

I. INTRODUCTION

At 1 AU, observations indicate that solar wind ions have nearly the same mean flow velocities and that their elemental abundances are very approximately the same as that of the photosphere (Formisano and Moreno, 1971). The abundances are such that the consideration of heavy ions may be expected to give thermal conductivities that are almost the same as those obtained for only protons and electrons. In their review, Formisano and Moreno find that many diffusive models give mean flow velocities for the heavy ions that are less than mean flow velocities of H^+ near the sun. If these differences in flow velocities and thus the enhancement of heavy ions in the corona should occur, thermal conductivities of the electrons may be modified enough to give appreciable changes in the temperature gradient near the sun. The purpose of this study is to examine the effects of such possible enhancements on electron temperatures in the corona and interplanetary space.

Many simplifying assumptions are made. Spherical symmetry and time independence are assumed. Heat sources are assumed only at the base of the corona. Collisional heat exchange between ions and electrons is usually neglected. Magnetic field and viscosity effects are not included.

In the following section, the electron equations and parameters are discussed. Results are given and discussed in Section III.

II. EQUATIONS

The electron energy equation that is appropriate for the assumed conditions is:

$$\frac{3}{2} k \frac{dT}{dr} - \frac{kT}{n} \frac{dn}{dr} - \frac{1}{nvr^2} \frac{d}{dr} \left(r^2 kT^{5/2} \frac{dT}{dr} \right) = CL' \quad (1)$$

where T , n , and v are the electron temperature, density, and mean flow velocity. CL' is a term that gives collisional heat transfer, with ions. The same equation is obtained for distribution functions expressed in terms of the mean mass flow velocity or mean flow velocity of the electrons (Burgers, 1969). The procedures for obtaining K are, however, quite different.

K values used here are obtained from an adaptation of the derivations by Burgers and are compared with values obtained by Spitzer and Harm(1953). Consider first the derivation where equations are given in terms of the mean mass flow velocities. These equations appear to be applicable, although differences in flow velocities of various ions relative to the mean mass flow velocity may be quite large, since these differences appear to be small compared with mean thermal velocities of electrons except for $T \ll 10^4 K$.

In his treatment of heat flow, Burgers uses the momentum, heat flow, and electric current equations for a plasma with a single ion species with charge, Z , but omits the pressure gradient. For velocities relative to the mean mass flow velocity, heat transfer by ions is considered small compared to that by electrons. With no net current flow, the following K may

be obtained from his results:

$$K = \frac{15}{16} \left(\frac{2}{\pi} \right)^{1/2} \frac{k^{7/2}}{m_e^{1/2} e^4} \frac{1}{\left(\frac{2\sqrt{Z}}{5} + \frac{13Z}{10} \right) \ln \Lambda} \quad (2)$$

The inclusion of the pressure gradient appears to give the same result. For the same assumptions, this result may be extended to mixtures of ions by replacing Z with $\sum n_i Z_i^2 / \sum n_i Z_i$ where the summations are over the ions.

For equations based on the mean flow velocity of the electrons, the derivation is somewhat simpler. The assumptions of little heat transport by ions for velocities relative to the mean mass flow velocity, and no net current flow may be used with Burgers' Equations (2.17a) and (32.13) to give the same K as above.

Spitzer and Harm appear to have used methods equivalent to the ones based on the mean mass flow velocity. Their value of K (a similar K is used by Hartle and Sturrock, 1968) is about 2.4 times larger for $Z=1$. A part of the difference may be due to the smaller thermoelectric field that Spitzer and Harm obtained compared to that obtained by Chapman (1958) and Burgers. Decreases in K due to increases in Z are somewhat greater for Burgers' K than for Spitzer and Harm's.

