFICANTLY LESS IF THERMAL MISMATCH
C     1  BETWEEN ADHERENDS IS SEVERE
C REDUCTION IN LOAD TO ACCOUNT FOR LIMITED ADHEREND STRENGTH IS
C     1  ACCOMPLISHED LATER IN PROGRAM
C PROVIDE OUTER LOOP TO ADJUST ADHESIVE PEAK SHEAR STRESS AT START OF
C     1  JOINT FOR CASES IN WHICH EITHER ADHESIVE IS MORE CRITICAL AT
C     2  OTHER END OF JOINT OR ADHERENDS ARE MORE CRITICAL THAN ADHESIVE.
      TAUUPP = 2. * TAUMAX
      TAULWR = 0.
C NOTE THAT PROGRAM IS PREVENTED FROM HANDLING PROBLEM IN WHICH SHEAR
C     1  STRESS IN ADHESIVE REVERSES SIGN, WHEN COMPUTATIONS START FROM
C     2  THE LESS CRITICAL END.  SOLUTION IS OBTAINABLE FROM OTHER END.
C NOTE ALSO THAT, IF THE MAXIMUM SHEAR STRESS AND APPLIED LOADS HAVE
C     1  OPPOSITE SIGNS, JOINT MUST BREAK APART UNDER RESIDUAL THERMAL
C     2  STRESS ALONE WITHOUT ANY EXTERNALLY APPLIED LOAD, SO NO CASES OF
C     3  REAL CONCERN ARE EXCLUDED BY THE RESTRICTION ABOVE
      DO 290 I = 1, 50
      TAU(1) = (TAUUPP + TAULWR) / 2.
      IF (TAU(1) .GT. TAUMAX) TAU(1) = TAUMAX
      IF (I .EQ. 1) TAU(1) = TAUMAX
C SET INITIAL CONDITIONS
      GAMMA(1) = TAU(1) / G
      TOUTER(1) = 5. * PBOND
      TINNER(1) = 0.
      DELTAO(1) = 0.
      DELTAI(1) = SGNLD * GAMMA(1) * ETA
      TMAX = 10. * PBOND
      TMIN = 0.
C     TLOAD = 5. * PBOND
C OPERATE ON THE LOAD LEVEL IN INTERMEDIATE LOOP
C LEAVE ADJUSTMENT OF TAUMAX FOR OUTER LOOP
      TCHECK = 0.
      DO 190 IFLAG = 1, NSTEPS
      SCHECK = 1000000000000000.
      DO 150 NCOUNT = 1, 100
C CONVERGENCE NEARLY ALWAYS OCCURRED BETWEEN 20 AND 30 CYCLES IN TEST
C     1  CASES, BUT THERE WERE SOME EXCEPTIONS
C INTERMEDIATE LOOP ADJUSTS LOAD LEVEL
C     TLOAD = TOUTER(IFLAG)
C CHECK ON CONVERGENCE OF TOUTER(IFLAG)
      P = TOUTER(IFLAG) / SCHECK
      IF ( (1.000000001 .GT. P) .AND. (0.999999999 .LT. P) ) GO TO 160
      DO 100 N = IFLAG, NSTEPS
C INNER LOOP COMPUTES ELASTIC JOINT STRENGTH
      A = TAU(N)
      ALAMDA = DSQRT(C2 * (1. / ETINR(N) + 1. / ETOTR(N)))
      B = (TINNER(N) / ETINR(N) - TOUTER(N) / ETOTR(N) + C5 * SGNLD)
     1  * C1 / ALAMDA
C NOTE THAT SGNLD SIGNIFIES WHETHER SHEAR LOAD IS TENSILE OR COMPRESSIVE
      C = STEPL(N)
      D = ALAMDA * C
      E = DSINH(D)
      F = DCOSH(D)
      TAU(N+1) = A * F + B * E
      DELT = (2. / ALAMDA) * (A * E + B * (F - 1.))
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      TOUTER(N+1) = TOUTER(N) - DELT
      TINNER(N+1) = TINNER(N) + DELT
      IF (N .EQ. NSTEPS) GO TO 90
      IF (TINNER(N+1) .LT. (-1. * TOUTER(1))) GO TO 130
      IF (TOUTER(N+1) .LT. (-1. * TOUTER(1))) GO TO 140
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
C IF THE FACTOR -1 IS EITHER TOO LARGE OR TOO SMALL, CONVERGENCE FAILS
   90 DELDLT = (2. / (ALAMDA**2)) * (A * (F - 1.) + B * (E - D))
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      DELTAO(N+1) = DELTAO(N) + C3 * C + SGNLD * (TOUTER(N) * C -
     1  DELDLT) / ETOTR(N)
      DELTAI(N+1) = DELTAI(N) + C4 * C + SGNLD * (TINNER(N) * C +
     1  DELDLT) / ETINR(N)
      GAMMA(N+1) = TAU(N+1) / G
  100 CONTINUE
C CHECK WHETHER OR NOT PRECISELY 100 PERCENT OF LOAD HAS TRANSFERRED
      R1 = TOUTER(1) / TINNER(MSTEPS)
C CHECK ALSO WHETHER OR NOT CONVERGENCE HAS BEEN OBTAINED
      R2 = TCHECK / TINNER(MSTEPS)
      IF ( (1.000001 .GT. R1) .AND. (0.999999 .LT. R1) .AND.
     1  (1.000001 .GT. R2) .AND. (0.999999 .LT. R2) ) GO TO 200
      IF (TOUTER(1) .LT. TINNER(MSTEPS)) GO TO 110
C IF SO, LOAD ESTIMATE IS TOO LOW
C IF NOT, LOAD ESTIMATE IS TOO HIGH
      GO TO 120
C R1 IS UNSUITABLE FOR A CONVERGENCE CHECK BECAUSE NEGATIVE VALUES OF R1
C     1 REPRESENT TOO HIGH A LOAD ESTIMATE, JUST LIKE THOSE VALUES IN
C     2  EXCESS OF UNITY
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```
  110 TMIN = TOUTER(IFLAG)
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
      TCHECK = TINNER(MSTEPS)
      GO TO 150
  120 TMAX = TOUTER(IFLAG)
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
      TCHECK = TINNER(MSTEPS)
      GO TO 150
C NOTE THAT LABELS 26 AND 7 GOVERN FINE ADJUSTMENTS TO THE JOINT LOADS,
C    1  WHILE LABELS 27 AND 28 REPRESENT COARSE ADJUSTMENTS
  130 TMAX = TOUTER(IFLAG)
      SCHECK = TOUTER(IFLAG)
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
      GO TO 150
  140 TMIN = TOUTER(IFLAG)
      SCHECK = TOUTER(IFLAG)
C IF ADHEREND, RATHER THAN ADHESIVE, LIMITS JOINT STRENGTH, NEED TO
C    1  BOOST PLOAD IN PROPORTION TO TAUMAX, EVEN IF IT MEANS EXCEEDING
C    2  ADHEREND STRENGTHS IN INTERMEDIATE COMPUTATIONS.  CORRECTIONS
C    3  ARE APPLIED LATER
      IF (TMIN .GE. TMAX) TMAX = 5. * TMAX
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
  150 CONTINUE
      IF (N .EQ. NSTEPS) GO TO 190
C CONVERGENCE WILL NOT PROCEED TO FAR END OF JOINT IN SINGLE PASS
C    1  BECAUSE OF NUMERICAL ACCURACY PROBLEMS.  REMEDY IS TO FREEZE
C    2  EARLIER VALUES, WHICH HAVE CONVERGED AND SLIGHTLY PERTURB
C    3  INTERMEDIATE VALUES, AND TO CHECK FOR CONVERGENCE AT THE FAR END
      TMAX = TOUTER(1)
      TMIN = -1. * TMAX
      GO TO 190
  160 ICOUNT = IFLAG + 1
      IF (TOUTER(ICOUNT) .GT. 0.) GO TO 170
      IF (TOUTER(ICOUNT) .LT. 0.) GO TO 180
      TMAX = TOUTER(1) / 10.
      TMIN = -1. * TMAX
      GO TO 190
  170 TMAX = 1.1 * TOUTER(ICOUNT)
      TMIN = 0.9 * TOUTER(ICOUNT)
      GO TO 190
  180 TMIN = 0.9 * TOUTER(ICOUNT)
      TMIN = 1.1 * TOUTER(ICOUNT)
C THE LIMITS ABOVE ARE CRITICAL IN ENSURING CONVERGENCE
C    1  THEY MUST BE NEITHER TOO LARGE NOR TOO SMALL
  190 CONTINUE
      NRVPS = 1
C
  200 IF ( (C5 .GT. 0.000001) .OR. (C5 .LT. -0.000001) ) GO TO 240
C IF NOT, FIRST SOLUTION MAY BE SCALED IN THE ABSENCE OF ANY THERMAL
C    1  MISMATCH BETWEEN ADHERENDS
C IF SO, SOLUTION MUST BE REFINED BY ITERATION, SINCE THERMAL STRESS
C    1  TERMS DO NOT SCALE LINEARLY, EVEN FOR ELASTIC ADHESIVE AND
C    2  ADHERENDS
C
C APPLY SCALE FACTOR TO SOLUTION FOR ONLY ADHEREND STIFFNESS IMBALANCE
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR
C    1  WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE
C PROGRAM ASSUMES ADHEREND ALLOWABLES HAVE SAME MAGNITUDE IN TENSION AS
C    1  IN COMPRESSION.  DISTINCTION IS USUALLY UNIMPORTANT SINCE, IN
C    2  PRACTICAL JOINTS, RESIDUAL THERMAL STRESSES ARE UNLIKLELY TO
C    3  BREAK ADHEREND(S) RATHER THAN ADHESIVE
      RSCALE = TOUTER(1) / STROTR(1)
      IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE
      RTAUMX = TAU(1) / TAUMAX
      IF (RTAUMX .LT. 0.) RTAUMX = -1. * RTAUMX
      DO 220 N = 2, MSTEPS
      RINR = TINNER(N) / STRINR(N)
      IF (RINR .LT. 0.) RINR = -1. * RINR
      IF (RINR .GT. RSCALE) RSCALE = RINR
      IF (N .EQ. MSTEPS) GO TO 210
      ROTR = TOUTER(N) / STROTR(N)
      IF (ROTR .LT. 0.) ROTR = -1. * ROTR
      IF (ROTR .GT. RSCALE) RSCALE = ROTR
  210 RTAU = TAU(N) / TAUMAX
      IF (RTAU .LT. 0.) RTAU = -1. * RTAU
      IF (RTAU .GT. RTAUMX) RTAUMX = RTAU
  220 CONTINUE
      RFCTR = RSCALE
C RFCTR IS PROPORTIONALITY CONSTANT GOVERNING ELASTIC SOLUTION
C IF RTAUMAX .GT. RSCALE, ADHESIVE PLASTICITY CAN INCREASE STRENGTH
C USUALLY ADHESIVE IS CRITICAL AT ONE END OF JOINT OR OTHER, SO RTAUMX
C    1  .GT. 1. MAY WELL JUST SIGNIFY THAT FAR END OF JOINT IS CRITICAL
C NOTE THAT PROGRAM ASSUMES THAT ANY INTERNAL ADHEREND STRESSES OF
C    1  REVERSED SIGN WITH RESPECT TO STRESS OUTSIDE THE JOINT ARE NOT
C    2  CRITICAL.  IF THEY ARE, IT MEANS THAT THE JOINT WILL FAIL DUE
C    3  TO RESIDUAL THERMAL STRESSES ALONE WITHOUT ANY MECHANICAL LOADS
      IF (RSCALE .LT. RTAUMX) RFCTR = RTAUMX
      DO 230 N = 1, MSTEPS
```

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      TOUTER(N) = TOUTER(N) / RFCTR
      TINNER(N) = TINNER(N) / RFCTR
      TAU(N) = TAU(N) / RFCTR
      GAMMA(N) = GAMMA(N) / RFCTR
      DELTAO(N) = DELTAO(N) / RFCTR
  230 DELTAI(N) = DELTAI(N) / RFCTR
      GO TO 310
C
C USE ITERATIVE SOLUTION WHEN ADHEREND THERMAL MISMATCH IS PRESENT
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR
C     1  WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE.
  240 RSCALE = TOUTER(1) / STROTR(1)
      IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE
      RTAUMX = TAU(1) / TAUMAX
      IF (RTAUMX .LT. 0.) RTAUMX = -1. * RTAUMX
      DO 260 N = 2, MSTEPS
      RINR = TINNER(N) / STRINR(N-1)
C NEED TO COMPARE LOAD WITH STRENGTH ON THIN SIDE OF STEP.  HENCE (N-1)
      IF (RINR .LT. 0.) RINR = -1. * RINR
      IF (RINR .GT. RSCALE) RSCALE = RINR
      IF (N .EQ. MSTEPS) GO TO 250
      ROTR = TOUTER(N) / STROTR(N)
      IF (ROTR .LT. 0.) ROTR = -1. * ROTR
      IF (ROTR .GT. RSCALE) RSCALE = ROTR
  250 RTAU = TAU(N) / TAUMAX
      IF (RTAU .LT. 0.) RTAU = -1. * RTAU
      IF (RTAU .GT. RTAUMX) RTAUMX = RTAU
  260 CONTINUE
C CHECK ON CONVERGENCE
      R = RTAUMX
      IF (RTAUMX .LT. RSCALE) R = RSCALE
      IF ( (1.00001 .GT. R) .AND. (0.99999 .LT. R) ) GO TO 310
      V = 2. * TAUMAX
      R = (V + TAUUPR) / (V + TAULWR)
      IF ( (1.00001 .GT. R) .AND. (0.99999 .LT. R) ) GO TO 310
C IF EITHER RTAUMX OR RSCALE .GT. UNITY, TAU(1) MUST BE DECREASED
      IF ( (RTAUMX .GT. 1.00001) .OR. (RSCALE .GT. 1.00001) ) GO TO 270
C IF BOTH RTAUMX AND RSCALE ARE .LT. UNITY, TAU(1) MUST BE INCREASED
      IF ( (RTAUMX .LT. 0.99999) .AND. (RSCALE .LT. 0.99999) ) GO TO 280
C IF NONE OF THE THREE CHECKS ABOVE IS MET, SOLUTION HAS FAILED TO
C     1 CONVERGE WITHIN SPECIFIED NUMBER OF ITERATIONS.  PRINT OUT ANSWER
C     GO TO 14
  270 IF (TAU(1) .GT. 0.) TAUUPR = TAU(1)
      IF (TAU(1) .LT. 0.) TAULWR = TAU(1)
      GO TO 290
  280 IF (TAU(1) .GT. 0.) TAULWR = TAU(1)
      IF (TAU(1) .LT. 0.) TAUUPR = TAU(1)
  290 CONTINUE
C IF PROGRAM GOES BEYOND PRECEEDING CONTINUE STATEMENT, SOLUTION HAS NOT
C     1  CONVERGED
      WRITE (6,300)
  300 FORMAT (1H1, 18HDIVERGENT SOLUTION)
C
C PRINT OUT RESULTS OF ELASTIC COMPUTATIONS
  310 WRITE (6,320) TOUTER(1), TAUMAX, SGNLD, DELTMP
  320 FORMAT (1H1/, 5(1H0/),
     1  39H ELASTIC JOINT STRENGTH, PLOAD (LBS) = , F10.1/,
     2  49H ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = , F8.1/,
     3  1H , 8HSGNLD = , F4.1, 54H   SGNLD = +1 FOR TENSILE SHEAR AND -1
     4 FOR COMPRESSIVE/, 36H TEMPERATURE DIFFERENTIAL (DEG F) = , F6.1//
     5,  1H , 5X, 1HN, 2X, 5HSTEPL, 1X, 6HTHICKO, 1X, 6HTHICKI, 3X,
     6  3HTAU, 4X, 5HGAMMA, 1X, 6HDELTAO, 1X, 6HDELTAI, 5X, 6HTOUTER,
     7  5X, 6HSTROTR, 5X, 6HTINNER, 5X, 6HSTRINR//)
      DO 340 N = 1, MSTEPS
      WRITE (6,330) N, STEPL(N), THICKO(N), THICKI(N), TAU(N), GAMMA(N),
     1    DELTAO(N), DELTAI(N), TOUTER(N), STROTR(N), TINNER(N), STRINR(N
     2)
  330 FORMAT (1H , 4X, I2, 1X, F6.4, 1X, F6.4, 1X, F6.4, 1X, F7.1, 1X,
     1  F6.3, 1X, F6.4, 1X, F6.4, 1X, F10.1, 1X, F10.1, 1X, F10.1, 1X,
     2  F10.1)
  340 CONTINUE
  350 CONTINUE
C
C RECOMPUTE SOLUTION FROM OTHER END OF JOINT, IF APPPROPRIATE
C NOTE THAT, IF COMPUTER PRINTS OUT TWO SOLUTIONS TO A GIVEN PROBLEM BY
C     1  REVERSING ENDS AND RE-ANALYZING, IT IS BECAUSE THE FIRST FAILED
C     2  TO CONVERGE, EVEN IF THE ANSWERS PRINTED SEEM TO SUGGEST
C     3  OTHERWISE.  THE SECOND SOLUTION IS TO BE PREFERRED, PARTICULARLY
C     4  IF IT STARTS AT THAT END OF THE JOINT AT WHICH THE ADHESIVE
C     5  SHEAR STRESS IS AT ITS HIGHEST.
C IDENTIFY CRITICAL END OF JOINT
C AVOID REVERSING ENDS BACK AGAIN
      IF (JFLAG .EQ. 2) GO TO 390
      IF (NRVRS .EQ. 1) GO TO 360
C IF SO, SOLUTION HAS FAILED TO CONVERGE, SO TRY AGAIN FROM OTHER END
C ACCURACY AT FAR END OF JOINT MAY BE POOR IF FAR END IS CRITICAL
      IF ((TAU(MSTEPS) .LE. TAU(1)) .AND. (TAU(MSTEPS) .GE.
     1  (-1. * TAU(1)))) GO TO 390
```

```
C  IF, AT FAR END OF JOINT, TAU(MSTEPS) .GT. TAU(1) AT NEAR END,
C     1  FAILURE TO CONVERGE MAY BE SIMPLY THE RESULT OF THE FAR END
C     2  OF THE JOINT BEING MORE CRITICAL THAN THE STARTING (NEAR) END
C   REVERSE DATA AND REANALYZE
  360 DO 370 N = 1, MSTEPS
      STEP(N) = STEPL(N)
      THCKNO(N) = THICKO(N)
      THCKNI(N) = THICKI(N)
      ETOUTR(N) = ETOTR(N)
      ETINNR(N) = ETINR(N)
      STRGTR(N) = STROTR(N)
  370 STRGNR(N) = STRINR(N)
      DO 380 N = 1, NSTEPS
      STEPL(N) = STEP(MSTEPS - N)
      THICKO(N) = THCKNI(MSTEPS - N)
      THICKI(N) = THCKNO(MSTEPS - N)
      ETOTR(N) = ETINNR(MSTEPS - N)
      ETINR(N) = ETOUTR(MSTEPS - N)
      STROTR(N) = STRGNR(MSTEPS - N)
  380 STRINR(N) = STRGTR(MSTEPS - N)
      STEPL(MSTEPS) = STEP(MSTEPS)
      THICKO(MSTEPS) = 0.
      THICKI(MSTEPS) = THCKNO(1)
      ETOTR(MSTEPS) = 0.
      ETINR(MSTEPS) = ETOUTR(1)
      STROTR(MSTEPS) = 0.
      STRINR(MSTEPS) = STRGTR(1)
      V = ALPHAO
      ALPHAO = ALPHAI
      ALPHAI = V
      JFLAG = 2
      NRVRS = 0
      GO TO 50
  390 CONTINUE
      STOP
      END
```

