

WAVE PROPAGATION AND EARTH SATELLITE RADIO  
EMISSION STUDIES

K. C. Yeh, C. H. Liu and B. J. Flaherty

January 1974

Final Report

July 1, 1959 to December 31, 1973

Prepared for  
National Aeronautics and Space Administration  
Washington, D. C. 20546

Grant NGR 14-005-002



Ionosphere Radio Laboratory  
Department of Electrical Engineering  
University of Illinois at Urbana-Champaign  
Urbana, Illinois 61801

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WAVE PROPAGATION AND EARTH SATELLITE RADIO  
EMISSION STUDIES

1. Introduction

This is the final report of the subject grant. The research has been concerned with radio propagation studies of the ionosphere using satellite radio beacons. The ionosphere is known as a dispersive, inhomogeneous, irregular and sometimes even nonlinear medium. After traversing through the ionosphere the radio signal bears signatures of these characteristics. A study of these signatures will be helpful in two areas: (i) It will assist in learning the behavior of the medium, in this case the ionosphere. (ii) It will provide information of the kind of signal characteristics and statistics to be expected for communication and navigational satellite systems that use the similar geometry. This grant is mainly concerned with the basic research i.e., area (i) above, although the information obtained has applications in area (ii).

Under the support of this grant many interesting and important contributions have been made. We only attempt to summarize these results here. For details interested readers should consult our publication list shown in Appendix I.

It should be mentioned that our NASA support is drawing to an end just at a time NASA is launching (April 1974) ATS-F which contains a radio beacon experiment. Actually we are prepared to carry out these experiments, if given a chance.

## 2. Summary of Experiments Carried Out Under the Grant

### 2.1 Introduction

Throughout the course of this grant our experimental program has been concentrated in two areas of ionosphere research. The first area has been the investigation of the ionosphere columnar electron content, hereafter called the total electron content (TEC). A measure of the TEC is obtained by the observation on the ground of the Faraday rotation (papers 4, 7, 9 of the Publication List in Appendix I) of the planes of polarization of radio signals transmitted by artificial earth satellites through the ionosphere. The second experimental area has been the investigation of small scale ionospheric irregularities by means of scintillation in the amplitude of the received satellite radio signals (papers 5, 13, 14).

Initially these observations were made at one observing station at the University of Illinois in Urbana, but as our knowledge of the ionosphere increased, the experiments became more complex and often required the setting up of additional satellite receiving stations. In the course of this grant stations were set up at many locations in the Northern Hemisphere. The extreme East and West stations being Norfolk, Virginia and Adak, Alaska while the extreme North and South stations being Baker Lake, Northwest Territories and Turkey Point Florida.

Each of the two experimental areas discussed above may further be divided into two classes of experiments. The first experimental class is concerned with periodic diurnal measurements using low orbiting satellites, and the second experimental

class is concerned with continuous diurnal measurements using geostationary satellites. Within each of these classes there are groups of measurements made at closely spaced stations and groups of measurements made at largely separated stations. In general the closely spaced measurements are used to study local ionosphere effects such as the heights of scintillation producing regions or the study of traveling ionosphere disturbances (TID). The largely spaced stations, on the other hand, are used to study latitude effects such as the latitude dependence of scintillation or the magnetic conjugate behavior of TEC and scintillation.

In section 2.2 a brief discussion of the major experiments using low orbiting satellites is discussed while in section 2.3 a brief discussion of the major experiments performed using geostationary satellites is presented. In section 2.4 several experiments that were performed with the joint support of this grant and other contracting agencies is presented. Section 2.5 presents the major items of equipment that were designed and developed either solely under support of this grant or in conjunction with other agencies.

## 2.2 Ionosphere Measurements Using Low Orbiting Satellites

In the early years of this grant, experimental data was collected at four stations. These stations were Urbana, Illinois; Houghton, Michigan; Baker Lake, Northwest Territories; and Adak, Alaska. The equipment at these stations was very simple. It consisted of linearly polarized antennas for Faraday rotation measurements and/or circular polarized antennae for scintillation studies, superheterodyne receivers and a recording device (paper 11).

The major satellites used for these measurements were Sputnik 3, Explorer 7, Transit 4A, and Discoverers 32 and 36. These satellites transmitted on frequencies from 20 to 54 MHz. The Faraday rotation data collected from these measurements were used in our early TEC investigations (papers 4, 9, and 22) and early scintillation studies (papers 5, 15 and 18). During this same period of high sun spot number several closely-spaced stations were installed around the Urbana station to investigate the heights and sizes of the scintillation producing regions of the ionosphere (papers 13 and 14).

With the launching of Explorer 22 (BE-B) in October 1964 and Explorer 27 in April 1965, ionosphere researchers had two beacon satellites that were designed especially for ionosphere research. These satellites transmitted on several coherent frequencies from 20 to 360 MHz. Using these satellites, we were able to make several different types of measurements. The Faraday rotation and scintillation measurements mentioned above were continued with minor modifications to the receiving equipment. Since these satellites transmitted coherent signals, it was possible to make dispersive phase measurements between the various carriers. The equipment to make these measurements was much more complex hence, they were only made at the Urbana station (for some of the results of these measurements see paper 31). Several interesting experiments resulted because of the fortuitous geometry between the orbit of the BE-C satellite and the mid-latitude location of our Urbana receiving stations. As BE-C passed over our stations, it traveled at a near constant  $\bar{M}$  value for a relatively long (a few minutes) period of time. This being so, we were able to make measurements

in the gradients of TEC and to study ionosphere irregularities (papers 26, 34, 36, 38 and 39). For these studies three stations were installed in the form of an equilateral triangle with sides equal to 50 KM. These stations were located at Urbana, Danville, and Brocton, Illinois.

The last major experiment performed with BE-B/BE-C was the repeating of the scintillation measurements made at closely-spaced stations. This study was performed during a period of low sunspot number and hence, it, in conjunction with the previous study, yielded height measurements over half the solar cycle (paper 51).

### 2.3 Ionospheric Measurements Using Geostationary Satellites

With NASA's launching of geostationary satellites our experimental program took a new direction. Since these satellites appeared relatively stationary in respect to the observing station, we were now able to make continuous diurnal TEC, and scintillation measurements. For these measurements a simple polarimeter was developed at Urbana using the Applications Technology Satellite-I (ATS-I) as a signal source. This satellite transmitted several VHF telemetry carriers that were suitable for Faraday rotation and scintillation studies. The system worked well and several other polarimeters were built as NASA launched ATS-3 and ATS-5. The three stations of the equilateral triangle used with BE-B/C were changed over to polarimeter operation for continuous measurements of TEC. From measurements at these three stations several studies of TID's were conducted. Some of the results of these studies have been published in the open literature (papers 58, 64, 66, 69). The last of the data from this study is presently being analyzed

and the results will be published shortly.

In addition to the three station experiment, several other experiments have been carried out with other groups using the Urbana TEC values. These experiments have shown the variability of the ionosphere during disturbed conditions (papers 60, 88, 89). An atlas of TEC values for Urbana was prepared for the period of December 1967 to December 1970 (Tech. Report #47), and an atlas for the period of January 1970 to June 1973 is presently being prepared.

## 2.4 Experiments Performed Under Joint Support of this Grant and Other Grants or Contracts

### 2.4.1 Solar Eclipse Experiments

In the period covered by this grant two experiments were undertaken to investigate the effect in the ionosphere caused by a total eclipse of the sun. The first experiment funded primary by NSF was in 1963. In this experiment linearly polarized high power HF radio signals were transmitted toward the moon from two locations in the United States. The signals were received at several other locations in the United States and Canada after they were reflected from the moon. By measuring the Faraday rotation of the received signal the changes in the TEC were measured. This experiment was only marginally successful because of the weak nature of the received signal. For the results of this study see paper 19. The second solar eclipse study funded principally by a contract with the Air Force Cambridge Research Laboratories took place in 1970. In this experiment VHF signals from the ATS-3

satellite were used. Faraday rotation measurements were made at two locations along the path of totality and at the three Urbana stations. Excellent data was recorded at all locations, and the results of this experiment appear in paper no. 52.

#### 2.4.2 Conjugate Behavior of TEC

To study conjugate ionospheric effects in the TEC especially during disturbed conditions, a one year cooperative experiment was undertaken by us and the University of Auckland in New Zealand. This experiment was funded mainly by the Air Force Cambridge Research Laboratories. For this experiment, VHF signals from ATS-I were used to make Faraday rotation measurements with a polarimeter we installed at Cold Bay, Alaska. This data along with the reduced data from New Zealand was used in the study (paper no. 78).

#### 2.4.3 The Prediction of MUF based on TEC Measurements

Under a contract with the Air Force Research Laboratories a preliminary experiment in trying to predict the Maximum Usable Frequency (MUF) based on measurements of the TEC at either the midpoint or the end of a propagation path was undertaken. The HF transmissions from WWV at Boulder, Colorado were recorded at Danville, Illinois. A polarimeter to make Faraday rotation measurements was installed in Sioux City, Iowa, the midpoint of the path. Data was collected for a period of one year. The results of this study appear in paper no. 76.

## 2.5 Equipment Developed Under Joint Support of this Grant and Other Grants and Contracts

Throughout the course of this grant, several major items of equipment have been developed for use in conducting our experimental program. Some of the major equipment items are discussed below.

In ionosphere research it is often desirable to have available a means of monitoring the earth's magnetic field as an indicator of electromagnetic activity in the ionosphere. Toward this end, a nuclear magnetometer was built (Tech. Report #12). This instrument was in operation for a few years at the Urbana station. Its output data was used in several studies (see for example paper no. 19).

For the 1963 solar eclipse study two items of equipment were designed and built. The first item, totally supported by this grant was a low loss capacitive goniometer (paper 12). The second item, sponsored mainly by the National Science Foundation, was a low noise, solid state HF receiver, called a "Model B". The receiver was designed by us and built commercially for our use in the solar eclipse study. After the 1963 eclipse these receivers and the gonimeters were modified for use in our TEC measurements.

To make the continuous diurnal measurements of TEC using geostationary satellite transmissions, a novel polarimeter was built. This device is modeled after the design by Titheridge with one major difference. In our instrument the physical rotation of the antenna is replaced by a rotating gonimeter. We have built six of these instruments, and they have been in operation for approximately six years with very good results. For a description of the polarimeter see Technical Report No. 37.

In April 1974 NASA is scheduled to launch the ATS-F satellite. This satellite will be the first geostationary satellite that will have a beacon transmitter designed for ionosphere research. We are presently in the final stages of constructing a phase-locked receiving system for use with this satellite. The receiver will be capable of making Faraday rotation measurements at 140 MHz and dispersive group delay measurements between modulation sidebands at 140 and 360 MHz. To facilitate handling the large volume of received data, a analog-to-digital data-logging system using magnetic tape has been built (Technical Report No. 49). This system can accommodate sixteen channels of input analog data, and it can sample each input at various rates.

### 3. Summary of Research Carried Out under the Grant

The research results obtained under NASA support are all published as either technical reports or technical papers in the open literature. For those interested in knowing our research results a complete publication list is attached as Appendix I of this report. As can be seen in Appendix I, under the support of this grant, there appeared 41 technical reports and 82 technical papers. Moreover, many papers have been presented at the national and international meetings, symposia, seminars and workshops. It is impossible to describe in detail all our results in this report. We shall only give a brief summary.

#### 3.1 Source of Publication

In Table 1 is listed the source of publication. On the list are 17 different journals, plus 7 journals in which only one paper

Table 1

SOURCE OF PUBLICATION

<u>Name of Journal</u>	<u>No. of Articles</u>
Journal of Geophysical Research	18
Radio Science (or Journal of Research/NBS- D. Radio Propagation)	12
Journal of Atmospheric and Terrestrial Physics	6
IEEE Transactions on Antennas and Propagation	5
Planetary and Space Science	3
Nature	3
Physics of Fluids	3
Annales de Geophysique	2
Reviews of Geophysics and Space Physics	2
Journal of Mathematical Physics	2
Journal of Plasma Physics	2
Tellus	2
Applied Science Research	2
Space Research	2
Journal of Physics A (General Physics)	2
Journal of Franklin Institute	2
AGARDography	2
Other journals, one in each journal	7
Symposium Proceedings, etc.	<u>5</u>
TOTAL	82

appeared. It is clear that the publication record is international in scope.

