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RECOMMENDED REFERENCE FIGURES FOR GEOPHYSICS AND GEODESY

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ABSTRACT

Specific reference figures are recommended for consistent use in geophysics and geodesy. The selection of appropriate reference figure for geophysical studies suggests a relationship between the Antarctic negative gravity anomaly and the great shrinkage of the Antarctic ice cap about 4-5 million years ago. The depression of the south polar regions relative to the north polar regions makes the southern hemisphere flatter than the northern hemisphere, thus producing the third harmonic (pear-shaped) contribution to the earth's figure.

RECOMMENDED REFERENCE FIGURES
FOR GEOPHYSICS AND GEODESY

The advent of artificial earth satellites made possible the determination of independent values for the geometric flattening of the earth on the one hand and the flattening of the corresponding equilibrium figure on the other hand. Since that time, there have been frequent debates over the question which flattening should be used for each particular purpose. Even the question of adopting a common reference figure both in geodesy and geophysics has been raised (see, for example, Moritz, 1973). The desirability of having a common reference figure for both of these disciplines stems from the overlap of dynamic geodesy with geophysics, particularly in relation to the geogravity field. The determination and description of the geopotential lie in the domain of dynamic geodesy, but its interpretation in terms of subsurface mass distribution lies properly within the domain of geophysics. But the possibility of using a common figure has to be examined rather carefully. As a first step, we define the purposes requiring a reference figure in each of these disciplines.

In geodesy, we study (a) departures of the geoid from a reference figure; (b) deflections of vertical defined by this reference figure; (c) the locations of various points on the actual surface of the earth (station locations); (d) the distances between two selected points and similar problems. All these quantities

are essentially geometric in character, though they may be definable and determinable by dynamic methods. Thus, in geodesy we deal with quantities which, although they are often determined dynamically are essentially geometric in nature. Consequently, in geodesy we need a reference figure which best fits the actual earth in the geometric sense. It is the shape one would find if one fitted a long wire along a meridian of the earth, at the level of the geoid.

The best way to determine this geometric shape would indeed be just that i.e., to fit long pieces of wire along the various meridians at the level of the geoid and take their average shape. A slightly easier alternative in practice would be the geodetic survey of the entire earth. But a complete geodetic survey coverage is not possible for obvious reasons of inaccessibility (70% of earth's surface is oceans), expense, and even political reasons. It was, however, precisely these scattered survey data which were used to determine the pre-satellite ellipsoidal flattening of 1/297.0 - a figure which was used in the now outdated International Gravity Formula.

The artificial earth satellites provided an excellent and much more accurate alternative. The second harmonic coefficient J_2 in the geopotential is directly related to the earth's flattening f by

$$f = \frac{3}{2} J_2 + \frac{1}{2} m + \frac{9}{8} J_2^2 + \frac{3}{56} m^2 + \frac{15}{28} J_2 m + O(f^3) \quad (1)$$

where: $m = \omega^2 a_e^3 (1-f)/GM$. In this relation, ω is earth's rate of rotation, a_e , its equatorial radius, GM , the product of its mass and gravitational constant. The quantity $O(f^3)$ denotes quantities of the order of f^3 . The coefficient J_2 can best be determined from rate of regression of the nodes of a close earth satellite orbit. The flattening f in the above equation is in fact, what we can call dynamic flattening. The regression of the node is produced by the second harmonic of the earth's gravitational field. This takes account of mass at any depth within the earth, as is clear from the well-known relation between the moments of inertia of a body and the second degree harmonics of the gravitational field.

To a rather coarse approximation, we could say that since the geoid is an equipotential surface of the geopotential, its shape is also a manifestation of the same field and hence the geometric definition of the second harmonic is the same from the geometric standpoint as from the dynamical. This, however, does not allow for the effects of mass above sea level. If we actually should bore holes through the earth's crust everywhere and determine the actual level physically corresponding to mean sea level, this level would clearly be affected by the hundreds, or thousands of feet of rock above it and we could not relate the two without taking account of the continental masses.

To a much better approximation, we can say that we don't care what the water would really do down there, a mile below Denver, Colorado, for instance. What we are really interested in is the co-geoid; this is a surface which is so

arranged that it does correspond to the external gravity field. It turns out that depth of the co-geoid, below the land surface is just about equal to the height as determined by the ordinary processes of surveying on the earth's surface, especially when account is taken of the variations in gravity. It is the co-geoid again that is determined by the processes of astronomical levelling.

But even here we are not out of the woods. It turns out that there is no unambiguous way to produce an imaginary continuation of the external gravitational field down into the earth. To see this, imagine a gigantic ball of rock suspended above the earth's surface. Clearly some lines of gravitational force will radiate from it. It follows that no distribution of mass which is confined to the interior of the earth can represent the effect of this ball on the outside. Thus we see that the concept of the co-geoid must necessarily be ambiguous in reality.

Fortunately, these problems are well below the precision to which we are working here; the same is true of the effects of the atmosphere, and hence f can be treated as the geometric flattening of the earth. The term geometrical flattening is preferable also because the term dynamic flattening has been used by some in another sense (see, for example, Jeffreys, 1962). The most recent number for f is $f = 1/298.255$, believed to be accurate to 5 parts in 10^8 .

The geometrical flattening is the one preferred by geodesists because it gives the smallest corrections to reduce baseline lengths from the co-geoid to the ellipsoid.