INPUT DATA

ALPHAO = 0.0 (PER DEG. F) ALPHAI = 0.0000050 (PER DEG. F) DELTMP = 0.0 (DEG. F)

N	STEPL	THICKO	THICKI	STROTR	STRINR	ETOTR	ETINR
1	0.0750	0.2640	0.0300	18216.0	3900.0	2112000.0	480000.0
2	0.0750	0.2640	0.0300	18216.0	3900.0	2112000.0	480000.0
3	0.0750	0.2640	0.0300	18216.0	3900.0	2112000.0	480000.0
4	0.0750	0.2640	0.0300	18216.0	3900.0	2112000.0	480000.0
5	0.0750	0.2640	0.0300	18216.0	3900.0	2112000.0	480000.0
6	0.0750	0.2200	0.0740	15180.0	9620.0	1760000.0	1184000.0
7	0.0750	0.2200	0.0740	15180.0	9620.0	1760000.0	1184000.0
8	0.0750	0.2200	0.0740	15180.0	9620.0	1760000.0	1184000.0
9	0.0750	0.2200	0.0740	15180.0	9620.0	1760000.0	1184000.0
10	0.0750	0.2200	0.0740	15180.0	9620.0	1760000.0	1184000.0
11	0.1250	0.1760	0.1180	12144.0	15340.0	1408000.0	1888000.0
12	0.1250	0.1760	0.1180	12144.0	15340.0	1408000.0	1888000.0
13	0.1250	0.1760	0.1180	12144.0	15340.0	1408000.0	1888000.0
14	0.1250	0.1760	0.1180	12144.0	15340.0	1408000.0	1888000.0
15	0.1250	0.1760	0.1180	12144.0	15340.0	1408000.0	1888000.0
16	0.1250	0.1320	0.1620	9108.0	21060.0	1056000.0	2592000.0
17	0.1250	0.1320	0.1620	9108.0	21060.0	1056000.0	2592000.0
18	0.1250	0.1320	0.1620	9108.0	21060.0	1056000.0	2592000.0
19	0.1250	0.1320	0.1620	9108.0	21060.0	1055000.0	2592000.0
20	0.1250	0.1320	0.1620	9108.0	21060.0	1056000.0	2592000.0
21	0.1250	0.0880	0.2060	6072.0	26780.0	704000.0	3296000.0
22	0.1250	0.0880	0.2060	6072.0	26780.0	704000.0	3296000.0
23	0.1250	0.0880	0.2060	6072.0	26780.0	704000.0	3296000.0
24	0.1250	0.0880	0.2060	6072.0	26780.0	704000.0	3296000.0
25	0.1250	0.0880	0.2060	6072.0	26780.0	704000.0	3296000.0
26	0.1250	0.0440	0.2500	3036.0	32500.0	352000.0	4000000.0
27	0.1250	0.0440	0.2500	3036.0	32500.0	352000.0	4000000.0
28	0.1250	0.0440	0.2500	3036.0	32500.0	352000.0	4000000.0
29	0.1250	0.0440	0.2500	3036.0	32500.0	352000.0	4000000.0
30	0.1250	0.0440	0.2500	3036.0	32500.0	352000.0	4000000.0
31	******	0.0	0.2500	0.0	32500.0	0.0	4000000.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 7828.0
ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 6000.0
N = 1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE
TEMPERATURE DIFFERENTIAL (DEG F) = 0.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.0750	0.2640	0.0300	6000.0	0.100	0.0	0.0005	7828.0	18216.0	0.0	3900.0
2	0.0750	0.2640	0.0300	3534.6	0.059	0.0003	0.0006	7132.9	18216.0	695.2	3900.0
3	0.0750	0.2640	0.0300	2324.7	0.039	0.0005	0.0007	6705.6	18216.0	1122.4	3900.0
4	0.0750	0.2640	0.0300	1940.7	0.032	0.0007	0.0009	6394.6	18216.0	1433.5	3900.0
5	0.0750	0.2640	0.0300	2245.9	0.037	0.0010	0.0012	6089.3	18216.0	1738.7	3900.0
6	0.0750	0.2200	0.0740	3349.0	0.056	0.0012	0.0015	5681.3	15180.0	2146.7	9620.0
7	0.0750	0.2200	0.0740	2359.1	0.039	0.0014	0.0016	5259.9	15180.0	2568.1	9620.0
8	0.0750	0.2200	0.0740	1826.4	0.030	0.0016	0.0018	4950.9	15180.0	2877.1	9620.0
9	0.0750	0.2200	0.0740	1647.6	0.027	0.0019	0.0020	4694.4	15180.0	3133.6	9620.0
10	0.0750	0.2200	0.0740	1798.0	0.030	0.0020	0.0022	4440.8	15190.0	3387.2	9620.0
11	0.1250	0.1760	0.1180	2274.9	0.038	0.0022	0.0024	4140.8	12144.0	3687.2	15340.0
12	0.1250	0.1760	0.1180	1225.1	0.020	0.0026	0.0027	3719.5	12144.0	4108.5	15340.0
13	0.1250	0.1760	0.1180	767.4	0.013	0.0029	0.0029	3479.6	12144.0	4348.4	15340.0
14	0.1250	0.1760	0.1180	680.6	0.011	0.0032	0.0032	3305.3	12144.0	4522.7	15340.0
15	0.1250	0.1760	0.1190	922.6	0.015	0.0035	0.0035	3112.4	12144.0	4715.6	15340.0
16	0.1250	0.1320	0.1620	1610.6	0.027	0.0037	0.0039	2807.4	9108.0	5020.6	21060.0
17	0.1250	0.1320	0.1620	855.2	0.014	0.0040	0.0041	2511.4	9108.0	5316.6	21060.0
18	0.1250	0.1320	0.1620	545.4	0.009	0.0043	0.0044	2343.3	9108.0	5484.7	21060.0
19	0.1250	0.1320	0.1620	519.8	0.009	0.0046	0.0046	2215.4	9108.0	5612.6	21060.0
20	0.1250	0.1320	0.1620	765.0	0.013	0.0049	0.0049	2061.2	9108.0	5766.8	21060.0
21	0.1250	0.0880	0.2060	1408.6	0.023	0.0051	0.0052	1800.3	6072.0	6027.7	26780.0
22	0.1250	0.0880	0.2060	674.8	0.011	0.0054	0.0054	1553.0	6072.0	6275.0	26780.0
23	0.1250	0.0880	0.2060	401.3	0.007	0.0056	0.0057	1425.3	6072.0	6402.7	26780.0
24	0.1250	0.0880	0.2060	401.5	0.007	0.0059	0.0059	1330.0	6072.0	6498.0	26790.0
25	0.1250	0.0880	0.2060	675.4	0.011	0.0061	0.0062	1202.2	6072.0	6625.8	26780.0
26	0.1250	0.0440	0.2500	1410.0	0.024	0.0063	0.0064	954.7	3036.0	6873.3	32500.0
27	0.1250	0.0440	0.2500	513.0	0.009	0.0065	0.0066	735.2	3036.0	7092.8	32500.0
28	0.1250	0.0440	0.2500	270.4	0.005	0.0068	0.0069	645.7	3036.0	7182.3	32500.0
29	0.1250	0.0440	0.2500	372.6	0.006	0.0071	0.0071	572.3	3036.0	7255.7	32500.0
30	0.1250	0.0440	0.2500	350.1	0.016	0.0072	0.0073	421.3	3036.0	7406.7	32500.0
31	******	0.0	0.2500	2739.5	0.046	0.0073	0.0075	0.0	0.0	7828.0	32500.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 6764.2
ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 9000.0
N = 1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE
TEMPERATURE DIFFERENTIAL (DEG F) = 0.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.0750	0.2640	0.0300	9000.0	0.045	0.0	0.0002	6764.2	18216.0	0.0	3900.0
2	0.0750	0.2640	0.0300	3133.7	0.016	0.0002	0.0003	5932.4	18216.0	831.8	3900.0
3	0.0750	0.2640	0.0300	1232.3	0.006	0.0004	0.0005	5633.1	18216.0	1131.1	3900.0
4	0.0750	0.2640	0.0300	889.9	0.004	0.0006	0.0006	5487.7	18216.0	1276.5	3900.0
5	0.0750	0.2640	0.0300	1673.5	0.008	0.0008	0.0009	5311.9	18216.0	1452.3	3900.0
6	0.0750	0.2200	0.0740	4574.4	0.023	0.0010	0.0011	4883.6	15180.0	1880.6	9620.0
7	0.0750	0.2200	0.0740	2158.4	0.011	0.0012	0.0012	4403.8	15180.0	2360.4	9620.0
8	0.0750	0.2200	0.0740	1199.9	0.006	0.0014	0.0014	4165.3	15180.0	2598.9	9620.0
9	0.0750	0.2200	0.0740	1016.1	0.005	0.0016	0.0016	4008.2	15180.0	2756.0	9620.0
10	0.0750	0.2200	0.0740	1524.2	0.008	0.0017	0.0018	3927.1	15180.0	2937.1	9620.0
11	0.1250	0.1760	0.1180	3053.7	0.015	0.0019	0.0020	3500.9	12144.0	3263.3	15340.0
12	0.1250	0.1760	0.1180	894.2	0.004	0.0022	0.0022	3062.6	12144.0	3701.6	15340.0
13	0.1250	0.1760	0.1180	309.3	0.002	0.0024	0.0024	2929.0	12144.0	3835.2	15340.0
14	0.1250	0.1760	0.1180	268.9	0.001	0.0027	0.0027	2864.8	12144.0	3899.4	15340.0
15	0.1250	0.1760	0.1180	701.9	0.004	0.0029	0.0030	2757.0	12144.0	4007.2	15340.0
16	0.1250	0.1320	0.1620	2370.6	0.012	0.0032	0.0032	2415.9	9108.0	4348.3	21060.0
17	0.1250	0.1320	0.1620	663.3	0.003	0.0034	0.0034	2081.8	9108.0	4682.4	21060.0
18	0.1250	0.1320	0.1620	223.0	0.001	0.0037	0.0037	1984.2	9108.0	4780.0	21060.0
19	0.1250	0.1320	0.1620	208.9	0.001	0.0039	0.0039	1936.7	9108.0	4827.5	21060.0
20	0.1250	0.1320	0.1620	594.0	0.003	0.0041	0.0041	1848.3	9108.0	4916.0	21060.0
21	0.1250	0.0880	0.2060	2114.0	0.011	0.0043	0.0044	1550.0	6072.0	5214.2	26780.0
22	0.1250	0.0880	0.2060	492.8	0.002	0.0046	0.0046	1272.4	6072.0	5491.8	26780.0
23	0.1250	0.0880	0.2060	138.6	0.001	0.0048	0.0048	1205.1	6072.0	5559.1	26780.0
24	0.1250	0.0880	0.2060	140.7	0.001	0.0050	0.0050	1175.3	6072.0	5589.9	26780.0
25	0.1250	0.0880	0.2060	504.3	0.003	0.0052	0.0052	1106.6	6072.0	5657.6	26780.0
26	0.1250	0.0440	0.2500	2164.5	0.011	0.0054	0.0054	822.3	3036.0	5941.9	32500.0
27	0.1250	0.0440	0.2500	304.8	0.002	0.0056	0.0056	585.5	3036.0	6178.8	32500.0
28	0.1250	0.0440	0.2500	54.3	0.000	0.0058	0.0058	551.0	3036.0	6213.2	32500.0
29	0.1250	0.0440	0.2500	90.3	0.000	0.0060	0.0060	537.1	3036.0	6227.1	32500.0
30	0.1250	0.0440	0.2500	603.4	0.003	0.0062	0.0062	470.6	3036.0	6293.6	32500.0
31	******	0.0	0.2500	4301.8	0.022	0.0063	0.0064	-0.0	0.0	6764.2	32500.0

A.5 Computer Program A4EG For Elastic-Plastic Strength of Stepped-Lap Bonded Joints

The elastic-plastic strength of stepped-lap joints is covered by the analysis in Section 6. The digital computer program A4EG has been prepared as a design tool for the analysis of such joints. By printing out detailed internal stresses, the program can serve to aid in design improvement by changing the joint proportions in such a manner as to reduce the load transfer in the more critical regions and to increase it in those less severely loaded areas.

In addition to those features of the elastic solution A4EF, this elastic-plastic program A4EG seeks the existence and extent of any plastic adhesive zones within any step or step portion. The convergence of the nested iterative do loops is complicated by the addition of an extra loop accounting for the maximum adhesive shear strain. This is only rarely a known quantity for ductile adhesives because the end step of the stiffer adherend is usually the most critical detail.

