STATIC DC TO DC POWER CONDITIONING-ACTIVE RIPPLE FILTER, 1 MHZ DC TO DC CONVERSION, AND NONLINEAR ANALYSIS

by

William August Sander, III

Prepared under Research Grant No. NGL 34-001-001, Supplement 13

March 23, 1973
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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ABSTRACT

Dc to dc static power conditioning systems on unmanned spacecraft have as their inputs highly fluctuating dc voltages which they condition to regulated dc voltages. These input voltages may be less than or greater than the desired regulated voltages. Also, these systems should have high reliability, small size and weight, and high efficiency. This dissertation discusses the design of two circuits which address specific problems in the design of these power conditioning systems and a nonlinear analysis of one of the circuits.

One area of concern in spacecraft power conditioning systems is the low reliability of large-capacitance capacitors commonly used in the L-section or pure-capacitance output filters of dc to dc converters. In spacecraft applications these capacitors are usually tantalum electrolytic capacitors and are normally the least reliable electronic component. The first circuit design discussed is for a nondissipative active ripple filter for dc supplies which uses an operational amplifier to amplify the sensed ripple voltage. The output of the amplifier is connected to the primary winding of a mutual inductor which has its secondary connected in series with the main power path. The closed-
loop gain and phase shift of the amplifier are designed so that the voltage on the secondary of the mutual inductor is of the same amplitude and phase as the ripple voltage at the input to the filter. The secondary voltage has the effect of cancelling the ripple voltage. One configuration of the ripple filter requires only 4.87 μF and has approximately the same attenuation as a low pass L-section filter with a break frequency of 100 Hz requiring 84 times more capacitance. The small required capacitance for this filter can be obtained using the more reliable ceramic capacitors.

Normal optimum switching for dc to dc power conditioning circuits is in the range of 2 to 50 kHz. However, in the future, experiments for spacecraft are anticipated which will make measurements in this frequency range. Therefore, a dc to dc converter operating at a switching frequency of 1 MHz is the second circuit discussed. The type of circuit chosen is an energy-storage converter using an isolated two-winding mutual inductor as the energy storage element and a triangular waveform clock signal to produce a constant switching frequency of 1 MHz. This circuit is designed to demonstrate the feasibility of operation at 1 MHz; therefore, the design does not include self-starting and requires several power supplies. However, the circuit operates at an output power of 20 watts with a peak efficiency of 79 percent and output-voltage regulation of approximately 1.5 percent. Because of the high switching frequency of 1 MHz, this circuit also requires less filter capacitance and should have smaller size and
weight than lower frequency converters.

The third subject of this dissertation is a nonlinear analysis of the type of dc to dc converter utilized in designing the 1 MHz converter. This type of circuit adds the output voltage to the triangular clock signal and subtracts this sum from the reference voltage resulting in an error voltage. The analysis is performed on this error voltage assuming instantaneous switching. The period of oscillation is divided into segments described by linear differential equations which are represented in a limit cycle in a normalized phase plane of the time derivative of the error voltage versus the error voltage. The theoretical phase plane compares closely with an experimental phase plane determined from a 2 kHz converter. Piecewise linear transformations are made around the limit cycle to obtain two sequence functions defining the time periods of a cycle. The accuracy of these sequence functions is verified using data from the experimental 2 kHz converter.
ACKNOWLEDGMENTS

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My supervisor Dr. Thomas G. Wilson deserves special acknowledgment for many hours on weekends and evenings which he gladly gave to the goal of a timely completion of this research and dissertation. The experience of working with him has taught me much practical knowledge which could not be learned from any other source. I am very grateful for all that he has given me.

These acknowledgments are not complete without expressing my appreciation to my wife for her love, her patience, and her understanding of the many hours of work required for the completion of this dissertation.

William A. Sander, III
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STATIC DC TO DC POWER CONDITIONING-ACTIVE RIPPLE FILTER, 1 MHZ DC TO DC CONVERSION, AND NONLINEAR ANALYSIS
Chapter I

INTRODUCTION

The subject of this dissertation is static electrical dc to dc power conversion for spacecraft power conditioning systems. The most commonly used power system on spacecraft has been and continues to be a system which consists of a dc source and a battery. The battery is used to store energy during periods of high output from the source and to supply energy during periods of low output from the source. Typical sources are fuel cells, solar cells, thermoelectric sources, and thermionic sources. The battery, by its nature, acts as a filter; however, the battery voltage fluctuates depending on whether it is in a fully charged or discharged state and whether it is being charged or discharged. The power conditioning system uses this battery as its source and conditions the battery voltage by conversion, regulation, and filtering to voltages which can be utilized by the various electrical equipment aboard the spacecraft.

Unmanned scientific satellites are continually being developed for a variety of scientific experiments. The power conversion equipment on each of these satellites generally has three common requirements which are high reliability, high
efficiency, and small size and weight. Other characteristics of the conversion equipment such as voltage levels, ripple voltage, switching frequency, and regulation may vary depending on the specific tasks or experiments which must be performed. Many of these requirements are becoming more stringent requiring continued applied research to develop equipment which provides the characteristics required by the increasingly more sophisticated experiments.

Scope

One area of concern in spacecraft power conditioning systems is the low reliability of large-capacitance capacitors commonly used in the L-section or pure-capacitance output filters of dc to dc converters. Capacitors greater than a few microfarads in value require the use of electrolytic capacitors. In spacecraft applications, tantalum electrolytic capacitors are used. These capacitors are normally the least reliable electronic component on the spacecraft and thus affect the overall reliability of the power conditioning system significantly. Chapter II discusses the design of a nondissipative active filter which uses an operational amplifier to amplify the sensed ripple voltage. The output of the amplifier is connected to the primary winding of a mutual inductor which has its secondary connected in series with the main power path. The closed-loop gain and phase shift of the operational amplifier are
designed so that the voltage on the secondary of the mutual inductor is of the same amplitude and phase as the ripple voltage at the input to the filter. The voltage on the secondary of the mutual inductor thus has the effect of cancelling the ripple voltage. This filter requires only a few microfarads of capacitance and uses the more reliable ceramic type of capacitor achieving approximately the same attenuation characteristic as an L-section filter requiring hundreds of microfarads. The design procedures and experimental characteristics are given in detail in Chapter II.

The normal optimum switching frequency for dc to dc electrical power conversion considering size and weight, reliability and efficiency is in the approximate range of two to fifty kHz. In the near future, however, it is anticipated that spacecraft experiments will include electromagnetic measurements at frequencies up to hundreds of kilohertz. In order that the spacecraft power conditioning equipment not interfere with the experimental measurements, it becomes necessary to design the switching frequencies of the power conditioning equipment at least as high as 1 MHz. This design presents many problem areas such as obtaining semiconductor diodes and transistors with adequately fast switching speeds and designing circuits which can effectively utilize these switching speeds. Core materials must be found which can be used to design transformers that operate efficiently at switching frequencies at least as high as 1 MHz. These problems are discussed in Chapter III where the design of
a basic dc to dc converter operating at a constant frequency of 1 MHz is discussed in detail. The design does not include an automatic starting circuit and requires several auxiliary supplies; however, the basic objective to operate a converter at a switching frequency of 1 MHz with reasonable efficiency is demonstrated. This converter is designed with electrical isolation between the source and the load and operates with good efficiency and a maximum output power of 20 watts. The output power can be increased simply by increasing the base drive to the power transistor. This converter provides a regulated output voltage of 28 volts while operating with a source voltage variation from 21 to 31 volts. The high switching frequency of this converter enables the designer to use the more reliable capacitors of only a few microfarads, therefore offering a second possible means for reducing the output filter capacitance in dc to dc converters.

Every new spacecraft seems to generate a new set of requirements for its power conditioning system. The challenge to design these systems is greatly aided by an analytical ability to predict the performance of the circuits used in the systems prior to designing them. There are a number of different types of circuits used for dc to dc energy conversion and only a small number of these have been thoroughly analyzed. A complete analysis of converters which utilize switching techniques usually requires nonlinear analysis. Chapter IV of this dissertation contains a nonlinear analysis of the error voltage for the type of
dc to dc converter circuit used for the 1 MHz converter in Chapter III. The basic regulation concept of the converter is verified by the analysis and by an experimental confirmation of the analysis using a converter operating at a frequency of 2 kHz. The analysis assumes instantaneous switching and divides the full period of oscillation into segments which are described by linear differential equations. These equations are represented in the phase plane by segments of continuous curves and instantaneous switching is represented by jumps. This converter has a triangular waveform as a clock signal which results in nonautonomous differential equations which cannot be directly plotted in the phase plane; however, an approximation is used which makes the equations autonomous. These equations and analytic expressions for the jumps at switching are used to perform piecewise linear transformations from point to point around the limit cycle resulting in a pair of sequence functions. These sequence functions are expressed in terms of the known circuit parameters and unknown time periods of the limit cycle. An experimental limit cycle for a circuit operating at 2 kHz and the theoretical phase-plane trajectory compare very closely. The accuracy of the sequence functions is also verified by the experimental data. The analysis thus shows that systems with triangular driving functions can be analyzed using the method of piecewise linear transformations in the phase plane.

Chapter V provides a summary of the work described in the dissertation and describes certain conclusions concerning this work.
Chapter II

NONDISSIPATIVE ACTIVE FILTER FOR DC SUPPLIES

Power sources used on spacecraft typically produce widely-fluctuating dc voltages which cannot be used directly to supply electronic equipment. Consequently, dc to dc converters are employed to provide the various regulated dc supply voltages required by the spacecraft equipment. These converters must be capable of meeting very stringent requirements such as tightly-regulated output voltages with small ripple voltage, high operating efficiency, high reliability, small size, and low weight. Nondissipative switching-mode converters instead of series-dissipative or shunt-dissipative regulators are commonly used because they are more efficient when applied to systems with widely-fluctuating input voltages and because they can be used to obtain regulated voltage levels higher than the source voltage [1]. The output current of a switching-mode converter usually includes rapid large-amplitude changes which require a significant amount of filtering. Dc to dc converters commonly operate at switching frequencies in the range of two to fifty kHz and, in order to achieve high efficiencies, require output filters with nondissipative dc paths. Nondissipative L-section or pure-capacitance filters are commonly used to provide the
filtering required to produce smooth dc voltages. In the frequency range of two to fifty kHz, these filters require large amounts of capacitance.

In order to provide these large amounts of capacitance the designer is required to use electrolytic capacitors. These large-microfarad dc filter capacitors are less than ideal in satellite static power converter applications. They occupy a large volume and have a relatively low reliability compared to low-microfarad capacitors, such as ceramic, mica, and solid tantalum, and other circuit components. In applications such as spacecraft where size and weight are important factors, the type of capacitor which is usually chosen from among the many different types currently available is the dry slug or wet slug tantalum electrolytic capacitor which has large capacitance per unit volume. However, these capacitors tend to fail due to inherent dielectric breakdown, internal heating, and electrolyte leakage particularly in the severe operating conditions of outer space [2,3,4]. The low-microfarad types of capacitors have better reliability but if used to provide hundreds of microfarads of capacitance would occupy an exorbitantly large volume due to their small capacitance per unit volume. Consequently, there is a definite need to reduce the amount of capacitance required in the output filters of dc supplies, even by increasing circuit complexity, so that the more reliable low-microfarad capacitors can be used to improve filter reliability.
Various techniques for reducing the required capacitance have been developed; however, they generally do not have dc paths or they have dissipative elements in the dc paths [5,6]. As is later explained in Chapter III, the required capacitance can also be reduced at the expense of other circuit performance characteristics, such as efficiency, by going to a much higher switching frequency in the vicinity of one MHz.

Scope

The active ripple filter configuration discussed in this chapter has a non-dissipative dc path and operates in the frequency range of two to fifty kHz. It utilizes an operational amplifier, a mutual inductor, and more reliable low-microfarad capacitors to provide the same ripple attenuation that is obtained using conventional L-section or pure-capacitance filters.

The basic principles of L-section and pure-capacitance filters and of the active ripple filter configuration are explained using block diagrams of the converter system. Then, the complete schematic diagram for the active ripple filter configuration is given along with a brief description of the purpose of each component. The transfer characteristic of the operational amplifier is derived separately and then combined with the remainder of the circuit to obtain an analytical transfer function. Some circuit principles, or design considerations, are given next which should be considered when designing an
active ripple filter. Three types of active ripple filters all derived from the same basic configuration are discussed. The special case of the basic configuration which yields an active filter having a narrow bandwidth of attenuation and its transfer function are discussed first. Next, the special case of a wide-bandwidth filter and its transfer function are discussed. Feedback is then added from the output of the wide-bandwidth filter and a new transfer function is derived which is the third special case with an attenuation characteristic significantly better than the attenuation characteristic of the wide-bandwidth filter without feedback. The theoretical and experimental characteristics of each of these three active ripple filters are discussed individually. In addition, the capacitance advantage of the wide-bandwidth filter with feedback as compared to an L-section filter is discussed; and some possible applications and additional conclusions are given.

Circuit Principles

Background. Before an active filter can be designed to attenuate the load ripple voltage generated by nondissipative switching dc to dc converters, the manner in which this voltage is generated should be fully understood. Fundamental examples of three types of dc to dc switching converters are shown in Fig. 1. Figs. 1(a) and 1(b) show voltage step-up and voltage step-down energy-storage converters, respectively [7,8,9,10].
Fig. 1. Basic converter configurations.
In both cases, when the switch is closed, energy from the source $E$ is stored in the inductor $L$; and when the switch is opened, the energy is released through diode $D$ forming the current waveforms shown. These waveforms are subsequently filtered by capacitance $C'$ and the L-section filter made up of $L_f$ and $C_f$ to produce the ripple voltage on the load $R_L$. Fig. 1(c) employs two controlled switches which switch in such a manner as to apply the source $E$ alternately to the primary windings of the non-saturating transformer $T$. The voltage on the secondary is rectified to obtain the voltage $v$ which is filtered by $C'$ and the L-section filter $L_f$ and $C_f$ to obtain the ripple voltage on the load $R_L$ [11]. The waveform for $v$ in Fig. 1(c) is the full-wave rectified voltage without any filtering due to $C'$, $L_f$, or $C_f$; and the voltage is shown to dip to zero during the finite switching times. Whereas the circuits in Figs. 1(a) and 1(b) generate current waveforms which require a capacitive input filter, the circuit in Fig. 1(c) generates a full-wave rectified voltage waveform for which a capacitance $C'$ may not be required. In either case, it is possible to remove the L-section filter, made up of $L_f$ and $C_f$, and have a pure-capacitive filter utilizing a single large-microfarad capacitance $C'$.

The nature of the ac voltage on $C'$ is of significance when designing the filter. This ac voltage is generated when the capacitance charges and discharges during the switching cycle of the converter. The converter is basically a nonlinear circuit. A Fourier analysis of the switching waveform would
yield a large amplitude fundamental component with the same fre-
quency as the switching frequency. The nonlinear nature of the
circuit would also produce numerous large-amplitude higher-order
harmonics. The most desirable filter is, therefore, a lowpass
filter which must be capable of passing a large direct current
and which must have a cutoff frequency less than the switching
frequency.

A basic dc to dc converter system is shown in Fig. 2(a). The dc
to dc converter is represented by the block appropriately
labelled. Capacitance C' is included as part of the converter
since many types of converters do require some output capaci-
tance. The source E is shown as well as its internal resistance
r_e. Inductor L_f and capacitor C_f make up the conventional L-
section filter and R_L is the load resistance. The direct volt-
age on C' is V'; and the converter output ripple voltage is v_C.
V' + v_C is the complete converter output voltage. After passing
through the filter, the direct voltage at the load R_L is V; and
the ripple voltage is v_L. In the following development of the
active filter, it is important to remember that the objective of
the active filter is to replace the conventional L-section fil-
ter with circuitry that: (1) passes the required direct current
with high efficiency, (2) requires only small-microfarad filter
capacitance so as to improve the filter reliability, (3) does
not significantly increase the size and weight of the filter,
and (4) achieves approximately the same ripple-voltage attenua-
tion as the L-section filter. One of the changes which must be
Fig. 2. (a) DC to dc converter using a conventional L-section filter. (b) DC to dc converter using the ripple filter configuration.
made to the filter shown in Fig. 2(a) in order to meet the objectives is that the capacitance requirements for $C'$ and $C_f$ must either be reduced or removed.

The basic configuration for the active ripple filter which is discussed in this chapter is shown in Fig. 2(b). Capacitor $C_f$ is removed and inductor $L_f$ is replaced by mutual inductor $M$. Capacitor $C$ may be only a few microfarads instead of the hundreds of microfarads required for $C'$ in Fig. 2(a). The effect of the capacitance reduction is an increase in the magnitude of the converter output ripple $v_c$. The active ripple filter uses the principles of Kirchhoff's Law to reduce $v_c$ to an acceptable small amplitude for $v_L$. The converter output ripple voltage $v_c$ in Fig. 2(b) is sensed and regenerated, by the block so labelled, on the primary winding $N_p$ of the mutual inductor so as to induce a voltage $v_s$ on the secondary winding $N_s$. By Kirchhoff's Law, $v_L$ is equal to the difference between $v_c$ and $v_s$ as

$$v_L = v_c - v_s$$  \hspace{1cm} (1)

If the waveform of $v_s$ is exactly the same as the waveform of $v_c$, then $v_L$ is zero. However, this condition is impractical to obtain; and $v_L$ always has some magnitude. The following discussion is concerned with the schematic diagram of one example of this type of active filter and a discussion of some of the design considerations.
The active filter circuit. The complete schematic diagram for the active filter is given in Fig. 3 and the component values are given in Table 1. In this diagram, a battery is used to represent the dc component $V'$ of the converter output voltage; and an ac voltage source is used to represent the converter output ripple voltage $v_c$. Resistor $r_c$ is the equivalent series output resistance of the converter. The dc load-current path through the mutual inductor $M$ and load $R_L$ is indicated by heavy lines. The operational amplifier with its associated components is employed as the ripple sensor and regenerator referred to in Fig. 2(b). Capacitor $C_1$ is the coupling capacitor which blocks the dc voltage $V'$ and should have a negligible effect on the ac input signal to the operational amplifier. Resistor $R_S$ is the series resistance of the operational amplifier and $R_F$ is the feedback resistor. The closed-loop gain of the amplifier is therefore approximately $R_F/R_S$. Resistor $r_p$ is the equivalent series resistance of the primary winding $N_p$ of the mutual inductor $M$ which has its secondary winding $N_S$ in series with the dc path. $L_p$, $L_s$, and $M$ are the primary, secondary, and mutual inductance, respectively. It should be noted that the negative input to the amplifier is used which puts the amplifier in the inverted mode and that the polarities of the primary and secondary windings of the mutual inductor are therefore reversed. Bias resistor $R_Z$ and zener diodes $D_{Z1}$ and $D_{Z2}$ provide the plus and minus voltage supplies for the amplifier. Capacitors $C_S$
Fig. 3. Schematic diagram for the active ripple filter.
Table I. Components list for the active filter.

<table>
<thead>
<tr>
<th>Component</th>
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<tr>
<td>OP AMP</td>
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<tr>
<td>C₁</td>
<td>4.77 μF</td>
</tr>
<tr>
<td>D₁, D₂</td>
<td>1N964A</td>
</tr>
<tr>
<td>Rₚ</td>
<td>28 kΩ fixed + 5 kΩ trimpot</td>
</tr>
<tr>
<td>Rₛ</td>
<td>5110 ohms</td>
</tr>
<tr>
<td>Rₗ</td>
<td>100 ohms</td>
</tr>
<tr>
<td>rₚ</td>
<td>52.22 ohms</td>
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<tr>
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<td>source internal resistance</td>
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<tr>
<td>Lₚ</td>
<td>203 mH (1000 turns AWG no. 36)</td>
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<tr>
<td>Lₛ</td>
<td>6.2 mH (176 turns AWG no. 22)</td>
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<td>M</td>
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<td>Core</td>
<td>Magnetics, Inc. no. 55928</td>
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<tr>
<td>Narrow bandwidth</td>
<td>147 pF</td>
<td>67 pF</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wide bandwidth</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wide-bandwidth with feedback</td>
<td>0</td>
<td>0</td>
<td>0.1 μF</td>
<td>5 kΩ trimpot</td>
</tr>
</tbody>
</table>
and $C_p$ are used to compensate the transfer characteristic of the amplifier. They are shown dashed because it is not always necessary to utilize them. Likewise, $R_1$ and $C_2$ may not always be used so they are shown dashed. The functions of these components are explained in more detail later in the chapter.

**Active ripple filter power dissipation.** It is important that the active filter be essentially nondissipative. For this reason, it is necessary to show that the load on the operational amplifier is essentially non-dissipative and that all other dissipation is very small compared to the power passing through the filter to the load. First, the load ripple voltage $v_L$ should be very small so that $v_s$ essentially cancels $v_c$ and negligible load is reflected to the primary winding $N_p$. Second, the primary inductance $L_p$ must have a high $Q$ to minimize phase shift as is explained later. This means that $r_p$ must be small and the dissipation in $r_p$ is negligible. Under these two conditions the load on the output of the operational amplifier is essentially a pure inductance $L_p$. The output stage of most operational amplifiers is a class A amplifier; and, when a class A amplifier is used to drive a pure inductive load such as $L_p$, there is no real power delivered from the output of the amplifier. This means that the power dissipated in the active ripple filter in Fig. 3 that is not present in a conventional L-section filter is: (1) the negligible power dissipation in $R_L$ due to a nonzero $v_L$ (2) the negligible dissipation due to $r_p$ and (3) the quiescent power required by the operational amplifier and its bias sources. The
first and second of these are usually much smaller than the quiescent dissipation, and the quiescent dissipation is usually much less than the dc power passing through the filter to the load.

**Derivation of the transfer function.** The purpose of the following analysis is to derive the analytical expression for the transfer function $V_L(s)/V_C(s)$ between the input voltage $v_C$ and the output voltage $v_L$ of the filter. The analysis is primarily an ac analysis and does not include any dc or transient analyses. The transfer function $K_{op}(s)$ for the operational amplifier is found first. Then a circuit analysis is performed using $K_{op}(s)$ to obtain the overall transfer function $V_L(s)/V_C(s)$ for the active filter.

When the operational amplifier has an open loop gain significantly higher than the closed loop gain at all frequencies of interest, the closed loop gain is assumed to be equal to the feedback impedance $Z_F(s)$ divided by the series impedance $Z_S(s)$ [12]. In the diagram of Fig. 3, $Z_S$ is the combination of $C_1$, $R_S$, and $C_S$; and $Z_F$ is the combination of $C_F$ and $R_F$ so that

$$Z_S(s) = \frac{1}{sC_1} + \frac{R_S}{1+R_SC_F s}$$  \hspace{1cm} (2)

and

$$Z_F(s) = \frac{R_F}{1+R_FC_F s}.$$  \hspace{1cm} (3)

Using (2) and (3), the transfer function $K_{op}(s)$ of the
operational amplifier is

\[
K_{op}(s) = \frac{V_p(s)}{V_c(s)} = -\frac{Z_F(s)}{Z_S(s)} = -\frac{\left[1 + R_S C_S s\right] R_F C_1 s}{\left[1 + R_F C_F s\right]\left[1 + R_S \left(C_1 + C_S\right) s\right]}.
\] (4)

Since \(C_1\) is a coupling capacitor, it normally has a much larger value compared with \(C_S\); therefore \(C_S\) may be neglected in the denominator of (4) yielding

\[
K_{op}(s) \approx \frac{\left[1 + R_S C_S s\right] R_F C_1 s}{\left[1 + R_F C_F s\right]\left[1 + R_S C_1 s\right]}.
\] (5)

As mentioned earlier, the transfer function for the operational amplifier is derived on an assumption that the loop gain, which is the difference between the open-loop gain and the closed-loop gain of the operational amplifier is so large that the closed-loop gain can be assumed to be equal to \(Z_F(s)/Z_S(s)\). This assumption limits the frequency of validity of (5) to frequencies well below the unity-gain-bandwidth frequency of the amplifier. At high frequencies, the loop gain of the operational amplifier becomes small; and the ideal approximation \(K_{op}(s) = Z_F(s)/Z_S(s)\) is no longer accurate. The Analog Devices 148A operational amplifier used in Fig. 3 has a unity-gain-bandwidth product of at least 10 MHz. Since most all converters operate at switching frequencies well below this frequency the approximation is acceptable.

The operational amplifier gain \(K_{op}(s)\) can be used to relate \(V_p\) and \(V_c\) as
Knowing this, Fig. 3 can be analyzed to obtain the transfer function $V_L(s)/V_C(s)$. The internal resistance of $L_s$ and $r_C$ are in series with $R_L$ along the dc path. These resistances are considered to be negligible compared to $R_L$ for all reasonable loads. Also, $R_Z$ is large with respect to typical values of $R_L$ so the current $i_s$ which flows through the secondary winding $N_s$ is assumed to flow entirely through $R_L$ which yields

$$\frac{V_L}{V_C} = \frac{i_S R_L}{V_C} . \quad (7)$$

The loop equations for the secondary and primary circuits of the mutual inductor are

$$V_C(s) = \left(sL_s + R_L\right)I_s(s) + sMI_p(s) \quad (8)$$

and

$$V_p(s) = -sMI_s(s) - \left(sL_p + r_p\right)I_p(s) \quad (9)$$

where $V_C(s)$, $V_p(s)$, $I_p(s)$, and $I_s(s)$ are the Laplace forms of the converter output voltage $v_C$, the primary voltage $v_p$, the primary current $i_p$, and the secondary current $i_s$. Solving (8) and (9) by Cramer's rule yields

$$I_s(s) = \frac{\begin{vmatrix} V_C(s) & sM \\ V_p(s) & -\left(sL_p + r_p\right) \end{vmatrix}}{\begin{vmatrix} sL_s + R_L & sM \\ -sM & -\left(sL_p + r_p\right) \end{vmatrix}} . \quad (10)$$
Reducing the determinants in (10) yields

\[
I_s(s) = \frac{-[sL_p + r_p]V_c(s) - sMV_p(s)}{-[sL_s + R_L][sL_p + r_p] + s^2M^2} \quad . \quad (11)
\]

Substituting for \(V_p(s)\) by using (6) yields

\[
I_s(s) = \frac{sL_p + r_p + SMK_{op}(s)}{[sL_s + R_L][sL_p + r_p] - s^2M^2} V_c(s) \quad . \quad (12)
\]

Rearranging (12) yields

\[
I_s(s) = \frac{1 + \left( \frac{L + K_{op}(s)M}{r_p} \right)s}{s^2L_p + \left( \frac{L_p + L_s}{r_p + R_L} \right)s + 1} \cdot \frac{V_c(s)}{R_L} \quad . \quad (13)
\]

As previously given in (7), the load ripple voltage can be expressed as \(V_L = i_sR_L\) in the time domain and as \(V_L(s) = I(s)R_L\) in the Laplace form. Therefore the Laplace form of the overall filter transfer function is

\[
\frac{V_L(s)}{V_c(s)} = \frac{I_s(s)R_L}{V_c(s)} \quad . \quad (14)
\]

By substituting (13) into (14) and cancelling \(V_c(s)\) and \(R_L\) in the numerator and denominator, the complete transfer function for the active ripple filter is obtained in terms of the circuit parameters and \(K_{op}(s)\) as
\[
\frac{V_L(s)}{V_C(s)} = \frac{1 + \left(\frac{L_p + K_{op}(s)M}{r_p}\right)s}{L_sL_p - M^2 \left(\frac{L_p}{r_pR_L}\right)s^2 + \left(\frac{L_p}{r_p} + \frac{L_s}{r_p} - \frac{L_p}{r_pR_L}\right)s + 1}
\]  

(15)

In (15) the coefficient \(\frac{L_sL_p - M^2}{r_pR_L}\) is much smaller than \(\frac{L_p}{r_p} + \frac{L_s}{R_L}\) and \(L_s/R_L << L_p/r_p\). Also it was determined experimentally that \(V_L(s)/V_C(s)\) is independent of the magnitude of \(R_L\) except when \(R_L\) takes values less than ten ohms. So if \(R_L\) is allowed to approach infinity (15) yields

\[
\frac{V_L(s)}{V_C(s)} = \frac{1 + \left(\frac{L_p + K_{op}(s)M}{r_p}\right)s}{1 + \frac{L_p}{r_p}s}
\]  

(16)

Equation (5) is now substituted into (16) to obtain the approximate transfer function as

\[
\frac{V_L(s)}{V_C(s)} = 1 - \frac{MR_FC_1}{1 + \frac{L_p}{r_p}s} \cdot \frac{s^2 + 1}{s^2 + 1 + R_FCFS}
\]  

(17)

Since \(V_L = V_C - V_s\), then

\[
\frac{V_L(s)}{V_C(s)} = \frac{V_C(s) - V_s(s)}{V_C(s)} = 1 - \frac{V_s(s)}{V_C(s)}
\]  

(18)

A comparison of (17) and (18) yields

\[
\frac{V_s(s)}{V_C(s)} = \frac{MR_FC_1}{1 + \frac{L_p}{r_p}s} \cdot \frac{s^2 + 1}{s^2 + 1 + R_FCFS}
\]  

(19)
Equation (17) is the theoretical transfer function for the circuit shown in Fig. 3 excluding the components $R_1$ and $C_2$. The purpose of $R_1$ and $C_2$ is discussed later and another transfer function is derived which includes these components. Fig. 3 and Equation (17) will be used in the following discussion.