In geophysical studies, except probably in regional and local studies such as geophysical prospecting the purpose is to use the gravity anomalies to study the state of stress distribution in the earth's interior arising from an anomalous mass distribution. Hence a logical reference figure is a figure of zero stress. Such a figure is called the equilibrium or hydrostatic figure of the earth because it is the shape the earth would have assumed at its present rotational velocity if it were in a fluid state. The flattening f_h of such a figure depends principally on its mean or polar moment of inertia and its rotational velocity as illustrated by the first order relation:

$$f_h = \frac{5}{2} m / \left[1 + \left(\frac{5}{2} - \frac{15}{4} \frac{C}{Ma^2} \right)^2 \right] + O(f^2) \quad (2)$$

where C is the polar moment of inertia. The rotational velocity enters Equation 2 via m . The mathematical and computational details of the theory have been given by O'Keefe (1959, 1960), Henriksen (1960), Jeffreys (1962) and Khan (1969, 1973). This hydrostatic flattening f_h , derived from the second order theory (Khan, 1969) is f_h 1/299.75. The refinements of various parameters involved in the computation or of the theory are not likely to change this figure by more than a few parts in 10^8 .

A geometric figure for the earth can give misleading ideas about the state of stress in the earth's interior. This is demonstrated in Figure 1 and 2 which show a typical satellite-determined geoid based on a recent geopotential solution (Lerch, et al. 1973) referred to the geometric flattening of 1/298.255 and the

equilibrium flattening of 1/299.75 respectively. The geometric picture is correctly represented in Figure 1. It is clear that this picture is significantly different from that referred to the equilibrium flattening, as in Figure 2. The maximum effect of the difference caused by each reference figure occurs at the poles; accordingly polar regions are shown separately in Figure 3A and 3B and Figure 4A and 4B.

The most striking effect of the use of the hydrostatic figure is the emphasis which it lays on the great Antarctic negative anomaly. Clearly this is one of the dominant features of the earth's gravitational field as a whole, it is clearly the principal reason for the size of the third harmonic of the earth's field.

Comparison of geoidal undulations over the north and south polar regions (Figs. 3A, 3B and 4A, 4B), either with respect to the geometric flattening or equilibrium flattening, shows that the south polar regions are depressed more than 60 meters relative to the north polar regions. This makes the southern hemisphere flatter than the northern hemisphere.

With this striking representation before one, it is natural to try to see for example a connection between the Antarctic negative anomaly (also see Kaula, 1972 and Wang, 1966) and the great shrinkage of the Antarctic ice cap in the Pliocene which seems to be indicated by the preliminary conclusions based on Leg 28 of the Deep Sea Drilling Project aboard Glomar Challenger (Hayes, et al., 1973). The geological evidence uncovered in this cruise seems to indicate that extensive glaciation began on the Antarctic about early Miocene, resulting in a possible subsidence of

the continent though some local tectonism as cause of this subsidence cannot be discounted entirely. The glacial cycle reached a peak about 4 to 5 million years ago, followed by extensive melting and retreat of ice, probably without subsequent uplift of the Antarctic. Independent stratigraphic studies (for example, marine rock types in Yorktown-Duplin formation on the East Coast of the United States of America which is of Pliocene age and other marine rocks of same age in California, Florida and Europe) indicate the possibility of a major sea level transgression of about 30 meters about 4 million years ago (Hazel, personal communication). If this transgression were to be ascribed to Antarctic ice melting, it would generate a mass deficiency of approximately 1.1×10^{22} grams in the south polar regions. The satellite-determined gravity anomaly referred to the equilibrium figure is shown in Figure 5. Integration of this anomaly over areas with more than 10 milligals gravity deficiency yields a mass deficiency of approximately 1.3×10^{22} grams. The agreement is as good as could be expected.

It is recognized that in the absence of some local tectonism, the hypothesis suggested above is difficult to reconcile with recent estimates of viscosity for the upper and lower mantle (see for example, McConnell, 1968 and O'Connell, 1971) as well as with the work of Goldreich and Toomre (1969). It may suggest that the response of the earth to loading is more complex than simple viscous models indicate; and there are wide areas (e. g. the Hawaiian Islands) where the response is orders of magnitude slower than in the classical Fennoscandian uplift; though local tectonic processes have been evoked to explain the slow response in some of these cases.

Thus it is clearly demonstrated that the geophysical significance of the great Antarctic feature is more clearly seen on a map referred to the hydrostatic figure of the earth than on one referred to the best-fitting ellipsoid.

Similarly the hydrostatic representation brings out the importance of the geoidal trough in Canada as well as a lesser negative anomaly in the Siberia.

Outside the polar areas, the hydrostatic representation points to the significance of the East Indies and the Andes, while the older representation laid emphasis only on the Indian Ocean, the Eastern Pacific, the Bermudas and the North Atlantic.

The changes which are introduced by changing the reference figure from a geometric flattening of $f = 1/298.255$ to an equilibrium flattening of $f = 1/299.75$ are illustrated in Figure 6 for the geoid and Figure 7 for the gravity anomalies.

CONCLUSIONS

1. Recommended Figures:

Geodetic uses: $f = 1/298.255$

Geophysical uses: $f_h = 1/299.75$

Since both the geometric and equilibrium flattenings recommended above are correct to a few parts in 10^8 , it is recommended for the sake of consistency and facility of intercomparison within each field, that the values given here should be adopted and each should be used as a fixed value for the appropriate discipline.

