

THE EFFECT OF AERODYNAMIC PARAMETERS ON  
POWER OUTPUT OF WINDMILLS

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SUMMARY

This paper reports the aerodynamic results of a study on wind power generation the Boeing Vertol Company has conducted during this past year. Windmill power output is presented in terms that are commonly used in rotary wing analysis, namely, power output as a function of drag developed by the windmill. Effect of tip speed ratio, solidity, twist, wind angle, blade setting and airfoil characteristics are given.

INTRODUCTION

Several papers have been written on the performance of windmills, but none, to the author's knowledge, have systematically examined the effects on power output of the various aerodynamic parameters. Therefore, the Boeing Vertol Company formulated a theory and computer program to accept the various parameters that can be studied such as tip speed ratio, solidity, twist, wind angle, blade angle, airfoil characteristics, etc. The purpose of this paper is to present the effect of such variations on power output of a windmill. All combinations possible are not the intent of this study, but it is believed that an initial understanding of how these parameters effect power can be obtained from the data herein.

METHOD OF ANALYSIS

The program used is our SR1BR Rotor Research Program which calculates 10 points along the blade at 12 evenly spaced positions of rotation and sums the individual points to give the usual rotor parameters such as thrust, drag, lift and power. The program is set up to calculate induced velocity based on the disk loading at the particular point of calculation and thus uses a nonuniform downwash program. The data resulting from these calculations are all referenced to the product of wind dynamic pressure, wind velocity, and windmill rotor solidity ratio.

SYMBOLS

d diameter of rotor, ft

P power output

$q$  wind dynamic pressure,  $(1/2) \rho V^2$   
 $R$  blade radius, ft  
 $V$  wind velocity, ft/sec  
 $X$  rotor drag, lb  
 $\alpha_s$  stall angle of airfoil  
 $\alpha_{0.90}$  aerodynamic angle of attack at blade element at 0.90 blade radius  
 $C_{D_0}$  profile drag coefficient of blade at blade angle of attack =  $0^\circ$   
 $\theta_t$  blade twist (linear)  
 $\theta_{0.70}$  blade incidence at 0.70 blade radius  
 $\rho$  air mass density  
 $\sigma$  solidity, ratio of blade area to disk area  
 $\mu$  tip speed ratio, ratio of wind velocity to rotational tip velocity of rotor ( $V/WR$ )  
 $\omega$  angular velocity of rotor

## RESULTS

### Comparison with Test Results

Figure 1 shows that the SR1BR Program calculations compare very favorably with the power output measured values published by the Brace Research Institute for their 32-foot-diameter, three bladed windmill.

### Effect of Blade Angle

Figure 2 shows how power output  $P$  and drag  $X$  vary with blade angle at 0.7 of the blade radius for the baseline windmill chosen for this paper. This baseline<sup>1</sup> performance is based on a blade airfoil section that stalls at a  $14^\circ$  angle of attack, a tip speed ratio of 0.30, and a solidity ratio of 0.20. It will be noted that there are two values of blade pitch where the power is zero. Point (A) is where the relative velocity is in line with the blade elements. Point (B) is where the blade elements are fully stalled as will be noted by the value of angle of attack

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<sup>1</sup>Baseline parameters were arbitrarily established for example only. They should not be accepted as optimum parameters. Future studies should be conducted to determine optimum aerodynamic parameters for specific applications.

at the blade tip of  $19.3^\circ$ , which is beyond the  $14^\circ$  stall angle of the airfoil. The maximum power output  $P$  occurs at a blade angle of  $16^\circ$ , but the important item to note is that at maximum power the blade angle of attack at the 0.90 radius point is  $11.4^\circ$  - about  $3^\circ$  less than the stall angle. Point (C) is maximum efficiency point (that where the power for a given drag is greatest). However, windmills should operate at Point (D) because that is where the power output is greatest. The tower structure must be designed for the drag at Point (B), unless great care is expended in developing a drag limiting governor.

#### Effect of Blade Twist

Figure 3 shows that, for the baseline windmill operating at a tip speed ratio of 0.30 with solidity of 0.20, the best linear blade twist is  $30^\circ$ . Such twist will produce 25 percent more power than an untwisted blade. It should be noted that optimum twist<sup>2</sup> will vary with tip speed ratio.

#### Effect of Solidity

Figure 4 shows that power output normalized to  $(qVd^2\sigma)$  decreases with solidity, but figure 5 (based on the maximum power line of fig. 4) shows that power output is maximum at a solidity between 0.20 and 0.40 for the baseline conditions of  $\theta_t = 30^\circ$ ,  $\alpha_s = 14^\circ$ , and  $\mu = 0.30$ .