A complete listing of the program A4EG follows. Precisely the same input data is used as for program A4EF and the output format is the same except inasmuch as A4EG prints out separate elastic and elastic-plastic solutions.

```
CDECK A4EG
C STEPPED-LAP ADHESIVE-BONDED-JOINTS
C ELASTIC-PLASTIC SOLUTIONS
C JOINT ANALYSIS PROGRAM
C PROGRAM CAN BE USED TO OPTIMIZE JOINT DESIGN PROPORTIONS
C SOLUTION EXAMINES ADHESIVE SHEAR STRESS AND ADHEREND NORMAL (AXIAL)
C     1  STRESS BUT OMITS CONSIDERATION OF ADHESIVE PEEL STRESS ON THE
C     2  GROUND THAT OUTER END STEP IS USUALLY SUFFICIENTLY THIN FOR
C     3  PEEL STRESS PROBLEMS NOT TO ARISE
C NOTE THAT CONVERGENCE PROBLEM IS ACUTE FOR STEPPED-LAP JOINTS, EVEN
C     1  WITH DOUBLE-PRECISION.  STEPS TAKEN HERE TO CONSTRAIN TENDENCY
C     2  TO DIVERGE (BY FREEZING SOLUTION ONE STEP AT A TIME) HAVE BEEN
C     3  ADOPTED AFTER TRYING BOTH MORE AND LESS STRINGENT TECHNIQUES
C NOTE ALSO THAT CONVERGENCE DIFFICULTIES ARE PROBLEM DEPENDENT, BEING
C     1  MORE SEVERE FOR BRITTLE (HIGH MODULUS) ADHESIVES.  LOW MODULUS
C     2  ADHESIVES PROVED AMENABLE TO A CONVERGENT SOLUTION IN ONLY A
C     3  SINGLE PASS STRAIGHT THROUGH THE JOINT FROM END TO END, IN
C     4  SINGLE-PRECISION, WITH ONLY A SMALL LOSS OF ACCURACY IN LATER
C     5  STEPS.
C THE UNDERLYING DIFFICULTY IS ONE OF NUMERICAL ACCURACY LOSS IN THE
C     1  PRESENCE OF EXTREMELY HIGH ADHESIVE SHEAR STRESS GRADIENTS AT
C     2  BOTH ENDS OF EACH OF THE OUTER STEPS.
C PROGRAM HAS BEEN ADAPTED TO RUN ON CDC COMPUTERS IN SINGLE PRECISION
C     1 BUT ONLY WORKS INTERMITTENTLY IN SINGLE PRECISION ON IBM MACHINES
      DIMENSION TOUTER(150), TINNER(150), GAMMA(150), TAU(150),
     1  DELTAO(150), DELTAI(150), STEPL(50), THICKO(50), THICKI(50),
     2  ETOTR(50), ETINR(50), STROTR(50), STRINR(50), STEP(150),
     3  THCKNO(150), THCKNI(150), ETOUTR(150), ETINNR(150), STRGTR(150),
     4  STRGNR(150)
      DOUBLE PRECISION TOUTER, TINNER, GAMMA, TAU, DELTAO, DELTAI,
     1  TMAX, TMIN, A, B, C, D, E, F, ALAMDA, DELT, DELOLT, C1, C2, C3,
     2  C4, C5, C7, C10, V, V4, V5, V6, V7, V8, V9, V10, STEP, TCHECK,
     3  THCKNO, THCKNI, ETOUTR, ETINNR, STRGTR, STRGNR, STEPL, THICKO,
     4  THICKI, ETOTR, ETINR, STROTR, STRINR, TAUUPR, TAULWR, GAMUPR,
     5  GAMLWR, ELSTR, XP, EL, ELMAX, ELMIN
      READ (5,10) M
   10 FORMAT (I2)
C M .EQ. NUMBER OF JOINT CONFIGURATIONS TO BE SOLVED
C READ IN MATERIAL PROPERTIES
      DO 820 MCOUNT = 1, M
      NRVRS = 0
      JFLAG = 1
C JFLAG IDENTIFIES END OF JOINT FROM WHICH ANALYSIS COMMENCES
      READ (5,20) TAUMAX, G, GAMMAX, GAMMAE, ETA, ALPHAO, ALPHAI,
     1   DELTMP, SGNLD, ANSTEP
   20 FORMAT (8F10.3)
      NSTEPS = ANSTEP
      MSTEPS = NSTEPS + 1
      LMAX = 3 * NSTEPS
      MCHECK = LMAX + 1
C READ IN JOINT GEOMETRY
      READ (5,30) ((THICKO(N), THICKI(N), STEPL(N), ETOTR(N), ETINR(N),
     1   STROTR(N), STRINR(N)), N = 1, MSTEPS)
   30 FORMAT (7F10.3)
C CHECK ON CONSISTENCY OF ADHESIVE DATA
      VCHECK = G * GAMMAE
      R = TAUMAX / VCHECK
      IF ((1.001 .LT. R) .OR. (0.999 .GT. R)) GO TO 820
      IF (GAMMAX .GE. GAMMAE) GO TO 40
C IF NOT, REDUCE PEAK SHEAR STRESS TO LESS THAN MAXIMUM ELASTIC VALUE
      TAUMAX = G * GAMMAX
      GAMMAE = GAMMAX
C SUM LAP LENGTHS
   40 OLAP = STEPL(1)
      DO 50 N = 2, NSTEPS
   50 OLAP = OLAP + STEPL(N)
C A CHECK ON THE CONSTANCY OF THE TOTAL THICKNESS OF THE STEPPED-LAP JOI
C     1 NT ADHERENDS IS NOT PROVIDED BECAUSE STRONGER JOINTS ARE OBTAINED
C     2 BY MATCHING THE ADHEREND EXTENSIONAL STIFFNESSES AT THE ENDS OF
C     1 THE JOINT AND MAINTAINING THIS TOTAL APPROXIMATELY CONSTANT THROU
C     1 GHOUT THE LENGTH OF THE JOINT.  THE OMISSION OF A CHECK ON THE
C     1 ADHEREND THICKNESSES MAKES THE PROGRAM MORE VERSATILE.
C SET UP RECURRING CONSTANTS
   40 C1 = G / ETA
      C2 = 2. * C1
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      C7 = -1. * GAMMAE
      C10 = -1. * TAUMAX
   60 C3 = ALPHAO * DELTMP
      C4 = ALPHAI * DELTMP
      C5 = C4 - C3
C
C PREPARE FOR SEPARATE COMPUTATION OF POTENTIAL ADHESIVE SHEAR STRENGTH
C     1  ON ASSUMPTION OF ACTUAL ADHEREND STIFFNESSES & INFINITE STRENGTH
      KADHSV = 0
C
C PRINT OUT INPUT DATA
```

```
   70 WRITE (6,80) ALPHAO, ALPHAI, DELTMP
   80 FORMAT (1H1/, 5(1H0/), 11H INPUT DATA//, 10H ALPHAO = , F10.7,
     1  13H (PER DEG. F), 3X, 9HALPHAI = , F10.7, 13H (PER DEG. F),
     2  3X, 9HDELTMP = , F6.1, 9H (DEG. F)//,
     3  1H , 5X, 1HN, 2X, 5HSTEPL, 1X, 6HTHICKO, 1X, 6HTHICKI, 4X,
     4  6HSTROTR, 5X, 6HSTRINR, 5X, 5HETOTR, 5X, 5HETINR//)
      DO 100 N = 1, MSTEPS
      WRITE (6,90) N, STEPL(N), THICKO(N), THICKI(N), STROTR(N),
     1 STRINR(N), ETOTR(N), ETINR(N)
   90 FORMAT (1H , 4X, I2, 3(1X, F6.4), 2(1X, F10.1), 2(1X, F10.1))
  100 CONTINUE
      IF (KADHSV .EQ. 1) GO TO 410
C
C START WITH ELASTIC ANALYSIS
C PROCEED TO ELASTIC-PLASTIC ANALYSIS ONLY IF ADHESIVE IS MORE CRITICAL
C     1  THAN ADHEREND(S)
C NEED ELASTIC SOLUTION TO IDENTIFY CRITICAL END FOR ELASTIC-PLASTIC
C     1   SOLUTION WHEN BOTH THERMAL AND STIFFNESS ADHEREND MISMATCHES
C     2   ARE PRESENT
C ESTIMATE MAXIMUM POSSIBLE BOND CAPACITY FOR FULLY-PLASTIC ADHESIVE
      PBOND = TAUMAX * OLAP * 2.
C NOTE FACTOR 2. INCLUDED FOR DOUBLE-SIDED JOINT
C REDUCE TO 1. IF JOINT HAS ONLY ONE SIDE BONDED
C PERFECTLY-ELASTIC BOND CAPACITY WILL BE CLOSER TO ASYMPTOTE OF SCARF
C     1   JOINT SOLUTION AND IS SIGNIFICANTLY LOWER THAN PLASTIC ESTIMATE
C SCARF JOINT STRENGTH ESTIMATE WOULD BE THE LESSER OF PBOND = 2.*TAUMAX
C     1 *OLAP*(E1T1)/(E2T2) AND PBOND = 2.*TAUMAX*OLAP*(E2T2)/(E1T1)
C NOTE, HOWEVER, THAT STEPPED-LAP JOINTS EXHIBIT CHARACTERISTICS OF
C     1 DOUBLE-LAP JOINTS TO THE EXTENT THAT THE LOAD TRANSFERRED ON ANY
C     2 ONE STEP IS INDEPENDENT OF THAT STEP LENGTH ONCE THE LENGTH
C     3 EXCEEDS A TRANSITIONAL VALUE.  LIKEWISE, THE TOTAL LOAD TRANSFER
C     4 BECOMES INDEPENDENT OF EACH AND EVERY (LONG) STEP IN THE JOINT
C ACTUAL LOAD CAPACITY MAY BE SIGNIFICANTLY LESS IF THERMAL MISMATCH
C     1  BETWEEN ADHERENDS IS SEVERE
C REDUCTION IN LOAD TO ACCOUNT FOR LIMITED ADHEREND STRENGTH IS
C     1  ACCOMPLISHED LATER IN PROGRAM
C PROVIDE OUTER LOOP TO ADJUST ADHESIVE PEAK SHEAR STRAIN AT START OF
C     1   JOINT FOR CASES IN WHICH EITHER ADHESIVE IS MORE CRITICAL AT
C     2   OTHER END OF JOINT OR ADHERENDS ARE MORE CRITICAL THAN ADHESIVE.
      TAUUPR = 2. * TAUMAX
      TAULWR = 0.
C NOTE THAT PROGRAM IS PREVENTED FROM HANDLING PROBLEM IN WHICH SHEAR
C     1   STRESS IN ADHESIVE REVERSES SIGN, WHEN COMPUTATIONS START FROM
C     2   THE LESS CRITICAL END.  SOLUTION IS OBTAINABLE FROM OTHER END.
C NOTE ALSO THAT, IF THE MAXIMUM SHEAR STRESS AND APPLIED LOADS HAVE
C     1   OPPOSITE SIGNS, JOINT MUST BREAK APART UNDER RESIDUAL THERMAL
C     2   STRESS ALONE WITHOUT ANY EXTERNALLY APPLIED LOAD, SO NO CASES OF
C     3   REAL CONCERN ARE EXCLUDED BY THE RESTRICTION ABOVE
      DO 310 I = 1, 50
      TAU(1) = (TAUUPR + TAULWR) / 2.
      IF (TAU(1) .GT. TAUMAX) TAU(1) = TAUMAX
      IF (I .EQ. 1) TAU(1) = TAUMAX
C SET INITIAL CONDITIONS
      GAMMA(1) = TAU(1) / G
      TOUTER(1) = 5. * PBOND
      TINNER(1) = 0.
      DELTAO(1) = 0.
      DELTAI(1) = SGNLD * GAMMA(1) * ETA
      TMAX = 10. * PBOND
      TMIN = 0.
C OPERATE ON THE LOAD LEVEL IN INTERMEDIATE LOOP
C LEAVE ADJUSTMENT OF TAUMAX FOR OUTER LOOP
      TCHECK = 0.
      DO 210 IFLAG = 1, NSTEPS
      SCHECK = 1000000000000000.
      DO 170 NCOUNT = 1, 100
C CONVERGENCE NEARLY ALWAYS OCCURRED BETWEEN 20 AND 30 CYCLES IN TEST
C     1  CASES, BUT THERE WERE SOME EXCEPTIONS
C INTERMEDIATE LOOP ADJUSTS LOAD LEVEL
      V10 = -1. * TOUTER(1)
C CHECK ON CONVERGENCE OF TOUTER(IFLAG)
      R = TOUTER(IFLAG) / SCHECK
      IF ( (1.000000001 .GT. R) .AND. (0.999999999 .LT. R) ) GO TO 180
      DO 120 N = IFLAG, NSTEPS
C INNER LOOP COMPUTES ELASTIC JOINT STRENGTH
      A = TAU(N)
      ALAMDA = DSQRT(C2 * (1. / ETINR(N) + 1. / ETOTR(N)))
      B = (TINNER(N) / ETINR(N) - TOUTER(N) / ETOTR(N) + C5 * SGNLD)
     1   * C1 / ALAMDA
C NOTE THAT SGNLD SIGNIFIES WHETHER SHEAR LOAD IS TENSILE OR COMPRESSIVE
      C = STEPL(N)
      D = ALAMDA * C
      E = DSINH(D)
      F = DCOSH(D)
      TAU(N+1) = A * F + B * E
      DELT = (2. / ALAMDA) * (A * E + B * (F - 1.))
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
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      TOUTER(N+1) = TOUTER(N) - DELT
      TINNER(N+1) = TINNER(N) + DELT
      IF (N .EQ. NSTEPS) GO TO 110
      IF (TAU(N+1) .LT. C10) GO TO 150
      IF (TOUTER(N+1) .LT. V10) GO TO 160
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
C IF THE FACTOR -1. IS TOO LARGE OR TOO SMALL, CONVERGENCE FAILS
  110 DELDLT = (2. / (ALAMDA**2)) * (A * (E - 1.) + B * (E - D))
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      DELTAO(N+1) = DELTAO(N) + C3 * C + SGNLD * (TOUTER(N) * C -
     1 DELDLT) / ETOTR(N)
      DELTAI(N+1) = DELTAI(N) + C4 * C + SGNLD * (TINNER(N) * C +
     1 DELDLT) / ETINR(N)
      GAMMA(N+1) = TAU(N+1) / G
  120 CONTINUE
C CHECK WHETHER OR NOT PRECISELY 100 PERCENT OF LOAD HAS TRANSFERRED
      R1 = TOUTER(1) / TINNER(MSTEPS)
C CHECK ALSO WHETHER OR NOT CONVERGENCE HAS BEEN OBTAINED
      R2 = TCHECK / TINNER(MSTEPS)
      IF ( (1.000001 .GT. R1) .AND. (0.999999 .LT. R1) .AND.
     1 (1.000001 .GT. R2) .AND. (0.999999 .LT. R2) ) GO TO 220
      IF (TOUTER(1) .LT. TINNER(MSTEPS)) GO TO 130