**Design considerations.** This subsection discusses certain principles of design which must be considered when designing the active filter. The discussion of these principles should contribute toward the comprehension of the circuit functions.

The coupling capacitor $C_1$ which is the largest capacitor in the active filter is used to block the dc voltage $V'$ of the converter output and should have negligible effect on the ac input signal to the operational amplifier. In order to accomplish this, the reactance of $C_1$ must be much smaller than the input impedance of the amplifier. The closed-loop input impedance of the inverting operational amplifier is approximately the input series resistance $R_s$ [13]. Since it is desired to keep the total capacitance in the ripple filter small, $R_s$ should be as large as possible within the limits of amplifier performance so that $C_1$ may be small. $C_1$ is the largest capacitance in the active filter; and yet with $R_s = 5110$ ohms, $C_1$ is less than 5 μF compared with hundreds of microfarads commonly used in filters in dc to dc converters and power supplies. Therefore $C_1$ may be a ceramic or mica capacitor instead of the electrolytic capacitor.
required when hundreds of microfarads are needed.

A variety of amplifiers can be used for the amplifier function. The operational amplifier is selected because of its versatility and because its characteristics can be easily described analytically. When the amplifier is designed, its gain should be equal to the voltage ratio $\frac{v_p}{v_s}$ of the mutual inductor since the overall gain from $v_c$ to $v_s$ should be unity. The output voltage swing of the operational amplifier is limited by the supply voltage. Therefore, for a given voltage $v_c$, the voltage transformation ratio $\frac{v_p}{v_s}$ of the mutual inductor must be small enough so that the amplifier gain does not have to be so large that the output voltage limit is exceeded. The ripple amplitude $v_c$ at the output of the converter in Fig. 2(b) depends on $R_L$, $C$, the switching frequency, and the duty cycle. The converter output ripple amplitude must be determined before the closed-loop operational amplifier and the mutual inductor can be designed. If difficulty is encountered in designing the operational amplifier and mutual inductor so that the amplitude of $v_c$ does not saturate the amplifier, the converter output capacitance $C$ may be increased in order to reduce the amplitude of $v_c$.

The operational amplifier should have a wide bandwidth with the highest possible cutoff frequency. Since it is desirable that $R_S$ is large, the feedback resistance $R_F$ must also be large. These large resistors make the stray feedback capacitance in parallel with $R_F$ and the stray input capacitance which
is effectively in series with $R_s$ more significant especially at high frequencies. For these reasons, it may be necessary to add capacitors $C_F$ or $C_S$ in order to cancel, or balance, the effects of one or the other stray capacitances.

As mentioned earlier, the operational amplifier should have a high open-loop gain to minimize the closed-loop gain variation with frequency and should have a low quiescent power dissipation to maximize the filter efficiency. The operational amplifier may be either the integrated-circuit type or the module type. The integrated circuit is smaller and has less quiescent dissipation but the module type has higher open-loop gain to lower and higher frequencies.

All amplifiers have a limit, or peak, output current at which the amplifier is said to saturate. If it is assumed that the maximum output voltage for the amplifier has been determined, then the output current limit determines a minimum value for the primary inductance $L_p$ at the lowest frequency of operation. For this reason, the design of the mutual inductor is discussed next keeping in mind that a minimum inductance for $L_p$ must be established.

There is a nonlinear variation of inductance with dc bias which causes the inductance of a given winding on a core to decrease as the dc bias due to a direct current in any winding increases. This decrease in inductance affects both windings of the mutual inductor and requires that more alternating output
current be delivered by the operational amplifier. For this reason, the minimum inductance discussed in the last paragraph should be defined for full-load direct current flowing in $N_s$. With this condition met, the variation of the direct current does not significantly affect the performance of the filter. The inductance change due to a change in current is so small that the coefficient of coupling changes very little. It is also desirable that $r_p$ be as small as possible so that the mutual inductor has minimum phase shift at lower frequencies.

There are two additional factors to be considered in the design of the mutual inductor. One is leakage inductance and the other is winding capacitance. Leakage inductance is important because it affects the coupling coefficient of the mutual inductor. The coupling coefficient is a maximum, near to unity, when the leakage inductance is a minimum. The design techniques used to minimize leakage inductance increase the winding capacitance. For example, the technique of bifilar winding a mutual inductor puts each turn of one winding in close proximity to each turn of the other winding and thus increases the winding capacitance. If the windings are wound separately, separated by a layer of insulation, the winding capacitance is minimized; but the leakage inductance increases due to the physical separation of the windings. It is a known fact that leakage inductance is a much more significant parameter at low frequencies than is winding capacitance [14]. Since the frequency range of interest for most dc to dc converters in which large microfarad
capacitors are used is approximately two to fifty kHz, the mutual inductor should be designed to minimize the leakage inductance of the mutual inductor.

This section has discussed some of the properties of the components of the active filter, particularly the operational amplifier and the mutual inductor. The effect that these properties have on the performance of the active filter were discussed and design techniques which can be used to minimize these effects were given. An analysis to obtain the transfer function of the basic filter configuration has already been given. The next three sections of this chapter are devoted to the derivation of the transfer functions of three specialized configurations for a narrow-bandwidth filter, a wide bandwidth filter, and a wide-bandwidth filter with feedback. In each of these sections, the theoretical transfer characteristic is compared with the experimental transfer characteristic for the filter shown in Fig. 3.

Narrow-bandwidth Filter

The characteristics of the narrow-bandwidth filter are discussed first because they best illustrate the effects of the important parameters of phase shift and gain. As the name implies, this filter has a narrow frequency bandwidth over which the attenuation is significant. This filter does not have as important applications with regard to dc supplies as the wide-
bandwidth filter. Some examples of applications are a filter for a constant frequency dc to dc converter or a notch filter in a system which must pass direct current with little or no attenuation.

The circuit for the narrow-bandwidth filter is the circuit in Fig. 3 with \( C_F \) and \( C_S \) included but with \( R_1 \) and \( C_2 \) excluded. This circuit is the same one for which the approximate transfer function (17) has been derived. It can be determined by examining (18) that the attenuation of the filter approaches a maximum when \( V_s(s)/V_c(s) \) as defined in (19) approaches unity with zero phase shift. When \( C_S \) and \( C_F \) are included in the circuit, their values are chosen to obtain maximum attenuation at a chosen frequency. Then \( R_F \) is adjusted to obtain a maximum attenuation at this frequency. The calculation of the break frequencies of (19) and the magnitude of (19) at a frequency of one Hz are given in Table II. The break frequencies due to the time constants \( R_S C_1 \) and \( L_p/r_p \) are at low frequencies because \( C_1 \) and \( L_p \) have relatively large values. Also \( C_S \) and \( C_F \) are relatively small values of capacitance and the break frequencies due to \( R_S C_S \) and \( R_F C_F \) are high.

A sketch of the Bode diagram for (19) and its asymptotic approximation are plotted in Fig. 4 for the component values given in Table I. From these curves, it can be seen that \( |V_s(s)/V_c(s)| \) approaches unity over a finite band of frequencies. If only the magnitude of \( V_s(s)/V_c(s) \) were considered, it would
Table II. Calculation of the break frequencies of (19).

\[
\frac{V_s(s)}{V_c(s)} = \frac{\frac{M R_F C_1}{r_p}}{1 + R_S C_S s^2} \left( 1 + \frac{L_p}{r_p} s \right) \left[ 1 + R_F C_F s \right]
\]

\[
20 \log \frac{M R_F C_1}{r_p} \omega^2 = 20 \log \frac{35 \times 10^{-3} \times 29 \times 10^3 \times 4.7 \times 10^{-6}}{52} (2\pi)^2
\]

\[= -48.84 \text{ db}\]

\[
\omega_1 = \frac{1}{R_S C_1} = \frac{1}{5.1 \times 10^3 \times 4.7 \times 10^{-6}} = 41.7 \text{ rad/sec}
\]

\[
\omega_2 = \frac{r_p}{L_p} = \frac{52}{203} = 256 \text{ rad/sec}
\]

\[
\omega_3 = \frac{1}{R_F C_F} = \frac{1}{30 \times 10^3 \times 67 \times 10^{-12}} = 4.98 \times 10^5 \text{ rad/sec}
\]

\[
\omega_4 = \frac{1}{R_S C_S} = \frac{1}{5.1 \times 10^3 \times 1.47 \times 10^{-10}} = 1.33 \times 10^6 \text{ rad/sec}
\]

\[
f_1 = \frac{\omega_1}{2\pi} = 6.64 \text{ Hz}
\]

\[
f_2 = \frac{\omega_2}{2\pi} = 40.8 \text{ Hz}
\]

\[
f_3 = \frac{\omega_3}{2\pi} = 79.2 \text{ kHz}
\]

\[
f_4 = \frac{\omega_4}{2\pi} = 212 \text{ kHz}
\]
Fig. 4. The theoretical transfer characteristic for $\frac{v_s}{v_c}$ in (19) with $R_L = \infty$, $C_S = 147$ pF, and $C_P = 67$ pF.
be expected that the filter attenuation would reach a maximum and stay constant over the same finite bandwidth over which (19) is equal to unity. However, the phase shift in the mutual inductor in the vicinity of \( f_2 \), and the phase shift in the operational amplifier in the vicinity of \( f_3 \) have a significant effect. When \( v_c \) and \( v_s \) have very large amplitudes, such as a few volts, compared to the millivolts of ripple desired for \( v_L \), a tenth of a degree of phase shift between \( v_c \) and \( v_s \) can result in a significant amplitude for \( v_L \).

Since small amounts of phase shift are so significant, the phase shifts of the operational amplifier and mutual inductor are examined separately. The 180 degree phase shifts due to inversion in the operational amplifier and the mutual inductor offset each other and therefore are not considered to be part of either phase shift. The phase shift of the operational amplifier in the frequency range of normal interest from two kHz to fifty kHz can be found by examining the expression for \( K_{op}(s) \) in (5). Since \( f_1 = 1/(2\pi R_SC_1) = 6.64 \text{ Hz} \) is very much less than the frequency range of interest, \( R_SC_1 >> 1 \) and the term \( 1 + R_SC_1s \) in (5) can be approximated by \( R_SC_1s \) with (5) becoming

\[
K_{op}(s) \approx \frac{1+R_SC_s}{1+R_FC_Fs} \cdot \frac{R_F}{R_S}, \tag{20}
\]

Since the break frequency \( f_3 = 1/(2\pi R_FC_F) = 79.2 \text{ kHz} \) is much lower than \( f_4 = 1/(2\pi R_SC_S) = 212 \text{ kHz} \), the phase shift of the
The phase shift of the mutual inductor is found by deriving the transfer function of the mutual inductor with the secondary open-circuited. The secondary voltage is

\[ V_s(s) = M s I_p(s) \]  \hspace{1cm} (21)

where \( I_p(s) \), the current in the primary winding \( N_p \), is

\[ I_p(s) = \frac{V_p(s)}{r_p + sL_p} \]  \hspace{1cm} (22)

where \( V_p(s) \) is the output voltage of the operational amplifier. Substituting (22) into (21) and dividing by \( V_p(s) \) yields the transfer function

\[ K_M(s) = \frac{V_s(s)}{V_p(s)} = \frac{M}{r_p} \cdot \frac{s}{1 + \frac{L_p}{r_p} s} \]  \hspace{1cm} (23)

It is obvious from (23) that the mutual inductor has approximately zero phase shift at high frequencies but has an increasing leading phase shift as the frequency is decreased toward \( f_2 = r_p/(2\pi L_p) = 40.8 \text{ Hz} \). The experimental phase characteristic for the mutual inductor is also given in Fig. 5. The total phase shift which is the sum of the operational amplifier and mutual inductor phase shifts is shown dashed.
Fig. 5. Experimental phase characteristics for the narrow-bandwidth filter with $R_L = \infty$, $C_S = 147 \, \text{pF}$, and $C_P = 67 \, \text{pF}$. 

PHASE SHIFT (degrees)

FREQUENCY (Kiloherz)

-20
-10
0
10
20

0.1
1
10
100

Operational Amplifier

Total of $\frac{V(s)}{V_i(s)}$

Mutual Inductor

$\frac{V(s)}{V_i(s)}$
Equation (15) is the complete transfer function for the narrow-bandwidth filter including $R_L$. Equation (17) is the approximate transfer characteristic for the narrow-bandwidth filter under the approximation that $R_L$ can be assumed to approach infinity. Equation (17) is evaluated using the component values given in Table I and is plotted in Fig. 6. In order to verify the accuracy of the assumption that $R_L$ may be assumed to approach infinity, transfer curves were calculated using (15) for a wide range of values of $R_L$. Only when $R_L$ is 10 ohms or less, is there any significant deviation from the theoretical curve calculated using (17). Since these curves agree so closely, only (17) is plotted. Experimental data for the same circuit conditions with $R_L = \infty$ are also plotted in Fig. 6. It can be seen that the theoretical and experimental curves agree quite closely. The only significant difference being in the value of the maximum attenuation. By examining Fig. 5, it can be seen that the frequency at which the total phase shift from $v_C$ to $v_s$ equals zero is 2.5 kHz which is also the frequency at which maximum attenuation occurs in Fig. 6.

In this section, the configuration for the narrow-bandwidth filter has been discussed. The circuit configuration and analytical transfer function are given; and the role that the phase shifts in the operational amplifier and mutual inductor have in producing the null in the gain curve is discussed. The experimental and theoretical gain characteristics compare very closely. This configuration is discussed primarily to show the
Fig. 6. Attenuation characteristics for the narrow-bandwidth filter.
effect of phase shift on the performance of the filter. The next section is concerned with a filter in which the phase shift is minimized to obtain a wide-bandwidth of attenuation.

Wide-bandwidth Filter

The wide-bandwidth filter has a wide range, or bandwidth of frequencies over which the attenuation is below a certain specified value. The particular case which this dissertation is concerned with is the case of a low-pass filter. A circuit configuration is described in this section which achieves a close approximation to the gain characteristic of such a filter.

In the section on the narrow-bandwidth filter, it was shown that the dependence of the attenuation on frequency is related to the variation of the phase shifts of the operational amplifier and mutual inductor with frequency. If the total phase shift from these two components can be made zero over the bandwidth desired for the filter, then the result is a wide-bandwidth filter with a constant attenuation in that bandwidth.

The approximate transfer function for the operational amplifier is given in (20). This equation indicates that if \( C_s = 0.0 \) and \( C_F = 0.0 \) then the transfer function \( K_{op}(s) \) of the amplifier becomes a constant. Since \( K_{op}(s) \) is then independent of \( s \), the amplifier should have no phase shift. The experimental phase characteristic for the operational amplifier with \( C_s \)
and \( C_F \) removed is shown in Fig. 7. The small variation from zero for the phase shift could be caused by stray parameters in the wiring or in the amplifier, or it could be instrument error especially since the angle is so small.

The transfer function \( K_M(s) \) for the mutual inductor is given in (23). An examination of this function reveals that the phase shift of the mutual inductor is zero when the resistance \( r_p \) of the primary winding \( N_p \) approaches zero. However, there is a practical limit to the minimum value of \( r_p \) which makes the low-frequency attenuation dependent on the time constant \( L_p/r_p \) with a break frequency \( f_2 = r_p/(2\pi L_p) = 40.8 \) Hz. The experimental phase characteristic for the mutual inductor is shown in Fig. 7. This mutual inductor is the same one listed in Table I and used in the narrow-bandwidth filter. The phase shift of the mutual inductor and the operational amplifier with \( C_S = 0.0 \) and \( C_F = 0.0 \) are summed, and the total phase shift is shown dashed in Fig. 7. It can be seen that the total phase shift remains fairly close to zero over a relatively wide bandwidth compared to the total phase shift in Fig. 5.

From the preceding discussion, \( C_S = 0.0 \) and \( C_F = 0.0 \) for the wide-bandwidth filter; therefore the theoretical transfer function can be obtained by letting \( C_S = 0.0 \) and \( C_F = 0.0 \) in the transfer function (17) for the narrow-bandwidth filter yielding
Fig. 7. Experimental phase characteristics for the wide-bandwidth filter with $R_L = \infty$, $C_S = 0$ and $C_F = 0$. 
\[ \frac{V_L(s)}{V_c(s)} = 1 - \frac{MRFC_1}{rp} \cdot \frac{s^2}{\left(1 + \frac{Lp}{rp}s\right)\left(1 + RSC_1s\right)} \] (24)

The theoretical gain characteristic calculated using (24) and the experimental gain characteristic are shown in Fig. 8. The two curves agree closely except above thirty kHz as the phase shift increases above zero causing the experimental attenuation to decrease. This phase shift is probably caused by stray capacitance which is not included in the theoretical curve (24). It is also noted that the slope of these characteristics at low frequencies is -20 dB per decade. This slope is only half that of a low-pass L-section filter. It is desirable therefore that the filter be improved in order to increase its attenuation which is the purpose of the filter configuration discussed in the next section.

Wide-bandwidth Filter with Feedback

The wide-bandwidth filter with feedback is the one out of the three filters that is the most important from the viewpoint of an application as a filter for a dc to dc converter because its transfer characteristic has the greatest attenuation. The circuit configuration is the same as the wide-bandwidth filter with \( C_S = 0.0 \) and \( C_F = 0.0 \) in Fig. 3 except that a second input is added to the operational amplifier. This input is feedback from the load ripple voltage \( v_L \) to the input of the
Fig. 8. Attenuation characteristics for the wide-bandwidth filter.
The feedback path that is added is made up of $R_1$ and $C_2$ shown dashed in Fig. 3 so that the wide-bandwidth filter with feedback is the circuit in Fig. 3 including $R_1$ and $C_2$ but not including $C_S$ or $C_F$.

The feedback works in the following manner. The load ripple voltage $v_L$ is non-zero when $v_C$ and $v_S$ are not exactly equal in magnitude and phase. Since $v_L$ is the difference $v_C - v_S$, $v_L$ can be thought of as an error voltage. The converter output voltage $v_C$ is the known voltage to be eliminated; therefore, if $v_L$ is nonzero then $v_S$ is the voltage in error. The addition of feedback through $R_1$ and $C_2$ adds the error voltage $v_L'$ times a gain factor, to $v_S$ in such a manner as to decrease the amplitude of $v_L'$. This qualitative explanation may be better understood by examining the following derivation of the transfer function of the wide-bandwidth filter with feedback.

The transfer function $K_{op}(s)$ in (5) for the operational amplifier and the transfer function $K_M(s)$ in (23) for the mutual inductor have been derived. At this time, a new transfer function $K_F(s)$ for the feedback path through $R_1$ and $C_2$ is derived from the ratio of the impedance of the feedback elements $R_F$ and $C_F$ and the impedance of the series elements $R_1$ and $C_2$ as

$$K_F(s) = \frac{V_F(s)}{V_L(s)} = \frac{-R_F C_2 s}{(1 + R_F C_F s)(1 + R_1 C_2 s)}.$$

Since the two inputs are summed through the operational amplifier.
amplifier, the output voltage $V_p(s)$ of the amplifier is

$$V_p(s) = K_{op}(s)V_c(s) + K_F(s)V_L(s). \quad (26)$$

The secondary voltage $V_s(s)$ of the mutual inductor is obtained by using $K_M(s)$ in (23) and $V_p(s)$ in (26) as

$$V_s(s) = K_M(s)V_p(s) = K_M(s)[K_{op}(s)V_c(s) + K_F(s)V_L(s)]. \quad (27)$$

Starting with

$$V_L(s) = V_c(s) - V_s(s) \quad (28)$$

and substituting $V_s(s)$ from (27) into (28) yields

$$V_L(s) = V_c(s) - K_M(s)K_{op}(s)V_c(s) - K_M(s)K_F(s)V_L(s) \quad (29)$$

which can be reduced into the form

$$\frac{V_L(s)}{V_c(s)} = \frac{1 - K_M(s)K_{op}(s)}{1 + K_M(s)K_F(s)}. \quad (30)$$

Equation (30) is the transfer function for the wide-bandwidth filter with feedback. When $K_F(s) = 0$, (30) becomes the transfer function for the wide-bandwidth filter without feedback. Also, it can be seen that the attenuation of the filter with feedback is greater than the attenuation of the filter without feedback whenever $|1 + K_M(s)K_F(s)| > 1$. When $K_{op}(s)$ from (3), $K_M(s)$ from (23), and $K_F(s)$ from (25) are substituted in (30), (30) becomes
Using the values of the circuit parameters given in Table I, the theoretical characteristic curve for (31) is calculated; and the resulting theoretical curve as well as the experimental curve are shown in Fig. 9. Both curves have a steep initial negative slope; and at approximately 2 kHz, the slope begins to increase toward zero more sharply for the experimental curve than for the theoretical curve. At high frequencies, the experimental attenuation decreases while the theoretical attenuation reaches a constant level. The experimental curve for the wide-bandwidth filter without feedback showed the same property which is attributable to stray parameters in the circuit of the amplifier. The initial slopes of the two curves are approximately -40 db per decade which is the slope of the characteristic for an L-section filter. A comparison of these two filters is given later.

By comparing Figs. 6, 8, and 9, it can be seen that the attenuation characteristic for the wide-bandwidth filter with feedback reaches a greater attenuation at a lower frequency and maintains a large attenuation for at least a decade of frequencies. The wide-bandwidth filter with feedback is therefore the
Fig. 9. Attenuation characteristics for the wide-bandwidth filter with feedback.
the most logical choice for use with dc to dc converters because practically all of these circuits either have a variable frequency of operation or a highly nonlinear output ripple waveform which contains a number of large-amplitude harmonics. This filter is slightly more complex than the narrow-bandwidth filter or the wide-bandwidth filter; nevertheless, it is still the best choice. For this reason, the emphasis in the next section is on the wide-bandwidth filter with feedback.

Conclusions

This chapter begins with background information on the necessity for reducing the hundreds of microfarads of capacitance needed in the output filters for switching dc to dc converters. A brief description is given describing the basic principles of switching dc to dc converters. The function of the passive L-section output filter and the basic principle of the active filter are then given using block diagrams. The circuit diagram of the active filter is given next and its transfer function is derived. Principles which should be followed when designing the active filter are given and each of the three filter configurations are then discussed. The narrow-bandwidth filter configuration is discussed first. The narrow-bandwidth of attenuation is shown to be a function of the phase characteristics of the operational amplifier and the mutual inductor. The wide-bandwidth filter configuration is developed next by
minimizing the phase shift in the operational amplifier. Finally, feedback is added from the load to the input of the operational amplifier to produce the wide-bandwidth filter with feedback. This last filter is the most important in applications with dc to dc converters because it has the widest bandwidth of attenuation and its attenuation characteristic most nearly approximates that of an L-section filter. The theoretical attenuation characteristics obtained from the derived transfer functions and the experimental characteristics for each configuration are plotted. The theoretical and experimental curves compare favorably in each case.

An attempt is not made to analyze the active filter in a closed-loop converter system. The mathematical relations in such a system would be at least fourth order relations. The transient performance and the ripple attenuation of the active filter in Fig. 3, however, were tested by replacing the existing passive filter in a breadboard converter with the wide-bandwidth filter with feedback. The ripple attenuation was found to be commensurate with the characteristics given in Fig. 9 and the output ripple amplitude was essentially the same as the ripple amplitude when the original passive filter is used. The transient response of the output voltage to step changes in source voltage and load was tested using the active filter and was found to be essentially the same as the transient response when the original passive filter was used. An intuitive justification of the close comparison of the transient response may be
that the attenuation characteristic of the passive L-section filter has a -40 db per decade slope and the attenuation characteristic for the wide-bandwidth filter with feedback in Fig. 9 also has a -40 db per decade slope. The original L-section filter with large capacitance has therefore been replaced by the active filter which even though it requires only small capacitance has approximately the same characteristic as the L-section filter.

The similarity between the attenuation characteristic of an L-section filter and that of the wide-bandwidth filter with feedback in Fig. 9 suggests comparing the required capacitance for each filter that yields approximately the same attenuation characteristic. The slope of the experimental curve in Fig. 9 is -40 db per decade from 0.1 to 4 kHz. If the slope is extended toward lower frequencies, it intersects the zero db axis at 100 Hz. An L-section filter with the same attenuation characteristic over that frequency range requires that \( L_f C_f = \frac{1}{2 \pi (100)} \). The same value of secondary inductance \( L_s = 6.2 \) mH used in the mutual inductor of the active filter is used in the L-section filter so that \( L_f = 6.2 \) mH. Then it follows that \( C_f = 408.5 \) \( \mu \)F which can be compared with a total capacitance in the active filter of \( C_1 + C_2 = 4.87 \) \( \mu \)F. This represents an 84 to 1 saving in capacitance.

The discussion that was given on the effect that the phase shift of the operational amplifier and the mutual inductor have on the attenuation characteristic suggests possible
improvements that might be made to the filter. The high-frequency portion of the attenuation characteristic can be improved by using an amplifier which has a higher frequency response and smaller magnitudes of stray parameters. Stray parameters might also be reduced if the filter were built on a printed circuit board instead of a breadboard to minimize the lengths of connecting paths. The high-frequency attenuation of the filter can also be improved by adding a small capacitor on the order of $C_1$ in parallel with the load. The low frequency performance is determined by break frequency $f_2 = L_p/(2\pi r_p)$. This indicates that the attenuation can be improved by designing the primary inductance $L_p$ with a higher $Q = \omega L_p/r_p$. The discussion in this paragraph has indicated that improvement in the attenuation characteristic of the active filter can also be obtained by using more exact phase compensation.

In summary, it is known that the active filter uses a mutual inductor which provides a highly efficient dc path and that the operational amplifier dissipates only a small amount of fixed quiescent power so that the active filter has a high efficiency especially at high dc power levels. Also it is obvious from Table I that the circuit requires only small amounts of capacitance. The secondary winding $N_s$ and the core of the mutual inductor are essentially the same size as an L-section filter using an inductance $L_f$ equal to the secondary inductance $L_s$ of the mutual inductor. The additional size and weight of the primary winding $N_p$ and the operational amplifier with components
is no more than the additional size and weight of the tantalum capacitors required in the L-section filter; therefore the active filter does not have a greater size and weight than the L-section filter. The theoretical and experimental attenuation characteristics for the wide-bandwidth filter with feedback indicate that the wide-bandwidth filter with feedback achieves approximately the same attenuation as an L-section filter; and a comparison of the capacitances required for the active and L-section filters shows a significant capacitance advantage for the wide-bandwidth filter with feedback. These characteristics satisfy all of the objectives given in the first part of the chapter.