#### Effect of Wind Velocity

Figure 6 shows drag and horsepower output for a 100-foot-diameter windmill using the Points (A) and (B) design values from figure 5 for solidity = 0.20. It will be noted that a 45-foot per second wind will generate 800 horsepower or 0.10 horsepower per square foot of rotor swept area.

#### Effect of Airfoil Section

Figure 7 shows that if the airfoil stall angle is doubled without any other changes, that the power output increase over the baseline configuration with  $\alpha_s = 14^\circ$  is 20 percent while the windmill drag force  $X$  is increased 25 percent at maximum power output. The maximum drag that can be developed is almost doubled so windmills with high life airfoils will necessarily have to be provided with stronger towers at an increase in cost.

#### Effect of Wind Angle

Figure 8 shows that output power decreases rapidly if the wind angle with respect to the shaft is very much greater than  $12^\circ$ . Thus, windmills must be provided (as they have through the ages) with some means to point into the wind for maximum power generation.

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<sup>2</sup>Not shown in this paper.

## Effect of Tip Speed Ratio

Figure 9 shows that power output is dependent on tip speed ratio and that for each operating tip speed ratio there will be an optimum solidity to produce maximum power output. Boeing studies are continuing in the effort to quantify such effects.

### CONCLUDING REMARKS

(1) Boeing SR1BR Program gives results that compare favorably with test data.

(2) Maximum power output of windmill occurs when a blade-element angle of attack near tip of blade is about  $3^\circ$  less than airfoil stall angle for the baseline case considered.

(3) Selection of the proper blade twist will increase power output.

(4) For a given blade twist, airfoil and tip speed ratio, there is a range of solidity ratios that will produce nearly the same power.

(5) Power output varies approximately as the cube of the wind velocity. Drag of the windmill varies as the square of the velocity

(6) Higher lift airfoils for windmills will increase the power output, but greatly increase the design drag value for tower design.

(7) Windmills should be designed such that shaft should point into wind with tolerance no greater than  $\pm 12^\circ$  for good power output.

(8) Solidity will vary with operating tip speed ratio to produce maximum power output.

### DISCUSSION

Q: In your computer program can you use that blade element theory?

A: Yes, it is very similar to the vortex theory or ones that are developed on helical vortex analysis.

COMMENT: Certainly you have the solution where you have the slow rotating shaft speed into a high speed by putting some small rotors on the tip of the blades. This is the only position of application where you have rated the towing efficiencies. If you have a fixed wing on an aircraft, you want to give out from the resistance or the drag of the wind the maximum power. This is your efficiency. This efficiency is in another sense involving the ordinary windmill. But the towing efficiency is important for the transformation of speed.