C IF SO, LOAD ESTIMATE IS TOO LOW
C IF NOT, LOAD ESTIMATE IS TOO HIGH
      GO TO 140
C R1 IS UNSUITABLE FOR A CONVERGENCE CHECK BECAUSE NEGATIVE VALUES OF R1
C     1 REPRESENT TOO HIGH A LOAD ESTIMATE, JUST LIKE THOSE VALUES IN
C     2  EXCESS OF UNITY
  130 TMIN = TOUTER(IFLAG)
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
      TCHECK = TINNER(MSTEPS)
      GO TO 170
  140 TMAX = TOUTER(IFLAG)
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
      TCHECK = TINNER(MSTEPS)
      GO TO 170
C NOTE THAT LABELS 26 AND 7 GOVERN FINE ADJUSTMENTS TO THE JOINT LOADS,
C     1  WHILE LABELS 27 AND 28 REPRESENT COARSE ADJUSTMENTS
  150 TMAX = TOUTER(IFLAG)
      SCHECK = TOUTER(IFLAG)
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
      GO TO 170
  160 TMIN = TOUTER(IFLAG)
      SCHECK = TOUTER(IFLAG)
C IF ADHEREND, RATHER THAN ADHESIVE, LIMITS JOINT STRENGTH, NEED TO
C     1  BOOST PLOAD IN PROPORTION TO TAUMAX, EVEN IF IT MEANS EXCEEDING
C     2  ADHEREND STRENGTHS IN INTERMEDIATE COMPUTATIONS.  CORRECTIONS
C     3  ARE APPLIED LATER
      IF (TMIN .GE. TMAX) TMAX = 5. * TMAX
      TOUTER(IFLAG) = (TMIN + TMAX) / 2.
  170 CONTINUE
      IF (N .EQ. NSTEPS) GO TO 210
C CONVERGENCE WILL NOT PROCEED TO FAR END OF JOINT IN SINGLE PASS
C     1  BECAUSE OF NUMERICAL ACCURACY PROBLEMS.  REMEDY IS TO FREEZE
C     2  EARLIER VALUES, WHICH HAVE CONVERGED AND SLIGHTLY PERTURB
C     3  INTERMEDIATE VALUES, AND TO CHECK FOR CONVERGENCE AT THE FAR END
      TMAX = TOUTER(1)
      TMIN = -1. * TMAX
      GO TO 210
  180 ICOUNT = IFLAG + 1
      IF (TOUTER(ICOUNT) .GT. 0.) GO TO 190
      IF (TOUTER(ICOUNT) .LT. 0.) GO TO 200
      TMAX = TOUTER(1) / 10.
      TMIN = -1. * TMAX
      GO TO 210
  190 TMAX = 1.1 * TOUTER(ICOUNT)
      TMIN = 0.9 * TOUTER(ICOUNT)
      GO TO 210
  200 TMIN = 0.9 * TOUTER(ICOUNT)
      TMIN = 1.1 * TOUTER(ICOUNT)
C THE BOUNDS ABOVE ARE CRITICAL IN ENSURING CONVERGENCE
C THEY MUST BE NEITHER TOO LARGE NOR TOO SMALL
  210 CONTINUE
      NRVRS = 1
C
  220 IF ( (C5 .GT. 0.000001) .OR. (C5 .LT. -0.000001) ) GO TO 260
C IF NOT, FIRST SOLUTION MAY BE SCALED IN THE ABSENCE OF ANY THERMAL
C     1  MISMATCH BETWEEN ADHERENDS
C IF SO, SOLUTION MUST BE REFINED BY ITERATION, SINCE THERMAL STRESS
C     1  TERMS DO NOT SCALE LINEARLY, EVEN FOR ELASTIC ADHESIVE AND
C     2  ADHERENDS
C
C APPLY SCALE FACTOR TO SOLUTION FOR ONLY ADHEREND STIFFNESS IMBALANCE
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR
C     1  WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE
C PROGRAM ASSUMES ADHEREND ALLOWABLES HAVE SAME MAGNITUDE IN TENSION AS
C     1  IN COMPRESSION.  DISTINCTION IS USUALLY UNIMPORTANT SINCE, IN
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C     2  PRACTICAL JOINTS, RESIDUAL THERMAL STRESSES ARE UNLIKLELY TO     A4EG2650
C     3  BREAK ADHEREND(S) RATHER THAN ADHESIVE                            A4EG2660
      RSCALE = TOUTER(1) / STROTR(1)                                      A4EG2670
      IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE                           A4EG2680
      RTAUMX = TAU(1) / TAUMAX                                            A4EG2690
      IF (RTAUMX .LT. 0.) RTAUMX = -1. * RTAUMX                           A4EG2700
      DO 240 N = 2, MSTEPS                                                A4EG2710
      RINR = TINNER(N) / STRINR(N-1)                                      A4EG2720
C NEED STRENGTH ON THIN SIDE OF STEP.  HENCE THE (N-1) IN COMPARISON      A4EG2730
      IF (RINR .LT. 0.) RINR = -1. * RINR                                 A4EG2740
      IF (RINR .GT. RSCALE) RSCALE = RINR                                 A4EG2750
      IF (N .EQ. MSTEPS) GO TO 230                                        A4EG2760
      ROTR = TOUTER(N) / STROTR(N)                                        A4EG2770
      IF (ROTR .LT. 0.) ROTR = -1. * ROTR                                 A4EG2780
      IF (ROTR .GT. RSCALE) RSCALE = ROTR                                 A4EG2790
  230 RTAU = TAU(N) / TAUMAX                                              A4EG2800
      IF (RTAU .LT. 0.) RTAU = -1. * RTAU                                 A4EG2810
      IF (RTAU .GT. RTAUMX) RTAUMX = RTAU                                 A4EG2820
  240 CONTINUE                                                            A4EG2830
      RECTR = RSCALE                                                      A4EG2840
C RECTR IS PROPORTIONALITY CONSTANT GOVERNING ELASTIC SOLUTION            A4EG2850
C IF RTAUMAX .GT. RSCALE, ADHESIVE PLASTICITY CAN INCREASE STRENGTH       A4EG2860
C USUALLY ADHESIVE IS CRITICAL AT ONE END OF JOINT OR OTHER, SO RTAUMX    A4EG2870
C     1  .GT. 1. MAY WELL JUST SIGNIFY THAT FAR END OF JOINT IS CRITICAL  A4EG2880
C NOTE THAT PROGRAM ASSUMES THAT ANY INTERNAL ADHEREND STRESSES OF        A4EG2890
C     1  REVERSED SIGN WITH RESPECT TO STRESS OUTSIDE THE JOINT ARE NOT   A4EG2900
C     2  CRITICAL.  IF THEY ARE, IT MEANS THAT THE JOINT WILL FAIL DUE    A4EG2910
C     3  TO RESIDUAL THERMAL STRESSES ALONE WITHOUT ANY MECHANICAL LOADS  A4EG2920
      IF (RSCALE .LT. RTAUMX) RECTR = RTAUMX                              A4EG2930
      DO 250 N = 1, MSTEPS                                                A4EG2940
      TOUTER(N) = TOUTER(N) / RECTR                                       A4EG2950
      TINNER(N) = TINNER(N) / RECTR                                       A4EG2960
      TAU(N) = TAU(N) / RECTR                                             A4EG2970
      GAMMA(N) = GAMMA(N) / RECTR                                         A4EG2980
      DELTAO(N) = DELTAO(N) / RECTR                                       A4EG2990
  250 DELTAI(N) = DELTAI(N) / RECTR                                       A4EG3000
      GO TO 330                                                           A4EG3010
C                                                                         A4EG3020
C USE ITERATIVE SOLUTION WHEN ADHEREND THERMAL MISMATCH IS PRESENT        A4EG3030
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR   A4EG3040
C     1  WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE.        A4EG3050
  260 RSCALE = TOUTER(1) / STROTR(1)                                      A4EG3060
      IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE                           A4EG3070
      RTAUMX = TAU(1) / TAUMAX                                            A4EG3080
      IF (RTAUMX .LT. 0.) RTAUMX = -1. * RTAUMX                           A4EG3090
      DO 280 N = 2, MSTEPS                                                A4EG3100
      RINR = TINNER(N) / STRINR(N)                                        A4EG3110
      IF (RINR .GT. RSCALE) RSCALE = RINR                                 A4EG3120
      IF (N .EQ. MSTEPS) GO TO 270                                        A4EG3130
      ROTR = TOUTER(N) / STROTR(N)                                        A4EG3140
      IF (ROTR .GT. RSCALE) RSCALE = ROTR                                 A4EG3150
  270 RTAU = TAU(N) / TAUMAX                                              A4EG3160
      IF (RTAU .LT. 0.) RTAU = -1. * RTAU                                 A4EG3170
      IF (RTAU .GT. RTAUMX) RTAUMX = RTAU                                 A4EG3180
  280 CONTINUE                                                            A4EG3190
C CHECK ON CONVERGENCE                                                    A4EG3200
      R = RTAUMX                                                          A4EG3210
      IF (RTAUMX .LT. RSCALE) R = RSCALE                                  A4EG3220
      IF ( (1.00001 .GT. R) .AND. (0.99999 .LT. R) ) GO TO 330            A4EG3230
      V = 2. * TAUMAX                                                     A4EG3240
      R = (V + TAUUPR) / (V + TAULWR)                                     A4EG3250
      IF ( (1.00001 .GT. R) .AND. (0.99999 .LT. R) ) GO TO 330            A4EG3260
C IF EITHER RTAUMX OR RSCALE .GT. UNITY, TAU(1) MUST BE DECREASED         A4EG3270
      IF ( (RTAUMX .GT. 1.00001) .OR. (RSCALE .GT. 1.00001) ) GO TO 290   A4EG3280
C IF BOTH RTAUMX AND RSCALE ARE .LT. UNITY, TAU(1) MUST BE INCREASED      A4EG3290
      IF ( (RTAUMX .LT. 0.99999) .AND. (RSCALE .LT. 0.99999) ) GO TO 300A4EG3300
C IF NEITHER CHECK IS MET, SOLUTION HAS CONVERGED                         A4EG3310
      GO TO 330                                                           A4EG3320
  290 TAUUPR = TAU(1)                                                     A4EG3330
      GO TO 310                                                           A4EG3340
  300 TAULWR = TAU(1)                                                     A4EG3350
  310 CONTINUE                                                            A4EG3360
C IF PROGRAM GOES BEYOND PRECEEDING CONTINUE STATEMENT, SOLUTION HAS NOTA4EG3370
C     1  CONVERGED                                                        A4EG3380
      WRITE (6,320)                                                       A4EG3390
  320 FORMAT (1H1, 18HDIVERGENT SOLUTION)                                 A4EG3400
C                                                                         A4EG3410
C PRINT OUT RESULTS OF ELASTIC COMPUTATIONS                               A4EG3420
  330 WRITE (6,340) TOUTER(1), TAUMAX, SGNLD, DELTMP                      A4EG3430
  340 FORMAT (1H1/, 5(1H0/),                                              A4EG3440
