ANALYSIS OF STALL FLUTTER OF A HELICOPTER ROTOR BLADE

by Peter Crimi

Prepared by
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# 16. Abstract
A study of rotor blade aeroelastic stability was carried out, using an analytic model of a two-dimensional airfoil undergoing dynamic stall and an elastomechanical representation including flapping, flapwise bending and torsional degrees of freedom. Results for a hovering rotor demonstrated that the models used are capable of reproducing both classical and stall flutter. The minimum rotor speed for the occurrence of stall flutter in hover was found to be determined from coupling between torsion and flapping. Instabilities analogous to both classical and stall flutter were found to occur in forward flight. However, the large stall-related torsional oscillations which commonly limit aircraft forward speed appear to be the response to rapid changes in aerodynamic moment which accompany stall and unstall, rather than the result of an aeroelastic instability. The severity of stall-related instabilities and response was found to depend to some extent on linear stability. Increasing linear stability lessens the susceptibility to stall flutter and reduces the magnitude of the torsional response to stall and unstall.

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SUMMARY

A study of rotor blade aeroelastic stability was carried out, using an analytic model of a two-dimensional airfoil undergoing dynamic stall and an elastomechanical representation including flapping, flapwise bending and torsional degrees of freedom. Results for a hovering rotor demonstrated that the models used are capable of reproducing both classical and stall flutter. The minimum rotor speed for the occurrence of stall flutter in hover was found to be determined from coupling between torsion and flapping. Instabilities analogous to both classical and stall flutter were found to occur in forward flight. However, the large stall-related torsional oscillations which commonly limit aircraft forward speed appear to be the response to rapid changes in aerodynamic moment which accompany stall and unstall, rather than the result of an aeroelastic instability. The severity of stall-related instabilities and response was found to depend to some extent on linear stability. Increasing linear stability lessens the susceptibility to stall flutter and reduces the magnitude of the torsional response to stall and unstall.
Aerodynamic stability of a helicopter rotor blade is a multifaceted problem because of the extreme variations of the aerodynamic environment within the flight envelope of the aircraft. In hovering flight, a blade can undergo classical binary flutter (Ref. 1) or stall flutter (Ref. 2). In forward flight, the linear instability experienced by systems with periodically varying parameters can occur (Ref. 3). While these types of instability are not normally encountered with blades of current design, due to the relatively low disc loading and weak coupling of translational and rotational degrees of freedom, they are certainly not precluded from new designs, particularly those intended to extend present performance capabilities. Of immediate concern, however, in both design and operation, is the occurrence of large-amplitude torsional oscillations and excessive control-linkage loads associated with blade stall on the retreating side of the rotor disc at high forward speed or gross weight, effectively limiting aircraft performance. This problem has prompted a number of recent studies of dynamic stall and the effects of stall on blade dynamics (Refs. 4-8).

While stall has been identified as a causal element of the problem, the nonlinearity of the stall process, coupled with the unsteady aerodynamic environment, has precluded an analysis to the depth required to gain a thorough understanding of the mechanisms involved. In particular, it has not been clear whether the blade undergoes a true aerelastic instability, a simple forced response, or some hybrid phenomenon which takes on the character of one or the other extreme, depending on flight conditions and blade vibrational characteristics.

Stall flutter for axial flight is amenable to analysis by empirical methods similar to those developed for analyzing stall flutter in cascades (Ref. 9). The flutter mechanism for that case has been identified as deriving from the extraction of energy from the free stream by the periodic variation of the aerodynamic moment. Analogous methods applied to the forward-flight problem (Refs. 10 and 11) have been inconclusive, however, the primary difficulty possibly being in applying empirical methods without a clear definition of the underlying mechanism of the problem.

A method was recently developed for analyzing dynamic stall of an airfoil undergoing arbitrary pitching and plunging motions which provides an ideal tool for analyzing the stall problem in forward flight. The method, which is described in detail in Ref. 7, employs models for each of

INTRODUCTION
the basic flow elements contributing to the unsteady stall of a two-dimensional airfoil. Calculations of the loading during transient and sinusoidal pitching motions are in good qualitative agreement with measured loads. Dynamic overshoot, or lift in excess of the maximum static value, as well as unstable moment variation, are in clear evidence in the computed results.

This study was directed to analyzing the aeroelastic stability of a helicopter rotor, particularly as it relates to stall, using the method of Ref. 7 to compute aerodynamic loading. The representation of the elastomechanical system includes flapping and flapwise bending degrees of freedom as well as torsion. A listing of the computer program used to perform the calculations is given in Appendix A.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$b$</td>
<td>blade semichord, m</td>
</tr>
<tr>
<td>$C_L$</td>
<td>mean lift coefficient, ratio of time average of 1 to $\rho \Omega^2 R^2 b$</td>
</tr>
<tr>
<td>$C_{1}$</td>
<td>lift coefficient, $C_{1} = C_{1} / (\rho U^2 b)$</td>
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<tr>
<td>$C_{m \cdot c/4}$</td>
<td>moment coefficient referred to quarterchord, $C_{m \cdot c/4} = m_{c/4} / (2 \rho U^2 b^2)$</td>
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<tr>
<td>$c$</td>
<td>blade chord, m</td>
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<td>$m$</td>
<td>blade mass per unit span, kg/m</td>
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<td>Symbol</td>
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<td>$r_R$</td>
<td>aerodynamic reference radius, m</td>
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<td>$U$</td>
<td>instantaneous free-stream speed at aerodynamic reference section, m/sec</td>
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<tr>
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<td>reference speed, $U_o = \Omega \ r_R$, m/sec</td>
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<tr>
<td>$x_m$</td>
<td>distance aft of elastic axis of blade section mass center, m</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>distance aft of pitch axis of mass center of $m_1$, m</td>
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<tr>
<td>$Z_\beta$</td>
<td>generalized coordinate of 2-D system, equivalent to $h_\beta$, semichords</td>
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<td>$Z_\phi$</td>
<td>generalized coordinate of 2-D system, equivalent to $h_\phi$, semichords</td>
</tr>
<tr>
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<td>angle of attack, deg</td>
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<td>flapping hinge offset, m</td>
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<td>$\Theta_o$</td>
<td>collective pitch angle, deg or rad</td>
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<td>$\Theta_l$</td>
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<td>$\tilde{\Theta}$</td>
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<td>$\mu$</td>
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<tr>
<td>$\rho$</td>
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<tr>
<td>$\tau$</td>
<td>dimensionless time, $\tau = U_o \ t/b$</td>
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<tr>
<td>$\psi$</td>
<td>blade azimuth angle measured from downwind direction, deg or rad</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>rotor rotational speed, rad/sec</td>
</tr>
<tr>
<td>$\Omega^*$</td>
<td>dimensionless rotor speed, $\Omega^* = \Omega \ R/(\omega \Theta_o \ b)$</td>
</tr>
<tr>
<td>$\omega_f$</td>
<td>flutter frequency, rad/sec</td>
</tr>
</tbody>
</table>
\( \omega_{\theta_0} \) frequency of first uncoupled, nonrotating torsion mode, rad/sec

\( \omega_{\phi_0} \) frequency of first uncoupled, nonrotating flapwise bending mode, rad/sec
PROBLEM FORMULATION

Aerodynamic Loading

In the flutter analysis, only leading-edge stall was considered, so the following relates specifically only to that type, even though the basic method can treat trailing-edge stall as well. When the airfoil is not stalled, the flow elements represented are (see Figure 1a): (1) the laminar boundary layer from the stagnation point to separation near the leading-edge, (2) the small leading-edge separation bubble; and, (3) a potential flow, including a vortex wake generated by the variation with time of the circulation about the airfoil. When the airfoil is stalled, as indicated in Figure 1b, the flow elements are: (1) the laminar boundary layer, (2) a dead-air region extending from the separation point to the pressure recovery point; and, (3) a potential flow external to the airfoil and dead-air region, again including a vortex wake. The analytic representations of these elements are described briefly below. Details are given in Ref. 7.

Potential Flow.—Given the airfoil section characteristics and motions, together with the distribution of pressure in the dead-air region if the airfoil is stalled, the flow and pressure over the airfoil must be determined to compute the integrated load and analyze the boundary layer. The problem was formulated by imposing linearized boundary conditions of flow tangency and pressure, using a perturbation velocity potential derived from source and vortex distributions. The resulting coupled set of singular integral equations is solved by casting the singularity distributions in series form and solving for the unknown coefficients by imposing boundary conditions at prescribed points.

Boundary Layer.—Because the relative importance of the individual elements of the boundary layer flow as they affect dynamic stall could not be established in advance, the representation in Ref. 7 was made as general as possible. The method of finite differences for unsteady flows with variable step size in both streamwise and normal directions, was employed, with the error in each finite-difference approximation the order of the square of the step size. It was determined from preliminary calculations performed for this study that, at least for leading-edge stall, results are virtually unaffected by assuming quasi-steady flow in the boundary layer. That assumption was therefore employed for all flutter computations, to take advantage
Figure 1 FLOW ELEMENTS
of the resulting substantial savings in computer storage requirements and computing time.

Dead-Air Region.—The function of the model of the dead-air region is to define the streamwise distribution of pressure in that region, given the locations of the separation and recovery points and the pressure at the recovery point. The dead-air region is assumed to consist of a laminar constant-pressure free shear layer from separation to transition, a turbulent constant-pressure mixing region, and a turbulent pressure-recovery region. The laminar shear layer is analyzed by the method of Ref. 12, assuming quasi-steady flow. The turbulent mixing and pressure-recovery regions are analyzed using the steady-flow momentum integral and first moment equations. Profile parameters in these regions are assumed to be universal functions of a dimensionless streamwise coordinate, with those functions derived from an exact viscous-inviscid interaction calculation. Matching of approximate solutions for the mixing and pressure-recovery regions at their interface completes the analysis.

Leading-Edge Bubble.—The leading-edge bubble on an unstalled airfoil is analyzed using the same basic relations employed for the dead-air region. Given the boundary-layer parameters at separation, the length of the bubble and the amount of pressure rise possible, for that length, in the pressure recovery region, are computed. That pressure rise is compared with the rise in pressure in the potential flow over the length of the bubble. If the latter is greater than the former, the bubble is assumed to have burst, and the stall process is initiated.

Loading Calculation Procedure.—Calculations proceed by forward integration in time, using the blade motions derived by integrating the equations of motion of the elastomechanical system. If, at a given instant, the airfoil is not stalled, the potential flow is computed, and the boundary layer and leading-edge bubble are analyzed to check for bubble bursting. If the airfoil is stalled, the pressure distribution in the dead-air region is computed, the potential flow evaluated, and the boundary layer is analyzed to locate the separation point. The last two steps are repeated iteratively until assumed and computed separation points agree. Rate of growth of the dead-air region is determined from an estimate of the rate of fluid entrainment derived from the potential-flow solution. Unstall is determined by first postulating its occurrence and analyzing the leading-edge bubble which would then form to ascertain whether that event did in fact occur.
During unstall, the dead-air region is washed off the airfoil at the free-stream speed.

Elastomechanical Representation

The equations of motion for a rotor blade with flapping, flapwise bending and torsional degrees of freedom can be written in the form (Ref. 3)

\[
\frac{d^2 h_\beta}{d \tau^2} + \frac{R}{b} \frac{M_{\beta \theta}}{M_{\beta \beta}} \frac{d^2 \theta_1}{d \tau^2} + \bar{\omega}_\beta^2 h_\beta - \frac{R}{b} \Omega^2 \frac{T_{\beta \theta}}{M_{\beta \beta}} \theta_1
\]

\[
= \frac{R b}{U_0^2} \frac{F_\beta}{M_{\beta \beta}}
\]

\[
\frac{d^2 h_\phi}{d \tau^2} + \frac{M_{\phi \theta}}{b M_{\phi \phi}} \frac{d^2 \theta_1}{d \tau^2} + \bar{\omega}_\phi^2 h_\phi - \Omega^2 \frac{T_{\phi \theta}}{M_{\phi \phi}} \theta_1
\]

\[
= \frac{b}{U_0^2} \frac{F_\phi}{M_{\phi \phi}}
\]

\[
\frac{d^2 \theta_1}{d \tau^2} + \frac{b}{R} \frac{M_{\beta \theta}}{M_{\theta \theta}} \frac{d^2 h_\beta}{d \tau^2} + \frac{b}{R} \frac{M_{\phi \theta}}{M_{\theta \theta}} \frac{d^2 h_\phi}{d \tau^2} + \bar{\omega}_\theta^2 \theta_1
\]

\[
- \frac{b}{R} \Omega^2 \frac{T_{\beta \theta}}{M_{\theta \theta}} h_\beta - \Omega^2 \frac{b}{R} \frac{T_{\phi \theta}}{M_{\theta \theta}} h_\phi
\]

\[
= \frac{b^2}{U_0^2} \frac{F_\theta}{M_{\theta \theta}}
\]
where $h_\beta$ and $h_\phi$ are tip displacements due to flapping and bending, respectively, in semichords, $\theta_1$ is torsional displacement at the blade tip and the frequencies* are the following functions of rotational speed:

\[
\overline{\omega}_\beta^2 = -\frac{\Omega^2}{M_{\beta\beta}} T_{\beta\beta}, \quad \overline{\omega}_\phi^2 = \overline{\omega}_\phi^2 - \frac{\Omega^2}{M_{\phi\phi}} T_{\phi\phi},
\]

\[
\overline{\omega}_\theta^2 = \overline{\omega}_{\theta_0}^2 - \frac{\Omega^2}{M_{\theta\theta}} T_{\theta\theta}
\]

The inertial and centrifugal-force coefficients are given by

\[
M_{\beta\beta} = \int_\delta^R (r + \delta)^2 m dr, \quad M_{\phi\phi} = \int_\delta^R m f_\phi^2 dr,
\]

\[
M_{\theta\theta} = \int_\delta^R I_\theta f_\theta^2 dr,
\]

\[
M_{\beta\theta} = -\int_\delta^R m x_m (r - \delta) f_\theta dr,
\]

\[
M_{\phi\theta} = -\int_\delta^R m x_m f_\phi f_\theta dr,
\]

\[
T_{\beta\beta} = -\int_\delta^R (r (r - \delta)) m dr.
\]

*Barred quantities are dimensionless frequencies, $U_o/b$ being reference frequency; e.g., $\overline{\Omega} = \Omega b/U_o$.  

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The complexity of the aerodynamic representation precludes evaluation of the generalized forces $F_{\phi}$, $F_{\theta}$, and $F_{\beta}$ by the usual strip approximation. It was felt essential, however, to retain both translational degrees of freedom in the investigation of the forward-flight problem, so a simple two-dimensional model of the dynamics could not be used. Therefore, a two-dimensional airfoil suspended in such a way as to have three degrees of freedom was analyzed. Inertial and stiffness parameters were assigned to make the coupled natural frequencies of the two-dimensional system match those of the rotor blade.

The system analyzed is shown schematically in Figure 2. The matching of the two-dimensional system with the blade dynamics proceeds as follows. Three generalized coordinates are first defined to correspond to those of the blade. Clearly, angular displacement $\theta_1$ should correspond to blade torsional displacement at the blade tip. The counterparts of flapping and bending, $Z_{\beta}$ and $Z_{\phi}$, respectively, are defined by

$$Z_{\beta} = A_1 h_1 + Bh_2, \quad Z_{\phi} = A_2 h_1 - Bh_2$$

where

$$A_1 = \frac{\omega_\beta^2 - \omega_2^2}{\omega_\phi^2 - \omega_\beta^2}, \quad A_2 = \frac{\omega_2^2 - \omega_\phi^2}{\omega_\phi^2 - \omega_\beta^2},$$

$$B = \frac{(\omega_2^2 - \omega_\phi^2)(\omega_2^2 - \omega_\beta^2)}{(\omega_\phi^2 - \omega_\beta^2)\omega_2^2} \quad (1)$$
Figure 2 TWO-DIMENSIONAL ELASTOMECHANICAL SYSTEM
and \( \bar{\omega}_1^2 = \left( \frac{k_1}{m_1} \right) \left( \frac{b}{U_0} \right)^2, \quad 1 = 1, 2. \)

With the above definitions, \( Z_\beta + Z_\theta = -h_1 \), to give the correct translational correspondence. It can further be shown that the uncoupled natural frequencies of the two-dimensional system match those of the blade, provided

\[
\left( \frac{k_\theta + k_1 l_{s1}^2 + k_2 l_{s2}^2}{I_o} \right) \left( \frac{b}{U_0} \right)^2 = \bar{\omega}_\theta^2
\]

while \( \bar{\omega}_1^2 \) and \( \bar{\omega}_2^2 \) satisfy

\[
\bar{\omega}_1^2 \bar{\omega}_2^2 = \bar{\omega}_\theta^2 \bar{\omega}_\beta^2,
\]

\[
\bar{\omega}_1^2 + (1 + m_2/m_1) \bar{\omega}_2^2 = \bar{\omega}_\theta^2 + \bar{\omega}_\beta^2 \quad (2)
\]

By comparing the generalized masses of the two systems, it follows that

\[
m_1 \frac{b^2}{I_o} = -A_1 M_{\beta\beta} \frac{b^2}{(M_{\theta\theta} R^2)}
\]

\[
A_2/A_1 = M_{\beta\beta} / (M_{\theta\theta} R^2) \equiv \lambda_m
\]

The last relation, together with Eqs. (1) and (2), fixes \( m_2/m_1 \):

\[
m_2/m_1 = \frac{(1 + \lambda_m)(\bar{\omega}_\theta^4 + \lambda_m \bar{\omega}_\beta^4)}{(\lambda_m \bar{\omega}_\beta^2 + \bar{\omega}_\theta^2)^2} - 1
\]

Equating the corresponding coefficients of the characteristic equations of the two systems provides three additional relations, which can be solved for the coupling parameters \( \bar{x}, l_{s1}, l_{s2} \). That calculation is outlined in Appendix B.
To complete the matching, quasi-steady approximations to the damping terms of the flapping equations are equated with the result that

\[ m_1 \frac{R}{(-A_1)} = 4 \frac{r_R}{R} \frac{M_{\beta\beta}}{R^2 \left[ 1 - (r_o/R)^4 \right]} \]

\[ U/U_o = 1 + \frac{4}{3} \left[ \frac{1 - (r_o/R)^2}{1 - (r_o/R)^4} \right] \mu \sin \psi \]

where \( \Omega r_R = U_o \). The aerodynamic reference radius \( r_R \) was selected to be \( .75R \).

The angle of zero restraint in torsion was varied periodically to approximate the effects of cyclic pitch variation in forward flight, according to the formula

\[ \widetilde{\theta} = \theta_o \left[ 1 - 2 \frac{R}{r_R} \mu \sin \psi \right] \]

This variation gives nominally constant lift.

The equations of motion were solved by integrating analytically, using linear extrapolations to approximate the variation of lift and aerodynamic moment over the interval of integration. This scheme was found to give satisfactory results, provided the time interval of integration is no longer than about one fifth of the period of the coupled mode having the highest natural frequency.
RESULTS OF COMPUTATIONS

Configurations Analyzed

Vibrational and aerodynamic characteristics of the blade analyzed were selected to correspond to those of the model rotor blade described in Ref. 2. That blade is untwisted, of constant chord, with offset flapping hinge. Pertinent dimensionless parameters of the model blade are listed in Table 1.

TABLE 1

<table>
<thead>
<tr>
<th>BLADE PARAMETERS FOR NOMINAL CONFIGURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>b/R</td>
</tr>
<tr>
<td>δ/R</td>
</tr>
<tr>
<td>ro/R</td>
</tr>
<tr>
<td>ωθo/ωφo</td>
</tr>
<tr>
<td>ρ R b²/Mb</td>
</tr>
<tr>
<td>x_m/b</td>
</tr>
<tr>
<td>m R/Mb</td>
</tr>
<tr>
<td>I'θ/Mb R</td>
</tr>
</tbody>
</table>

Two elastomechanical configurations in addition to the nominal one were analyzed. One of these had ωθo/ωφo = 2.5, with all other parameters as listed in Table 1. The third configuration had x_m/b = .108, with the remaining parameters as listed in Table 1.

The bending mode shape, which was computed by a finite-element method, was found not to vary appreciably over the range of rotational speeds of interest. The mode shape for ωφo/Ω = 1.26, which is plotted in Figure 3, was used for all computations. The torsional mode shape for the nonrotating blade, also shown in Figure 3, was used to evaluate torsional inertia parameters.
Figure 3  BENDING AND TORSION MODE SHAPES
The test blade had a NACA 23012 section. The variation of static lift and moment coefficients with angle of attack for this section were computed from a series of transient pitch calculations, and are shown in Figure 4, together with the measured section characteristics, from Ref. 13. The aerodynamic model is seen to give nearly the correct maximum lift, but at a slightly lower angle of attack, and, as indicated from the variation of $C_m c/4$, the computed center of pressure is somewhat further aft than that of the actual airfoil section below the stall angle.

Stability in Hover

Initial calculations were performed for hovering flight, with the nominal configuration, to allow a direct comparison with the test results of Ref. 2. First, rotor speed was varied parametrically, with the collective pitch at a value well below the stall incidence. A classical bending-torsion instability was encountered at $\Omega^* = \Omega R/(\omega_0 b) = 5.3$ with $\omega_f/\omega_0 = .803$. The variation of bending, flapping, and torsional displacements with azimuth angle at flutter onset are shown in Figure 5. By way of comparison, tests (Ref. 2) yielded classical flutter at about $\Omega^* = 7.1$ with $\omega_f/\omega_0 = .72$.

It should be noted that since the system stability was analyzed by direct simulation, a precise point of linear instability was not computed. The values of $\Omega^*$ at onset of a linear instability, both for hover and forward flight, were obtained by successively increasing or decreasing rotor speed, in small steps, until the transient response changed from convergent to divergent, or visa versa. The maximum error in the value of flutter speed, for the results presented here, is estimated to be about three percent.