### 3.2 Summary of Research in Terms of Topics

Our research can be broadly classified into 8 areas. They are (1) scintillation, (2) electron content, (3) acoustic-gravity waves, (4) plasma physics, (5) thermospheric dynamics, (6) propagation effects, (7) instrumentation and measurements, and (8) satellite drag. In the following we shall give only highlights in terms of these areas. The numerals following each area refer to the paper number in our publication record listed in Appendix I.

3.2.1 Scintillation (Experimental: 5, 13, 14, 15, 18, 23, 33, 51.  
Theoretical: 10, 16, 17, 20, 27, 28, 30, 32,  
37, 41, 42, 48, 53, 65, 67, 75, 79, 80, 82,  
84, 90)

Soon after the launch of the first artificial satellite it was learned that the radio signals transmitted by the satellite went into violent fading at times. The fading was caused by ionospheric irregularities and the phenomenon is referred to as scintillation. The occurrence of fading might be very sudden, indicating the presence of scintillation boundary. One of the first reports on satellite scintillation and scintillation boundary was made in paper 5. Subsequent studies were made in papers 15 and 23. By using spaced receiver techniques the irregularity heights were measured in papers 14 and 51. Using scintillation observations at Adak, Alaska and Wellington, New Zealand a correlation study has been carried out (see paper 33). The locations of these two

stations are sufficiently close to being a geomagnetic conjugate pair. Cross-correlation studies indicated that the conjugate effect was statistically significant at night, but not in daytime. There was also an indication of slight asymmetry with respect to L shells.

The theoretical work on scintillation can be classified as a part of studies of wave propagation in random media. The random medium can be composed of randomly positioned discrete scatterers (papers 16, 17) or it can be a random continuum. Most of our work has been concerned with random continuum approach. In this approach our work has progressed from cases in which the background medium is homogeneous and isotropic (papers 10, 20, 53) to cases in which the background medium is either inhomogeneous (papers 28, 32) or anisotropic (papers 27, 48, 65, 79). The mathematical techniques used in solving these problems were approximate. Generally some kind of perturbation method was used and the results could be interpreted as single scattering solution or Born solution. Iteration to get higher order Born solution has been found very difficult. Yet when the random fluctuations are strong these multiple scattering effects must be included. Some multiple scatter results have been obtained (papers 30, 42, 48, 67, 75, 84, 90). Our recent work has been concerned with pulse distortion, coherent bandwidth, spatial decorrelation and communication channel modeling.

Another approach is to use ray theory. In this case it is possible to derive a Fokker-Planck equation which describes the probability of finding a ray at a given position and a given ray direction (papers 37, 41). Since ray equations can be integrated numerically on computers fairly efficiently a Monte Carlo type

of computation was simulated on computers. These Monte Carlo results are described in papers 80 and 82.

3.2.2 Electron Content (Experimental: 9, 22, 25, 26, 31, 36, 38, 45, 78. Theoretical: 4, 6, 7, 11, 50, 71, 74)

When the transionosphere radio wave is received on the ground, the signal has already undergone a transformation in its wave properties. Wave properties such as frequency, amplitude, phase, polarization, direction of arrival, etc. would go through systematic changes. Since the signal traverses the complete ionosphere, the information that can be obtained is integrated. In the literature the integrated electron density is often referred to as the electron content.

Diurnal, seasonal and solar cycle variations in electron content have been reported in papers 9, 22, 25, 26, and 45. Comparison studies were conducted in terms of measurement techniques (paper 31) or in terms of locations (paper 36). Observations at geomagnetically conjugate stations (paper 78) showed that the day-to-day variations were negatively correlated at extreme magnetic quiet and increased with magnetic activities to positive values during magnetic storms. The electron content value and its gradient have been used to predict the direction of arrival of a ground-to-ground signal via an ionospheric reflection. The predicted results agreed with experimental results very closely (paper 38).

To assist the electron content experimental program some parallel theoretical investigations have to be carried out. Some deal with various propagation techniques (papers 4, 11, 74), others

deal with propagation or geomagnetic geometry (paper 7), or high order effects (paper 6), or computer simulations (papers 50, 71). In each case care was exercised to make sure that the interpretation of experimental results was a correct one. This assurance came from theoretical considerations and computer simulations.

3.2.3 Acoustic-Gravity Waves (Observational: 34, 39, 64, 66, 69, 70  
Theoretical: 40, 44, 46, 47, 58, 59,  
62, 83, 87)

It is now fairly well accepted that traveling ionospheric disturbances observed by many workers by many investigative techniques are excited by acoustic-gravity waves. In order to confirm the traveling properties it is necessary to set up spaced stations. For this purpose three stations at the vertex of an approximate equilateral triangle were set up. The first report of results was given in paper 34 using low orbiting satellite. Later a very unusual event was observed in collaboration with other workers (paper 39). More recently observations have been made on signals transmitted by geostationary satellites. The emphasis of these observations was on thermospheric dynamic aspect of acoustic-gravity waves (paper 64, 69, 70). In this regard the strong neutral winds in the thermosphere were expected to modify the acoustic-gravity wave propagation. Model computations (paper 66) have revealed a curious result in that most of Muro's average acoustic-gravity waves took a minimum time to reach the thermosphere.

On theoretical investigations we have found that our experience in electromagnetic wave propagation was extremely helpful. Based on this experience we were able to investigate a large class of

problems. One area of interest, especially in connection of dynamic heating, was wave damping. The damping could come about because of dissipative processes such as viscosity, thermal conduction and ion drag (paper 40). It could also come about through randomization through scattering (paper 46). These waves could be ducted, such as because of the presence of a vertical wind shear (paper 47). The behavior of the group ray, especially in the presence of winds, has been investigated in paper 58. The nonlinear behavior has been discussed from the point of view of wave-wave interaction (paper 44) or from the point of view of nonlinear dispersion relation (paper 83). The problem of generation of acoustic-gravity waves has been formulated and treated in paper 59 in a fairly general fashion. The processes involved in the interaction of these waves with the ionosphere were thoroughly studied in paper 62. Recently a review (paper 87) on this subject has been written (see section on Recent Developments and Appendix II).

#### 3.2.4 Plasma Physics (21, 24, 35, 43, 55)

In the course of studying low frequency waves in the ionosphere it was discovered that the ionosphere would become unstable if the electron density gradient became too large (papers 21 and 24). Such an instability was called the gradient instability by us.

The plasma is a collection of charged particles of which ionosphere is an example. The time evolution of plasma behavior is generally described statistically. The kinetic equation is but one example. An alternate method of description is to use the set of generalized stochastic equations (paper 35). Using such a set of generalized stochastic equations the dressed particle

description in plasma kinetic theory has been derived (paper 44). Another area we worked at was the subject of ion acoustic waves in a gravitational field (paper 55). The subject has application in the heating of solar corona.

### 3.2.5 Thermospheric Dynamics (29, 56, 60, 88, 89)

The terrestrial thermosphere is subject to continuous agitation. Such agitations can come from many sources such as tides (paper 29), the dynamo electric field combined with pressure gradients due to heating to produce thermospheric winds (paper 56), magnetic storm phenomenon (papers 60 and 89), and the solar radiation connected with solar flares (paper 88).

### 3.2.6 Propagation Effects (Experimental: 2, 8

Theoretical: 57, 61, 68, 81)

In early satellite days there was controversy about possibilities of reflecting radio signals off of satellite plasma wake, especially by using signals transmitted by WWV. An experiment was therefore performed (paper 2). It was found that WWV signal bursts were not correlated with the satellite transits. Another experiment of interest was the behavior of signal strength near the skip distance (paper 8).

Theoretically many problems were investigated. The problem of wave-wave interaction involving two electromagnetic waves and one ion sound wave was investigated (paper 57). It was found that through nonlinear resonant interaction the electromagnetic wave could be trapped. In the ionospheric research the Luxembourg effect has received appreciable attention. The problem is difficult

because of nonlinear characters. However, often in the interaction region, there may also present irregularities. Therefore, it is of interest to investigate the Luxembourg effect when there are ionospheric irregularities. This problem was treated in paper 81. Another problem received our attention was the reflection of signals from randomly oscillating conducting plane (paper 61).

At the invitation of the editor of Reviews of Geophysics and Space Physics a 78-page review on ionospheric propagation was published (paper 68).

### 3.2.7 Instrumentation and Measurements (12, 54)

In order to make Faraday rotation measurements a low insertion-loss capacitively scanned goniometer was invented (paper 12). This device was made widely available to many research groups (e.g. Stanford University) and has worked successfully to this day.

### 3.2.8 Satellite Drag (1, 3)

In the first few years of this project astronomers were involved in the computation of satellite orbits and the study of drag effects (papers 1 and 3).

## 3.3 Recent Developments

We wish to describe our recent work on solar flare effects on the ionosphere and on our review of acoustic-gravity waves.

### 3.3.1 Flare Effects

In collaboration with many observatories around the world the behavior of the ionosphere during a solar flare was studied. The paper was submitted and accepted for publication in JGR.

In the following the abstract is reproduced.

THE BEHAVIOR OF THE IONOSPHERIC F-REGION DURING  
THE GREAT SOLAR FLARE OF 7 AUGUST 1972

M. Mendillo, J. A. Klobuchar, R. B. Fritz,  
A. V. da Rosa, L. Kersley, K. C. Yeh, B. J. Flaherty  
S. Rangaswamy, P. E. Schmid, J. V. Evans,  
J. P. Schodel, D. A. Matsoukas, J. R. Koster,  
A. R. Webster, P. Chin

ABSTRACT

The response of the ionospheric F-region to the large solar flare which occurred near 15:00 U.T. on 7 August 1972 has been monitored by means of Faraday rotation measurements made at 17 stations in North America, Europe and Africa. With observations spanning over ten hours in local time and over 70 degrees in latitude, the first truly global morphology of a flare-induced F-region event was obtained. The sizes of the individual sudden increases in the total electron content (SITEC's) ranged from  $1.8$  to  $8.6 \times 10^{16}$  el/m<sup>2</sup>; on a percentage basis, all of the SITEC's fell within the 15 to 30% range. No obvious relationship was found between the sizes of the increases and the solar zenith angles at the various sub-ionospheric points, nor between the observed SITEC's and the sudden flare effects (SFE's) noted on nearby magnetometer recordings. The latitudinal behavior provided the only simple ordering parameter found in the data, with the lower latitudes having larger observed increases than the higher latitudes. Millstone Hill incoherent scatter data showed that nearly 40% of the total content enhancement observed at that site came from heights above 300 km. All of the SITEC's has a rise-time of about

10 minutes during which the TEC rate of change showed an excellent correlation with the time development of the solar radio burst monitored at 35,000 MHz.

### 3.3.2 Review on Acoustic-Gravity Waves

At the invitation of the editor of Reviews of Geophysics and Space Physics a review on acoustic-gravity waves was prepared. The complete paper has been included in this report as Appendix II. Interested readers should consult that Appendix. A somewhat condensed version is tentatively scheduled to appear in the May 1974 issue of Reviews of Geophysics and Space Physics.

## 4. List of Graduates

In the following we give a list of graduates. Unless otherwise specified the degree awarded will be from the Department of Electrical Engineering, University of Illinois.

### 4.1 M.S. Degree

Before about 1964 the Department of Electrical Engineering did not have M.S. thesis requirements. Therefore, only names will be listed for those graduated before 1965. For those graduated after 1965, names as well as M.S. theses are given.

J. P. McClure  
William Butler  
Henri Robe (Astronomy)  
R. A. Schorn (Astronomy)  
William Johnston  
Henry Chow  
Thomas Ronald Pound  
William Greig (Astronomy)  
Carl Steubenrauch  
Bernard J. Flaherty  
Richard Eckhouse  
Paul Kranz, Scintillation observations of satellite signals, 1965.

- J. R. Cornellier, Ionospheric electron density during solar eclipse of 20 July 1963, 1966.
- A. P. Weise, Latitude dependence of ionospheric electron content, 1967.
- S. Chalasani, A solid-state programmable digital clock, 1967.
- M. Y. Youakim, Study of ionospheric electron content from observations at different stations, 1968.
- James H. Fritz, An investigation of F2 region ionospheric behavior by computer methods, 1968.
- Lawrence M. Paul, Measurement of irregularity heights by the spaced receiver technique, 1968.
- Han R. Cho, Neutral winds and the behavior of the ionospheric F2 region, 1969.
- Homayoun Nomani, A study of ionospheric electron content at geomagnetically conjugate locations, 1970.
- Bradford Joseph Bolfig, A study of the relation between the maximum usable frequency and electron content in the ionosphere, 1971.
- Robert Edward Terry, A very high frequency radio interferometer for investigating ionospheric disturbances using geostationary satellites, 1972.
- Richard Edward DuBroff, Determination of changes in exospheric electron content by a comparison of group delay and Faraday rotation, 1972.
- Amaro Lopes DeAbreu Neto, Generation of acoustic-gravity waves in an isothermal atmosphere by moving sources, 1972.
- Thomas C. Lloyd, A slow-speed multiple-channel analog-to-digital data logging system, 1973.
- Allen Lee Hearn, Study of the response of the ionospheric F region to magnetic storms, 1973.
- Edward Shi Chen, Generation of acoustic-gravity waves in an isothermal atmosphere by impulse sources, 1973.
- Scott Franklin Wetenkamp, A phase-lock receiver for measuring Faraday rotation and dispersive group delay on geostationary satellite signals, 1973.