2. The mass deficiency inferred from the satellite-determined gravity negative over Antarctic region is approximately equal to the ice load removed from the region during the Pliocene ice-melting episode.
3. The third harmonic in earth's shape clearly stems from the Antarctic negative anomaly which makes the southern hemisphere flatter than the northern hemisphere.

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Figure 1. A typical satellite-derived geoid referred to a flattening $f = 1/298.255$.

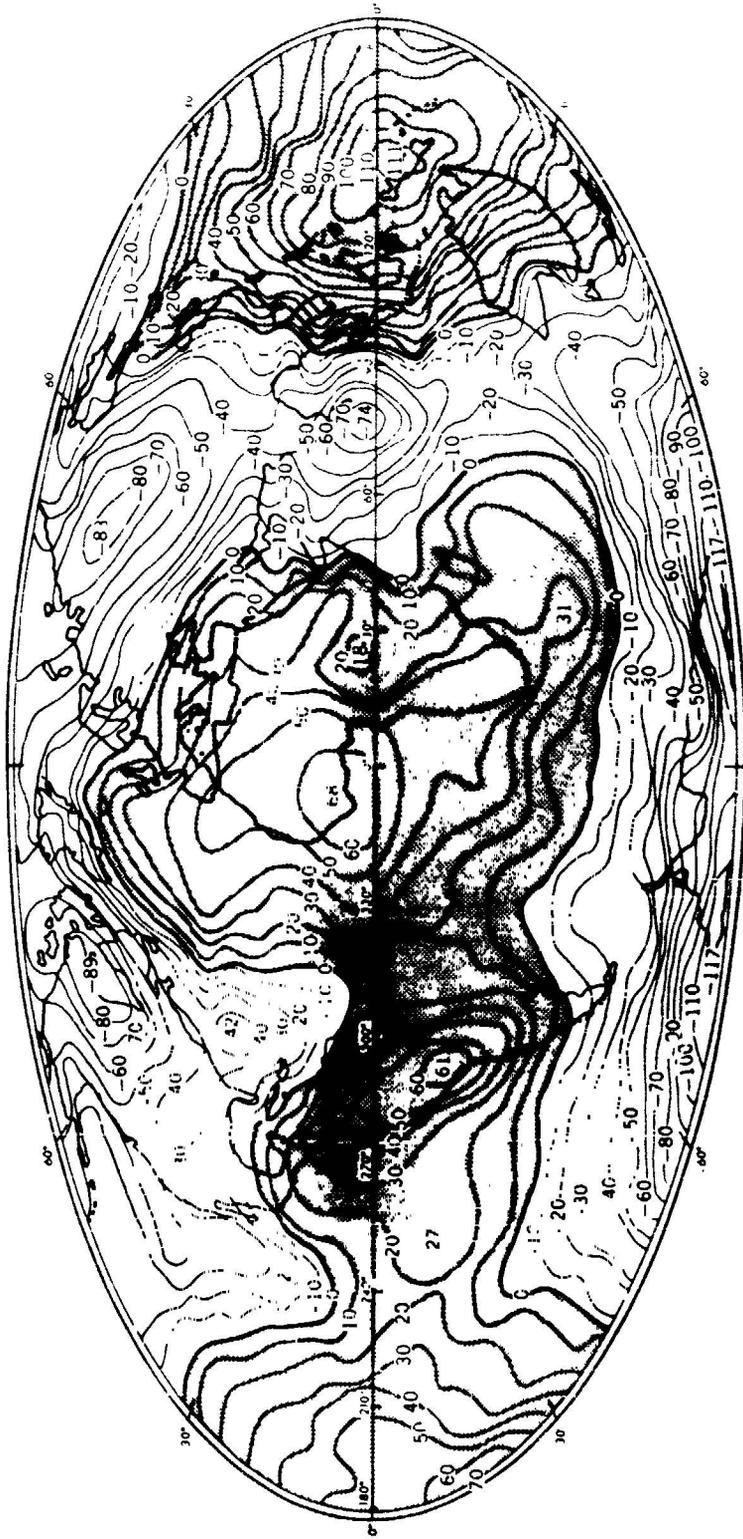


Figure 2. A typical satellite-derived geoid referred to the equilibrium flattening $f = 1/299.75$.