Susceptibility of the system to stall flutter was investigated next. It was found that a torsional limit cycle, at approximately the highest coupled natural frequency of the system, could be triggered for $\Omega^*$ as low as 3.4. Computed blade motions for stall flutter at $\Omega^*$ of 3.5 are shown in Figure 6.

For $\Omega^*$ below 3.4, a limit cycle could not be set up, regardless of the initial conditions or the collective pitch angle. Severe oscillations involving repeated stall and unstall could be made to occur by imposing a large initial bending deflection. However, the flapping response modulated the torsional response, and caused continuous stall and/or unstall of the blade over a significant portion of
Figure 4  AIRFOIL SECTION CHARACTERISTICS FOR NACA 23012
Figure 5  DISPLACEMENT TIME HISTORIES AT CLASSICAL FLUTTER ONSET
\[ \Omega^* = 5.3, \theta_0 = 11 \text{ deg}, \mu = 0 \]
Figure 6 DISPLACEMENT TIME HISTORIES FOR STALL FLUTTER

$\Omega^* = 3.5, \theta_0 = 15.0 \text{ deg, } \mu = 0$
a revolution, due to the large plunging rate generated by the flapping motion. An example of this occurrence is shown in Figure 7. Thus, while stall flutter involves only the rotational degree of freedom, the results obtained indicate that the minimum speed for its occurrence is determined by coupling with a translational degree of freedom.

Results for the hovering case are summarized in Figure 8, which compares computed and measured flutter speed and frequency, plotted against collective pitch angle. No upper limit in collective pitch angle for the occurrence of stall flutter was calculated, since that limit would depend strongly on initial conditions, and so would be arbitrary. Quantitative differences between the computed and measured stability boundaries of Figure 8 can be attributed in large part to the use of a two-dimensional aerodynamic model, which cannot precisely reproduce the aerodynamic coupling between the rotational and translational degrees of freedom.

From the basic similarity of the computed and measured stability boundaries and the character of the computed instabilities (Figures 5 and 6) it can be concluded that the aerodynamic and dynamic models formulated are capable of reproducing both classical and stall flutter as experienced by a rotor blade, and so can be employed to investigate the forward-flight problem.

Stability in Forward Flight

The nominal configuration was analyzed next for an advance ratio of 0.1. Computations were carried out in the same sequence as for hovering. First, the rotational speed at which classical flutter occurs was determined. Then, stall-related instabilities were investigated.

A linear bending-torsion instability of the Floquet type (Ref. 14) was encountered at $\Omega^* = 5.2$. Blade motions as a function of azimuth angle at flutter onset are shown in Figure 9. The torsional and bending displacements are seen to display the aperiodic character typical of this type of instability. The flapping motion is the steady-state response to the cyclic pitch variation.

An instability analogous to stall flutter in hover was found to occur for $\Omega^*$ as low as about 4.4, with collective pitch angle greater than 12 deg. Blade motions for $\Omega^* = 4.8$ are shown in Figure 10. The torsional displacement time history, while not strictly periodic, is nonetheless
Figure 7 BLADE RESPONSE BELOW STALL FLUTTER BOUNDARY

$\Omega^* = 3.1, \theta_0 = 15.0 \text{ deg}, \mu = 0$
Figure 8 FLUTTER SPEED AND FREQUENCY VARIATION WITH COLLECTIVE PITCH ANGLE FOR A HOVERING ROTOR
Figure 9 DISPLACEMENT TIME HISTORIES AT LINEAR INSTABILITY ONSET
\( \Omega^* = 5.2, \theta_0 = 6\ \text{deg}, \mu = 0.1 \)
Figure 10 DISPLACEMENT TIME HISTORIES FOR STALL FLUTTER

\[ \Omega^* = 4.8, \theta_0 = 13 \text{ deg, } \mu = 0.1 \]
brought about by successive stall and unstall. The azimuth positions at which those events occur are marked by (S) and (U), respectively, on the ψ-scale.

The blade motions for the type of instability shown in Figure 10 are not of the same character as those of particular concern in the limiting of helicopter performance, in that the excessive torsional displacements shown in Figure 10 persist over a complete revolution of the blade. The control load time history, taken from flight test (Ref. 6), shown in Figure 11 illustrates the type of stall-related blade motions usually encountered at a thrust level or forward speed near the upper limit of an aircraft. Large oscillations in the control loads, presumably deriving from blade torsional oscillations, are seen from Figure 11 to persist only between about ψ = 270 deg and ψ = 400 deg, rather than throughout a complete revolution of the blade.

A torsional displacement time history closely resembling the variation of control loads in Figure 11 was obtained for Ω* less than 4.4, for collective pitch angles between 12 and 13 deg. Results for two typical cases are shown in Figures 12 and 13. The occurrences of stall and unstall are indicated on the abscissas. The large oscillations in torsion are clearly related to stall, but their persistence is not the result of successive stalling and unstalling, as would be the case for true stall flutter. The blade appears to be responding to the sudden changes in aerodynamic moment at stall onset and unstall, as can be seen by comparing the variation of moment coefficient shown in Figures 12 and 13 with that of torsional displacement, and noting the azimuth positions at which stall and unstall occur. There is some cyclic stall-unstall within the stall zone evident in the results, particularly at the higher rotor speed (Ω* = 4.15, Figure 13). However, the major contributors to the oscillations appear to be the initial and final pulses associated with stall and unstall upon entering and leaving that zone. There are, in general, two cycles of torsional oscillation of excessive amplitude after the blade unstalls the last time on a given revolution. The response can be regarded as transient, on a localized time scale, or forced, when viewed on a scale of several rotor revolutions. The severity of the response is apparently due in part to the suddenness of load changes at stall and unstall, and partly to the relative lack of aerodynamic damping in pitch, particularly when the blade is not stalled.

If the collective pitch angle is increased, the blade does undergo stall flutter, as seen from the time history plotted in Figure 14. These results are for the same rotor
Figure 11 VARIATION OF PITCH LINK LOAD IN FLIGHT TEST OF CH-47 AT 123 KNOTS
(from Ref. 6)
Figure 12 DISPLACEMENT AND MOMENT TIME HISTORIES FOR EXCESSIVE TORSIONAL RESPONSE

$\Omega^* = 3.89$, $\theta_0 = 12$ deg, $\mu = 0.1$
Figure 13 DISPLACEMENT AND MOMENT TIME HISTORIES FOR EXCESSIVE TORSIONAL RESPONSE

$\Omega^* = 4.15, \theta_0 = 12 \text{ deg, } \mu = 0.1$
Figure 14 DISPLACEMENT TIME HISTORIES FOR STALL FLUTTER AT LOW ROTOR SPEED

\[ \Omega^* = 3.89, \theta_0 = 14.3 \text{ deg}, \mu = 0.1 \]
speed as those of Figure 12, but with $\theta_0$ increased from 12 deg to 14.3 deg. Successive stall and unstall persists over the whole revolution of the blade for this case.

It could be argued that the blade torsional oscillations of Figures 12 and 13 are still a manifestation of stall flutter, even though successive stall and unstall is not taking place, since the aerodynamic moment can undergo unstable variations when the blade remains stalled throughout a cycle (Ref. 4). It may, in fact, be the case that the large deflections do result partly from that effect, so choosing to term them as simply a response may be somewhat misleading. On the other hand, the solutions are distinctly different from what is definitely stall flutter obtained both in hover (Figure 6) and in forward flight (Figures 10 and 14) so that label would seem to be even less appropriate. Further, the persistence of the oscillations after exit from the stall zone is clearly symptomatic of a response, so, for lack of a more precise term, solutions of the type shown in Figures 12 and 13 are identified in what follows as excessive response.

Linear Stability Boundaries

The value of $\Omega^*$ at the onset of linear instability was determined for the three configurations considered, for advance ratios of 0, .1, .2, and .3. The effects of advance ratio and torsion-bending frequency ratio on linear stability are shown in Figure 15, where $\Omega^*$ is plotted against $\mu$ for two different frequency ratios. Increasing advance ratio is seen to cause some decrease in flutter rotational speed, with most of the decrease occurring between advance ratios of .1 and .2. The substantial decrease in frequency ratio, from 3.69 to 2.5, caused only about a 4 percent reduction in flutter speed over the range of advance ratios considered. The insensitivity to frequency ratio can be attributed to the large chordwise mass imbalance, which produces the same effect in classical binary flutter of a wing (Ref. 15).

The effect of chordwise mass imbalance on linear stability is shown in Figure 16, where $\Omega^*$ at flutter onset is plotted against $\mu$ for values of $x_m$ of .216 and .108 semichords. As one would expect, the reduction in $x_m$, and hence in the coupling between bending and torsion, causes a substantial increase in the flutter rotational speed.
Figure 15 EFFECT OF ADVANCE RATIO AND TORSION-BONDING FREQUENCY RATIO ON LINEAR STABILITY - $X_nV_b = 0.216$
Figure 16 EFFECT OF $X_m$ ON LINEAR STABILITY -
$\omega_{\theta_0}/\omega_{\phi_0} = 3.69$
Stall Flutter and Response Boundaries

The effect of forward speed on stall-related instabilities for the three configurations was investigated by systematically varying the collective pitch angle and advance ratio, with $\Omega^* = 3.89$. In order to relate the results to rotor performance, a mean lift coefficient $\overline{C_L}$ is defined, according to

$$\overline{C_L} = \frac{\overline{1}}{\rho \Omega^2 R^2 b}$$

where $\overline{1}$ is the time-averaged lift per unit span at the aerodynamic reference radius. This coefficient is, to a good approximation, directly proportional to the thrust coefficient (see Ref. 16). The two-dimensional aerodynamic model does not provide a good measure of $\overline{C_L}$ when the rotor is partially stalled, so $\overline{C_L}$ was computed assuming it varies linearly with the collective pitch angle, using the formula

$$\overline{C_L} = a(\mu)(\theta_o + 0.217)$$

The slope $a$ and zero-lift collective pitch angle of $-0.0217$ rad were obtained from calculations of $\overline{C_L}$ for the nominal configuration with stall precluded. The variation of $a$ with $\mu$ is shown in Figure 17.

The results obtained for the nominal configuration are summarized in Figure 18 as a plot of $\overline{C_L}$ vs $\mu$. As thrust is increased at a given $\mu$, the rotor is seen to first encounter a region of excessive response, of the type discussed previously, and then, for $\mu$ of .2 or less, a region where stall flutter occurs. Increasing advance ratio has the effect of suppressing the tendency for stall flutter. At $\mu = .2$, stall flutter occurs at $\overline{C_L} = .85$, but a further increase in $\overline{C_L}$ results in excessive response again. At $\mu = .3$ a limit-cycle type of oscillation could not be triggered at all. As a result, stall flutter is confined to a region somewhat as indicated by the shaded area in Figure 18.

The suppression of stall flutter at high advance ratio is apparently caused by an effect similar to the one encountered at low rotor speed in hover, whereby the flapping motion prevented a limit cycle from occurring. This can be seen from the blade motions obtained for $\mu = .3$ and
Figure 17 VARIATION OF $a = d\bar{C}_L/d\theta_o$ WITH ADVANCE RATIO
Figure 18 STALL STABILITY BOUNDARIES FOR $\Omega^* = 3.89$, $\omega_{\theta_0}/\omega_{\theta_0} = 3.69$ AND $X_m/b = 0.216$
$\bar{C}_L = .78$, plotted in Figure 19. On the first revolution, as the blade enters the stall zone on the retreating side, it appears that a limit cycle is being set up, with repeated stall and un stall occurring. However, at about $\psi = 420$ deg, the flapping motion has built up in response to the large cyclic pitch changes, producing a negative plunging rate sufficient to keep the blade un stalled over the remainder of its passage on the advancing side. Then, when the blade again enters the stall zone, the large positive flap-induced plunging rate precludes unstall until exit from the stall zone at about $\psi = 670$ deg. As a result, the blade subsequently undergoes excessive torsional response, rather than stall flutter.

The effect of torsion-bending frequency ratio on stall-related instabilities can be seen from Figure 20, where $\bar{C}_L$ is plotted against $\mu$ for $\omega_\theta/\omega_\phi = 2.5$. No instance of excessive torsional response occurred with this configuration for an advance ratio of .2 or less. Instead, limit-cycle type oscillations were set up, with almost no evidence of suppression by the flapping motion, even at relatively high values of $\bar{C}_L$ with $\mu = .2$. At $\mu = .3$, however, only excessive response was obtained, similar to the results for $\omega_\theta/\omega_\phi = 3.69$.

The marked deterioration in stability at the lower frequency ratio is apparently associated with the lessened linear stability of the system. The configuration with $x_m/b = .108$, which is more stable, in the linear sense, than the nominal one, exhibited a trend opposite to the one resulting from a decrease in frequency ratio. The results for the smaller mass center offset, shown in Figure 21, are similar to those of the nominal configuration, Figure 18, but the region in which stall flutter occurs is somewhat reduced, there being no occurrence of stall flutter at an advance ratio of .2. Also, the amplitude of the torsional oscillations in the region of excessive response is considerably reduced, as evidenced by comparing the blade motions plotted in Figure 22, which are for $\mu = .1$, $\bar{C}_L = .95$ and $x_m/b = .108$, with those of the nominal configuration plotted in Figure 12.
Figure 19 DISPLACEMENT TIME HISTORIES AT HIGH ADVANCE RATIO –
\( \Omega^* = 3.89, \, \bar{C}_L = 0.78, \, \mu = 0.3 \)
Figure 20 STALL STABILITY BOUNDARIES FOR $\Omega^* = 3.89$, $\omega_{\theta_0}/\omega_{\phi_0} = 2.5$
AND $X_m/b = 0.216$
Figure 21 STALL STABILITY BOUNDARIES FOR $\Omega^* = 3.89$, $\omega_\theta / \omega_\phi = 3.69$ AND $X_m/b = 0.108$
Figure 22 DISPLACEMENT TIME HISTORIES FOR EXCESSIVE TORSIONAL RESPONSE.
$\Omega^* = 3.89$, $\bar{C}_L = 0.95$, $\mu = 0.1$, AND $X_m/b = 0.108$
CONCLUSIONS

An analysis has been performed of the aeroelastic stability of a helicopter rotor blade in hovering and forward flight. An analytical model of an airfoil undergoing unsteady stall and an elastomechanical representation including flapping, flapwise bending and torsional degrees of freedom were employed in the study. The following conclusions can be drawn from the results obtained.

1. Analysis of aeroelastic stability for a hovering rotor demonstrated that the aerodynamic and dynamic representations developed are capable of reproducing classical and stall flutter.

2. While stall flutter is an instability involving a single rotational degree of freedom, the minimum rotational speed for its occurrence, in hover, is determined from coupling with a translational degree of freedom.

3. In forward flight, the rotor can undergo a linear instability analogous to classical flutter and a stall-induced flutter which, while not manifested by a strictly periodic limit cycle, has the same basic mechanism for its occurrence as stall flutter of a hovering rotor.

4. The large stall-related torsional oscillations which limit forward speed and thrust are primarily the response to the rapid changes in aerodynamic moment which accompany stall and unstall, rather than the result of an aeroelastic instability.

5. Linear stability is relatively insensitive to advance ratio for advance ratios as large as .3.

6. While excessive response due to stall occurs at high advance ratio, stall flutter is precluded by the large flap-induced plunging rates.
7. The severity of stall-related instabilities and response depends to some extent on linear stability. Increasing linear stability lessens the susceptibility to stall flutter and reduces the magnitude of the torsional response to stall and unSTALL.
APPENDIX A

PROGRAM LISTING
APPENDIX A

PROGRAM LISTING

A listing of the FORTRAN coding of the computer program follows. The program was written in FORTRAN IV for use on an IBM 360/75 computer.
PROGRAM TO ANALYZE UNSTEADY AIRFOIL STALL

COMMON /BL1/, NTIME, NDIMC, ISTD

COMMON /CLCMHL / CLVB, CMVR, CMPAVB

COMMON /INPTVB/, TVB(64), FPVB(64), FPRVRB(64), DIDPRVB(64), SETUPS17
A X4VB(64), DELVB, XMUVB, FOVB, XMUAVB, SETUPS18
B ATOVB, ATCVB, ATSVB, ROVB, RVB(64), SETUPS19
C MV8(64), NVR

COMMON /INPUTS/, NSBL, NZ, NOFF, NGAM, NSIG, SETUPS20
A NC01, NCORD, LOWER, MSTOP, MAXT, MATR, SETUPS21
B NOTBL, INDV, ELSIG, DXI, REB, ROAB, SETUPS22
C FRZ, AR, AMPLU, FREQU, ALPH1, ALPH2, SETUPS23
D HEAVE, AROT, FREQF, PHIH, NY, RV1, SETUPS24
E DRY, Y(100), TEST, UPRIM, XU(30), YU(30), SETUPS25
F XL(30), YL(30), ER1, ER2, ER3, BD3R, SETUPS26
G RRBR, SETUPS27

H, CMPA, CMPAS, BARG, EMI, HVOR, NV0R, SSPA, SVOR, TDRF, X1VOR
I, PLOTOP, PSTLOW, PSTUP
J, NOTU

COMMON/ Z/ ZZ/(3)

DIMENSION USAV(300,100), SCALS(300)

DIMENSION USAV(1), SCALS(300)

DIMENSION CAMBR(24), THICK(24)

DIMENSION XGAM(30), XSIG(100), XSIGA(100), XSIGB(100), X(300), X(300), MAIN 7
LSBL(300), XSIG(100)

DIMENSION ACAP(30,3), BCAP(100,31), AS2(30), AS(30,30), BS(30, 30), ASHB21, MAIN 9
1(100), ASH(30, 30), SHF(30, 30), AR(30), ARH(100), UE(300, 3)
DIMENSION ALAM(30), VZIFI(30), FPRES(100), GAMMA(1000), X1W(1000)

DIMENSION BLAM(30), FLAM(10), XFLAM(10)

DIMENSION SCAL300, 2), U(1,1,1), U(100,3), V(100,2)

1, P(200,7)

DOUBLE PRECISION CMAT(60,60), RMAT(130)

DATA IN, MOUT, NF/ 5, 6, 24/
DATA PI, TIME, UINF, RENEH, USTOP/3.14159, 0.0, 1.0, 9.47594, 2.8/
DATA FLAM /1.75, 1.75, 1.724, 1.527, 1.354, 1.0, 663, 1.452, 25/
DATA XFLAM /-100.0, -11.26, -7.01, -3.48, -1.766, 0.0, 1.888, 4.0,
DATA DEGREES /1.74 53292 51994 3300-2/

EQUIVALENT (CMAT(1), USAV(1), ASH(1), SCALS(1))