#### 4.2 Ph.D. Degrees

- Victor H. Gonzalez, A method of determination of ionospheric electron density profiles from Faraday rotation of satellite born radio signals, 1962.
- N. C. Mathur, Multiple scattering of electromagnetic waves by random scatterers of finite size, 1964.
- J. P. McClure, Beacon satellite studies of small, scale ionospheric inhomogeneities, 1964.
- T. R. Pound, Response of ionospheric electron density to a change of electron temperature, 1966.
- Kam-Chuen So, Generalized stochastic equations and their applications to plasmas, 1967.
- David H. Cowling, A study of traveling disturbances in the ionosphere, 1970.

Robert Morris Clark, Gravity waves in the ionosphere, 1970.  
Maurice Yacoub Youakim, A numerical study of rays in random  
media, 1970.  
Dale Martin Simonich, Radio wave scattering from random  
fluctuations in an anisotropic medium, 1971.  
Han-Ru Cho, A study of nonlinear atmospheric waves, 1972.

## APPENDIX

## Publication List of Ionosphere Radio Laboratory\*

Papers in Scientific Journals

1. I. R. King, G. C. McVittie, G. W. Swenson, Jr., and S. P. Wyatt, "Further Radio Observations of the First Satellite," Nature, 180, 943, 1957.
2. C. D. Hendricks, Jr., G. W. Swenson, Jr., and R. A. Schorn, "Radio Reflections from Satellite-Produced Ion Columns," Proc. I.R.E., 1763, October, 1958.
3. S. P. Wyatt, "Solar Effects in the Motion of Vanguard," Nature, 184, 351, 1959.
4. G. W. Swenson, Jr., "Radio Observations of Earth Satellites," Advances in Astronautical Sciences, 4, 393-397, 1959.
5. K. C. Yeh and G. W. Swenson, Jr., "The Scintillation of Radio Signals from Satellites," J. Geophys. Res., 64, 2281-2286, 1959.
6. K. C. Yeh, "Second-Order Faraday Rotation Formulas," J. Geophys. Res., 65, 2548-2550, 1960.
7. K. C. Yeh and V. H. Gonzalez, "Note on the Geometry of the Earth Magnetic Field Useful to Faraday Effect Experiments," J. Geophys. Res., 65, 3209-3214, 1960.
8. K. C. Yeh and G. W. Swenson, Jr., "Observed Field Strength in the Neighborhood of the Skip Distance," J. Geophys. Res., 66, 654-656, 1961.
9. K. C. Yeh and G. W. Swenson, Jr., "Ionospheric Electron Content and Its Variations Deduced from Satellite Observations," J. Geophys. Res., 66, 1061-1067, 1961.
10. K. C. Yeh, "Propagation of Spherical Waves Through an Ionosphere Containing Anisotropic Irregularities," J. Research, 66D, No. 5, 621-636, September-October 1962.
11. G. W. Swenson, Jr., "The Utilization of Ionosphere Beacon Satellites," NASA Technical Note, D-1669, May 1962.
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Appendix II

ACOUSTIC-GRAVITY WAVES IN THE UPPER ATMOSPHERE

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## ACOUSTIC-GRAVITY WAVES IN THE UPPER ATMOSPHERE

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### Abstract

In this paper we review the theory of acoustic gravity waves, the interaction of such waves with the ionosphere, the experimental support for the existence of such waves in the upper atmosphere, and their implication on the thermospheric dynamics. After a thorough discussion on the properties of acoustic gravity waves in an ideal isothermal atmosphere, the effects produced by horizontal winds, sharp boundary discontinuities, and dissipative processes are discussed. The generation of these waves by stationary or moving sources is then treated.

It is shown that the atmospheric response to an impulse source can be described by the emission of three waves: acoustic, buoyancy and gravity. These discussions are then followed by reviewing propagation effects in a realistic atmosphere for both free waves and guided waves. Recent numerical results are given. When acoustic gravity waves propagate through the ionosphere, interaction between the wave and the ionosphere will take place. The physical processes involved in such an interaction are examined. The response of the ionosphere to acoustic gravity waves can be fairly complex but its understanding is necessary to interpret various experimental data. We then review the existing experimental data on traveling disturbances and find, with very few exceptions, that most of the observed data in the upper atmosphere and ionosphere can be interpreted in terms of acoustic gravity waves, in agreement with Hines. The existence of such waves throughout the atmosphere implies coupling between the lower atmosphere and the upper atmosphere. Both momentum and energy are transported by the wave process. The implication of such coupling on thermospheric dynamics is discussed.

# ACOUSTIC-GRAVITY WAVES IN THE UPPER ATMOSPHERE

K. C. Yeh and C. H. Liu

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## A. INTRODUCTION

The earth's atmosphere is capable of sustaining a large number of wave phenomena. Based on human experience the most familiar wave process is the propagation of sound. The sound propagates as acoustic waves whose wave motion arises from interchange between the kinetic energy of the fluid and the potential energy stored in compression. But in the terrestrial environment gravity is an important force. The role of gravity in the propagation of atmospheric waves was demonstrated by Lamb [1908, 1910]. The atmospheric stratification in the presence of gravity gives rise to a restoring force known as the buoyancy force. Under certain conditions this restoring force is responsible for atmospheric oscillations at a characteristic frequency known variously as the buoyancy frequency or the Brunt-Väisälä frequency [Väisälä, 1925]. When the frequency is much higher than the buoyancy frequency as in the ordinary acoustic wave, the gravitational effects can be ignored. As the frequency is gradually lowered to a value comparable to the buoyancy frequency the gravitational potential energy associated with the wave motion may have an equal order of magnitude as the kinetic energy and the compressional energy. When this is the case the wave is called the acoustic gravity wave. Since the gravity acts in one direction the related wave process is expected to be highly anisotropic. The subject of the propagation of acoustic gravity waves has been previously treated and reviewed [Gossard and Munk, 1954; Eckart, 1960; Tolstoy, 1963]. There have been several special symposia and conferences on acoustic gravity waves, where interested readers may find detailed information [Special Issue, 1968; Georges, 1968a; AGARD, 1972].

The existence of acoustic gravity waves at thermospheric and ionospheric heights is inferred mainly by indirect ground based measurements on ionization. The early systematic study was made by Munro [1950, 1958] and others [e.g., Toman, 1955; Heisler, 1958; Valverde, 1958] who presented data to show properties of traveling ionospheric disturbances. Subsequent experimentation showed such disturbances in ionization could be detected by a large number of radio experiments. Martyn [1950] attempted to explain these disturbances in terms of cellular atmospheric waves. This theory has

since been refined and developed fully by Hines [1960]. Currently most investigators accept Hines theory although with additional refinements and modifications.

This paper reviews various aspects of acoustic gravity waves in the upper atmosphere and their interaction with the ionosphere. The topics discussed have been given in the Table of Contents.

## B. ATMOSPHERIC ENVIRONMENT

In order to study the propagation of atmospheric waves it is desirable to know about the environment within which these waves are going to propagate. The primary properties of the atmosphere are its density, pressure, temperature, composition, and motion. These properties are highly variable in time and in geographic location. In the following we will discuss some of these properties relevant to us in terms of model atmospheres, keeping in mind that the real atmosphere may depart from these models appreciably.

### 1. Structure of the Atmosphere

One of the most striking features of the earth's atmosphere is its vertical diminution of density. The cause of this rapid decrease with height is the strong gravitational force which is balanced by the pressure gradient force. This force balance is described by the hydrostatic equation

$$dp = - \rho g dz \quad (1)$$

where  $p$  is the pressure,  $g$  the gravity,  $\rho$  the mass density and  $z$  the height. For an ideal gas, which we assume, the pressure and density are related by

$$p = \rho KT/m \quad (2)$$

where  $T$  is the temperature,  $K$  Boltzmann's constant and  $m$  the mean molecular mass. Eliminating  $\rho$  between (1) and (2) and integrating the resultant equation from a reference height  $z_0$  at which  $p = p_0$  to an arbitrary height  $z$ , we obtain

$$p = p_0 \exp\left(-\int_{z_0}^z dz/H\right) \quad (3)$$

The scale height  $H$  in (3) is defined by

$$H = KT/mg \quad (4)$$

If  $H$  is given as a function of  $z$ , then (3) can be used to find the pressure at any height. The values of  $H$  are tabulated in almost all model atmospheres [e.g. CIRA, 1965], and its structure is rather complex. For the special case of an isothermal atmosphere

$$p = p_0 \exp(-z/H) \quad (5)$$

which describes an exponentially distributed pressure.

The speed of sound  $c_0$  is given by  $c_0^2 = (dp/d\rho)_{\text{adiabatic}}$ . For an ideal gas it reduces to

$$c_0^2 = \gamma p / \rho = \gamma H g \quad (6)$$

Taking values given by the U.S. Standard Atmosphere 1962 [Valley, 1965] for heights below 100 km and values given by CIRA [1965] for heights above 100 km, the speed of sound can be computed as shown in Fig. 1. Note the presence of sound ducts where the speed of sound is a minimum. The possibilities of such ducts to serve as waveguides of acoustic energy have been discussed by several authors [e.g. Pfeffer and Zarichny, 1962]. In the thermosphere the speed of sound varies diurnally and with solar activities.

## 2. Characteristic Frequency of Oscillation

The gravitational field is responsible for making the atmosphere inhomogeneous as discussed in section 1 and consequently for making the propagation of atmospheric waves anisotropic. In an ideal fluid, a small parcel of air can move with a constant velocity along the equipotential surface without doing any work. However, if the displacement is away from the equipotential surface, such as in a vertical direction, the fluid parcel experiences a buoyancy force which tends to restore the equilibrium configuration. The speed with which the equilibrium is restored characterizes the response time of the atmosphere.

Let us apply an external force which displaces a small parcel of air vertically upward a small distance  $\zeta$  in an atmosphere in hydrostatic equilibrium. In its new position the air parcel experiences the buoyancy force. Now if the external force is removed, the air parcel will move according to

$$\rho \ddot{\zeta} = -g \Delta \rho \quad (7)$$

We note that  $\Delta \rho = (\Delta \rho)_{\text{inside}} - (\Delta \rho)_{\text{outside}}$ . Let the density change inside the air parcel take place adiabatically, then  $(\Delta \rho)_{\text{inside}} = \Delta p / c^2 = -(\rho g / c^2) \zeta$ . Due to inhomogeneous nature of the atmosphere, the background density at the new position is different and it is changed by an amount  $(\Delta \rho)_{\text{outside}} = -\zeta d\rho / dz$ . Taking both contributions into account, (7) can be recast into

$$\ddot{\zeta} = -\omega_b^2 \zeta \quad (8)$$

If  $\omega_b^2$  is positive, the air parcel oscillates with a characteristic buoyancy frequency (also referred to as the Brunt-Väisälä frequency) given by [Väisälä, 1925; Eckart, 1960; Tolstoy, 1963]

$$\omega_b^2 = (\gamma - 1)g^2 / c^2 + (g / c^2) dc^2 / dz \quad (9)$$

If  $\omega_b^2$  as given by (9) is negative, the initial perturbation will grow exponentially with time and the atmosphere becomes unstable. The condition for marginal stability is  $\omega_b^2 = 0$  or

$$dT/dz = -mg(\gamma - 1) / \gamma K$$

which is just the lapse rate of an isentropic atmosphere.

In an isothermal atmosphere with a constant mean molecular mass (9) reduces to

$$\omega_b^2 = (\gamma - 1)g^2 / c^2 \quad (10)$$

In the terrestrial atmosphere the buoyancy period  $T_b = 2\pi / \omega_b$  has a height structure given by Fig. 2.