Equation (1) may be transformed by using:

$$\frac{d \log T}{d \log r} = - \delta \quad (3)$$

$$\frac{d \log n}{d \log r} = - \epsilon \quad (4)$$

to give:

$$\frac{d\delta}{d\log r} = \delta\left(\frac{7}{2}\delta-1\right) + \left(\delta-\frac{2}{3}\varepsilon+CL\right) \frac{3k(nvr^2)}{2kRT^{5/2}} - \frac{\delta d\log K}{d\log r} \quad (1a)$$

CL gives heat transfer due to collision with ions and for T in 10^6 K, R in solar radii, and average conditions in the solar wind is approximately:

$$CL = \frac{T_p - T}{RT^{5/2}} \cdot \frac{0.9}{(v \times 10^{-7})^2} \quad (5)$$

CL is not included in the calculations below; some effects due to its inclusion are discussed.

For the solar wind electron flux in Equation (1a), a median value of 2.6×10^8 per cm^2sec at 1 AU is used (Hundhausen et al., 1970). Various values of $\varepsilon(R)$ are used and are discussed below. $\varepsilon(R)$ that is compatible with electron density measurements is used. From the continuity equation ε is:

$$\varepsilon = 2 + \frac{d\ln v}{d\ln R} \quad (6)$$

For

$$\frac{d\ln v}{d\ln R} = \frac{7.0}{R} \quad (7)$$

n values of 3.2×10^8 per cm^3 at $R=1$, 6.5 per cm^3 at 1 AU and values at other R that are near measured values are possible. T results appear to be quite insensitive to the $\frac{d\ln v}{d\ln R}$ that is assumed provided that it gives n that are near coronal and interplanetary densities.

Values of the electric field are also approximately evaluated. Leaving out the small terms in the electron momentum equation, the electric field, E , is given by:

$$\frac{d \ln(nT)}{dr} = - \frac{eE}{kT} \quad (8)$$

eE may be expressed in terms of the gravitational force:

$$eE = m^* m_p g$$

where m^* is in units of the proton mass. With T in $10^6 K$ and R in solar radii, m^* is:

$$m^* = \frac{TR}{23.2} (\delta + \epsilon) \quad (9)$$

Near the sun where flow velocities are small, hydrostatic equilibrium is approached; m^* is then approximately the average particle mass including electrons. m^* values near the sun should be 0.5 for a H^+ and e corona and should be >0.5 when other heavy ions are present.

III. RESULTS

Once $Z(R)$ is specified, simultaneous solutions to Equations (1a) and (3) are most readily obtained by specifying T and δ at some large R and integrating to $R=1$. For a given δ at the outer boundary, T at the boundary is obtained from Equation (1a) with the two derivative terms, which may be expected to be quite small, set equal to zero.

A number of solutions, obtained through a computer for $Z=1$ (two fluid model), are shown in Figure 1. The various solutions are labeled with δ at the outer boundary. The x symbols give the range of T that have been measured by Montgomery et al. (1968); the circles give the range of T that Serbu (1972) has measured.

The curve with initial $\delta=0.30$ is quite near results obtained by Hartle and Sturrock (1968). This T result and their T_p result may be expected to give near minimum solar wind velocities. Solutions with higher T than this one may be expected to give solar wind velocities with T_p values near those of Hartle and Sturrock but tend to give rather high values of T near 1 AU. Solutions with lower T probably require T_p values that are higher than those of Hartle and Sturrock in order to give solar wind velocities.

Solution above the $\delta=0.4$ one are concave upward and may be expected to approach $\delta=2/7$ at very large R . Solutions below the $\delta=0.4$ solution are concave downward for $R>10$. If the collision term is not considered, δ is expected to increase

with R for these solutions and T can become zero at some finite R . As T becomes small, however, the collision term (Equation (5)) can become appreciable; T solutions that approach T_p at large R appear to be possible. At these small T and n values, other considerations such as viscosity and deviations from spherical symmetry may be the important ones in determining T .