The primary advantage of this active filter is the reduction in the microfarads of capacitance required in output filters for dc power supplies. The reduction in capacitance enables the designer to use capacitors such as ceramic or mica. These capacitors are much more reliable than the electrolytic types of capacitors such as tantalum. The active filter is essentially non-dissipative requiring only a small amount of quiescent power for the bias of the amplifier. Because the attenuation characteristic of this filter is so similar to that of an L-section filter, it can be used in applications where highly reliable power supplies are required. One of these applications is in dc to dc converters especially those used in outer space where the vacuum and the extreme variations in temperature are known to cause failures in electrolytic capacitors. They can be
used in other dc supplies where long life is essential such as in unmanned Arctic and oceanic stations. The active filter may also be used in dc to dc converters and power supplies in computers to improve their reliability. Reliability is important to the operating expense of large computers since improved reliability results in less maintenance which is expensive. The wide-bandwidth filter with feedback can be used in virtually any dc to dc converter or power supply to reduce the required capacitance and in so doing to improve the reliability.
Chapter III

DESIGN OF A MEGAHERTZ DC TO DC CONVERTER

This chapter discusses the design of a 20 watt dc to dc converter operating with high efficiency at a switching frequency of approximately 1 MHz and a regulated output voltage of 28 volts. The output power and voltage are typical values used in unmanned spacecraft and some other aerospace applications; but the switching frequency of 1 MHz is unusually high. DC to dc converters are normally designed to operate in the frequency range of 2 to 20 kHz in order to accomplish a compromise between high efficiency and small size and weight. There are several reasons for designing this converter at a switching frequency of 1 MHz. The first is that an increase in frequency is usually accompanied by a reduction in the size and weight of the circuitry. This size and weight reduction is especially pronounced in the output filter where large amounts of capacitance and inductance are required when switching frequencies are from 2 to 20 kHz. A second reason is that the reduced amount of capacitance required to filter the 1 MHz switching waveform can result in eliminating the need for relatively low reliability electrolytic filter capacitors. Ceramic or mica capacitors could be used to improve the reliability of the converter. Reduction in
size and weight and improved reliability are desirable properties for any converter if an unacceptable loss in efficiency is not created by increasing the switching frequency. The third and most important reason for designing a 1 MHz converter is that there are experiments planned for satellites to measure electromagnetic radiation in outer space within the frequency range of 0.2 to 100 kHz [15]. A converter operating at a switching frequency of 1 MHz would not emit electromagnetic radiation that would interfere with these experiments. An additional advantage of the 1 MHz converter is that since the operating frequency of the converter is so high the system should be capable of responding faster to source voltage and load transients than the low-frequency converters. The major reason dc to dc converters have not been designed to operate at frequencies approaching 1 MHz is that components have not been available that would operate at that frequency at a sufficiently high efficiency. These components are becoming available but the task remains to devise adequate design techniques at these frequencies.

This chapter discusses the design of a 1 MHz energy-storage type converter. The discussion begins with information on the properties of the components to be used and design considerations at 1 MHz. Reasons for choosing an energy-storage type circuit and the design of the converter are given next. Finally some experimental results are given followed by the conclusions.
Design Principles at High Switching Frequencies

The design of a 1 MHz converter begins with a study of circuit components in order to determine the type of each component needed to obtain adequate efficiency and switching times. Each of these components has an upper frequency limit below which it must operate. The lowest of these frequencies is the highest frequency at which the converter may operate.

Capacitors. At frequencies near 1 MHz the lead inductance and internal resistance of electrolytic capacitors become significant. One MHz can easily exceed the resonant frequency of a large-microfarad electrolytic capacitor so that it appears inductive [16]. The internal power loss of electrolytic capacitors increases significantly at high frequencies [17]. Both of these factors cause the filtering ability and response of the capacitor to be impaired. For these reasons, high-Q capacitors such as ceramic, mica, or solid tantalum must be used. These capacitors are only available in values up to several microfarads; but as mentioned in Chapter II, they are significantly more reliable than electrolytic capacitors. Since the converter frequency is so high, the required microfarads for filter, bypass, and coupling capacitors are small; and, consequently, the microfarads of the low-capacitance high-Q capacitors are adequate.
Magnetic components. If the capacitors must be high-Q, then the magnetic components such as inductors, mutual inductors, and transformers must also be low-loss, or high-Q. Experimental data in manufacturers' catalogs indicates that total magnetic material losses, including hysteresis and eddy-current losses, increase significantly with frequency. For this reason, eddy-current losses are more significant at very high frequencies. Typical converters have used square B-H characteristic materials which are either tape-wound in toroids or stacked laminations. These components have high eddy-current losses at high frequencies and are not recommended for use above 500 kHz [8]. Another commonly used magnetic core material is powdered permalloy which has a linear B-H characteristic; however, this material has high losses above 200 kHz [19]. There are two materials remaining to choose. One is air and another is ferrite. Both of these materials exhibit linear B-H characteristics and are used to design high-frequency inductors and mutual inductors in the megahertz range. The high reluctance and low permeability of air result in a large amount of leakage inductance which makes it undesirable for use in switching applications. The energy stored in the leakage inductance must be dissipated in the circuit components during switching. These circuit losses easily become much greater than the core loss which would be present if a ferrite material were used. Also, the stray magnetic field associated with the leakage inductance by coupling with other parts of the circuit can cause false triggering of
switching circuits. Therefore, an air core inductor would have to be shielded. A ferrite pot core has less leakage flux and inherent shielding as a result of its physical configuration.

Winding capacitance also may have a detrimental effect on the high frequency performance of an inductor. Unfortunately, most of the design principles used to reduce leakage inductance bring the windings in closer proximity which causes an increase in winding capacitance. However, when large currents are switched at high frequencies and the source impedance and load are small, the effect of leakage inductance is much more significant than the effect of winding capacitance [14]. This effect is directly related to the inductive time constants found by dividing the leakage inductance by the source resistance or load. Since large currents are switched in the inductors of energy-storage converters, the design is primarily concerned with reducing the leakage inductance, or maximizing the coupling coefficient.

Wire. Frequencies near 1 MHz also produce significant skin effect. Litz wire may be used up to sine-wave frequencies of 0.5 to 2 MHz to reduce skin effect. However, as the frequency increases above these frequencies, irregularities in stranding and capacitance between individual strands causes Litz wire to become progressively less advantageous [20]. Switching at a rate near 1 MHz requires a much faster response than a sinusoidal signal at 1 MHz does, due to the rapid rise and fall
times of the waveform. For this reason, Litz wire is not any better than solid copper wire at switching frequencies near 1 MHz.

**Semiconductors.** Now that the considerations for the passive components have been studied, it is necessary to find semiconductors which operate efficiently at the required voltage, current, and frequency. Semiconductors are required that switch several amperes with very fast rise and fall times and block at least thirty to fifty volts reverse voltage. Since a switching element must switch at least once in every half cycle, a rough estimate of the switching times for turning-on and turning-off can be obtained as fifty nsec by taking ten percent of a half period of 1 MHz. This means that the rise and fall times of a transistor or diode must be less than fifty nsec when switching several amperes if efficient switching is to be accomplished.

A comprehensive search of conventional diode specifications reveals that ion-implanted diodes are available which have reverse recovery times of 9 nsec, with a forward current of 0.6 amperes (Isofilm International 10A26) and 10 nsec with a forward current of 3 amperes (Isofilm International 10R2). They have forward recovery times of 0.5 nsec and 1.0 nsec respectively and peak reverse voltages PRV of 100 volts. These diodes are more than adequate for use in the converter.
An investigation of controlled switching devices quickly reveals that the response of devices such as unijunction transistors and thyristors are well below that required for operation near 1 MHz. There are field-effect transistors with very fast switching speeds but they are not capable of processing adequate power levels. The remaining devices to be examined are the other types of transistors. Unfortunately, transistor technology is lagging diode technology in the area of switching time; and the switching time of the transistor is the factor which determines the upper limit on the switching frequency of the converter. Manufacturers are developing ion-implanted transistors which should be very useful in increasing the switching frequencies of converters. A continuing search of transistor specifications yields several transistors designed for computer circuits which can switch up to approximately 1.5 amperes efficiently at a frequency of 1 MHz. However, these transistors are not designed for power applications. Further searching has uncovered two Solitron NPN power transistor switches. One (SDT6106) switches 2 amperes on or off in 50 nsec, and has a maximum collector current rating of 5 amperes and a collector to emitter blocking voltage of 50 volts. This transistor is ideal as the driver for the second transistor (SDT6116) which switches 5 amperes on or off in 75 nsec and has a maximum collector current rating of 10 amperes and a collector to emitter blocking voltage of 50 volts. The saturation voltages of these transistors are sufficiently low (0.8 to 1.0 volts) to permit efficient power conversion.
In any regulated energy storage dc to dc converter, it is necessary to generate a controlled duty cycle. At a frequency near 1 MHz, problems with stray pickup between parts of the circuit would be expected if the circuit is constructed of discrete components. Particular care would have to be taken in planning the physical layout of such a circuit. However, since the duty cycle generator is normally required to deliver a low power signal, this problem can be minimized if integrated circuits are used. There are a number of circuits available that are designed for use in computer logic circuits which are ideal for a duty cycle generator at 1 MHz. One of these is the 710 series of differential voltage comparators. These devices have an output current capability of 10 mA and dissipate typically only 110 mW. These circuits can operate as hysteretic or Schmitt trigger circuits capable of producing an output voltage which switches from -0.5 volts to -3.2 volts with switching times on the order of 40 nsec.

Physical layout. The physical layout of a circuit operating near 1 MHz is very important. As has been mentioned before, it is easy to have stray magnetic induction which results in undesirable coupling causing false-triggering, oscillations, or other deterioration of the performance of the circuit. In order to avoid these problems, shielded cables should be used whenever possible and especially when long leads are required, all connecting wires both shielded and unshielded should be kept as short as possible, and circuit complexity should be minimized.
Other shielding besides shielded cables should be used to isolate the sensitive small-signal parts of the circuit from the high-current switching parts of the circuit. Ground return paths should be well planned to avoid having high-current paths near the sensitive circuits and also to minimize the lengths of the high-current switching paths which are most likely to have stray magnetic fields which could interfere with the normal operation of the circuit. In addition, all points in the circuit easily affected by stray noise such as the input to an amplifier having a high input impedance should be bypassed with high-Q high frequency capacitors.

So far in this chapter, background reasons for designing a 1 MHz dc to dc converter and some principles of design at this high frequency have been discussed. In addition, the transistors, diodes, and magnetic material have been selected in accordance with these principles. The components and principles discussed in this section are utilized in the next section to obtain the complete circuit design.

Block Diagram of a Regulated 1 MHz DC to DC Power Converter

The design is presented by dividing the converter into functional circuits and discussing each separately. Initially a block diagram of the converter is given and discussed to provide a better understanding of the function of each circuit and its
contribution to the operation of the whole converter. Each circuit is discussed in greater detail in subsections of the dissertation. These subsections contain theoretical explanations, design procedures, and design examples yielding the actual components used for each circuit.

The block diagram. The block diagram is shown in Fig. 10. The dc source in an unmanned spacecraft is normally a battery charged from solar cell panels. The voltage of this source fluctuates significantly. For experimental measurements for this dissertation a regulated power supply is used for the battery and solar cells. The power circuit block contains the power conversion circuits. In order to accomplish a change in dc voltage, the dc energy must be converted to ac energy and rectified [21]. The power circuit block performs these functions using a saturating power transistor as an oscillating power switch to generate the ac and a diode to rectify.

The dc or rectified waveform from the power circuit must be filtered in order to obtain a low-ripple output voltage. This function is performed by the output filter block which is normally either a pure capacitance or a pi-network low-pass filter. The load in this circuit is simply a pure resistive element.

Regulation is obtained by a combination of the functions of the three blocks in the lower part of the diagram. The differential voltage comparator circuit has two direct-voltage input thresholds with a difference of only millivolts. When the
Fig. 10. Block diagram of the converter.
applied direct voltage rises above the higher threshold, the comparator output voltage is positive and remains in this state until the applied dc voltage falls below the lower threshold when its output voltage either becomes zero or negative.

In order to obtain a constant frequency of oscillation of 1 MHz, a 1 MHz triangular waveform generator is added to the input of the comparator circuit as shown in Fig. 10. The output of the comparator becomes a pulsed voltage with a constant frequency of 1 MHz. The duty cycle of the pulsed voltage which controls the power switch in the power circuit varies inversely with the dc voltage applied to the input of the comparator circuit and thus maintains regulation.

The comparator circuit does not supply enough current to drive the power transistor in the power circuit. The drive circuit is designed, therefore, to provide sufficient forward bias to hold the power transistor in saturation and to provide a rapidly switched reverse bias to enable the power transistor to switch rapidly and efficiently.

It can be seen that the block diagram in Fig. 10 forms a closed loop capable of maintaining a regulated output voltage. A complete schematic diagram is shown in Fig. 11 and the component parts list is given in Table 3. Dotted lines are used to separate each section of the schematic diagram corresponding to a block in Fig. 10 and each section is labelled. Figs. 10 and 11 can be used to relate the schematic diagrams of the circuit
Fig. 11. Schematic diagram of the converter.
Table III. Components list for the converter.

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1, C2</td>
<td>0.5 (\mu F) 100 V disc ceramic</td>
</tr>
<tr>
<td>C3, C4, C5</td>
<td>4.7 (\mu F) 50 V solid tantalum (Erie)</td>
</tr>
<tr>
<td>C6, C7, C8, C9</td>
<td>0.1 (\mu F) 500 V disc ceramic</td>
</tr>
<tr>
<td>C10</td>
<td>1500 pF 1000 V disc ceramic</td>
</tr>
<tr>
<td>C11</td>
<td>33 pF 1000 V disc ceramic</td>
</tr>
<tr>
<td>D1</td>
<td>10R2 (Isofilm International)</td>
</tr>
<tr>
<td>D2, D3</td>
<td>10A26 (Isofilm International)</td>
</tr>
<tr>
<td>DZ1</td>
<td>1N750</td>
</tr>
<tr>
<td>DZ2</td>
<td>1N751</td>
</tr>
<tr>
<td>I1</td>
<td>SN72710L Differential Comparator (TI)</td>
</tr>
<tr>
<td>I2</td>
<td>CA3000 DC Amplifier (RCA)</td>
</tr>
<tr>
<td>L1</td>
<td>33 (\mu H)</td>
</tr>
<tr>
<td>L2</td>
<td>131.5 (\mu H)</td>
</tr>
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<td>L4, L5</td>
<td>1 m(H)</td>
</tr>
<tr>
<td>N1</td>
<td>4 turns #19 AWG</td>
</tr>
<tr>
<td>N2</td>
<td>8 turns #21 AWG</td>
</tr>
<tr>
<td>N4, N5</td>
<td>49 turns #26 AWG bifilar wound</td>
</tr>
<tr>
<td>Q1</td>
<td>SDT6115 (Solitron Devices, Inc.)</td>
</tr>
<tr>
<td>Q2</td>
<td>SDT6106 (Solitron Devices, Inc.)</td>
</tr>
<tr>
<td>R1</td>
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</tr>
<tr>
<td>R2</td>
<td>22 (\Omega)</td>
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</tr>
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</tr>
<tr>
<td>R7</td>
<td>69.8 k(\Omega)</td>
</tr>
<tr>
<td>R8</td>
<td>14.3 k(\Omega)</td>
</tr>
<tr>
<td>R9, R10</td>
<td>1.0 k(\Omega)</td>
</tr>
<tr>
<td>R_{L}</td>
<td>26 (\Omega) to (\infty)</td>
</tr>
<tr>
<td>T1</td>
<td>2616P-3D3 UG (Ferroxcube, Corp.)</td>
</tr>
<tr>
<td>T2</td>
<td>768T188/3D3 (Ferroxcube, Corp.)</td>
</tr>
</tbody>
</table>
blocks which are discussed separately in the following subsections.

The Power Circuit

The first step in designing a nondissipative switching converter that operates near a frequency of 1 MHz with a relatively high efficiency is to decide which of the current types of power converters is best suited for operation at these high frequencies. From among the possible dc to dc converter configurations including energy-storage converters [22] and parallel inverters with voltage-feedback [11] or current-feedback [23], the energy-storage converter is chosen for the following reasons: (1) The circuit is less complex requiring a minimum number of components including only one transistor switch used in conjunction with a switching diode. (2) Power dissipation should be high in the switching power transistor for any converter. Since the energy-storage converter requires only one power transistor, it is most likely to have smaller total switching losses than the parallel inverter which requires two switching transistors and, additionally, some form of full-wave rectification. (3) The energy-storage converter uses inductive transformation which does not require driving the magnetic material to saturation and uses a core material with a linear B-H characteristic which is available in materials such as ferrite which are designed for efficient operation at frequencies near 1
MHZ. (4) Since the transistor and diode currents are transferred one for the other during the switching times in the energy-storage converter, delay time and storage time in the power switching transistor do not produce switching loss but are important only so far as they are concerned with the duty cycle, the time of a period of the oscillation, and the equilibrium, or steady-state, of the closed-loop system. The conclusion is that the energy-storage converter is the most versatile, most efficient, and least complex of dc to dc power converters and is best suited for use at frequencies near 1 MHz.

There are three basic types of dc to dc energy-storage converters from which to choose. One is the voltage step-up converter shown in Fig. 1(a); a second is the voltage step-down converter shown in Fig. 1(b); and the third, often called an isolated energy-storage converter, is shown in Fig. 12. There are two basic disadvantages to the first two circuits. One is that they can perform only a singular function which is either voltage step-up or voltage step-down. Another is that they do not provide electrical isolation between the source and the load. The third circuit uses a mutual inductor which provides isolation between the source and the load and, in addition, provides the capability of operating as a voltage step-up or step-down converter. The isolated energy-storage converter is chosen because it can perform all of these functions.
Fig. 12. Schematic diagram of the power circuit and output filter.
**Design considerations.** The source voltage $E_S$ variation of 21 to 31 volts commonly occurs in orbiting satellites whose batteries are charged from solar cell panels. The load voltage $V_O$ on the load $R_L$ was chosen to be 28 volts because this is a typical source voltage for aerospace electrical equipment.

Capacitors $C_1$ and $C_2$ are high-$Q$ high-frequency capacitors which have several uses. The low impedance of these high-frequency capacitors shunts any spurious high-frequency signal from the source and also provides a path for current spikes generated during the switching of $Q_1$ so that they do not return through the source. Since $C_1$ and $C_2$ are mounted inside the converter on the converter end of the line from the source, they enable the collector current of $Q_1$ to rise faster when turn-on occurs by shunting the inductance of the source line during the switching time.

The actual isolated energy-storage converter consists of transistor $Q_1$, mutual inductor $R_1$, and diode $D_1$ and is shown in Fig. 12 as the main part of the power circuit. It should be noted that there are different grounds for the source side and the output side of $T_1$ to achieve isolation. Because the isolated energy-storage converter has been described previously in the open literature, the operation and design of the circuit is only summarized in this text. The design procedures given use relations which were derived previously [22].
Circuit functioning. The discussion of the sequence of circuit operation begins by assuming that the circuit has reached a steady-state condition and that Q1 has just reached saturation. While Q1 is on, the source voltage $E_S$ is applied to winding N1 of mutual inductor T1. The total series resistance around the path through $E_S$, N1, and Q1 is negligible. Therefore, if $E_S$ is constant during the time Q1 is on, the flux rate of change $\frac{d\phi}{dt}$ is constant; and the current in N1 increases linearly. During this time, diode D1 is reverse-biased by the voltage on N2; and the current supplying the load comes from energy stored in capacitors C3 and C4. When Q1 turns off, the voltage polarities of N1 and N2 reverse; and the energy stored in T1 during the on-time is discharged through D1 into C3, C4, and $R_L$. C3 and C4 are chosen so that $V_O$ does not vary more than the desired ripple voltage amplitude. They then provide an effective filter for the current pulses passing through D1 to yield a constant dc voltage $V_O$ on the load resistance $R_L$.

Circuit design. In order to choose components for Q1 and D1 and design T1, the operating currents and voltages for these elements must be known. To compute these, it is necessary to start with the desired output voltage $V_O$, output power $P_O$, source voltage $E_S$, and the efficiency $\eta$. The largest currents exist when the circuit is delivering the maximum output power $P_O(\text{max})$ of 20 watts and the source voltage $E_S$ is a minimum of 21 volts. If $\eta$ is assumed to be 80 percent, then the maximum input power $P_{\text{in}(\text{max})}$ can be computed as
\[ P_{\text{in}}(\text{max}) = 100 \frac{P_0(\text{max})}{n} = 100 \times \frac{20}{80} = 25 \text{ watts} \]  

(32)

Maximum average primary current \( I_1(\text{max}) \) occurs when \( P_{\text{in}} \) is a maximum and \( E_S \) is a minimum; therefore

\[ I_1(\text{max}) = \frac{P_{\text{in}}(\text{max})}{E_S(\text{min})} = \frac{25}{21} = 1.19 \text{ amperes} \]  

(33)

\( I_1(\text{max}) \) is the source current averaged over the entire period in which time \( i_1(t) \) is zero during the off-time \( t_{\text{off}} \) and finite during the on-time \( t_{\text{on}} \) as is shown in Fig. 13(a). The current \( i_2(t) \) in winding \( N2 \) is zero during \( t_{\text{on}} \) and finite during \( t_{\text{off}} \) as shown in Fig. 13(b). Since \( V_0 \) is regulated and constant, \( \frac{d\phi}{dt} \) is constant during \( t_{\text{off}} \); and since the energy stored in \( T1 \) is being discharged, \( \frac{d\phi}{dt} \) is negative. It follows that \( \frac{di_2(t)}{dt} \) is constant and negative.

Since the physical parameters of \( T1 \) are constant, the current waveforms for \( i_1(t) \) and \( i_2(t) \) are linearly related to the field intensity \( H \). Also it is known that under steady-state conditions the increment \( \Delta \phi \) in flux \( \phi \) during \( t_{\text{on}} \) must equal the decrement in flux during \( t_{\text{off}} \). These ideas are represented in Fig. 13(c) which shows the idealized linear B-H characteristic of the core used in \( T1 \). The subscripts \( l, a, \) and \( h \) in Fig. 13 correspond to the lowest, average, and highest flux levels respectively in the core. The saturation flux level and field intensity are \( \phi_S \) and \( H_S \), respectively. Therefore, during \( t_{\text{on}} \) the flux level goes from \( \phi_l \) to \( \phi_h \); and during \( t_{\text{off}} \) the flux level goes from \( \phi_h \) to \( \phi_l \).
Fig. 13. Current waveforms and B-H characteristic for T1.
Design of mutual inductor T1. The first step in the design of T1 is to determine the turns ratio $N_1/N_2$. The limiting factor on the turns ratio is the breakdown voltage $BV_{CEO}$ of Q1 during $t_{off}$ when $E_S$ is maximum. $BV_{CEO}$ is expressed as

$$BV_{CEO} > V_O \times N_1/N_2 + E_{S(max)} \ . \tag{34}$$

From (34) the maximum turns ratio is

$$N_1/N_2 < \left[ BV_{CEO} - E_{S(max)} \right] / V_O = \frac{50 - 31}{28} = 0.68 \ . \tag{35}$$

where $BV_{CEO} = 50$ volts for Solitron transistor SDT6115 which is chosen for Q1. A turns ratio of 0.5 is used in order to allow for a margin of safety.

Before the core for T1 can be selected, point a or the bias point in Fig. 13(c) must be determined. First the expression for maximum $t_{on}/t_{off}$ which is derived in Reference 23 is given as

$$t_{on} / t_{off} = \frac{V_O}{E_{S(min)}} \times \frac{N_1}{N_2} = \frac{28 \times 1}{21 \times 2} = \frac{2}{3} = 0.67 \ . \tag{36}$$

From this expression, the maximum duty cycle D.C. can be found as

$$D.C. = \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}/t_{off}}{\left(t_{on}/t_{off}\right) + 1} = \frac{2/3}{(2/3) + 1} = \frac{2}{5} = 0.40 \ . \tag{37}$$

Using this duty cycle and $I_{1a(max)}$ from (33), maximum $I_{1a}$ which is the maximum average current in N1 during $t_{on}$ can be obtained as
\[ I_{1a} = \frac{\bar{I}_1(\text{max})}{D.C.} = \frac{1.19}{0.40} = 2.98 \text{ amperes} \quad (38) \]

Since \( \frac{di_1}{dt} \) is constant during \( t_{on} \) and \( t_{off} \), the approximate formula for \( \Delta i_1 \) during \( t_{on} \) as shown in Fig. 13(a) can be obtained from Lenz's Law as

\[ E_s = L_1 \frac{di_1}{dt} = L_1 \frac{\Delta i_1}{\Delta t} \quad (39) \]

Which when rearranged yields \( \Delta i_1 \) as

\[ \Delta i_1 = E_s(\text{min}) t_{on}/L_1 \quad (40) \]

From Ampere's Law, where \( N \) is the number of turns, \( I \) is the current, and \( l \) is the mean-free-path length

\[ H = 0.4\pi NL/l \quad (41) \]

\( \Delta H \) shown in Fig. 13(c) can be expressed as a function of \( \Delta i_1 \) as

\[ \Delta H = 0.4\pi N_1 \Delta i_1/l \quad (42) \]

also

\[ H_a = 0.4\pi N_1 I_{1a}/l \quad (43) \]

In Fig. 13(c), it is easy to determine that

\[ H_h = H_a + \frac{1}{2} \Delta H \quad (44) \]

which after substituting (42) for \( \Delta H \) and (43) for \( H_a \) becomes

\[ H_h = 0.4\pi N_1 \left( I_{1a} + \frac{1}{2} \Delta i_1 \right)/l \quad (45) \]

Substituting (40) into (45) yields
The inductance factor $A_L$ for a given linear core is usually given as mH for 1000 turns which yields

$$L_1 = \frac{N_1^2}{(1000)^2} \times A_L \times 10^{-3} = N_1^2 \times A_L \times 10^{-9} \text{ henries} \quad (47)$$

and substituting (47) into (46) yields

$$H_h = 0.4\pi \frac{I_{1a} N_1 + \frac{E_S(\text{min}) t_{on}}{2L_1}}{\frac{N_1}{2} A_L \times 10^{-9}} \quad (48)$$

The first criterion which must be met when designing $T_1$ is that $H_h$ does not reach saturation magnetic intensity $H_S$ or

$$H_h < H_S \quad (49)$$

Before $H_h$ in (48) can be used in (49), the on-time $t_{on}$ must be computed using $D.C. = 0.40$ from (37) and $f = 1$ MHz as

$$t_{on} = D.C. \times T = D.C./f = 0.40 \times 10^{-6} \text{ seconds} \quad (50)$$

Substituting $I_{1a}$ from (38), $t_{on}$ from (50), and $E_S(\text{min}) = 21$ volts into (48) yields

$$H_h = 0.4\pi \left\{ 2.98 N_1 + \frac{4.2 \times 10^3}{N_1 A_L} \right\} \quad (51)$$

Equations (49) and (51) are used in a trial and error procedure to determine the core and the number of turns for $N_1$. This is done by choosing a core, substituting for $l$ and $A_L$ in (51), and solving (51) for $N_1$. Then $N_2$ must be calculated.
knowing \( \frac{N_1}{N_2} = 0.5 \), the wire size for \( N_1 \) and \( N_2 \) must be chosen, and \( N_1 \) and \( N_2 \) must be checked for fit in the window area of the core. This procedure follows.

It has already been determined that for purposes of reducing leakage flux and consequently leakage inductance an ungapped pot core should be chosen. One such core is pot core #2616P-3D3 UG manufactured by Ferroxcube Corp. which has \( A_L = 2340 \text{ mH/1000 turns} \), \( l = 3.73 \text{ cm} \), and \( H_S = 5 \text{ oersteds} \). Substituting \( H_h = H_S \), \( l \), and \( A_L \) into (51) yields

\[
N_1^2 - 3.31N_1 + 0.407 = 0.
\] (52)

The roots of (52) are \( N_1 = 0.125 \) or 3.13 turns. The maximum turns for \( N_1 \) is then 3 turns and for \( N_2 \) is 6 turns using \( \frac{N_1}{N_2} = 0.5 \) chosen from (35). If \( N_1 = 3 \) turns, \( l = 3.73 \text{ cm} \), and \( A_L = 2340 \text{ mH/1000 turns} \) is substituted into (51), \( H_h \) is found to be 3.24 oersteds which is less than \( H_S = 5 \text{ oersteds} \).

From (33), the full-cycle average current in \( N_1 \) is 1.19 amperes which requires \#19 AWG wire based on 1000 circular mils per ampere. The maximum average current in \( N_2 \) is simply

\[
P_0(\text{max})/V_0 = 20/28 = 0.714 \text{ amperes which requires \#21 AWG wire based on 1000 circular mils per ampere. Now it is necessary to check } N_1 \text{ and } N_2 \text{ for fit in the cross sectional area } A_{CB} \text{ of the core through which the windings must pass. This area for core 2616P-3D3-UG is 0.406 in}^2. \text{ In order to allow for the space and insulation between conductors, the total cross-sectional area}
$A_{cu}$ of the turns should not exceed 40 percent of $A_{CB}$ which is 0.163 in$^2$. AWG #19 wire has a cross-sectional area $A_{N1}$ of 0.00101 in$^2$ and AWG #21 wire has a cross-sectional area $A_{N2}$ of 0.000638 in$^2$. $A_{cu}$ can now be calculated as

$$A_{cu} = 3A_{N1} + 6A_{N2} = 3(0.00101) + 6(0.000638) = 0.006912 \text{ in}^2.$$  \hspace{1cm} (53)

Since $A_{cu} = 0.006912 \text{ in}^2$ is less than $0.4A_{CB} = 0.163 \text{ in}^2$, $N_1 = 3$ turns AWG #19 and $N_2 = 6$ turns AWG #21 can be wound; and the mutual inductor is designed. The 3 turns of $N_1$ are bifilar wound with the first 3 turns of $N_2$. The remaining 3 turns of $N_2$ are wound normally.