The preceding analysis assumes that the oscillation amplitude is so small that linearization procedure is valid. The nonlinear theory predicts that the oscillation frequency will be shifted [Cho, 1972]. For an isothermal atmosphere it reduces to

$$\omega = \omega_b - \frac{1}{6} \left( \frac{\gamma - 1}{\gamma} \right)^2 \frac{\varepsilon^2}{H^2} \omega_b + \dots \quad (11)$$

As seen from (11) the frequency shift is unimportant if the oscillation amplitude  $\epsilon$  is small in comparison with the scale height. Otherwise, the nonlinear effects must be included.

### C. PROPAGATION OF ACOUSTIC-GRAVITY WAVES IN AN ISOTHERMAL ATMOSPHERE

We assume that the atmospheric behavior can be described by a set of hydrodynamic equations. These are macroscopic equations that are based on conservation laws [e.g. Landau and Lifshitz, 1963]. For our purpose they take the following form:

$$\partial \rho / \partial t + \nabla \cdot (\rho \vec{v}) = q_1 \quad (12)$$

$$\rho D\vec{v}/Dt + \nabla p - \rho \vec{g} + 2\rho \vec{\Omega} \times \vec{v} = \vec{f} \quad (13)$$

$$\rho TDs/Dt = Q \quad (14)$$

In the continuity equation (12),  $q_1$  is the rate of production of mass per unit volume and is assumed to be localized. In the equation of motion (13)  $\vec{v}$  is the fluid velocity,  $\vec{g}$  the gravitational acceleration,  $\vec{\Omega}$  the Coriolis vector with a direction parallel to the earth's rotational axis toward Polaris and with a magnitude equal to  $7.28 \times 10^{-5}$  rad/sec. The vector  $\vec{f}$  is the resultant of all other forces, including momentum sources and processes related to the viscous stresses. It can be represented by

$$\vec{f} = \vec{q}_2 + \nabla \cdot \vec{\tau} - v_{in} \rho_i (\vec{v} - \vec{v}_i) \quad (15)$$

The localized momentum source  $\vec{q}_2$  contains the external momentum source as well as the component connected with the mass production. The viscous stress tensor is given by

$$\vec{\tau} = \eta [\nabla \vec{v} + (\nabla \vec{v})^T] - (2/3) \eta \vec{I} \nabla \cdot \vec{v} \quad (16)$$

In (16), a superscript T is used to denote the transpose of a tensor,  $\eta$  the coefficient of viscosity and  $\vec{I}$  the identity tensor. The second coefficient of viscosity is always ignored in gravity wave theory, perhaps because it is largely unknown and also because the gravity waves under Boussinesq approximation are nearly incompressible so that the second viscosity does not contribute. The last term of (15) represents transfer of momentum due to collisions with the ions. Equation (14) is the heat transfer

equation. The change in entropy per unit mass  $s$  for an ideal gas can also be written in alternate forms,

$$\begin{aligned} Ds/Dt &= (Dp/Dt - c^2 D\rho/Dt)/(\gamma-1)\rho T \\ &= (c_v/T)DT/Dt - (p/\rho^2 T)D\rho/Dt \end{aligned} \quad (17)$$

where  $\gamma$  is the ratio of specific heats and  $c_v$  the specific heat capacity at constant volume. The quantity  $Q$  of (14) is given by

$$Q = q_3 + \nabla \cdot (\kappa \nabla T) + \tilde{\tau} : \nabla \vec{v} + v_{in} \rho_i (\vec{v} - \vec{v}_i) \cdot (\vec{v} - \vec{v}_i) \quad (18)$$

In (18)  $q_3$  is the heat source which includes heat production due to mass source and momentum source as well as the external heat input and  $\kappa$  is the thermal conductivity.

The propagation of waves in a lossy atmosphere is accompanied by dissipation. The total rate of energy dissipation is

$$W_t = T_0 \frac{d}{dt} \int \rho s dV$$

where integration is carried out over all sourceless region. The temperature  $T_0$  is the temperature of the equivalent thermodynamic system with the same entropy. Making use of (12) and (14), the total dissipation can be computed. On a unit volume basis the dissipation due to thermal conductivity, viscosity and ion drag can be shown to be given by [Landau and Lifshitz, 1963, p. 298]

$$W = \kappa (\nabla T)^2 / T_0 + \tilde{\tau} : \nabla \vec{v} + v_{in} \rho_i (\vec{v} - \vec{v}_i)^2 \quad (19)$$

We note that the set of equations (12) through (14) are nonlinear. For small perturbations we may adopt the linearization procedure. Use a subscript 0 for unperturbed quantities and a prime for perturbed quantities. The linearized equations take the form

$$\partial \rho' / \partial t + \vec{v}' \cdot \nabla \rho_0 + \rho_0 \nabla \cdot \vec{v}' = q_1' \quad (20)$$

$$\rho_0 \partial \vec{v}' / \partial t + \nabla p' - \rho' \vec{g} + 2\rho_0 \vec{\Omega} \times \vec{v}' = \vec{f}' \quad (21)$$

$$\partial p' / \partial t + \vec{v}' \cdot \nabla p_0 - c_0^2 \partial \rho' / \partial t - c_0^2 \vec{v}' \cdot \nabla \rho_0 = (\gamma-1)Q' \quad (22)$$

where the unperturbed atmosphere is assumed to be stationary and time independent. Based on this set of equations the following conservation law can be derived in the sourceless region for an inviscid atmosphere originally in hydrostatic equilibrium.

$$\frac{\partial}{\partial t} \left[ \frac{1}{2} \rho_0 \vec{v}'^2 + \frac{p'^2}{2\rho_0 c_0^2} + \frac{g}{2\rho_0 \beta c^2} (p' - c_0^2 \rho')^2 \right] + \nabla \cdot (\vec{v}' p') = 0 \quad (23)$$

where  $\beta = (\gamma - 1)g + dc^2/dz$ . We can identify  $\rho_0 \vec{v}'^2/2$  as the kinetic energy,  $p'^2/2\rho_0 c_0^2$  as the elastic energy of acoustics and  $g(p' - c_0^2 \rho')^2/2\rho_0 \beta c_0^2$  as the thermobaric energy [Eckert, 1960, p. 54]. Both kinetic energy and elastic energy are familiar forms of energy. The thermobaric energy comes about through departures of the fluid from its naturally buoyant state. This is because the time rate of change of the thermobaric energy can be put in the form  $\vec{v}' \cdot \vec{g} \Delta \rho$  which is just the work done against the buoyancy force (compare with eq. (7)). The second order energy flux is given by  $\vec{v}' p'$ . It should be mentioned that even though all terms of (23) are of second order, there are second order energy and flux terms not contained in (23) [see discussion by Eckert, 1960, p. 54]. But (23) does contain all energy and flux terms involving the first order quantities. Consequently, they have been called the pseudo-energy and the pseudo-energy flux by some authors [Sturrock, 1962]. Since only these pseudo quantities are discussed in this paper we shall omit the prefix "pseudo" for simplicity.

Our interest in this chapter is to solve the set (20)-(22) in a lossless isothermal background atmosphere for which many simplifications will result in all the equations discussed so far in this chapter.

### 1. Adiabatic Process

It is customary to assume that atmospheric perturbations take place adiabatically. The assumption is valid if the speed of sound is much greater than the speed of heat conduction. Heat conducts with a speed given by  $k/\rho c_V$  where  $k$  is the wave number. Numerically, the heat conduction speed is estimated to be  $4 \times 10^{-9}$ , 8, 50 and 150 m/sec at the earth surface, 200 km height, 300 km height and 400 km height respectively. When compared with the

speed of sound shown in Fig. 1, the effect of heat conduction is expected to be negligible up to a height of 200 km, above which it will serve as a damping mechanism for the wave.

When  $\kappa = 0$ , the process is adiabatic and we may take  $\gamma = 1.4$  for air. When  $\kappa = \infty$ , the process is isothermal, for which  $\gamma = 1$ . As we will see later the internal gravity wave branch disappears if the process is isothermal. It has been shown [Grigor'ev, 1967; Francis and Kahn, 1970] that for finite  $\kappa$  the quantity  $\gamma$  in the adiabatic theory should be replaced by  $\gamma_{\text{eff}}$  which is given by

$$\gamma_{\text{eff}} = (1 - i\gamma\omega\tau) / (1 - i\omega\tau)$$

where  $\tau = \rho c_v / k^2 \kappa$  and the time dependence  $\exp(-i\omega\tau)$  is assumed.

## 2. Dispersion Relation

The pressure and density in an isothermal atmosphere (more accurately, an atmosphere with a constant scale height) are both exponentially distributed as can be seen from (5). The perturbation equations are given by (20)-(22). If we ignore all sources and loss processes the right hand sides of (20)-(22) can be set to zero. For plane geometry with Coriolis effect ignored, the set (20)-(22) can be written as a matrix equation [Liu and Yeh, 1971]

$$\tilde{D} \cdot \vec{F} = 0 \quad (24)$$

where the matrix operator  $\tilde{D}$  is given by

$$\tilde{D}(v, a/\partial t) = \begin{pmatrix} \partial/\partial t & 0 & \partial/\partial x & \partial/\partial y & \partial/\partial z - 1/2H \\ 0 & \partial/\partial x & \partial/\partial t & 0 & 0 \\ 0 & \partial/\partial y & 0 & \partial/\partial t & 0 \\ g & \partial/\partial z - 1/2H & 0 & 0 & \partial/\partial t \\ -c^2 \partial/\partial t & \partial/\partial t & 0 & 0 & (\gamma-1)g \\ 0 & & & & \end{pmatrix} \quad (25)$$

and the field vector is

$$\vec{F} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{pmatrix} = \begin{pmatrix} \rho'/\rho^{1/2} \\ 0 \\ p'/\rho^{1/2} \\ 0 \\ v'_{x\rho}{}^{1/2} \\ 0 \\ v'_{y\rho}{}^{1/2} \\ 0 \\ v'_{z\rho}{}^{1/2} \\ 0 \end{pmatrix} \quad (26)$$

The convenience of the assumed isothermal background atmosphere is clear since  $\tilde{D}$  has constant coefficients. Its determinant is given by

$$D(\nabla, \partial/\partial t) = \det \tilde{D} = \partial^4/\partial t^4 - c_0^2(\nabla^2 - 1/4H^2) - \omega_b^2 c_0^2 \nabla_h^2 \quad (27)$$

where  $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$  and the buoyancy frequency  $\omega_b$  is given by (10). The differential operator (27) may be called the acoustic-gravity wave operator. For plane waves, we may replace  $\partial/\partial t$  by  $-i\omega$ ,  $\partial/\partial x$  by  $ik_x$ ,  $\partial/\partial y$  by  $ik_y$  and  $\partial/\partial z$  by  $ik_z$  in (27). Then setting  $D = 0$  yields directly the dispersion relation [Eckart, 1960; Hines, 1960]

$$k_h^2(1 - \omega_b^2/\omega^2) + k_z^2 = k_0^2(1 - \omega_a^2/\omega^2) \quad (28)$$

In (28),  $k_0 = \omega/c_0$ , and  $\omega_a = c_0/2H$  is known as the acoustic cutoff frequency. For  $\gamma = 1.4$ ,  $\omega_b = 0.904\omega_a$ . In this case (28) gives two branches of propagation for internal waves for which  $\omega$  and  $\vec{k}$  are real. These two branches are shown hatched in Fig. 3. The high frequency branch is the acoustic branch ( $\omega > \omega_a$ ). The dispersion surface given by (28) is the surface of an ellipsoid with its major axis in the horizontal direction, having a magnitude  $k_0(1 - \omega_a^2/\omega^2)^{1/2}/(1 - \omega_b^2/\omega^2)^{1/2}$  and with its minor axis in the vertical direction having a magnitude  $k_0(1 - \omega_a^2/\omega^2)^{1/2}$ . Since the surface is closed, there is no resonance. The phase speed of acoustic waves is always greater than the sound speed. In the limit  $\omega \rightarrow \infty$ , (28) reduces to

$$k^2 = k_0^2 \quad (29)$$

This limiting wave propagates just like the ordinary sound and is isotropic. The region  $\omega_b < \omega < \omega_a$  is the cutoff region in which there

are no free waves. The low frequency branch in the region  $\omega < \omega_b$  is the gravity branch. The dispersion surface given by (28) is the surface of revolution of a hyperbola about the vertical axis. The surface intersects the horizontal axis at  $k_0 (\omega_a^2/\omega^2 - 1)^{1/2} / (\omega_b^2/\omega^2 - 1)^{1/2}$ . The resonance at which  $\vec{k} \rightarrow \infty$  occurs when the propagation has a polar angle  $\theta_r$  given by

$$\sin \theta_r = \omega/\omega_b \quad (30)$$

In the gravity wave branch,  $k$  is always greater than  $k_0$ , showing that the phase speed of the wave is always less than the speed of sound. A sample set of dispersion surfaces is shown in Fig. 4. The importance of dispersion surfaces in the study of wave propagation has been thoroughly discussed [e.g. Lighthill, 1960; Lighthill, 1965].

