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**MODIFICATION OF A THREE-DIMENSIONAL
SUPERSONIC NOZZLE ANALYSIS AND
COMPARISON WITH EXPERIMENTAL DATA**

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MODIFICATION OF A THREE-DIMENSIONAL SUPERSONIC NOZZLE ANALYSIS AND COMPARISON WITH EXPERIMENTAL DATA

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SUMMARY

A computer program previously developed to analyze three-dimensional supersonic nozzles by the method of characteristics has been modified to study less restrictive nozzle geometries and nonuniform inlet conditions. An example which is discussed indicates that averaging a nonuniform initial profile for one-dimensional calculation may generate significant errors. The analytical method and the data from a three-dimensional experiment are compared. The results show generally good agreement between analysis and experiment.

INTRODUCTION

There have been several methods developed to analyze three-dimensional supersonic nozzle flows by the method of characteristics (refs. 1 to 4). Moretti (ref. 2), Rakich (ref. 3), and Dash and Del Giudice (ref. 4) use various approximations to the characteristic equations to produce a calculation accurate to first order in the grid spacing. The scheme of Ransom, Cline, Hoffman, and Thompson (ref. 1) is an exact solution to the characteristic equations and is accurate to second order in the grid spacing. This report describes modifications to the computer code of reference 1 that allow it to accept a less restrictive wall geometry and initial profile.

In a finite difference analysis it is often very convenient to specify the boundary conditions in an algebraic form. The values are usually continuous and well behaved, and differentiation and integration are straightforward. The computer program in reference 1 allows the use of only a limited class of algebraic expressions for the solid-wall boundary; the axial variation in the throat is specified as a circular arc matched to a quadratic form for the expansion region of the nozzle. The cross sections are specified as superellipses, which may be different in each quadrant.

There are, however, many nozzle geometries that are not easily specified in an algebraic form. Nozzles designed for optimum performance or corrected for boundary layer growth are usually described by a series of data points. Experimental nozzles are often designed for convenience in manufacturing rather than to fit any algebraic equations. Therefore, an alternate solid-boundary routine has been developed which retains the superelliptical cross sections but allows the axial geometry variation to be specified on a point-by-point basis. With this form of input a much more arbitrary geometry can be analyzed and complex nozzle geometries can be conveniently specified.

At the initial station in the duct, an initial flow field must be specified. In reference 1 an initial profile with radial variations in total pressure, total temperature, Mach number, and so forth, could be analyzed; but no azimuthal variation was permitted. This restriction has been relaxed to allow a general two-dimensional variation of flow properties at the inlet station. The initial flow field is specified on a point-by-point basis, and a two-dimensional interpolation scheme is used to determine intermediate points. While this procedure permits a very general initial flow field to be considered, some care must be taken to select conditions that are consistent with the boundary conditions.

The first half of this report documents these modifications and gives the results for a few sample cases, while the second half describes a three-dimensional supersonic flow experiment and compares the experimental data with the three-dimensional analysis.

PROGRAM MODIFICATIONS

Changes in Geometry Specification

The solid-boundary routines in the computer program of reference 1 have been modified to accept axial variations in the body coordinates specified on a point-by-point basis. The superelliptical cross sections have been retained, but the intercepts of the superellipses with the coordinate planes and the superelliptical exponents are specified as a series of points. The exponents need not be specified at the same locations as the intercepts. A variable centerline coordinate has been added, so that curved or S-shaped ducts may be studied. This variation is also specified as a series of points.

Interpolation method. - Whenever a curve is specified by a series of points, an interpolation scheme is required to determine intermediate values. The method developed for the modified nozzle computer program uses Stirling's three-point interpolation twice: once with the center point behind the new point (as in region I of fig. 1) and again with the center point ahead of the new point (as in region II of fig. 1). A weighted average is taken of the two results, the weighting factor being the distance along the abscissa

from the new point to the center point of the interpolation region. With this method of interpolation the function values and the first derivatives are continuous. This routine is particularly good near the junction of two regions with a discontinuity in curvature, such as a circular arc changing to a straight line. Splined interpolation will oscillate about the true value for some distance beyond the junction, while the method used herein provides a much smoother transition.

Within the computer program the interpolation is performed in a separate subroutine and therefore may be easily replaced. Since the interpolation uses points on both sides of the unknown point, it is advisable to supply additional points on either side of the region of interest. While the subroutine will interpolate between the last two points and, in fact, will extrapolate beyond the last point, the accuracy is necessarily reduced in these regions.

Example solutions. - To illustrate the new input scheme and to demonstrate its capabilities, two representative nozzles have been analyzed. A simple axisymmetric nozzle with a straight centerline is used for example I. This example serves as a check between the new and old input methods. A three-dimensional nozzle, demonstrating the flexibility of the new input, is used for example II.

The nozzle used in example I is axisymmetric with a circular-arc throat and a 10° conical exit section. Figure 2 is a sketch of this nozzle, and the coordinates are given in table I. The uniform initial profile at the entrance station (throat) has a Mach number of 1.1. Figure 3 is a representation of the Mach number profile at the exit station. One-quarter of the exit plane is shown with the nozzle centerline at the origin of the coordinate system. The curves in figure 3 are elements of the surface formed when the Mach number is plotted as a function of position in the exit plane. This type of "surface" presentation is not necessary for axisymmetric flow, but it is very useful in visualizing three-dimensional flow and is used in subsequent figures in this report. No differences between the results obtained from the old and new methods of specifying geometry were observed.