FIG. 1 COMPARISON OF BRACE INSTITUTE WINDMILL POWER TO VALUES CALCULATED BY SRIBR

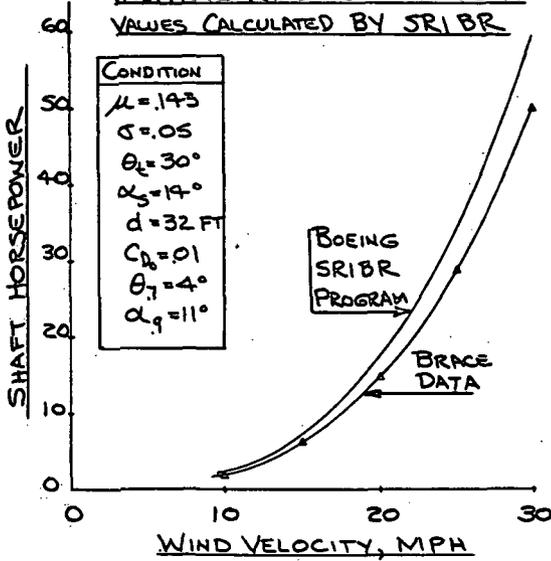


FIG. 2 EFFECT OF BLADE PITCH ON WINDMILL POWER OUTPUT AND DRAG

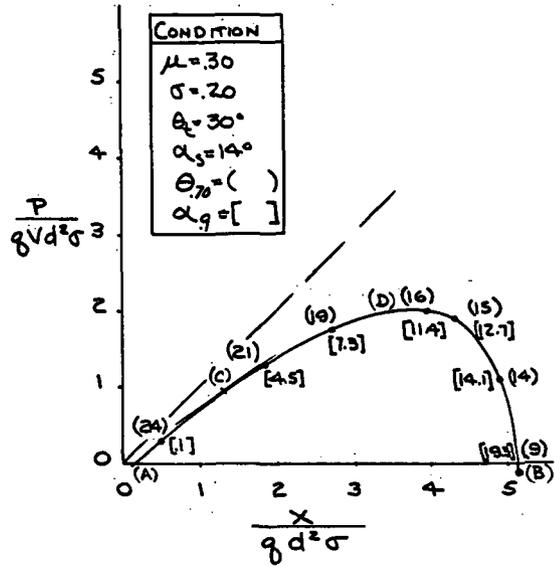


FIG. 3 EFFECT OF BLADE TWIST ON WINDMILL POWER AND DRAG

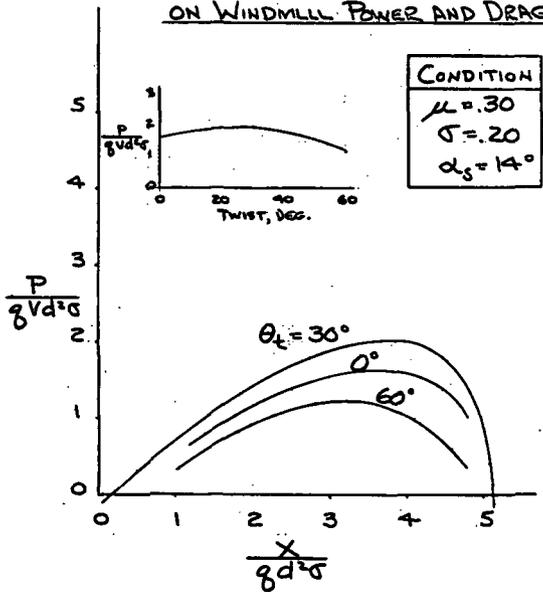
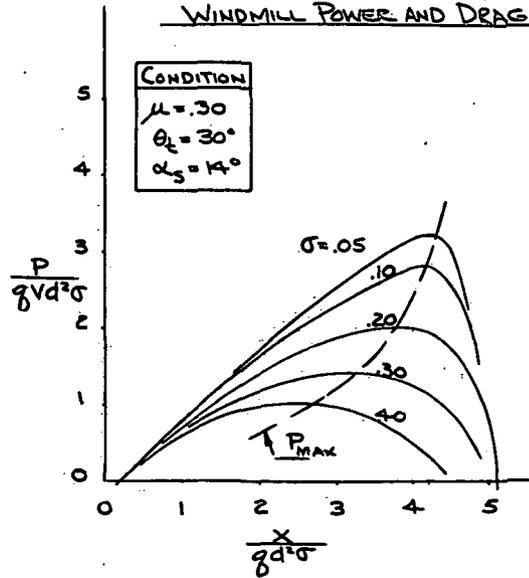


FIG. 4 EFFECT OF SOLIDITY ON WINDMILL POWER AND DRAG



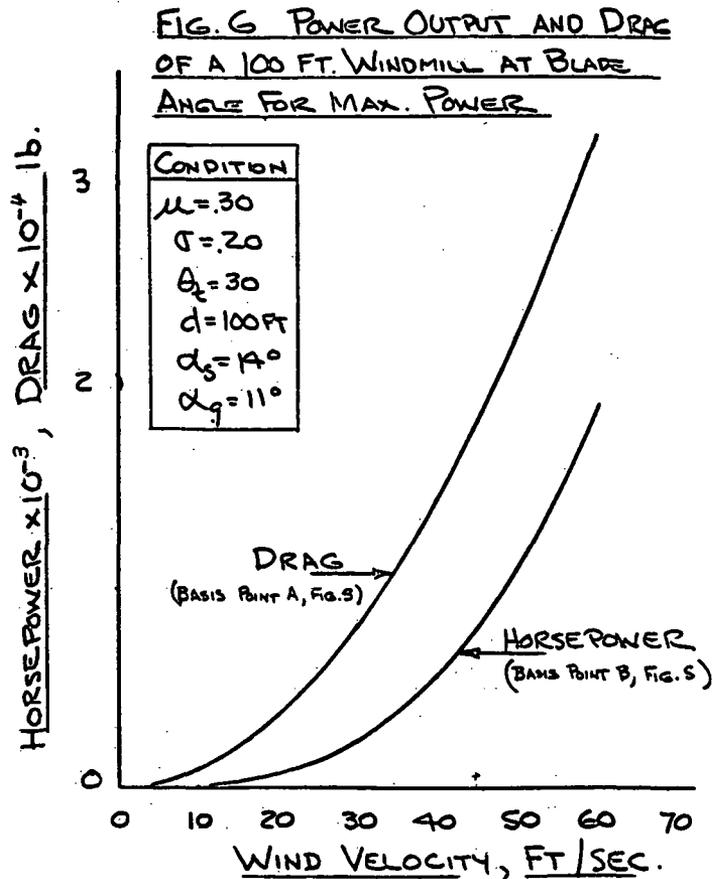
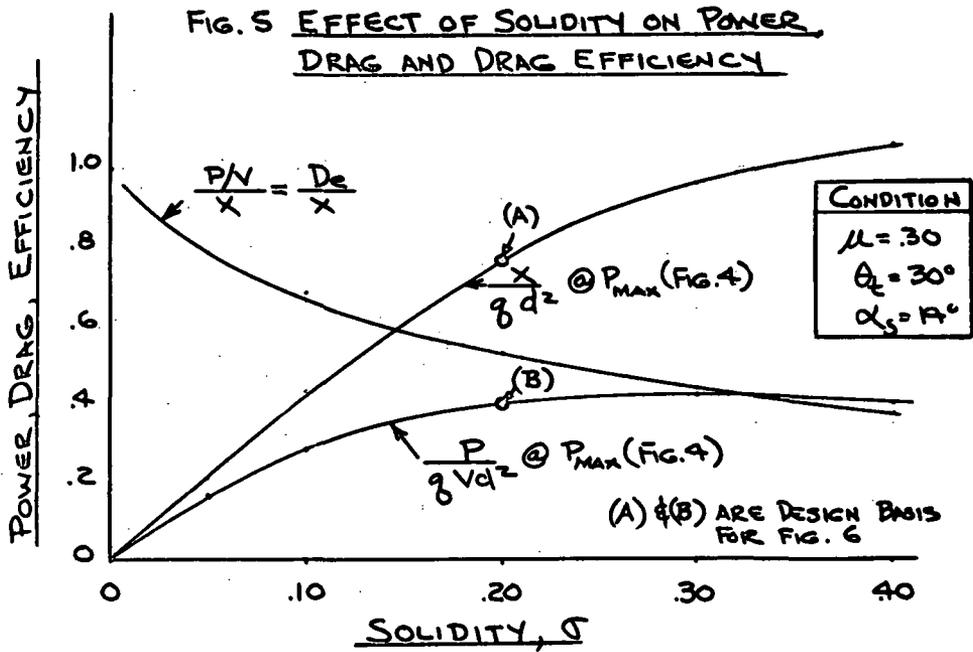