IF ISTO = -1 TIME DERIVATIVES NOT USED
ISTD = 1
RAD = 180. / PI
IL = 8888
NDMC = 60
CALL SETUPS
IF (1STD .EQ. 1) GO TO 40
DO 100 J = 1, 1300
SCAL = 0.
DO 100 I = 1, 1100
100 USAV (I, J) = 0
CONTINUE
C
CALL READIN (IL, IL)
C
NOTE OFFSETS ARE PUT IN AS LISTED IN THEORY OF WING SECTIONS, I.E. MAIN 59
C
AS A FRACTION OF TOTAL CHORD, XI BEING MEASURED FROM THE LEADING EDGE. MAKE SURE NF IS AN EVEN NUMBER.
C
TIME = 0.
NTIME = 0.
NMAKE = 999
ISEP = 0
ISEPY = 0
IWASH = 2
UNF = 1.
C
IND = IND+1
WRITE(MOUT, 6)
PITCH = ALPH1
IF (IND + MODR .LE. 2) PITCH = PITCH - ALPH2
IF (IND .EQ. 2)
X = AMPLU = 1.33333*XMUAVB*(1.-ROVB**3)/(1.-ROVB**4)
IF (IND .EQ. 2) FREQU = BDBR/RDBDR
IF (IND .GE. 2) GO TO 343
WRITE(MOUT, 23)
RY = RV1
HVO = HVR**2
BARG = BARG/6.2832
CALL SECT(XU, YU, XL, YL, NOFF, NF, RDBB, TMDDB, CHDDB, THICK, CAMBR)
DO 7875 N = 1, NF
CAMB(N) = CAMBR(N) + CHDDB
7875 THICK(N) = THICK(N) + TMDDB
WRITE(MOUT, 4)
WRITE(MOUT, 7) AMPLU, FREQU, ALPH1, ALPH2, HEAVE, AROT, FREQF, RDBB, REB
WRITE(MOUT, 8)
WRITE(MOUT, 9) (N, CAMBR(N), THICK(N), NF)
MX = NSVLNZ = 1
CALL SCAL(SBL, NSBL, FRZ, ARR, RDBB)
CALL CORDX(NSBL, NZ, RDBB, SBL, X, XC)
DO 2420 N = 1, MX
IF (XCM) - 1.7 2420, 2419, 2419
2419 MEND = M - 1
GO TO 2421
2420 CONTINUE
2421 MX = MEND
MXM1 = MX - 1
LE(MX+1,1) = 1.
EPSLE = 2.*(XNZ) - X(NZ-1))
PSSTF = XM1 - XM2
ALTG = 8.364/ SQRT(REB)
IF ( ISTD.EQ. 1 ) GO TO 50
DO 2422 M = 1, MX
SCALE(M,1) = 0.
SCALE(M,2) = 0.
DO 2422 N = 1, NY
U(M,N,1) = 0.
2422 U(M,N,2) = 0.
50 CONTINUE
NSIGA = NSIG
NSIG = NSIG
NSIG1 = NSIG + 1
MOTR = NOTR + 1
NOTBL = NOTBL + 1
XMAX = 1. - ELSIG
CCNA = 3.75*PI/DXI
ANGS = PI/ FLOAT(NSIG)
CALL SETSX(NSIG1,1,1,2,*,XSIG, ANGS)
XSEP = 1.1
DO 2430 N = 1, NSIG1
XSIGB(N) = XSIG(N)
2430 XSIGA(N) = XSIG(N)
DO 2431 N = 1, NSIG
DO 2431 NU = 1, 3
2431 BCAPIN, NUM = 0.
PINT = 2. / FLOAT(NCORD)
NC1 = NCORD + 1
THXI = 1.5/ DXI
NG1 = NGAM + 1
NW1 = NWAKE - 1
COUNT = 0.
DO 8456 N = 1, NWAKE
GAMM(N) = 0.
XIW(N) = 1. + COUNT
8456 COUNT = COUNT + DXI
ANGLE = PI/ FLOAT(NGAM)
COUNT = 0.
DO 1002 M = 1, NGP1
PHIM = COUNT + ANGLE
XGAM(M) = COS( PHIM)
COUNT2 = 2.
DO 1001 N = 2, NGAM
( M,N) = COS(DO)T*PHIM)
1001 COUNT = COUNT + 1.
1002 COUNT = COUNT + 1.
CALL WASH1(XGAM,NGAM,TIME,ALPH1, ALPH2, HEAVE, ARCT, FREQ, PHIM, UINF, CAMAIN)
IMBR, NP, VIP(I,1,1)
DO 8458 N = 1, NGP1
LMAT(N,1) = 1.
TEMP = 2.* VIP(M)
RMAT(M) = TEMP
CMAT(M,2)=XGAM(M)
DO 8457 N=3,Ngp1
8457 CMAT(M,N)=AS(M,N-1)
8458 CONTINUE
CALL ALSOL(NGPL,CMAT,RMAT)
DO 8459 N=1,Ngp1
ACAP(N,1)=RMAT(N)
ACAP(N,3)=RMAT(N)
8459 ACAP(N,2)=ACAP(N,1)
DO 2784 M=1,MAX
SIGN=1.
IF(M-N)/2 2774,2775,2776
2774 SIGN=-SIGN
2775 CALL QCALL (ISEP,NGAM,NSIG,NF,EXP,ACAP,BCAP,THICK,RBB,GAMAN1,UIMAIN)
1NF,NCX(N),UF(N),SIGNAL)
2784 UFM=M=M-EWN
1008 BLAM(M)=(1.125*XGAM(M)+1.875*(1.+XGAM(M)))/(1.-3.*XGAM(M))
1+XGAM(M)))/(1.-XGAM(M))/DXI
BLAM(NGPL)=-1.125/DRX
CALL CLCMT(NC,TSFP,NGAM,NSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,ACAP,RCMAIN)
1AP,THICK,RRBB,GAMAN,UINF,UDOT,DXI,AROT,CMP4)
IF (INDV .EQ. 2)
CALL SUPPL
C INDEXING IN TIME IS CARRIED OUT AT THIS POINT.
C
9559 CONTINUE
CALL ACUCPU( IACU )
IF (IACU .LT. 35000 ) GO TO 99
C
C NOTE - FOR READ-IN OF FCIL MOTIONS, MAKE ALPHI = ALPHA,
C ALPH2 = ALPHA-DOT, AND HEAVE = H-DOT.
C
C
IF (NCTR .EQ. 2)
XREAD(IN,2,END=9898) ALPH1,ALPH2,HEAVE
158 NITM=1
TIME=TIME+DXI
NTIME=NTIME+1
NWAKE=NTIME+2
IF (NWAKE=-998) 202,201,201
201 NWAKE=-998
202 IF (MAXNTIME) 9898,8800,8800
8800 SAVF=UINF
L=1+1
PL(l,1) = BCBR / RRBB * TIME * RAD
PSI360= AMODT (PL(l,1), 360.1
UINF=1.+AMPLU*SIN(FREQF*TIME)
IF (INDV .EQ. 2)
XCALL SUPPL(UINF)
PITCH = ALPH1
IF (INDV .EQ. 2) PITCH = PITCH - ALPH2*COS(FREQF*TIME)
UDOT=FREQF*AMPLU*COS(FREQF*TIME)
STEPX=50*DXI*(UINF+SAVEU)
DO 1003 J=2,NWAKE

MAIN 151
MAIN 152
MAIN 153
MAIN 154
MAIN 155
MAIN 156
MAIN 157
MAIN 158
MAIN 159
MAIN 160
MAIN 161
MAIN 162
MAIN 163
MAIN 164
MAIN 165
MAIN 166
MAIN 167
MAIN 168
MAIN 504
MAIN 505
 MAIN 169
MAIN 170
MAIN 171
MAIN 172
MAIN 173
MAIN 174
MAIN 175
MAIN 176
MAIN 177
MAIN 178
MAIN 179
MAIN 180
MAIN 181
MAIN 182
MAIN 183
MAIN 184
MAIN 185
MAIN 186
MAIN 187
MAIN 188
MAIN 189
MAIN 190
MAIN 191
MAIN 192
MAIN 193
51
JC = NWAKE - J + 2  
GAMAW(JC) = GAMAW(JC-1)  
1003 XIW(JC) = XIW(JC-1) + STFPX  
IF (ISEP) 2009, 2009, 2007  
2007 DC 2008 N=1, NSIG  
RCAP(N, 3) = RCAP(N, 2)  
2008 RCAP(N, 2) = RCAP(N, 1)  
DO 4433 N=1, NSIG  
XSIGR(N) = XSIG(N)  
4433 XSIGA(N) = XSIG(N)  
GO TO 2010  
2009 DEADL = 0.  
E LDN = UNF  
2010 DO 1014 M=1, MX  
UE(M, 3) = UE(M, 2)  
1014 UE(M, 2) = UE(M, 1)  
DEADL = DEADL  
E LDN = E LDN  
1105 ALAM(1) = (1.125+ 0.75*ALO((STEPX+6))/DXI  
DO 1005 M=2, NGP1  
2006 ACAP(M, 3) = ACAP(M, 2)  
2006 ACAP(M, 2) = ACAP(M, 1)  
ALPHS = VZIP(1)  
CALL WASH(XGAM, NGAP, TIME, ALPH1, ALPH2, HEAVE, ART, FREQF, PHIH, UNINF, CAMAX)  
IMBR, NF, VZIP, MTR, INDV)  
DO 1006 M=1, NGP1  
ASZ(M) = 1 + 2 * ALAM(M)  
AS(M, 1) = XGAM(M) + ALAM(M)  
SUM=0.  
DO 4343 J=2, NWMK  
4343 SUM=SUM + (GAMAW(J+1) - GAMAW(J)) * (XGAM(M) - XIW(J)) / XIW(J+1)  
1-XIW(J+1)) * ALOG(XIW(J+1) - XGAM(M)) / XIW(J) - XGAM(M))  
1-XIW(J+1)) * ALOG(XIW(J+1) - XGAM(M)) / XIW(J) - XGAM(M))  
ELX=1., -XGAM(M)  
IF (M=1) 1006, 2130, 1006  
2130 ELX=1.  
1006 AR(M) = 2 * VZIP(M) + ALAM(M) * AFACT/3. + (SUM-GAMAW) * (1 - XGAM(M)) * LOG(M)  
3335 I=1, STEPX-XGAM(M)/ELX/STEPX/PI  
C THE FOLLOWING CALCULATIONS, THROUGH STATEMENT 4444, ARE PERFORMED  
C ONLY IF THE AIRFOIL IS STALLED. THE AIRFOIL IS DESIGNATED TO BE  
C STALLED IF INTEGER ISEP IS NONZERO.  
C  
C IF (ISEP) 3247, 4444, 3247  
3247 GO TO (3344, 3345), I WASH  
3344 KSXP=KSEP+DXI  
4S (KSXP-XMAX) 3248, 3347, 3347  
3347 I=WASH=2  
ISEP=0  
XSEP=1, 1  
DO 3015 K=1, 3  
DO 3015 N=1, NSIG
3015 RCAP(N,K)=0.
GO TO 4444

3345 IF(INOT) 3349,3348,3248
3349 IF(NITS=1) 3248,3349,3248

3349 IF(INOT.EQ.2) GO TO 6349
IF(VIP(1)-ALPHS) 6 349,6348,6348

6348 NITS=2
GO TO 3248

6349 CALL UPPINGAM,AR,ALAM,AFACT,RMAT,HAT,CMAT,GAM,AS,ACAP,MX,NZ,IF,XSIG

1,RCAP,THICK,ROBB,UNIF,XC,UE)
GO TO 2785

3248 XATT=XSEP+DEADL+.5*(ELD1+ELDOT)*NI
DFAIL=XATT-XSEP
DIFF=1.-XATT

XTEST = XSEP + 3.* EPSL
CALL SETS(NSIG,XSEP,XATT,XSIG,ANGS)
DO 4434 N=1,NSIG

4434 XBSIG(N)=.5*(XSIG(N)+XSIG(N+1))
DO 3086 M=1,NGPI
DO 3086 N=1,NSIG

3C66 RMS(N)=0.
DO 3087 M=1,NGPI
IF(XGAM(M)-XSEP) 3088,3088,3089

3C89 IF(XATT-XGAM(M)) 3107,3087,3091

3C91 IF(XGAM(M)-XSIG(1)) 3093,3092,3092

3093 MARK=1
GO TO 3094

3C92 CONTINUE

3C54 WIDE S=XSIG(MARK)-XSIG(MARK-1)
RS[M,MARK-1]=(XSGT(MARK)-XGAM(M))/WIDES
BS[M,MARK]=(XGAM(M)-XSIG(MARK-1))/WIDES
RS[M,1]=SQR((XGAM(M)-XSEP)/(XATT-XGAM(M)))

3088 IF(DIFF-1.E-6) 3087,3098,3098

3C58 BS[M,1]=RS[M,1]+DIFF.5*SQR(DEADL)*(2.*DIFF*SQR((1.-XGAM(M))/XATT-XGAM(M))
GO TO 3087

3167 BS[M,1]+DIFF.5*SQR(DEADL)*3.*XATT-4.*XGAM(M))
3087 CONTINUE

C
C SET-UP OF THE SECOND SET OF EQUATIONS STARTS HERE

C
DO 4350 K=1,NSIG
IF(XRSIG(K)-1.) 4348,4349,4349

4348 COSK=XRSIG(K)
SINK=SQR((1.-COSK*COSK)
THETK=ARCTCOSK)
TANT=SIGN(K)*THETK/COS(.5*THETK)
ASHZ(K)=TANT+CONA*(1.+COSK)*K*.3*COSK*/UNIF*THX*P*THETK+SIN
ICONA*1.+COSK*SINK*0.)/UNIF
ASHK,K=SIGN(K)*TANT+SIGNK
COUNT=1.
DO 4355 N=2,NGAM
COUNT=COUNT+1.

4355 ASHK,NI=SIGN(COUNT+THETK)+.75*SIGN(COUNT+1.)*THETK/(COUNT+1.)-SIN MAIN 304
195 \times (COUNT-1) \times \text{THETK} / (COUNT-1) / (DxI \times UINF) 

GO TO 4350

4349 \text{ASHZ(K)} = 0.

DO 4359 \text{N} = 1, \text{NGAM}

4355 \text{ASHZ(N)} = 0.

4350 \text{CONTINUE}

IF (DIFF 

50C5 \text{PREC} = 0.

GO TO 50C7

50C6 \text{CALL ATTTR (PREC, XSIG, NSIG, ASZ, AS, AR, CMAT, RMAT, NGAM, NF, ACAP, THICK, RMAT, IOM, INOT, DEL1, THEM, NXBS, LNSIG, NSIG, IND, IOT, DOR, CORD, VAL1, CORD, VAL2, N0, N, UINF, MOD2, IOM, RMAT, IOM, INOT, DEL1, THEM, NXBS, LNSIG, NSIG, IND, IOT, DOR, CORD, VAL1, CORD, VAL2))

50C7 \text{CALL MIXERF (PREC, UINF, JDOT, THICK, NF, XBSIG, NSIG, INOT, DEL1, THEM, NXBS, LNSIG, NSIG, IND, IOT, DOR, CORD, VAL1, CORD, VAL2, N0, N, UINF, MOD2, IOM, RMAT, IOM, INOT, DEL1, THEM, NXBS, LNSIG, NSIG, IND, IOT, DOR, CORD, VAL1, CORD, VAL2))

C \text{CPC} = \text{CPC} + 1

DO 4800 \text{K} = 1, \text{NSIG}

CORD = XBSIG(K)

4808 \text{BSH(K, I) = 1 \times \text{THXI} \times \text{INT}(XSEP, XATT, CORD) / UINF}

DO 4808 \text{N} = 2, \text{NSIG}

C \text{BSH(K, N) = F } \times \text{(XSIG(N-1), XSIG(N), XSIG(N+1), CORD) \times \text{THXI} \times \text{GB}(XSIG(N-1), XSIG(N), XSIG(N+1), CORD) / UINF}

CALL ESIGI (2, NSIG, XSIG, BCAP, CORD, VAL1)

CALL ESIGI (3, NSIG, XSIG, BCAP, CORD, VAL2)

ARH(K) = FPREST(K) \times (2 * VAL1 - 5 * VAL2) / (DxI \times UINF)

IF (CORD < 1) 5009, 4800, 4800

50C8 \text{CALL EGAMI (2, NGAM, ACAP, BCAP(1, 2), XSIGA(1), XSIGA(NSIGA+1), GAMAW(2), MAIN, CORD, VAL1, CORD, VAL2)

C \text{CALL EGAMI (3, NGAM, ACAP, BCAP(1, 3), XSIGB(1), XSIGB(NSIGB+1), GAMAW(3), MAIN, CORD, VAL2)

ARH(K) = ARH(K) + (2 * VAL1 - 5 * VAL2) / (DxI \times UINF) \times 0.625 \times FACT \times PI \times (1 + CORD) \times (CORD + \text{THXI} \times (1 - CORD) / (DxI \times UINF))

4484 \text{CONTINUE}

C \text{CALCULATIONS FROM THIS POINT ON COMBINE THE CASES OF STALLED AND UNSTALLED AIRFOILS.}

C \text{CONTINUE}

DO 6500 \text{N} = 1, \text{NGP1}

RMA = ARH(M)

CMAT(M, I) = ASZ(M)

DO 6485 \text{N} = 1, \text{NGAM}

6485 \text{CMAT(M, N+1) = ASZ(M, N)

IF (ISEP) 6486, 6500, 6486

6486 \text{DO 6499 \text{N} = 1, \text{NSIG}

NGG = N + NP1

6459 \text{CMAT(M, N+1) = BSN(M, N)

6500 \text{CONTINUE}

TF (ISEP) 6502, 6501, 6502

6501 \text{NTOT = NGP1}

GO TO 6751

652 \text{DO 6750 \text{K} = 1, \text{NSIG}

KK = N + NP1

RMA = ARH(K)

CMAT(KK, I) = ASHZ(K)

DO 6748 \text{N} = 1, \text{NGAM}

6748 \text{CMAT(KK, N+1) = ASH(K, N)
DO 6750 N=1,NSIG
   NGG=N+NGP1
6750 CMAT(KK,NGG)=RSH(K,N)
   NTO=NSIG+NGP1
6751 CALL ALSOL(NTOT,CMAT,RMAT)
   DO 6800 N=1,NGP1
6800 AACAP(N,1)=RMAX(N)
   IF (ISFPI) 6805,6920,6805
6805 CALL QECAL(ISEP,NGAM,NSIG,NF,XSIG,ACAP,BCAP,THICK,RDRR,GAMA4(1),UTIV)
   INF,XC(M),UE(M,1),SIGN)
2145 DO 8886 IS=1,N
   US2=UE(M,1)
   DO 8886 M=1,MAX1
   US1=UE(M,1)
   UET(M,1)=(US1+US2+UE(M+1,1))/3.
8866 US2=US1
   GO TO (8351,8353,1735)
8351 DO 8352 M=1,4X
   GO TO 1786
8352 SCALSM=M=0.
   GO TO 1786
8353 CALL YSET(RY1,Y2,N,Y)
   RY=RY1
   GO 8354 M=1,MAX1
8354 SCALSM=M=0.
   IF(INDV=EQ.2) GO TO 8370
   IF(ISEP.EQ.0.AND.VZIP(1),ALPHS) GO TO 1786
8370 CALL QECALMX,N,STAGMX,NST,DXR,DRY,X,Y,UE,U,USAV,SCALSM,ISEP)
   XMEM=1
   XSEP=XSEP
   DXX=DXI
   IF(ISEP.EQ.1.AND.ISEP.EQ.0.AND.NITS.EQ.1) DXX=1.E+30
   8367 CALL BLX(X,Y,MST,CHNO,NY,DRY,Y,DXI,REB,UPRIM,FLAM,XFLAM,TESTU,SC)
   IALE,UE,U,V,XSEP,USEP,DSEP,THETA,LOWER,LAMQ,HEEP,LEX,USAV,SCALSM,NIT
   15,NITMAX,NOTBL,XTST,NZ,NOUT)
   IF(XSEP=MAX1) 7736,7735,7735
7735 IF(ISEP) 1786,1786,7736
7736 DELL=DSEP
   THETA=THETA
   INDT=O-LAND
   IF(INDT=EQ.1.AND.NTOTAL.EQ.2) GO TO 1786
   WRITE(MOUT,23) XSIG(1),CPLT,XSEP
   IF(INDU) 8462,8462,8463
8462 IF(ISEP) 8562,8562,8562
8563 IF(NITS=1) 8562,8562,8562
8562 IF(ISEP) 7742,7742,8562
55
CALL RUBD(DEL,THET,RF8,XSEP,USEP,XC,DEL5,X,XC,NC,XS,UMAIN 416
IF(ALC,RF8,USEP+0.02046#USEP**3
USEP=USEP-USEP+0.02046#USEP+0.05
WRITE(#DUT,22)PDIF,DCP
IF(DCP-PDIFF) 8263,8366,8366
8263 ISEP=0
GO TO 8463
8366 IF(ISEP) 8368,8369,8369
8369 IF(ISEP) 8467,8467,8368
8467 IWASH=1
NITS=2
GO TO 3344
8368 GO TO (8168,1786),NOTBL
8168 CALL RFATTUC,V,X,Y,M,N,R,PRE,USEP+DEL5,MST,REB)
LAM=0
GO TO 8367
8463 IF(ISEP) 7741,7741,7742
7741 ISEP=1
NITS=NITS+1
IF(NITS=7743,7743,7643
7643 ISEP=1
DXSEP=1.-XSEP
XSEP=6*XSEP+.4
CALL CPC(ISEP,NGAM,NF,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,MAIN 440
LTHICK,READ,HAMAM,UNINF,UDOT,1.,XSEP,DX1,CPL)
GO TO 3248
7742 CALL FLDER(BCAP,XSIG,NSIG,UNINF,ELDCT,SIGSUM,YMX1)
IF(ISEP.EQ.1. AND. ISEP.EQ.0. AND. NITS.EQ.1) GO TO 9210
IF(ISEP+.5) 7841,7842,7842
7841 EPS=EPSLE
GO TO 7843
7842 EPS=EPSLE
GO TO 7843
7843 EPS=ABS(XSEP-XSEPS)
IF(DXSEP-EPS) 7836,7834,9210
7834 IF(DXSEP-XMAX) 1786,1786,7835
7835 ISEP=0
ISEP=0
GO 7836 K=1,3
GO 7836 N=1,NSIG
7836 BCAP(N,K)=0.
GO TO 1786
9210 NITS=NITS+1
IF(NITS.EQ.2. AND. NITS.EQ.0) XSEPS=XSEP
IF(NITS=4) 9211,9211,1786
9211 IF(ISEP-XSEPS) 9335,9305,9306
9305 XSEP=6*XSEP+.4*XSEP
GO TO 9307
9307 XSEP=6*XSEP+.4*XSEP
9307 IF(ISEP-XMAX) 9212,9212,7835
9212 CALL CPC(ISEP,NGAM,NF,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,MAIN 466
LTHICK,READ,HAMAM,UNINF,UDOT,1.,XSEP,DX1,CPL)
TF(NOTBL,NEQ.2. AND. XSEP.GT.0.1 XSEP=-.98
GO TO 3248
7743 TF(NITS-1) 7737,7737,3248
7737
SCALE(M,1)=SCALS(M)
DC 7950 N=1,NY
U(M,N,2)=U(M,N,1)
7550 U(M,N,1)=USA(V(M,N))
GO TO 9999
8589 CONTINUE

CONTINUE
CALL PLOTSB(PLOTOP,P,L)
CALL ACUCPU(IACU)
IF(IACU.LT.35000)GO TO 60
GO TO 40
60 CONTINUE
IF(PLTOP.EQ.0.)CALL EXIT
CALL PLTND
CALL EXIT
RETURN

C C C
1 FORMAT(1315)
2 FORMAT(3F10.4)
3 FORMAT(2F10.4)
4 FORMAT(H1//)
5 FORMAT(6F10.4)
6 FORMAT(I1,H1,50X,34HANALYSIS OF UNSTEADY AIRFOIL STALL///

7 FORMAT(5X,6HUBAR=1E13.5/7X,7HUFREQ=1E13.5/3X,11HALPHA ONE =1E13.5/MAY

13X,11HALPHA TWO =1E13.5/8X,6HUBAR =1E13.5/11X,3HA =1E13.5/8X,6HUFREQ =MAIN

28 LE13.5/7X,6HRO/9 =1E13.5//9X,5HREB =1E13.5///)
8 FORMAT(29X,1HN,25X,4HCN(1),26X,4HT(1)/)
9 FORMAT(30,2E30.5)
10 FORMAT(5X,3HT=1E13.5/5X,3HU=1E13.5/4X,4HXS=1E13.5/4X,4HXO=1E13.5/4MAIN