Solutions for other solar wind fluxes may be obtained from those of Figure 1 by moving solutions up or down. This is possible since the last term of Equation (1a) and effects due to variations in $\ln \Lambda$ are small. Solutions with increases (decreases) in fluxes by some factor are approximately given by shifting the curves up (down) by the (factor)^{2/5}. Solutions with a factor of 2 increase in flux are approximately given by moving all curves to higher T by $2^{2/5}$ or 1.32. Similarly solutions for a constant composition with K decreased because of $Z > 1$ may be obtained by a similar translation. For $Z = 1.21$, which is near the value for the average solar wind at 1 AU the solutions are approximately obtained by shifting the solutions of Figure 1 upward by $(2\sqrt{2}/5 + 1.3 \times 1.21)^{2/5} / (2\sqrt{2}/5 + 1.3 \times 1)^{2/5}$ or 1.05. This is such a small amount that it appears to be quite certain that solutions for the average composition of the solar wind at 1 AU with no enhancement should be very near those for two fluid models.

The changes in the solution owing to enhancements of heavy ions near the sun are investigated by using the following:

$$Z(R) = 1.21 + \frac{Z(1) - 1.21}{1 + (\log R / \log 4)^3} \quad (10)$$

Solutions for $\delta=0.5$ at $R=10^4$ and $Z(1)$ values of 1.21 (lowest curve), 2, 4, and 8 (highest curve) are given in Figure 2. Other solutions with different δ at the outer boundary give similar curves; increases in T at $R=1$ are by almost the same factors.

It appears that reductions in the thermal conductivities due to the enhancement of heavy ions in the corona can contribute to the reduction of calculated electron temperatures at 1 AU while permitting high temperatures near the sun. The magnitude of these effects can be much less than or much greater than is shown in Figure 2 depending on the R dependence of Z .

The amount of enhancements of heavy ions that are necessary to give $Z(1)$ values are illustrated in Table 1. The 0 in column 1 represents elements from C through Mg; S is for all elements heavier than Al. Column 2 gives assumed average charges. Column 3 gives assumed relative densities at 1 AU. Columns 4-7 give assumed enhancements at $R=1$. Below each of the last 4 columns are $Z(1)$ and hydrostatic m^{**} values for each set of assumed enhancements. These assumed enhancements are large; however, enhancements derived from mean flow

velocities of heavy ions relative to hydrogen have been found to be quite large for diffusive coronal models (Geiss et al, 1970, Nakada, 1970). The conditions assumed in their models, however, may be quite different from the real corona. In a different diffusive model, Alloucherie (1970) does not find appreciably different mean flow velocities near the sun.

The m^{**} in Table 1 is the approximate hydrostatic value and depends only on the composition at the base of the corona. It is approximately:

$$m^{**} = \frac{\sum n_i Z_i m_i}{\sum n_i Z_i^2} \quad (11)$$

where the summations are over the ions and electrons and m_i are in units of m_p . This m^{**} is compared below with the m^* (Equation (9)) that is obtained from the electron distribution.

In Figure 1, the solution with the $\delta=0.30$ label and which is near results of Hartle and Sturrock has $m^*(1)=1.02$. This could indicate the presence of a considerable amount of heavy ions for the electron distribution that was assumed. m^* is, however, very sensitive to the ϵ or the electron distribution that is assumed. By keeping the density at 1 AU the same and reducing the density at the base of the corona, it is possible to obtain $m^*(1)=0.5$ and thus consistency with the two fluid models with only minor changes in the $T(R)$ solution.

m^* values for the solutions of Figure 2 are shown in Figure 3. Only near the sun where ion velocities may be

expected to be small is there a meaningful comparison with the $m^{**}(1)$ of Table 1. Because of the sensitivity of the m^* results to the electron distribution, the rather weak conclusion that can be drawn from this comparison is that it appears possible to obtain consistency between m^{**} and m^* with considerable enhancements of heavy ions and still have an electron distribution that is near measured values.