Example II demonstrates the flexibility of the new input geometry. Figure 4 is a sketch of the nozzle used in example II, and the coordinates are given in table II. This nozzle has a varying cross section that changes from circular at the throat to almost rectangular at the exit. The variation is made by changing the superelliptic exponent of the z-coordinate from 2 at the throat to 20 at the exit. The exponent of the y-coordinate is constant at 2. In addition, the centerline of the nozzle has a slight S-shaped bend in the x-y plane. The uniform initial profile at the entrance station (throat) has a Mach number of 1.02. Figure 5 shows the Mach number distribution at the exit station for example II. One-half of the exit plane is shown, and the nozzle centerline is offset from the origin of the coordinate system because of the bend of the nozzle. The asymmetry in the exit Mach number is caused primarily by the curvature in the nozzle centerline.

Initial Profile Modifications

A three-dimensional calculation is necessary when an axisymmetric nozzle has a nonuniform inlet flow. The original code described in reference 1 would accept only a radial distortion of the inlet profile. This restriction has been relaxed to allow a general two-dimensional variation of flow properties at the inlet station. The initial profile is specified on a point-by-point basis, each point given by its coordinates and the flow properties at that point. There are no restrictions on the spacing or the distribution of points, with the following two exceptions: all points must have the same axial coordinate and the points must be transformable into a rectangular grid. The points must be arranged in rows and columns corresponding to this grid, with every row having the same number of points and every column the same number of points. This latter restriction is a requirement of the interpolation scheme which is used to find intermediate points on the initial-value surface.

Two-dimensional interpolation method. - A two-dimensional interpolation scheme for the initial datum surface is necessary because the computer program chooses its own starting grid. The method used in the modified program determines the two triangular regions, I and II in figure 6, enclosing the new point (y_1, z_1) . A plane is fitted through the three corners of each region to determine the values of a flow variable as a function of y and z in each region. A typical flow variable $S(y_1, z_1)$ is found on each plane, and the two values are averaged to find the final result. This method has been used successfully in several other problems. A continuous functional value is generated with a minimum of curve fitting and mathematical calculation. This interpolation is performed in a separate subroutine within the program, so that another interpolation routine may be substituted.

The flexibility used in the construction of the grid for the initial datum surface allows for a wide variety of initial inlet flow fields to be specified. However, care must be taken to match the flow field properties with the boundary conditions at the solid wall (velocity tangent to wall, etc.). A provision is made within the program to alter the wall velocities slightly to match exactly the tangent condition at the solid boundary.

Example solution. - To demonstrate the initial-profile scheme developed a third example is presented. Example III is an axisymmetric nozzle with a nonuniform inlet flow profile. Both Mach number and total pressure have a linear variation with the z -coordinate, giving the entrance flow both radial and circumferential distortion with symmetry about both the y and z axes. Table III gives the geometry of the nozzle, and table IV the nonuniform initial profile. A sketch of the nozzle used in example III is shown in figure 7.

The Mach number distributions at the entrance and exit stations are shown in figure 8. One-quarter of the exit plane is presented, with the nozzle centerline at the ori-

gin of the coordinate system. The distributions are not to the same geometric scale, since the exit radius is 4.8 times as large as the entrance radius. The initial and final Mach number distributions are readily apparent in these two plots.

A distorted Mach number profile at the exit station is not the only effect of initial-profile distortion. The performance parameters of the nozzle, such as mass flow and thrust, are also changed. In many situations involving airflow characteristics within nozzles, it is not convenient to consider local flow variations within the flow field. Therefore, the properties of the flow at one station are treated as though they are uniformly distributed, so that one-dimensional equations can be used to predict the flow at other stations of interest. The use of averaged values, however, can produce significant errors in mass flow and momentum flux when compared to the actual integrated values. Table V shows such a comparison between the results of the three-dimensional analysis of example III and the results of a one-dimensional analysis that used several different averaging methods. A simple average, an area-weighted average, and a mass-flow-weighted average were used to determine a single value for Mach number and total pressure at the entrance station. These values were used in a one-dimensional analysis to find the mass flow and thrust.

The three one-dimensional methods give different mass flows and thrusts and are in error from the three-dimensional values by varying amounts.¹ All three one-dimensional methods predict thrust values that are above the value for the three-dimensional calculation. This means that a one-dimensional calculation with an averaged profile at the initial station may give an optimistic value of thrust, and that any distortion of the initial profile may tend to degrade the performance of the nozzle.

COMPARISON WITH EXPERIMENT

Description of Experiment

Some three-dimensional flow data have been obtained as part of an experiment in the 10- by 10-Foot Supersonic Wind Tunnel at the Lewis Research Center. A series of plates were placed in the throat of the nozzle to generate distorted flow fields in the test section. A calibration rake was also installed in the test section, permitting a limited survey of the three-dimensional flow. Data from this experiment will be compared with the results of an analysis of the same geometry that were obtained by using the modified three-dimensional method-of-characteristics program.

¹The simple average agrees best with the three-dimensional analysis for this case because of the initial profile considered. For an arbitrary initial profile a simple average would be expected to be least accurate.

A sketch of the experimental geometry is shown in figure 9. The floor and ceiling of the nozzle were fixed, and the side walls were adjustable to produce various test-section Mach numbers. As indicated in figure 9, the disturbance generator was a flat plate placed on the floor of the tunnel at the throat. Plates of two different sizes were used with several different wall settings to vary the Mach number and the strength of the disturbance in the test section. Near the entrance to the test section a multiple-wedge calibration rake was used to survey the flow properties at several axial stations.

Description of Analysis

Several approximations were made in analyzing this geometry. The rectangular cross section of the actual nozzle was approximated by a superellipse with exponents of 20. Since the program cannot analyze a separated region, a triangular-shaped solid boundary replaced the vertical flat plate (fig. 9). The height of the triangle was the same as that of the plate. A uniform initial profile with a Mach number of 1.02 was assumed to exist at the plate location. The boundary layer and other dissipative effects were not considered in the analysis.