Before the design is complete maximum instantaneous current $I_{1h}$ must be compared with the maximum ratings for $Q_1$.

Since field intensity $H$ is linearly related to $I_{1h}$

$$I_{1h} = I_{1a} + \frac{1}{2} \Delta i_1.$$  \hspace{1cm} (54)

The inductance $L_1$ is calculated from (47) as

$$L_1 = N_1^2 A_L \times 10^{-9} = 3^2 \times 2340 \times 10^{-9} = 21.06 \text{ \mu H}.$$  \hspace{1cm} (55)

Substituting $E_{S(min)} = 21$ volts from Fig. 12, $t_{on} = 0.40 \times 10^{-6}$ seconds from (50), and $L_1 = 21.06 \text{ \mu H}$ from (55) into (40) yields

$$\Delta i_1 = E_{S(min)} t_{on} / L_1 = 21 \times 0.40 \times 10^{-6} / (21.06 \times 10^{-6}) = 0.40 \text{ amperes}.$$  \hspace{1cm} (56)

Substituting $\Delta i_1 = 0.40 \text{ amperes}$ from (56) and $I_{1a} = 2.98 \text{ amperes}$ from (38) yields
\[ I_{1h} = 2.98 + \frac{0.40}{2} = 3.18 \text{ amperes} \quad (57) \]

Since \( I_{1h} \) is less than the rated 10 amperes maximum collector current for Q1, the design is complete.

The output filter capacitors C3 and C4 are chosen experimentally from non-electrolytic high-frequency capacitors. This capacitance is increased until the load ripple voltage amplitude is within acceptable limits resulting in values of 4.7 \( \mu \text{F} \) each for C3 and C4.

The Control Circuits

The control circuits consist of the differential voltage comparator and the triangular waveform generator as shown in Fig. 10. The purpose of the control circuits is to sense the output voltage and provide a duty-cycle signal which when used to control Q1 results in a regulated output voltage \( V_0 \). Integrated circuits are used where possible in the design of the control circuits because they are small, efficient, and switch very fast. The control circuits can be built with discrete components; however, shielding, grounding, pickup, and delay time at this high frequency become greater problems than with integrated circuits. One disadvantage of using integrated circuits is the need for multiple low-voltage power supplies. These supplies should be readily available in most applications; but if they are not, a system of Zener diodes or small high-frequency
switching regulators would provide the necessary voltages.

The schematic diagram of the control circuits is shown in Fig. 14. They consist of a threshold circuit, which is built around the integrated circuit I1 and is commonly called a differential voltage comparator circuit, and a triangular waveform generator which is built around integrated circuit I2. A brief description of the means by which these circuits generate a duty cycle is given first, then the design of each circuit is discussed.

**Functioning of the control circuits.** The voltage on DZ2 in Fig. 14 acts as the reference voltage $V_R$ for the voltage comparator I1 and is applied to the positive input at pin 2 of I1 through R6. The direct output voltage $V_O$ is applied to the negative input at pin 3 of I1 by means of a voltage divider made up of R7 and R8. In addition, a 1 MHz triangular waveform $v_t$ is obtained by integrating the output of I2 which is utilized as a 1 MHz square wave oscillator. Integration is accomplished through the RC combination of the output resistance of I2, R7, R8, and C10. C9 has negligible effect because the time constant resulting from the product of the parallel combination of R7 and R8 and C9 is approximately three orders of magnitude longer than the period of 1 MHz. The resulting $v_t$ on C10 is coupled through capacitor C9 and is added to the fraction of $V_O$ on pin 3 of I1 resulting in a voltage $v_A$ at pin 3 of I1 given by
Fig. 14. Schematic diagram of the differential voltage comparator and the triangular waveform generator.
The differential input to voltage comparator II is $V_R - V_A$; therefore whenever $V_A$ is less than $V_R$ the differential input is positive so there is a positive output signal at the comparator output terminal, pin 7 and vice versa. An examination of Fig. II reveals that Q1 is turned on when Q2 is turned on, and Q2 is turned on when the voltage at pin 7 is positive. This means that Q1 turns on whenever $V_A$ is less than $V_R$. For purposes of simplifying this explanation, it is assumed that the voltage comparator has no hysteresis. It is also assumed that the ac ripple voltage component of $V_O$ is negligible compared to $v_t$; and the frequency $f$ is constant at 1 MHz so the period $T = t_{on} + t_{off}$ is constant.

Keeping these factors in mind, the manner in which a duty cycle capable of maintaining a regulated output voltage is obtained can be explained. Fig. 15 shows three examples of $V_A$ each at different direct voltage levels. It has already been established that when $V_A < V_R$ the power transistor Q1 is on; and when $V_A > V_R$, Q1 is off. So in Fig. 15, the times during which $V_A$ is above the dashed line at $v = V_R$ correspond to $t_{off}$ and the times $V_A$ is below this line correspond to $t_{on}$ as it is so labelled. In Fig. 15, the voltage $V_A$ at pin 3 of II is assumed to consist of two components, the direct voltage $V_A = V_O R_8 / (R_7 + R_8)$ and the alternating triangular voltage $v_t$. 

\[ V_A = \frac{R_8}{R_7 + R_8} V_O + v_t \] 

(58)
Fig. 15. Voltage waveforms of $v_A$ at different duty cycles.
In Fig. 15(a), $V_A$ equals the direct reference voltage $V_R$, $t_{on} = t_{off}$, and the duty cycle D.C. = 0.50. If the load is decreased or $E_S$ is increased, $V_A$ tends to increase to a condition shown in Fig. 15(b) in which $t_{on}$ is shortened lowering the duty cycle and tending to decrease $V_A$ to its original value. If the load is increased or $E_S$ is decreased, $V_A$ tends to decrease creating the condition shown in Fig. 15(c) in which $t_{on}$ is lengthened increasing the duty cycle and tending to increase $V_A$ to its original value. This system operating in a closed-loop condition tends to maintain a regulated output voltage. A much more detailed non-linear analysis of this type of regulating mechanism is given later in Chapter IV which includes the effects of hysteresis and the ripple voltage component of $V_O$. Now that a brief explanation has been given on the operation of the control circuits the design procedures are discussed.

**Design of the comparator.** The integrated circuit chosen for II in Fig. 14 is the 710 series of differential voltage comparators. This device is chosen because it dissipates typically only 110 mw internally while supplying a +3.2 volt and 10 ma positive output that switches on or off in 40 nsec. II requires two power supplies of +12 volts and -6 volts. Capacitors C6, C7, and C8 are placed between the power sources and ground in order to bypass any noise or voltage spikes that may adversely affect the operation of the comparator and to provide a low impedance source during the transient time when the comparator switches. These capacitors are chosen experimentally to provide
optimum comparator performance. In this example, C6, C7, and C8 are 0.1 \( \mu F \) as given in Table III. Resistor R5 is used to bias Zener diode DZ2, and DZ2 provides the reference voltage for voltage comparator II. The value of R5 is determined by finding a bias point with good regulation on the characteristic of DZ2. It is desirable that this bias point have a minimum current level in order to minimize power dissipation; and it must have a voltage value of less than 5 volts which is the maximum input voltage for II. From these requirements, Zener diode 1N751 is chosen for DZ2. From an examination of the characteristic of DZ2 on a transistor curve tracer, a bias point at 4.5 volts and approximately 2.8 ma. is chosen. The current through R6 is negligible since the input impedance to the comparator is high; therefore R5 is computed as

\[
R_5 = \left( 12 - V_{\text{DZ2}} \right) / I_{\text{DZ2}} = (12 - 4.5) / (2.8 \times 10^{-3}) = 2.7 \text{ k\( \Omega \)}
\]  

as is given in Table III. R7 and R8 are determined by assuming that the comparator is regulating and in a balanced state. Under these conditions, \( V_R \) equals the dc component \( V_A \) of \( v_A \) given as

\[
V_A = V_O R_8 / \left( R_7 + R_8 \right) = V_R.
\]

Substituting \( V_R = 4.5 \) volts and \( V_O = 28 \) volts yields

\[
R_7 = \frac{23.5}{4.5} R_8.
\]

Since the input to II at pin 3 has a very high impedance and draws negligible current, the current through R7 and R8 may be
very small. It is still desirable to minimize power dissipation so an arbitrary negligible power dissipation \( P_d \) of 10 mw is chosen. This yields a total resistance of

\[
R_7 + R_8 = \frac{V_0^2}{P_d} = \frac{28^2}{10^{-2}} = 78.4 \, \text{k}\Omega \quad . \quad (62)
\]

Substituting \( R_7 \) from (61) into (62) yields \( R_8 = 12.6 \, \text{k}\Omega \) and substituting \( R_8 \) into (62) yields \( R_7 = 65.8 \, \text{k}\Omega \). These resistors should be temperature compensated to keep \( V_0 \) from drifting with changes in temperature. Values as close as possible to those calculated were chosen from stock and adjusted experimentally to obtain an output voltage \( V_0 \) of 28 volts. The resulting final values were \( R_7 = 69.8 \, \text{k}\Omega \) and \( R_8 = 14.3 \, \text{k}\Omega \) as given in Table III. In order to obtain the best possible regulation, the accuracy of the comparator must be maximized. This is done by making the source resistance of the reference voltage \( V_R \) equal to the dc source resistance of the signal \( v_A \). With equal source resistances, the bias currents at the inputs create nearly equal voltage drops minimizing the offset voltage. The dc source resistance for \( v_A \) is the parallel combination of \( R_7 \) and \( R_8 \) so

\[
R_6 = \frac{R_7 R_8}{R_7 + R_8} = \frac{14.3 \times 69.8}{14.3 + 69.8} \times 10^3 = 11.9 \, \text{k}\Omega \quad . \quad (63)
\]

The nearest discrete resistor to this value in stock is 12.0 k\( \Omega \) which is used for \( R_6 \) as given in Table III. This completes the design of the differential voltage comparator.
Design of the triangular waveform generator. The triangular waveform generator is built around the dual operational amplifier I2 shown in Figs. 11 and 14. I2 is a type CA3000 IC differential amplifier by RCA. This amplifier has a typical internal power dissipation of 30 mw which is desirable from the viewpoint of maximizing efficiency. Also it is capable of operating at frequencies to 30 MHz which is well beyond the switching frequency of the converter. I2 requires power supplies of +6 volts at pin 9 and -6 volts at pin 3 with respect to pin 2. These supplies are obtained in Fig. 14 by using the existing +12-volt supply on pin 9, grounding pin 3, and providing an additional +6 volt supply so that pin 9 is +6 volts and pin 3 is -6 volts with respect to the +6 volt supply at pin 2.

A square wave oscillator circuit is formed with I2 using R9, R10, and C11 as given in the manufacturer's application manual. This oscillator produces a square wave at the output at pin 10. The frequency of oscillation is adjusted to 1 MHz by experimentally varying C11 until it is 33 pF. R9 and R10 are chosen to be 1 kΩ because these are typical values used by the manufacturer [24].

The open-circuit output $v_{10}$ at pin 10 switches from $V_{10}' = 11.2$ volts to $V_{10}'' = 5.0$ volts and vice versa. When pin 10 is connected to C10, this voltage produces a dc voltage $V_{dc} = 8.1$ volts on C10. Since $V_{dc}$ does not normally equal the dc component of $v_A$, coupling capacitor C9 is required to keep
\( V_{dc} \) from upsetting the balance of I1. In order that \( v_t \) is coupled to R8 without any significant distortion, C9 must be chosen so that

\[
\frac{1}{2\pi f C_9} << \frac{R_7 R_8}{R_7 + R_8}.
\]

This relation is satisfied at \( f = 1 \) MHz when \( C_9 = 0.1 \) \( \mu F \). The triangular waveform \( v_t \) is generated by the charging and discharging of C10 during alternate half cycles of the square-wave output on pin 10 of I2. The time constant of this circuit is found by assuming that \( V_0 \) and C9 are shorted, that pin 10 returns to the neutral through an internal resistance \( r_o \), and that the input impedance at pin 3 of I1 is large enough to be assumed an open circuit. These assumptions mean that \( R_7, R_8, r_o, \) and C10 are effectively in parallel which yields a time constant \( \tau \) of

\[
\tau = \frac{R_7 R_8 r_o}{R_7 R_8 + r_o R_7 + r_o R_8} C_{10} = RC_{10}
\]

where \( R = R_7 R_8 r_o / \left( R_7 R_8 + r_o R_7 + r_o R_8 \right) \) is the parallel combination of \( R_7, R_8, \) and \( r_o \).

With these assumptions, the load on pin 10 of I2 can be thought of as the parallel combination of \( R \) and C10. Across C10 there is an average dc bias of \( V_{dc} \). If it is assumed that the open circuit voltage \( v_{10} \) on pin 10 of I2 switches to \( V'_{10} = 11.2 \) volts at \( t = 0 \), the equation for \( v_t(t) \) during the half cycle, \( t = 0 \) to \( t = 1/2f \), when C10 is charging is
\[ v_t(t) = \left( V'_{10} - V_{dc} + V_{tm} \right) \left( 1 - e^{-\frac{t}{\tau}} \right) + V_{dc} - V_{tm} \]  

(66)

Where at \( t = 0 \), (66) yields \( v_t(0) = V_{dc} - V_{tm} \); and at \( t = \infty \), (66) yields \( v_t(\infty) = V'_{10} \). The peak value of \( v_t(t) \) occurs at \( t = \frac{1}{2\tau} \) and is \( V_{dc} + V_{tm} = v_t(1/2\tau) \). The peak value \( V_{tm} \) of \( v_t(t) \) can be computed by substituting \( t = \frac{1}{2\tau} \) and \( \tau = \frac{RC_{10}}{10} \) into (66) and evaluating

\[ V_{tm} = \frac{1}{2} \left[ v_t(1/2\tau) - v_t(0) \right] = \frac{1}{2} \left( V'_{10} - V_{dc} + V_{tm} \right) \left( 1 - e^{-\frac{1}{2fRC_{10}}} \right) \]  

(67)

Rearranging, taking the natural logarithm of both sides of (67), and solving for \( C_{10} \) yields

\[ C_{10} = \frac{-1}{2fR \ln \left( 1 - \frac{2V_{tm}}{V'_{10} - V_{dc} + V_{tm}} \right)} \]  

(68)

which is the value of \( C_{10} \) which yields \( V_{tm} \).

For I2, the typical value of \( r_o \) is 8 k\( \Omega \); and R7 and R8 have been determined as 69.8 k\( \Omega \) and 14.3 k\( \Omega \) respectively. Substituting into (65) yields

\[ R = \frac{69.8 \times 14.3 \times 8.0 \times 10^9}{(69.8 \times 14.3 + 8.0 \times 69.8 + 8.0 \times 14.3)10^6} = 4.78 \text{ k}\Omega \]  

(69)

Substituting \( f = 10^6 \text{ Hz} \), \( R = 4.78 \text{ k}\Omega \), \( V_{tm} = 0.1 \text{ volts} \), \( V'_{10} = 11.2 \text{ volts} \), and \( V_{dc} = 8.1 \text{ volts} \) in (68) yields
\[ C_{10} = \frac{-1}{2 \times 10^6 \times 4.28 \times 10^3 \times \ln \left(1 - \frac{0.2}{\frac{11.2}{8.1} + 0.1}\right)} = 1620 \text{ pF} \]  

(70)

A 1500 pF disc ceramic capacitor as given in Table III was chosen from stock for C10.

This concludes the design of the control circuits. The final section of the converter to be discussed next is the drive circuit.

The Drive Circuit

This part of the converter boosts the power available from the duty cycle output of differential comparator II to a power sufficient to control the power switching transistor Q1. The schematic diagram is shown in Fig. 16. The output from pin 7 of the differential comparator II is used to drive the base of Q2 through R2. Bias resistor R4 and Zener diode DZ1 supplied by the -6 volts on pin 4 of II provide a negative voltage at b which serves to reverse bias the emitter-base junction of Q2 through R3 during the off-time. When Q2 is on, the +6 volt supply is applied to the primary winding N5 so that the dotted ends of windings N5 and N4 are positive. The voltage coupled to the secondary winding N4 during the time Q2 is on is of such a polarity to drive Q1 through R1 so that when Q2 turns on, Q1 turns on. The time that Q1 is on is designated \( t_{on} \), and the time that Q2 is on is designated \( t'_{on} \).
Fig. 16. Schematic diagram of the drive circuit.
The time $t'_\text{on}$ may not be the same as $t_{\text{on}}$ because the response times of Q2 and Q1 are normally different. The voltage applied to N5 during the on-time of Q2 causes energy to be stored in T2. When Q2 turns off, the energy stored during the on-time of Q2 is discharged through R1, D2, D3, and N4; and the voltage drop on D2 and D3 provides a reverse bias on the emitter-base junction of Q1 to reduce the storage and fall times of Q1. It should be noted that T2 serves three functions. One is to provide isolation between the source and the load. The second function is to provide base drive to Q1. The third function is to store energy during the on-time of Q2 which can be used to reverse-bias Q1 during the off-time of Q2 to speed turn-off of Q1.

Primary and secondary current relations for T2. The design of the drive circuit begins with an analysis of the current waveforms of mutual inductor T2. T2 is a mutual inductor wound on a toroidal ferrite core with a linear B-H characteristic. In the following analysis, it is assumed that Q1 and Q2 switch instantaneously with the exception of the storage time of Q1 and that the primary inductance $L_5$ is large enough so that the variation of the primary current is linear. Fig. 17(a) shows the waveform for the primary current $i_5(t)$ and Fig. 17(b) shows the waveform for the secondary current $i_4(t)$. Since the resistance in the primary circuit is negligible, the voltages $v_5(t)$ on N5 and $v_4(t)$ on N4 are constant during $t'_\text{on}$; consequently, $i_4(t)$ is constant and equal to $-I_4$ given by
Fig. 17. Current waveforms for T2.
\[ i_{4n}(t) = -i_4 = -\frac{N_4 \left(6 - V_{CE2}\right)/N_5}{R_1} - V_{BE1} \quad 0 \leq t \leq t'_{on} \]

(71)

where the subscript \( n \) designates \( i_4(t) \) during \( t'_{on} \), \( V_{CE2} \) is the collector-emitter saturation voltage on \( Q2 \) during \( t'_{on} \), and \( V_{BE1} \) is the base-emitter saturation voltage on \( Q1 \) during \( t'_{on} \). Symbols such as \( V_{CE2} \) and \( V_{BE1} \) are used in lieu of \( V_{CE(sat)2} \) and \( V_{BE(sat)1} \) to represent the saturation voltages of transistors in order to eliminate lengthy and cumbersome subscripts on these variables. Also during \( t'_{on} \), the primary current \( i_{5n}(t) \) consists of a constant component \( N_5 i_4/N_4 \) which is \( I_4 \) reflected to the primary and a component which represents the energy stored in \( T2 \) and increases approximately linearly during \( t'_{on} \) by \( \Delta i_{5n} \) as shown in Fig. 17(a). Assuming that \( i_{5n}(t) \) varies linearly during \( t'_{on} \), the voltage of self induction is

\[ V_{N5} = L_5 \frac{\Delta i_{5n}}{t'_{on}}. \]

(72)

Since \( V_N \) must equal \( 6 - V_{CE2} \), the slope \( \frac{\Delta i_{5n}}{dt} = \frac{\Delta i_{5}}{t'_{on}} \) of \( i_{5n}(t) \) during \( t'_{on} \) must be \( \frac{\left(6 - V_{CE2}\right)}{L_5} \) which yields

\[ i_{5n}(t) = \frac{6 - V_{CE2}}{L_5} t + K \quad 0 \leq t \leq t'_{on} \]

(73)

where \( K \) is the value of \( i_{5n}(t) \) at \( t = 0 \) as shown in Fig. 17(a).

The subscript \( f \) designates variables during the time period \( t'_{off} \). During \( t'_{off} \), \( i_{5f}(t) = 0 \); but \( i_{4f}(t) \) is nonzero flowing into the dotted end of \( N4 \) and passing through \( D2, D3, \) and \( R1 \). The differential equation for \( i_{4f}(t) \) around this loop is
where $V_{FD}$ is the forward voltage on diode D2 or D3 and the number of diodes $m$ is as yet undetermined. Solving (74) for $i_{4f}(t)$ yields

$$i_{4f}(t) = A e^{ - \frac{R_1}{L_4} \left( t - t_{on} \right) } - \frac{mV_{FD}}{R_1} \ t_{on} \leq t \leq T . \quad (75)$$

Expressions for the primary and secondary currents during both $t'_{on}$ and $t'_{off}$ have been derived. During $t'_{on}$, $i_{4n}(t)$ is given by (71) and $i_{5n}(t)$ by (73). During $t'_{off}$, $i_{5f}(t) = 0$ and $i_{4f}(t)$ is given by (75).

There are two unknown constants, $K$ in (73) and $A$ in (75). Their values are related to the steady-state energy levels in mutual inductor $T_2$. One set of conditions is found by equating the total ampere turns just prior to turn-on to the total ampere turns just after turn-on to obtain

$$\frac{N_5 i_{5n}(0)}{N_4} + i_{4n}(0) = i_{4f}(t) . \quad (76)$$

Another set of conditions is obtained by equating the ampere turns just prior to turn-off to the ampere turns just after turn-off to obtain

$$\frac{N_5 i_{5n}(t_{on})}{N_4} + i_{4n}(t_{on}) = i_{4f}(t_{on}) . \quad (77)$$
Substituting (71), (73), and (75) into (76) yields

$$\frac{N_5 K}{N_4} - I_4 = A e^{-\frac{R_1 t'}{L_4 \cdot \text{on}}} - \frac{mV_{FD}}{R_1}. \quad (78)$$

Substituting (71), (73), and (75) into (77) yields

$$\frac{N_5}{N_4} \left[ \frac{6 - V_{CE2}}{L_5} t'_\text{on} + K \right] - I_4 = A - \frac{mV_{FD}}{R_1}. \quad (79)$$

Solving (78) for K and substituting into (79) yields

$$A = \frac{N_5 \left( \frac{6 - V_{CE2}}{L_5} t'_\text{on} \right)}{\frac{R_1}{L_4 t'_\text{off}} - 1 - e^{-\frac{R_1 t'}{L_4 \cdot \text{on}}}}. \quad (80)$$

Substituting (80) into (78) and solving for K yields

$$K = \frac{6 - V_{CE2}}{L_5} t'_\text{on} - \frac{N_4 \left( \frac{mV_{FD}}{R_1} - I_4 \right)}{\frac{R_1}{L_4 t'_\text{off}} - 1 - e^{-\frac{R_1 t'}{L_4 \cdot \text{on}}}}. \quad (81)$$

The constant K given in (81) can be substituted into (73) to find $i_{5n}(t)$ during $t'_\text{on}$. During $t'_\text{off}$, $i_{5f}(t) = 0.0$. The base drive $i_4(t)$ for Q1 during $t'_\text{on}$ is given by (71), and the constant A in (80) can be substituted into (75) to find $i_{4f}(t)$ during $t'_\text{off}$. Now that relations for the currents in T2 have been determined, the design of the drive circuit can continue.
Base drive for Q1. The next step in the design is to determine the magnitude of the base current required by Q1 as

\[ i_{1b(max)} = \frac{i_{1C(max)}}{h_{1fe}} \tag{82} \]

where \( i_{1b(max)} \) is the maximum instantaneous base current corresponding to the maximum instantaneous collector current \( i_{1C(max)} \) in Q1. A current gain \( h_{1fe} \) of 10 is assumed, as used in the manufacturer's specification sheet for switching five amperes, to insure that Q1 is driven into saturation. The maximum instantaneous collector current occurs when the output power \( P_0 \) is a maximum at 20 watts, the source voltage \( E_S \) is a minimum at 21 volts, and \( t = t'_{on} \). It has been calculated in (57) as 3.18 amperes. Using (82), the maximum base current \( i_{1b(max)} \) for Q1 is

\[ i_{1b(max)} = \frac{3.18}{10} = 0.318 \text{ amperes} \tag{83} \]

The drive current \( I_i \) given in (71) must have a magnitude at least as large as \( i_{1b(max)} \) in (83) so that

\[ \frac{N_s (6 - V_{CE2})}{N_s - V_{BE1}} \geq \frac{0.318}{R_1} \tag{84} \]

From the specification sheets for Q1 and Q2, \( V_{CE2} = 0.8 \) volts and \( V_{BE1} = 1.7 \) volts.

At this time, it is necessary to determine the turns ratio for T2. It is desired that \( R_1 \) be as small as possible to minimize power dissipation; consequently, the voltage on \( N4 \) during \( t'_{on} \) should be small. Also, the same principles of design are
applied to T2 as were applied to T1. Because the source imped-
ances of T2 are small, leakage inductance degrades the switching
performance of T2 more than winding capacitance so T2 should be
bifilar wound to reduce leakage inductance. In consideration of
these principles, a unity turns ratio is chosen for T2 so that
the voltage on N4 is $6 - V_{CE2}$ volts during $t'_\text{on}$ and N4 and N5 are
wound turn for turn to minimize leakage inductance.

Solving for $R_1$ in (84) and substituting for $V_{CE2}$, $V_{BE1}$,
$I_n$, and $n$ yields

$$R_1 = \frac{N_a (6 - V_{CE2})}{N_b - V_{BE1}} \cdot \frac{1}{I_n} = \frac{(6 - 0.8) - 1.7}{0.318} = 10.1 \ \Omega . \ (85)$$

The value chosen from stock for $R_1$ is 10 $\Omega$ as given in Table III.

**Reverse bias.** The next step in the design is to consider
$i_{4f}(t)$ during $t'_{\text{off}}$. This current should be sufficient during the
first part of $t'_{\text{off}}$ to provide the current to sweep the charge
stored during $t'_{\text{on}}$ from the emitter-base junction of Q1 so that
storage time and fall time are minimized. The current $i_{4f}(t)$
should also be nonzero during all of $t'_{\text{off}}$ since reverse-biased
transistors can block greater collector voltage spikes. The
current $i_{4f}(t)$ is obtained from the energy stored in T2 during
$t'_{\text{on}}$. This stored energy should not be too great since $i_{5n}(t)$
must increase in magnitude proportionately. When $i_{5n}(t)$ in-
creases, $i_{b2}(t)$ must increase to maintain Q2 in saturation re-
sulting in more power dissipation and possibly a requirement for
another stage of amplification. This means that the energy
stored in T2 should be minimized to maintain a high efficiency, but enough energy must be stored to reverse bias Q1 and reduce the storage time of Q1.