There are cases of interest in which  $\omega$  and  $\vec{k}$  in the dispersion relation (28) are not all real. For example, for surface waves  $k_z$  is purely imaginary, i.e.  $k_z = ik_z''$ . For this case (28) becomes

$$k_h^2 (1 - \omega_b^2/\omega^2) - k_z''^2 = k_0^2 (1 - \omega_a^2/\omega^2) \quad (31)$$

In this case the field vector  $\vec{F}$  of (26) decays exponentially with height. (The growing solution is discarded for finiteness of energy flux  $\vec{v} \cdot \vec{p}'$  as  $z \rightarrow \infty$ ). A special case studied by Lamb [1932] known as Lamb waves can be obtained from (31) as

$$\begin{aligned} k_x^2 + k_y^2 &= k_0^2 \\ k_z'' &= -(2-\gamma)/2\gamma H \end{aligned} \quad (32)$$

This case corresponds to the 45° line of Fig. 3. Lamb waves propagate horizontally with a speed  $c_0$ . Since  $v_z' = 0$ , Lamb waves can be supported by a rigid ground.

The effect of Coriolis force can be studied by retaining the term  $2\rho_0 \vec{\Omega} \times \vec{v}'$  in the equation of motion. The differential operator  $\vec{D}$  given by (25) should be appropriately modified. For the special case of vertical axis of rotation, the dispersion relation becomes [Eckart, 1960]

$$k_h^2 (1 - \omega_b^2/\omega^2) / (1 - 4\Omega^2/\omega^2) + k_z^2 = k_0^2 (1 - \omega_a^2/\omega^2) \quad (33)$$

With this modification, the lower boundary of gravity wave propagation region is changed from  $\omega = 0$  as was the case in (28) to  $\omega = 2\Omega$ . The rotational correction is of the order  $\Omega^2/\omega^2$  on  $\rho'$  and  $p'$  and of the order  $\Omega/\omega$  on  $\vec{v}$ . Therefore, even for waves having a one-hour period, the effect of rotation on  $\rho'$  and  $p'$  will be only about one per cent.

For different applications it is sometimes desirable to make approximations. Some of these are tabulated in Table 1. In reference to Table 1 we note that the Boussinesq approximation assumed (i) the fluid is incompressible and (ii) variations in the fluid density are neglected in the inertia term and are taken into account in the gravitational term. Assumption (ii) is valid when the vertical wavelength is small in comparison with the scale height (compare with Hines' asymptotic limit given in the footnote of Table 1). Because of assumption (i) the Boussinesq fluid cannot support acoustic waves. To remedy this situation the assumption (i) can be removed, resulting in the semi-Boussinesq approximation for which the acoustic branch reappears.

### 3. Properties of Internal Waves

We have seen that the dispersion surface given by (28) is not spherical, indicating anisotropic nature of these waves. Since the group velocity is in a direction normal to the dispersion surface, the phase progression and the wave packet will generally propagate in different directions [Yeh and Liu, 1972a]. By differentiating (28) we can obtain the group velocity

$$\vec{v}_g = [\hat{x}k_x(\omega^2 - \omega_b^2) + \hat{y}k_y(\omega^2 - \omega_b^2) + \hat{z}k_z\omega^2] \omega c_0^2 / (\omega^4 - \omega_b^2 k_x^2 c_0^2) \quad (34)$$

where  $\hat{x}$ ,  $\hat{y}$  and  $\hat{z}$  are unit vectors along respective coordinate axes. For acoustic waves ( $\omega > \omega_a$ ) the x-, y- and z- components of  $\vec{v}_g$  all have the same sign as  $k_x$ ,  $k_y$  and  $k_z$  respectively. However, for gravity waves ( $\omega < \omega_b$ ) the x- and y- components of  $\vec{v}_g$  still have the same sign as  $k_x$  and  $k_y$ , but z- component of  $\vec{v}_g$  and  $k_z$  have opposite signs.

The system of equations (24) has a unique solution except for a constant multiplier. Therefore, we may solve four of the five components of  $\vec{F}$  in terms of the remaining. Experience has shown it is convenient to express all other components in terms of  $F_2$ , resulting in the following polarization relations

$$\begin{aligned} F_2 &= F_1 c_0^2 (\omega^2 - \omega_b^2) / \omega^2 = F_3 \omega / k_x = F_4 \omega / k_y \\ &= F_5 (\omega^2 - \omega_b^2) / \omega [k_z + i\omega_a (\gamma - 2) / \gamma c_0] \end{aligned} \quad (35)$$

Table 1. Dispersion relations for different fluids

Fluid Property	Dispersion Relation	Comments	References
Incompressible and homogeneous	$k_z^2 = -k_x^2$	No free waves. Boundaries are necessary to support surface waves	
Compressible and homogeneous	$k_z^2 = k_0^2 - k_x^2$	Longitudinal acoustic waves	e.g. Landau and Lifshitz (1963)
Compressible and isothermal but with zero gravity	$k_z^2 = k_0^2 - k_x^2 - (1/2H)^2$	Free waves for $\omega > \omega_a = \gamma g / 2c_0$ Has both longitudinal and transverse components	Bergmann (1946)
Incompressible and isothermal	$k_z^2 = -k_x^2 (1 - \omega_b^2 / \omega^2) - (1/2H)^2$	Free waves for $\omega < \omega_b = \gamma^{1/2} g / c_0$	Tolstoy (1963)
Boussinesq fluid*	$k_z^2 = -k_x^2 (1 - \omega_b^2 / \omega^2)$	Free waves for $\omega < \omega_b = (\gamma - 1)^{1/2} g / c_0$ Has only transverse components	Chandrasekhar (1961)
Semi-Boussinesq fluid	$k_z^2 = k_0^2 - k_x^2 (1 - \omega_b^2 / \omega^2) - 1/\gamma H^2$	Free waves for $\omega > \omega_a = \gamma^{1/2} g / c_0$ or for $\omega < \omega_b = (\gamma - 1)^{1/2} g / c_0$	Cho (1972)
Compressible and isothermal	$k_z^2 = k_0^2 - k_x^2 (1 - \omega_b^2 / \omega^2) - (1/2H)^2$	Free waves for $\omega > \omega_a = \gamma g / 2c_0$ or for $\omega < \omega_b = (\gamma - 1)^{1/2} g / c_0$	Eckart (1960) Hines (1960)
Compressible, isothermal and rotation about vertical axis	$k_z^2 = k_0^2 - \frac{k_x^2 (1 - \omega_b^2 / \omega^2)}{(1 - 4\Omega^2 / \omega^2)} - (1/2H)^2$	Free waves for $\omega > \omega_a = \gamma g / 2c_0$ or for $2\Omega < \omega < \omega_b = (\gamma - 1)^{1/2} g / c_0$	Eckart (1960)

\*Also called the asymptotic limit which obtains from the compressible and isothermal case when  $k_z^2 \gg \omega_a^2 / c_0^2$ , and  $k_z^2 \gg \omega^2 / c_0^2$  (Hines, 1960)

The three components of the total energy density are kinetic, elastic and thermobaric as given by (22). In the complex notation used here, the time averaged total energy density can be computed by using the polarization relations (35) and the dispersion relation, obtaining

$$\begin{aligned} E &= \frac{1}{4} \left( |F_3|^2 + |F_4|^2 + |F_5|^2 + |F_2|^2/c_0^2 + |F_2 - c_0^2 F_1|^2/(\gamma-1)c_0 \right) \\ &= |F_2|^2 (\omega^2 - \omega_b^2 k_x^2 c_0^2 - \omega_b^2 k_y^2 c_0^2) / 2\omega^2 c_0^2 (\omega^2 - \omega_b^2) \end{aligned} \quad (36)$$

It can be shown that the potential energy of the wave is mainly contained in the elastic energy if  $\omega \gg \omega_b$  and mainly in the thermobaric energy if  $\omega \ll \omega_b$ . The averaged energy flux can also be computed similarly, giving

$$\begin{aligned} \vec{\Gamma} &= (1/2) \operatorname{Re} F_2 (F_3^* + F_4^* + F_5^*) \\ &= (|F_2|^2/2) [\hat{x}k_x/\omega + \hat{y}k_y/\omega + \hat{z}\omega k_z/(\omega^2 - \omega_b^2)] \end{aligned} \quad (37)$$

By comparing (34), (36) and (37), we obtain

$$\vec{\Gamma}/E = \vec{v}_g \quad (38)$$

Therefore, the energy is transported with the group velocity, a very satisfying result.

#### 4. Effect of Horizontally Stratified Winds

There is plenty of evidence to indicate the presence of winds in the atmosphere. In the thermosphere the wind may be as high as 200 m/sec [See review by Rishbeth, 1972] and its effect on the propagation of acoustic gravity waves can be very important [Cowling et al., 1971]. Several studies have been made [e.g. Hines and Reddy, 1967; Pierce, 1965]. Basically, the effect of a constant horizontal background wind can be taken into account relatively simply by introducing a co-moving coordinate system in which the background atmosphere is stationary. Since wave properties in the stationary atmosphere are known, the effect of wind can be found by coordinate inversion. In this section let us use a prime to denote those quantities in the rest frame of the medium and those in the laboratory frame by not using any prime. Then the Galilean space-time transformation valid for non-relativistic horizontal velocity  $\vec{v}_0$  is

$$\begin{aligned}\vec{r}' &= \vec{r} - \vec{v}_0 t & \vec{k}' &= \vec{k} \\ t' &= t & \omega' &= \omega - \vec{k} \cdot \vec{v}_0\end{aligned}\quad (39)$$

We note that (39) implies

$$\omega' t' - \vec{k}' \cdot \vec{r}' = \omega t - \vec{k} \cdot \vec{r} \quad (40)$$

That is, a plane wave in the rest frame of the atmosphere is also a plane wave in the laboratory frame relative to which the atmosphere is moving with  $\vec{v}_0$ . Further, if we let  $\vec{r}$  and  $\vec{r}'$  be the location of a wave packet in these two frames, the first of (39) implies

$$\vec{r}' = \vec{r} - \vec{v}_0 t, \text{ or}$$

$$\vec{v}_g = \vec{v}_g' + \vec{v}_0 \quad (41)$$

Therefore, the group velocity in the laboratory frame is equal to the group velocity in the rest frame of the medium plus the wind velocity [Bretherton, 1966; Jones, 1969; Cowling et al., 1971]. Since  $\vec{v}_0$  is in the horizontal direction, the z-components of  $\vec{v}_g$  and  $\vec{v}_g'$  are equal.

The rules for Galilean transformation of the energy-momentum tensor also implies that the time averaged energy density will transform according to [Sturrock, 1962]

$$E = (\omega/\omega') E' \quad (42)$$

We note that since  $E'$  is the energy density in the rest frame of the medium it is therefore always positive. However, the energy density in the laboratory frame can be negative even for positive  $\omega$  if the Doppler shift is large enough to make  $\omega'$  negative. The quantity  $E'/\omega'$  has been called the wave action by Garrett [1968] and Bretherton and Garrett [1969]. It has been proven by them that for a slowly varying wavetrain in various lossless media in motion there exists a conservation law

$$\frac{\partial}{\partial t} \left( \frac{E'}{\omega'} \right) + \nabla \cdot \left( \frac{E'}{\omega'} \vec{v}_g' \right) = 0 \quad (43)$$

The energy flux given by  $\vec{f}' = E' \vec{v}_g'$  in the rest frame of the medium becomes

$$\vec{F} = E\vec{V}_g = (\omega/\omega')E'(\vec{V}'_g + \vec{V}'_0) \quad (44)$$

in the laboratory frame.

The effect of a continuous slowly varying wind shear on an incident acoustic gravity wave can be studied by noting that  $\vec{k}_h =$  a constant vector owing to the Snell's law. As a result an otherwise penetrating wave can, because of horizontal winds, either penetrate the atmosphere, or be reflected from the atmosphere, or be critically coupled [Booker and Bretherton, 1967]. Reflection occurs at the level at which  $k_z$  vanishes due to Doppler shift, and critical coupling occurs at the level where the Doppler shifted frequency vanishes. Tracing in the model atmospheres show that reflected rays have a characteristic loop structure and the critically coupled rays become asymptotically trapped and dissipated [Cowling et al., 1971].