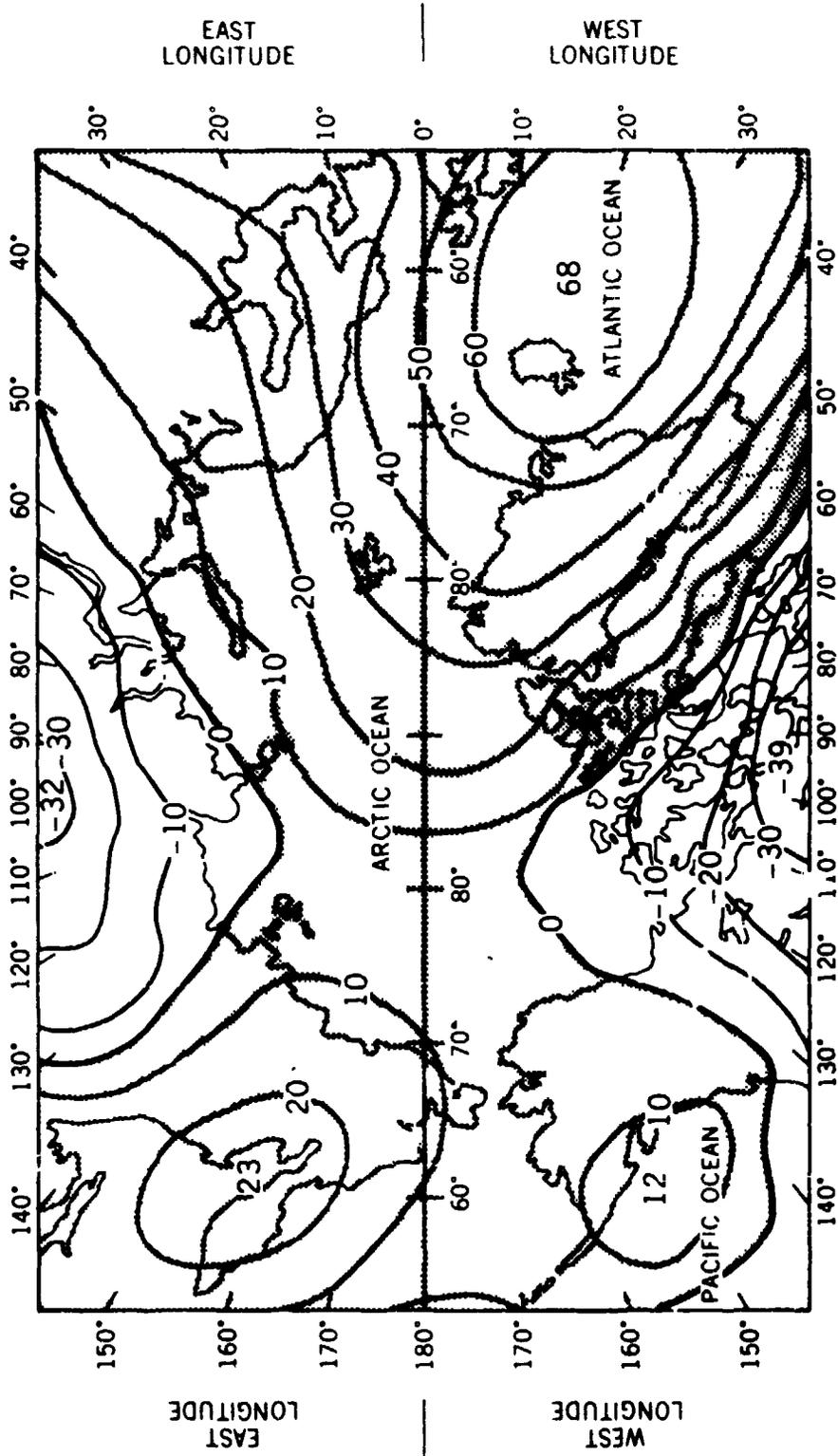


Figure 3A. A typical satellite-derived geoid with reference to $f = 1/298.255$. North Polar Regions.