**Determination of L4, L5, and m.** Under the condition of maximum source current, \( i_{sn}(0) \) must be at least equal to \( I_u/n \) if Q1 is to be driven into saturation. In order to determine \( L_4, L_5, \) and the number of diodes \( m, t = 0 \) and \( K \) from (81) are substituted into (73) to obtain

\[
i_{sn}(0) = K = \frac{6 - V_{CE2} \cdot t'_on}{L_5} - \frac{N_4 \left( \frac{mV_{FD}}{R_1} - I_u \right)}{N_5} \geq \frac{N_4 I_u}{N_5} \quad (86)
\]

where \( V_{CE2} = 0.8 \) volts, \( R_1 = 10 \) ohms, \( V_{FD} = 0.66 \) volts at 200 ma, \( N_4/N_5 = 1, L_4 = L_5, \) and \( I_u = 0.318 \) amperes from (84). If switching response times are neglected, then \( t'_on = t_{on} = 0.4 \) \( \mu \)sec from (50) and \( t'_off = t_{off} = 0.6 \) \( \mu \)sec. Substituting these values into (86) yields

\[
i_{sn}(0) = 2.08 \times 10^{-6} \left( \frac{6 \times 10^{-6}}{L_4} - 1 \right)^{-1} - 0.066 m + I_u \geq I_u \quad (87)
\]

which can be solved by trial and error for \( L_4 = L_5 \) and \( m \). The number of diodes \( m \) must be less than 6 since the maximum emitter-base voltage \( mV_{FD} \) for Q1 is 4 volts.
Since $K$ is constant, the expression for $\Delta i_{sn}$ is derived as

$$\Delta i_{sn} = \frac{6 - V_{CE2}}{L_5} t'_{on}. \quad (88)$$

If it is assumed that $\Delta i_{sn} = 2$ ma to minimize the energy stored in $T_2$, then solving (88) for $L_5$ yields

$$L_5 = \frac{6 - V_{CE2}}{\Delta i_{sn}} t'_{on} = \frac{6 - 0.8}{2 \times 10^{-3}} \times 0.4 \times 10^{-6} = 1.04 \text{ mH}. \quad (89)$$

Letting $L'_s = L_s = 1$ mH and substituting into (87) yields

$$2.08 \times 10^{-6} \left[ \frac{6 \times 10^{-6}}{e^{10^{-3}}} - 1 \right]^{-1} \geq 0.066 \text{ m} \quad (90)$$

and solving for $m$ yields

$$m \leq 5.25 \text{ diodes}. \quad (91)$$

If it is assumed that the current $i_{sf}(t'_{on})$ at turn-off is 0.2 amperes, then the number of diodes needed to obtain this current can be computed by substituting $t = t'_{on}$ into (75) to obtain

$$i_{sf}(t'_{on}) = 0.2 \leq A - \frac{mV_{FD}}{R_1}. \quad (92)$$

Solving (92) for $m$ yields
\[
m \leq \frac{R_1}{V_{FD}} (A - 0.2) = \frac{R_1}{V_{FD}} \left[ \frac{N_5}{N_4} \left( \frac{6 - V_{CE2}}{L_5} t'_\text{on} \right) \right]
\]

Substituting known values into (93) yields

\[
m \leq \frac{10}{0.66} \left[ \frac{6 - 0.8(0.4 \times 10^{-6})}{10^{-3}} - \frac{10}{10^{-3}(0.6 \times 10^{-6})} \right] = 2.24 . \quad (94)
\]

The lesser of the two values for \( m \) in (91) and (92), or \( m = 2 \), is chosen. Two 10A26 diodes are used for D2 and D3. The last step in the design of the drive circuit is to design \( T_2 \) so that \( L_4 \) and \( L_5 \) are 1 mH.

**Mutual inductor \( T_2 \).** As mentioned earlier a toroidal core configuration with \( N_4 = N_5 \) and bifilar winding is chosen to reduce the leakage inductance. In addition, Ferroxcube Corp. type 3D3 ferrite material is chosen because it has the lowest loss factor at a frequency of 1 MHz. The final procedure is to choose a core size, compute the number of turns which is required to make 1 mH, check the mmf to be certain that the core is not saturated, determine the wire size, and check to be certain that the turns for \( N_4 \) and \( N_5 \) fit in the window area. This procedure is repeated by trial and error until the smallest core that meets the requirements is found. For this converter, Ferroxcube Corp. toroidal core 768T188/3D3 is chosen. This core has an inductance factor \( A_L \) of 4.5 mH/1000 turns, a mean free
The number of turns for N4 and N5 is calculated as

\[ N_4 = N_5 = \left[ \frac{L_5(1000)^2}{415} \right]^{1/2} = 10^3/(415)^{1/2} = 49.1 \text{ turns} \] (95)

N4 and N5 are therefore chosen to be 49 turns. The mmf corresponding to \( i_{nf}(t') \text{on} \) is calculated as

\[
H = 0.4\pi \frac{N_4 i_{nf}(t')\text{on}}{L_e} = 0.4\pi \frac{N_4 \left( A - \frac{mV_{FD}}{R_1} \right)}{L_e} \\
= 0.4\pi \frac{49 \left[ 0.348 - \frac{2(0.66)}{10} \right]}{3.03} = 4.39 \text{ Oersteds} \] (96)

Since the saturation mmf for 3D3 material is 5 Oersteds, the core is not driven to saturation. In order to determine the required wire size, the maximum full-cycle average value of the absolute value of \( i_4(t) \) and \( i_5(t) \) must be determined. The maximum values of \( i_4(t) \) and \( i_5(t) \) occur when \( t' = 0.4 \mu\text{sec} \) and \( t' = 0.6 \mu\text{sec}. \) From Fig. 17 knowing that \( \Delta i_{5n} = 2 \text{ ma} \), \( i_5(t) \) is

\[
\frac{i_5(t)}{T} = \frac{t'}{T} \left( K + \frac{\Delta i_{5n}}{2} \right) = \frac{0.4}{1.0} \left( 0.532 + \frac{0.002}{2} \right) = 0.213 \text{ Amperes} \] (97)

Again using Fig. 17
\[
\frac{1}{i_4(t)} = \left[ I_4 \frac{t'}{t_{on}} + \frac{1}{2} \left( A - \frac{mV_{FD}}{R_1} + \frac{N_5 K}{N_4} - I_4 \right) \frac{t'}{t_{off}} \right] / T \\
= \left\{ 0.318(0.4) + \frac{1}{2} \left[ 0.348 \frac{2(0.66)}{10} + 0.532 - 0.318 \right] 0.6 \right\} / 1.0 \\
= 0.1272 + \frac{1}{2}(0.430)0.6 = 0.256 \text{ Amperes} . \quad (98)
\]

Based on 1000 circular mils per ampere N4 and N5 can be wound using AWG 26 wire. This wire with heavy formvar insulation can be wound with 2932 turns per in². Assuming a winding factor of 0.4, the number of turns N of AWG 25 that can be wound on core 768T188/3D3 is

\[
N = 0.4(A_C)2932 = 0.4(0.4)2932 = 469 \text{ turns} . \quad (99)
\]

Since N4 + N5 = 98 is much less than 469, the design of T2 and the converter has been completed. The next section gives the experimental results and some conclusions.

Experimental Results

This section discusses experimental data obtained from the 1 MHz dc to dc converter shown in Fig. 11. Regulation and efficiency characteristics for the converter are given. Photographs of selected waveforms in the circuit are used to show properties such as voltage and current amplitudes and switching times. Throughout the section, design techniques which could improve the performance of the converter are discussed.
Fig. 18 shows the output voltage $V_o$ regulation characteristics for load conditions from light to heavy. The converter should not be operated with no load because the voltage on C3 and C4 could exceed safe operation limits causing the destruction of one or more components. A 50 to 100 KΩ resistor could be added in parallel with $R_L$ to prevent no-load conditions. The addition of this resistor would cause additional power loss of only 8 to 16 mw. Fig. 18 shows a family of three curves; one each for the minimum source voltage $E_s$ of 21 volts, an $E_s$ of 26 volts, and the maximum $E_s$ of 31 volts. Over the full range of data, $V_o$ varied from 27.48 to 28.30 volts for a mean value of 27.89 volts. The output voltage regulation can be expressed as ±1.47 percent about the mean of 27.89 volts. When $P_o$ increases, the average source current must increase. Consequently, output voltage $V_L$ decreases as the output power $P_o$ increases and as the source voltage $E_s$ decreases because losses in the main power path caused by greater currents are not fully compensated by the converter. Regulation can be improved by decreasing the amplitude of the triangular waveform $v_t$. However, when this is done, it may also be necessary to reduce the comparator hysteresis for example by using a feedback resistor from the output to the positive input of the comparator. Additionally, it may be necessary to reduce the magnitude of the output ripple voltage at the input to the comparator if it is on the order of the new magnitude of $v_t$. 
Source Voltage, $E_S$
A = 31 volts
B = 26 volts
C = 21 volts

Fig. 18. Converter regulation characteristics.
The converter efficiency characteristics shown in Fig. 19 were obtained by measuring the current and voltage of each source $E_s$, +12 volts, +6 volts, and -6 volts and computing the input power to the converter as the sum of the power from these four sources. The output power $P_o$ was computed as the square of the output voltage $V_o$ divided by the load $R_L$. The efficiency shown is the ratio of these power computations. Three curves are shown; one each for $E_s = 21$, 26, and 31 volts. Each curve has the same general shape. Efficiency is lower for higher source voltages and is a minimum at light load. Efficiency increases as load increases passing a maximum at $P_o \approx 10$ watts and decreasing slightly as $P_o$ increases.

Power losses occur in several major areas of the converter. These are semiconductor static and switching losses, core losses, and power transistor $Q_1$ drive circuit losses. Other areas where power losses are not as significant are the power supplied to the comparator and triangular waveform generator, the copper loss in the circuit wiring, and the filter capacitor series and shunt resistances.

Transistor $Q_1$ static power loss includes the product of collector current $I_1$ during the on time, saturation voltage $V_{CE(sat)}$, and the duty cycle. This loss increases when the collector current or duty cycle increases, that is when $P_o$ increases or $E_s$ decreases. Static loss also includes the product of the blocking voltage and leakage current during the off time.
Fig. 19. The collector-emitter voltage $v_{CE1}(t)$ and the collector current $i_1(t)$ when $E_S = 21$ volts and $R_L = 100$ ohms.
multiplied by 1-duty cycle and this loss increases when $E_S$ increases or the duty cycle decreases. Dynamic losses occur during the switching times and are the integrals of the products of $v_{CE}(t)$ and $i_C(t)$ during turn-on and turn-off multiplied by the respective time periods of the integrations and divided by $T$. Dynamic losses also increase as $E_S$ and $P_o$ increase. Static and dynamic losses in diode D1 behave the same as the losses in Q1 except that D1 is off when Q1 is on. This means that static power loss when D1 is on decreases with duty cycle and vice versa.

Core losses are highly dependent on frequency which is constant in this converter; however, hysteresis losses increase as the flux excursion increases and copper losses increase as the current levels increase. This means that core loss increases as $P_o$ increases.

Power required to supply the base drive to Q1 is the third major area of power dissipation. The drive circuits dissipate more power when Q1 and Q2 are on so the power dissipated in these circuits increases as the duty cycle increases, or as $P_o$ increases and $E_S$ decreases.

Figs. 19 and 20 show the collector-emitter voltage $V_{CE1}$ and the collector current $i_C$ waveforms of power switching transistor Q1. The waveforms in Fig. 19 are for the conditions $E_S = 21$ volts and $R_L = 100$ ohms. In both figures, (a) shows a full period of the waveform, (b) shows the turn-on portion
Fig. 20. The collector-emitter voltage $v_{CE1}(t)$ and the collector current $i_1(t)$ when $E_S = 31$ volts and $R_L = 100$ ohms.
expanded, and (c) shows the turn-off portion expanded. By examining Figs. 19(a) and 20(a), it is determined that the time T for a full period is 0.8 μsec resulting in a switching frequency of 1.25 MHz. The frequency was increased slightly above 1 MHz so that the entire period would occupy slightly less than the full width of one oscilloscope photograph.

In the beginning of this chapter, the fact that the switching speed of semiconductors is the limiting factor for designing switching converters at a frequency as high as 1 MHz was discussed. This limitation shows up in the experimental efficiency curves in Fig. 21. It can be seen that efficiency is lower at higher values of $E_s$. This phenomena occurs because switching loss in D1 and especially Q1 are the major power losses in the converter and these losses increase as $E_s$ increases. The waveform in Fig. 20(c) for $E_s = 31$ volts shows a significant increase in the switching time as well as the magnitude of voltage switched in comparison to the waveform in Fig. 19(c) for $E_s = 21$ volts. The increase in semiconductor switching losses and semiconductor static losses during the off time exceeds the decrease in saturated semiconductor losses, drive circuit losses, and core loss when $E_s$ increases. The fact that the majority of the converter power loss is in Q1 is supported by the fact that this component operates at a higher temperature than any other in the converter even though it is mounted on a heavy copper heat sink. Fig. 19(b) indicates that $v_{CE1}$ switches from 90 to 10 percent of the blocking voltage in 25 nsec. The
Source Voltage, \( E_S \)
- \( A = 31 \) volts
- \( B = 26 \) volts
- \( C = 21 \) volts

Fig. 21. Converter efficiency characteristics.
collector current $i_{C1}$ switches from 10 to 90 percent of the linearly extrapolated initial current in 10 nsec. The collector current appears to switch much faster than $v_{CE1}$ and reaches a peak much higher than the initial current. This peak is coupled through $T1$ and is inverted in the waveform for $i_2$. In addition, the peak increases if external capacitance is added in parallel with $D1$; therefore, it is assumed that the current peak is the current required to charge the junction capacitance of $D1$ when $Q1$ turns on. This suggests that switching losses in $Q1$ during turn-on may be reduced if the junction capacitance of $D1$ can be reduced.

In general, the performance characteristics of this 1 MHz dc to dc converter are good. Regulation is less than 1.5 percent and efficiency approaches 80 percent at power outputs from 5 to 20 watts. Switching in the main power path is accomplished in less than 10 percent of the full period at a switching frequency of 1 MHz. Some conclusions about the 1 MHz converter are given in the next section.

Conclusion

Most dc to dc converters are designed with typical operating frequencies from 2 to 20 kHz and maximum operating frequencies in the vicinity of 100 kHz. The design of a 1 MHz converter was begun because of requirements for satellites to measure electromagnetic radiation in outer space within the
frequency range of 0.2 to 100 kHz. In order that the power conditioning equipment on the satellites does not interfere with the measurements, the switching frequencies must be well above the 0.2 to 100 kHz range, for example 1 MHz. The converter design discussed in this chapter demonstrates the feasibility of designing converters at megahertz switching frequencies.

The chapter begins with a discussion of design principles which must be considered when designing circuits to switch at frequencies near 1 MHz. In general, stray inductance, capacitance, and coupling must be kept small by shielding and keeping circuit lead length to a minimum. Also, the circuit components must meet special high-frequency requirements. Magnetic materials with low loss factors at 1 MHz and high-frequency capacitors must be used. Leakage inductance in the mutual inductors has to be minimized to prevent large voltage spikes on the switching elements. In addition, high-frequency bypass capacitors are required at various points in the circuit to reduce switching spikes. Integrated circuits are needed in the control circuits for the differential voltage comparator and the triangular waveform generator in order to reduce the effects of stray parameters and to obtain faster switching speeds. The power switching diodes and transistors used in the design need to switch the required voltage and current at least as fast as 50 nsec. The switching speed of the power transistor is considered to be the most significant factor limiting an additional increase in switching frequency and only a few diodes and
transistors are available which switch the required voltage and current as fast as 50 nsec.

The design of the converter is approached by dividing the converter into five basic circuits excluding the DC Source and the Load. These circuits are shown in Fig. 10 as the Power Circuit, Output Filter, Drive Circuit, Differential Voltage Comparator, and Triangular Waveform Generator. The Triangular Waveform Generator oscillates at a constant frequency of 1 MHz which determines the operating frequency of the converter and, in conjunction with the Comparator, determines the duty cycle required to maintain regulation. The Drive Circuit amplifies the duty cycle signal from the Comparator to a sufficient power level to drive the power transistor in the Power Circuit. The Power Circuit converts the dc power from the Source to ac power and back to dc power with a regulated average direct voltage level which can be at either a higher or lower voltage level than the source. The Output Filter averages the switched direct voltage from the Power Circuit to a constant value on the Load with sufficiently small ripple voltage.

The particular circuit configuration utilized for this converter was chosen because it provides electrical isolation between the source and the load and a regulated output voltage which may be either greater than or less than the source voltage. The choice of a megahertz switching frequency reduces the values of required capacitors, especially the filter capacitors, and
permits the use of ceramic capacitors instead of electrolytic capacitors. This is an advantage since electrolytic capacitors are considerably less reliable than ceramic capacitors as was discussed in the previous chapter. The high switching frequency reduces the size and weight of the circuitry especially in the areas of magnetic components and capacitors in the output filter and power circuit.

The power output of the converter was designed to be 20 watts; however, the power switching transistor $Q_1$ is capable of processing more power. The converter power output can be increased by redesigning mutual inductor $T_1$ to process the additional power and by increasing the output capability of the Drive Circuit so that more base drive is provided to $Q_1$. The output voltage regulation is $\pm 1.47$ percent and might be able to be improved by reducing the hysteresis of the Comparator and the magnitude of the triangular waveform $v_T$. The maximum efficiency of 79 percent occurs at $P_0 = 10$ watts and decreases to 72 percent at the full power output of 20 watts. The efficiency is not as high as efficiencies of the lower-switching-frequency converters which are in the range of 90 percent but should be considered good for a switching frequency of 1 MHz. The majority of the power losses are the static and switching losses of transistor $Q_1$. Therefore, the efficiency of this converter will be improved with the availability of power transistors with faster switching speeds, lower saturation voltages, and higher current gains.
This converter is not inherently self-starting. By the nature of the regulation means, $Q_1$ remains on as long as $V_A$ is below its level at regulation. This means that $Q_1$ will remain on continuously as long as $V_A$ is less than $V_{tm}$ plus its value when the converter is regulating. To make the converter self-starting, a circuit which switches $Q_1$ at a fixed duty cycle adequate to achieve at least $V_o = 28$ volts under the conditions of minimum $R_L$ and $E_S$ must be included in the design. The starting circuit must also be designed to turn off when $V_o$ reaches 28 volts. Starting is an important subject but this dissertation does not address it. The emphasis is on designing a converter to operate at a switching frequency of 1 MHz with reasonable efficiency.

The next chapter of this dissertation is purely analytical. The method of closed loop regulation with a triangular waveform and a voltage comparator as used in the converter discussed in this chapter is analyzed using the technique of nonlinear analysis which employs the phase plane and sequence functions.
Chapter IV

PHASE-PLANE ANALYSIS OF A SWITCHING REGULATOR
SYSTEM DRIVEN BY A SMALL-AMPLITUDE
TRIANGULAR WAVEFORM

Because of the mathematical difficulties encountered the nonlinear analysis of switching systems is usually restricted out of necessity to those systems which yield autonomous equations. The equations of these systems depend on time only through the derivative with respect to time and on the circuit parameters which must be independent of time. Often, however, one encounters systems which have one or more properties that are dependent on a driving function.

The last chapter presented the design of a circuit which uses a triangular driving function in a dc to dc converter to determine a duty cycle which, in the closed-loop system, maintained a regulated output voltage. This system serves as a model of the type of circuit which is the subject of the nonlinear analysis presented in this chapter. The system is a switching system which is linear during each state of the switch.

The nonlinear analysis of switching systems may be performed by the method of point transformations in piece-wise
linear systems [25]. In this method, each mode of the switching system is considered a linear system. A set of initial conditions are assumed and are transformed using the linear system equations into the final conditions of the first mode. These final conditions are assumed as the initial conditions of the second mode and the second mode final conditions are derived. This process continues until the final conditions of the final mode are determined. These conditions may then be equated to the assumed first set of initial conditions to determine the system equations for the steady-state system. A total phase plane can then be drawn which consists of segments which are defined by the linear systems' equations for each mode of the switching system. The analysis performed in this chapter shows that a periodic triangular driving function in a regulator system can be analyzed using the method of point transformations in piece-wise linear systems. The application of this method to a regulator system utilizing a triangular driving function is shown to be dependent on the amplitude voltage of the triangular function being small compared to the reference voltage and on the fact that the time derivative of a triangular function is constant during fixed periods of time.

The first section of this chapter develops the model for the analysis and gives the assumptions which are made. In addition, the means by which regulation is maintained is briefly discussed to expand the explanation given in Chapter III to include hysteresis. The next section is devoted to developing the
system equations for each linear state of the system. These equations are used to sketch the limit cycle in the phase-plane for the system, and the method of point transformations is applied to obtain the sequence functions defining the steady-state conditions. The final section is devoted to conclusions which can be made about the analysis. An experimental limit cycle for a converter operating at a switching frequency of 2 kHz is compared to the theoretical limit cycle and the accuracy of the sequence functions are experimentally verified in Appendix C. Parameters and circuit conditions for this converter are shown in Table C.2.

Development of the System Model

The circuit for which the analysis of this chapter is performed is a regulated dc to dc converter using inductive-energy storage and having electrical isolation between the source and load. The 1 MHz converter discussed in detail in the previous chapter is an example of a converter of this type. However, the high operating frequency prohibits a direct application of the analysis to this specific converter since a frequency of 1 MHz would require the inclusion of effects such as stray capacitance, leakage inductance, and finite switching times which would overly complicate the analysis but which can be neglected at lower operating frequencies. It is for this reason that the experimental verification in Appendix C is made
using a 2 kHz converter.

In order to perform the analysis, a model must be established and assumptions must be made. Fig. 22 shows a diagram of the model which is to be used. This diagram is used in the next section of this chapter to develop the system equations. In order to develop the model depicted in Fig. 22, it is assumed that the mutual inductor is unity coupled so that it has no leakage inductance and that it has no distributed capacitance. Also, transistor Q and diode D are assumed to switch instantaneously. Any internal series resistance in the source E and primary inductance \( L_p \) is combined with the effective saturation resistance of power transistor Q to form \( R_p \). Similarly, the internal series resistance of the secondary inductance \( L_s \) is combined with the forward resistance of diode D to form \( R_s \). It is also necessary to include in the analysis the internal series resistance \( R_C \) of filter capacitor C. The load resistance is \( R \). The primary current is \( i_p \), the secondary current is \( i_s \), the load current is \( i_L \), and the output current of the hysteresis block which drives Q is \( i_H \). The collector-emitter voltage on Q is \( V_{CE} \), the anode-cathode voltage on D is \( V_D \), and the output, or load, voltage is \( v \).

The system operation is described in detail in Chapter III and is described only briefly here. Energy from the source E is stored in \( L_p \) as \( i_p \) increases during the time \( t_{on} \) when Q is in saturation. It should be observed that the relative
Fig. 22. The system model.
polarities of the voltages across $L_p$ and $L_s$ are such that diode $D$ is reverse-biased during $t_{on}$ and $i_s$ must be zero; however, $C$ is discharging into $R$ so that the output voltage $v$ is maintained. During $t_{off}$ when $Q$ is blocking and $D$ is conducting, $i_p$ is zero and the energy stored in the mutual inductor is discharging through $L_s$ into $C$ and $R$ by means of $i_s$. In this manner, capacitance $C$ and $L_s$ act as a filter which smooths the secondary current pulses $i_s$ producing a dc voltage with small amplitude ripple on $R$.

The summer shown in Fig. 22 is used to add a triangular waveform $v_t$ to the output voltage $v$ and to compare this sum with the reference voltage $V_R$. The difference of $V_R$ and $v + v_t$ is the error voltage $\epsilon$ which is given by

$$\epsilon(t) = V_R - v(t) - v_t(t) \quad \text{(100)}$$

In typical applications, the summer is part of a differential voltage comparator. This circuit switches between two modes whenever $\epsilon$ exceeds a certain voltage level or falls below another voltage level. The two modes are forward current $i_H$ and reverse current $-i_H$. Diode $D'$ provides a path for the reverse current during $t_{off}$. This type of switching has a hysteresis characteristic as shown in Fig. 22 where $\delta_1$ and $-\delta_2$ are the switching levels for $\epsilon$.

The means of regulation can best be understood by examining Fig. 23. This figure shows the time variation of $v + v_t(t)$
Fig. 23. Utilization of the triangular waveform to determine duty cycle.
where \( v \) is assumed to have a constant value \( V \) which is equivalent to assuming negligible ripple voltage on the load \( R \) and steady-state conditions. This assumption is made only to simplify the explanation of regulation and is not required in the analysis. The error voltage \( \varepsilon \) at the input terminals of the hysteresis block is portrayed as the difference between the dotted reference line \( V_R \) and any arbitrary point on the \( V + v_t(t) \) waveform. \( \varepsilon \) is positive when \( V_R \) is greater than \( V + v_t(t) \) and is negative when \( V + v_t(t) \) is greater than \( V_R \). The period of \( v_t(t) \) is shown to be \( T \). The instant when turn-off occurs is marked by a cross where \( V + v_t(t) \) is greater than \( V_R \) by \( \delta_2 \) and must always occur on the positive slope of \( v_t(t) \). If the waveform \( V + v_t \) always lies below \( V_R + \delta_2 \), i.e., if \( V + v_t < V_R + \delta_2 \) or \( \varepsilon = V_R - V - v_t < -\delta_2 \), then \( Q \) will never turn off. The instant of turn-on is marked by a circle and occurs when \( V + v_t(t) \) is less than \( V_R \) by \( \delta_1 \) and must always occur on the negative slope of \( v_t(t) \). If the waveform \( V + v_t \) always lies above \( V_R - \delta_1 \), i.e., if \( V - v_t > V_R - \delta_1 \) or \( \varepsilon < \delta_1 \), then \( Q \) will never turn on. Upon studying Fig. 23 further, it can be seen that as \( V \) increases \( t_{on} \) decreases and \( t_{off} \) increases. This is the same phenomenon explained in Chapter III using Fig. 15 where the hysteresis was neglected. The approximate dc or steady-state relationship between the source voltage \( E \) and the output voltage \( V \) can be derived by equating the expressions for the flux excursions during \( t_{on} \) and \( t_{off} \) as [22]
\[ V = \left[ \frac{N_s}{N_p} \right] \left( \frac{t_{on}}{t_{off}} \right) E \]  

(101)

The derivation of this equation assumes that there is negligible resistance in the dc current paths for \( i_p \) and \( i_s \), that \( V \) remains constant, that \( v_{CE} \) during \( t_{on} \) and \( v_D \) during \( t_{off} \) are negligible compared respectively to \( E \) and \( V \), and that switching occurs instantly.

Formulation of the System Equations

The objective of this section is to develop differential equations which can be used to describe trajectories on the phase plane for which the method of point transformations can be applied. The development begins with the differential equations for the output voltage \( v \) for the two states, \( t_{on} \) and \( t_{off} \), of the system derived for the circuit in Fig. 22 assuming instantaneous switching and a linear B-H characteristic for the mutual inductor. During \( t_{on} \), diode \( D \) is reverse-biased and \( i_s = 0 \); therefore, \( v \) is described by a simple first-order differential equation for the current loop for \( i_L \) where \( v = Ri_L \) as

\[ \frac{dv}{dt} + \frac{1}{(R + R_C)C} v = 0 \]  

(102)

During \( t_{off} \), \( D \) is forward-biased and \( i_s \) is nonzero. Since \( i_L \) is now dependent on two energy-storage components, \( L_s \) and \( C \), the derived equation for \( v \) during \( t_{off} \) is a second-order differential equation given by
\[ \frac{d^2 v}{dt^2} + \left( \frac{1}{RC} + \frac{R_C + R_s}{L_s} \right) \frac{dv}{dt} + \frac{1}{L_s C} v = -\frac{V_D}{L_s C} \]  

(103)

where it has been assumed that \( V_D \) is constant and \( R_C \) and \( R_s \) are negligible compared to \( R \). Equations (102) and (103) are derived in detail in Appendix A.

If a phase plane for \( \frac{dv}{dt} \) versus \( v \) is considered, the switching points must be described by solving (100) for \( v(t) \) as

\[ v(t) = V_R - v_0(t) - \xi(t). \]  

(104)

The switching points for \( v(t) \) are found by substituting \( \xi(t) = \delta_1 \) and \( -\delta_2 \). When this is done, it is obvious that the switching lines drawn in the \( \frac{dv}{dt} \) versus \( v \) phase plane do not have fixed positions since \( v \) is still a function of the time-varying triangular waveform \( v_t \). In order to obtain fixed switching lines in the phase plane, a change of dependent variable from \( v \) to \( \xi \) is made using (104). Substituting \( v(t) \) from (104) into (102) yields

\[ \frac{d\xi}{dt} + \frac{1}{T_C} \xi = \frac{V_R - v_t}{T_C} - \frac{dv_t}{dt} \]  

(105)

where \( T_C = (R + R_C)C \approx RC \); and substituting \( v(t) \) into (103) yields

\[ \frac{d^2 \xi}{dt^2} + \left( \frac{1}{RC} + \frac{R_C + R_s}{L_s} \right) \frac{d\xi}{dt} + \omega_0^2 \xi = \omega_0^2 \left( V_D + V_R - v_t \right) - \left( \frac{1}{RC} + \frac{R_C + R_s}{L_s} \right) \frac{dv_t}{dt} \]  

(106)
where $\omega_0^2 = \left( L_S C \right)^{-1}$. Since switching occurs whenever $\xi = \delta_1$ or $-\delta_2$ which are constants, switching points are located on fixed lines in the phase plane.

