ON THE USE OF THICK-AIRFOIL THEORY TO DESIGN AIRFOIL FAMILIES IN WHICH THICKNESS AND LIFT ARE VARIED INDEPENDENTLY

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A method has been developed for designing families of airfoils in which the members of a family have the same basic type of pressure distribution but vary in thickness ratio or lift, or both. Thickness ratio and lift may be prescribed independently. The method which is based on the Theodorsen thick-airfoil theory permits moderate variations from the basic shape on which the family is based.
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SUMMARY

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INTRODUCTION

In the design of airfoil sections, it is often desirable to vary the thickness or lift of a basic airfoil having favorable characteristics while maintaining as nearly as possible the same type of pressure distribution. A rough approach (ref. 1, ch. 4) to this problem is to design the mean line by thin airfoil theory and then superimpose on it the thickness distribution of a symmetric airfoil whose pressure distribution is known.

Another approach is to start with an airfoil having desirable characteristics and scale the ordinates by a constant factor to obtain an airfoil with a different thickness. Airfoils designed in this manner have several shortcomings, as described in reference 2. Reference 2 then describes a different procedure which utilizes the thick-airfoil theory of Theodorsen. This procedure produces airfoils that differ from the basic shape both in thickness and in lift by essentially the same ratio. However, it is sometimes desirable to change the thickness of the airfoil without changing the lift, or to change both lift and thickness but in different proportions. The purpose of this paper is to describe a method for generating such variations within the context of the Theodorsen theory.

SYMBOLS

a arbitrary scale parameter
C constant
c  airfoil chord

c_\ell  airfoil lift coefficient

c_p  pressure coefficient

R,\phi  polar coordinates of exact-circle transformation of airfoil

r,\theta  polar coordinates of near-circle transformation of airfoil

V  undisturbed free-stream velocity

v  local velocity

X = x - x_o

x,y  rectangular coordinates of airfoil in physical plane

x_o  value of x at nose of airfoil

\alpha  angle of attack

\epsilon  function relating angular coordinates of near-circle and exact-circle airfoil transformations

\epsilon_N  value of \epsilon at airfoil nose

\epsilon_T  value of \epsilon at airfoil tail

\theta^*,\phi^*  dummy variables of integration

\psi  function relating radial coordinates of near-circle and exact-circle airfoil transformations

\psi_a  arbitrary small constant

\psi_o  average value of \psi

Prime superscript indicates derivative with respect to \theta.
THEORETICAL CONSIDERATIONS

The Theodorsen airfoil theory (ref. 3) involves a conformal transformation of the airfoil into a shape approximating a circle. (See fig. 1.) In the transformed plane this near circle is described by polar coordinates \( r, \theta \). The approximate circle is then transformed into an exact circle having coordinates \( R, \phi \). The function \( \psi \) and the constant \( \psi_0 \) are defined by \( r = a \epsilon^{i} \psi \) and \( R = a \epsilon^{i} \psi^{0} \), respectively. Theodorsen shows that \( \psi_0 \) is the average value of \( \psi \),

\[
\psi_0 = \frac{1}{2\pi} \int_{0}^{2\pi} \psi \, d\phi
\]  

(1)

and that the functions \( \psi - \psi_0 \) and \( \epsilon = \phi - \theta \) are related by the equations

\[
\epsilon = -\frac{1}{2\pi} \int_{0}^{2\pi} \psi \cot \frac{\phi^* - \phi}{2} \, d\phi^*
\]  

(2)

\[
\psi - \psi_0 = \frac{1}{2\pi} \int_{0}^{2\pi} \epsilon \cot \frac{\phi^* - \phi}{2} \, d\phi^*
\]  

(3)

The function \( \psi \) is directly related to the airfoil coordinates in the physical plane by

\[
x = 2a \cosh \psi \cos \theta
\]  

(4a)

\[
y = 2a \sinh \psi \sin \theta
\]  

(4b)

In practice \( \psi \) is obtained as a function of \( \theta \) from equations (4), and then a function \( \epsilon(\theta) \) is obtained from \( \psi(\theta) \) by replacing \( \phi^* \) and \( \phi \) in equation (2) with \( \theta^* \) and \( \theta \), respectively. Henceforth in this paper \( \epsilon \) will refer to this function \( \epsilon(\theta) \) which is a close approximation to the exact function. The error involved in this approximation arises primarily as a tiny adjustment in \( \epsilon \) (but not in \( \epsilon' \)) in the velocity equation:

\[
\frac{v}{V} = \frac{\sin(\alpha + \epsilon + \theta) + \sin(\alpha + \epsilon_T)}{\sqrt{(\sinh^2 \psi + \sin^2 \theta)(1 + \psi^2)}}
\]  

(5)

This discrepancy is normally very small compared with that arising from viscous effects, and so the approximation is considered to be adequate for the purposes of this analysis.

Reference 3 also demonstrates that the angle of zero lift is \( -\epsilon_T \) and the ideal angle of attack is \( -(\epsilon_N + \epsilon_T)/2 \). These fundamental relations from Theodorsen's analysis yield an interpretation of the airfoil family design procedure described in reference 2 which consists of simply multiplying the function by a constant \( C \). According to equation (3), \( \psi \) is thereby changed by a factor of \( C \). Expanding equation (4b) in powers of \( \psi \) yields
\[ y = 2a \sin \theta \left( \psi + \frac{\psi^3}{3} + \ldots \right) \]  \hspace{1cm} (6)

Inasmuch as \( \psi \) consists of \( \psi_0 \) plus small perturbations about \( \psi_0 \), it is seen that the primary effect on the geometry of the airfoil is to change its thickness.