RESULTS AND DISCUSSION

Figure 10 presents the static pressure distribution along the tunnel ceiling when the sidewalls were positioned for a nominal test-section Mach number of 2.4 with no plate present. The static pressure ratio (static to total) on the ceiling centerline is plotted as a function of the dimensionless axial distance from the throat (actual distance divided by tunnel height). Both the experimental data and the results of the analysis are plotted in figure 10 for comparison.

The agreement between data and experiment is very good. The Mach number profiles (not shown) are very uniform, the variation being less than 1 percent at any station. These results are also in good agreement with the experimental data.

Figure 11 shows the disturbance generated when a plate 30.5 centimeters high was placed on the floor at the throat. Again the static pressure ratio is plotted against axial location. The floor and ceiling centerline values predicted analytically, as well as the experimental data for the ceiling, are plotted. The strong compression created by the plate can be easily traced as it was reflected between the ceiling and the floor. The agreement between data and analysis is good except near the throat. The experimental data indicate that subsonic flow was present downstream of the geometric throat, which was at the axial location of the plate. Apparently, a vena contracta was present in this

region, so that the minimum flow area did not coincide with the minimum geometric area.

Figure 12 is a plot of Mach number against axial location in the region surveyed by the calibration rake. The four plots represent the data from four wedges near the center of the tunnel. The vertical positions were at 30, 43, 50, and 70 percent of the tunnel height. The Mach number predicted by analysis is presented for comparison. Both the data and the analysis indicate a strong compression crossing this region. The compression is shown more clearly in figure 13, which shows the predicted Mach number distribution at several axial locations in the region of figure 12. The comparison in figure 12 indicates that the analysis predicted the magnitude of the compression fairly well but that the compression occurred over a larger axial distance in the analysis than in the experimental data. This is probably the result of interpolation in the analysis, which tends to smooth any rapid changes.

Figure 14 is a plot of static pressure ratio against axial location but with the sidewalls positioned for a Mach number of 2.0. The same 30.5-centimeter-high plate used for the previous data (wall Mach number of 2.4) was placed at the throat. Because the sidewalls at the throat are further apart for a Mach number of 2.0, a gap of 30.1 centimeters existed between each end of the plate and the sidewalls. Both the floor and ceiling centerline static pressure values from the analysis are plotted, along with the experimental data on the ceiling. As with the Mach 2.4 wall setting, the subsonic flow near the throat is apparently caused by a vena contracta downstream of the plate. The deviation of the experimental data from the analysis in the dimensionless axial region from 1 to 2 may result from local separation caused by the strong compression reflected from the ceiling. Kuehn (ref. 5) indicates that the predicted pressure rise occurring near an axial location of 2 is enough to cause the flow to separate. The experimental data also indicate that the last reflection of the compression wave from the ceiling occurred upstream of the predicted location. The reason for this is not clear; but the separated region previously mentioned, the gap between the plate and the sidewall, or the boundary layer effects neglected in the analysis are all possible explanations.

The data in figure 15 are for a wall setting which gives a nominal test-section Mach number of 3.1. A plate only half as high as previously used, 15.2 centimeters, was placed at the throat. The static pressure ratio is plotted as a function of dimensionless axial distance. In figure 15 the analysis is in better agreement with the data, particularly in the region near the throat. The cause may be the weaker compression which was present because of the smaller plate.

The variations in Mach number at each axial station for the Mach 2.0 and Mach 3.1 wall settings are similar to that presented for the Mach 2.4 wall setting, except that the compressions occur at different axial locations, as indicated by the pressure distributions. In general, the distributions predicted analytically agree with the results of the

experiment, except for a smoothing and stretching out of the compression in the analysis.

SUMMARY OF RESULTS

The modifications to the original three-dimensional method-of-characteristics program permit the analysis of a wider variety of nozzle geometries and nonuniform inlet conditions. The results for the sample case with a nonuniform inlet profile indicate that a one-dimensional calculation may have significant errors, no matter how the non-uniformity is averaged.

The comparison between experimental data and the three-dimensional analysis shows good agreement, especially since the method of characteristics assumes an inviscid flow field. While the analysis smooths out the sharp changes across a shock wave and ignores all dissipative effects, it does predict the location and magnitude of strong compressions fairly well.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 4, 1973,
501-24.

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1. Ransom, W. H.; Cline, M. C.; Hoffman, J. D.; and Thompson, H. D.: A Second-Order Numerical Method of Characteristics for Three-Dimensional Supersonic Flow. Vol. 2. Purdue Univ. (AFAPL-TR-69-98, vol. 2), Jan. 1970.
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4. Dash, S. M.; and Del Guidice, P. D.: Three-Dimensional Nozzle-Exhaust Flow Field Analysis by a Reference Plane Technique. Paper 72-704, AIAA, June 1972.
5. Kuehn, Donald M.: Experimental Investigation of the Pressure Rise Required for the Incipient Separation of Turbulent Boundary Layers in Two-Dimensional Supersonic Flow. NASA Memo 1-21-59A, 1959.