Equations (105) and (106) can be put in a more compact form which is more easily analyzed by normalizing the variables. In this case, time is normalized with respect to the period of the natural frequency $\omega_0$ as

$$\tau = \omega_0 t. \quad (107)$$

Also, the error voltage $\xi$ is normalized to the reference voltage $V_R$ as

$$x = \xi/V_R. \quad (108)$$

When the changes in variables defined in (107) and (108) are made in (105) and (106), they become

$$\frac{dx}{d\tau} + \frac{1}{\tau_C} x = \frac{V_R - v_t}{V_R \tau_C} - \frac{dv_t}{d\tau}, \quad (109)$$

$$\frac{d^2 x}{d\tau^2} + 2h \frac{dx}{d\tau} + x = \frac{V_D + V_R - v_t}{V_R} - 2h \frac{dv_t}{d\tau} \quad (110)$$

where $\tau_C$ is the normalized time constant of the $i_L$ current loop during $t_{on}$ and is defined by

$$\tau_C = \omega_0 \left( R + R_C \right) C = \omega_0 T_C \quad (111)$$

$v_t = v_t/V_R$ is the normalized triangular waveform, $dv_t/d\tau$ is the normalized derivative of the triangular waveform defined by
\[
\frac{dv_t}{dt} = \frac{1}{w_0 VR} \frac{dv_t}{dt}
\]

(112)

and \( h \) is the normalized damping factor during \( t_{\text{off}} \) defined by

\[
h = \frac{1}{2w_0} \left( \frac{1}{RC} + \frac{R_C + R_S}{L_s} \right).
\]

(113)

It can be seen that (109) and (110) are the generalized first and second order linear differential equations in terms of the normalized error voltage \( x \) with time-dependent driving functions on the right-hand sides of the equations. These driving functions result from the addition of the time-dependent periodic triangular voltage \( v_t(t) \) to the output voltage \( v \) at the summer shown in Fig. 22. Equations (109) and (110) are nonautonomous equations which are difficult to analyze, especially using phase-plane techniques. In the \( dx/dt \) versus \( x \) phase plane, (109) is represented as a straight line with a slope of \(-1/\tau_C\) and the ordinate intercept equal to the value of the right-hand side; therefore this line moves parallel to itself with the ordinate and abscissa intercepts varying as \( v_t \) and \( dv_t/dt \) vary.

If \( 0 < h < 1 \) in (110), the solution of this equation in the phase plane is a stable focus whose equilibrium point is located on the abscissa at the value of \( x \) equal to the right-hand side of (110); therefore the equilibrium point of this focus moves along the \( x \)-axis as \( v_t \) and \( dv_t/dt \) vary with time.
Equations (109) and (110) can be simplified if the nature of \( v_t(t) \) is examined. The peak value \( V_{tm} \) of \( v_t(t) \) can always be much smaller than \( V_R \) so that \( V_{tm} \ll V_R \). Because of this, \( v_t \) in the numerators of the first terms on the right-hand sides of (109) and (110) can be neglected. Additionally, by the nature of a triangular waveform, \( \frac{dv_t}{dt} \) and \( \frac{dv_t}{d\tau} \) are constant and have equal magnitudes but opposite signs during half of the total time period \( T \) of \( v_t \) so that for \( 0 \leq t \leq T/2 \) using the time reference given in Fig. 23

\[
\frac{dv_t}{dt} = \frac{4V_{tm}}{T} \tag{114}
\]

and for \( T/2 \leq t \leq T \)

\[
\frac{dv_t}{dt} = \frac{-4V_{tm}}{T} \tag{115}
\]

Substituting (114) and (115) respectively into (112) yields the normalized derivatives for \( 0 \leq t \leq T/2 \) as

\[
\frac{dv_t}{d\tau} = \frac{4V_{tm}}{\omega_0 \sqrt{V_R T}} \tag{116}
\]

and for \( T/2 \leq t \leq T \) as

\[
\frac{dv_t}{d\tau} = \frac{-4V_{tm}}{\omega_0 \sqrt{V_R T}} \tag{117}
\]

By neglecting \( v_t \) and substituting (116) and (117) into (109) and (110) the following equations are obtained

DURING \( t_{on} \):
\[
\frac{dx}{dt} + \frac{1}{\tau_C} x = \frac{1}{\tau_C} - \frac{4V_{tm}}{\omega_0 V_R T} \quad 0 \leq t \leq T/2
\]
(118)

\[
\frac{dx}{dt} + \frac{1}{\tau_C} x = \frac{1}{\tau_C} + \frac{4V_{tm}}{\omega_0 V_R T} \quad \frac{T}{2} \leq t \leq T
\]
(119)

DURING \(t_{\text{off}}\):

\[
\frac{d^2x}{dt^2} + 2h \frac{dx}{dt} + x = 1 + \frac{V_D}{V_R} - \frac{8hV_{tm}}{\omega_0 V_R T} \quad 0 \leq t \leq T/2
\]
(120)

\[
\frac{d^2x}{dt^2} + 2h \frac{dx}{dt} + x = 1 + \frac{V_D}{V_R} + \frac{8hV_{tm}}{\omega_0 V_R T} \quad \frac{T}{2} \leq t \leq T
\]
(121)

It can be seen that \(t\) has been eliminated from the right-hand sides of these equations so that they are constants. The elimination of \(t\) has made the equations autonomous so that they can be plotted on the phase plane as is done in the next section.

Plotting in the Phase Plane \(dx/dt\) Versus \(x\)

The first step in determining the phase plane trajectory is to determine the nature of (118) through (121) in the phase plane. Equations (118) and (119) are linear functions of \(dx/dt\) and \(x\). This means that the phase plane trajectory during \(t_{\text{on}}\) is made up of straight lines with slopes equal to \(-1/\tau_C\) and \(dx/dt\) intercepts equal to the right-hand sides of (118) and (119).

Equations (120) and (121) are second order differential equations in \(x\) whose solutions are damped sinusoidal waveforms.
which are represented in the phase plane by spirals with focuses located on the x-axis at values of x equal to the right-hand sides of (120) and (121).

It is concluded that there are four segments of the phase-plane trajectory. During $t_{on}$, there are two linear segments which are portions of the lines defined by (118) and (119). During $t_{off}$, there are two segments of spirals defined by (120) and (121). The positions of the two linear segments defined by (118) and (119) are always fixed being defined by their slopes $1/\tau_C$ and their $dx/dt$ intercepts which do not change. The positions of the two spiral segments defined by (120) and (121) are not fixed and depend on the phenomena which occur at switching. By reason of the hysteresis, it is known that turn-on occurs when $\xi = \delta_1$ and turn-off occurs when $\xi = -\delta_2$. These values of $\xi$ normalized with respect to $V_R$ are at turn-on

$$x = \lambda_1$$

and at turn-off

$$x = -\lambda_2$$

where

$$\lambda_1 = \delta_1/V_R$$

$$\lambda_2 = \delta_2/V_R$$

Since $\delta_1$, $\delta_2$, and $V_R$ are constants, (122) and (123) are vertical lines in the phase plane perpendicular to the x-axis and, because of the nature of hysteresis, are confined to the first
and third quadrants of the phase plane. These lines are called switching lines; and whenever the phase plane trajectory crosses one, switching must occur. For $x = \lambda_1$, turn-on occurs; and for $x = -\lambda_2$, turn-off occurs.

A representation of the phase-plane trajectory is shown in Fig. 24, and Fig. 25 contains sketches of the normalized waveforms for $v_t(\tau), v(\tau), x(\tau)$, and $dx(\tau)/d\tau$ where $\tau_{on} = \omega_0 t_{on}$ and $\tau_{off} = \omega_0 t_{off}$. The $dx/d\tau$ intercepts for practical circuit parameters have much larger numerical values than the extremities of the $x$-axis excursions. The magnitude of the output ripple voltage component of $v(t)$ is normally much smaller than the peak voltage $V_{tm}$ of the triangular waveform; therefore, the positive and negative $x$-axis extremities of the phase-plane trajectory should be approximately equal to the normalized peak-to-peak magnitude $2V_{tm}/V_R$ of $v_t(t)$. The values of $\delta_1$ and $\delta_2$ are usually much smaller than $V_{tm}$; therefore $\lambda_1$ and $\lambda_2$ for practical circuits are extremely close to the $dx/d\tau$ axis. These characteristics result in a phase-plane trajectory drawn to scale that is very tall and narrow. For example, the ratio of height to width for the case of the 2 kHz converter described in Appendix C is approximately 30 to one. For this reason, the $dx/d\tau$ axis in Fig. 24 is compressed and the values of $\lambda_1$ and $\lambda_2$ are shown to be larger than typical so that the behavior of the trajectory is clearly depicted.
n = 0,1,2,3,.....

Fig. 24. The phase plane trajectory.
Fig. 25. Normalized waveforms for $v_1(\tau), v(\tau), x(\tau)$ and $dx(\tau)/d\tau$. 

---

V(t), $\frac{V(t)}{V_R}$

$\frac{V_{tm}}{V_R}$

$\frac{V_{tm}}{V_R}$

$0$

$\frac{\omega_0 T}{2}$

$\omega T$

$\frac{3\omega_0 T}{2}$

$2\omega_0 T$

$\tau$

$\text{turn-on}$

$\text{turn-off}$

$\tau_{on}$

$\tau_{off}$

$\omega_0 (\frac{1}{2} - \alpha)$

$\omega_0 \alpha$

$\omega_0 \gamma$

$\omega_0 (\frac{1}{2} - \gamma)$

$\frac{\omega_0 T}{2}$

$\omega_0 T$

$\frac{3\omega_0 T}{2}$

$2\omega_0 T$

$x(\tau)$

$P_0$

$P_1$

$P_2, 3$

$P_4$

$P_5$

$P_6, 7$

$0$

$\frac{\omega_0 T}{2}$

$\omega_0 T$

$\frac{3\omega_0 T}{2}$

$2\omega_0 T$

$\text{dx}(\tau)/d\tau$

$P_0, 1$

$P_2$

$P_7$

$P_3$

$P_5$

$P_6$

$0$

$\frac{\omega_0 T}{2}$

$\omega_0 T$

$\frac{3\omega_0 T}{2}$

$2\omega_0 T$
The solid-line portions of the trajectory are determined by (118) through (121). The dotted-line portions are used to indicate jumps in the coordinates at times when switching occurs [26]. The magnitudes of these jumps are derived in the next section. These jumps represent changes in conditions which occur during real switching and must be included in the phase-plane trajectory when instantaneous switching is assumed. The alternative to assuming instantaneous switching is to assume a linear, exponential, or another appropriate function for the variable during switching. However, these assumptions increase the complexity of the expressions which must be manipulated in this example to such an extent that instantaneous switching is the preferable assumption.

A brief explanation of the trajectory is given now by taking one segment at a time, beginning at point $P_0$ which is at the time immediately before turn-on. At the intersection $P_0$ of the $\lambda_1$ line, turn-on occurs and a jump in the direction indicated occurs. The direction and magnitude of this jump are dependent on the circuit conditions just prior to turn-on and the value of $R_C$. During $\tau_{\text{off}}$, $i_S$ is greater than $i_L$ since $C$ is charging. During $\tau_{\text{on}}$, $i_S$ is zero and $C$ is discharging by $i_L$. This means that at turn-on the change in current $\Delta(i_S - i_L)$ in $R_C$ is negative so that a negative jump appears in the output voltage $v$. The magnitude of the jump is dependent on the value of $R_C$ and the jump $\Delta(i_S - i_L)$. From (100) and (108), the corresponding jump in $x$ at turn-on is positive as shown in Fig. 24 from $P_0$ to $P_1$. 
The dependence of the jump in $dx/d\tau$ on circuit conditions, such as load and source voltage, is such that it cannot be easily described in a general qualitative manner. The segment of the trajectory from $P_1$ to $P_2$ is defined by (119) and occurs during the first portion of $t_{on}$ called $\alpha$. Points $P_2$ and $P_3$ occur at the time $\omega_0 nT$ where $n = 0, 1, 2, 3, \ldots$ at which the slope of $v_t(\tau)$ changes instantaneously from a negative to a positive value. This instantaneous change in slope accounts for the negative jump in the $dx/d\tau$ direction while $x$ remains constant. The segment from $P_3$ to $P_4$ is defined by (118) and occurs during the second part of $t_{on}$ called $\omega_0 \gamma$. When the trajectory reaches the $-\lambda_2$ line at $P_4$, turn-off occurs. At turn-off, a jump to $P_5$ occurs in the direction indicated. At turn-off, the status of $C$ changes from discharging to charging which is opposite to the change at turn-on. As a result, there is an instantaneous increase in $\left[ i_S - i_L \right]$ through $R_C$ which causes an instantaneous increase in $v$. Therefore from (100) and (108), the jump in $x$ at turn-off is negative as shown in Fig. 24 from $P_4$ to $P_5$. As in the case of the jump at turn-on, the jump in $dx/d\tau$ at turn-off is dependent on circuit conditions and cannot easily be described in a general qualitative manner. The trajectory segment from $P_5$ to $P_6$ occurs during $t_{off}$ for the normalized time interval of $\omega_0 (T/2 - \gamma)$ and is defined by (120). Points $P_6$ and $P_7$ occur at $\omega_0 (2n + 1)T/2$ when the slope of $v_t(\tau)$ changes again this time from positive to negative so that there is a positive jump in $dx/d\tau$ while $x$ remains constant. The trajectory segment
from $P_7$ to $P_0$ occurs during $t_{off}$ for the time interval of $\omega_0(T/2-\alpha)$ and is defined by (121). This completes the motion about the phase-plane trajectory for one complete limit cycle.

The next section of this chapter applies the mathematical expressions for the segments to a point transformation, or sequence function, analysis of the limit cycle.

The Sequence Function Analysis of the Phase Plane Trajectory

This section describes the point transformation, or sequence function, analysis of the limit cycle trajectory in the phase plane given in Fig. 24. The analysis begins by giving the solutions to the differential equations for the various segments of the phase plane and by determining analytical expressions for the jumps at the switching points. These expressions are then used to derive the sequence function using a modified version of the method of point transformation in the phase plane.

A normal point transformation analysis begins with an assumed initial point such as $P_0(x_0, y_0)$ and transforms the coordinates of this point in sequence to each succeeding point around the limit cycle until the final point is reached. The expressions for the coordinates of the final point are then equated to $x_0$ and $y_0$, which must be the same point if a limit cycle exists, to determine the unknown parameters which in this case are the time intervals $\omega_0\alpha$ and $\omega_0\gamma$. The system being analyzed in this
chapter has certain properties which can be used to simplify the analysis. The nature of the switching which occurs between points $P_2$ and $P_3$ and points $P_6$ and $P_7$ requires that the $x$-coordinates of these points are equal, respectively, so that $x_2 = x_3$ and $x_6 = x_7$. In addition, the $x$ and $dx/dt$ coordinates of points on the segments between points $P_1$ and $P_2$ and between points $P_3$ and $P_4$ are linearly related by (119) and (118) respectively; and $x_0 = \lambda_1$ at $P_0$ and $x_4 = -\lambda_2$ at $P_4$. The jumps from $P_0$ to $P_1$ and from $P_4$ to $P_5$ are derived in Appendix B from linear differential circuit equations. For these reasons, expressions for $x_3$ and $x_6$ in terms of $\omega_0 \gamma$ are determined by point transformations from $P_4$; and expressions for $x_2$ and $x_7$ in terms of $\omega_0 \alpha$ are determined by point transformations from $P_0$. The expressions for $x_2$ and $x_3$ are equated and the expressions for $x_6$ and $x_7$ are equated to obtain the sequence functions determining the limit cycle. This procedure simplifies the mathematical manipulations required to obtain the sequence functions. The simplification is important in the analysis of this system since the expressions are complex.

The solutions of the differential equations. The general solution for (118) is

$$x(\tau) = \left(x_3 + \frac{\tau}{C}\right) e^{-\tau/\tau_C \cdot \tau_C} \tag{126}$$

$$y(\tau) = \left(-\frac{x_3}{C} + \frac{\tau}{\tau_C}\right) e^{-\tau/\tau_C} \tag{127}$$

and for (119) is
\[ x(\tau) = \left( x_1 - \tau C \xi \right) e^{-\tau/\tau C} + \tau C \xi \quad (128) \]

\[ y(\tau) = \left[ \frac{x_1}{\tau C} \right] e^{-\tau/\tau C} \quad (129) \]

where \( y(\tau) \) is used to represent \( \frac{dx}{d\tau} \), \( x_0 \) is the x-coordinate of \( P_0 \), \( x_3 \) is the x-coordinate of \( P_3 \).

\[ -\eta = \frac{1}{\tau C} - \frac{4V_{tm}}{\omega_0 VR T} \quad (130) \]

and

\[ \xi = \frac{1}{\tau C} + \frac{4V_{tm}}{\omega_0 VR T} \quad (131) \]

It can be seen that \(-\eta\) and \(\xi\) are the \( \frac{dx}{d\tau} \) axis intercepts of (118) and (119) respectively in the phase plane.

The general solution for (120) is

\[ x(\tau) = e^{-h\tau} \left[ \frac{y_5 + h(x_5 - \delta)}{k} \sin k\tau + \left( x_5 - \delta \right) \cos k\tau \right] + \delta \quad (132) \]

\[ y(\tau) = e^{-h\tau} \left[ \frac{-hy_5 - x_5 + \delta}{k} \sin k\tau + y_5 \cos k\tau \right] \quad (133) \]

where

\[ \delta = 1 + \frac{V_D}{V_R} - \frac{8h V_{tm}}{\omega_0 VR T} \quad (134) \]

and

\[ k = (1 - h^2)^{1/2} \quad (135) \]

The general solution for (121) is
\[ x(\tau) = e^{-h\tau} \left( \frac{y_{\gamma} + h(x_{\gamma} - \rho)}{k} \sin k\tau + (x_{\gamma} - \rho)\cos k\tau \right) + \rho \]  

\[ y(\tau) = e^{-h\tau} \left( \frac{-hy_{\gamma} - x_{\gamma} + \rho}{k} \sin k\tau + y_{\gamma} \cos k\tau \right) \]  

where

\[ \rho = 1 + \frac{V_D}{V_R} + \frac{8hV_{tm}}{\omega_0 V_R T} \]  

Equations (120) and (121) are represented as spirals in the phase plane with focuses on the x axis at values equal to the right-hand sides of (120) and (121). It should be observed that the parameters \( \delta \) and \( \rho \) are the x values of the focuses and that these focuses are located symmetrically on either side of \( 1 + V_D/V_R \).

Determination of expressions for the jumps. The jumps which occur from \( P_2 \) to \( P_3 \) and \( P_6 \) to \( P_7 \) are a result of the instantaneous change in the slope of \( v(t) \) at \( t = nT \) and \( t = (2n + 1)T/2 \). The slope of \( v(t) \) is

\[ \frac{dv_{t}(t)}{dt} = \pm \frac{4V_{tm}}{T} \]  

which is positive when \( nT < t < (2n + 1)T/2 \) and negative when \( (2n + 1)T/2 < t < nT \). The normalized slope is

\[ \frac{dv_{t}(\tau)}{d\tau} = \pm \frac{4V_{tm}}{\omega_0 V_R T} \]  

which is positive when \( \omega_0 nT < \tau < \omega_0 (2n + 1)T/2 \) and negative when \( \omega_0 (2n + 1)T/2 < \tau < \omega_0 nT \).
Solving (104) for $\xi$ and substituting into (108) yields

$$x = \frac{\xi}{V} = \frac{V_R - v_{t}(t) - v(t)}{V_R}, \quad (141)$$

Differentiating with respect to $\tau$ yields

$$\frac{dx}{d\tau} = -\frac{dv_{t}(\tau)}{d\tau} + \frac{dv(\tau)}{d\tau} \quad (142)$$

which is the ordinate coordinate in the phase plane and is a function of the normalized derivatives of the triangular waveform $v_{t}(t)$ and the output voltage $v(t)$. Whenever the derivative of $v_{t}(\tau)$ switches instantaneously, the converter is between turn-on and turn-off, or vice versa. At these times, $v(\tau)$ is continuous and $dv(\tau)/d\tau$ cannot change instantaneously. Since $dv_{t}(\tau)/d\tau$ changes from positive to negative or vice versa at the instant of the jump, the magnitude of the jump in the phase plane is twice the magnitude of $dv_{t}(\tau)/d\tau$ which is $8v_{tm}/\omega V_R$ as shown in Fig. 24. Since (142) has a negative sign on the right-hand side, the jump is negative from $P_2$ to $P_3$ at $\tau = \omega_n T$ and positive from $P_6$ to $P_7$ at $\tau = \omega_n (2n + 1) T/2$ when $v_{t}(\tau)$ is as shown in Fig. 25(a). The jumps from $P_2$ to $P_3$ and $P_6$ to $P_7$ have been determined. Now the jumps from $P_0$ to $P_1$ at turn-on and from $P_4$ to $P_5$ at turn-off will be determined.

Fig. 26(a) shows $v_{t}(t)$ and the location of typical turn-on and turn-off points. Fig. 26(b) shows the approximate output voltage waveform $v(t)$ for the same time periods and at a scale
Fig. 26. Converter voltage waveforms (a) $v_t(t)$, (b) $v(t)$, (c) $v(t)$ during $t_{on}$, (d) $v(t)$ during $t_{off}$.
five to six times greater than \( v_0(t) \) in Fig. 26(a). Equation (103) yields a simple exponentially decaying voltage for \( v(t) \) during \( t_{on} \). This voltage is shown in Fig. 26(c) and the segment marked heavy is the portion of the whole that is typically \( v(t) \) during \( t_{on} \) in Fig. 26(b). Equation (103) results in an exponentially damped sinusoidal general solution for \( v(t) \) as shown in Fig. 26(d). The portion marked heavy is that part that is typically \( v(t) \) during \( t_{off} \). These segments of \( v(t) \) during \( t_{on} \) and \( t_{off} \) are depicted by solid lines in Fig. 26(b). The jumps in \( v(t) \) at switching, depicted by dotted lines, result as the current through \( R_C \) changes directions. The following analysis concerns the current \( i_S - i_L \) through \( R_C \) and \( C \) shown in Fig. 22. During \( t_{on} \), \( i_S - i_L = -i_L < 0 \); and during \( t_{off} \), \( i_S - i_L > 0 \). Therefore at turn-on, \( \Delta (i_S - i_L) < 0 \); and the jump \( \Delta v \) is negative. Similarly at turn-off, \( \Delta (i_S - i_L) > 0 \); and the jump \( \Delta v \) at turn-off is positive. Since

\[
x = \frac{\xi}{V_R} = \frac{V_R - v - v_t}{V_R}
\]

the jumps in \( v \) appear in \( x \) with opposite signs and reduced amplitude. These jumps can be seen in Fig. 24 where at turn-on from \( P_0 \) to \( P_1 \) the jump in \( x \) is positive and at turn-off from \( P_4 \) to \( P_5 \) the jump in \( x \) is negative. The directions of the jumps in \( dx/dt \) can be explained qualitatively by examining Fig. 26(b). At turn-on, \( dv/dt \) changes very little as shown in Fig. 26(b); therefore, at turn-on in Fig. 24, the jump from \( P_0 \) to \( P_1 \) is
represented as being approximately zero in \( \frac{dx}{dt} \). The slope \( \frac{dv}{dt} \) is more negative at the end of \( t_{on} \) in Fig. 26(b) than it is at the beginning of \( t_{off} \). This results in a positive jump in \( \frac{dv}{dt} \) and a negative jump in \( \frac{dx}{dt} \) as shown in Figs. 24 and 25(d). There is no reason, however, that under the right circuit conditions, the jump in \( \frac{dx}{dt} \) at turn-off cannot be positive. The quantitative expressions for the jumps in \( x \) and \( \frac{dx}{dt} \) at turn-on and turn-off are derived in Appendix B. At turn-on, from (B.55) and (B.35)

\[
\Delta x_0 = \frac{-\Delta v_0}{V_R} = \left[ 1 - \frac{V_{tm}(4\gamma - 1)}{V_R} \right] e^{\left( \frac{\alpha + \gamma}{R + RC} \right) C} - \left[ 1 - \frac{V_{tm}(4\alpha - 1)}{V_R} \right] - \lambda_1
\]

(144)

and from (B.59) and (B.52)

\[
\Delta y_0 = \frac{dx}{dt} \bigg|_0 = - \frac{1}{\omega V_R} \Delta \frac{dv}{dt} \bigg|_0
\]

\[
= - \frac{1}{\omega V_R} \left\{ \frac{RR_C}{L_s[R + RC]} \left( V_D + v_{o-} \right) + \left[ \frac{1}{RC C} - \frac{R_s}{L_s} - \frac{1}{(R + RC) C} \right] \Delta v_0 \right\}
\]

(145)

where \( v_{o-} \) is given in (B.30). The quantitative expressions for the jumps at turn-off are from (B.56) and (B.51)

\[
\Delta x_f = -\Delta v_f / V_R = -\frac{\Delta v_0}{V_R} e^{-\frac{R_{p \, on}}{L_s}} + \frac{RR_C}{R + RC} \frac{N_{p \, CE}}{N_s} \frac{E - V_{CE}}{V_R \, p} \left[ 1 - e^{-\frac{R_{p \, on}}{L_s}} \right]
\]

(146)

where \( \Delta v_0 \) is given in (B.35), and from (B.60) and (B.53)
\[ \Delta y_f = \Delta \frac{dx}{dt} \bigg|_f = \frac{-1}{\omega_0 V_R} \Delta \frac{dv}{dt} \bigg|_f \]

\[ = \frac{-1}{\omega_0 V_R} \left\{ -\frac{R R_C}{(R + R_C) L_s} (v_D + v_{f+}) + \left[ \frac{R^2}{(R + R_C)^2 C} - \frac{R R_C R_s}{(R + R_C) L_s} \right] i_{S_0-} \right\} \]

\[ - \frac{N_p}{N_s} \frac{E-V_{CE}}{R_p} \left[ e^{-\frac{R_p \text{ ton}}{L_p}} + \frac{N_p}{N_s} \frac{E-V_{CE}}{R_p} \right] \] (147)

where \( \Delta (dv/dt) \bigg|_f \) is given in (B.53), \( v_{f+} \) is given in (B.15), and \( i_{S_0-} \) is given in (B.38). The four jumps in the normalized phase plane shown in Fig. 24 have now been defined. The final derivation of the sequence function expressions is given in the next section.

**The sequence functions.** This section utilizes the solutions of the differential equations and the expressions for the jumps given in the previous two sections to derive the sequence functions for the normalized phase plane in Fig. 24. From all the circuit parameters, there are two that are undetermined; \( \alpha \) and \( \gamma \). Two unknown circuit parameters require two independent relations to adequately define the behavior of the circuit. These independent relations are called the sequence functions because they are normally derived by transforming in sequence the coordinates of the points of the phase-plane trajectory such as shown in Fig. 24. The transformations begin at an assumed initial point and continue to the final point of one cycle of operation for the circuit. If it is assumed that the circuit has
reached steady-state conditions, the final point should be the same as the initial point; therefore the expressions for the coordinates of these two points can be equated to obtain two sequence functions defining the nature of the circuit.

The nature of the phase plane in Fig. 24 allows for the sequence functions to be determined in a simpler manner. The x-coordinates of points $P_0$ and $P_4$ are always fixed at $\lambda_1$ and $\lambda_2$, respectively. In addition, the x-coordinates of $P_2$ and $P_3$ and the x-coordinates of $P_6$ and $P_7$ must be equal due to the assumed instantaneous switching as explained previously. For these reasons, $P_2$ and $P_7$ can be expressed in terms of $\omega_0\alpha$ and the coordinates of $P_0$; and $P_3$ and $P_4$ can be expressed in terms of $\omega_0\gamma$ and the coordinates of $P_4$. The sequence functions can then be found by equating $x_2$ and $x_3$ and by equating $x_6$ and $x_7$. This procedure is performed in the remainder of this section.