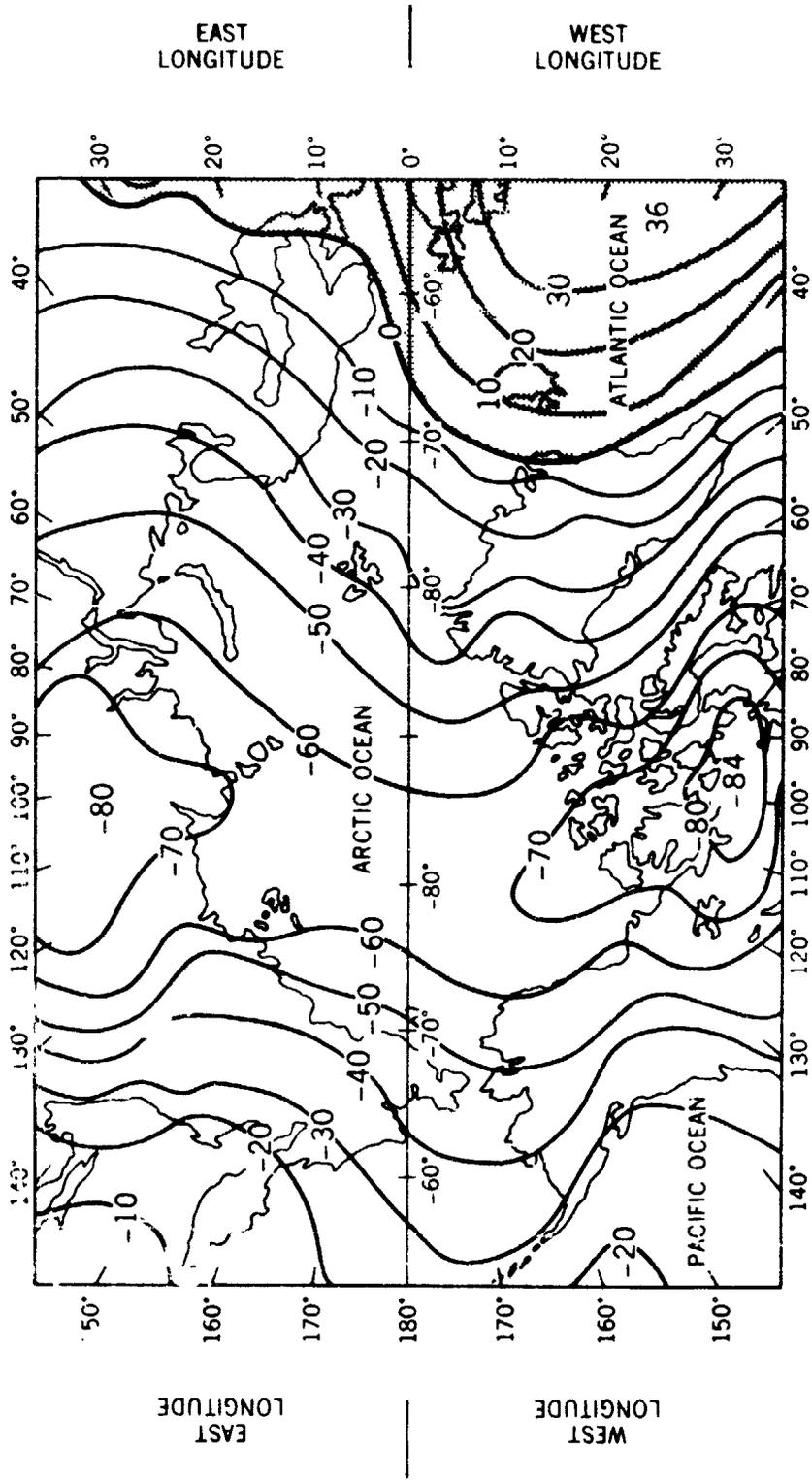


Figure 3B. A typical satellite-derived geoid with reference to $f = 1/299.75$. North Polar Regions.

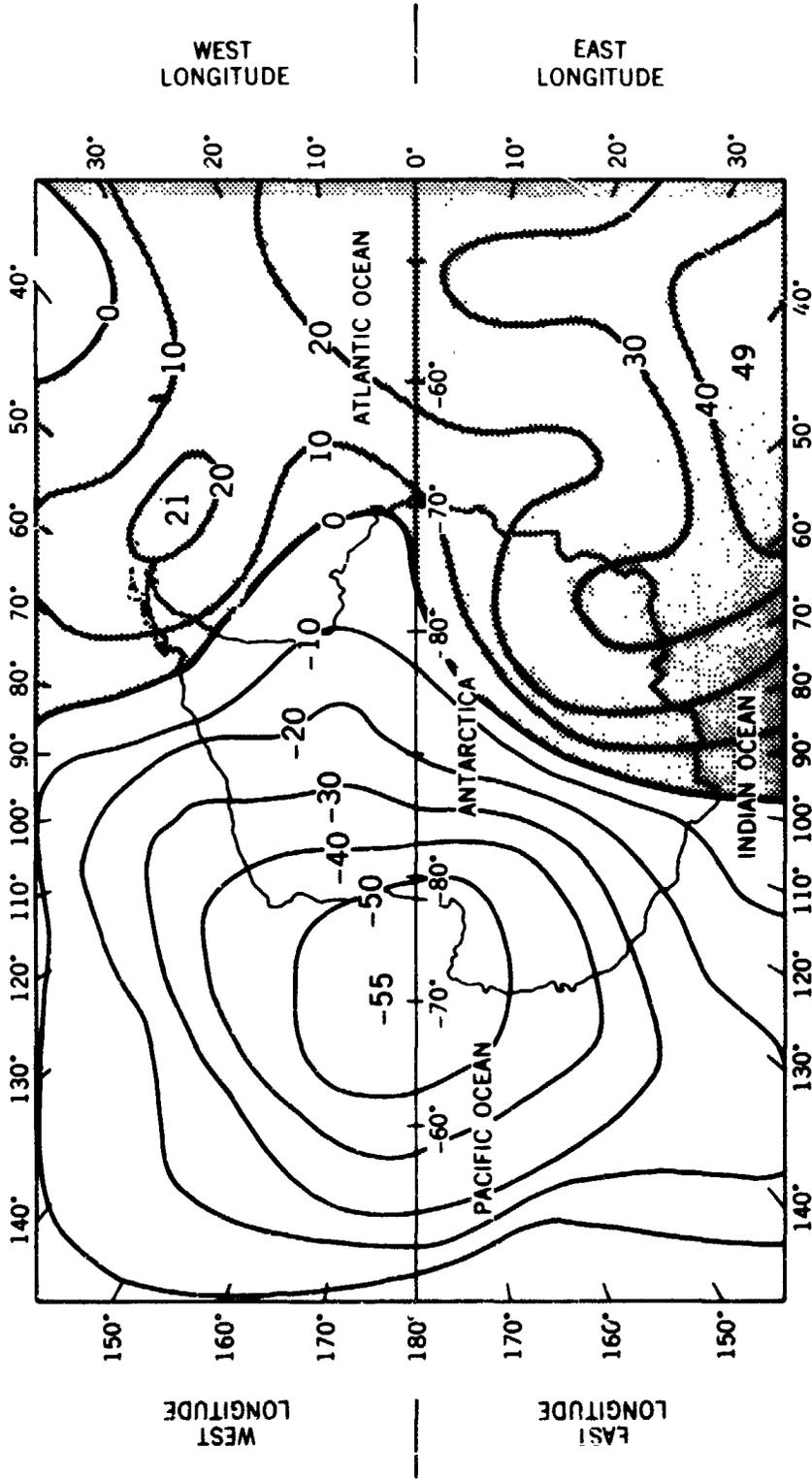


Figure 4A. A typical satellite-derived geoid with reference to $f = 1/298,255$. South Polar Regions.

