AN EFFICIENCY STUDY ON OBTAINING THE MINIMUM WEIGHT OF A THERMAL PROTECTION SYSTEM
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S. D. Williams
Lockheed Electronics Company, Inc.
Houston, Texas

D. M. Curry
Lyndon B. Johnson Space Center
Houston, Texas
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By S. D. Williams* and D. M. Curry
Lyndon B. Johnson Space Center

SUMMARY

This report results from an efficiency study conducted to obtain the minimum weight of a thermal protection system. Based on previous experience with the adaptive creep-pattern search method for the evaluation of thermal protection system models, this study sought more efficient methods for reducing computer usage time; these methods were a numerical optimization method and two nonlinear least-squares methods. Three minimizing techniques were evaluated in an effort to solve the minimum-weight problem involving multimaterials and two or more constraints. The techniques were the Powell optimization routine, the Powell least-squares routine, and the Peckham least-squares routine. Final analysis of data showed that the Peckham method of least squares was the most efficient minimizing technique investigated, whereas a simple quadratic fit was the most efficient method for solving the one material and one constraint problem.

INTRODUCTION

The design of a minimum-weight thermal protection system (TPS) for the Space Shuttle is desirable to allow the maximum payload capability. This requires an analysis involving material selection, arrangement, and thickness subject to design constraints of the materials. In a previous study (ref. 1), the adaptive creep-pattern search method was found effective in determining the minimum weight of a multilayer insulative TPS with airgaps and two or more constraints. By using this analytical approach, the TPS minimum-weight problem is simplified to correlating optimum solutions for different TPS configurations.

Experience with the adaptive creep-pattern search method for the evaluation of many TPS models, materials, and trajectories has shown that while the analyst's

*Lockheed Electronics Company, Inc.
effort is greatly reduced, the computer time requirements may be excessive. An analysis was therefore conducted to determine a more efficient method for reducing the computer usage time and cost. The methods used in this investigation were a numerical optimization method developed by Powell (ref. 2) and two nonlinear least-squares methods developed by Powell (ref. 3) and Peckham (ref. 4). As with the adaptive creep-pattern search, none of the methods required the formal evaluation of derivatives. Finally, so that a meaningful comparison could be made, the same TPS models used for the efficiency study in reference 1 were used for the efficiency study in this investigation.

As an aid to the reader, where necessary the original units of measure have been converted to the equivalent value in the Système International d'Unités (SI). The SI units are written first, and the original units are written parenthetically thereafter.

**ANALYSIS**

The TPS weight optimization problems can be divided into two categories: those in which only one material thickness is to be optimized and those in which more than one material thickness is to be optimized. The current work showed that for the one material and one temperature constraint problem, a simple quadratic fit of temperature as a function of thickness is the most efficient technique. Desired temperatures are calculated for three different trial thicknesses of the material to be optimized, and a quadratic curve fit is used to predict thickness as a function of temperature. This allows for the direct estimation of a new thickness that will produce the desired temperature. The technique is iterative in that, if the temperature obtained from the predicted thickness is not within the desired tolerance, one of the old thickness values is thrown away and a new thickness is predicted from the three remaining values.

The case for more than one material to be optimized can be further subdivided into two classes: when the number of materials to be optimized is greater than the number of constraints and when the number of constraints equals or exceeds the number of materials to be optimized. The first class requires the use of true optimization techniques, but, because of TPS design criteria, this class seldom arises in practice. The second class is exact or overdetermined, and the solution can be made tractable by a least-squares method. In a least-squares approach, no attempt is made to achieve a minimum weight. Rather, the problem is to satisfy all constraints; these constraints are temperature constraints imposed on the backwall of
the materials in the TPS. In the least-squares formulation, a vector of independent variables \( x = x_1, x_2, \ldots, x_n \) is defined for which a minimum value of \( S \) is to be determined.

\[
S = \sum_{k=1}^{n} \left[ \psi_k(x) \right]^2
\]

with \( \psi_k = T_j - T^*_k \) where \( T_j \) is the temperature at the backwall of material \( j \) and \( T^*_k \) is the desired temperature for the \( k \)th constraint.

The three minimizing techniques used in this investigation were Powell's VA04A optimization routine (ref. 2), which is a method for finding the minimum of a function of several variables without calculating derivatives; Powell's VA02A least-squares routine (ref. 3), which is a method for minimizing a sum of squares of nonlinear functions without calculating derivatives; and Peckham's MINSQ least-squares routine (ref. 4), which is also a method for minimizing a sum of squares without calculating gradients.

Three specific TPS configurations were considered for this study: configuration 1 — Dynaflex/TG-15 000/aluminum; configuration 2 — Dynaflex/airgap/TG-15 000/aluminum; and configuration 3 — Dynaflex/TG-15 000/airgap/aluminum. (Dynaflex and TG-15 000 are product names of thermal insulation material.) The three configurations, shown schematically in figure 1, are the same TPS models used for the efficiency study in reference 1.