Fig. 5 shows some samples of the dispersion surface in the presence of background wind. When compared with Fig. 4, the transformation due to winds is seen to be very drastic. The effect of winds on the dispersion surface has been examined by Pierce [1966b].

##### 5. Surface Waves Supported by a Temperature Discontinuity

We have seen from Fig. 1 a sharp increase in the speed of sound near the base of the thermosphere (approximately at 130 km). To the first order the atmosphere may therefore be approximated by a temperature discontinuity. It is then of interest to look for possibilities of boundary waves supported by such a discontinuity. This problem was first treated by Thome [1968].

Let subscripts 1 and 2 denote the quantities respectively below and above the discontinuity which is assumed to occur at  $z = 0$ . The boundary conditions are continuity of pressure and normal velocity. For a stationary background atmosphere they reduce to the following jump conditions.

$$\begin{aligned} [\rho_0 c_0^2]_1^2 &= 0 & [v'_z]_1^2 &= 0 \\ [-i\omega p' - g\rho_0 v'_z]_1^2 &= 0 & & \end{aligned} \quad (45)$$

Making use of the polarization relations (35) and jump conditions (45), the following relation can be derived.

$$\frac{k_{z_1}'' - 1/2H_1 + gk_h^2/\omega^2}{\omega^2 - k_h^2 c_{01}^2} = \frac{k_{z_2}'' - 1/2H_2 + gk_h^2/\omega^2}{\omega^2 - k_h^2 c_{01}^2} \quad (46)$$

where owing to Snell's law  $k_{h_1} = k_{h_2} = k_h$ . Boundary waves are characterized by  $k_z = \pm ik_z$  where proper sign must be chosen to assure that the wave energy is confined near the boundary. The relation (46) together with the two dispersion relations (one for the medium below the discontinuity and one for above) provide three relations for four variables ( $\omega$ ,  $k_h$ ,  $k_{z_1}''$ , and  $k_{z_2}''$  or their equivalent). When any one of these four is given (usually  $\omega$  or period) these equations can be used to find remaining wave properties. Studies have shown that at long periods these waves are nondispersive and propagating with a speed near the speed of sound in the upper layer [Thome, 1968]. The kinetic energy for these long waves resides mainly above the discontinuity. As the period decreases through the Brunt-Väisälä period of the lower region, the horizontal phase velocity falls sharply and its kinetic energy is redistributed to lie below the discontinuity [Francis, 1973]. Some of these features are shown in Fig. 6. Also shown is the Lamb wave which is supported by the solid ground (See (32) and its associated discussion.). The kinetic energy of the Lamb wave is confined near the surface of the ground as it decreases exponentially with height as shown. In a model atmosphere with both the temperature discontinuity and the solid ground the boundary wave (labeled  $G_0$ ) and the Lamb wave (labeled  $L_0$ ) can both exist with slight modifications, but the atmosphere can also support an additional spectrum of higher order modes. These modes have been discussed extensively [Pfeffer and Zarichny, 1963; Harkrider and Wells, 1968; Volland, 1969; Tolstoy and Pan, 1970]. The similarities and some differences of modes  $G_0$  and  $L_0$  under this drastic simplification and modes in a more realistic lossy model atmosphere have been discussed thoroughly by Francis [1973].

## 6. Reflection and Refraction at a Sharp Boundary

The presence of a boundary can support surface waves but also can give rise to the phenomena of reflection and refraction. Consider an interface separating two isothermal atmospheres in relative motion. Due to <sup>the</sup> presence of waves this interface is perturbed from  $z = 0$  to  $z = \zeta$ . When evaluated at  $z = 0$ , the position of the interface is related to the vertical velocity through  $v'_z = \partial \zeta / \partial t + \vec{v}_0 \cdot \nabla \zeta$  or  $\zeta = -v'_z / i\omega'$  where  $\omega'$  is the Doppler shifted frequency given by (39). Since the interface position approached from below (medium 1) must be equal to that approached from above (medium 2),  $\zeta$  and hence  $v'_z / i\omega'$  must be continuous across the interface. The second boundary condition comes from continuity of pressure at the distorted boundary. When evaluated at  $z = 0$ , the continuity condition becomes  $p' + \zeta dp_0 / dz$  which reduces to the continuity of  $p' + g\rho_0 v'_z / i\omega'$ . Apply these two boundary conditions to the case in which a wave with energy incident from below is reflected from and transmitted through the interface we may derive the following reflection coefficient  $R$  and transmission coefficient  $T$  [Revah, 1969; McKenzie, 1972].

$$R = \left. \frac{v'_z(r)}{v'_z(i)} \right|_{z=0} = \frac{\rho_1 (\omega'_1 P^{(i)} - ig) - \rho_2 (\omega'_2 P^{(t)} - ig)}{-\rho_1 (\omega'_1 P^{(r)} - ig) + \rho_2 (\omega'_2 P^{(t)} - ig)} \quad (47)$$

$$T = \left. \frac{v'_z(t)}{v'_z(i)} \right|_{z=0} = (\omega'_2 / \omega'_1) (1+R) \quad (48)$$

Here  $P = (\omega'^2 - \omega_b^2) / \omega' [k_z + i\omega_a (\gamma - 2) / \gamma c_0]$  which comes from the polarization relation (35) and must be applied to the incident wave or reflected wave or transmitted wave. The mass densities  $\rho_1$  and  $\rho_2$  are those at the interface for the atmosphere below and above the interface respectively.

The reflection coefficient (47) simplifies, in the acoustic approximation ( $\omega' \gg \omega_a$ ,  $g \rightarrow 0$ ), to

$$R = \frac{1 - Z_a}{1 + Z_a}, \quad Z_a = \frac{\rho_2 \omega'^2 k_z^{(i)}}{\rho_1 \omega'^2 k_z^{(t)}} \quad (49)$$

which agrees with those derived by Miles [1957] and Ribner [1957]. On the other hand, under the Boussinesq approximation with  $\rho_1 = \rho_2$ , we obtain

$$R = \frac{1 - Z_b}{1 + Z_b}, \quad Z_b = \frac{\omega_2'^2 k_z^{(t)}}{\omega_1'^2 k_z^{(i)}} \quad (50)$$

For the more general case one has to work with (47). Two cases have been studied by McKenzie [1972]: reflection at a density (or temperature) discontinuity and wave amplification at a shear layer. He has found that the density discontinuity acts like a perfect reflector for those gravity waves with  $k_h \gg 1/2H$ . At the shear layer,  $|R| > 1$  if  $k_x > \omega/v_0$  where  $v_0$  is the wind speed along x-axis in the upper medium.

The reflection coefficient (47) was derived by requiring the continuity of the distorted interface and the continuity of pressure. It is not clear whether the expression so derived would also conserve energy flux at the interface [Hines and Reddy, 1967]. The complication arises from relative motion so that proper Galilean transformation discussed in section C.4 is required. When such care is taken the vertical energy flux given by the z-component of (44) is found continuous at the interface [McKenzie, 1972]. However, it should be remembered that the proof is carried out by using the pseudo-energy concept. It is still not clear whether the complete second order vertical energy flux is conserved at the interface. In order to show that, computations must be carried out to the second order and it does not seem to have been done.

An examination of (47) shows that it is possible that  $|R| > 1$  which corresponds to wave amplification. The condition for wave amplification has been shown to be [McKenzie, 1972]

$$\omega_1'/\omega_2' < 0 \quad (51)$$

Because of (42) the condition (49) can also be written as

$$E_1^{(i)}/E_2^{(t)} < 0 \quad (52)$$

Therefore, for amplification, it is necessary that in the laboratory frame the energy density of the incident wave be of the opposite sign from that of the transmitted wave. In the acoustic approximation amplification occurs when  $Z_a < 0$  as can be seen from (49). This condition is equivalent to

$$k_z^{(i)}/k_z^{(t)} < 0 \quad (53)$$

Because of the similarity between (50) and (49), the condition (53) also applies under Boussinesq approximation.

Fig. 7 illustrates several possible cases of reflection and transmission at a wind sheared interface by making use of the dispersion surfaces.

In the study of acoustic gravity waves propagating in a realistic atmosphere it is desirable to approximate such a model atmosphere by multiple isothermal layers. The justification of such a procedure for an inviscid atmosphere has been given [Pierce, 1966a; see further discussion in Chapter E.].

## 7. Effect of Ion Drag

In previous sections wave properties have been deduced in an isothermal atmosphere by ignoring all damping mechanisms. We wish now to examine the damping produced by various loss processes in this section and the next. The effects of damping have been studied by many authors [e.g. Pitteway and Hines, 1963; Liu and Yeh, 1969; Thome and Rao, 1969].

The presence of ionization provides a mechanism whereby wave energy is dissipated. At F region heights the ion gyrofrequency (approximately 300 rad/sec) is much larger than the ion-neutral collision frequency (approximately  $1 \text{ sec}^{-1}$ ) which is in turn much larger than the frequency of acoustic-gravity waves. Consequently, the ions would respond to neutral motion in a quasi-equilibrium way along the steady magnetic field, i.e.

$$\vec{v}_i = \vec{v}' \cdot \hat{B}_0 \hat{B}_0 \quad (54)$$

where  $\vec{v}_i$  is the ionic velocity,  $\vec{v}'$  the wave associated neutral velocity and  $\hat{B}_0$  a unit vector in the direction of the steady magnetic

field. By keeping the ion drag term in the equation of motion (13) it is possible to derive a dispersion relation in the same manner as the lossless case if the neutral-ion collisional frequency  $\nu = \nu_{in} \rho_i / \rho_0$  is a constant [Liu and Yeh, 1969]. For the special case of propagation in the magnetic meridian plane at the magnetic equator, the dispersion relation reduces to

$$k_z^2 = k_0^2 (1 + i\nu/\omega - \omega_a^2/\omega^2) - k_x^2 (1 + i\nu/\omega - \omega_b^2/\omega^2) \quad (55)$$

If  $\nu$  is not a constant there will be in general partial reflections. Again for propagation in the magnetic meridian plane at the magnetic equator the differential equation for  $F_5 = \nu_z^1 \rho_0^{1/2}$  can be derived as

$$d^2 F_5 / dz^2 + k_z^2(z) F_5 = 0 \quad (56)$$

where  $k_z$  is given by (55) and is a function  $z$  through its dependence on  $\nu$ . For the special case of exponential collisional frequency

$$\nu = \nu_0 \exp \alpha(z - z_0) \quad (57)$$

the differential equation (56) can be transformed to the standard Bessel's equation and solved. The results give the reflection coefficient [Liu and Yeh, 1969]

$$|R| = \exp \left[ -\pi |k_z(-\infty)| / \alpha \right] \quad (58)$$

for both acoustic and gravity waves. In (58)  $k_z(-\infty)$  is the vertical wave number as  $z \rightarrow -\infty$ . As seen from (58)  $|R|$  vanishes if  $\alpha = 0$  which corresponds to constant  $\nu$ . The reflection is appreciable if the collision frequency varies considerably in one vertical wavelength.

#### 8. Attenuation due to Loss Processes

As seen from (23) the three processes that convert wave energy into heat are ion drag, thermal conduction and viscosity. Let the acoustic-gravity wave incident from below in a lossless atmosphere. Because of horizontal stratification and the Snell's law,  $k_h$  must be real and constant at any height. In this case the wave dissipation is reflected by having  $k_z$  complex. If the dissipation is small

then the imaginary part of  $k_z$  can be computed by using

$$k_z'' = W/2\Gamma_z \quad (59)$$

where  $W$  is the time averaged dissipation per unit volume and  $\Gamma_z$  is the vertical component of energy flux in an inviscid atmosphere as given by (37). Since  $W$  due to three loss processes is additive [see eq. (19)], their effects on  $k_z''$  will also enter additively in our perturbation computation given by (59) and therefore it is convenient to compute  $k_z''$  separately.

For the purpose of computing energy loss due to ion drag, let us orient the coordinate axes so that the x-axis is eastward, y-axis is northward and z-axis is upward. Let  $I$  be the magnetic dip. The average energy loss per unit volume due to ion drag is then

$$\begin{aligned} W_{id} &= \frac{1}{2} v_{in} \rho_i (\vec{v}' - \vec{v}_i) \cdot (\vec{v}' - \vec{v}_i)^* \\ &= \frac{1}{2} v |F_2|^2 \{ (k_x^2 + k_y^2 \sin^2 I) / \omega^2 + [(\omega^2 - k_h^2 c_0^2) \cos^2 I + 2k_y k_z c_0^2 \sin I \cos I] / c_0^2 (\omega^2 - \omega_b^2) \} \end{aligned}$$

The corresponding  $k_z''$  can be found easily by using (59)

$$(k_z'')_{id} = \frac{v (\omega^2 - \omega_b^2)}{2\omega k_z} \left[ \frac{k_x^2 + k_y^2 \sin^2 I}{\omega^2} + \frac{(\omega^2 - k_h^2 c_0^2) \cos^2 I + 2k_y k_z c_0^2 \sin I \cos I}{c_0^2 (\omega^2 - \omega_b^2)} \right] \quad (60)$$

For the special case of propagation at <sup>the</sup> equator in a magnetic meridian plane (60) reduces to a formula that checks with (55).