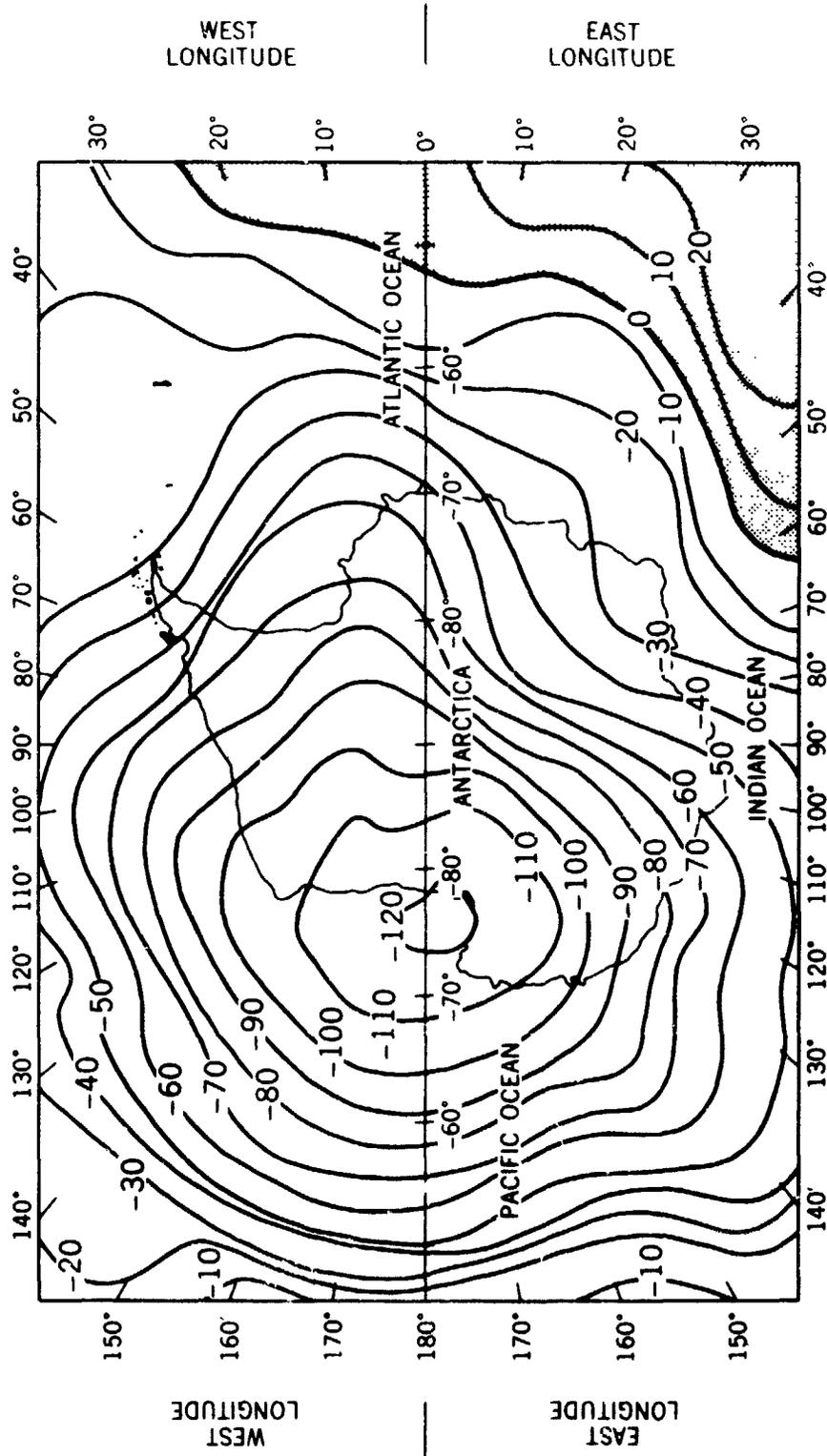


Figure 4B. A typical satellite-derived geoid with reference to $f = 1/299.75$. South Polar Regions.