FIG. 7 EFFECT OF BLADE AIRFOIL STALL ANGLE ON POWER AND DRAG

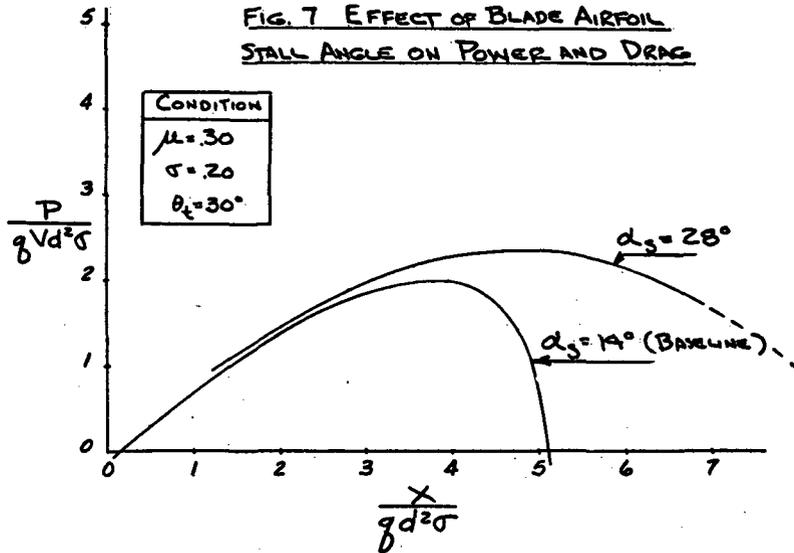


FIG. 8 EFFECT OF WIND ANGLE ON WINDMILL POWER OUTPUT

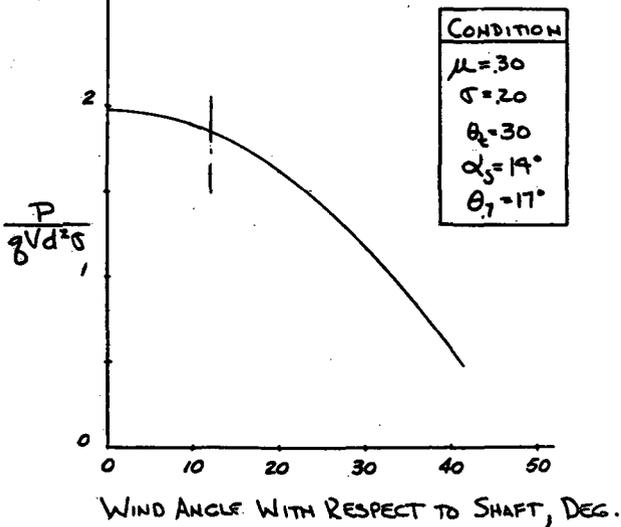


FIG. 9 EFFECT OF TIP SPEED RATIO ON POWER OUTPUT OF WINDMILL