The temperature perturbation associated with the wave propagating in xz-plane can be computed by making use of the perturbed ideal gas law (2). Expressing all perturbed quantities in terms of  $F_2$  by using the polarization relation (35), the average energy loss per unit volume due to thermal conduction can be found as

$$\begin{aligned} W_{tc} &= (\kappa/2T_0) (\nabla T') \cdot (\nabla T')^* \\ &= (|F_2|^2 \kappa / 2c_p^2 T_0 \rho_0) (k^2 + 1/4H^2) [\gamma - \omega^2 / (\omega^2 - \omega_b^2)]^2 / (\gamma - 1)^2 \end{aligned}$$

The attenuation of the wave due to thermal conduction is then given by

$$(k_z'')_{tc} = \frac{\kappa(\omega^2 - \omega_b^2)(k^2 + 1/4H^2)}{2c_0^2 \rho_0 \omega k_z} \left( \frac{1}{c_v} - \frac{1}{c_p} \right) \left[ \frac{\gamma}{\gamma-1} - \frac{\omega^2}{(\gamma-1)(\omega^2 - \omega_b^2)} \right]^2 \quad (61)$$

of  
In the absence/gravity (62) reduces to that given by Landau and Lifshitz [1963, p. 300] for a homogeneous atmosphere. In the Boussinesq fluid (61) reduces to

$$(k_z'')_{tc} = - \frac{\kappa \omega_b^2 k^2}{2c_0^2 \rho_0 \omega k_z} \frac{M\gamma}{R} \quad (62)$$

where M is the molecular weight and R the universal gas constant.

The attenuation caused by viscous dissipation can be computed similarly. The algebra involved is somewhat tedious and it can be arranged into

$$(k_z'')_{vd} = \frac{\eta}{6\rho_0 \omega^3 c_0^2 k_z} \{ 4k^2 \omega^4 + \omega^2 \omega_a^2 [(20/\gamma - 11)k_h^2 + 4k_z^2 + 1/H^2] - k_h^2 \omega_b^2 (3k^2 c_0^2 + 7\omega_a^2) \} \quad (63)$$

In the acoustic limit ( $\omega \rightarrow \infty, H \rightarrow \infty$ ) the first term in the braces dominates and (63) reduces to the form given by Landau and Lifshitz [1963, p. 300]. In the Boussinesq fluid (63) reduces to

$$(k_z'')_{vd} = -\eta k_h^4 \omega_b^4 / 2\rho_0 \omega^5 k_z \quad (64)$$

which is identical to that given by Thome and Rao [1969]. Note that for gravity waves incident from below  $k_z$  is negative and consequently both  $(k_z'')_{tc}$  and  $(k_z'')_{vd}$  are positive, indicating vertical attenuation of the wave. Fig. 8 shows the extinction distance defined as  $(k_z'')_{tc}^{-1}$  for internal gravity waves at a height of 200 km in the mean CIRA [1965] atmosphere.

## D. GENERATION OF ACOUSTIC- GRAVITY WAVES IN ISOTHERMAL ATMOSPHERE

Up to this point, while concentrating on the properties of acoustic-gravity waves, the problem of how these waves are generated in the atmosphere has not been discussed. We shall now address ourselves to this subject in this chapter.

Experimental observations have suggested many possible generation mechanisms, both natural and artificial (see Chap. G). In general these mechanisms can be classed as due to one or a combination of three types of sources: mass production, momentum production and heat production. Various authors have studied the source problem for different cases (Pierce, 1963; Row, 1967; Cole and Greifinger, 1969; Dickinson, 1969; Chimones and Hines, 1970). It turns out that for an isothermal atmosphere, the transient response of the atmosphere due to excitation can be discussed in a fairly general fashion (Liu and Yeh, 1971).

### 1. Formulation

The starting point is again the set of equations (20)-(22). Since we are dealing with a non-rotating, lossless atmosphere, the Coriolis and loss terms in the equations can be neglected. Putting the set into matrix form, an equation similar to (24) is obtained, except now the equation is inhomogeneous because of the presence of sources.

$$\tilde{\mathbf{D}} \cdot \vec{\mathbf{F}} = \vec{\mathbf{Q}}(\vec{\mathbf{r}}, t) \quad (65)$$

Here  $\tilde{\mathbf{D}}$  and  $\vec{\mathbf{F}}$  are given by (25) and (26) respectively and  $\vec{\mathbf{Q}}$  is the source vector

$$\vec{\mathbf{Q}} = \begin{pmatrix} q_1' / \rho_0^{1/2} \\ q_{2x}' / \rho_0^{1/2} \\ q_{2y}' / \rho_0^{1/2} \\ q_{2z}' / \rho_0^{1/2} \\ (\gamma - 1) q_3' / \rho_0^{1/2} \end{pmatrix} \quad (66)$$

The  $q_i$ 's are the source terms defined in (12), (15) and (18).  
Eq. (65) can be recast into a scalar equation

$$D(\nabla, \partial/\partial t)\vec{F} = \vec{S}(\vec{r}, t) \quad (67)$$

where  $D$  is the acoustic-gravity wave operator defined in (27)  
and  $\vec{S}$  is the equivalent source vector given by

$$\vec{S} = \hat{D} \cdot \vec{Q} \quad (68)$$

Here  $\hat{D}$  is the adjoint of  $\tilde{D}$ .

To solve (67), we apply Fourier transform both in time and  
in space and obtain

$$D(\vec{k}, \omega)\vec{F}(\vec{k}, \omega) = \vec{S}(\vec{k}, \omega) \quad (69)$$

where we have used the same symbols to denote Fourier transform  
pairs so that  $\vec{F}(\vec{k}, \omega)$  is the transform of  $\vec{F}(\vec{r}, t)$ , and  $\vec{S}(\vec{k}, \omega)$  the  
transform of  $\vec{S}(\vec{r}, t)$ . In the transformed domain  $D$  becomes

$$D(\vec{k}, \omega) = \omega^2 (\omega^2 - \omega_a^2) - \omega^2 c_0^2 [(1 - \omega_D^2/\omega^2)k_h^2 + k_z^2] \quad (70)$$

We note that by setting  $D(\vec{k}, \omega) = 0$ , we obtain the dispersion relation  
(28). Solving (69) and going back to the space and time domain,  
we have the formal solution to (67) in terms of the inverse transform

$$\vec{F}(\vec{r}, t) = \frac{1}{2\pi} \int_{\Gamma} \vec{F}(\vec{r}, \omega) \exp(-i\omega t) d\omega \quad (71)$$

where

$$\vec{F}(\vec{r}, \omega) = \frac{1}{(2\pi)^3} \int \frac{\vec{S}(\vec{k}, \omega)}{D(\vec{k}, \omega)} \exp(i\vec{k} \cdot \vec{r}) d\vec{k} \quad (72)$$

The integration path  $\Gamma$  in (71) is taken parallel to the real  $\omega$ -axis  
and above all singularities of the integrand so that the principle  
of causality is satisfied.

The inverse transforms (71) and (72) in general are difficult  
to carry out exactly. However, if our interests are in the radiation  
fields far away from the localized source, certain asymptotic  
techniques can be used to yield physically meaningful and relatively  
simple expressions. Lighthill (1960) has shown that in the far

field approximation, for integrals of the type of (72), the contributions to the radiation field comes only from those rays whose group velocity vectors lie in the same direction as the observation point  $\vec{r}(r, \theta, \phi)$ . These contributing rays can be found from the dispersion surface  $D(\vec{k}, \omega) = 0$ . In the present case, the asymptotic expression of (72) is found to be (Liu and Yeh, 1971)

$$\vec{F}(\vec{r}, \omega) \sim \frac{-1}{4\pi c_0^2 r} \frac{\vec{S}[\vec{k}(\omega), \omega]}{\sqrt{(\omega^2 - \omega_b^2)(\omega^2 - \omega_c^2)}} \exp[i r \xi(\omega)] \quad (73)$$

where

$$\omega_c = \omega_b \cos \theta \quad (74)$$

$$\xi(\omega) = (1/c_0) \sqrt{(\omega^2 - \omega_a^2)(\omega^2 - \omega_c^2)/(\omega^2 - \omega_b^2)} \quad (75)$$

and the dependence on  $\vec{k}$  of  $\vec{S}[\vec{k}(\omega), \omega]$  is given by

$$\begin{aligned} k_h^2 &= (\omega^4 \sin^2 \theta / c^2) (\omega^2 - \omega_a^2) / (\omega^2 - \omega_b^2) (\omega^2 - \omega_c^2) \\ k_z^2 &= (\cos^2 \theta / c^2) (\omega^2 - \omega_a^2) (\omega^2 - \omega_b^2) / (\omega^2 - \omega_c^2) \end{aligned} \quad (76)$$

$$k_y / k_x = \tan \phi$$

For a given  $\omega$ , (76) gives the wave vector  $\vec{k}$  that has the associated group velocity vector lying in the same direction as the observation point  $\vec{r}(r, \theta, \phi)$ .

Substituting (73) into (71), we obtain the transient response

$$\vec{F}(\vec{r}, t) \sim \frac{-1}{8\pi^2 c_0^2 r} \int_{\Gamma} \frac{\vec{S}[\vec{k}(\omega), \omega]}{\sqrt{(\omega^2 - \omega_b^2)(\omega^2 - \omega_c^2)}} \exp[iq(\omega)] d\omega \quad (77)$$

where

$$q(\omega) = r\xi(\omega) - \omega t \quad (78)$$

To evaluate (77), the method of saddle point can be used (Yeh and Liu, 1972). For large  $r$ , the contribution to the integral comes mainly from the vicinity of saddle points  $\omega_s$  of the exponent determined by

$$dq/d\omega = 0$$

$$\text{or } \xi'(\omega_s) = d\xi/d\omega|_{\omega_s} = t/r \quad (79)$$

Fig. 9 shows a typical plot of  $\xi'c_0$  vs.  $\omega$ . The saddle points are found by intersections of a horizontal line with these curves. The line is drawn for a given  $\tau = c_0 t/r$ . No signal arrives at the observation point for  $t < r/c_0$ . As  $\tau \rightarrow 1^+$ , one pair of saddle points at high frequency appears and contributes to the acoustic precursors. This is the acoustic mode and its frequency decreases at the observation point as time increases until it approaches  $\omega_a$  asymptotically. Sometime after the first arrival of the acoustic precursors, as  $\tau \rightarrow \tau_0$ , a signal with frequency  $\omega_0$  begins to build up corresponding to the arrival of the low frequency contribution. This portion of the signal is decomposed into two components as time increases, one at frequency  $\omega_0 < \omega < \omega_b$  and the other at  $\omega_c < \omega < \omega_0$ . In the literature they have been referred to as buoyancy mode and gravity mode respectively (Dickinson, 1969; Liu and Yeh, 1971). Therefore, for  $\tau > \tau_0$ , three waves exist at the observation point and their respective frequencies approach  $\omega_a$ ,  $\omega_b$  and  $\omega_c$  asymptotically. Analytic asymptotic expressions for the field  $\vec{F}(\vec{r}, t)$  in different regions of  $\tau$  can be obtained using the saddle point method but will not be given here. Interested readers are referred to the original paper (Liu and Yeh, 1971). In the following, a special example will be discussed using the results of this section.