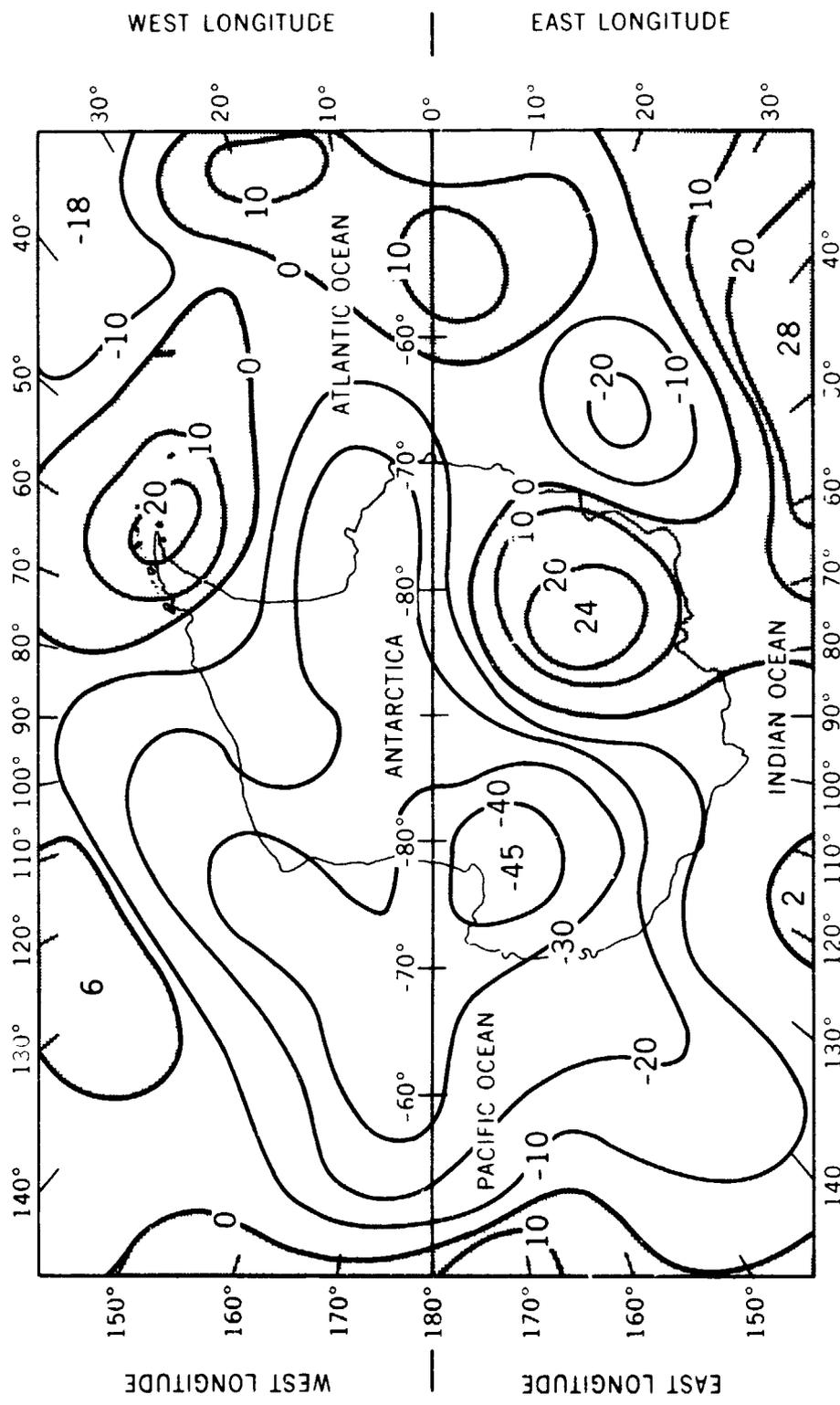


Figure 5. Satellite-derived gravity anomalies with reference to $f = 1/299.75$. South Polar Regions.