It is seen that multiplying \( \epsilon \) by \( C \) automatically multiplies \( \epsilon_T \) and \( \epsilon_N \) by \( C \). Consequently, both the angle of zero lift and the ideal angle of attack are multiplied by \( C \) and thus the design lift coefficient is changed by a factor of \( C \).

A second type of change is now considered: that of adding a constant \( \psi_a \) to \( \psi \). The constant \( \psi_0 \) is thereby changed by \( \psi_a \) but \( \psi - \psi_0 \) is unaffected, as is its conjugate function \( \epsilon \). Therefore, the thickness of the airfoil will be changed but not its lift parameters. This possibility was mentioned in reference 4.

Finally, consider the case where it is desirable to change both the thickness and the lift but in different proportions. This variation may be accomplished by combining the two procedures: first change the lift and thickness in the same ratio by multiplying \( \psi \) and \( \epsilon \) by the same constant, and then add a constant to \( \psi \) to adjust the thickness to the desired value.

**EXAMPLES**

In order to check the usefulness of the methods described in the previous section, several calculations were performed. The basic airfoil on which the family is based was very similar to that described in reference 5. The favorable characteristics of this airfoil are predicted by the method of reference 6 which includes a boundary-layer calculation. The well-known effect of the boundary layer is to reduce the lift, as can be seen by the comparison in figure 2 of the viscous pressure distribution with the inviscid pressure distribution. In this case \( c_\ell \) was reduced from 0.455 to 0.381.

It appears reasonable that although the predicted lift for the viscous case is significantly less than that for the inviscid case, a prescribed change in the inviscid lift will change the lift for the viscous case in approximately the same ratio. The assumption that is made here is that the boundary layer has approximately the same effect for all members of a family. Of course, a slight error results from the assumption. The performance calculations in the following examples have been computed by the viscous theory in order to give a more realistic assessment of the usefulness of the design theory.

Figure 3 shows the calculations for a family in which both the lift and the thickness were varied in the same ratio. The 17-percent-thick \( \frac{\text{Thickness}}{\text{Chord}} = 0.17 \) airfoil is the basic model. For the 19-percent-thick airfoil the design calculation predicts a lift coefficient of 0.426 compared with the value of 0.429 obtained with the viscous calculation.
For the 15-percent-thick airfoil the design and viscous calculations yield 0.336 and 0.338, respectively. The difference amounts to less than 1 percent in both cases. The $\psi$ and $\epsilon$ functions for this family are shown in figure 4.

The airfoils shown in figure 5 vary in thickness from 15 percent to 19 percent but are all designed to have the same viscous lift coefficient 0.381. In this case the viscous calculation gives, for the 19-percent-thick airfoil, $c_\ell = 0.388$, and for the 15-percent-thick airfoil, $c_\ell = 0.373$, a discrepancy of about 2 percent in each case. These airfoils all have the same $\epsilon$ function as the 17-percent-thick airfoil. (See fig. 4(b).) The $\psi$ functions for the 19- and 15-percent-thick airfoils are displaced by $\pm 0.021$, respectively, from that of the 17-percent-thick airfoil.

Figure 6 shows another variation based on the same 17-percent-thick airfoil. In this case the thickness was increased by a factor of 1.059 to 18 percent, whereas the lift coefficient was designed to increase by 15 percent to 0.438. The lift coefficient actually obtained is 0.447, an increase of 17 percent.

Naturally, it is not possible to design for a much larger increase in lift than thickness. Such an attempt results in an airfoil with zero thickness in the cusp region near the trailing edge.

CONCLUDING REMARKS

A method for designing families of airfoils has been described. These families are based on utilizing an initial airfoil with a desirable type of pressure distribution and then varying the thickness and/or lift while maintaining the same basic shape of the pressure distribution. The method is based on the Theodorsen transformation and so is not limited by the approximations of thin-airfoil theory. However, attempts to design large variations from the original airfoil tend to result in unsatisfactory shapes.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., February 14, 1974.
REFERENCES


4. Theodorsen, Theodore: Airfoil Contour Modifications Based on $c$-Curve Method of Calculating Pressure Distribution. NACA WR L-135, 1944. (Formerly ARR L4G05.)


Figure 1.- Transformations used to derive airfoils and calculate pressure distributions.
Figure 2.- Comparison of viscous and inviscid pressure distributions for the basic 17-percent-thick airfoil.
Figure 3.- Pressure distributions for an airfoil family for which the thickness and lift are varied in the same ratio.
Figure 4. - Transformation functions for airfoil family in which thickness and lift are varied in the same ratio.
Figure 4.- Concluded.

(b) $\epsilon$ function.
Figure 5.- Pressure distributions for an airfoil family designed for no lift variation.
Figure 6.- Pressure distribution for an 18-percent-thick airfoil design from the basic 17-percent-thick airfoil with a design lift increase of 15 percent.