The control variables for optimization were the thicknesses of the Dynaflex and the TG-15 000. A thin metallic skin was assumed to form the aerodynamic surface to which the Dynaflex was attached; however, this thin skin was ignored in the thermal model. The thickness of the aluminum was held to a constant value of 0.18 centimeter (0.07 inch). The maximum temperature at the backwall of the Dynaflex was constrained to \( 922 \pm 0.28 \) K (1660\(^\circ\) ± 0.5\(^\circ\) R), whereas the backwall of the aluminum was constrained to \( 366 \pm 0.28 \) K (660\(^\circ\) ± 0.5\(^\circ\) R). No temperature constraint was imposed on the backwall of the TG-15 000. The emissivity for the airgap was held at 0.2 on both the front and back of the materials. An emissivity of 0.8 was used at the face of the Dynaflex for radiation to space.

Because the purpose of this effort was to evaluate minimizing techniques, a trajectory entry time of 500 seconds was used for the thermal model. The convective heating rate history given in figure 2 is for a straight-wing shuttle vehicle entering at an angle of attack of 60\(^\circ\). The heating rate shown is for a 0.305-meter-radius vehicle.
(1-foot-radius) sphere. This heating rate was multiplied in the numerical program by a heating factor of 0.41 to provide surface temperature typical of the leading edge and fuselage nose areas. Although this is no longer the design entry trajectory for the Space Shuttle, it is adequate for the comparisons required in this study.

RESULTS

The most efficient minimizing technique for all three TPS models was Peckham's method of least squares. For configuration 1 (no airgap), 12 function evaluations were required for convergence. For both configuration 2 (airgap between the Dyna-flex and the TG-15 000) and configuration 3 (airgap between the TG-15 000 and the aluminum), 11 function evaluations were required for convergence. This represents a reduction in computer time by a factor from 13 to 16 when compared to the adaptive creep-pattern search method. The results for all cases are summarized in table I. The results for the adaptive creep-pattern search (ref. 1) are included in table I to assist in comparing the relative efficiency of each method.

An ablative TPS model was also considered to determine whether Peckham's technique would also be effective for ablators. The ablative TPS composite model consisted of four materials: the Apollo ablator (Avco 5026/39/HCG), TG-15 000, titanium, and aluminum. The ablative TPS model was constructed as follows: ablator/titanium/TG-15 000/airgap/aluminum. The control variables for optimization were the thicknesses of the ablator and the TG-15 000. For this case, the thickness of the titanium was 0.02 centimeter (0.008 inch) and the thickness of the aluminum was 0.18 centimeter (0.07 inch). The maximum temperature at the backwall of the ablator was constrained to $922 \pm 1.1 \text{ K (1660}^\circ \text{F} \pm 2 \text{ R)}$, whereas the backwall of the TG-15 000 was constrained to $422 \pm 1.11 \text{ K (760}^\circ \text{F} \pm 2 \text{ R)}$. No temperature constraint was imposed on the other two materials. The convective heating rate used was the same as for the passive TPS given in figure 1. This rate was modified by a factor of 0.8 to provide surface temperatures typical of the area of attachment of the wing to the fuselage. The emissivity for the airgap was held at 0.8 on both the front and back of the materials. (The analytical ablation model and thermal properties are analyzed in reference 5.) For this case, 10 function evaluations were required for convergence, indicating that the efficiency of the least-squares routine for an ablating system is about the same as for a passive TPS system.

The efficiency of using the quadratic fit for solving the one material and one constraint problem for both passive and ablative TPS problems was analyzed. In general, for a passive system, the solution was obtained with four function evaluations, whereas six or seven function evaluations were required for the ablative system. Because three function evaluations are required to provide the initial data for determining the coefficients of the quadratic $^*$ the fourth evaluation for the passive system is used to confirm that the pre-determined thickness does produce the...
desired temperature. The additional evaluations required for the ablative TPS are due to the nonlinearity induced by mass loss and its effect on the temperature distribution through the material.

CONCLUSIONS

The quadratic fit was the most efficient method for solving the one material and one constraint minimum weight problem for a thermal protection system. For the general problem involving more than one material and two or more constraints, Peckham's method of least squares was the most efficient minimizing technique investigated.

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REFERENCES


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\textsuperscript{a} Data are derived from reference 1.

\textsuperscript{b} Corrections became too small; converged in 15 more function evaluations for a total of 45.
Configuration 1:

Heat in

Thin metallic skin (not included in thermal model)

Dynaflex  TG-15 000  Aluminum

Adiabatic backwall

Configuration 2:

Heat in

Thin metallic skin (not included in thermal model)

Dynaflex  Airgap  TG-15 000  Aluminum

Adiabatic backwall

Configuration 3:

Heat in

Thin metallic skin (not included in thermal model)

Dynaflex  TG-15 000  Airgap  Aluminum

Adiabatic backwall

Figure 1. - The three different TPS models used in the efficiency study.
Figure 2. - Convective heating rate for the TPS model.