     1  39H ELASTIC JOINT STRENGTH, PLOAD (LBS) = , F10.1/,               A4EG3450
     2  49H ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = , F8.1/,      A4EG3460
     3  1H , 8HSGNLD = , F4.1, 54H    SGNLD = +1 FOR TENSILE SHEAR AND -1A4EG3470
     4 FOR COMPRESSIVE/, 36H TEMPERATURE DIFFERENTIAL (DEG F) = , F6.1//A4EG3480
     5,  1H , 5X, 1HN, 2X, 5HSTEPL, 1X, 6HTHICKO, 1X, 6HTHICKI, 3X,       A4EG3490
     6  3HTAU, 4X, 5HGAMMA, 1X, 6HDELTAO, 1X, 6HDELTAI, 5X, 6HTOUTER,     A4EG3500
     7  5X, 6HSTROTR, 5X, 6HTINNER, 5X, 6HSTRINR//)                       A4EG3510
      DO 360 N = 1, MSTEPS                                                A4EG3520
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      WRITE (6,350) N, STEPL(N), THICKO(N), THICKI(N), TAU(N), GAMMA(N),
     1  DELTAO(N), DELTAI(N), TOUTER(N), STROTR(N), TINNER(N), STRINR(N
     2)
  350 FORMAT (1H , 4X, I2, 1X, F6.4, 1X, F6.4, 1X, F6.4, 1X, F7.1, 1X,
     1  F6.3, 1X, F6.4, 1X, F6.4, 1X, F10.1, 1X, F10.1, 1X, F10.1, 1X,
     2  F10.1)
  360 CONTINUE
C
C RECOMPUTE SOLUTION FROM OTHER END OF JOINT, IF APPROPRIATE
C NOTE THAT, IF COMPUTER PRINTS OUT TWO SOLUTIONS TO A GIVEN PROBLEM BY
C    1  REVERSING ENDS AND RE-ANALYZING, IT IS BECAUSE THE FIRST FAILED
C    2  TO CONVERGE, EVEN IF THE ANSWERS PRINTED SEEM TO SUGGEST
C    3  OTHERWISE.  THE SECOND SOLUTION IS TO BE PREFERRED, PARTICULARLY
C    4  IF IT STARTS AT THAT END OF THE JOINT AT WHICH THE ADHESIVE
C    5  SHEAR STRESS IS AT ITS HIGHEST.
C IDENTIFY CRITICAL END OF JOINT
C AVOID REVERSING ENDS BACK AGAIN
      IF (JFLAG .EQ. 2) GO TO 400
      IF (NRVRS .EQ. 1) GO TO 370
C IF SO, SOLUTION HAS FAILED TO CONVERGE, SO TRY AGAIN FROM OTHER END
C ACCURACY AT FAR END OF JOINT MAY BE POOR IF FAR END IS CRITICAL
      IF ((TAU(MSTEPS) .LE. TAU(1)) .AND. (TAU(MSTEPS) .GE.
     1  (-1. * TAU(1)))) GO TO 400
C IF, AT FAR END OF JOINT, TAU(MSTEPS) .GT. TAU(1) AT NEAR END,
C    1  FAILURE TO CONVERGE MAY BE SIMPLY THE RESULT OF THE FAR END
C    2  OF THE JOINT BEING MORE CRITICAL THAN THE STARTING (NEAR) END
C  REVERSE DATA AND REANALYZE
  370 DO 380 N = 1, MSTEPS
      STEP(N) = STEPL(N)
      THCKNO(N) = THICKO(N)
      THCKNI(N) = THICKI(N)
      ETOUTR(N) = ETOTR(N)
      ETINNR(N) = ETINR(N)
      STRGTR(N) = STROTR(N)
  380 STRGNR(N) = STRINR(N)
      DO 390 N = 1, MSTEPS
      STEPL(N) = STEP(MSTEPS - N)
      THICKO(N) = THCKNI(MSTEPS - N)
      THICKI(N) = THCKNO(MSTEPS - N)
      ETOTR(N) = ETINNR(MSTEPS - N)
      ETINR(N) = ETOUTR(MSTEPS - N)
      STROTR(N) = STRGNR(MSTEPS - N)
  390 STRINR(N) = STRGTR(MSTEPS - N)
      STEPL(MSTEPS) = STEP(MSTEPS)
      THICKO(MSTEPS) = 0.
      THICKI(MSTEPS) = THCKNO(1)
      ETOTR(MSTEPS) = 0.
      ETINR(MSTEPS) = ETOUTR(1)
      STROTR(MSTEPS) = 0.
      STRINR(MSTEPS) = STRGTR(1)
      V = ALPHAO
      ALPHAO = ALPHAI
      ALPHAI = V
      JFLAG = 2
      NRVRS = 0
      GO TO 60
C
C BYPASS ELASTIC-PLASTIC COMPUTATIONS IF ADHERENDS ARE MORE CRITICAL
C    1  THAN ADHESIVE
  400 IF (PSCALE .GE. PTAUMX) GO TO 820
C RECORD ELASTIC JOINT STRENGTH
      ELSTR = TOUTER(1)
C
C START ELASTIC-PLASTIC SOLUTION
C ELASTIC SOLUTION HAS IDENTIFIED CRITICAL END OF JOINT, AND REVERSED
C    1  ORDER OF DATA IF NECESSARY, SO THERE IS NO NEED FOR SUCH
C    2  CAPABILITY IN THE ELASTIC-PLASTIC SOLUTION
C ADD EXTRA LOCATIONS INSIDE STEPS TO ACCOUNT FOR POTENTIAL PLASTIC-TO-
C    1  ELASTIC AND ELASTIC-TO-PLASTIC TRANSITIONS IN ADHESIVE
  410 DO 420 N = 1, MSTEPS
      L = 3 * N
      THCKNO(L-2) = THICKO(N)
      THCKNO(L-1) = THICKO(N)
      THCKNO(L) = THICKO(N)
      THCKNI(L-2) = THICKI(N)
      THCKNI(L-1) = THICKI(N)
      THCKNI(L) = THICKI(N)
      ETOUTR(L-2) = ETOTR(N)
      ETOUTR(L-1) = ETOTR(N)
      ETOUTR(L) = ETOTR(N)
      ETINNR(L-2) = ETINR(N)
      ETINNR(L-1) = ETINR(N)
      ETINNR(L) = ETINR(N)
      STRGTR(L-2) = STROTR(N)
      STRGTR(L-1) = STROTR(N)
      STRGTR(L) = STROTR(N)
      STRGNR(L-2) = STRINR(N)
      STRGNR(L-1) = STRINR(N)
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  420 STRGNR(L) = STRINR(N)
      THCKNO(MCHECK) = 0.
      THCKNI(MCHECK) = THICKI(MSTEPS)
      ETOUTR(MCHECK) = 0.
      ETINNR(MCHECK) = ETINR(MSTEPS)
      STRGTR(MCHECK) = 0.
      STRGNR(MCHECK) = STRINR(MSTEPS)
C USE OUTER LOOP TO ADJUST MAXIMUM ADHESIVE SHEAR STRAIN LEVEL
      GAMUPR = GAMMAX
      GAMLWR = GAMMAE
C GAMUPR AND GAMLWR SERVE AS BOUNDS ON SHEAR STRAIN ACTUALLY DEVELOPED
C SET UPPER AND LOWER BOUNDS ON STRENGTH
      TUPPER = TAUMAX * OLAP * 2.
C NOTE THAT THE FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF
C     1  ADHERENDS.  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      TLOWER = ELSTR
C TMAX .EQ. FULLY-PLASTIC JOINT STRENGTH
C TMIN .EQ. PERFECTLY-ELASTIC JOINT STRENGTH
      ICHECK = 0
      JCHECK = 10
C NEED TO SET DIFFERENT VALUES FOR ICHECK AND JCHECK WITH NEITHER
C     1  EQUAL TO EITHER 1 OR 2
      DO 760 I = 1, 50
      IF ( (KADHSV .EQ. 1) .AND. (I .GT. 1) ) GO TO 770
C THIS INSTRUCTION PRINTS OUT SOLUTION FOR POTENTIAL BOND SHEAR STRENGTH
      TMAX = TUPPER
      TMIN = TLOWER
      GAMMA(1) = (GAMUPR + GAMLWR) / 2.
      IF ( (KADHSV .EQ. 1) .OR. (I .EQ. 1) ) GO TO 440
      IF (ICHECK .NE. JCHECK) GO TO 450
      IF (ICHECK .EQ. 1) GO TO 430
C IF NOT, ICHECK .EQ. 2 AND LOAD HAS BEEN TOO HIGH FOR TWO CONSECUTIVE
C     1  ITERATIONS
      GAMMA(1) = (GAMMA(1) + GAMLWR) / 2.
      ICHECK = 2
      GO TO 450
C IF ICHECK .EQ. 1, LOAD HAS BEEN TOO LOW FOR TWO CONSECUTIVE ITERATIONS
  430 GAMMA(1) = (GAMMA (1) + GAMUPR) / 2.
      ICHECK = 1
      GO TO 450
  440 GAMMA(1) = GAMMAX
C SET INITIAL CONDITIONS
  450 TOUTER(1) = (TUPPER + TLOWER) / 2.
      TINNER(1) = 0.
      TAU(1) = TAUMAX
      DELTAO(1) = 0.
      DELTAI(1) = SGNLD * GAMMA(1) * ETA
      DO 700 IFLAG = 1, NSTEPS
      J1 = 3 * IFLAG - 2
      J2 = J1 + 3
      SCHECK = 1000000000000000.
      DO 660 NCOUNT = 1, 50
C USUALLY 20 CYCLES OF ITERATION WERE SUFFICIENT AT THIS POINT
C MIDDLE LOOP ADJUSTS LOAD LEVEL
C CHECK ON CONVERGENCE OF TOUTER(J1)
      R = TOUTER(J1) / SCHECK
      IF ( (1.000000001 .GT. R) .AND. (0.999999999 .LT. R) ) GO TO 670
      V11 = TOUTER(1)
      V10 = -1. * V11
      DO 600 N = IFLAG, NSTEPS
C INNERMOST LOOP COMPUTES JOINT STRENGTH
      L = 3 * N - 2
      STEP(L) = STEPI(N)
      STEP(L+1) = 0.
      STEP(L+2) = 0.
C IF ADHESIVE IS NOT LOADED INTO PLASTIC ZONE IN LATER STEPS OF JOINT,
C     1  BYPASS SUCH COMPUTATIONS AND PROCEED TO PERFECTLY-ELASTIC SOLUTI
      V8 = GAMMA(L)
      IF ( (V8 .LE. GAMMAE) .AND. (V8 .GE. C7) ) GO TO 510
      XP = 10000000000.
      MFLAG = 0
C SOLVE FOR MAXIMUM POSSIBLE EXTENT OF PLASTIC ADHESIVE ZONE
C     1  AND COMPARE WITH STEP LENGTH
      V4 = ETOTR(N)
      V5 = ETINR(N)
      V6 = TOUTER(L)
      V7 = TINNER(L)
      V9 = STEP(L)
      V = (1. / V4 + 1. / V5) / ETA
      A = TAUMAX * V
C IF BONDED ON ONE SIDE OF JOINT ONLY, DIVIDE A BY 2.
      B = (C5 * SGNLD - V6 / V4 + V7 / V5) / ETA
C NOTE THAT B SHOULD BE NEGATIVE AT AND NEAR MORE CRITICAL END OF JOINT
C IF NOT, SOLUTION IS PROCEEDING FROM WRONG END OF JOINT
C HENCE NEED FOR PRIOR ELASTIC SOLUTION TO IDENTIFY CRITICAL END
      IF (V8 .LT. 0.) GO TO 460
C PROCEDURE FOR POSITIVE PLASTIC ADHESIVE SHEAR STRAINS
      IF (B .GE. 0.) GO TO 470
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      C = V8 - GAMMAE
      D = B**2 - 4. * A * C
      IF (D .LT. 0.) GO TO 470
C IF SO, PLASTIC ZONE IS UNBOUNDED, AS AT FAR END OF JOINT
      XP = (-1. * B - DSQRT(D)) / (2. * A)
      GO TO 470
C PROCEDURE FOR NEGATIVE PLASTIC ADHESIVE SHEAR STRAINS
  460 IF (B .LE. 0.) GO TO 470
      C = V8 + GAMMAE
      D = B**2 + 4. * A * C
      IF (D .LT. 0.) GO TO 470
C IF SO, PLASTIC ZONE IS UNBOUNDED, AS AT FAR END OF JOINT
      XP = (B - DSQRT(D)) / (2. * A)
  470 IF (XP .GE. V9) GO TO 480
C IF SO, ADHESIVE IS FULLY-PLASTIC THROUGHOUT THAT STEP
C IF NOT, BREAK UP STEP INTO PLASTIC AND ELASTIC PORTIONS
      MFLAG = 1
      STEP(L) = XP
      STEP(L+1) = V9 - XP
      V9 = XP
C MAY HAVE TO DECREASE STEP(3*N-1) AND ADD TO STEP(3*N) LATER
C
C PROCEDURE FOR FULLY-PLASTIC STEP OR STEP PORTION
C THIS SERIES OF EQUATIONS HOLDS REGARDLESS OF SIGN OF SHEAR STRESS
C     1  GRADIENT AT START OF STEP
  480 DELT = 2. * TAU(L)   * V9
C THE USE OF TAU(L) INSTEAD OF TAUMAX COVERS REVERSAL OF SIGN
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      L = L + 1
      TOUTER(L) = V6 - DELT
      TINNER(L) = V7 + DELT
      GAMMA(L) = GAMMA(L-1) + B * V9 + TAU(L-1) * V * (V9**2)
      TAU(L) = TAU(L-1)
      DELTAO(L) = DELTAO(L-1) + C3 * V9 + SGNLD * (V6 * V9 - TAU(L) *
     1  (V9**2) / 2.) / V4
      DELTAI(L) = DELTAI(L-1) + C4 * V9 + SGNLD * (V7 * V9 + TAU(L) *
     1  (V9**2) / 2.) / V5
C NOTE THAT USE OF TAU(L) INSTEAD OF TAUMAX AUTOMATICALLY ACCOUNTS FOR
C     1  SIGN OF ADHESIVE SHEAR STRESS
      IF (MFLAG .EQ. 1) GO TO 490
C IF NOT, STEP IS PLASTIC THROUGHOUT
      L1 = L + 1
      L2 = L + 2
      TOUTER(L1) = TOUTER(L)
      TOUTER(L2) = TOUTER(L)
      TINNER(L1) = TINNER(L)
      TINNER(L2) = TINNER(L)
      TAU(L1) = TAU(L)
      TAU(L2) = TAU(L)
      GAMMA(L1) = GAMMA(L)
      GAMMA(L2) = GAMMA(L)
      DELTAO(L1) = DELTAO(L)
      DELTAO(L2) = DELTAO(L)
      DELTAI(L1) = DELTAI(L)
      DELTAI(L2) = DELTAI(L)
  490 IF (N .EQ. NSTEPS) GO TO 500
      IF (TOUTER(L) .LT. V10) GO TO 620
      IF (TINNER(L) .LT. V10) GO TO 610
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