Some trends in proton temperatures near the sun due to the presence of heavy ions may be obtained from the proton energy equation. Neglecting certain small terms Burgers' Equation (33.8) may be written:

$$\frac{5}{2} k \frac{dT_p}{dr} - \frac{1}{nvr^2} \frac{d}{dr} \left(r^2 K_p T_p^{5/2} \frac{dT_p}{dr} \right) = m_p g (-1 + m^* + W) \quad (12)$$

W is a term that gives heat transfer between various species and protons; for this discussion, the main contribution to W is considered to be due to differences in mean flow velocities of protons and heavy ions and not due to temperature differences between the ions.

For the two fluid models, W is zero and the electric field is given by $m^* = 0.5$. The main reason for the drop in T_p near the sun is work done against the gravitational field minus the work done on the protons by E . If W were to remain zero and m^* increase, a slower drop in T_p compared to the two fluid models may be expected. Consider, for example, the situation with a near average solar wind composition and no enhancement in the corona (column 4 of

Table 1.). The flow velocities of all ions would be the same so $W=0$. But with $m^* = 0.70$, the net work done per unit distance would be reduced by 40% so that T_p may be expected to drop more slowly with radial distance than it does for two fluid models. If $m^*(1)$ were as large as 1.0 with small W , T_p may be expected to remain almost constant with distance near the sun.

For enhancements that give $m^*(1)$ near 1.0, it is likely that W should be appreciable and negative. Two assumptions are made to illustrate the effects of appreciable W . The proton momentum equation is used to evaluate a term that is almost W with the assumption that n_p/n_e is constant with r . This term is substituted into the proton energy equation with the assumption that the flow velocity of H^+ is large compared with flow velocities of He^{++} and other heavy ions. This gives the result that T_p can be constant with radial distance near the sun for m^* between 1.1 and 1.4. This range is due to assumptions about the predominant heavy ion. Enhancements of heavy ions thus appear to be able to give T_p near the sun that are considerably higher than is given by the two fluid models. T_p greater than electron T for some distance near the sun appear to be possible. For sufficiently large m^* , T_p may increase with distance near the sun.

The results of this study are that enhancements of heavy ions in the corona tend to give lower electron temperatures and higher proton temperatures at 1 AU when compared with Hartle and Sturrock's two fluid models of the solar wind. Considering the many simplifications and

that the momentum and energy equations for all of the ions are not yet solved and may even be insoluble, it can only be concluded that enhancements may contribute to the understanding of solar wind temperature measurements but are not necessarily required.

TABLE 1

Average Charges, $Z(l)$, and Electric Fields Expressed in Terms of Near Average Particle Masses, $m^{**}(l)$, for Assumed Enhancements Near the Sun.

Element	Charge Z_i	Relative Densities at 1 AU	Assumed Enhancements			
H	1	1.0	1	1	1	1
He	2	.05	1	10	20	40
O	8	.002	1	10	100	400
S	11	.0002	1	10	100	1000
		$Z(l)$	1.21	2.07	4.2	6.2
		$m^{**}(l)$	0.70	1.07	1.66	1.94

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FIGURE CAPTIONS

- Figure 1. Solutions to the electron energy equation for two fluids for various values of $\delta = -d\ln T/d\ln R$ at the outer boundary.
- Figure 2. Solutions to the electron energy equation with the thermal conductivity modified by enhancement of heavy ions near the sun. Assumed values of average charge, $Z(1)$, at the base of corona are 1.21 (lowest curve), 2, 4, and 8 (highest curve).
- Figure 3. Electric fields that are obtained from the electron density distribution and the solutions of Figure 2. m^* gives the ratio of the electric field force to the magnitude of the gravitation force on protons.