The first step in finding the first sequence function is to find $x_2$ starting with

$$x_0 = \lambda_1.$$  \hfill (148)

The transformation from $P_0$ to $P_1$ yields

$$x_1 = x_0 + \Delta x_0 = \lambda_1 + \Delta x_0$$  \hfill (149)

where $\Delta x_0$ is the jump in $x$ at turn-on given by (144). During the normalized time period $\omega_0\alpha$, $x$ is governed by (119) whose solution is (128). Substituting $\tau = \omega_0\alpha$ and $x_1$ from (149) into (128) yields
The next step is to find an expression for $x_3$. The expression for $x$ during the normalized time period $\omega_0 \gamma$ from $P_3$ to $P_4$ is given in (126). Substituting $\tau = \omega_0 \gamma$ and $x_4 = -\lambda_2$ into (126) and solving for $x_3$ yields

$$x_3 = \left( \tau C - \lambda_2 \right) e^{\frac{-\omega_0 \alpha}{\tau C}} + \tau C \xi \ . \ (151)$$

Since the $x$-coordinate does not change from $P_2$ to $P_3$, the first sequence function can be obtained by equating $x_2$ and $x_3$ from (150) and (151), respectively, yielding

$$x_2 = \left( \lambda_1 + \Delta x_0 - \tau C \xi \right) e^{\frac{-\omega_0 \alpha}{\tau C}} + \tau C \xi \ . \ (152)$$

The first step in finding the second sequence function is to find $x_6$. To do this the coordinates of $P_4$ must be transformed to $P_6$. The coordinates of $P_4$ in Fig. 24 are

$$x_4 = -\lambda_2 \quad \text{ (153)}$$

$$y_4 = \frac{\lambda_2}{\tau C} - \eta \quad \text{ (154)}$$

where (154) is derived by substituting $x_4$ from (153) and $-\eta$ from (130) into (118) and solving for $dx_4/d\tau = v_4$. The jump from $P_4$ to $P_5$ at turn-off is defined by (B.56) and (B.60) in
the x and y-coordinate directions, respectively. Therefore, the coordinates of $P_5$ are

$$x_5 = x_4 + \Delta x_f = -\lambda_2 + \Delta x_f$$

$$y_5 = y_4 + \Delta y_f = \frac{\lambda_2}{\tau} - \eta + \Delta y_f$$  \hspace{1cm} (155)

During the time period from $P_5$ to $P_6$, the x and y-coordinates of the phase plane are governed by differential equation (120). The solution to (120) is given in (132) and (133). Substituting $x_5$ from (155), $y_5$ from (156), and $\tau = \omega_0 (T/2 - \gamma) = \beta$ into (132) yields

$$x_6 = e^{-h\beta} \left[ \frac{\lambda_2 - \eta + \Delta y_f + h \left( -\lambda_2 + \Delta x_f - \delta \right)}{k} \right] \sin k\beta$$

$$+ \left( -\lambda_2 + \Delta x_f - \delta \right) \cos k\beta + \delta$$  \hspace{1cm} (157)

Now an expression for $x_\gamma$ must be found. The differential equation defining the phase-plane trajectory from $P_\gamma$ to $P_0$ during $\omega_0 (T/2 - \alpha)$ is (121). The solution to (121) is given in (136) and (137). Substituting $\tau = \omega_0 (T/2 - \alpha) = \theta$ yields

$$x_\gamma = e^{-h\theta} \left[ \frac{y_\gamma + h (x_\gamma - \rho)}{k} \sin k\theta + \left( x_\gamma - \rho \right) \cos k\theta \right] + \rho$$

$$y_\gamma = e^{-h\theta} \left( \frac{-hy_\gamma - x_\gamma + \rho}{k} \sin k\theta + y_\gamma \cos k\theta \right)$$  \hspace{1cm} (158)

Substituting
\[ y_0 = y_1 - \Delta y_0 = \xi - \frac{\lambda_1 + \Delta x_0}{\tau_C} - \Delta y_0 \quad (160) \]

Into (159) and solving for \( y_\gamma \). Then substituting \( y_\gamma \) and \( x_\delta = \lambda_1 \) into (158) and solving for \( x_\gamma \) yields

\[
x_\gamma = \left[ \frac{\left( \xi - \frac{\lambda_1 + \Delta x_0}{\tau_C} - \Delta y_0 \right) + \left( h - \frac{k}{\tan k\theta} \right) \left( \lambda_1 - \rho \right)}{-k\left( \sin k\theta + \frac{\cos^2 k\theta}{\sin k\theta} \right)} \right] e^{h\theta + \rho} . \quad (161)\]

Equating \( x_\delta \) in (157) and \( x_\gamma \) in (161) yields the second sequence function as

\[
\frac{\lambda_2}{\tau_C} - \eta + \Delta y_f + h\left( -\lambda_2 + \Delta x_f - \delta \right) \sin k\beta + \\
\left( -\lambda_2 + \Delta x_f - \delta \right) \cos k\beta \] e^{-h\beta} + \delta = \\
\left[ \frac{\xi - \frac{\lambda_1 + \Delta x_0}{\tau_C} - \Delta y_0}{-k\left( \sin k\theta + \frac{\cos^2 k\theta}{\sin k\theta} \right)} \right] e^{h\theta + \rho} . \quad (162)\]

Conclusion

The regulated switching converter analyzed in this chapter is of the type discussed in Chapter III. The converter uses
a triangular driving function to obtain regulation at a constant
switching frequency. The first section of this chapter contains
the development of the system model for the converter as shown
in Fig. 22 and explains the means by which regulation is ob-
tained. In the next section, the total period is divided into
segments during which the system is linear; and normalized lin-
ear differential equations are derived to describe the circuit
operation during each segment. In the derivation, it is neces-
sary to assume that the amplitude $V_{tm}$ of the triangular waveform
is negligible compared to the reference voltage $V_R$ and that the
derivative of $v(t)$ is constant for half cycle periods. These
equations are then used to plot the phase plane shown in Fig. 24
and to sketch the waveforms in Fig. 25 where switching is assumed
to occur instantaneously creating jumps whenever power transistor
Q switches or the derivative of $v(t)$ changes sign. In the final
section, solutions are obtained for the phase plane equations and
expressions for the jumps which are used to derive the sequence
functions given in (152) and (162) which completely describe the
steady-state conditions of the converter by defining the unknown
time periods $\alpha$ and $\gamma$. Therefore, by assuming that $V_{tm} \ll V_R$ and
that $dv(t)/dt = \text{constant}$, equations describing the steady-state
operation of a nonlinear system with a triangular driving func-
tion have been derived.

Since $\alpha$ and $\gamma$ are defined, numerical values can be as-
signed to the circuit parameters; and they can be substituted
into (152) and (162) to find $\alpha$ and $\gamma$. Since $t_{on} = \alpha + \gamma$ and $T$
is fixed and known, the duty cycle $\frac{t_{on}}{T}$ can then be computed from the values of $\alpha$ and $\gamma$ determined from the sequence functions.

Appendix C contains an experimental verification of the theoretical limit cycle computed using the equations of a converter operating at a frequency of 2 kHz. Photographs of experimental waveforms and an experimental limit cycle are given. The experimental limit cycle is normalized and plotted on the same set of axes with the theoretical limit cycle. These two limit cycles are shown to compare closely. Also, experimental circuit conditions are substituted into (152) and (162) and the accuracy of the sequence function is verified.

The sequence functions given in (152) and (162) are highly complex transcendental equations. Before this analysis can be practically expanded to be quantitative, the values of the circuit parameters must be known and approximations must be sought which simplify the complexity of the sequence functions so that they can be solved for $\alpha$ and $\gamma$. It is also possible, knowing values of the circuit parameters, to use a digital computer to calculate and plot characteristic curves involving $\alpha$ and $\gamma$. These analyses are suggested as extensions of the work presented in this chapter whose scope is to show that regulated dc to dc energy storage converters using triangular driving functions can be analyzed using piecewise linear analysis in the phase plane.
Chapter V
CONCLUSION

This dissertation presents techniques for improving spacecraft power conditioning systems and for meeting new requirements for these systems. All spacecraft equipment generally needs to have minimum size and weight, maximum efficiency, and maximum reliability. Research and development for spacecraft equipment must continually strive to improve these characteristics. In addition, space technology is advancing continuously. Each new spacecraft has more sophisticated equipment designed to perform more sophisticated experiments. This sophistication and advanced technology is transferred down to the most basic electronic system on the spacecraft. The one without which none of the other systems can function is the power conditioning system. Chapter I begins the dissertation by providing the background and scope of the research to be presented.

In order to achieve low output ripple voltage for dc to dc converters operating at typical switching frequencies of 2 to 50 kHz, the designer is required to use the large-microfarad electrolytic capacitors. Unfortunately these capacitors have large size and weight and are the least reliable component when...
compared to the other components used on spacecraft. Chapter II discusses an active ripple filter designed to replace the pi-network, L-section, and pure capacitance output filters for dc to dc converters and to reduce the amount of capacitance required so that more reliable capacitors can be used. The filter schematic diagram and the circuit design are given first. The filter consists basically of a mutual inductor and an operational amplifier. The mutual inductor secondary winding is in series with the main power path and permits dc current to flow through the filter with little or no dissipation. The operational amplifier is used to amplify the ripple voltage sensed at the input to the filter. This signal is conditioned by the amplifier and impressed on the primary so that the secondary voltage is identical in magnitude and phase to the ripple voltage at the input, thus cancelling the ripple. The transfer function for this filter is derived next and is shown to be equivalent over the frequency range of interest to the transfer function for an L-section filter. The result of this research is an active ripple filter with a nondissipative dc path which filters ripple voltages from 2 to 50 kHz. The ripple filter requires a total of only 4.87 μF which can be obtained using the more reliable ceramic and mica capacitors. The equivalent L-section filter using the same inductance as the secondary inductance of the active filter requires 408.5 μF to achieve the same attenuation characteristic. Thus the active filter essentially eliminates the need for the less reliable electrolytic filter capacitors by
requiring approximately 84 times less capacitance than an equivalent L-section filter.

Chapter III discusses the design of a 1 MHz dc to dc switching converter which was designed to provide 20 watts of output power at a regulated output voltage of 28 volts operating from a dc source voltage varying from 21 to 31 volts. Operation at 1 MHz is desirable for several reasons. The high frequency will result in a reduction in the size and weight of the transformers, inductors, and capacitors. The amount of capacitance required in the output filter is greatly reduced thereby permitting the use of the more reliable mica and ceramic capacitors as was done with the ripple filter in Chapter II. A more specific reason is that experiments are planned for measuring electromagnetic radiation in outer space in the frequency range from 0.2 to 100 kHz which require the design of the system to use switching frequencies of at least 1 MHz in order to avoid interference between the switching in the power conditioning system and the experiments. The first part of Chapter III gives the background and reasons for choosing the energy-storage type of circuit with electrical isolation. In addition, the choice of ferrite magnetic material and high-speed transistors, diodes, and integrated circuits is discussed. Considerations to be used in designing power switching circuits at high frequencies are then given before the actual converter circuit design is discussed. Experimental results in the form of the regulation and efficiency characteristics and oscillograms of the switching waveforms are
given. The converter operates over the full range of source and load conditions with the regulation better than 1.5 percent of the mean output voltage and a maximum efficiency of 79 percent. The majority of the power dissipation was found to be in the power switching transistor and diode. The feasibility of such circuits to operate in the 1 MHz frequency range has been demonstrated by the circuit discussed in Chapter III. Before such a circuit can be considered for practical application, additional consideration needs to be given to providing a reliable means for self-starting and to providing a means to return the circuit to normal regulating operation if a disturbance large enough to cause the circuit to leave the normal operating range is encountered.

Regulated inverters and converters constitute a highly complex group of closed-loop nonlinear feedback control systems. For this reason, there has been only a small amount of analytical work on these circuits. The analysis performed in Chapter IV illustrates the complexity that the expressions can attain when analysis is attempted. Chapter IV contains an analysis of the type of circuit presented in Chapter III. Because operation at frequencies of 1 MHz require analytical work to consider stray parameters not considered in this analysis, the results of Chapter IV may not be directly applicable to the 1 MHz converter of Chapter III. For this reason, an experimental verification of the analysis is made using a converter designed to operate at a switching frequency of 2 kHz. Both circuits use a constant
frequency triangular waveform as a driving function to generate the duty cycle needed for regulation. The analysis uses the method of point transformations in the phase plane. The system differential equations contain time dependent functions resulting from the triangular waveform driving function which make these equations nonautonomous. However, approximations can be made which make the differential equations autonomous. These approximations are that the magnitude of the triangular waveform is much smaller than the value of the reference voltage and that the slope, or time derivative, of the triangular waveform is constant.

Having made these approximations, system equations are developed and normalized. These normalized differential equations are used to sketch the phase plane trajectory in the normalized phase plane. To minimize the complexity of the analysis, instantaneous switching is assumed for the power transistor at turn-on and turn-off. Additionally, it is assumed that the slope of the triangular waveform changes instantaneously. Since instantaneous switching is assumed for all of these cases, there are four jumps in the phase plane represented by dotted lines. Expressions for these jumps are derived and solutions for the differential equations describing the linear segments of the phase plane trajectory are obtained. The validity of these equations is verified experimentally by comparing experimental and theoretical limit cycles.
These equations are also used to make point transformations around an assumed limit cycle to obtain two equations called the sequence functions. The sequence functions define the relationship between the known circuit parameters and the two unknown time parameters of the total period of the full cycle which have to be known to define the limit cycle. The accuracy of the equations defining the sequence functions is verified experimentally using the circuit conditions of the 2 kHz converter. The sequence functions can be used to fully define the nature of the oscillations for a set of circuit parameters. Therefore, the original objective, to accomplish a nonlinear analysis of a regulated dc to dc closed-loop switching converter with isolation which utilizes a triangular waveform driving function, has been accomplished.
Appendix A
DERIVATION OF EQUATIONS (102) AND (103)

Equation (102) for the time period \( t_{on} \). Since \( i_s = 0 \) during \( t_{on} \), the voltages summed around the \( i_L \) current loop in Fig. 22 yield

\[
\frac{1}{C} \int i_L \, dt + \left( R + R_C \right) i_L = 0 . \tag{A.1}
\]

When differentiated with respect to \( t \), (A.1) becomes

\[
\frac{di_L}{dt} + \frac{1}{(R + R_C)C} i_L = 0 . \tag{A.2}
\]

(A.2) is multiplied by \( R \) and \( Ri_L = v \) is substituted to obtain

\[
\frac{dv}{dt} + \frac{1}{(R + R_C)C} v = 0 \tag{A.3}
\]

which is the differential equation describing the variation of \( v \) during \( t_{on} \) and is identical to (102).

Equation (103) for the time period \( t_{off} \). During \( t_{off} \), \( i_p = 0 \) and \( i_s \) and \( i_L \) are nonzero. This means that there are two current loops with a common branch yielding two linear first order differential equations which can be combined to obtain one second order differential equation in \( i_L \) or \( v = Ri_L \).

(159)
The sum of the voltages around the $i_S$ current loop in Fig. 22 during $t_{off}$ can be written as

\[ L_s \frac{di_s}{dt} + R_s i_s + V_D + v = 0 \quad \text{(A.4)} \]

Dividing (A.4) by $L_s$ yields the first differential equation

\[ \frac{di_s}{dt} + \frac{R_s}{L_s} i_s = \frac{-v - V_D}{L_s} \quad \text{(A.5)} \]

The sum of the voltages around the $i_L$ current loop in Fig. 22 is

\[ \frac{1}{C} \int \left( i_L - i_S \right) dt + R_C \left( i_L - i_S \right) + R_i L = 0 \quad \text{(A.6)} \]

Differentiating (A.6) and letting $R_i L = v$ yields the second differential equation

\[ \frac{di_s}{dt} + \frac{1}{R_C C} i_s = \frac{R + R_C}{R R_C} \frac{dv}{dt} + \frac{1}{R R_C C} v \quad \text{(A.7)} \]

Equation (A.7) is solved for $i_s$ to obtain

\[ i_s = \frac{\left( R + R_C \right) C}{R} \frac{dv}{dt} + \frac{1}{R} v - R_C C \frac{di_s}{dt} \quad \text{(A.8)} \]

Equation (A.5) is solved for $di_s/dt$ to obtain

\[ \frac{di_s}{dt} = -\frac{1}{L_s} \left[ R_s i_s + V_D + v \right] \quad \text{(A.9)} \]

Equation (A.9) is substituted in (A.8) which is then solved for $i_s$ to obtain
\[ i_S = \frac{1}{\frac{1}{C} - \frac{R_C R_S}{L_S}} \left[ \frac{R + R_C}{R} \frac{dv}{dt} + \left( \frac{1}{R C + \frac{R_C}{L_S}} \right) v + \frac{R_C}{L_S} V_D \right] \quad (A.10) \]

Equation (A.10) is differentiated to obtain

\[ \frac{di_S}{dt} = \frac{1}{\frac{1}{C} - \frac{R_C R_S}{L_S}} \left[ \frac{R + R_C}{R} \frac{d^2v}{dt^2} + \left( \frac{1}{R C + \frac{R_C}{L_S}} \right) \frac{dv}{dt} \right] \quad (A.11) \]

Equations (A.10) and (A.11) are now substituted into (A.5) to obtain

\[ \frac{d^2v}{dt^2} + \left[ \frac{1}{(R + R_C) C} + \frac{R R_C}{(R + R_C) L_S} + \frac{R_S}{L_S} \right] \frac{dv}{dt} + \left[ \frac{R + R_S}{(R + R_C) L_S C} \right] v \]

\[ = -\frac{R}{(R + R_C) L_S C} V_D \quad (A.12) \]

Equation (A.12) can be simplified by assuming that \( R_C \ll R \) and \( R_S \ll R \) to obtain

\[ \frac{d^2v}{dt^2} + \left( \frac{R_C + R_S}{R C + \frac{R_C}{L_S}} \right) \frac{dv}{dt} + \frac{1}{L_S C} v = -\frac{V_D}{L_S C} \quad (A.13) \]

which is the linear second order differential equation describing \( v \) during \( t_{\text{off}} \) and is identical to (103).
Appendix B
DERIVATION OF THE EXPRESSIONS FOR THE JUMPS
AT TURN-ON AND TURN-OFF IN FIG. 24

Using unnormalized variables the error voltage is

\[ \varepsilon(t) = V_R - v_t(t) - v(t) \]  \hspace{1cm} (B.1)

Since switching at turn-on and turn-off occurs instantaneously, \( v_t(t) \) is constant during switching and

\[ \Delta \varepsilon = - \Delta v \]  \hspace{1cm} (B.2)

Differentiating (B.1) yields

\[ \frac{d\varepsilon(t)}{dt} = - \frac{dv_t(t)}{dt} - \frac{dv(t)}{dt} \]  \hspace{1cm} (B.3)

Since \( dv_t(t)/dt \) is constant during switching

\[ \Delta \frac{d\varepsilon}{dt} = - \Delta \frac{dv}{dt} \]  \hspace{1cm} (B.4)

For these reasons, this appendix derives the end points of the jumps in terms of \( v(t) \) and \( dv(t)/dt \).

**Turn-off.** During \( t_{on} = \alpha + \gamma \), diode D is reverse-biased and the secondary circuit of Fig. 22 is equivalent to Fig. B.1(a).
Fig. B.1. The secondary circuits of Fig. 22 during (a) \( t_{on} \) and (b) \( t_{off} \).
The output voltage $v$ is defined by

$$v(t) = V_R - v(t) - \varepsilon(t) \quad \text{(B.5)}$$

At a time immediately prior to turn-off, at $P_4$

$$\varepsilon(\gamma) = -\delta_2 \quad \text{(B.6)}$$

$$v_t(\gamma) = V_{tm}\left(\frac{4\gamma}{T} - 1\right) \quad \text{(B.7)}$$

Using the subscript $f$- to denote the conditions at $P_4$ in Fig. 24, $v(t)$ at $P_4$ is derived from (B.5) substituting (B.6) and (B.7) as

$$v_{f-} = V_R - V_{tm}\left(\frac{4\gamma}{T} - 1\right) + \delta_2 \quad \text{(B.8)}$$

The differential equation for $v(t)$ in Fig. B.1(a) during $t_{on}$ is derived as

$$\frac{dv(t)}{dt} + \frac{1}{(R + R_C)C} v(t) = 0 \quad \text{(B.9)}$$

Substituting $v_{f-}$ from (B.8) for $v(t)$ in (B.9) yields

$$\left.\frac{dv}{dt}\right|_{f-} = -\frac{1}{(R + R_C)C} \left[V_R - V_{tm}\left(\frac{4\gamma}{T} - 1\right) + \delta_2\right] \quad \text{(B.10)}$$

During $t_{off} = T - \alpha - \gamma$, diode D is forward-biased and the secondary circuit of Fig. 22 is equivalent to Fig. B.1(b).

The instantaneous addition of $i_s$, which normally is greater than $i_L$, at turn-off causes a positive jump as a result of the instantaneous change $\Delta i_s$ in the current through $R_C$. Using
the subscript \( f^+ \) to denote conditions at \( P_s \), \( i_L \) at \( P_s \) is
\[
i_{Lf^+} = v_{f^+}/R.
\]  \hspace{1cm} (B.11)

Also, \( v(t) \) during \( t_{off} \) is
\[
v(t) = R_C \left[ i_S(t) - i_L(t) \right] + v_C(t).
\]  \hspace{1cm} (B.12)

Therefore \( v_{f^+} \) just after turn-off can be expressed as
\[
v_{f^+} = R_C \left( i_{sf^+} - i_{Lf^+} \right) + v_{Cf}.
\]  \hspace{1cm} (B.13)

Where \( v_{Cf} \) is the capacitor voltage during turn-off. Since switching is instantaneous \( v_{Cf} \) is constant from \( P_s \) to \( P_s \) and is expressed by
\[
v_{Cf} = \frac{R + R_C}{R} v_{f^-} = \frac{R + R_C}{R} \left[ v_R - v_{tm} \left( \frac{4\gamma}{T} - 1 \right) + \delta_2 \right].
\]  \hspace{1cm} (B.14)

Substituting (B.11) and (B.14) into (B.13) yields
\[
v_{f^+} = \frac{R}{R + R_C} \left( R_C i_{sf^+} + v_{Cf} \right) = \frac{R R_C}{R + R_C} i_{sf^+} + v_{f^-}.
\]  \hspace{1cm} (B.15)

Taking the derivative of (B.12) evaluated just after turn-off yields
\[
\left. \frac{dv}{dt} \right|_{f^+} = R_C \left( \left. \frac{di_S}{dt} \right|_{f^+} - \left. \frac{di_L}{dt} \right|_{f^+} \right) + \left. \frac{dv_C}{dt} \right|_{f^+}.
\]  \hspace{1cm} (B.16)

During \( t_{off} \), \( v_C(t) \) is expressed by
\[
v_C(t) = \frac{1}{C} \int \left[ i_S(t) - i_L(t) \right] dt + \frac{Q_0}{C}.
\]  \hspace{1cm} (B.17)
where \( Q_0 \) is the charge on \( C \) at the beginning of \( t_{off} \). Substituting \( i_L(t) = v(t)/R \) in (B.17) and taking the derivative of (B.17) evaluated just after turn-off yields

\[
\left. \frac{dv}{dt} \right|_{f^+} = \frac{1}{C} \left( i_{Sf^+} - \frac{1}{R} v_{f^+} \right) .
\]  

(B.18)

Substituting \( v_{f^+} \) in (B.15) for \( v \) in (B.18) yields

\[
\left. \frac{dv}{dt} \right|_{f^+} = \frac{1}{C} \left( \frac{R}{R + R_C} i_{Sf^+} - \frac{v_f^-}{R} \right) .
\]  

(B.19)

Taking the derivative of \( i_L(t) = v(t)/R \) evaluated just after turn-off yields

\[
\left. \frac{di}{dt} \right|_{f^+} = \frac{1}{R} \left. \frac{dv}{dt} \right|_{f^+} .
\]  

(B.20)

Substituting (B.19) and (B.20) into (B.16) and rearranging yields

\[
\left. \frac{dv}{dt} \right|_{f^+} = \frac{RR_C}{R + R_C} \left. \frac{di}{dt} \right|_{f^+} + \frac{R^2 i_{Sf^+}}{\left( R + R_C \right)^2 C} - \frac{v_f^-}{\left( R + R_C \right) C} .
\]  

(B.21)

The jump in \( v(t) \) at turn-off is

\[
\Delta v_f = v_{f^+} - v_{f^-} = \frac{RR_C}{R + R_C} i_{Sf^+}
\]  

(B.22)

where \( v_{f^+} \) and \( v_{f^-} \) are defined by (B.15) and (B.8), respectively.

The jump in \( dv(t)/dt \) at turn-off is

\[
\Delta \left. \frac{dv}{dt} \right|_f = \left. \frac{dv}{dt} \right|_{f^+} - \left. \frac{dv}{dt} \right|_{f^-} = \frac{RR_C}{R + R_C} \left. \frac{di}{dt} \right|_{f^+} + \frac{R^2 i_{Sf}}{\left( R + R_C \right)^2 C} .
\]  

(B.23)
The jumps in (B.22) and (B.23) are expressed in terms of the circuit parameters and $i_{Sf^+}$ and $[di_S/dt]_{f^+}$.

**Turn-on.** The secondary circuit acts as a simple RC circuit discharging $C$ during $t_{on}$. So that the voltage $v_{Cf}$ on $C$ at turn-off can be related to the voltage $v_{C_0}$ on $C$ at turn-on by

$$v_{Cf} = v_{C_0} e^{-\frac{\alpha + \gamma}{(R + R_C)C}}.$$  

(B.24)

Solving for $v_{C_0}$ and substituting $v_{Cf}$ from (B.14) into (B.24) yields

$$v_{C_0} = v_{Cf} e^{\frac{\alpha + \gamma}{(R + R_C)C}} = \frac{R + R_C}{R} \left[ V_R - V_{tm} \left( \frac{4\gamma}{T} - 1 \right) + \delta_2 \right] e^{\frac{\alpha + \gamma}{(R + R_C)C}}.$$  

(B.25)

The voltage on $C$ is constant during turn-off and turn-on and is defined by (B.14) and (B.25) respectively.

Just after turn-on, before $C$ begins to discharge, the output voltage $v_{o^+}$ can be obtained from voltage division as

$$v_{o^+} = \frac{R}{R + R_C} v_{C_0} = \left[ V_R - V_{tm} \left( \frac{4\gamma}{T} - 1 \right) + \delta_2 \right] e^{\frac{\alpha + \gamma}{(R + R_C)C}}.$$  

(B.26)

where $v_{C_0}$ in (B.25) has been substituted into (B.26). Solving (B.9), the differential equation for $v(t)$ during $t_{on}$, for $dv(t)/dt$ and evaluating $dv(t)/dt$ just after turn-on yields
The conditions of \( v(t) \) just after turn-on have been determined in (B.26) and (B.27). The conditions of \( v(t) \) just prior to turn-on will now be derived.

The output voltage just prior to turn-on can be found by substituting \( v(t) \) and \( \xi(t) \) at the end of \( t_{\text{off}} \) into (B.5).

At the end of \( t_{\text{off}} \), \( t = T - \alpha \) so that

\[
v_t(T - \alpha) = V_{tm} \left( \frac{4\alpha}{T} - 1 \right) \quad \text{(B.28)}
\]

and from Fig. 23

\[
\xi(T - \alpha) = \delta_1 \quad \text{(B.29)}
\]

Substituting (B.28) and (B.29) into (B.5) yields

\[
v_0^- = V_R - V_{tm} \left( \frac{4\alpha}{T} - 1 \right) - \delta_1 \quad \text{(B.30)}
\]

To compute \( dv(t)/dt \) evaluated just prior to turn-on, the derivative of (B.12) evaluated just prior to turn-on at \( P_0 \) is computed as

\[
\left. \frac{dv}{dt} \right|_{0^-} = R_C \left( \left. \frac{di_S}{dt} \right|_{0^-} - \left. \frac{di_L}{dt} \right|_{0^-} \right) + \left. \frac{dv_c}{dt} \right|_{0^-} \quad \text{(B.31)}
\]

The derivative of \( v_C(t) \) given by (B.17) during \( t_{\text{off}} \) evaluated just prior to turn-on is

\[
\left. \frac{dv}{dt} \right|_{0^-} = \frac{1}{(R + R_C)C} \left[ V_R - V_{tm} \left( \frac{4\alpha}{T} - 1 \right) + \delta \right] e^{\frac{\alpha + \gamma}{(R + R_C)C}} \quad \text{(B.27)}
\]
Taking the derivative of \( i_L(t) = \frac{v(t)}{R} \) evaluated just prior to turn-on yields

\[
\left. \frac{di_L}{dt} \right|_{0^-} = \frac{1}{R} \left. \frac{dv}{dt} \right|_{0^-} .
\] (B.33)

Substituting (B.32) and (B.33) into (B.31) yields

\[
\left. \frac{dv}{dt} \right|_{0^-} = \frac{RR_C}{R+RC} \left. \frac{di_S}{dt} \right|_{0^-} + \frac{R}{(R+RC)C} \left( i_{S0^-} - \frac{v_0^-}{R} \right) .
\] (B.34)

The jump in \( v(t) \) at turn-on is

\[
\Delta v_0 = v_{0+} - v_{0-} = \left[ V_R - V_{tm} \left( \frac{4\gamma}{T} - 1 \right) + \delta_2 \right] e^{\frac{\alpha + \gamma}{(R+RC)C}}
\]

\[
\left[ V_R - V_{tm} \left( \frac{4\alpha}{T} - 1 \right) - \delta_1 \right] .
\] (B.35)

The jump in \( \frac{dv(t)}{dt} \) at turn-on is

\[
\Delta \left. \frac{dv}{dt} \right|_0 = \left. \frac{dv}{dt} \right|_{0+} - \left. \frac{dv}{dt} \right|_{0-} = -\frac{v_{0+}}{(R+RC)C} - \frac{RR_C}{R+RC} \left. \frac{di_S}{dt} \right|_{0^-} - \frac{Ri_{S0^-} - v_0^-}{(R+RC)C}
\]

\[
= -\frac{RR_C}{R+RC} \left. \frac{di_S}{dt} \right|_{0^-} - \frac{Ri_{S0^-} + \Delta v_0}{(R+RC)C} .
\] (B.36)

The jump from \( P_4 \) to \( P_5 \) at turn-off has been derived and is defined by (B.22) and (B.23) and the jump from \( P_0 \) to \( P_1 \) is
defined by (B.35) and (B.36). Four unknown parameters remain to be defined. These are the conditions of \( i_s(t) \) just after turn-off, \( i_{Sf^+} \) and \( \left[ \frac{d i_s}{d t} \right]_{f^+} \), and just prior to turn-on, \( i_{S_0^-} \) and \( \left[ \frac{d i_s}{d t} \right]_{0^-} \). Expressions for these conditions are derived in the next section.