1X,4HPA=1E13.5//7//)
11 FORMAT(///4X,1HN,7I1X,1HX,14X,5HVZ(X),12X,5HRN(X),12X,4HAIN(X),21X,3HMAIN

36 IXIXW,14X,5HGMAMA)
12 FORMAT(15,4E17.5,8X,2E17.5)
13 FORMAT(I1H1,8X,1HN,2OX,1HX,21X,5HFP(X),22X,5HRH(N),21X,4HBN(1))
14 FORMAT(///5X,9H,L-DCT =1E13.5///5X,27HPRESURES IN SEPARATED FLOWMAIN

40 1//55X,1HX,19X,2HCP/)
15 FORMAT(///H1,11X,1HX,16X,3HQEL,15X,3HCPL,15X,3HEQEU,15X,3HCPU,13X,9HCMAIN

42 1PL = CPU/)
16 FORMAT(6E18.5)
17 FORMAT(10,4E25.5)
18 FORMAT(40X,2E20.5))
19 FORMAT(I1H1,5OX,12HTIME STEP NO3/)
20 FORMAT(I1H1,50X,12HTIME STEP NO3/)
21 FORMAT(///4OX,26HINCRED IN CP REQUIRED IS1E13.5///4OX,26HINCREASE MAIN

49 N1N CP POSSIBLE IS1E13.5)
22 FORMAT(///46X,26HPOTENTIAL FLOW XS=1E12.4//60X,8HCP(XS)=E12.4

51 1/45X,23BOUNDARY LAYER XS=E12.4)
23 FORMAT(///45X,23BPOTENTIAL FLOW XS=1E12.4/60X,8HCP(XS)=E12.4

51 1/45X,23BOUNDARY LAYER XS=E12.4)
24 FORMAT(15,5F10.4)
25 FORMAT(12X,4HVW=12.3X,3HS=1E12.4,3X,3HH=1E12.4,3X,3HG=1E12.4,3X

54 INX=1E12.4//7X,4HMI=1E12.4,3X,4HM=1E12.4,3X,4HPI=1E12.4//7/)
26 FORMAT(4X,4XW=1E13.5)
27 FORMAT(401 FORMAT”0”, 750, “EQUIVALENT ROTOR BLADE RESPONSE”)
9CC1A // T 5, 'FLAP DISP =', G14.5
9CC1P , T47, 'BENDING DISP =', G14.5
9CC1C , T39, 'TORSIONAL DISP =', G14.5
9CC1D / T38, 'SECTION PITCH ANGLE =', F9.3, ' DEGREES OR ',
9CC1E / F9.4, ' RADIANS ' 
9CC1F / T21, 'SECTION PITCH RATE =', G14.5
9CC1G , T71, 'SECTION PLUNGING RATE =', G14.5 //)
END
SUBROUTINE SUPPL
IMPLICIT REAL*8 (A-H,P-F,I-2)
REAL*8 FR1S, FR2S, FR3S, ANST, OMS

REAL*8 CLVB, CMVB, CMPAVB
1. DUMMY, PLOTOP
REAL FTVB, FPVB, FPPRVB, DIDRVB, XMVB, DELVB, XMUVB,
A FOVB, XMUVAR, ATOVB, ATCVB, ATSVB, ROVA, RVB, MV3,
C WOXI, PSTI, UINF
REAL ELSIG, DXI, REBI, RDRB, FRZ, ARR, AMPLU, FREQU,
A ALPHI, ALPH2, HEAVE, ARCT, FREQM, PHIH, RY1, DRY,
B X, TEST, UPRIM, XU, YU, XL, YL, ER1, ER2, ER3, RDBR,
C RDBR
REAL SUM(8), YCLUD(8), YNEW(8), DEL(3,3), CMPA(3), CL(3), S(3)
A 2, ZPR(3), SMALLG(3), Y(3,3), YPR(3,3), GCAP(3,3)
COMMON /BL1/ NTIME, NDIMC
    COMMON /CLCMRL/ CLVB, CMVB, CMPAVB
    COMMON /Z2/ Z(3)
    COMMON /INPTVB/ FTVB(64), FPVB(64), FPPRVB(64), DIDRVB(64),
A XMVB(64), DELVB, XMUVB, FOVB, XMUVAR,
B ATOVB, ATCVB, ATSVB, ROVA, RVB;
C MV3(64), NVB
    COMMON /INPUTS/ NSBL, NZ, NOFF, NGAM, NSIG,
A NCOI, NCOORD, LOWER, MSTOP, MAXT, MTR,
B NOTBL, INDV, ELSIG, DXI, REB, ROBA,
C FRZI, ARR, AMPLU, FREQU, ALPHI, ALPH2,
D HEAVE, ARCT, FREQM, PHIH, NY, RY1,
E DRY, X1(100), TEST, UPRIM, XU(30), YU(30),
F XL(30), YL(30), ER1, ER2, ER3, B03R,
G RDBR
H, DUMMY(10), PLOTOP
DIMENSION DELTA(3,3)
DIMENSION ALPHAI(3,3), BETAI(3,3), GAMMAI(3,3), OMS(3), OMEGA(3), CM(3)
DIMENSION AA(10), AB(10), ANB(20), ANT(20), AAX(10), ANS(20), SORT(3)
1, TOT(2)
CF4(X) = Fb - Bb+(Rb*C6-C41*X*X)
Z1(X) = Hb*(CF4(X)/GB)**2+(CF4(X)*FR1S+1.-C6*X**X)*B2-F2*X**X
Z2(X) = (F2/FR1S+FR1S*CF4(X)-F2+1.-C6*X**X)*(R2-BZ/FR1S)**X
SI(X) = (Z1(X)*GB)**2+(FR1S+FR2S)**X*GA
S2(X) = (FR1S+FR2S)**X
FUNTX1 = (R1+Z1(X)-R2*Z1(X)**2+R1*S2(X)-R2*S1(X)**2+Z2(X)**S1(X)-Z1(X)**S2(X))
DATA BBS, REL, NPOD/1.E-7, 1.E-6, 3/
DO 66 K = 1, 8
  YOLD(K) = YNEW(K)
  CALL YVATYNEW,II
  IF (II .LE. 1) GO TO 69
DO 67 K = 1, 8
  SUM(K) = (YNEW(K) + YOLD(K)) * (RVR(I) - RV(I-1)) / 2. + SUM(K)
  CONTINUE
  FM11 = SUM(1)
  EM22 = SUM(2)
  EM33 = SUM(3)
  EM13 = SUM(4)
  EM23 = SUM(5)
  H11 = SUM(6)
  H22 = SUM(7)
  H33 = -EM33
  H13 = -EM13
  H23 = SUM(8)
  BDBRR = BDRR / RDBRR
  BDS = BBRR**2
  T11 = H11 * BDS
  T22 = H22 * BDS
  T33 = H33 * BDS
  T13 = H13 * BDS
  T23 = H23 * BDS
  FR1S = BDS*ER1**2 - T11/EM11
  FR2S = ER2**2 * BDS - T22/EM22
  FR3S = FR3**2 * BDS - T33/EM33
  FR1 = DSQR(TFR1S)
  FR2 = DSQR(TFR2S)
  FR3 = DSQR(TFR3S)
  RATH = EM11/EM22
  ZETA = (1. + RATH) * (RATH*FR1S**2 + FR2S**2)/(RATH*FR1S + FR2S)**2
  RM = ZETA - 1.
  SUMS = FR1S + FR2S
  HIGHS = (SUMS + DSQR(SUMS**2 - 4.*ZETA*FR1S*FR2S)/(2.*ZETA))/(SUMS + FR1S + FR2S)*HIGHS
  SMALS = FR1S*FR2S/HIGHS
  DEN = FR2S - FR1S
  A1 = -(HIGHS - FR1S)/DEN
  A2 = 1. - A1
  B = A1 * DEN/HIGHS
  SLAM1 = EM11 * BDBR**2/EM33
  SLAM2 = SLAM1
  SUM3 = SUMS + FR3S
  ADDZ = FR1S*FR2S*FR3S
  ADDZ = FR1S*FR3S
  BBAR = 1. - (EM1**2*EM11 + EM2**2*EM22 + EM3**2)*EM11
  B4 = SUM3 + EM23*T23/EM22 + EM11*FR1S*EM23**2 + EM22*FR2S*EM33
  IEM1**2*EM11 + EM2**2*EM22 + EM3**2*EM33
  B4 = B4 + .04/BBAR
  BZ = DD2*T3 + .04*FR2S*EM11 + 2.*FR1S*EM23**2 + EM22*FR2S*EM33
  IEM1**2*EM11 + EM2**2*EM22 + EM3**2*EM33
  BZ = BZ/BBAR
  BZ = ADDZ - (FR2S*T13**2 + EM11 + FR1S*T3*EM23**2 + EM22 + EM33
  BZ = BZ/BBAR

61
\[
\begin{align*}
S1L &= (XI - RM*HIGHs*S2L)*HIGHs/(FR1S*FR2S) \\
WRI T &= (6,4) FR1,FR2,EF3,RM \\
WRI T &= (6,721) FR1,FR2,FR3,ALoW,BL0W \\
WRI T &= (6,5) EM1,EM22,EM33,EM13,EM23 \\
WRI T &= (6,6) H11,H22,H33,H13,H23 \\
C13 &= ALoW/BoDR \\
C23 &= BLoW/BoDR \\
XRAB &= XPAR/RdBR \\
SILR &= SIL/BoDR \\
S2L &= S2L/BoDR \\
WRI &= WRI (5,4) Rm1 \\
E3 &= E82 \\
E63 &= E63 \\
RYS &= RYS \\
SFR &= SFR \\
STUPPL &= 1162 \\
WRI &= WRI (6,91) FR1,FR2,FR3,ALO,W,BL0W \\
E3 &= E82 \\
E63 &= E63 \\
RYS &= RYS \\
SFR &= SFR \\
STUPPL &= 1162 \\
\end{align*}
\]
71  
11=2
12=3
GO TO 74

72  
11=1
12=3
GO TO 74

73  
11=1
12=2
GO TO 75

74  
IF IOMS(11).GT.OMS(12) GO TO 75
MINI=1
MIDI=12
GO TO 76

75  
MINI=1
MIDI=1
SORT(1)=OMS(MINI)
SORT(2)=OMS(MIDI)
SORT(3)=OMS(MAXI)
DO 77 1=1,3
OMS(1)=SORT(1)

77  
OMEGA(1)=DSQRT(CMS(1))
DO 302 I=1,3

302  
ALPHA(1,1)=1.
DENB(1)=BETA(2,1)*BETA(3,2)-BETA(3,1)*BETA(2,2)-OMS(1)
ALPHA(1,2)=BETA(1,1)*BETA(3,1)-BETA(1,2)*BETA(2,2)-OMS(1)
ALPHA(1,3)=BETA(1,1)*BETA(1,2)-BETA(2,2)-OMS(1)
OMEGA(1)=DSQRT(CMS(1))
DO 302 I=1,3

WRITE(6,488)
WRITE(6,489) 1,OMEGA(1),BETA(1,1),BETA(1,2),BETA(1,3),ALPHA(1,1),SUPPL221

488  
1=1,3

489  
SUPPL222
SUPPL223
SUPPL224
SUPPL225
SUPPL226
SUPPL227
SUPPL228
SUPPL229
SUPPL230
SUPPL231
SUPPL232
SUPPL233
SUPPL234
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SUPPL251
SUPPL252
SUPPL253
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SUPPL255
SUPPL256
SUPPL257
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SUPPL261
SUPPL262
SUPPL263
SUPPL264
SUPPL265
SUPPL266
SUPPL267
SUPPL268
SUPPL269
SUPPL270
SUPPL271
WRITE(6,11)
WRITE(6,12) (I,GAMMA(I,1),GAMMA(I,2),GAMMA(I,3),I=1,3)
AMPLU = XMUAVB * (1. - ROVB**3) / (1. - ROVB**4) * 1.3333333333D0
SA = SMALS * SILA + RM * S2LB + HIGHS
SB = SMALS * SILA**2 + RM + S2LR**2 * HIGHS
DEL(I,1) = XMUAVB * (1. - ROVB**4) / (4. * (1. - SLAMZ * XBAB**2))
A = RROBR * EM11
DEL(I,2) = 2. * SLAMZ * XBAB + DEL(I,1)
DEL(I,3) = A1 * (SLAMZ * XBAB + SB - SA) / (1. - SLAMZ * XBAB**2)
A + B * HIGHS + S2LB
DEL(2,1) = A2 / A1 * DEL(1,1)
DEL(2,2) = A2 / A1 * DEL(1,2)
DEL(2,3) = A2 * (SLAMZ * XBAB + SB - SA) / (1. - SLAMZ * XBAB**2)
A - R * SMALS - S2LB
DEL(3,1) = - SLAMZ * XBAB + DEL(1,1) / A1
DEL(3,2) = -2. * SLAMZ + DEL(1,1) / A1
DEL(3,3) = (BDRR / RROBR)**2 + SLAMZ + (XBAB + SA - SP) / A
431 GAMMA(I,J)=SORT(I)
432 CONTINUE
\begin{verbatim}
CMAP(1) = 2. * CMAP(2) - CMAP(3)  
CMAP(2) = CL(2)  
CMAP(3) = CLV0  
CMAP(I) = 2. * CMAP(2) - CMAP(3)  
PSI = (BDAR / RRDRR) * NTIME * DXI  
SIN PSI = SIN(PSI)  
COS PSI = COS(PSI)  
TOT(2) = TOT(1)  
TO = ATOV@ + ATCVB * COS PSI + ATSVB * SIN PSI  
TOT(1) = TO - ATOVB  
TO PR = (BDAR / RRDRR) * (ATSVB * COS PSI - ATCVB * SIN PSI)  
DO 60 K = 1, 2  
DO 64 I = 1, 3  
64 SMALL G(I) = UiNF **2 * (DEL(I,1) + CL(K) + DEL(I,2) * CMAP(K))  
A + DEL(I,3) * TOT(K)  
DO 65 I = 1, 3  
G CAP(I, K) = 0.  
DO 65 J = 1, 3  
65 G CAP(I, K) = GCAP(I, K) + ALPHA(I,J) * SMALLG(J)  
60 CONTINUE  
DO 62 I = 1, 3  
Y(I+1) = Y(I+1)  
YPR(I+2) = YPR(I+1)  
WDXI = OMEGA(I) * DXI  
SWDXI = SIN(WDXI)  
CDWDXI = COS(WDXI)  
Y(I+1) = Y(I+2) - CDWDXI * YPR(I+2) - SWDXI / OMEGA(I)  
A + (GWCAP(I,2) - GCAP(I,1)) * (SWDXI - WDXI * CDWDXI) / WDXI  
B + GCAP(I,1) * (SWDXI / OMEGA(I)**2  
62 YPR(I,1) = YPR(I,2) * CDWDXI - OMEGA(I) ** Y(I,2) / SWDXI  
SWDXI / WDXI + GCAP(I,1) * SWDXI / OMEGA(I)  
DO 61 I = 1, 3  
Z(I) = 0.  
ZPR(I) = 0.  
DO 61 J = 1, 3  
61 Z(I) = Z(I) + GAMMA(I,J) * Y(J,1)  
SWDXI = ZPR(I) + GAMMA(I,J) * YPR(J,1)  
ALPH1 = TO / Z(I)  
ALPH2 = TO / ZPR(I)  
HEAVE = ZPR(I) - ZPR(2)  
IF (PLOTOP .LT. 0.)  
1 WRITE( 6,9000) TO, Z, TOPR, ZPR, Y, YPR, DEL, SMALLG  
2 , TOT  
RETURN  
1 FORMAT(5F10.4)  
2 FORMAT(5F10.4)  
3 FORMAT(1H1,10X,*ITERATION FOR XBAR DIVERGED*)  
4 FORMAT(1H1,5X,4HF1 =E13.5,5X,4HF2 =E13.5,5X,4HF3 =E13.5/5X,4HRM =E13.5////)  
5 FORMAT(5X,5HM1 =E13.5,5X,5HM22 =E13.5,5X,5HM33 =E13.5,5X,5HM13 =E13.5////)  
6 FORMAT(5X,5HM11 =E13.5,5X,5HM23 =E13.5////)  
7 FORMAT(20X,6HXB/R =E13.5,10X,6HXB/B =E13.5/20X,6HL1/R =E13.5,10X,6SUPPL361
\end{verbatim}
1H1/8 =E13.5,10X,6HL2/R =E13.5/9X,TH/1/M1 =E13.5
1/9X,74K2/M2 =E13.5

41 FORMAT(//10X,5HR/R =E13.5,20X,6M/R/R =E13.5//)

44 FORMAT(1H1,20X,*POLYNOMIAL COEFFICIENTS*///7X,5HP/R/R,R=12X,5+BLADE, SUPPL365

126X,3H2-D//)

46 FORMAT(10,2030,9)

47 FORMAT(1H1,20X,*ROOTS OF POLYNOMIALS*///30X,*BLADE*,60X,*2-1*20X, SUPPL368

14HREAL,21X,4HIMAG,31X,6HREAL,21X,4HIMAG//)

49 FORMAT(2D25.9,10X,2D25.9)

11 FORMAT(///10X,1H1,15X,10HSGMA(I,1),15X,10HSGMA(I,2),15X,10HSGMAI, SUPPL371

1A(I,3)//)

12 FORMAT(10,3E25.5)

488 FORMAT(1H1,8X,1H1,7X,5HWEKA,4X,9HBETAI,1/I,4X,9HETAI,2,4X,9HBE SUPPL374

I1A(I,3),3X,10HALPHA(I,1),3X,10HALPHA(I,2),3X,10HALPHA(I,3),9X,3HC/ SUPPL375

1K//)

489 FORMAT(10,8E13.5)

721 FORMAT(77/10X,5HFR =E13.5,10X,5HFR =E13.5//10X, SUPPL378

14HSA =E13.5,10X,4HSB =E13.5//1)

END
CALL SETUP(*XMUVR , 1, 4, XMUVR )
CALL SETUP(*XMUAVB , 1, 4, XMJAVB )
CALL SETUP(*XU   , 1, 4, XU,  30 )
CALL SETUP(*Y    , 1, 4, Y,   100 )
CALL SETUP(*YL   , 1, 4, YL,  30 )
CALL SETUP(*YU   , 1, 4, YU,  30 )

C
C
C
C
C
C

PSILOW = 1.1E10
PSIUP = -1.1E10
PLOTOP = 1.

NOUT = 0

RETURN

C
C

END
SUBROUTINE BLCX, Y, MST, MEND, NY, RCY, DXI, RER, JPREM, FLAM, XFLAM, TESRLC
IT, U, SCALF, JIC, UC, VXSE, USEP, DISJ, THET5, LAMQ, MSEP, XC, USAV, SCALC
1LS, NITS, NTIME, NCTSL, XTEST, NZ, NOUT }

C PROGRAM FOR ANALYZING LAMINAR AND TURBULENT BOUNDARY LAYERS
C BY THE METHOD OF FINITE DIFFERENCES. IF THE INTEGER LAMQ
C IS GREATER THAN ZERO, THE BOUNDARY LAYER IS LAMINAR.
C
COMM / RAL1/ NOUMMY, NOINC, ISTD
DIMENSION USAV(300,100), SCALS(300)
DIMENSION X(300), Y(100), UC(300,3), UC(100,3), V(100,2), XC(300)
DIMENSION SD(100), SE(100), SF(100), VISC(100,2), GRAD(100)
DIMENSION AL(100), BL(100), C(100), OI(100), FI(100)
DIMENSION ALPHA(100), ETA(100), GAMMA(100), DELTA(100)
DIMENSION SCAL(300,2), VAR1(100), VAR2(100)
DIMENSION FLAM(10), XFLAM(10), YR1(100), YR2(100)
DIMENSION U(300,110,2)
DIMENSION CAPG(100), CAPI(100), CPJ(100), CAPK(100)
DOUBLE PRECISION AP(100), BP(100), CP1(100), DP1(100), FP1(100), UP(100)
10 FORMAT(IH1, 41X, 36H ANALYSIS OF LAMINAR BOUNDARY LAYER///5X, 12HTRLC
11 FORMAT(IH1, 41X, 36H ANALYSIS OF TURBULENT BOUNDARY LAYER///5X, 12HTRLC
3'1/)
12 FORMAT(15,8(14.,4.,13))
20 FORMAT(1H1, 2X, 3HM =14//2X, 3HX =145/2X, 4HUE =145, 10X, 17+11/RRLC
24 FORMAT(1X, 3HM =14//2X, 3HX =145/2X, 3HY =145, 10X, 17+11/RRLC
28 FORMAT(1H1, 2X, 3HM =14//2X, 3HX =145/2X, 3HY =145, 10X, 17+11/RRLC
32 FORMAT(1H1, 2X, 3HM =14//2X, 3HX =145/2X, 3HY =145, 10X, 17+11/RRLC
36 FORMAT(1H1, 2X, 3HM =14//2X, 3HX =145/2X, 3HY =145, 10X, 17+11/RRLC
810 FORMAT(1H1, 2X, 3HM =14//2X, 3HX =145/2X, 3HY =145, 10X, 17+11/RRLC
BCON = 1., 570UXI
FCON = 1.//2X, DXI)
1F1( ISTD, NE. 11 GO TO 900
DXI = 1.30
BCON = 0.
FCON = 0.
900 CONTINUE
NOUT = NOUT + 1
MST2 = MST - 2
MST1 = MST - 1
MST MD = MOD(MST1, NOUT1)
MAXIT = 0
GO TO (543, 550), L3

543 IF(LAMQ) 544, 544, 545
WRITE(MOUT, 11) NTIME, NITS
GO TO 550

545 WRITE(MOUT, 10) NTIME, NITS
CONTINUE
YTR = SQRT(RET)
UC(1, 1) = 0.
V(1, 1) = 0.
NV = NY - 2
NVM1 = NV - 1
NVP1 = NV + 1
CALL YDIFF(NY, ALPHA, BETA, GAMMA, DELTA, SD, SE, SF, C2, C3, C4, Y)
DO 41 N = 1, NVP1
VISC(N, 1) = 1.