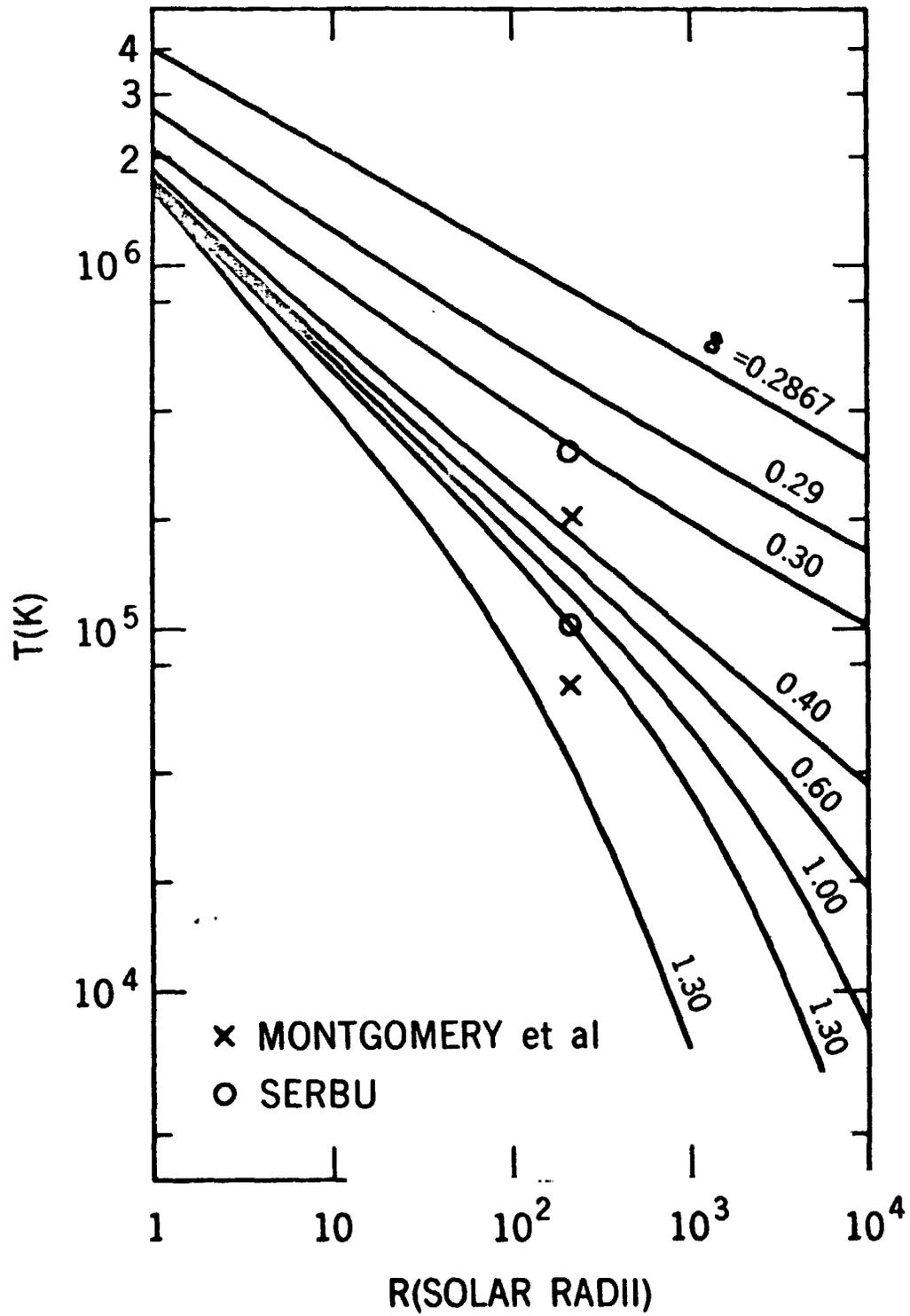


Fig. 1