TABLE I. - COORDINATES OF AXISYMMETRIC
NOZZLE USED FOR EXAMPLE I

Axial coordinate direction, X	Coordinate perpendicular to axial coordinate, Y
-0.05234	1.00137
-.03490	1.00061
-.01745	1.00015
0	1.00000
.01745	1.00015
.03490	1.00061
.05234	1.00137
.06976	1.00244
.08716	1.00381
.10453	1.00548
.12187	1.00745
.13917	1.00973
.15643	1.01231
.17365	1.01519
.18000	1.01631
.20000	1.01984
.30000	1.03747
.50000	1.07274
1.00000	1.16090
2.00000	1.33723
4.00000	1.68988

TABLE II. - COORDINATES OF THREE-DIMENSIONAL NOZZLE USED FOR EXAMPLE II

(a) Solid wall - coordinate plane intercept (y and z values are identical)

Dimensionless axial distance from throat, X	Coordinate perpendicular to axial coordinate, Y
-9.30000	1.81850
-5.90000	1.49250
-2.95000	1.31630
0	1.25300
2.95000	1.31140
6.00000	1.48700
9.40000	1.79790
12.90000	2.21750
16.20000	2.67970
18.84999	3.08010
21.39999	3.47680
23.95000	3.87120
26.50000	4.24650
29.25000	4.61110
32.20000	4.94850
35.50000	5.26050
38.89999	5.51030
42.29999	5.69070
45.70000	5.81270
49.09999	5.88920
52.59999	5.93330
56.09999	5.95870
59.59999	5.97950
63.09999	6.00000
66.00000	
70.00000	

(b) Superelliptical exponent of z-coordinate

Dimensionless axial distance from throat, X	Superelliptical exponent of z-coordinate
-3.00000	2.00000
-1.50000	2.00000
0	2.00000
5.00000	2.32000
10.00000	3.21000
15.00000	4.57000
20.00000	6.28000
25.00000	8.24000
30.00000	10.34000
35.00000	12.47000
40.00000	14.53000
45.00000	16.40999
50.00000	17.98999
55.00000	19.18999
60.00000	19.87000
63.09999	20.00000
65.00000	20.00000
70.00000	20.00000

(c) Coordinate of nozzle centerline (z = 0)

Dimensionless axial distance from throat, X	Coordinate perpendicular to axial direction, Y
-3.00000	0
-1.50000	
0	
5.00000	
10.00000	
15.00000	.10000
20.00000	.20000
25.00000	.40000
30.00000	.60000
35.00000	.80000
40.00000	1.00000
45.00000	1.10000
50.00000	1.20000
55.00000	
60.00000	
63.09999	
65.00000	
70.00000	

TABLE III. - COORDINATES OF AXISYMMETRIC

NOZZLE USED FOR EXAMPLE III

Dimensionless axial distance from throat, X	Coordinate perpendicular to axial coordinate, Y
-9.30000	1.81850
-5.90000	1.49250
-2.95000	1.31630
.00000	1.25300
2.95000	1.31140
6.00000	1.48700
9.40000	1.79790
12.90000	2.21750
16.20000	2.67970
18.84999	3.08010
21.39999	3.47680
23.95000	3.87120
26.50000	4.24650
29.25000	4.61110
32.20000	4.94850
35.50000	5.26050
38.89999	5.51030
42.29999	5.69070
45.70000	5.81270
49.09999	5.88920
52.59999	5.93330
56.09999	5.95870
59.99999	5.97950
63.09999	6.00000
64.00000	↓
66.00000	
70.00000	

TABLE IV. - INITIAL-VALUE SURFACE DATA USED FOR EXAMPLE III

Coordinates perpendicular to axial coordinate		Mach number	Total pressure
Y	Z		
0	0	1.1	1000
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓
0	.21429	1.2	1100
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓
0	.42857	1.3	1200
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓
0	.64286	1.4	1300
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓
0	.85714	1.5	1400
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓
0	1.07143	1.6	1500
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓
0	1.28571	1.7	1600
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓
0	1.50000	1.8	1700
.21429	↓	↓	↓
.42857	↓	↓	↓
.64286	↓	↓	↓
.85714	↓	↓	↓
1.07143	↓	↓	↓
1.28571	↓	↓	↓
1.50000	↓	↓	↓

TABLE V. - THREE-DIMENSIONAL EFFECTS IN CALCULATING
NOZZLE PERFORMANCE

Type of calculation	Entrance Mach number	Entrance total pressure, N/m^2	Mass flow, kg/sec	Thrust, N
Three-dimensional flow analysis	Variable	Variable	4.72	24.2
Average of initial conditions, one-dimensional flow analysis	1.31	8.32×10^4	4.72	24.6
Area-weighted average of initial conditions, one-dimensional flow analysis	1.35	8.61×10^4	4.81	24.8
Mass-flow-weighted average of initial conditions, one-dimensional flow analysis	1.35	8.63×10^4	4.80	25.3

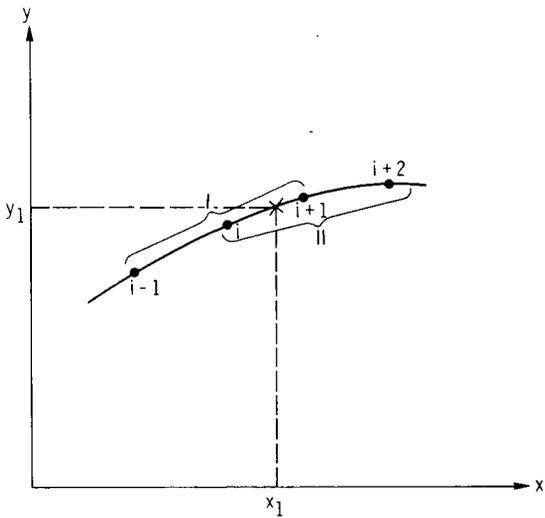


Figure 1. - Diagram of interpolation scheme used to find intermediate points on solid boundaries.