41 VISC(N, 2) = 1.
DO 42 M = MST2, MST1
L = MST1 - M + 2
DO 50 N = 1, NV

50 GRAD(N+1) = SD(N+1)*UC(N+2, L) + SE(N+1)*UC(N+1, L) - SF(N+1)*UC(N, L)
GRAD(1) = C2*UC(2, L) + C3*UC(3, L) + C4*UC(4, L)
MM = M - 1
CALL PGRAD(MM, M, N, X, Y, UX, UY, PRESS, GRAD, DELT, DISP, THETA, VISC, MTLB3)
ITER = 0.
GO TO 46
M = M + 1

46 IF(M MOD(N, M)) GO TO 225
CALL SETIT(LAMQ, M, NV, REB, X, Y, UX, UY, PRESS, GRAD, DELT, DISP, THETA, VISC, MTLB3)
MEND1 = MEND - 1
GRADS = GRAD(1)
GRADSS = GRAD(1)

C THE MAIN CALCULATION STARTS HERE.

C

DO 99 M = MST1, MEND1
ITER = 0.
WALLG = 0.
MPI = M + 1
DELTP = DELT/YTR
DISPT = DISP*YTR
THETT = THETA*YTR
SHEAR = GRAD(1)/YTR

C IF(MOD(M, NOUT1).NE. MSTD1) GO TO 225

99 CONTINUE
GO TO (561, 562), lower

561 WRITE(MOUT, 12) M, XM(N), XC(M), UE(M, 1), PRESS, DELTP, DISPT, THETA, SHEAR, MAXIT

1, MAXIT
GO TO 225

562 WRITE(MOUT, 20) M, XM(N), UE(M, 1), PRESS, REB, UPRIM
WRITE(MOUT, 24) DELTP, DISPT, THETA, DELT, DISPT, THETT
WRITE(MOUT, 21)
WRITE(MOUT, 22) (YIN, UC(N, 2), VIN, 1), GRAD(N), VISC(N, 1), N = 1, NVP1
WRITE(MOUT, 25) SHEAR

225 IF(GRADS-GRADS-1.E-6) 229, 229, 408
408 XX = XM(N) + (XM(N) - XM(N-1)), GRADS/1GRADS-GRADS
IF(XX-XX) 409, 409, 229

409 XX = XX
GO TO 225

90 END
WFS = (XSY - X(M-1)) / (X(M) - X(M-1))

GO TO 224

229 IF (GRAD(1)) 227, 227, 273
273 IF (NISP .GT. 0. AND. THETA .GT. 0.) GO TO 273
283 CONTINUE
XSEP = XC(M-1)
USEP = UE(M-1,1)
XR X(M-1)
WRITE (MOUT,21) XBL, XSEP
RETURN

227 WFS = GRADS / (GRADS - GRAD(1))
224 WFS = 1. - WFS
XSEP = WFS * XC(M-1) + WFS * XC(M)
XR X = WFS * X(M-1) + WFS * X(M)
USEP = WFS * UE(M-1,1) + WFS * UE(M,1)
WP F = (XRL - X(M-2)) / (X(M-1) - X(M-2))
WPL = 1. - WP F
DISS = DISS * WP F + DISS * WP F
THET S = T HET S * WP F + T HET S * WP F
WRITE (MOUT, 23) XBL, XSEP
IF (LAMQ .EQ. 0. AND. M .LT. MTRAN + 5) LAMQ = 1
GO TO 222

223 CONTINUE
IF (NOTAL .EQ. 2 .AND. NITS .GT. 1 .AND. M .GT. NZ .AND.
1 XC(M) .GT. XTEST) GO TO 283
IF (LAMQ) 801, 801, 802
8C2 IF (NOTAL .EQ. 2) GO TO 801
CALL TRANS (UPRIM, PRESS, THETA, REB, UC, NY, FLAM, XFLAM, LAMQ)
IF (LAMQ) 805, 805, 901
8C5 WRITE (MOUT, 30) X(M)
MTRAN = M+1
801 CONTINUE
IF (Y(NV) - DELT) 620, 641, 641
620 RY = RY + DRY
8C C RE SCALING CALCULATION STARTS HERE.

C DO 632 N = 1, NY
YBL(N) = Y(N)
VAR1(N) = UC(N,2)
632 VAR2(N) = UC(N,3)
CALL YSET (RY, YSUB2, NY, Y)
WRITE (MOUT, 351) YBL(NY), Y(NY)
DO 633 N = 2, NVP1
YIN = Y(N)
CALL T ERP (YIN, YBL, VAR1, NY, UPAS1)
UC(N,21) = UPAS1
CALL T ERP (YIN, YBL, VAR1, NY, UPAS2)
633 UC(N,3) = UPAS2
CALL YDIFF (NY, ALPH A, BETA, GAMMA, DELTA, SD, SE, SF, C2, C3, C4, Y)
IF (LAMQ) 700, 700, 701
700 DO 635 N = 2, NVP1
VAR1(N) = VISC(N,1)
635 VAR2(N) = VISC(N,2)
DO 636 N = 2, NVP1
636 N = 2, NVP1

73
YIN = Y(N)

CALL TESP(YIN,YB1,VAR1,NVP1,UPAS1)

VISC(N+1) = UPAS1

CALL TESP(YIN,YB1,VAR2,NVP1,UPAS2)

636 VISC(N+2) = UPAS2

761 DO 637 N=2,NVP1

VAR1(N) = V(N,1)

637 VAR2(N) = V(N,2)

DO 638 N=2,NVP1

YIN = Y(N)

CALL TESP(YIN,YB1,VAR1,NVP1,UPAS1)

V(N+1) = UPAS1

CALL TESP(YIN,YB1,VAR2,NVP1,UPAS2)

638 V(N,2) = UPAS2

641 CONTINUE

C

RESCALING CALCULATION ENDS HERE.

C

CALL PGRAD(M,X,UE,DXI,PRESS,SA,SB,SC,SR,SS)

C

RECURSION RELATIONS ARE SET UP HERE.

C

IF (IST0.EQ.1) GO TO 820

IF (SCALE(M+1,1)-1.) 522,522,521

521 IF (SCALE(M+1,2)-1.) 522,522,523

522 LACKU=1

FACUI=UE(M+1,2)/UE(M+1,1)

FACU2=UE(M+1,3)/UE(M+1,1)

GO TO 820

523 LACKU=2

GO TO 610 N=1,NY

VAR1(NN) = U(M+1,NN+1)

610 VAR2(NN) = U(M+1,NN+2)

CALL YSET(SCALE(M+1,1),YSUB2,NY,YB1)

620 CALL YSET(SCALE(M+1,2),YSUB2,NY,YB2)

820 DO 88 N=2,NV

CALL CAPS(TTERT,N,CAPG,CAPH,CAPJ,SR,SS,SD,SE,SF,VISC,V,UC)

A(N) = SF(N)*CAPG(N)-DELTA(N)*CAPH(N)+SF(N)*CAPJ(N)

R(N) = BCON*SA*CAPK(N)+SF(N)*CAPG(N)-GAMMA(N)*CAPH(N)-SE(N)*CAPJ(N)

C(N) = SD(N)*CAPG(N)-BETA(N)*CAPH(N)-SD(N)*CAPJ(N)

DT(N) = -ALPHA(N)*CAPH(N)

IF (IST0.EQ.1) GO TO 576

GO TO 574,575

574 UPAS1=FACU1*UC(N,1)

575 YIN = Y(N)

UPAS2=FACU2*UC(N,1)

GO TO 576

CONTINUE

576 F(N) = PRESS+FCON*(4.*UPAS1-UPAS2)*CAPK(N)*(SB*UC(N,2)-SC*UC(N,3))

88 CONTINUE

C

SOLUTION FOR VELOCITY PROFILE STARTS HERE.

C

DO 89 N=2,NV

C

.74
AP(N) = A(N)
RP(N) = B(N)
CP(N) = C(N)
DP(N) = D(N)
58 FP(N) = F(N)
DO 77 N=2,NVPL
CP(N) = CP(N)/RP(N)
DP(N) = DP(N)/RP(N)
FP(N) = FP(N)/RP(N)
RP(N+1) = RP(N+1) - CP(N) * AP(N+1)
CP(N+1) = CP(N+1) - DP(N) * AP(N+1)
77 FP(N+1) = FP(N+1) - FP(N) * AP(N+1)
UP(NY) = UE(N+1,1)
UP(NV1) = UP(NY)
UP(NV) = (FP(NV)-UP(NY)*(DP(NV) + CP(NV)))/RP(NV)
DO 56 N=3,NV
NN=NV+2-N
DO 66 N=2,NV
UC(N,1) = UP(N)
IF (ITER) 843,841,843
841 DO 842 N=2,NVPL
VIN(2) = VIN(1)
842 VISC(N,2)=VISC(N,1)
DISS=DISS
DISS=DISS
THETS=THETS
THETS=THETS
GRADSS=GRADS
GRADSS=GRADS(1)
DO 843 N=2,NVPL
55 VIN(1) = VIN(N-1,1) - S*(N(N-1)*UP(N+1)) + UC(N,1) + UC(N,1) - UC(N,1)
1N(N,2) + UC(N/1,1) + UC(N,1) + UC(N-1,1)
DO 56 N=1,NV
56 GRAD(N+1) = SD(N+1)*UC(N+1)/2.1 + SE(N+1)*UC(N+1,1)/2 - SF(N+1)*UC(N,1)
GRAD(I) = C2*UC(2,1) + C3*UC(3,1) + C4*UC(4,1)
CALL SFITL(TLAM, NPI, NP, REB, X, Y, UC, PRESS, GRAD, DELT, DISP, THETA, VISC, BLC, BLC)
1MTRAN
BLC 234
ITER=ITER+1
GOTO (330, 809), LOWER
809 WRITE(OUT,810) ITER,GRADS(1)
83C IF (ITER-9) 811, 811, 812
811 FPW=ABS(GRAD(I)-WALLG)
IF (WALLG=1.) 132, 130, 119
119 EPW=EPW+WALLG
120 IF (EPW-TEST) 812, 814, 814
814 WALLG=GRADS(1)
GOTO 820
812 DO 44 N=1,NV
UC(N,3) = UC(N,2)
UC(N,2) = UC(N,1)
44 CONTINUE
MAXT=ITER
IF (ISTD.EQ.1) GO TO 99
80 48 N=1,NV
75
48 USAV(M+1,N) = UC(N,1) 
SCALSM+1) = RY 
99 CONTINUE
XSEP=1.1
USEP=UE(MX,1)
222 CONTINUE
RETURN
END
SUBROUTINE PLOTS { PLOTOP, P, L )
REAL * 9 ORD(6)
DIMENSION P(200,7), TITL(56), NF(5,4)
1 1, NF(6)
DATA NL, N2, NO, N42
1/1, 2, 0, 42/
DATA ORD(7), THETA-P(.,.), TORS(.,.), FLAP-H(.,.), RENJ-H(.,.),
1 CL(.,.), CM-A(.,.)/
IF (PLOTOP .EQ. 0.) RETURN
IF (L .LT. 2) RETURN
IF (PLOTOP .EQ. 2.) GO TO 2
PLOTP = 2,
CALL INFOBM ( 'CRINP', 'P', '30', '5100'
) CONTINUE
3 NL = 1
DO 1 J = 1, 6
CALL E2PL0T(9, , N1, N1, P, P(1,J+1), L, -N1, N2
1, N42, 1, 12, 'PSI-DEGREES', 8, ORD(J)
2, N1, N1, XL, XU, N1, YL, YU, N1, NO, NL)
1 CONTINUE
NFP(1) = -1
NFP(2) = 66
NFP(3) = 50
NFP(4) = 50
NFP(5) = 50
NFP(6) = 50
CALL E2PL0T(9, , N1, N1, P, P(1,2), L, -N1, N2
1, N42, 1, 12, 'PSI-DEGREES', 8, ORD(1)
2, NFP, N1, XL, XU, N1, YL, YU, N1, NO, NL)
1 NFP(1) = -2
NFP(2) = 66
NFP(3) = 50
NFP(4) = 350
NFP(5) = 380
CALL E2PL0T(9, , N1, N1, P, P(1, 6), L, -N1, N2
1, N42, 1, 12, 'PSI-DEGREES', 8, ORD(5)
2, NFP, N1, XL, XU, N1, YL, YU, N1, NO, NL)
1 NFP(2) = 50
NFP(3) = 690
NFP(4) = 40
CALL E2PL0T(9, , N1, N1, P, P(1, 7), L, -N1, N2
1, N42, 1, 12, 'PSI-DEGREES', 8, ORD(6)
2, NFP, N1, XL, XU, N1, YL, YU, N1, NO, NL)
1 NFP(1) = -1
NFP(2) = 50
NFP(3) = 50
NFP(4) = 50
NFP(5) = 50
NFP(6) = 50
CALL E2PL0T(9, , N1, N1, P, P(1, 3), L, -N1, N2
1, N42, 1, 12, 'PSI-DEGREES', 8, ORD(2)
2, NFP, N1, XL, XU, N1, YL, YU, N1, NO, NL)
1 NFP(1) = -2
NFP(2) = 66
NFP(3) = 350
NFP(4) = 380
CALL E2PL0T(9, , N1, N1, P, P(1, 4), L, -N1, N2
1, N42, 1, 12, 'PSI-DEGREES', 8, ORD(3)
2 , NFP , N1 , XL , XU , N1 , YL , YU , N1 , N0 , N1)
NFP(2) = 50
NFP(4) = 690
NFP(5) = 49

CALL EZPLOT(9. , N1 , N1 , P , P(1,5) , L , -N1 , N2
1 , N42 , 1 , 12 , 8 , ORD(4)
2 , NFP , N1 , XL , XU , N1 , YL , YU , N1 , N0 , N1)
           RETURN
           END
SUBROUTINE STAG(X,Y,MS,TOP,MS,DX,Y,DY,E,V,EUSAV,SCAL,STAG)

115FP)

C PROGRAM FOR CALCULATING THE BOUNDARY LAYER PROFILE NEAR

C THE STAGNATION POINT

COMMON XH1/, NTIME, ADIMC, ISTDO
DIMENSION USAV(300,100), SCALS(300)
DIMENSION PHI(24), PHIP(24), FTAP(24)
DIMENSION X(300), Y(100), UE(300,3), UC(100,3), V(100,2)
DIMENSION FF(100), EFP(100)
DATA FTAP /0, 2, 4, 6, 8, 1, 2, 1, 4, 1, 6, 1, 8, 2, 2, 2, 2, 4, 2, 6, 2, 8, 3, 1,
13, 2, 3, 4, 6, 8, 9, 4, 2, 4, 4, 6/
DATA PHIP /0, 0.23, 0.38, 0.83, 0.18, 0.87, 0.31, 0.45, 0.62, 0.79, 0.97, 0.16, 0.91,
19, 1.36, 1.5, 1.557, 1.75, 1.95, 2.2, 1.53, 2.3, 3.5, 2.3, 3.52, 2.4, 3.75, 2.5, 2.95, 2.1,
1.21, 1.352, 1.452, 1.552, 1.752, 1.952, 1.91, 1.856, 1.68, 1.77, 1.846, 1.89, 1.93, 1.956,
1.8, 1.97, 2.035, 2.046, 2.097, 2.098, 2.098, 2.098, 2.098, 2.098, 2.098, 2.098, 2.098,
1, 1, 1, 1/
RAG = .08
IF(ISP) 10, 10, 5
BAG = .3
10 IF(1) = 0.
   EFP(i) = 0.
   DO 20 M=1, MX
      IF(UD(M,1)) 20, 20, 19
   15 MSP = M
   GO TO 21
20 CONTINUE
21 ASTAG = (UE(MSP+2,1)-UE(MSP+1,1))/(X(MSP+2)-X(MSP+1))
   IF(ASTAG) 22, 22, 23
22 ASTAG = (UE(MSP+1)-UE(MSP-1,1))/X(MSP)-X(MSP-1))
   SQAS = SORT(ASTAG)
   DELT = 2.67*SQAS
30 IF(DSPL) 311, 310, 310
310 RY = RY+DRO
   CALL YSET(RY, Y(2), NY, Y)
   GO TO 309
311 CONTINUE
   DC RO N=2, NY
   YET = Y(N)+SQAS
   DO 33 NN=1, 24
      IF(YET-ETAP(NN)) 408, 408, 33
   40 MSP = NN
   GO TO 410
33 CONTINUE
   FF(N) = YET-6/439
   EFP(N) = 1.
   GO TO 80
410 FRAC = (YET-ETAP(MARK-1))/(ETAP(MARK)-ETAP(MARK-1))
   FRAC = 1.-FRAC
   EF(N) = PHITZ(MARK-1)*FRAC+PHIZ(MARK)*FRAC
   EFP(N) = PHI(MARK-1)*FRAC+PHI(MARK)*FRAC
88 CONTINUE
   M1 = MSP-MS-TOP
   M2 = MSP+MS-TOP

79
M=M1-1
M=M+1
MST=M+1
SCALS(M)=RY
DO 71 N=1,NY
UC(N,2) = UC(N+2)
UC(N,2) = UE(M,1)*EFP(N)
V(N,2) = V(N,1)
V(N,1) = -SQAS*EF(N)
IF (STD.EQ. 1) GO TO 71
USA(M,N)=UC(N,2)
71 CONTINUE
IF (M=2) 50,55,55
55 IF (UF(M,1)=BAG) 50,50,81
81 CONTINUE
RETURN
END
SUBROUTINE ATTPR(PREC,XSIG,NSIG,ASZ,AR,CMAT,RMAT,NGAM,NF,ACAP,THICK,RAIATTPR)
DIMENSION XSIG(100),ASZ(30),AR(100),ACAP(100,3)
DIMENSION ACAP(30,3),THICK(24),GAMM(1000)
DECIMAL PRECISION CMAT(50,60),RMAT(130)
PI=3.14159
NGAM=NGAM+1
DO 50 M=1,NGAM
CMAT(M,1)=ASZ(M)
RMAT(M)=AR(M)
DC 25 N=1,NGAM
25 CMAT(M,N+1)=AS(M,N)
50 CONTINUE
CALL ALSOL(NGAM,CMAT,RMAT)
DO 75 M=1,NGAM
75 ACAP(M,1)=RMAT(M)
GAMM(1)=GAMM(ACAP,DXI,PI)
SAVE=XSIG(NSIG+1)
XSIG(NSIG+1)=2.
CALL CPC(0,NGAM,NF,XSIG,NSIG,ASIG,XSIG,NSIG,ACAP,BCAP,THICK,RAIATTPR)
100 RAIN=CMNF,UNIF,UDOT,1.,SAVE,DXI,PREC)
XSIG(NSIG+1)=SAVE
RETURN
END
SUBROUTINE UNPOP(NGAM, AR, ALAM, AFACT, RMAT, CMAT, XGAM, AS, ACAP, 4X, NZ, NUNPOP)
1F, XSIG, ACP, THICK, RDBB, UINF, XC, UF)
DIMENSION AR(30), ALAM(30), XGAM(30), AS(30, 30), ACAP(30, 3), XSIG(110), NUNPOP
1BCAP(100, 3), THICK(24), XC(300), JE(300, 3)
DOUBLE PRECISION RMAT(130), CMAT(60, 60)
NGP1=NGAM+1
DO 5 M=1, NGP1
SUB=AR(M) - ALAM(M)*AFACT/3.
RMAT(M)=SUB
CMAT(M, 11)=1.
CMAT(M, 2)=XGAM(M)
DO 5 N=2, NGAM
CMAT(M, N+1)=AS(M, N)
CALL ALSCM(NGP1, CMAT, RMAT)
DO 10 N=1, NGP1
ACAP(N, 1)=RMAT(N)
DO 15 M=1, MX
SIGN=1.
IF(M-NZ) 12, 14, 14
12 SIGN=-SIGN
14 CALL QECAL(0, NGAM, NGAM, NF, XSIG, ACAP, BCAP, THICK, RDBB, 0., UINF, XC(M), NUNPOP
1UE(M, 1), SIGN)
15 CONTINUE
RETURN
END
SURROUTINE ALSO\(1\)(NT, C, R)

DOUBLE PRECISION C\(1,\ldots, N\), \(R(1)\)

DOUBLE PRECISION CMAX, SAVE, SUM

COMMON /RL1/ NTIME, NDIMC

NT1 = NT - 1

DC 99 J=1,NT1

CMAX = C(NT,J)

L=NT

DC 10 I=J,NT1

IF (DABS(CMAX)-DABS(C(I,J))) 5,10,10

5 CMAX = C(I,J)

L=1

10 CONTINUE

DC 15 JJ=J,NT

SAVE = C(L, JJ)

C(L, JJ) = C(J, JJ)

15 C(J, JJ) = SAVE/CMAX

SAVE = R(L)

R(L) = R(J)

R(J) = SAVE/CMAX

JPI = J+1

DO 25 I=JPI, NT

20 JJ = JPI, NT

C(I, JJ) = C(I, JJ) - C(I, J)*C(J, JJ)

25 R(I) = R(I) - R(J)*C(I, J)

95 CONTINUE

R(NT) = R(NT)/C(NT,NT)

DO 150 K=1, NT1

I=NT-K

IF = I+1

SUM = 0.