  500 IF (MFLAG .EQ. 1) GO TO 510
      GO TO 590
C
C PROCEDURE FOR PERFECTLY-ELASTIC ZONE
C IDENTIFY WHETHER STEP IS ELASTIC-PLASTIC OR FULLY ELASTIC THROUGHOUT
  510 K = L - 3 * N + 2
C K .EQ. 0 CORRESPONDS TO NO PLASTIC ZONE AT NEAR END OF JOINT
C SET INITIAL CONDITIONS AT START OF STEP
      V4 = ETOTR(N)
      V5 = ETINR(N)
      V6 = TOUTER(L)
      V7 = TINNER(L)
      ALAMDA = DSQRT(C2 * (1. / V4 + 1. / V5))
      LFLAG = 0
C COMPUTE VALUES AT FAR END OF ELASTIC ZONE
      A = TAU(L)
      B = (V7 / V5 - V6 / V4 + C5 * SGNLD) * C1 / ALAMDA
C NOTE THAT SGNLD SIGNIFIES WHETHER SHEAR LOAD IS TENSILE OR COMPRESSIVE
      C = STEP(L)
      D = ALAMDA * C
      E = DSINH(D)
      F = DCOSH(D)
      TAU(L+1) = A * F + B * E
      IF (  (TAU(L+1) .LE. TAUMAX) .AND. (TAU(L+1)  .GE. C10) )
     1  GO TO 540
C IF NOT, ELASTIC STEP SIZE IS EXCESSIVE.  REDUCE BY ITERATION
      ELMAX = C
      ELMIN = 0.
```

```
      DO 520 ICOUNT = 1, 100
      EL = (ELMAX + ELMIN) / 2.
      D = ALAMDA * EL
      E = DSINH(D)
      F = DCOSH(D)
      TCHECK = A * E + B * F
      IF ( (TCHECK .GT. TAUMAX) .OR. (TCHECK .LT. C10) ) ELMAX = EL
      IF ( (TCHECK .LT. TAUMAX) .AND.  (TCHECK .GT. C10) ) ELMIN = EL
      R = ELMIN / ELMAX
      IF ( (1.000000001 .GT. R) .AND. (0.999999999 .LT. R) ) GO TO 530
  520 CONTINUE
  530 STEP(L) = EL
      STEP(L+1) = STEP(L+1) + C - EL
      LFLAG = 1
      IF (TCHECK .GT. 0.) TAU(L+1) = TAUMAX
      IF (TCHECK .LT. 0.) TAU(L+1) = C10
  540 DELT = (2. / ALAMDA) * (A * F + B * (E - 1.))
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      L = L + 1
      TOUTER(L) = V6 - DELT
      TINNER(L) = V7 + DELT
      DELDLT = (2. / (ALAMDA**2)) * (A * (F - 1.) + B * (E - D))
C FACTOR OF 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
      DELTAO(L) = DELTAO(L-1) + C3 * EL + SGNLD * (V6 * EL - DELDLT) /V4
      DELTAI(L) = DELTAI(L-1) + C4 * EL + SGNLD * (V7 * EL + DELDLT) /V5
      GAMMA(L) = TAU(L) / G
      IF (LFLAG .EQ. 1) GO TO 550
C IF NOT, THERE IS NO (SECOND) PLASTIC ZONE AT FAR END OF STEP
      L1 = L + 1
      TOUTER(L1) = TOUTER(L)
      TINNER(L1) = TINNER(L)
      TAU(L1) = TAU(L)
      GAMMA(L1) = GAMMA(L)
      DELTAO(L1) = DELTAO(L)
      DELTAI(L1) = DELTAI(L)
      IF (K .EQ. 1) GO TO 550
C IF NOT, THERE WAS NO (FIRST) PLASTIC ZONE AT NEAR END OF JOINT
      L2 = L + 2
      TOUTER(L2) = TOUTER(L)
      TINNER(L2) = TINNER(L)
      TAU(L2) = TAU(L)
      GAMMA(L2) = GAMMA(L)
      DELTAO(L2) = DELTAO(L)
      DELTAI(L2) = DELTAI(L)
  550 IF (N .EQ. NSTEPS) GO TO 560
      IF  (TOUTER(L) .LT. V10) GO TO 620
      IF (TINNER(L) .LT. V10) GO TO 610
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
C IF V10 IS EITHER TOO LARGE OR TOO SMALL, CONVERGENCE FAILS
  560 IF (LFLAG .EQ. 1) GO TO 570
      GO TO 590
C
C PROCEDURE FOR (SECOND) PLASTIC ZONE AT FAR END OF STEP
  570 V9 = STEP(L)
      DELT = 2. * TAU(L) * V9
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF  INNER  ADHEREND.
C     1  IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
C SET INITIAL CONDITIONS AT START OF STEP
      V4 = ETOTR(N)
      V5 = ETINR(N)
      V6 = TOUTER(L)
      V7 = TINNER(L)
      L = L + 1
      TOUTER(L) = V6 - DELT
      TINNER(L) = V7 + DELT
      A = (TAU(L-1) / ETA) * (1./V4 + 1./V5)
C NOTE THE USE OF TAU(L-1) INSTEAD OF TAUMAX IN ORDER TO ACCOUNT
C     1  AUTOMATICALLY FOR THE SIGN OF THE SHEAR STRESS
C IF BONDED ON ONE SIDE OF JOINT ONLY, DIVIDE A BY 2.
      B = (C5 * SGNLD - V6 / V4 + V7 / V5) / ETA
      GAMMA(L) = GAMMA(L-1) + B * V9 + A * (V9**2)
      TAU(L) = TAU(L-1)
      DELTAO(L) = DELTAO(L-1) + C3 * V9 + SGNLD * (V6 * V9 - TAU(L) *
     1  (V9**2) / 2.) / V4
      DELTAI(L) = DELTAI(L-1) + C4 * V9 + SGNLD * (V7 * V9 + TAU(L) *
     1  (V9**2) / 2.) / V5
C IF THERE HAS BEEN NO PLASTIC ZONE AT START OF STEP, TRANSFER VALUES
C     1  JUST COMPUTED ACROSS TO LAST SUBDIVISION IN STEP
C THIS IS NECESSARY TO PROVIDE INPUT DATA FOR START OF NEXT STEP
      IF (K .EQ. 1) GO TO 580
      L1 = L + 1
      TOUTER(L1) = TOUTER(L)
      TINNER(L1) = TINNER(L)
      TAU(L1) = TAU(L)
      GAMMA(L1) = GAMMA(L)
      DELTAO(L1) = DELTAO(L)
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      DELTAI(L1) = DELTAI(L)
  580 IF (N .EQ. NSTEPS) GO TO 630
      IF (TOUTER(L) .LT. V10) GO TO 620
      IF (TINNER(L) .LT. V10) GO TO 610
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
  590 IF (N .EQ. NSTEPS) GO TO 630
  600 CONTINUE
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO HIGH
  610 TMAX = TOUTER(J1)
      SCHECK = TOUTER(J1)
      TOUTER(J1) = (TMAX + TMIN) / 2.
      GO TO 660
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO LOW
  620 TMIN = TOUTER(J1)
      SCHECK = TOUTER(J1)
      TOUTER(J1) = (TMAX + TMIN) / 2.
      GO TO 660
C
C CHECK WHETHER OR NOT PRECISELY 100 PERCENT OF LOAD HAS TRANSFERRED
  630 R1 = TOUTER(1) / TINNER(MCHECK)
      R2 = TCHECK / TINNER(MCHECK)
      IF ( (1.000001 .GT. R1) .AND. (0.999999 .LT. R1) .AND.
     1  (1.000001 .GT. R2) .AND. (0.999999 .LT. R2) ) GO TO 710
      IF (TOUTER(1) .LT. TINNER(MCHECK)) GO TO 640
C NOTE THAT, HERE, LOAD IS TAKEN TO BE POSITIVE WHETHER TENSILE OR NOT
C     1  FOR A NEGATIVE LOAD, PRECEDING INSTRUCTION SHOULD BE
C     2  INTERCHANGED WITH THE FOLLOWING ONE
C IF SO, LOAD ESTIMATE IS TOO LOW
C IF REVERSE HOLDS, LOAD ESTIMATE IS TOO HIGH
      GO TO 650
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO LOW
  640 TMIN = TOUTER(J1)
      TOUTER(J1) = (TMAX + TMIN) / 2.
      TCHECK = TINNER(MCHECK)
      GO TO 660
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO HIGH
  650 TMAX = TOUTER(J1)
      TOUTER(J1) = (TMAX + TMIN) / 2.
      TCHECK = TINNER(MCHECK)
  660 CONTINUE
C CONVERGENCE WILL NOT PROCEED TO FAR END OF JOINT IN SINGLE PASS
C     1  BECAUSE OF NUMERICAL ACCURACY PROBLEMS.  REMEDY IS TO FREEZE
C     2  EARLIER VALUES, WHICH HAVE CONVERGED, AND TO PERTURB SLIGHTLY
C     3  INTERMEDIATE VALUES, AND TO CHECK FOR CONVERGENCE AT THE FAR END
      TMAX = TUPPER
      TMIN = -1. * TMAX
      GO TO 700
  670 IF (TOUTER(J2) .GT. 0.) GO TO 680
      IF (TOUTER(J2) .LT. 0.) GO TO 690
      TMAX = TOUTER(1) / 10.
      TMIN = -1. * TMAX
      GO TO 700
  680 TMAX = 1.1 * TOUTER(J2)
      TMIN = 0.9 * TOUTER(J2)
      GO TO 700
  690 TMAX = 0.9 * TOUTER(J2)
      TMIN = 1.1 * TOUTER(J2)
C THE BOUNDS ABOVE ARE CRITICAL IN ENSURING CONVERGENCE
C THEY MUST BE NEITHER TOO LARGE NOR TOO SMALL
  700 CONTINUE
C
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC-PLASTIC
C     1  ADHESIVE
  710 RSCALE = TOUTER(1) / STRGTR(1)
      IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE
      RGAMAX = GAMMA(1) / GAMMAX
      IF (RGAMAX .LT. 0.) RGAMAX = -1. * RGAMAX
      DO 730 N = 2, MCHECK
      RINR = TINNER(N) / STRGNR(N-1)
C AT EACH STEP TRANSITION NEED THE THINNER SECTION.  HENCE THE (N-1)
      IF (RINR .LT. 0.) RINR = -1. * RINR
      IF (RINR .GT. RSCALE) RSCALE = RINR
      IF (N .EQ. MCHECK) GO TO 720
      ROTR = TOUTER(N) / STRGTR(N)
      IF (ROTR .LT. 0.) ROTR = -1. * ROTR
      IF (ROTR .GT. RSCALE) RSCALE = ROTR
  720 RGAMX = GAMMA(N) / GAMMAX
      IF (RGAMX .LT. 0.) RGAMX = -1. * RGAMX
      IF (RGAMX .GT. RGAMAX) RGAMAX = RGAMX
  730 CONTINUE
C
C IF UPPER AND LOWER BOUNDS ON JOINT LOAD HAVE COALESCED,
C     1  NO MORE CONVERGENCE IS POSSIBLE.  PRINT OUT RESULTS.
      R = TUPPER / TLOWER
      IF ( (1.000000001 .GT. R) .AND. (0.999999999 .LT. R) ) GO TO 770
C ADJUST MAXIMUM ADHESIVE SHEAR STRAIN IF ADHEREND STRENGTH GOVERNS OVER
C     1  ADHESIVE STRENGTH CONSIDERATIONS
      IF ( (RSCALE .GT. 1.0001) .OR. (RGAMAX .GT. 1.0001) )
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    1 GO TO 740
      IF ( (RSCALE .LT. 0.9999) .AND. (RGAMAX .LT. 0.9999) )
    1 GO TO 750
C IF NEITHER OF THESE CHECKS IS MET, SOLUTION HAS CONVERGED.  PRINT OUT.
      GO TO 770
C IF NOT, REITERATE
  740 GAMUPR = GAMMA(1)
      TUPPER = TOUTER(1)
      JCHECK = 2
      GO TO 760
  750 GAMLWR = GAMMA(1)
      TLOWER = TOUTER(1)
      JCHECK = 1
  760 CONTINUE
C IF PROGRAM GOES PAST THIS CONTINUE STATEMENT, CONVERGENCE HAS FAILED
      WRITE (6,320)
C
C PRINT OUT RESULTS OF ELASTIC-PLASTIC COMPUTAIONS
  770 WRITE (6,780) TOUTER(1), GAMMAX, SGNLD, DELTMP
  780 FORMAT (1H1/, 5(1H0/),
     1  47H ELASTIC-PLASTIC JOINT STRENGTH, PLOAD (LBS) = , F10.1/,
     2  43H ALLOWABLE ADHESIVE SHEAR STRAIN, GAMMAX = , F6.3/,
     3  1H , 8HSGNLD = , F4.1, 54H   SGNLD = +1 FOR TENSILE SHEAR AND -1
     4 FOR COMPRESSIVE/, 36H TEMPERATURE DIFFERENTIAL (DEG F) = , F6.1//
     5,  1H , 5X, 1HN, 2X, 5HSTEPL, 1X, 6HTHICKO, 1X, 6HTHICKI, 3X,
     6  3HTAU, 4X, 5HGAMMA, 1X, 6HDELTAO, 1X, 6HDELTAI, 5X, 6HTOUTER,
     7  5X, 6HSTROTR, 5X, 6HTINNER, 5X, 6HSTRINR//)
      DO 800 N = 1, MCHECK
      WRITE (6,790) N, STEP(N), THCKNO(N), THCKNI(N), TAU(N), GAMMA(N),
     1   DELTAO(N), DELTAI(N), TOUTER(N), STRGTR(N), TINNER(N), STRGNR(N
     2)
  790 FORMAT (1H , 4X, I2, 1X, F6.4, 1X, F6.4, 1X, F6.4, 1X, F7.1, 1X,
     1  F6.3, 1X, F6.4, 1X, F6.4, 1X, F10.1, 1X, F10.1, 1X, F10.1, 1X,
     2  F10.1)
  800 CONTINUE
      IF (KADHSV .EQ. 1) GO TO 820
      IF (RSCALE .LT. RGAMAX) GO TO 820
C IF NOT, COMPUTE POTENTIAL BOND STRENGTH OF ADHESIVE
      KADHSV = 1
      DO 810 K = 1, MSTEPS
      STROTR(K) = 100000000000000000000.
  810 STRINR(K) = 100000000000000000000.
      GO TO 70
  820 CONTINUE
      STOP
      END
```