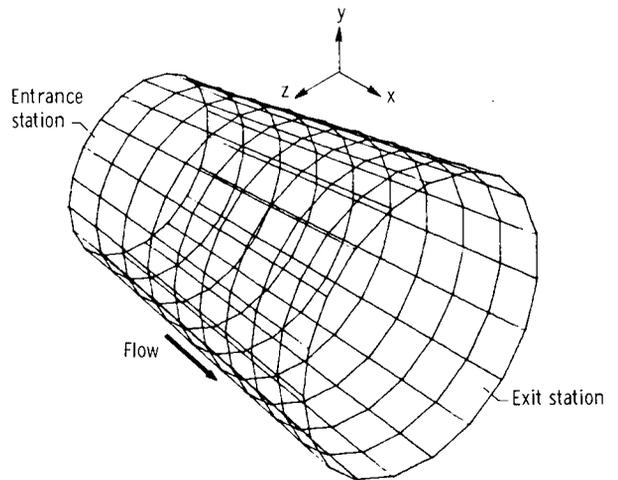


Figure 2. - Sketch of axisymmetric nozzle used for example I.

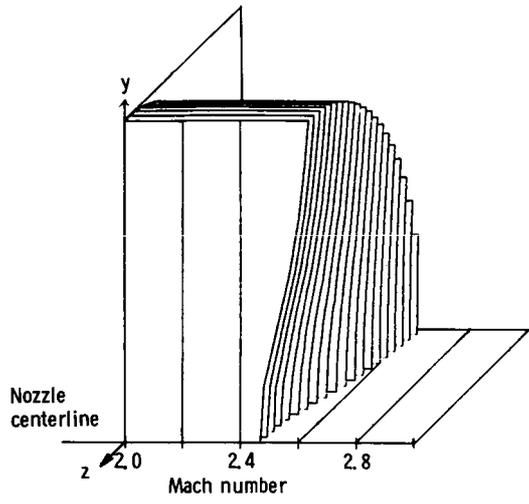


Figure 3. - Mach number distribution at exit station for axisymmetric nozzle of example I.

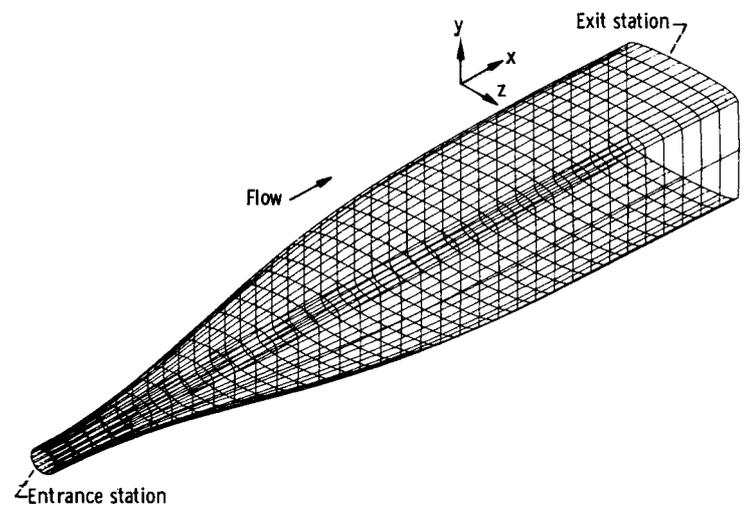


Figure 4. - Sketch of nozzle used for example II.

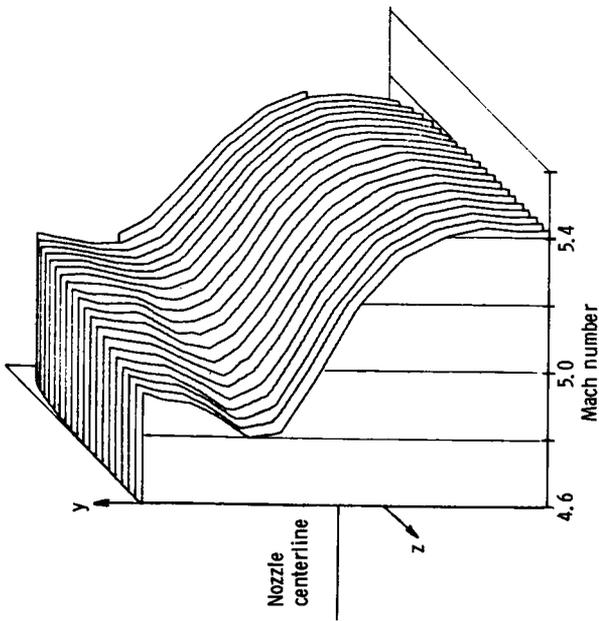


Figure 5. - Mach number distribution at exit station for example II.

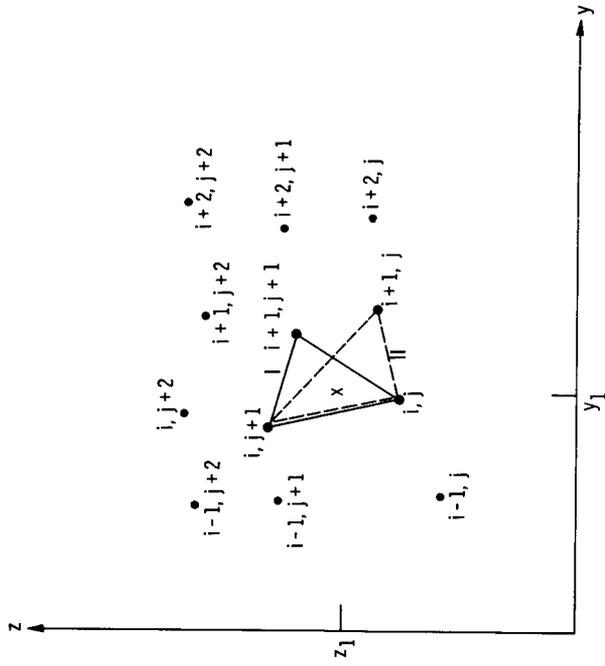


Figure 6. - Diagram of two-dimensional interpolation scheme used to find intermediate points on initial-value surface.