DO 125 J=IP1, NT

125 SUM = SUM + R(J)*C(I, J)

150 R(I) = R(I) - SUM

RETURN

END
SURPUTINFEPC[ISEP,NGAM,NSIG,NSIG,NSIGA,NSIGA,NSIG,NSIGB,NSIGB,ACAP,ACAP,ACAP,ACAP,ACAP,
BCAP,THICK,ROBB,GAMAW,UNIF,UDOT,USIGN,VC,DXI,CP]
DIMENSIONXSIG(100),XSIGA(100),XSIGB(100),ACAP(30,3),ACAP(100,3)
DIMENSIONGAMAW(1000),THICK(24)
THETA=ARCT(XC)
RECIPE=1./(USIGN*USIGN)
SUM=0.
ANGLE=0.
DC=5
N=1,NF
ANGLE=ANGLE+THETA
5
SUM=SUM+THICK(N)*COS(ANGLE)
CP=UDOT*RECIPE+THICK(1)+2.+(1.-XC)*SUM
CALLQECAL(ISEP,NGAM,NSIG,NSIG,ACAP,BCAP,THICK,ROBB,GAMAW(1),USIGN)
CP=CP+2.*(SIN*USIGN/USIGN-1.)
CALLEGAMI1(NGAM,ACAP,BCAP(1,1),XSIG(1),XSIG,VSIG(1),GAMAW(1),USIGN)
1VAL(1)
CALLEGAMI2(NGAM,ACAP,BCAP(1,2),XSIGA(1),XSIGA,NSIGA+1),GAMAW(2)
1XC,VAL2
CALLEGAMI3(NGAM,ACAP,BCAP(1,3),XSIGB(1),XSIGB,NSIGB+1),GAMAW(3)
1XC,VAL3
CP=CP-SIGN*RECIPE*(1.5*VAL1-2.*VAL2+.5*VAL3)/DXI
IF(ISSEP)20,20,10
10
CALLFSIGI(1,NSIG,NSIG,BCAP,XC,VAL1)
7CALLFSIGI2(2,NSIGA,NSIGA,BCAP,XC,VAL2)
CALLFSIGI3(3,NSIGB,NSIGB,BCAP,XC,VAL3)
CP=CP+RECIPE*(1.5*VAL1-2.*VAL2+.5*VAL3)/DXI
20
CP=CP
RETURN
END
SUBROUTINE CLCM(NC41,ISFP,NGAM,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM)
ICAP,BCAP,THICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
COMMON /CLCMAL/,CLVR,CMVR,CMVIA
DIMENSION ARG(21),ARG4(21)
DIMENSION GMAW(100),THICK(24)
DIMENSION XSIG(100),XSIGA(100),XSIGA(100),ACAP(30,3),HCAP(100,3)
FORMAT(//40X,4HCL=F13.5/40X,4HCM=F13.5,17H(ABOUT MIDCHORD))/40X,CLCM
4
NCLM=6
MCLO=8
SAVE=THICK(1)
THICK(1)=0.
D=3.14159/FLOAT(NC41)
CL=0.
CM=0.
XI=-1.
ANGLE=0.
FLI=0.
FM=0.
IF(ISFP) 5,5,7
XATT=XSIG(NSIGA+1)
IF(XATT=-95) 8,5,5
7
XATT=XSIG(NSIGA+1)
5
IF(ISFP) 5,5,7
XATT=XSIG(NSIGA+1)
IF(XATT=-95) 8,5,5
8
XATT=XATT+5,F-4
XAP=XAX+.025
C1=-.5*(1.+XATT)
C2=CM+XATT
C1P=-.5*(1.-XAP)
C2P=CM+XAP
DC 10 I=1,NC41
ANGLE=ANGLE+AT
XI1=CI*COS(ANGLE)+C2
CALL CPCITSEP,NGAM,1,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
CALL CPCITSEP,NGAM,1,XSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
FMI=FLIPFM+FLI
FMIP=FLIPFM+FLI
CLX=CM*(XI1-XI)*(FMI1+FLI)
FM=FMIP
XI=FI
FLI=FLI
FLI=FLI
FMI=FMIP
XI=1.
FLI=0.
FM=0.
ANGLE=0.
DC 15 I=1,NC41
ANGLE=ANGLE+AT
XI1=CI*COS(ANGLE)+C2
CALL CPCITSEP,NGAM,1,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
CALL CPCITSEP,NGAM,1,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
FMI=FLIPFM+FLI
FMIP=FLIPFM+FLI
CLX=CM*(XI1-XI)*(FLIPFM+FLI)
FMI=FMIP
FLI=FLI
FLI=FLI
FMI=FMIP
XI=1.
FLI=0.
FM=0.
ANGLE=0.
DC 15 I=1,NC41
ANGLE=ANGLE+AT
XI1=CI*COS(ANGLE)+C2
CALL CPCITSEP,NGAM,1,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
CALL CPCITSEP,NGAM,1,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
FMI=FLIPFM+FLI
FMIP=FLIPFM+FLI
CLX=CM*(XI1-XI)*(FLIPFM+FLI)
FMI=FMIP
FLI=FLI
FLI=FLI
FMI=FMIP
XI=1.
FLI=0.
FM=0.
ANGLE=0.
DC 15 I=1,NC41
ANGLE=ANGLE+AT
XI1=CI*COS(ANGLE)+C2
CALL CPCITSEP,NGAM,1,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
CALL CPCITSEP,NGAM,1,XSIG,NSIG,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,NSIGA,A4CLCM
I=HICK,ROBB,GMAW,UNIF,UDOT,DXI,AROT,CM4A)
FMI=FLIPFM+FLI
FMIP=FLIPFM+FLI
CLX=CM*(XI1-XI)*(FLIPFM+FLI)
FMI=FMIP
FLI=FLI
FLI=FLI
FMI=FMIP
XI=1.
FLI=0.
CM=CM-(XI Pl-XI)* (FMI Pl + FMI)
XI=XI Pl
FMI=FMI Pl
15
DO 16 I=1,21
CALL CPC(ISEP,NGAM,1, XI SIG, XI SIGA, NS IGA, XI SIGB, NS IGB, ACAP, BCAP, TCLCM)
1 HICK, RDBR, GAMMAW, UINF, UDOTT, 10, XI Pl, DXI, CPU)
17
CALL CPC(ISEP,NGAM,1, XI SIG, XI SIGA, NS IGA, XI SIGB, NS IGB, ACAP, RCAP, TCLCM)
CALL CPC(ISEP,NGAM,1, XI SIG, XI SIGA, NS IGA, XI SIGB, NS IGB, ACAP, BCAP, TCLCM)
1 HICK, RDBR, GAMMAW, UINF, UDOTT, 10, XI Pl, DXI, CPU)
ARGL(I)=CPL-CPU
ARGM(I)=XI Pl*ARGL(I)
16
XI Pl=XI Pl+.00125
SUML=0.
SUMM=0.
DO 17 I=1,19,2
SUML=SUML + 2.*ARGL(I)+4.*ARGL(I+1)
SUMM=SUMM + 2.*ARGM(I)+4.*ARGM(I+1)
CL=CL+0.833333F-.3*(SUML+ARGL(Zl)-ARGL(11))
CM=CM+XATT*RCFIN
17
CL=CL+(XI Pl-XI)* (FMI Pl + FMI)
CM=CH(TI Pl-XI)
FLI =FLI'Pl
100
CM=-.125*CM
CMPA=CM*XRTT*CL+.5
WRITE(MOUT,4) CL,CM,CMPA,AROT
THICK(I)=SAVE
CLVB = CL
CMVB = CM
CMPAVB = CMPA
RETURN
END
SUBRUTINE QECAL (ISEP, NGAM, NSIG, NF, XSIG, ACAP, RCAP, THICK, RDBA, GAMMA, QECAL 1
1, UINF, XC, U, SIGN)
DIMENSION ACAP(30, 31), RCAP(100, 31), XSIG(100)
DIMENSION THICK(24)
EPS = 1.5
CORR = 7.37107/(1. -.63662 * SQRT(RDBA) + .25 * RDBA)
SINT = SQRT(1. - XC + XC)
THETA = ARCT(XC)
COUNT = 0.
SUM = 0.
SINT2 = SINT(5 * THETA)
COST2 = COST(5 * THETA)
IF (5 * SINT - EPS) 4, 6, 6
4 FACT = THETA/5
GO TO 8
6 FACT = (1. - XC)/SINT
8 DO 10 N = 1, NF
COUNT = COUNT + 1.
ANGLE = THETA * COUNT
SUM = SUM + THICK(N) * (COUNT * FACT * SINT(ANGLE) - COS(ANGLE))
CONTINUE
U = 2. * SIGN * UINF * COST2 * SUM + ACAP(1, 1) * SINT2 + .25 * COST2 * (1. + XC) * (3. * XC - QECAL 22
1.1) * GAMMA
SUM = 0.
ANGLE = 0.
DO 12 N = 1, NGAM
ANGLE = ANGLE + THETA
12 SUM = SUM + ACAP(N, 1) * SINT(ANGLE)
U = U + COST2 * SUM
IF (ISEP) 25, 99, 25
25 SUM = 0.
XSEP = XSIG(1)
XATT = XSIG(1) NSIG
DO 40 N = 2, NSIG
40 SUM = SUM + BCAP(N, 1) * FBT(XSIG(N - 1), XSIG(N), XSIG(N + 1), XC)
IF (XC - XATT - EPS) 45, 44, 46
46 FACT = (1. - XATT)**(-1.5) * SQRT((XATT - XSEP) * (1. + XC) / (XC - XATT)) * (1. + 3. * QECAL 37
XATT)**(-2.5) - SIGN * (1. - SQRT((XSEP - XC) / (XATT - XC)))
GO TO 55
45 IF (XSEP - XC) 49, 49, 48
48 FACT = -SIGN * T - SQRT((XSEP - XC) / (XATT - XC))
GO TO 55
49 FACT = -SIGN
55 U = U + COST2 * (BCAP(N, 1) * FACT * SIGN * SJM)
99 U = T * SIGN * UINF * SQRT(1. + XC) + CORR * U15 / SQRT1 + XC + .5 * RDBA)
RETURN
END
SLBROUINE YVA(Y, I)

REAL Y(I)

REAL MV

COMMON /MTPTVA/ FTVB(64), FPVR(64), FPPRVA(64), DIDRVA(64), YVB

A X'MVB(64), DELVA, XMVB, FOVR, XMUVA, YVB

B ATOVR, ATCVB, ATSVR, ROVB, RVB(64), YVB

C MVY(64), YVB

Y(I) = (RVB(I) - DELVA)**2 * MV(1)

Y(2) = FPVB(I)**2 * MV(1)

Y(3) = FTVB(I)**2 * DIDRVA(I)

Y(4) = (DELVA - RVB(I)) * FTVB(I) * XMVB(I) * MV(1)

Y(5) = FPVB(I) * FTVB(I) * XMVB(I) * MV(1)

Y(6) = RVB(I) * (DELVA - RVB(I)) * MV(1)

Y(8) = (RVB(I) - DELVA) * FPPRA(V) * FTVB(I) * XMVB(I) * MV(1)

IP1 = 1 + I

IF(IP1 .GE. NVB) GO TO 12

SUM = 0.

DD 10 J = IP1, NVB

10 SUM = SUM - (RVB(I+1) - RVB(I)) * (RVB(I+1) * MV(J+1))

A = RVB(J) * MV(J)

12 Y(I) = FPVRV(1)**2 * SUM / 2.

RETURN

END
SUBROUTINE POLLY(N,ABS,REL,AN,A0)
IMPLICIT REAL*(A-H,O-Z)
C COMPLEX ROOTS OF A POLYNOMIAL BAIRSTOW'S METHOD
DIMENSION A(30),AN(60),C(26),ABAR(26),B(30),AA(30)
III=1
7 NPI=N+1
NPP1=N+2
DO 66 C1 I=1,NPI
LLL=NPP1-I
601 A(I)=AA(LLL)
13 DO 14 K=1,NPI
14 ABAR(K)=A(K)
ABSSQ=ABS**2
REL=REL*REL
NBAR=N
B(1)=A(1)
C(1)=A(1)
15 IF(NBAR-2).LT.200,210,17
17 P1=2
Q1=1
18 IFR=0
19 P1=P1+5,
Q1=Q1+10,
NP1=NBAR+1
34 L=1
LAST=NBAR
DTST=999999
C BAIRSTOW ITERATION
37 B(2)=ABAR(2T)-PB1
DO 40 K=3,NP1
40 B(K)=ABAR(K)-PBTK-1-Q*B(K-2)
45 C(2)=B(2)-P*C(1)
DO 50 K=3,LAST
50 C(K)=C(K-1)-P*C(K-1)-Q*C(K-2)
C(LAST)=C(LAST)-B(LAST)
D=C(LAST-1)*C(LAST-1)-C(LAST)*C(LAST-2)
DSQR=DSQR1*4
IF(DSQR-1.0).LT.36.0,19,19
19 DELP=TR(LAST)*C(LAST-1)-ATLAST+IT*C(LAST-2)/D
DELQ=(1/LAST+1)*C(LAST-1)-B(LAST)*C(LAST)/D
C TEST FOR CONVERGENCE
RELP=DEL/P
RELQ=DELQ/Q
RELP=RELS+RELP
RELQ=RELQ+RELQ
DELQ=RELS*RELQ
P=P+DELQ
Q=Q+DELQ
1F(RELS=RELS170.0,170.0,65
65 IF(Delp=DELQ=ABS)70,70,80
70 IF(RELS=RELQ120.0,120.0,75
75 IF(DELQ*DELQ=ABSQ120.0,120.0,80
80 GO TO 90
100 CONT
50 ITER=ITER+1
    IF (250-ITER) 310,37,37
100 IF (DTEST-DELSQ) 34,34,110
110 DTEST=DELSQ
    P(2)=A(2)-P*B(1)
    DO 115 K=3,NPI
115 R(K)=A(K)-P*R(K-1)-Q*A(B)(K-2)
    GO TO 45
C ITERATION HAS CONVERGED
120 GO TO (130,140),L
130 L=2
    LAST=N
    GO TO 110
C FACTOR OUT QUADRATIC
140 NBAR=NBAR-2
    NBP1=NBAR+1
    ABAR(2)=ABAR(2)-P*ARAR(1)
    DO 150 K=3,NBP1
150 ABAR(K)=ABAR(K)-P*ABAR(K-1)-Q*ABAR(K-2)
    GO TO 250
C SOLVE LINEAR EQUATION
200 NBAR=NBAR-1
    RI=ABAR(2)/ABAR(1)
    R2=0.
    GO TO 262
C NORMALIZE QUADRATIC
210 P=ABAR(2)/ABAR(1)
    Q=ABAR(3)/ABAR(1)
    NBAR=NBAR-2
C SOLVE NORMALIZED QUADRATIC
250 RI=-P/2.
    C1=RI*R1-Q
    IF (C1)*270,280,260
260 C1=DSQRT(C1)
    R2=RI-C1
    R1=RI+C1
    GO TO 290
262 C1=0.
    GO TO 290
270 C1=-C1
    C1=DSQRT(C1)
    R2=RI
280 R2=RI
290 C2=-C1
    AN(I+1)=C1
    AN(I+1+1)=RI
    AN(I+1+2)=C2
    AN(I+1+3)=R2
    C1=III+4
    IF (NBAR-1) 14,200,15
C SPECIAL CONDITIONS
310 WRITE (6,600)
600 FORMAT(1X,50HNO CONVERGENCE IN 250 ITERATIONS, POOLY HAS SP3KEN)
4 CONTINUE
    RETURN
END
SUBROUTINE SETIT(LGO,M,NV,REB,X,Y,UC,PRESS,GRAD,DELT,DISP,THETA,VISETUP)

C SUBROUTINE FOR CALCULATION OF BOUNDARY LAYER THICKNESS, DISPLACEMENT THICKNESS, MOMENTUM THICKNESS AND FLOW VISCOITY.

DIMENSION X(300),Y(100),UC(100,3),VISC(100,2),GRAD(100)

RTR=SQR(REB)
NY = NV + 2
UEDGE = .995*UC(NY,1)
DO 10 N=1,NV
IF(UEDGE-UC(N+1,1)) 41,41,10
41 NDELT = N
GO TO 20
10 CONTINUE
20 DELT = Y(NDELT)* (UEDGE-UC(NDELT,1))*(Y(NDELT+1)-Y(NDELT))/(Y(N+1)-Y(N))
SUM = 0.
DO 50 N=2,NV
50 SUM = SUM + (Y(N)-Y(N-1))*UC(N,1)*UC(N-1,1)
DISP = (Y(NV)-.5*SUM/UC(NV,1))/RTR
SUM = 0.
UEDGE = UC(NV,1)
DO 60 N=2,NV
60 SUM = SUM + (Y(N)-Y(N-1))* (UEDGE-UC(N,1))*UC(N,1)*(UEDGE-UC(N-1,1))
THEETA = .5*SUM/(RTR*UEDGE**2)
IF(LGO) 53,55,56
53 NVPI = NV+1
EASY = 1.
IF(M-MTRAN) 31,32,32
32 IF(MTRAN+5-M) 31,31,33
33 EASE = (X(M)-X(MTRAN))/(X(MTRAN+5)-X(MTRAN))
31 CONTINUE
INNER=0
FAC1 = -16*RTR*EASY
FAC2 = -016*UEDGE*DISP*REB*EASY
FFAC1 = -RTR/26.
FFAC2 = PRESS/RTR
TAUW = GRAD(1)/RTR
DO 160 N=2,NVPI
ALTER = 1.+FAC2/(1.+5.5*(Y(N)/DELT)**6)
IF(INNER) 402,401,402
402 VISC(N,1)=ALTFR
GO TO 160
401 CONTINUE
TAUW=TAUW-Y(N)*EFAC2
IF(TAUW) 701,701,702
7C1 VISC(N,1)=1.
GO TO 703
702 FX=Y(N)*EFAC1*SQRT(TAUW)
VISC(N,1) = 1.+FAC1*Y(N)*ABS(GRA(N))* (1.-EXP(EX))**2
7C3 IF(VISC(N,1)=ALTER) 150,160,521
521 VISC(N,1)=ALTER
INNER=1
16C CONTINUE
SAVE=1.
DO 162 N=2,NV
RAVE=VISC(N,1)
VISC(N,1)=(VISC(N+1,1)+RAVE+SAVE)/3.
162 SAVE=RAVE
56 CONTINUE
RETURN
END
SUBROUTINE MIXER(FPRESS,PRES,PRECI,INF,UDOT,THICK,NF,XBSIGNSIG,NDT,DEL/MIXER 1
11,THETL,REB,USEP,X4,P4)
DIMENSION FPRESS(100),THICK(24),XBSIG(100)
FCAP(X)=-19.556*X+107.535*X*X-336.33*X*X*X+308.1*X**4-295.96*X**5 MIXER 2
UI1(X)=-.46932*X+.68425*X*X-.45293*X**3+.6592*X**4 MIXER 3
UI2(X)=-.045929*X-.9165*X*X+2.91843*X**3-.542125*X**4 MIXER 4
DIST=.5*(XBSIG(2)-XBSIG(1)) MIXER 5
XSEP=XBSIG(1)-DIST MIXER 6
XATT=XBSIGNSIG+DIST MIXER 7

C IF INDT IS NONZERO, THE BOUNDARY LAYER IS TURBULENT MIXER 8
C AT SEPARATION.
C
CALL H4X4(INOT,XSEP,DEL1,THEL,XATT,REB,USEP,X3,X4,H4)
IF(XSF-1.1)24,25,25
25 CP4=0.
GO TO 27
24 UMAT=EXP(-.08712-U1(H4)-.24723*(.3255*UI2(H4))) MIXER 9
CP4=1.-(1.-PREC)/UMAT**2 MIXER 10
DEADL=XATT-XSEP MIXER 11
IF(DEADL-.1)5,6,6 MIXER 12
5 G=.5*DEADL**2 MIXER 13
GO TO 7
6 G=1. MIXER 14
7 CP4=PREC-(CP4-PREC)*(1.-G*XSEP) MIXER 15
27 CONTINUE
COEF=(PREC-CP4)/(XATT-X4)
C2=.2*UDOT/UINF MIXER 16
DO 20 H=1,NSIG MIXER 17
20 CONTINUE MIXER 18
CONTINUE MIXER 19

C & C - 2 * UINF
C - 20 M=1,NSIG MIXER 20
SUM=0.
COUNT=0.
X=XBSIG(M)
IF(X-1.)2,2,3
2 THETA = ARCTAN(X) MIXER 21
TANT = SIN(.5*THETA)/COS(.5*THETA) MIXER 22
C1 = -CZ*(1.-COS(THETA)) MIXER 23
DO 10 N=1,NF MIXER 24
COUNT=COUNT+1.* MIXER 25
ANGLE=COUNT*THETA MIXER 26
10 SUM=SUM+THICK(N)*(C1*COS(ANGLE)+C2*(COUNT*TANT*SIN(ANGLE)-C1*SIN(ANGLE)) MIXER 27
IF(-1.)41,25,25
41 SUM=SUM-.5*CZ*THICK(1)
GO TO 35
3 CI=CZ*(1.-X)
XRAD=1./X*SQRT(X*X-1.) MIXER 28
CI=CZ*(X-1.1) MIXER 29
RF=SQRT((X-1.)/X*1.)) MIXER 30
SUM=THICK(1)*XRAD*{(CZ*(RF-1.1)-CZ*(1.-.5*XRAD)) MIXER 31
FRAD=XRAD MIXER 32
COUNT=1.* MIXER 33
DO 30 N=2,NF MIXER 34
COUNT=COUNT+1.* MIXER 35
FRAD=FRAD*XRAD MIXER 36
30 SUM=SUM+THICK(N)*FRAD*(CZ*(COUNT*RF-1.)+CI) MIXER 37
35 END MIXER 38