INPUT DATA

ALPHAO = 0.0000050 (PER DEG. F) ALPHAI = 0.0 (PER DEG. F) DELTMP = -280.0 (DEG. F)

N	STEPL	THICKO	THICKI	STROTR	STRINR	ETOTR	ETINR
1	0.7500	0.2500	0.0440	32500.0	3036.0	4000000.0	352000.0
2	0.7500	0.2060	0.0880	26780.0	6072.0	3296000.0	704000.0
3	0.7500	0.1620	0.1320	21060.0	9108.0	2592000.0	1056000.0
4	0.7500	0.1180	0.1760	15340.0	12144.0	1888000.0	1408000.0
5	0.3750	0.0740	0.2200	9620.0	15180.0	1184000.0	1760000.0
6	0.3750	0.0300	0.2640	3900.0	18216.0	480000.0	2112000.0
7	******	0.0	0.2640	0.0	18216.0	0.0	2112000.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 10730.1
ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 6000.0
N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE
TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.7500	0.2500	0.0440	5696.0	0.095	0.0	-.0005	10730.1	32500.0	0.0	3036.0
2	0.7500	0.2060	0.0880	2568.5	0.043	-.0028	-.0030	8817.0	26780.0	1913.1	6072.0
3	0.7500	0.1620	0.1320	2243.7	0.037	-.0057	-.0059	7344.6	21060.0	3385.5	9108.0
4	0.7500	0.1180	0.1760	2239.0	0.037	-.0087	-.0099	5804.3	15340.0	4925.8	12144.0
5	0.3750	0.0740	0.2200	2797.8	0.047	-.0117	-.0119	4018.4	9620.0	6711.8	15180.0
6	0.3750	0.0300	0.2640	3722.2	0.062	-.0132	-.0135	2232.4	3900.0	8497.8	18216.0
7	******	0.0	0.2640	6000.0	0.100	-.0147	-.0152	-0.0	0.0	10730.1	18216.0

INPUT DATA

ALPHAO = 0.0 (PER DEG. F) ALPHAI = 0.0000050 (PER DEG. F) DELTMP = -280.0 (DEG. F)

N	STEPL	THICKO	THICKI	STROTR	STRINR	ETOTR	ETINR
1	0.3750	0.2640	0.0300	18216.0	3900.0	2112000.0	480000.0
2	0.3750	0.2200	0.0740	15180.0	9620.0	1760000.0	1184000.0
3	0.7500	0.1760	0.1180	12144.0	15340.0	1408000.0	1888000.0
4	0.7500	0.1320	0.1620	9108.0	21060.0	1056000.0	2592000.0
5	0.7500	0.0880	0.2060	6072.0	26780.0	704000.0	3296000.0
6	0.7500	0.0440	0.2500	3036.0	32500.0	352000.0	4000000.0
7	******	0.0	0.2500	0.0	32500.0	0.0	4000000.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 10730.1
ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 6000.0
N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE
TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.3750	0.2640	0.0300	6000.0	0.100	0.0	-.0005	10730.1	18216.0	0.0	3900.0
2	0.3750	0.2200	0.0740	3722.2	0.062	-.0017	-.0020	8497.8	15180.0	2232.4	9620.0
3	0.7500	0.1760	0.1180	2797.8	0.047	-.0033	-.0035	6711.8	12144.0	4018.4	15340.0
4	0.7500	0.1320	0.1620	2239.0	0.037	-.0064	-.0066	4925.9	9108.0	5804.3	21060.0
5	0.7500	0.0880	0.2060	2243.7	0.037	-.0093	-.0095	3385.5	6072.0	7344.6	26780.0
6	0.7500	0.0440	0.2500	2568.5	0.043	-.0122	-.0124	1913.1	3036.0	8817.0	32500.0
7	******	0.0	0.2500	5696.1	0.095	-.0147	-.0152	-0.0	0.0	10730.1	32500.0

ELASTIC-PLASTIC JOINT STRENGTH, PLOAD (LBS) = 16996.6
ALLOWABLE ADHESIVE SHEAR STRAIN, GAMMAX = 1.7
N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE
TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.1238	0.2640	0.0300	6000.0	0.218	0.0	-.0011	16996.6	18216.0	0.0	3900.0
2	0.2357	0.2640	0.0300	6000.0	0.100	-.0010	-.0014	15510.9	19216.0	1485.7	3900.0
3	0.0155	0.2640	0.0300	6000.0	0.100	-.0026	-.0030	13282.4	18216.0	3714.2	3900.0
4	0.0185	0.2200	0.0740	6000.0	0.110	-.0027	-.0031	13096.3	15180.0	3900.3	9620.0
5	0.3565	0.2200	0.0740	6000.0	0.100	-.0028	-.0032	12874.5	15180.0	4122.1	9620.0
6	0.0	0.2200	0.0740	4616.9	0.077	-.0077	-.0071	10040.8	15180.0	6955.8	9620.0
7	0.7500	0.1760	0.1180	4616.9	0.077	-.0077	-.0071	10040.8	12144.0	6955.8	15340.0
8	0.0	0.1760	0.1180	3483.0	0.058	-.0120	-.0114	7168.7	12144.0	9827.9	15340.0
9	0.0	0.1760	0.1180	3483.0	0.058	-.0120	-.0114	7169.7	12144.0	9827.9	15340.0
10	0.7500	0.1320	0.1620	3483.0	0.058	-.0120	-.0114	7168.7	9108.0	9827.9	21060.0
11	0.0	0.1320	0.1620	3340.8	0.056	-.0160	-.0154	4824.1	9108.0	12172.6	21060.0
12	0.0	0.1320	0.1620	3340.8	0.056	-.0160	-.0154	4824.1	9108.0	12172.6	21060.0
13	0.7500	0.0880	0.2060	3340.8	0.056	-.0160	-.0154	4824.1	6072.0	12172.6	26780.0
14	0.0	0.0880	0.2060	3679.3	0.061	-.0198	-.0193	2676.1	6072.0	14320.6	26780.0
15	0.0	0.0880	0.2060	3679.3	0.061	-.0198	-.0193	2676.1	6072.0	14320.6	26780.0
16	0.7135	0.0440	0.2500	3679.3	0.061	-.0198	-.0193	2676.1	3036.0	14320.6	32500.0
17	0.0365	0.0440	0.2500	6000.0	0.100	-.0233	-.0230	438.1	3036.0	16558.6	32500.0
18	0.0	0.0440	0.2500	6000.0	0.136	-.0233	-.0232	-0.0	3036.0	16996.6	32500.0
19	0.0	0.0	0.2500	6000.0	0.136	-.0233	-.0232	-0.0	0.0	16996.6	32500.0

INPUT DATA

ALPHAO = 0.0 (PER DEG. F) ALPHAI = 0.0000050 (PER DEG. F) DELTMP = -280.0 (DEG. F)

N	STEPL	THICKO	THICKI	STROTR	STRINR	ETOTR	ETINR
1	0.3750	0.2640	0.0300	**********	**********	2112000.0	480000.0
2	0.3750	0.2200	0.0740	**********	**********	1760000.0	1184000.0
3	0.7500	0.1760	0.1180	**********	**********	1408000.0	1888000.0
4	0.7500	0.1320	0.1620	**********	**********	1056000.0	2592000.0
5	0.7500	0.0880	0.2060	**********	**********	704000.0	3296000.0
6	0.7500	0.0440	0.2500	**********	**********	352000.0	4000000.0
7	******	0.0	0.2500	**********	**********	0.0	4000000.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 10730.1
ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 6000.0
N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE
TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.3750	0.2640	0.0300	6000.0	0.100	0.0	-.0005	10730.1	**********	0.0	**********
2	0.3750	0.2200	0.0740	3722.2	0.062	-.0017	-.0020	8497.8	**********	2232.4	**********
3	0.7500	0.1760	0.1180	2797.8	0.047	-.0033	-.0035	6711.8	**********	4018.4	**********
4	0.7500	0.1320	0.1620	2239.0	0.037	-.0064	-.0066	4925.8	**********	5804.3	**********
5	0.7500	0.0880	0.2060	2243.7	0.037	-.0093	-.0095	3385.5	**********	7344.6	**********
6	0.7500	0.0440	0.2500	2568.5	0.043	-.0122	-.0124	1913.1	**********	8817.0	**********
7	******	0.0	0.2500	5696.1	0.095	-.0147	-.0152	-0.0	**********	10730.1	**********

ELASTIC-PLASTIC JOINT STRENGTH, PLOAD (LBS) = 30568.5
ALLOWABLE ADHESIVE SHEAR STRAIN, GAMMAX = 1.7
N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE
TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.3750	0.2640	0.0300	6000.0	1.700	0.0	-.0085	30568.5	**********	0.0	**********
2	0.0	0.2640	0.0300	6000.0	1.151	-.0052	-.0099	26068.5	**********	4500.0	**********
3	0.0	0.2640	0.0300	6000.0	1.151	-.0052	-.0099	26068.5	**********	4500.0	**********
4	0.3750	0.2200	0.0740	6000.0	1.151	-.0052	-.0099	26069.5	**********	4500.0	**********
5	0.0	0.2200	0.0740	6000.0	0.669	-.0105	-.0122	21568.5	**********	9000.0	**********
6	0.0	0.2200	0.0740	6000.0	0.669	-.0105	-.0122	21568.5	**********	9000.0	**********
7	0.7500	0.1760	0.1180	6000.0	0.669	-.0105	-.0122	21568.5	**********	9000.0	**********
8	0.0	0.1760	0.1180	6000.0	0.133	-.0208	-.0177	12568.5	**********	18000.0	**********
9	0.0	0.1760	0.1180	6000.0	0.133	-.0208	-.0177	12568.5	**********	19000.0	**********
10	0.0520	0.1320	0.1620	6000.0	0.133	-.0208	-.0177	12568.5	**********	18000.0	**********
11	0.6980	0.1320	0.1620	6000.0	0.100	-.0214	-.0182	11944.1	**********	18624.4	**********
12	0.0	0.1320	0.1620	5744.6	0.096	-.0271	-.0240	7948.2	**********	22620.3	**********
13	0.7476	0.0880	0.2060	5744.6	0.096	-.0271	-.0240	7948.2	**********	22620.3	**********
14	0.0024	0.0880	0.2060	6000.0	0.100	-.0337	-.0306	4355.5	**********	26213.1	**********
15	0.0	0.0880	0.2060	6000.0	0.102	-.0337	-.0306	4326.7	**********	26241.9	**********
16	0.0018	0.0440	0.2500	6000.0	0.102	-.0337	-.0306	4326.7	**********	26241.9	**********
17	0.6194	0.0440	0.2500	6000.0	0.100	-.0337	-.0306	4305.3	**********	26263.3	**********
18	0.1288	0.0440	0.2500	6000.0	0.100	-.0389	-.0358	1545.5	**********	29023.0	**********
19	0.0	0.0	0.2500	6000.0	0.271	-.0393	-.0369	0.0	0.0	30568.5	**********