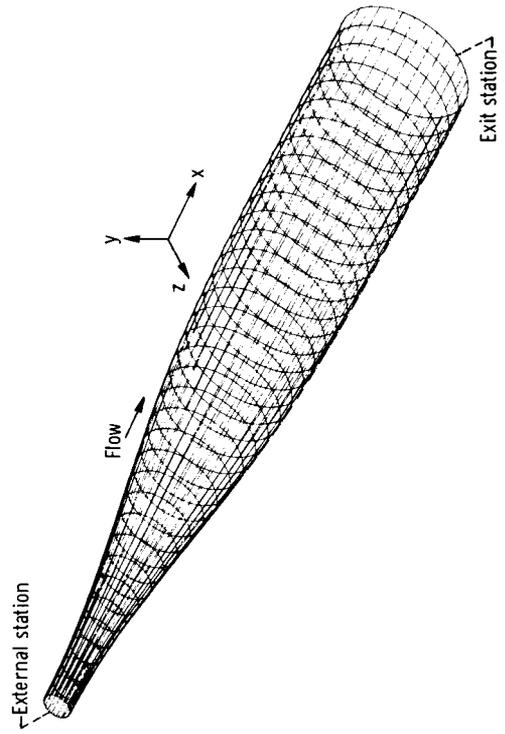


Figure 7. - Sketch of axisymmetric nozzle used for example III.

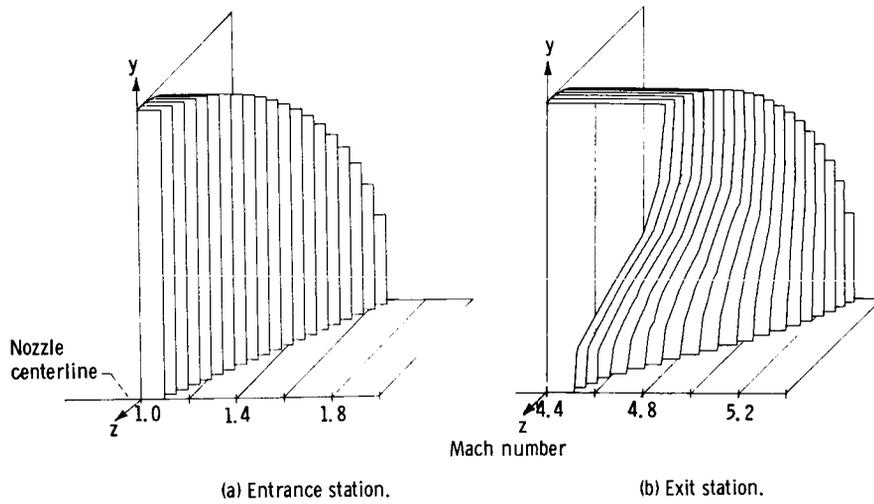


Figure 8. - Mach number distribution at entrance and exit stations for axisymmetric nozzle of example III.

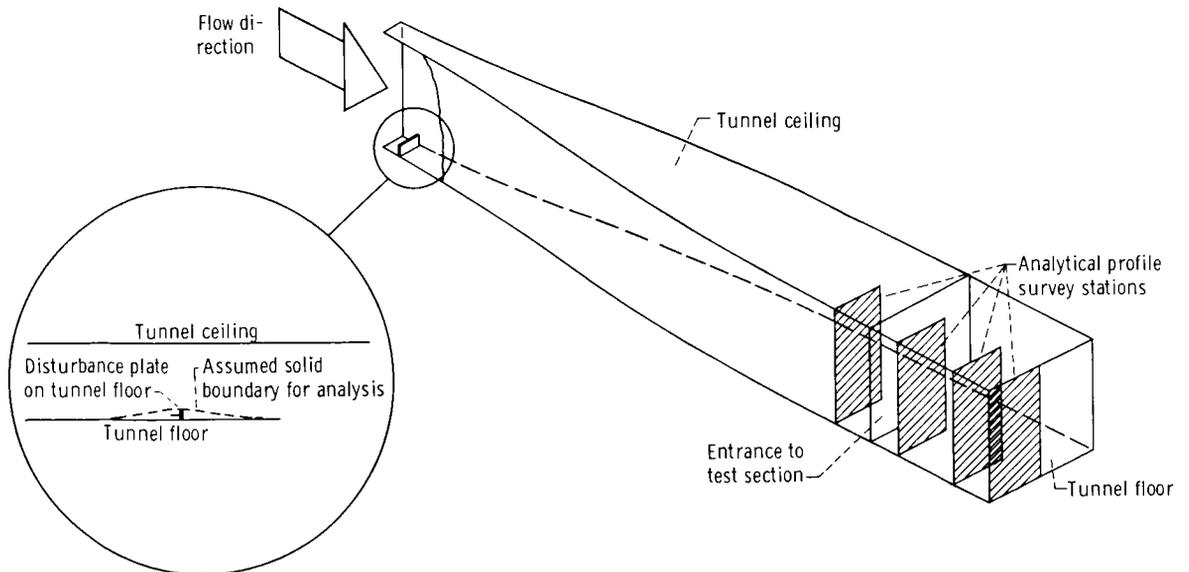


Figure 9. - Sketch of 10- by 10-Foot Supersonic Wind Tunnel showing disturbance generator.