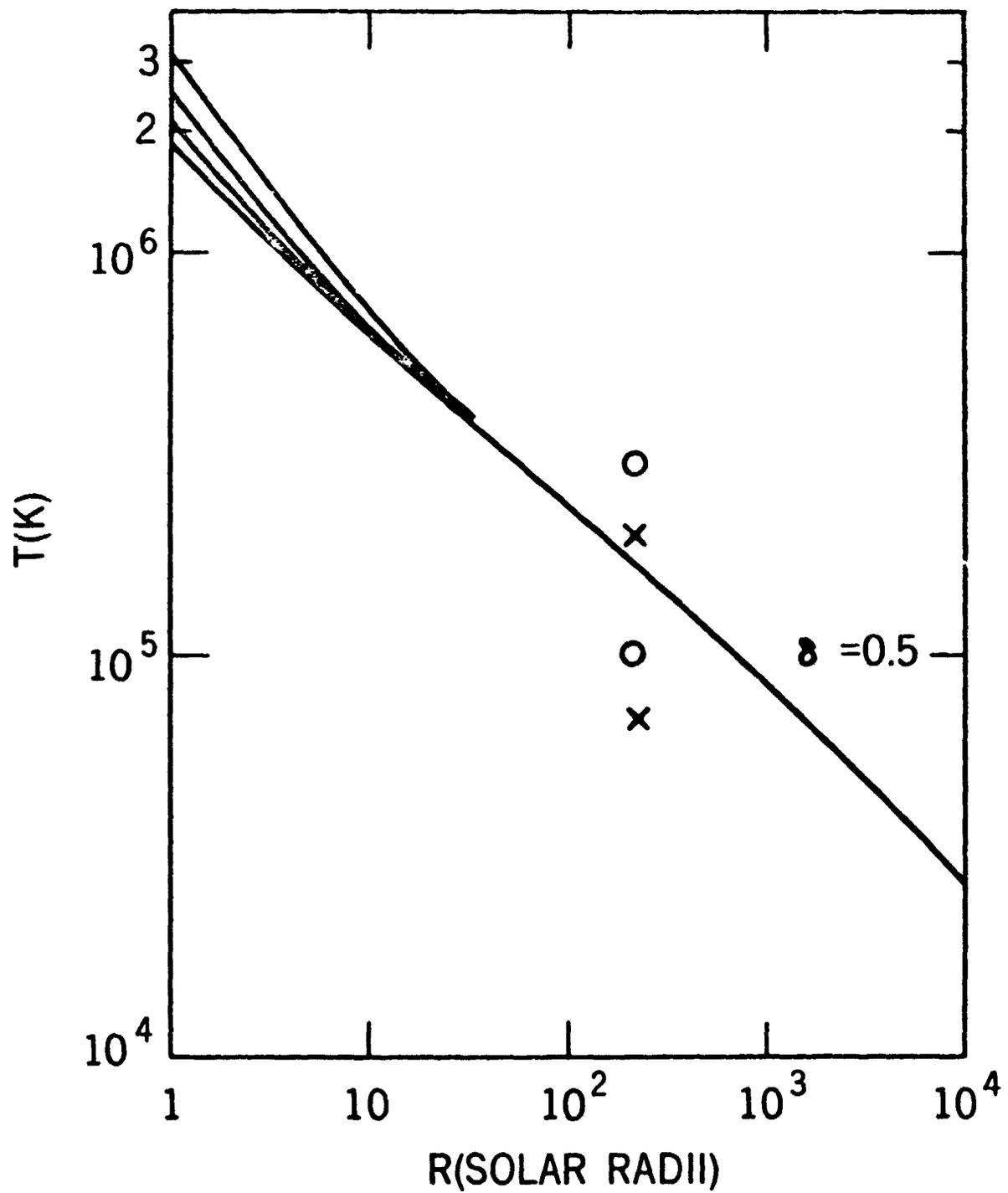


Fig. 2