93
35 CP = CP4
IF (X-X4) 55, 50, 50
50 CP = CP + (X-X4) * COEF
55 CONTINUE
FPRES(M) = UINF + CP + SUM
20 CONTINUE
RETURN
END
SUBROUTINE YSET(R,A, NY, Y)
DIMENSION Y(100)
RPI=1.+R
Y(1)=0.
Y(2)=A
DO 10 N=3, NY
10 Y(N)=RPI*Y(N-1)-A*Y(N-2)
RETURN
END
SURROUTINE H4X4(I, NOT, X1, DEL1, THET1, X5, RER, U1, X3, H3, X4, H4)
CURLF(H)=26.703/H+305.33*ALOG(H)-2111.3*H+3327.8*H*H-2403.9*H*H*H+3
FDDELTX4=EXP(2.5734+3*252*X-4379*X**X+.076511*X**X+.0039707*X**X)H4X4
FAITH(X)=EXP(-3.7481+.038772*X+.41967*X**X+.071046*X**X+.0032162*X*H4X4
1*4)

10 FORMAT(/20X,54HA SOLUTION FOR X4 COULD NOT BE OBTAINED IN 1000 TRIAN
11ALS)
MOUT=6

C IF NOT IS NONZERO, THE BOUNDARY LAYER IS TURRULFNT
C AT SEPARATION.
C
1 IF(NOT) 2,5,2
2 H3=THETI/DELI
X3=X1
DEL3=DELI
GO TO 20
5 X3=X1+5.F4/(U1*REB)
ARG=ALOG((X3-X1)/(REB*DELI*DELI))
H3=THET1*FAITH(ARG)/DELI
DEL3=50*FDDELTX4/DELI
IF(X3-X5) 20,15,15
15 H4=.429
X4=X5
GO TO 50
20 CONTINUE
IGN=0
DIST=X5-X1
UNDER=0
H4=H3+H3
COEF1=DEL3*H3
COEF2=10.5*DEL3*H3
SUB=X3-COEFL*CURLF(H3)
95 OVER=H4
H4=.5*(H4+UNDER)
X4=CURLF(H4)*COEF1+SUB
ALTER=X5-COEF2*(1.-H4/4.429)**21/H4
IGN=IGN+1
IF(X4-ALTER) 41,50,42
41 IF(IGN-1000) 95,61,61
42 IF(ABS(X4-ALTER)/DIST-.001) 50,50,43
43 CONTINUE
H4=.5*(OVER+H4)
X4=CURLF(H4)*COEF1+SUB
ALTER=X5-COEF2*(1.-H4/4.429)**21/H4
IGN=IGN+1
IF(X4-ALTER) 52,50,51
51 IF(IGN-1000) 43,61,61
52 IF(ABS(X4-ALTER)/DIST-.001) 50,50,95
61 H4=.429
X4=X5
WRITE(MOUT,10)
50 CONTINUE
RETURN
END
SUBROUTINE SFTSX(NSPI, XSEP, XATT, XSIG, ANGLE)
DIMENSION XSIG(100)

A = 5*(XSEP+XATT)
B = 5*(XATT-XSEP)
ARG = 0

DO 5 N=1,NSPI
XSIG(N) = A - B*COS(ARG)
ARG = ARG + ANGLE

RETURN
END

SFTSX 1
SFTSX 2
SFTSX 3
SFTSX 4
SFTSX 5
SFTSX 6
SFTSX 7
SFTSX 8
SFTSX 9
SFTSX 10
FUNCTION ARCT(X)

PI = 3.14159

IF (ABS(X) - 1.E-6) 1, 2, 2

1  ARCT = .5 * PI
    GO TO 6

2  IF (X + .99999) 3, 4, 4

3  ARCT = PI
    GO TO 6

4  ARCT = ATAN (SQR (1 - X * X) / X)
    IF (ARCT) 5, 6, 6

5  ARCT = ARCT + PI

6  CONTINUE
RETURN
END

ARCT 1
ARCT 2
ARCT 3
ARCT 4
ARCT 5
ARCT 6
ARCT 7
ARCT 8
ARCT 9
ARCT 10
ARCT 11
ARCT 12
ARCT 13
ARCT 14
FUNCTION GAM1(ACAP, DXI, PI)
DIMENSION ACAP(30,3)
GAM1=PI*(-1.5*ACAP(1,1)-.75*ACAP(2,1)+2.*ACAP(1,2)+ACAP(2,2)-.5*ACGAM1)
LAP(1,3)-.25*ACAP(2,3)) / DXI
RETURN
END
FUNCTION FB(X1, X2, X3, Y)
D1=1./ (X2-X1)
D2=1./ (X3-X2)
T1=ABS(Y-X1)
T2=ABS(Y-X2)
T3=ABS(Y-X3)
EPS=1./E-6
IF(T1<EPS) 2,3,3
2  F1=0.
   F2=ALOG(T2)
   F3=ALOG(T3)
   GO TO 10
3  F1=ALOG(T1)
   IF(T2<EPS) 4,5,5
4  F2=0.
   F3=ALOG(T3)
   GO TO 10
5  F2=ALOG(T2)
   IF(T3<EPS) 6,7,7
6  F3=0.
   GO TO 10
7  F3=ALOG(T3)
10  FB=(Y-X1)#F1+(D1+D2)#(X2-Y)#F2+(Y-X3)#F3*D2/3.14159
RETURN
END
SUBROUTINE EGAM1(NU,NG,A,B,XSEP,XATT,GAMMA,Y,G1)

DIMENSION A(30,3)
SINT=SQR1(1.-Y*Y)
THETA=ARCT(Y)
SUM=0.
COUNT=1.
DO 6 N=2,NG
COUNT=COUNT+1.
6 SUM=SUM+A(N+1,NU)*(SIN(COUNT*N)*THETA)/(COUNT+1.)*SIN((COUNT-1.)*THETA)
    GI=(3.1416-THETA*SINT)*(A(1,NU)+.5*A(2,NU))+.5*SUM-.25*GAMMA*(1.+EGAM1)
    IF(Y-XATT).LT.8.8.17
    CONTINUE
    RETURN
END

102
SUBROUTINE ESIGI(NY, NX, XS, A, Y, SI)
DIMENSION XS(100), A(3, 3)
SUM = 0.
DC 1C I = 2, NX
10 SUM = SUM + A(I, NY) * G9(XS(I-1), XS(I), XS(I+1), Y)
SI = A(I, NY) * AINT(XS(I), XS(NX+1), Y) + SUM
RETURN
END
FUNCTION GB(X1,X2,X3,X)
GB = ABINT(X1,X2,X) - ABINT(X3,X2,X)
GB = GB/3.14159
RETURN
END
FUNCTION ABINT(A,B,X)

ARGA=ABS(X-A)
ANCR=ABS(X-B)
COEF=2.*(B-A)
AP1=A+1.
BP1=B+1.

IF(ARGA-1.E-1) 2,3,3

2 CA=0.
GO TO 5

3 CA=ALOG(ARGA)
IF(ARGA-1.E-6) 4,5,5

4 CR=0.
GO TO 6

5 CR=ALOG(ARGA)

6 ABINT=(CA-.5)*ARGA**2-(CB-.5)*ARGB**2=(ALOG(AP1)-.5)*AP1**2+(ALOG(BP1)-.5)*BP1**2-CR
ABINT=(ABINT/COEF)
RETURN

FND

ABINT 1
ABINT 2
ABINT 3
ABINT 4
ABINT 5
ABINT 6
ABINT 7
ABINT 8
ABINT 9
ABINT 10
ABINT 11
ABINT 12
ABINT 13
ABINT 14
ABINT 15
ABINT 16
ABINT 17
ABINT 18
ABINT 19
FUNCTION BINT(XS,XZ,X)
RTS=SQRT(1.+XS)
RTZ=SQRT(1.+XZ)
BINT=-1.-X+RTS*RTZ
IF(XZ-X) 1,2,3,4
2 RTSX=SQRT(X-XS)
RTZX=SQRT(X-XZ)
BINT=BINT+(XZ-XS)*ALG(SQRT(1.+RTS)+(1.+RTZ))/RTS*RTZX
GO TO 50
BINT 1
BINT 2
BINT 3
BINT 4
BINT 5
3 IF(XZ-XS) 5,5,4
4 BINT=BINT+(XZ-XS)*ALG(SQRT(XZ-XS)/(1.+RTS))
GO TO 50
BINT 6
BINT 7
BINT 8
BINT 9
BINT 10
BINT 11
BINT 12
5 RTSX=SQRT(XS-X)
RTZX=SQRT(XZ-X)
BINT=BINT+(XS-XS)*ALG((1.+RTS)+(1.+RTZ))/RTS*RTZX
BINT 13
BINT 14
BINT 15
BINT 16
BINT 17
BINT 18
50 CONTINUE
RETURN
END
BINT 18
SUBROUTINE SCAL(SAL, NSAL, FRZ, ARR, RDBR)
DIMENSION SAL(300)
DELZ = FRZ*RDBR
EN = ARR/FRZ
DO 5 N=1,300
  IF (EN-N) 4,4,5
4  NE=N
  GO TO 6
5  CONTINUE
6  NG=NSAL-NE
  EN=FLOAT(NG)
  NGM1=NG-1
  SBL(1)=0.
  DO 7 N=2,NE
  7  SAL(N)=SAL(N-1)+DELZ
     FRAC=2.2/DELZ
     FRACT=FRACT-1.
     R=FRACT*(1./FLOAT(NGM1))
     SAVE=R
     R=R-(R**NG-FRACT*R+FRACT)/(EN*R**NGM1-FRACT)
     IF (ABS(SAVE-R)-1.E-6) 9,9,8
8  RPL=R+1.
  DO 10 N=NE,NSAL
  10  SBL(N+1)=RPL*SBL(N)-R*SBL(N-1)
  RETURN
END
SUBROUTINE TERPFP(XI,J,TA81,TA82,TA83,TA84,XITAR,FPr)
DIMENSION TAR1(24),TAB2(24),TAB3(24),TAB4(24),XITAR(24)
IF(XI-.0001) 2,2,10
2 GC TC (3,4,5,6),J
3 FP=2.53-2.439*ALG(XI)
GO TO 99
4 FP=3.54-1.725*ALG(.7071*XI)
GO TO 99
5 FP=4.58-1.2195*ALG(.5*XI)
GO TO 99
6 FP=10.12
GO TO 99
10 DO 12 N=1,24
IF(XI-XITAB(N),11,11,12
11 NX=N
GO TO 13
12 CONTINUE
13 TX=(XI-XITAB(NX-1))/(XITAB(NX)-XITAR(NX-1))
TX1=1.-TX
GO TO (14,15,16,17),J
14 FP=TX1*TAR1(NX-1)+TX*TA81(NX)
GO TO 99
15 FP=TX1*TAB2(NX-1)+TX*TA82(NX)
GO TO 99
16 FP=TX1*TAR3(NX-1)+TX*TA83(NX)
GO TO 99
17 FP=TX1*TAB4(NX-1)+TX*TA84(NX)
CONTINUE
RETURN
END
TERPF 1
TERPF 2
TERPF 3
TERPF 4
TERPF 5
TERPF 6
TERPF 7
TERPF 8
TERPF 9
TERPF 10
TERPF 11
TERPF 12
TERPF 13
TERPF 14
TERPF 15
TERPF 16
TERPF 17
TERPF 18
TERPF 19
TERPF 20
TERPF 21
TERPF 22
TERPF 23
TERPF 24
TERPF 25
TERPF 26
TERPF 27
TERPF 28
TERPF 29
TERPF 30
SUBROUTINE EVAL(NNF, XX, SSC, SST, CCA, TTB, CCM, TTM)
DIMENSION SSC(50), SST(50)
COST = 2.*XX - 1.
COSTS = COST**2
IF(COSTS-1.E-8) 303, 304, 304
304 TANT = SQRT(1./COSTS - 1.)
THF = ATAN(TANT)
GO TO 305
303 THE = 1.5708
305 IF(COST) 403, 404, 404
403 THE = 3.14159 - THE
404 ARG = 0.
SUM1 = 0.
SUM2 = 0.
DO 551 N=1, NNF
ARG = ARG + THE
SUM1 = SUM1 + SSC(N)*SIN(ARG)
551 SUM2 = SUM2 + SST(N)*SIN(ARG)
CCM = SUM1*SIN(THE)*CCM
TTB = (1. - COS(THE))*SUM2*TTM
RETURN
END

EVAL 1
EVAL 2
EVAL 3
EVAL 4
EVAL 5
EVAL 6
EVAL 7
EVAL 8
EVAL 9
EVAL 10
EVAL 11
EVAL 12
EVAL 13
EVAL 14
EVAL 15
EVAL 16
EVAL 17
EVAL 18
EVAL 19
EVAL 20
EVAL 21
EVAL 22
SUBROUTINE SIMP(NS, DX, ORD, FIND)  SIMP 1
DIMENSION ORD(50)  SIMP 2
C INTEGRATION OF NS + 1 EQUALLY SPACED ORDOinate VALUES  SIMP 3
C BY SIMPSON'S RULE. NS MUST BE EVEN  SIMP 4
SUM = 0.  SIMP 5
DC 88 I=2,NS+2  SIMP 6
88 SUM = SUM + 2.*ORD(I-1) + 4.*ORD(I)  SIMP 7
FIND = DX*(SUM - ORD(1) + ORD(NS+1))/3.  SIMP 8
RETURN  SIMP 9
END  SIMP 10
PROGRAM TO COMPUTE COEFFICIENTS TN AND CN OF THE FOURIER SERIES

DIMENSION XU(30), YU(30), XL(30), YL(30), YUC(30), YLC(30), ST(24), SC(24)

12 FORMAT(///47X,26HINPUT AND COMPUTED OFFSETS/) SECT 6
13 FORMAT(19X,4HX/1/C,12X,4HY/C,11X,5HYUC/C,23X,4H/C,12X,4HYL/C,11X) SECT 7
15, SHYLC/C/) SECT 8
14 FORMAT(3X,3F16.5,4X,3F16.5) SECT 9

NA=6
RNA=6.
RF=FLOAT(NF)
MCUT=6
PI = 3.14159
DELT = PI/(2.*RF)
NTC = 2*RF - 1
NINT = NTC + 2
NSIMP = NTC + 1
RDC=5*RDCBC
VARY = 0.
CA = 0.
TB = 0.

THEA = 0.
DO 89 K=1,NTC
THFTA = THEA + DELT
X1 = .5*(1. + COS(THTFA))
DO 90 LAM=2,NOFF

110 YUINT = YULAM-1) + (XI - XULAM-1))*YULAM) - YULAM-1))/XULAM) SECT 29
1) - XULAM-1)) SECT 30

GA TO 111
89 CONTINUE
SECT 31
111 DO 80 LAM=2,NOFF

IF(XI-XULAM) 110,90,90

210 YLINT = YL(LAM-1) + (XI - XLLAM-1))*YL(LAM) - YLLAM-1))/XLLAM) SECT 35
1) - XLLAM-1)) SECT 36

GA TO 112
80 CONTINUE
SECT 37
112 TRARTK+1) = .5*(YUINT - YLNT)
99 CHAR(K+1) = .5*(YUINT + YLINT)

TMAX = 0.
CMAX = 0.
DO 79 K = 2,NSIMP

IF(TBAR(K)=TMAX) 801,802,802

801 TMAX = TRARTK) SECT 45
802 IF(CCHAR(K)=CMAX) 79,702,702
702 CMAX = CHARTK) SECT 47
75 CONTINUE

IF(CMAX<=.5) 1201,1202,1202
1201 CMAX=1. SECT 49
1202 CONTINUE

IF(TMAX<=.5) 1140,1141,1141

1140 TMAX=1. SECT 53
1141 DO 69 K=2,NSIMP

TBART(K) = TRARTK)/TMAX

111
65  CBAR(K) = CRAR(K)/CMAX
    TBAR(1) = 0.
    CRAR(1) = 0.
    TRAR(NINT) = 0.
    CRAR(NINT) = 0.
    TTA = TRAR(NA)
    TTR = TRAR(NA+1)
    TTC = TRAR(NA+2)
    TAA = DELT*(RNA-1,1)
    TBB = TAA + DELT
    TCC = TBB + DELT
    KA = .5*COS(TAA)
    XB = .5*COS(TBB)
    XC = .5*COS(TCC)
    SLOPE = ((TTC-TTB)*(XB-KA)/(XC-XB) + (TTB-TTA)*(XC-XR)/(XB-XA))/XSEC 70
    IC=I 70
    THETA = 0.*
    COSR = COS(TBB)
    DO 456 I = 2,NA
    THEA = THETA + DELT
    COST = COS(THETA)
    456  TBAR(I) = (SQRT(1. - COST)/(1. - COSR)**1.5)*(TBB*(1. + COST-2.*COSB))/XSEC 77
    1. - COSR) + .5*SLOPE*(COST-COSB))
    NLE = 2*NF + 1 - NA
    COSR1 = 1. + COS(PI-RNA*DELT)
    THEA = PI
    SINA=SIN(RNA*DELT)**2
    COSA=COS(RNA*DELT)
    ANG=0.
    DO 457 I = 2,NA
    IND = 2*NF + 2 - 1
    THEA = THETA - DELT
    COST1 = 1. + COST*THETA
    ANG=ANG+DELT
    COEF=(SINAS-SIN(ANG)**2)/(COSR1*(COSI*ANG)+COSAS))
    457  TBAR(IND) = (SQRT(RDBC*COST)*COEF/TMAX*TBAR(NLE)*(COST1/COSR1)**1)
    1.5)/(2. - COST)
    THEA = TAA
    NAPI = NA + 1
    DO 458 I = NAPI,NLE
    THEA = THEA + DELT
    458  TBAR(I) = TRARI/I (1. - COS(THEA))
    THEA = 0.
    DO 459 I = 2,NSIMP
    THEA = THEA + DELT
    459  CBAR(I) = CBAR(I)/SIN(THEA)
    RKK = 0.
    DO 59 K = 1,NF
    RKK = RKK + 1.
    THEA = 0.
    DO 777 I = 1,NINT
    DUM(I) = TBARI/I*SWIN(THEA*RKK)
    777  THEA = THEA + DELT
    CALL SIMP1(NSIMP,DELT,DUM,VARY)
    STK(I) = 2.*VARY/PI
    112
THETA = 0.
DO 888 I=1,NINT
DUM(I) = CRAR(I)*SIN(THETA*RKK)
END

THETA = THETA + DELT
CALL SIMP(NSIMP,DELT,DUM,VARY)

59 SC(K) = 2.*VARY/PI
DO 969 I=1,NOFF
   X = XU(I)
   CALL FVAL(NF,X,SC,ST,CR,TR,CMAK,TMAX)
969 CONTINUE
END

569 YLC(I) = CB + TR
   X = XL(I)
   CALL FVAL(NF,X,SC,ST,CR,TR,CMAK,TMAX)

669 YLC(I) = CB - TR
   SUM1 = 0.
   COUNT = 0.
   DO 659 T=1,NF
      COUNT = COUNT + 1.
559 SUM1 = SUM1 - ST(T)*COUNT*(-1.)*T**2
   RCDRC = R*(TMAX*SUM1)**2
   RCDRC=2.*RCDRC
   TMAX=2.*TMAX
   CMAK=2.*CMAK
   WRITE(MOUT,12)
   WRITE(MOUT,13)
   WRITE(MOUT,14) (XI(I),YU(I),YUC(I),XL(I),YL(I),YLC(I),I=1,NFF)
   RF TURN
END
SUBROUTINE CORDX(NSRL,NZ,RDAB,SRL,XC)
C
C BOUNDARY LAYER COORDINATES AND CORRESPONDING CHORDAL
C COORDINATES ARE COMPUTED HERE.
C
DIMENSION SRL(300),X(300),XC(300)

FORMAT(10X,31HITERATION TO COMPUTE XC FOR M =15,32H DID NOT
ERGE IN 1000 STEPS.)