Conditions of \( i_s(t) \). Since transistor Q in the primary circuit is blocking for both sets of conditions of \( i_s(t) \), the primary current \( i_p(t) \) is zero. Since \( i_p(t) = 0 \), the flux \( \phi \) in the mutual inductor and the secondary current are linearly related. Also, since switching is instantaneous, \( \phi \) must be constant during switching and is equal to the value of \( \phi \) just prior to switching and just after switching.

Solving (B.12) for \( i_c(t) \) and evaluating \( i_c(t) \) just prior to turn-on yields

\[
i_{S_0^-} = \frac{v_0 - vC_0}{RC} + i_{L0^-}
\]  

(B.37)

Substituting \( i_{L0^-} = \frac{v_0}{R} \) into (B.37), yields

\[
i_{S_0^-} = \frac{R + RC}{RRC} v_0 - \frac{vC_0}{RC}
\]  

(B.38)

where \( v_0^- \) is defined in (B.30) and \( vC_0 \) is defined in (B.25).

The solution of the differential equation for the primary current \( i_p \) during \( t_{on} \) as shown in Fig. 22 is
The elapsed time from turn-on to turn-off is $t_{on} = \alpha + \gamma$ which when substituted into (B.39) for $t$ yields

$$i_p(t) = \left[ i_{p0}^+ - \frac{E-V_{CE}}{R_p} \right] e^{-\frac{R_p t}{L_p}} + \frac{E-V_{CE}}{R_p} .$$  \hspace{1cm} (B.39)

Since the flux level $\phi$ does not change during instantaneous switching,

$$i_{p0}^+ = \frac{N_i}{N_p} i_{S0}^-$$  \hspace{1cm} (B.41)

and

$$i_{Sf}^+ = \frac{N_i}{N_s} i_{pf}^- .$$  \hspace{1cm} (B.42)

Substituting (B.41) into (B.40) and (B.40) into (B.42) yields

$$i_{Sf}^+ = \left[ i_{S0}^- - \frac{N_i}{N_s} \frac{E-V_{CE}}{R_p} \right] e^{-\frac{R_p t_{on}}{L_p}} + \frac{N_i}{N_s} \frac{E-V_{CE}}{R_p} .$$  \hspace{1cm} (B.43)

where $i_{S0}^-$ is given in (B.38).

The differential equation around the secondary current loop for $i_s(t)$ is

$$L_s \frac{d i_s(t)}{dt} + R_s i_s(t) + v = 0 .$$  \hspace{1cm} (B.44)

Solving (B.44) for $d i_s/dt$ and evaluating just prior to turn-on yields
Substituting $i_{S0-}$ in (B.38) into (B.45) yields

$$\left. \frac{di_S}{dt} \right|_{0-} = \frac{-1}{L_s} \left( v_{0-} + V_D + R_s i_{S0-} \right). \quad (B.45)$$

Substituting $i_{S0-}$ in (B.38) into (B.45) yields

$$\left. \frac{di_S}{dt} \right|_{0-} = \frac{-1}{L_s} \left\{ \left[ 1 + \frac{R_s(R + R_C)}{R_C} \right] v_{0-} - \frac{R_s}{R_C} v_{C0} + V_D \right\} \quad (B.46)$$

where $v_{0-}$ is given in (B.30) and $v_{C0}$ is given in (B.25).

Solving (B.44) for $i_S(t)/dt$ and evaluating just after turn-off yields

$$\left. \frac{di_S}{dt} \right|_{f+} = \frac{-1}{L_s} \left( R_s i_{sf+} - V_D + v_{f+} \right) \quad (B.47)$$

where $i_{sf+}$ is defined by (B.43) and $v_{f+}$ by (B.15).

The jumps. The jump in $v(t)$ at turn-on is given explicitly by (B.35) as

$$\Delta v_0 = \left[ v_R - v_{tm} \left( \frac{4\alpha}{T} - 1 \right) + \delta_2 \right] e^{\frac{\alpha + \gamma}{(R + R_C)C}} - \left[ v_R - v_{tm} \left( \frac{4\alpha}{T} - 1 \right) - \delta_1 \right] \quad (B.35)$$

The jump in $v(t)$ at turn-off is obtained by substituting (B.43) into (B.22) to yield

$$\Delta v_f = \frac{R R_C}{R + R_C} \left[ \left( i_{S0} - \frac{N_P}{N_s} \frac{E-V_{CE}}{R_P} \right) e^{-\frac{R_{ton}}{L_p}} + \frac{N_P}{N_s} \frac{E-V_{CE}}{R_P} \right]. \quad (B.48)$$

Substituting (B.38) into (B.48) yields
\[ \Delta v_f = \frac{R R_C}{R + R_C} \left[ \left( \frac{R + R_C}{RR_C} v^0 - \frac{v_{C_0}}{R_C} - \frac{R}{N_s} \frac{E - V_{CE}}{R} \right) e^{-\frac{R_{P\text{on}}}{L_s}} + \frac{R}{N_s} \frac{E - V_{CE}}{R_P} \right] \]

Rearranging (B.49) gives

\[ \Delta v_f = \left\{ v^0 - \frac{R v_{C_0}}{R + R_C} \right\} e^{-\frac{R_{P\text{on}}}{L_s}} + \frac{R R_C}{R + R_C} \frac{R}{N_s} \frac{E - V_{CE}}{R_P} \left\{ -\frac{R_{P\text{on}}}{L_s} \right\} \]

where \( v^0 \) is given by (B.30) and \( v_{C_0} \) is given by (B.25). If substitution is made for \( v^0 \) and \( v_{C_0} \) in (B.50), \( \Delta v_f \) can be expressed as

\[ \Delta v_f = -\Delta v^0 e^{-\frac{R_{P\text{on}}}{L_s}} + \frac{R R_C}{R + R_C} \frac{R}{N_s} \frac{E - V_{CE}}{R_P} \left\{ -\frac{R_{P\text{on}}}{L_s} \right\} \]

where \( \Delta v^0 \) is given in (B.35).

The jump in \( dv(t)/dt \) at turn-on is obtained by substituting (B.38) and (B.46) into (B.36) and rearranging to obtain

\[ \Delta \frac{dv}{dt} \bigg|_{t=0} = \frac{R R_C}{L_s (R + R_C)} \left( V_D + v^0 + v_{C_0} \right) + \left[ \frac{1}{R C} \frac{R_S}{L_s} - \frac{1}{(R + R_C) C} \right] \Delta v^0 . \]

where again \( v^0 \) is given in (B.30) and \( v_{C_0} \) is given in (B.25).

The jump in \( dv(t)/dt \) at turn-off is obtained by substituting (B.43) and (B.47) into (B.23) to obtain
\[ \frac{\Delta v}{\Delta t} = \frac{-RR_C}{(R+R_C)C} \left( V_D + V_{f+} \right) + \left[ \frac{R^2}{(R+R_C)^2} - \frac{RR_C R_S}{(R+R_C)C L} \right] \left[ i_{S_0} - \right] \]

\[ \frac{N_p E-V_{CE}}{N_s} \left( \frac{R p \text{on}}{L_p} + \frac{N_p E-V_{CE}}{N_s R_p} \right) \]

where \( v_{f+} \) is given in (B.15) and \( i_{S_0} \) is given in (B.38).

The jumps are in the output voltage \( v(t) \) and are expressed in terms of unnormalized parameters. In order to utilize these expressions in the sequence function analysis, they must be transformed to jumps in the normalized error voltage.

The error voltage is
\[ \xi(t) = V_R - v_t(t) - v(t) \quad \text{(B.54)} \]

The reference voltage \( V_R \) is constant; and since switching is assumed to occur instantaneously \( v_t(t) \) is constant during switching. Therefore, the jump in \( \xi(t) \) is the negative of the jump in \( v(t) \) which when normalized results in
\[ \Delta x_0 = - \Delta v_0 / V_R \quad \text{(B.55)} \]
\[ \Delta x_f = - \Delta v_f / V_R \quad \text{(B.56)} \]

Since \( V_R \) is a constant the derivative of the error voltage is
\[ \frac{d\xi(t)}{dt} = - \frac{dv_t(t)}{dt} - \frac{dv(t)}{dt} \quad \text{(B.57)} \]
Since $\frac{dv(t)}{dt}$ is constant,

$$\frac{\Delta E(t)}{\Delta t} = -\Delta \frac{dv(t)}{dt} \quad (B.58)$$

which when normalized results in

$$\Delta y_o = \Delta \frac{dx}{dt} \bigg|_0 = -\frac{1}{\omega_o V_R} \Delta \frac{dv}{dt} \bigg|_0 \quad (B.59)$$

and

$$\Delta y_f = \Delta \frac{dx}{dt} \bigg|_f = -\frac{1}{\omega_o V_R} \Delta \frac{dv}{dt} \bigg|_f \quad (B.60)$$

where $\omega_o = \frac{1}{\sqrt{L_s C}}$. 


Appendix C  
EXPERIMENTAL CONFIRMATION OF THE  
ANALYSIS OF CHAPTER IV

Many effects such as switching speeds and stray capacitance exist at a frequency of 1 MHz which make the 1 MHz converter in Chapter III unsuitable to use in making an experimental verification of the analysis in Chapter IV. For this reason, a converter, to be used experimentally to verify the analysis in Chapter IV, is designed to operate at a much lower switching frequency of 2 kHz. The complete schematic diagram of this converter circuit is given in Fig. C.1 and the complete components list is given in Table C.1.

Simplified operational concept. The basic regulation concept for the 2 kHz converter is the same as that for the 1 MHz converter; however, the circuitry used to implement this concept is different. A simplified operational diagram of the circuit in Fig. C.1 is shown in Fig. C.2. In this diagram the dc output voltage $v(t)$ is reduced by OP AMP I3 to a level compatible to the reference voltage $V_R$ and is summed with the triangular voltage $-v_t(t)$ at the input to OP AMP I2. I2 is a unity gain inverting amplifier which returns $v(t)$ to the normal
Fig. C.1. Schematic diagram of the 2 kHz converter.
Table C.1. Components list for Fig. C.1.

<table>
<thead>
<tr>
<th>Component</th>
<th>Value/Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cl</td>
<td>1875 µF</td>
<td>6- 300 µF 30 V tantalum electrolytic</td>
</tr>
<tr>
<td>C2</td>
<td>0.00022 µF</td>
<td>1 kV disc ceramic</td>
</tr>
<tr>
<td>C3, C5</td>
<td>0.00025 µF</td>
<td>1 kV disc ceramic</td>
</tr>
<tr>
<td>C4, C6</td>
<td>0.005 µF</td>
<td>1 kV disc ceramic</td>
</tr>
<tr>
<td>D1</td>
<td>10R2 (Isofilm International)</td>
<td></td>
</tr>
<tr>
<td>D2-D5</td>
<td>UTR-12 (Unitrode Corp.)</td>
<td></td>
</tr>
<tr>
<td>DZ1</td>
<td>1N750A</td>
<td></td>
</tr>
<tr>
<td>DZ2, DZ3</td>
<td>1N751A</td>
<td></td>
</tr>
<tr>
<td>E_S</td>
<td>Source Voltage, 21-31 volts</td>
<td></td>
</tr>
<tr>
<td>I1, I4</td>
<td>SN72 710L Differential Comparator (TI)</td>
<td></td>
</tr>
<tr>
<td>I2, I3</td>
<td>μ709 Operational Amplifier (Fairchild)</td>
<td></td>
</tr>
<tr>
<td>N1</td>
<td>100 turns #19 AWG</td>
<td></td>
</tr>
<tr>
<td>N2</td>
<td>150 turns #21 AWG</td>
<td></td>
</tr>
<tr>
<td>N4</td>
<td>23 turns #25 AWG</td>
<td></td>
</tr>
<tr>
<td>N5</td>
<td>113 turns #25 AWG</td>
<td></td>
</tr>
<tr>
<td>N6</td>
<td>10 turns #25 AWG</td>
<td></td>
</tr>
<tr>
<td>Q1</td>
<td>DTS-410 (Delco)</td>
<td></td>
</tr>
<tr>
<td>Q2</td>
<td>2N3501 (Motorola)</td>
<td></td>
</tr>
<tr>
<td>Q3</td>
<td>2N3499 (Motorola)</td>
<td></td>
</tr>
<tr>
<td>R_C</td>
<td>0.145 Ω</td>
<td></td>
</tr>
<tr>
<td>R1</td>
<td>8.2 Ω</td>
<td></td>
</tr>
<tr>
<td>R2, R5, R19, R22</td>
<td>22 Ω</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>220 KΩ</td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>580 KΩ</td>
<td></td>
</tr>
<tr>
<td>R6, R21</td>
<td>470 Ω</td>
<td></td>
</tr>
<tr>
<td>R7, R8, R20</td>
<td>1.8 KΩ</td>
<td></td>
</tr>
<tr>
<td>R9</td>
<td>12 KΩ</td>
<td></td>
</tr>
<tr>
<td>R10, R16</td>
<td>10 KΩ trimpot</td>
<td></td>
</tr>
<tr>
<td>R11, R15</td>
<td>5110 Ω 1%</td>
<td></td>
</tr>
<tr>
<td>R12, R17</td>
<td>1.5 KΩ</td>
<td></td>
</tr>
<tr>
<td>R13, R14</td>
<td>10 KΩ</td>
<td></td>
</tr>
<tr>
<td>R18</td>
<td>71.5 KΩ 1%</td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>55436 (Magnetics, Inc.)</td>
<td></td>
</tr>
<tr>
<td>T2</td>
<td>52176-1A (Magnetics, Inc.)</td>
<td></td>
</tr>
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</table>
Fig. C.2 Simplified operational diagram of the 2 kHz converter.
polarity and acts as a summing junction for \(-v_t(t)\) and \(-Kv(t)\) .
Since \(v_t(t)\) is a symmetrical signal, its inversion does not modify circuit functioning. Voltage \(v_t(t)\) has a negative sign at its input to the converter to maintain correct mathematical representation. The output \(v_A(t)\) of I2 is thus \(v_t(t) + Kv(t)\).

Signal \(v_A(t)\) is the input to the comparator I1 which compares \(v_A(t)\) with the reference voltage \(V_R\). The inset shows the hysteresis and output voltage levels which control Q2 which, in turn, controls the power transistor Q1. Regulation is accomplished by the same manner as explained in Chapter III. When \(v_A\) is greater than \(V_R -5\, \text{mV}\), comparator I1 output is \(-0.5\, \text{V}\) and Q1 is OFF. When \(v_A\) is less than \(V_R -5\, \text{mV}\), comparator I1 output is \(+1.5\, \text{V}\) and Q1 is ON.

T2 is a saturable reactor and has to be reset when Q2 is OFF. Reset is accomplished by Q3 and comparator I4. The input to I4 is the output of I1 such that when I1 has a \(+1.5\, \text{V}\) output I4 has a \(-0.5\, \text{V}\) output and vice versa. This means that Q3 is ON when Q2 is OFF resetting the magnetic flux excursion which occurs in T2 when Q2 is ON.

**Experimental waveforms.** Pictures of experimental waveforms for the ac components of \(v(t)\) and \(v_A(t)\) are shown in Fig. C.3. These waveforms were taken under the circuit conditions of \(E_s = 21.0\, \text{V}\) and \(R_L = 40.155\, \text{ohms}\). Complete measured circuit conditions and parameters for the same operating conditions as existed for Fig. C.1 are shown in Table C.2. The general shape
Fig. C.3. Experimental waveforms of the ac components of $v(t)$ and $v_A(t)$.
Table C.2. Numerical values for the experimental 2 kHz converter used to evaluate the theoretical expressions.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_S$</td>
<td>21.0 volts</td>
</tr>
<tr>
<td>$V_O$</td>
<td>28.26 volts</td>
</tr>
<tr>
<td>$V_{tm}$</td>
<td>0.10 volts</td>
</tr>
<tr>
<td>$V_R$</td>
<td>4.926 volts</td>
</tr>
<tr>
<td>$V_D$</td>
<td>0.725 volts</td>
</tr>
<tr>
<td>$V_{CE}$</td>
<td>0.40 volts</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.005 volts</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.005 volts</td>
</tr>
<tr>
<td>$R$</td>
<td>40.155 ohms</td>
</tr>
<tr>
<td>$R_C$</td>
<td>0.145 ohms</td>
</tr>
<tr>
<td>$R_S$</td>
<td>0.432 ohms</td>
</tr>
<tr>
<td>$R_P$</td>
<td>0.175 ohms</td>
</tr>
<tr>
<td>$C$</td>
<td>1875 (\mu)F</td>
</tr>
<tr>
<td>$L_S$</td>
<td>7.65 mH</td>
</tr>
<tr>
<td>$L_P$</td>
<td>3.05 mH</td>
</tr>
<tr>
<td>$N_P$</td>
<td>100 turns</td>
</tr>
<tr>
<td>$N_S$</td>
<td>150 turns</td>
</tr>
<tr>
<td>$T$</td>
<td>0.500 msec</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.091 msec</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.155 msec</td>
</tr>
<tr>
<td>$t_{on}$</td>
<td>0.246 msec</td>
</tr>
<tr>
<td>$t_{off}$</td>
<td>0.254 msec</td>
</tr>
</tbody>
</table>
of $v(t)$ in Fig. C.3 agrees with the shape depicted in Fig. 26(b) except immediately after turn-on and turn-off where Fig. C.3 exhibits more curvature which is likely to be caused by parasitic capacitance or inductance.

Fig. C.4 is a photograph of the experimental limit cycle in the phase plane which is the time derivative of $v_A(t)$ versus $V_A(t)$ shown in Fig. C.3. This experimental limit cycle is normalized and shown in Fig. C.5 along with the normalized theoretical limit cycle.

Computation of the theoretical limit cycle. The theoretical limit cycle is determined by assuming an initial point $P_0$ just prior to turn-on to be the same as the corresponding experimental point. $P_1$ is determined by evaluating the magnitude of the jump from $P_0$ to $P_1$ by substituting the appropriate values from Table C.2 into (B.35) and (B.52). The jumps $\Delta V_0$ and $\Delta v/dt|_0$ are normalized and added to the $x$ and $y$ coordinates of $P_0$ to obtain $P_1$.

The trajectory from $P_1$ to $P_2$ is determined by solving (119) for $x(\tau)$ as

$$x(\tau_1) = A e^{-\tau_1/\tau_C} + 1 + \frac{4V_{tm}\tau_C}{\omega_0 V_R T}.$$

(C.1)

Time $t_1$ is the real time variable and $\tau_1$ is the normalized time variable during movement on the trajectory from $P_1$ to $P_2$. This notation is used throughout the following discussion. The
Fig. C.4. Experimental limit cycle \(-dv_A(t)/dt\) versus \(-v_A(t)\).
Fig. C.5. Experimental and theoretical limit cycles for the 2 kHz converter.
numerical subscript on \( t \) or \( \tau \) is the same as the subscript of the point at which the time variable is zero. The final value of the variable is reached when the next higher integer subscripted point is reached. The unknown constant \( A \) is evaluated by letting \( x(0) = x_1 \) where \( x_1 \) is the x-coordinate of \( P_1 \) at \( t_1 = 0.0 \), or \( \tau_1 = 0.0 \). The y-coordinate is simply the derivative of (C.1) which is

\[
y(\tau_1) = -\frac{A}{\tau_C} e^{-\tau_1/\tau_C}.
\]

Equations (C.1) and (C.2) are evaluated at increments of \( \tau_1 \) from \( \tau_1 = 0.0 \) to \( \tau_1 = 0.024 \) when \( P_2 \) is reached. The final value of \( \tau_1 \) is calculated by \( \tau_1 = \omega_0 t_1 = \omega_0 \alpha \) where \( \omega_0 \) is calculated using values from Table C.2 and \( \alpha \) is measured in Fig. C.3 and given in Table C.2.

The jump from \( P_2 \) to \( P_3 \) occurs at the instant when the slope of \( v_A(t) \) changes from negative to positive by an amount equal to twice the slope of \( v_t(t) \). At this jump, the x-coordinate and \( \tau \) do not change; and the y-coordinate changes by \(-8v_{tm}/\omega_0 V_R \tau \) as shown in Fig. 24. The jump in the y-coordinate is negative because \( y(\tau) = \left(-1/\omega_0 V_R\right) dv_A/dt \). \( P_3 \) is thus defined by \( x_3 = x_2 \) and \( y_3 = y_2 - 8v_{tm}/\omega_0 V_R \tau \).

The trajectory for \( P_3 \) to \( P_4 \) is determined by solving (118) for \( x(\tau_3) \) as
where $\tau_3$ is the normalized time variable during movement from $P_3$ to $P_4$. The unknown constant $B$ is evaluated by letting 

$$x(0) = x_3$$

where $x_3$ is the x-coordinate of $P_3$ at $t_3 = 0.0$, or $\tau_3 = 0.0$. The y-coordinate is simply the derivative of (C.3) which is

$$y(\tau_3) = -\frac{B}{\tau C} e^{-\tau_3/\tau C}$$  \hspace{1cm} (C.4)$$

Equations (C.3) and (C.4) are evaluated at increments of $\tau_3$ from $\tau_3 = 0.0$ to $\tau_3 = 0.041$ when $P_4$ is reached. The final value $\tau_3$ is determined by $\tau_3 = \omega_0 \gamma$ where $\omega_0$ has been previously computed and $\gamma$ is given in Table C.2.

The jump from $P_4$ to $P_5$ occurs at turn-off and is determined by evaluating (B.51) and (B.53) for $\Delta v_f$ and $\Delta v_f/dt|_{f}$, respectively. The jumps are then normalized to yield $\Delta x_f$ and $\Delta y_f$ which are added to $x_4$ and $y_4$, respectively, to yield the x and y-coordinates of $P_5$.

The segment of the limit cycle from $P_5$ to $P_6$ is defined by (120). The solution to (120) which defines the x-coordinate is

$$x(\tau_5) = G e^{-h\tau_5} \cos \left( \omega \tau_5 - \delta \right) + 1 + \frac{V_D}{V_R} - \frac{8hV_{tm}}{\omega_0 V_R T} \hspace{1cm} (C.5)$$

where $h$ is given in (113) and $\omega = (1 - h^2)^{1/2}$. The derivative of (C.5) is
\[ y(\tau_s) = -G e^{-\tau_s} [\omega \sin(\omega \tau_s - \delta) + h \cos(\omega \tau_s - \delta)] \]  

(C.6)

and (C.6) defines the \( y \)-coordinate. \( G \) and \( \delta \) are unknown constants which are evaluated by substituting \( x(0) = x_s \) and \( y(0) = y_s \) into (C.5) and (C.6). Equations (C.5) and (C.6) are evaluated for incremental values of \( \tau_s \) from 0.0 to 0.025 to obtain the trajectory from \( P_5 \) to \( P_6 \). The final value of \( \tau_s = 0.025 \) at \( P_6 \) is obtained from \( \tau_s = \omega_0 [(T/2) - \gamma] \).

The jump from \( P_6 \) to \( P_7 \) occurs at the instant when the slope of \( v_A(t) \) changes from positive to negative. There is no change in \( x \) and the change in \( y \) is of equal magnitude but opposite sign as the jump from \( P_2 \) to \( P_3 \). \( P_7 \) is thus defined as \( x_7 = x_6 \) and \( y_7 = y_6 + 8V_{tm}/\omega_0 V_R T \).

The segment of the limit cycle from \( P_7 \) to \( P_8 \) is defined by (121). The solution to (121) is

\[ x(\tau_7) = H e^{-\tau_7} \cos(\omega \tau_7 - \theta) + \frac{V_L}{V_R} - \frac{8hV_{tm}}{\omega_0 V_R T} \]  

(C.7)

which defines the \( x \)-coordinate. The derivative of (C.7) is

\[ y(\tau_7) = -H e^{-\tau_7} [\omega \sin(\omega \tau_7 - \theta) + h \cos(\omega \tau_7 - \theta)] \]  

(C.8)

and (C.8) defines the \( y \)-coordinate. \( H \) and \( \theta \) are unknown constants which are evaluated by substituting \( x(0) = x_7 \) and \( y(0) = y_7 \) into (C.7) and (C.8). Equations (C.7) and (C.8) are evaluated for incremental values of \( \tau_7 \) from \( \tau_7 = 0.0 \) to 0.042 to obtain the trajectory from \( P_7 \) to \( P_8 \). The final value of
\[ \tau_7 = 0.042 \text{ at } P_8 \] is obtained from \( \tau_7 = \omega_0 [(T/2) - \alpha] \). \( P_8 \) is the final point in the limit cycle and for steady-state conditions should have the same coordinates as \( P_0 \).

Comparison of the experimental and theoretical limit cycles. The experimental and theoretical limit cycles in Fig. C.5 compare very closely. The experimental jumps from \( P_0 \) to \( P_1 \) at turn-on and from \( P_4 \) to \( P_5 \) at turn-off are slightly greater than the theoretical jumps. This difference could be caused by inaccuracies in the measurements of the numerical values given in Table C.2 or by parasitic circuit elements that were present but not considered in the analysis. This difference may also be caused by the finite switching time which exists experimentally and for which the theoretical analysis assumes instantaneous switching. The effect of the parasitic circuit elements should be the greatest effect because the differences in the slopes of the trajectories from \( P_1 \) to \( P_2 \) and the general difference in the nature of the trajectories from \( P_5 \) to \( P_6 \) is the result of the unpredicted curvature in \( v(t) \) immediately after turn-on and turn-off as shown in Fig. C.3 and is attributed to parasitic circuit elements.

It is interesting to note that the theoretical trajectory passes directly through the initial point \( P_0 \) in attaining point \( P_8 \). This phenomenon is probably coincidental; however, if \( P_5 \) and the entire lift side of the limit cycle were displaced in the negative \( x \) direction until the theoretical \( x_5 \) equals the
experimental $x_5$, $P_8$ would be closer to $P_0$. This leads to the conclusion that the difference in the jump at turn-off may at least be partially responsible for $P_8$ not being the same point as $P_0$.

**Experimental verification of the sequence functions.**

There are two equations (152) and (162) in Chapter IV which are derived using sequence function analysis. These equations completely define the steady-state conditions of the converter analyzed in Chapter IV. Equation (152) is obtained by equating $x_2$ and $x_3$ from (150) and (151).

To verify (152), $x_2$ and $x_3$ are evaluated using (150), (151), and the conditions given in Table C.2 yielding $x_2 = 0.02209607$ and $x_3 = 0.02212789$. The difference, $x_3 - x_2 = 3.182 \times 10^{-5}$, is practically negligible.

Equation (162) is verified by substituting the circuit conditions from Table C.2 into (157) and (161) to obtain $x_6 = -0.023485052$ and $x_7 = -0.02603728$. The difference $x_6 - x_7$ is $2.518676 \times 10^{-3}$. The differences $x_3 - x_2$ and $x_6 - x_7$ are small enough to conclude that the sequence function equations are experimentally verified.
LIST OF REFERENCES
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BIOGRAPHY

Name: William A. Sander, III

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