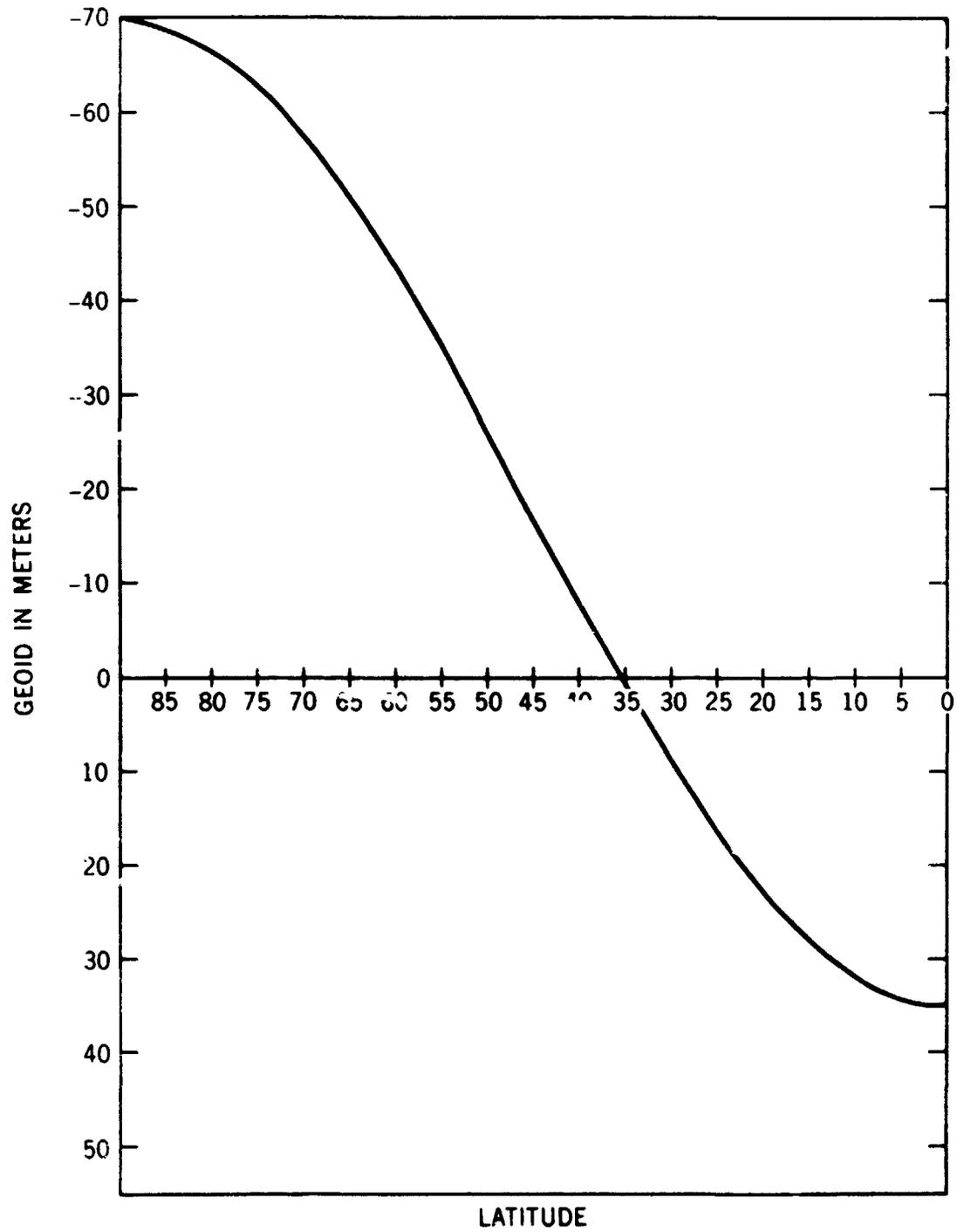


Figure 6. Geoidal differences between the geometric ($f = 1/298.255$) and equilibrium ($f = 1/299.75$) figures.

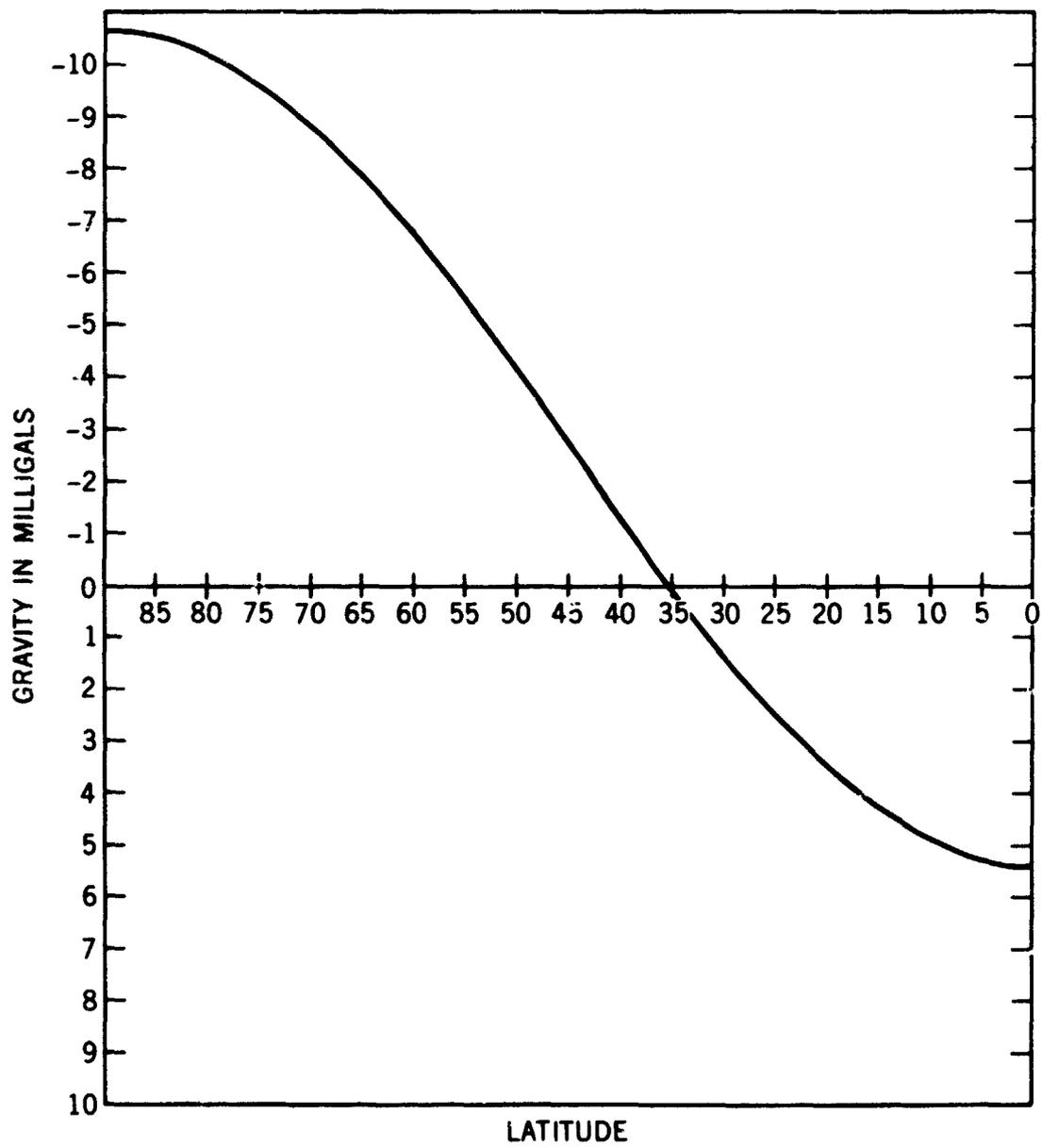


Figure 7. Gravity differences between the geometric ($f = 1/298.255$) and equilibrium ($f = 1/299.75$) figures.