FORMAT(1H1,25X,1H%M,20X,1HS,25X,1HX,24X,2HXC//)

FORMAT(22X,15,3E25.5)

MOUT=6
MX = NSBL + NZ - 1
RZERO = RDAB/2.
XC(NZ) = -1.
DO 255 M=1,NZ
MM = NZ + 1 - M
255 X(M) = SRL(NZ) - SRL(MM)
DO 256 M=MM,MX
MM = M + 1 - NZ
256 X(M) = SRL(NZ) + SRL(MM)
DO 257 M=1,MX
IF(NZ-M) 333,257,335
257 CONTINUE
X(M) = SBL(NZ) + SBL(MM)
DO 258 M=1,NZ
SAVE = XC(M)
CALC1 = SQRT((1. + X(M))/RZERO)
CALC2 = SQRT((1. + XC(M))/RZERO)
XC(M) = XC(M) + CALC1*SBL(K) - RZERO*(CALC1*CALC2 +
LOG(CALC1+CALC2))/CALC2
IF(ABS(SAVE-XC(M))<1.E-6) 257,257,258
258 CONTINUE
WRITE(MOUT,336) M
257 CONTINUE
WRITE(MOUT,337)
DO 264 M=1,MX
IF(NZ-M) 261,261,262
262 K=NZ-M+1
GO TO 263
261 K=M+1-NZ
263 CONTINUE
WRITE(MOUT,338) K,SBL(K),X(M),XC(M)
264 CONTINUE
RETURN
END

114
SUBROUTINE PGRAD(IX, X, UE, DXI, PRESS, SA, SR, SC, SS)
C
C SUBROUTINE FOR CALCULATION OF PRESSURE GRADIENT AND
C DERIVATIVE COEFFICIENTS.
C
DIMENSION X(300), UE(300, 3)
D1Z=X(M+1)-X(M)
D2Z=X(M+2)-X(M)
D21=X(M+2)-X(M+1)
D1M1=X(M+1)-X(M-1)
D2M1=X(M)-X(M-1)
XIM=D1Z/(D2Z*D21)
ETA=M1/D1Z-1./D21
ZETAM=D21/(D1Z*D2Z)
PRESS = (3. *UE(M+1, 1)-4. *UE(M+1, 2)+UE(M+1, 3))/(2. *DXI)+UE(M+1, 1)*PGRAD 15
XI=M1*UE(M+2, 1)+ETAM*UE(M+1, 1)-ZETAM*UE(M+1, 1))
SA=1./D1Z+1./D1M1
SR=D1M1/(D1Z*D2M1)
SC=D1Z/(D1M1*D2M1)
SR=D1M1/D2M1
SS=D1Z/D2M1
RETURN
END
SUBROUTINE TRANS (UPRIM, PRESS, THETA, REB, UC, NY, FLAM, XFLAM, LAM) TRANS 1
C C SUBROUTINE TO TEST FOR TRANSITION IN A LAMINAR BOUNDARY LAYER. TRANS 2
C DIMENSION UC(100,3), FLAM(10), XFLAM(10) TRANS 3
C F(X) = .11746 - 1.0582E-3*X - 1.1023E-4*X*X TRANS 4
C TKAY = PRESS*REP*THFTA**2/UC(NY,2) TRANS 5
C IF(TKAY-.077) 2,2,99 TRANS 6
C IF(ARS(TKAY)>.01011) 3,3,4 TRANS 7
C ARG = TKAY**7.2*49 TRANS 8
C GO TO 5 TRANS 9
C ARG = 0. TRANS 10
C DO 6 N=1,1000 TRANS 11
C SAVE = ARG TRANS 12
C ARG = ARG - (ARG**2(TKAY)**2/TKAY)/ (F(ARG)*(.11746-ARG**3.1746E-3 - TRANS 13
C IRG*ARG*5.5115E-4)) TRANS 14
C 6 IF(ARS(.SAVE/ARG)-1.E-6) 7,7,6 TRANS 15
C CONTINUE TRANS 16
C 7 IF(ARS**2.) 8,8,5 TRANS 17
C 8 EF = 1.75 TRANS 18
C 9 GO TO 10 TRANS 19
C 5 DO 15 N=1,10 TRANS 20
C CONTINUE TRANS 21
C 16 NBAR = N TRANS 22
C GO TO 16 TRANS 23
C 15 CONTINUE TRANS 24
C 10 R = .5*EF TRANS 25
C A = 3.36*UPRIM/UC(NY,2)**2 TRANS 26
C RTH = F(ARG)*(SQRTR((R*B+.9860.*A)-B)/A TRANS 27
C IF(REB*THETA-RTH) 99,50,50 TRANS 28
C LAMQ = 0 TRANS 29
C CONTINUE TRANS 30
C RRETURN TRANS 31
C END TRANS 32
SUBROUTINE CAPS(ITER, N, CAPG, CAPH, CAPJ, CAPK, SR, SS, SD, SE, SF, VISCI, V, UCAPS)
1C)
DIMENSION CAPG(100), CAPH(100), CAPJ(100), CAPK(100)
DIMENSION VISCI(100, 2), V(100, 2), UC(100, 3), SD(100), SE(100), SF(100)
IF (ITER) 4, 2, 4
2 CAPG(N) = SR * V(N, 1) - SS * V(N, 2)
CAPH(N) = SR * VISCI(N, 1) - SS * VISCI(N, 2)
CAPJ(N) = SR * (SD(N) * VISCI(N+1, 1) * SE(N) * VISCI(N-1, 1)) - SF * VISCI(N-1, 1)
CAPK(N) = SR * UC(N, 2) - SS * UC(N, 3)
GO TO 6
4 CAPG(N) = 5 * (CAPG(N) + V(N, 1))
CAPH(N) = 5 * (CAPH(N) + VISCI(N, 1))
CAPJ(N) = 5 * (CAPJ(N) + SD(N) * VISCI(N+1, 1) * SE(N) * VISCI(N-1, 1)) - SF * VISCI(N-1, 1)
CAPK(N) = 5 * (CAPK(N) + UC(N, 1))
6 CONTINUE
RETURN
END
SUBROUTINE TERPYIN,YBASE,VARY,NY,VALUE)
C
C SUBROUTINE FOR DETERMINING INTERPOLATED VALUE OF THE

C FUNCTION VARY AT Y = YIN.
C
DIMENSION YBASE(100),VARY(100)
IF(YIN-YBASE(NY-1)) 2,3,3
3 VALUE = VARY(NY)
GO TO 10
2 DO 15 N=1,NY
IF(YIN-YBASE(N)) 24,24,15
24 NRAR=N
GO TO 16
15 CONTINUE
16 D21=YBASE(NBAR)-YBASE(NBAR-1)
D31=YBASE(NRAR+1)-YBASE(NRAR-1)
D32=D31-D21
D3A=YBASE(NBAR+1)-YIN
DA1=YIN-YBASE(N3AR-1)
VALUE=D3A*D2A*VARY(NBAR-1)/(D21*D31)+D3A*DA1*VARY(NBAR)/(D21*D32)-TERP
1D2A*DA1*VARY(NBAR+1)/(D31*D32) TERP 21
10 CONTINUE
RETURN
END
SUBROUTINE YDIFF(NY, ALPHA, RETA, GAMMA, DELTA, SD, SE, SF, C2, C3, C4, Y)

DIMENSION ALPHA(100), RETA(100), GAMMA(100), DELTA(100)
DIMENSION SD(100), SE(100), SF(100), Y(100)

NV = NY - 2
NVPI = NY + 1
DC 40 N = 2, NV

ALPHA(N) = 2.*Y(N) - Y(N - 1) - Y(N + 1) / ((Y(N + 2) - Y(N - 1)) * (Y(N + 2) - Y(N))

DELTA(N) = 2.*Y(N + 2) + Y(N + 1) - 2.*Y(N) / ((Y(N + 2) - Y(N - 1)) * (Y(N + 1) - Y(N))

RETA(N) = (DELTA(N) * (Y(N) - Y(N - 1))) ** 3 - ALPHA(N) * (Y(N + 2) - Y(N)) ** 3)

GAMMA(N) = -ALPHA(N) - RETA(N) - DELTA(N)
CONTINUE

DC 39 N = 2, NVPI

SD(N) = (Y(N) - Y(N - 1)) / ((Y(N + 1) - Y(N - 1)) * (Y(N + 1) - Y(N)))
SE(N) = 1. / (Y(N - Y(N - 1))) - 1. / (Y(N + 1) - Y(N))
SF(N) = (Y(N + 1) - Y(N)) / ((Y(N) - Y(N - 1)) * (Y(N + 1) - Y(N - 1))
CONTINUE

C2 = Y(3) * Y(4) / (Y(2) * Y(3) * Y(4) - Y(2))
C3 = -Y(2) * Y(4) / (Y(3) * Y(4) - Y(3) - Y(2))
C4 = Y(2) * Y(3) / (Y(4) * Y(4) - Y(3) * Y(4) - Y(2))
RETURN
END
SUBROUTINE ELDER(BCAP,XSIG,NSIG,UINF,ELD,Y,YMAX)
DIMENSION RCAP(100,3),XSIG(100)
BCAP(NSIG+1,1)=0.
XS=XSIG(1)
XZ=XSIG(NSIG+1)
IF(XZ-1,16,16,1)
1 DEADL=XZ-XS
YMAX=1.0E-10
SUM=5.0*(XSIG(2)-XS)*RCAP(2,1)
DO 10 N=2,NSIG
  X=XSIG(N+1)
  SUM=SUM+.5*(X-XSIG(N))*RCAP(N+1,1)*BCAP(N,1))
10  IF(N-NSIG) 4,2,4
2  ANGLE=1.5708
  GO TO 6
4  ANGLE=ATAN(SQRT((X-XS)/(XZ-X)))
6  Y=SUM*BCAP(1,1)*DEADL*ANGLE-SQRT((X-XS)*(XZ-X)))
  IF(Y-YMAX) 10,10,8
8  YMAX=Y
10 CONTINUE
ELD=Y/YMAX
IF(ABS(ELD)-UINF) 20,20,12
12 IF(ELD) 14,16,16
14 ELD=-UINF
  GO TO 20
16 ELD=UINF
20 CONTINUE
RETURN
END
SUBROUTINE REATT (UC, VX, YX, VX, YX, VX, DRY, UE, X5, DELS, MST, REB)
DIMENSION UC(100,3), V(100,2), Y(100)
DIMENSION X(300), IF(300,3)
DIMENSION TARA(24), TAB2(24), TAB3(24), TAB4(24), XITAB(24)
DATA TARA /24.98, 23.97, 21.03, 19.33, 17.61, 15.29, 13.46, 11.54, 10.36, 9.00, 8.00
DATA TAB2 /20.05, 18.85, 17.25, 15.04, 14.13, 13.12, 12.11, 11.09, 10.08, 9.08, 8.08, 7.09, 6.17, 5.24, 4.32
DATA TAB3 /16.65, 15.84, 15.07, 14.28, 13.50, 12.72, 11.95, 11.19, 10.45, 9.72, 9.02
DATA TAB4 /10.12, 9.04, 8.08, 7.14, 6.22, 5.31, 4.42, 3.56, 2.73, 1.93, 1.16, 0.40, 0.00
DATA XITAB /3001, 3002, 3005, 3010, 3012, 3015, 3018, 3021, 3024, 3027, 3030, 3033, 3036, 3039, 3042

106..07, 08..09, 12..14, 16..18, 21..29, 33..35/

3 FORMAT(4/X, 5, HAT REATTACHMENT, BETA = 13.5)

MOUT=6
RTR=SCRV(REF)
UC(1,2)=0.
UC(1,3)=0.
V(1,1)=0.
V(1,2)=0.
DO 5 M=1, MX
IF(X5=X(TM)) 4, 4, 5
5
MST=M2
GO TO 6
CONTINUE

6 XA=X(MST-2)
XB=X(MST-1)
UA=UE(MST-2,1)
UB=UE(MST-1,1)
ZA=LOG((UA*DELS*REB)
PGRAD=2.0 * (UA-UB) / (UA+UB)*(XB-XA))
BETM2=1.0974-SORT((DELS*PGRAD) / (0.249+0.004657*TA)
IF(BETM2-1.) 8, 7, 7

7 BETM2=1.
GO TO 10
8 IF(RETM2-.3) 9, 9, 10
9 BETM2=3
10 BETA=1.7*(BETM2*BETM2)
WRITE(MOUT,3) BETA
AGAM=.0979*BETM2-.0249/BETA
9GAM=.00465/BETA
AH=1.-(5.3439+BETM2)*1.0974-.0249*BETM2)
BH=ETM2*(5.3439+BETM2)*.00465
GAMA=AGAM-BGAM*Z
DERIV=UA*FR*EXP(-ZA)*GAMA*GAMA*(1.+BETA*(1.+AH+BH*Z)) / (AH+BH+B*Z)
IF (DELL-Y(NY-3)) 14, 12, 12
12 RY=RY+DRY
CALL YSET(RY, Y(2), NY, Y)
GO TO 11
14 IF(BETA-4.) 102,101,101
101 TERP8=1.-4./BETA
INDEX=3
GO TO 110
102 IF(BETA-2.) 104,103,103
103 TERP8=.5*BETA-1.
INDEX=2
GO TO 110
104 TFRP8=BETA-1.
INDEX=1
110 K=0
TFRP1=1.-TERP8
50 K=K+1
GO TO (16,17,99),K
16 G=GAMA
DELTA=DELS
UEDGE=UA
L=3
GO TO 18
17 G=GAMB
DELTA=DELA
UEDGE=UA
L=2
18 XICQ=G/(DELTA*RTR*BEM2)
UCOM=RTR*(UEDGE*G)**2
EFCO=G*BEM2
NLAM=NY
DO 75 N=2,NY
X1=Y(N)*XICQ
IF(X1>35) 20,19,19
19 UC(N,L)=UEDGE
GO TO 75
20 CALL TERP8(X1,INDEX,TAB1,TAB2,TAB3,TAB4,XITAB,FP1)
INDEX=1
CALL TERP8(X1,INDEX,TAB1,TAB2,TAB3,TAB4,XITAB,FP2)
INDEX=1
FP=TERP1*FP1+TERP8*FP2
UC(N,L)=UEDGE*(1.-EFCO*FP)
IF(N-NLAM) 21,75,75
21 ALTER=UCOM*Y(N)
IF(ALTER=UC(N,L)) 33,33,32
32 UC(N,L)=ALTR
GO TO 79
33 NLAM=N
CONTINUE
GO TO 50
99 DO 60 K=2,3
SAVE2=0.
DO 60 N=3,NY
SAVE1=UC(N-1,K)
UC(N-1,K)=(SAVE2+SAVE1+UC(N,K))/3.
60 SAVE2=SAVE1
DUXD=0.
C0D=.5/(C0D-X1)
DO 65 N=2,NY
DUXDN=C0D*(UC(N,2)-UC(N,3))
95 CONTINUE
122.
\[ V(N,1) = V(N-1,1) - (Y(N) - Y(N-1)) \times (\text{DUDXP} + \text{DUDX}) \]

\[ V(N,2) = V(N,1) \]

\[ \text{DUDX} \times \text{DUDXP} \]

RETURN

END
SUBROUTINE ELPIT(ALPH1, ALPH2, EMI, TORF, THETZ, UINF, DXI, CMPA, CMPAS) ELPIT 1
SAVET=ALPH1                ELPIT 2
STEP=TORF*DXI               ELPIT 3
SINS=SIN(STEP)              ELPIT 4
C0SS=COS(STEP)              ELPIT 5
CONST=2.*EMI*(UINF/TORF)**2 ELPIT 6
ALPH1=THETZ+(ALPH1-THETZ)*C0SS+ALPH2*SINS/TORF+CONST*(2.*CMPA-CMPA) ELPIT 7
1S)*(1.0-C0SS)+CONST*(CMPAS-CMPA)*(SINS-STEP*C0SS)/(TORF*DXI) ELPIT 8
ALPH2=ALPH2*C0SS-TORF*SINS*(SAVET-THETZ)+CONST*(CMPA-CMPAS)*(1.-C0S ELPIT 9
1S)/DXI+CONST*CMPA*ICRF*SINS RETURN ELPIT 10
END ELPIT 11
SUBROUTINE VWASH(BARG, H, S, NVOR, X1, UNF, ZIP, XGA, NGPL, DXI)

DIMENSION ZIP(30), XGAM(3)

DO 10 N=1, NGPL
      DIFF = XGAM(N) - X1
      SUM = 0.
      DO 5 K = 1, NVOR
         SUM = SUM + DIFF/(DIFF*DIFF + H)
      5          DIFF = DIFF - S
   10 ZIP(N) = ZIP(N) + SUM*BARG

RETURN
END

VWASH 1
VWASH 2
VWASH 3
VWASH 4
VWASH 5
VWASH 6
VWASH 7
VWASH 8
VWASH 9
VWASH 10
VWASH 11
SLAROUTINE WASH(XGAM, NGAM, TIME, ALPH1, ALPH2, HEAVE, AROT, FREQ, PHIH, UWASH
1 INF, CAMRR, VF, VZIP, MOTR, INDV)

DIMENSION XGAM(30), VZIP(30), CAMRR(24)

NGP1 = NGAM+1

ANGLE = FREQ*TIME
GO TO (108, 120), INDV

108
GO TO (110, 120), MOTR

110
CONST = -ALPH2*COS(ANGLE)*UINF + HEAVE*COS(ANGLE+PHIH)*ALPH1*UINF
FACT = -ALPH2*FREQ*SIN(ANGLE)*JINF
GO TO 130

120
CONST = UINF*ALPH1*HEAVE
FACT = -UINF*ALPH2

130
DO 10 M = 1, NGP1

10
X = XGAM(M)
THETA = ARCT(X)
SUM = 0.
COUNT = 0.
DO 20 N = 1, NF
COUNT = COUNT + 1.
20
SUM = SUM + COUNT*CAMRR(N)*CCS(COUNT*THETA)
IF(M-1) 2 4, 2
2
IF(NGP1-M) 3 4, 3
4
SUM = SUM + SUM
GO TO 50
3
COUNT = 0.
COUNT = COUNT*THETA
DO 30 N = 1, NF
30
SUM = SUM + COUNT*CAMRR(N)*SIN(COUNT
50
VZIP = UINF*SUM + CONST + FACT*(AROT-X)
10
CONTINUE
RETURN
END
APPENDIX B

DETERMINATION OF COUPLING PARAMETERS
APPENDIX B

DETERMINATION OF COUPLING PARAMETERS

The characteristic equation for the rotor blade is

\[ \sum_{k=0}^{3} B_{2k} \lambda^{2k} = 0 \]

where

\[ B_0 = f_0 - \frac{\omega^2 T_\beta \theta^2}{M_{\beta \beta} M_{\theta \theta}} - \frac{\omega^2 T_\phi \theta^2}{M_{\phi \phi} M_{\theta \theta}} \]

\[ B_2 = f_2 + 2 \frac{\omega \phi M_\beta \theta T_\beta \theta}{M_{\beta \beta} M_{\theta \theta}} + 2 \frac{\omega^2 M_{\phi \theta} T_\phi \theta}{M_{\phi \phi} M_{\theta \theta}} - \frac{T_\beta \theta^2}{M_{\beta \beta} M_{\theta \theta}} - \frac{T_\phi \theta^2}{M_{\phi \phi} M_{\theta \theta}} \]

\[ B_4 = f_4 - \frac{\omega \phi M_\beta \theta}{M_{\beta \beta} M_{\theta \theta}} - \frac{\omega^2 M_{\phi \theta}^2}{M_{\phi \phi} M_{\theta \theta}} + 2 \frac{M_\beta \theta T_\beta \theta}{M_{\beta \beta} M_{\theta \theta}} + 2 \frac{M_{\phi \theta} T_\phi \theta}{M_{\phi \phi} M_{\theta \theta}} \]

\[ B_6 = 1 - \frac{M_\beta \theta^2}{M_{\beta \beta} M_{\theta \theta}} - \frac{M_{\phi \theta}^2}{M_{\phi \phi} M_{\theta \theta}} \]
in which

\[ f_0 = \bar{\omega}^2 \bar{\phi}^2 \bar{\theta}^2 \]

\[ f_2 = \bar{\omega}^2 \bar{\phi}^2 + \bar{\omega}^2 \bar{\theta}^2 + \bar{\phi}^2 \bar{\theta}^2 \]

\[ f_4 = \bar{\omega}^2 + \bar{\phi}^2 + \bar{\theta}^2 \]

The characteristic equation for the two-dimensional system is found to be

\[ \sum_{k=0}^{3} D_{2k} \lambda^{2k} = 0 \]

where

\[ D_0 = f_0 - \bar{\omega}^2 h_a a_1^2 - \bar{\omega}^2 h_b b_1^2 \]

\[ D_2 = f_2 - \bar{\omega}^2 g_a \bar{x} a_1 - \bar{\omega}^2 g_b \bar{x} b_1 - h_a a_1^2 - h_b b_1^2 \]

\[ D_4 = f_4 - c_4 \bar{x}^2 - g_a \bar{x} a_1 - g_b \bar{x} b_1 \]

\[ D_6 = 1 - c_6 \bar{x}^2 \]

in which

\[ h_a = \frac{M_{\bar{\phi}\bar{\phi}}}{R^2 M_{\theta\theta}} \quad h_b = \frac{M_{\bar{\phi}\bar{\phi}}}{M_{\theta\theta}} \]
\[ g_a = 2 h_a A_1 \quad g_b = 2 h_b A_2 \]

\[ c_4 = \bar{\omega}^2 h_a A_1^2 + \bar{\omega}^2 h_b A_2^2 \]

\[ c_6 = h_a A_1^2 + h_b A_2^2 \]

\[ a_1 = A_1 (\bar{\omega}^2 l_{s_1} + r_m \bar{\omega}^2 l_{s_2}) - B \bar{\omega}^2 l_{s_2} \]

\[ b_1 = A_2 (\bar{\omega}^2 l_{s_1} + r_m \bar{\omega}^2 l_{s_2}) + B \bar{\omega}^2 l_{s_2} \]

Equating \( D_0/D_6 \) to \( B_0/B_6 \), \( D_2/D_6 \) to \( B_2/B_6 \) and \( D_4/D_6 \) to \( B_4/B_6 \) provides three relations in the three unknowns \( \bar{x}, l_{s_1}, \) and \( l_{s_2} \). If \( a_1 \) and \( b_1 \) are eliminated, the following equation for \( \bar{x} \) is obtained:

\[(r_1 t_2 - r_2 t_1)^2 + (r_1 s_2 - r_2 s_1)(t_2 s_1 - t_1 s_2) = 0\]

where

\[ r_1 = -\left[ h_a + \frac{h_b g_a^2}{g_b^2} \right] \quad r_2 = \left[ \frac{\bar{\omega}^2}{\bar{\omega}^2} - 1 \right] h_a \]

\[ s_2 = (\bar{\omega}^2 - \bar{\omega}^2) g_a \bar{x}, \quad s_1 = s_2 + \frac{2 h_b g_a F}{g_b^2 \bar{x}} \]

\[ t_1 = (1 - c_6 \bar{x}^2) B_2/B_6 - f_2 + \bar{\omega}^2 F + \frac{h_b F^2}{g_b^2 \bar{x}^2} \]
$$t_2 = (1 - c_6 \bar{x}^2)(b_2 - b_0/\bar{w}^2)/b_6 - f_2 + \bar{w}^2 F + f_0/\bar{w}^2$$

in which

$$F = f_4 - b_4/b_6 + (b_4 c_6/b_6 - c_4) \bar{x}^2$$

With some algebraic manipulation, a polynomial of fourth degree in \(\bar{x}^2\) can be extracted from that equation. The value of \(\bar{x}\) is taken to be the square root of the smallest positive root of that polynomial. The original equations are then used to solve for \(a_1\) and \(b_1\), from which \(l_{s1}\) and \(l_{s2}\) are readily obtained.
REFERENCES


15. Theodorsen, T.; and Garrick, I. E.: Mechanism of Flutter--A Theoretical and Experimental Investigation of the Flutter Problem. NACA TR 685, 1940.