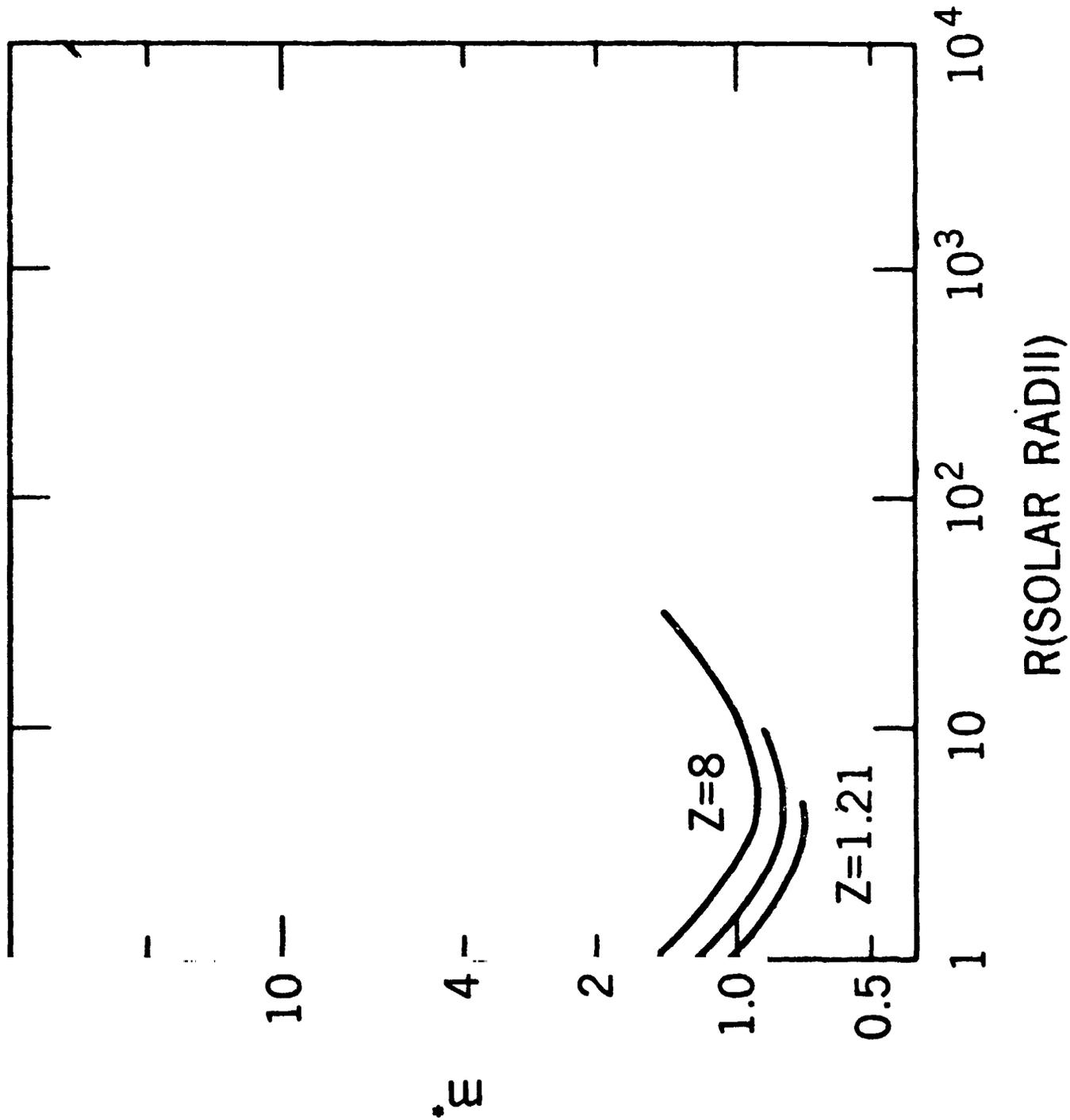


Fig. 3