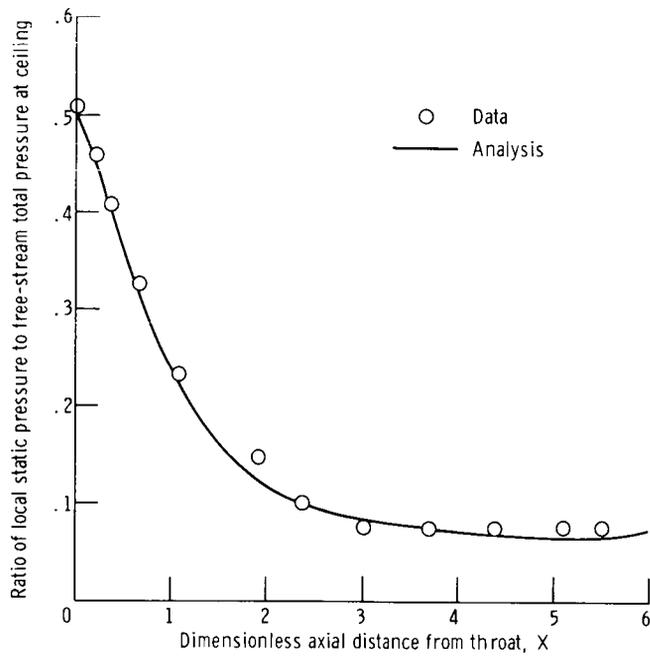


Figure 10. - Undisturbed static-pressure distribution in 10- by 10-Foot Supersonic Wind Tunnel. Wall setting, Mach 2.4.

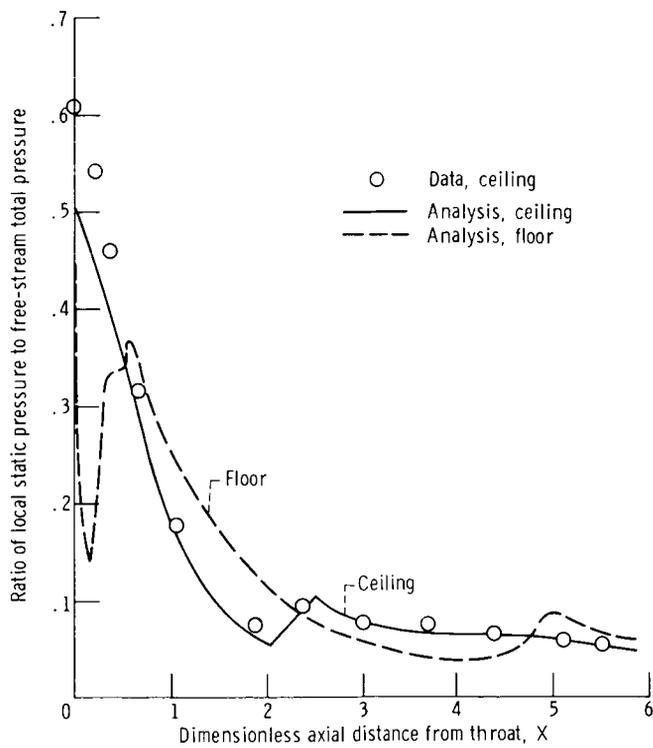


Figure 11. - Disturbed static-pressure distribution in 10- by 10-Foot Supersonic Wind Tunnel. Height of generator, 30.5 centimeters; wall setting, Mach 2.4.

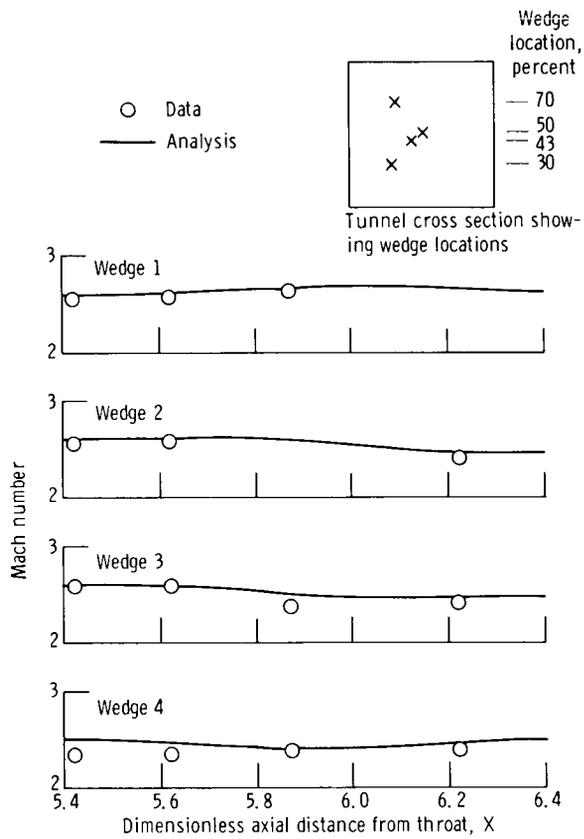
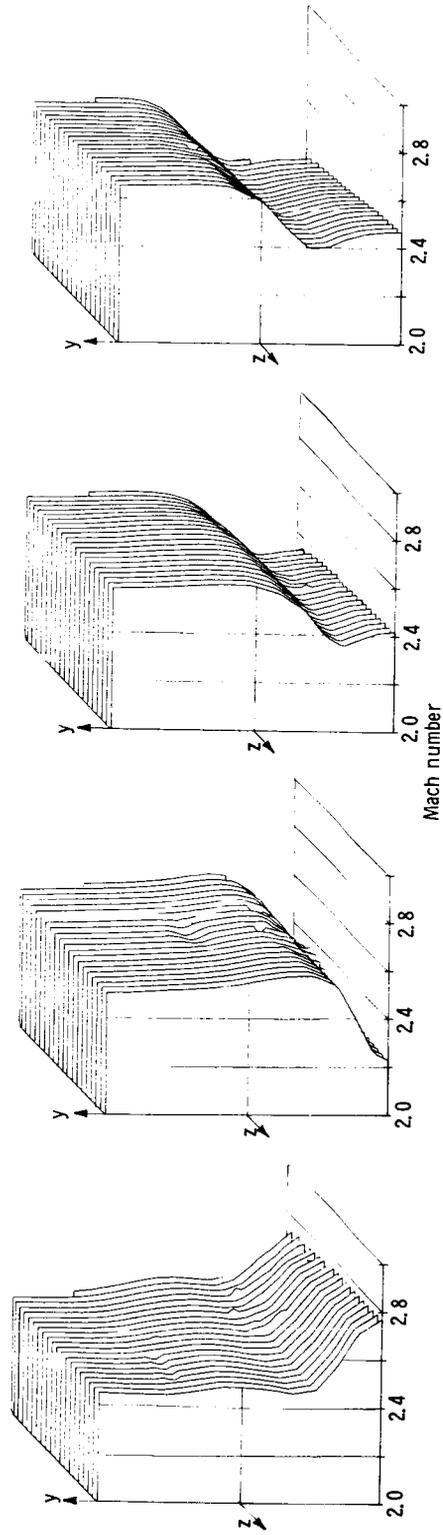


Figure 12. - Mach number as function of axial length for four wedges. Height of generator, 30.5 centimeters; wall setting, Mach 2.4.



(a) $X = 4.71$

(b) $X = 5.30$

(c) $X = 5.77$

(d) $X = 6.14$

Figure 13. - Disturbed Mach number distribution at various axial locations X . Height of generator, 30.5 centimeters; wall setting, Mach 2.4.

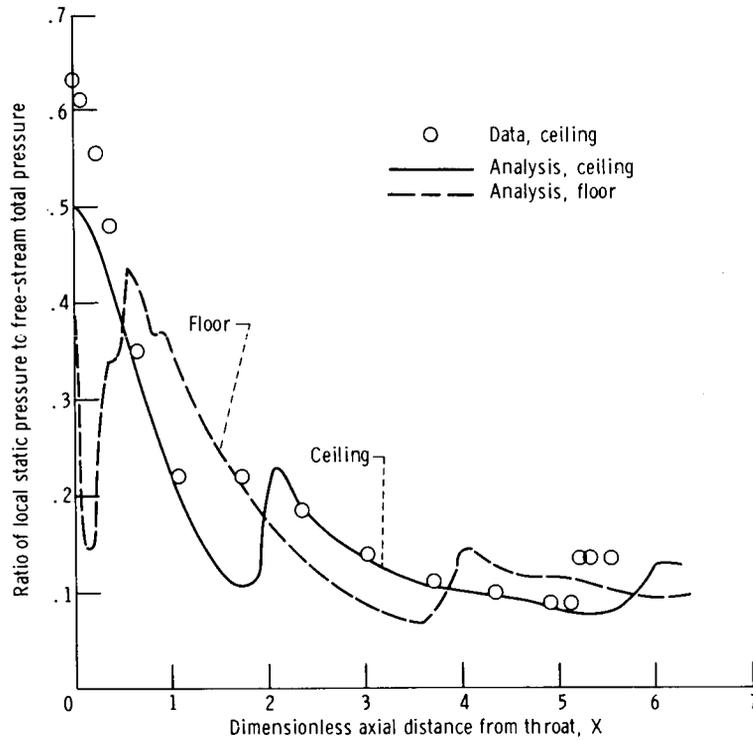


Figure 14. - Disturbed static-pressure distribution in 10- by 10-Foot Supersonic Wind Tunnel for 30.5-centimeter-high disturbance generator and wall setting of Mach 2.0.

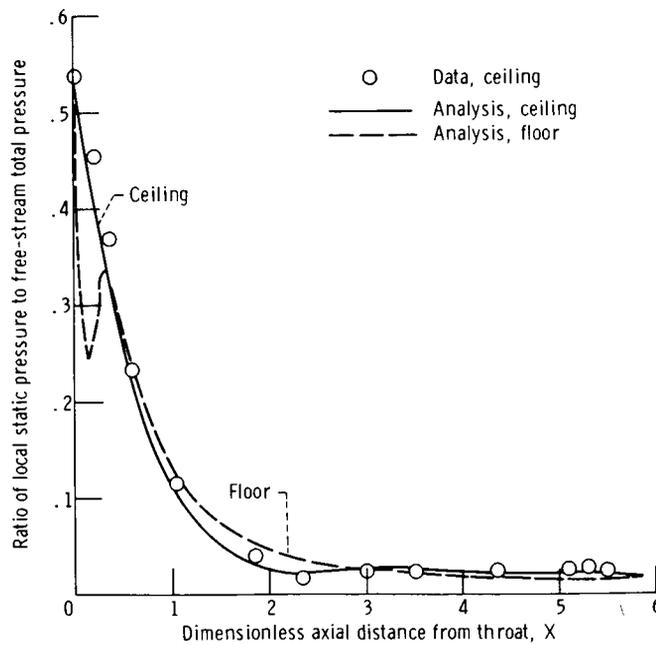


Figure 15. - Disturbed static-pressure distribution in 10- by 10-Foot Supersonic Wind Tunnel for 15.2-centimeter-high disturbance generator and wall setting of Mach 3.1.