## 2. Stationary Impulse Source

In the study of the response of the upper atmosphere to nuclear detonations, Row (1967) investigated the case of an impulsive point source. The response may be considered as a Green's function or a propagator (Dickinson, 1969). The equation satisfied by the Green's function, after Fourier transform in time, is of the form

$$\left[ \nabla_h^2 + \frac{\omega^2}{\omega^2 - \omega_b^2} \frac{\partial^2}{\partial z^2} + \frac{\omega^2 (\omega^2 - \omega_a^2)}{c_0^2 (\omega^2 - \omega_b^2)} \right] G(\vec{r}, \omega) = -\delta(\vec{r}) \quad (80)$$

Concentrating on the long period waves for  $z/r \ll 1$ , Row (1967) obtained an approximate solution for the Green's function:

$$G(\vec{r}, t) \sim \frac{\omega_b}{4\pi r} J_0(\omega_c \sqrt{t^2 - r^2/c^2}) U(t-r/c) \quad (81)$$

where  $J_0(x)$  is the Bessel's function of zeroth order and  $U(t)$  is the unit step function. For  $t \gg r/c$ , the asymptotic expression for the Green's function reduces to

$$G(\vec{r}, t) \sim \frac{\omega_b}{4\pi r} \sqrt{\frac{2}{\pi \omega_c t}} \cos(\omega_c t - \pi/4) \quad t \gg r/c_0 \quad (82)$$

Using the techniques described in the previous section, the complete Green's function due to impulsive point source can be obtained. The response includes first the acoustic precursor at  $t \approx r/c_0$ . Then the arrival of the gravity and buoyancy modes. For  $t \gg r/c_0$ , the response is given by (Liu and Yeh, 1971)

$$\begin{aligned} G(\vec{r}, t) \sim & \frac{b_1}{2\sqrt{\pi} t^{3/2}} \sqrt{\frac{\omega_a^2 - \omega_b^2}{\omega_a^2 - \omega_c^2}} \cos(\omega_a t + \pi/4 - b_1^2 r^2/4t) \\ & + \frac{(b_2 r/2)^{2/3}}{\pi \sqrt{6\pi} r t^{7/6}} \sqrt{\frac{\omega_b}{\omega_b^2 - \omega_c^2}} \cos[\omega_b t + \pi/4 - 3t^{1/3} (b_2 r/2)^{2/3}] \\ & + \frac{1}{4\pi r} \sqrt{\frac{2(\omega_b^2 - \omega_c^2)}{\pi \omega_c t}} \cos[\omega_c t - \pi/4 - b_3^2 r^2/4t] \end{aligned} \quad (83)$$

where

$$\begin{aligned} b_1 &= \frac{1}{c_0} \sqrt{\frac{2\omega_a (\omega_a^2 - \omega_c^2)}{\omega_a^2 - \omega_b^2}} \\ b_2 &= \frac{1}{c_0} \sqrt{\frac{(\omega_b^2 - \omega_c^2) (\omega_a^2 - \omega_b^2)}{2\omega_b}} \\ b_3 &= \frac{1}{c_0} \sqrt{\frac{2\omega_c (\omega_a^2 - \omega_c^2)}{\omega_b^2 - \omega_c^2}} \end{aligned}$$

We note in (83) that the contribution comes from the three modes at frequencies approaching  $\omega_a$ ,  $\omega_b$ , and  $\omega_c$  respectively. The last

term agrees with (82) for large time and  $z/r \ll 1$ , this is the contribution from the gravity mode. Fig. 10 shows a typical response for F-region parameters. The dominating contribution is the long period gravity mode. Superposed on it is the shorter period acoustic mode. As time increases the gravity mode becomes the only important one. It is interesting to note that at a given observation point, the frequency of the gravity mode approaches  $\omega_c = \omega_b z/r$ . This fact has been used by Row (1967) to interpret the experimental observations of ionospheric traveling disturbances caused by nuclear detonations. The interesting range dependence of  $\omega_c$  can be explained by using the group velocity concept. Since the group rays are straight lines in an isothermal atmosphere the position of a ray propagating in xz-plane is determined by, using equation (34),  $x/z = k_x(\omega^2 - \omega_b^2)/k_z \omega^2$ . Under Boussinesq approximation it reduces to  $x/z = \sqrt{\omega_b^2 - \omega^2}/\omega$  which when solved for  $\omega$  yields  $\omega = \omega_b z/r$ . This is just  $\omega_c$  discussed above.

### 3. Moving Sources.

Various authors have suggested the possibilities of generating acoustic-gravity waves by natural or artificial sources in motion (Wilson, 1960; Chimonas and Peltier, 1970; Chimonas and Hines, 1970; Balachandran and Donn, 1971). In this section, we shall consider the response of the atmosphere due to an impulsive point source moving with a constant speed. This again may be thought of as the Green's function or propagator from which response due to more general sources may be constructed.

Consider an impulsive point source moving in the horizontal direction with a constant speed  $u$ . In a coordinate system where the x-axis is along the trajectory of the source, the Green's function satisfies an equation similar to (80) with the source term replaced by  $(1/uc_0^2)\delta(y)\delta(z)\exp(i\omega x/u)$ . The formal solution is written in terms of an inverse transform (Liu and Yeh, 1972)

$$G(x, r_{\perp}, t) = \frac{i}{8\pi c_0^2 u} \int_{\Gamma} \frac{\sqrt{\omega^2 - \omega_b^2}}{\omega} H_0^{(1)}[\xi_1(\omega) r_{\perp}] \exp[-i\omega(t-x/u)] d\omega \quad (85)$$

where  $H_0^{(1)}$  is the Hankel function of first kind of zeroth order,

$r_{\perp} = (y^2+z^2)^{1/2}$  is the radial distance from the trajectory of the source and

$$\xi_1(\omega) = \frac{\sqrt{\beta^2-1}}{\beta c_0} \sqrt{\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_c^2)}{(\omega^2-\omega_b^2)}}$$

$$\omega_1 = \left(\frac{\beta^2\omega_a^2-\omega_b^2}{\beta^2-1}\right)^{1/2} \quad (86)$$

$$\beta = u/c_0$$

For a source traveling with a supersonic speed, it is easy to show from (85) that the response is confined within a conical region trailing behind the source with the apex angle

$$\theta_a = \sin^{-1}(1/\beta) \quad (87)$$

This corresponds to the well known Cérenkov cone. For this case, the cutoff frequency  $\omega_1$  is greater than  $\omega_a$ . The transient radiation field may be obtained from (85) by the saddle point method and is similar to that of the stationary source case with  $\omega_1$  replacing  $\omega_a$ . The detailed expressions can be found in the reference (Abreu Neto and Liu, 1972). Qualitatively, high frequency acoustic mode arrives at the observation point first, followed by the buoyancy and gravity mode. The gravity mode at frequency  $\omega_c$  dominates the response as time increases.

For the case where the source moves with a subsonic speed,  $\beta < 1$  and  $\omega_1 < \omega_b$ . It is easy to show that there is no response in front of the source ( $x > ut$ ). There is no high frequency acoustic mode in the response. In general, three modes may still appear in the response with asymptotic frequencies approaching  $\omega_b$ ,  $\omega_1$  and  $\omega_c$  respectively. The frequency of any mode at any time is always smaller than  $\omega_b$ . As time increases at a given observation point, the gravity mode with frequency  $\omega_c$  is by far the most dominant component in the response (Liu and Yeh, 1972).

Similar analysis and results have been obtained for the case where the source is moving in the vertical direction (Grigo'ev and Dokuchayev, 1970).

The Green's function obtained above may be used in principle to construct responses due to more realistic source models such as auroral arcs in supersonic motion (Chimonas and Peltier, 1970), moving solar shadow during eclipse (Chimonas and Hines, 1970), rocket launching (Balachandran and Donn, 1971), etc.

#### 4. Coupling with seismic waves.

Experimental evidence finds support in the generation of atmospheric acoustic-gravity waves by the traveling seismic waves along the surface of the earth (Donn and Posmentier, 1964; Yuen et al. 1969; Cook, 1971). In this section, the transient response of the atmosphere due to ground motion will be studied by using a simple model.

Let us assume a traveling seismic wave propagating in the x-direction with an associated vertical ground displacement

$$z(x,t) = z_0 \cos k_1 (x-ut) U[k_1 (ut-x)] \quad (88)$$

This movement forces the atmosphere into motion with the vertical velocity at  $z = 0$  approximately equal to

$$v_{z0} = \left. \frac{dz}{dt} \right|_{z=0} \quad (89)$$

where  $z$  is given by (88). The vertical component of the velocity in the atmosphere satisfies the equation

$$D(\nabla, \partial/\partial t) F_5 = 0 \quad (90)$$

where  $D(\nabla, \partial/\partial t)$  is the acoustic-gravity wave operator (27) and  $F_5 = \rho_0^{1/2} v_z'$ . The solution of (90) with the boundary condition given by (89) may be written as

$$v_z'(\vec{r}, t) = \frac{z_0}{2\pi} \exp(z/2H) \int_{\Gamma} \frac{\omega^2}{\omega^2 - k_1^2 u^2} \exp\{-i[\omega t - \xi_2(\omega)z]\} d\omega \quad (91)$$

where

$$\xi_2(\omega) = \sqrt{(\beta^2-1)(\omega^2-\omega_1^2)}/\beta c_0 \quad (92)$$

$\beta$  and  $\omega_1$  are given in (86).

For  $\beta > 1$ , it is easy to show that for a given  $t$ , the signal above the ground can be found only within the region bounded by the plane

$$x + \sqrt{\beta^2-1} z = ut \quad (93)$$

trailing behind the front of the surface wave on the ground. The transient response of the atmosphere can be obtained from (91) through saddle point integration. Detail expressions are given in the reference (Abreu Neta and Liu, 1972). At any observation point in the upper atmosphere (given  $x$  and  $z$ ), a high frequency precursor arrives first. Then at the time

$$t_0 = [x + z \sqrt{\beta^2-1} \sqrt{1-\omega_1^2/k_1^2 u^2}] / u \quad (94)$$

the main signal with frequency  $\omega = k_1 u$  arrives which dominates the response and is given by

$$v_z(\vec{r}, t) \sim k_1 z_0 u \exp(z/2H) \sin[\sqrt{(\beta^2-1)(k_1^2 u^2 - \omega_1^2)} z/u + k_1(x-ut)] \quad (95)$$

We note that this main signal travels in a direction that makes an angle  $\theta$  with the vertical, where

$$\theta = \tan^{-1} \left[ \frac{1}{\frac{\sqrt{1-\omega_1^2/k_1^2 u^2}}{\sqrt{\beta^2-1}}} \right] \quad (96)$$

For a wave to propagate upward, we must have  $k_1 u > \omega_1$ . But for  $\beta > 1$ ,  $\omega_1 > \omega_a$ . Therefore, in this case, the excited wave always belongs to the acoustic branch. In reality, for seismic waves,  $\beta$  is usually

of the order of 10 while  $k_1 u \gg \omega_1$ , therefore the angle  $\theta$  is very close to zero. The main signal travels almost vertically.

As a numerical example, we consider the atmospheric wave excited by a Rayleigh wave of period 25 seconds and speed 3.9 km/sec. The ground moves with a peak-to-peak displacement of 5mm ( $z_0 = 5\text{mm}$ ). For these parameters, the corresponding  $v_z$  at a height of 150 km is approximately 30 m/sec. Other parameters such as the pressure and the density perturbations may be obtained through the polarization relations [see Eq. (35)]. This kind of upper atmosphere disturbances due to ground traveling seismic waves have been observed experimentally (Yuen et al. 1969).

Similar analysis can be made for the case where  $\beta < 1$ . This may correspond to the coupling between the atmosphere waves and surface ocean waves (Golitsyn and Klyatskin, 1967).

#### E. PROPAGATION OF ACOUSTIC-GRAVITY WAVES IN A REALISTIC ATMOSPHERE

The temperature in a real atmosphere varies as a function of height (Fig. 1). Therefore, the results discussed in the previous chapters for acoustic-gravity waves in an isothermal atmosphere must be used with caution when applied directly to the realistic case. Furthermore, the various loss mechanisms discussed in Sections D.7 and D.8 and the background neutral wind all affect the propagation of acoustic-gravity waves in a very complicated way. To study these waves in a realistic atmosphere, these effects have to be considered simultaneously. The problem is immensely complex. Many authors have attempted to solve it under various approximations (Press and Harkrider, 1962; Pfeffer and Zarichny, 1963; Harkrider, 1964; Friedman, 1966; Midgley and Liemohn, 1966; Hines and Reddy, 1967; Volland, 1969a, b; Klost<sup>e</sup>rarmeyer, 1969, 1972a, b; Clark et al., 1971; Francis, 1973). In this chapter, some of these results will be discussed.

In general, two approaches have been adopted for the solution of the problem. One is the free wave approach where the

effect of the boundaries and discontinuities are neglected. The other is the guided mode analysis where boundaries play very important roles. For waves in the upper atmosphere where the ground reflection is not so important, the free wave approach usually is more convenient to use. For waves near the ground or supported by discontinuities, guided mode analysis is more appropriate.

### 1. Ray Theory

If the properties of the medium do not vary appreciably within a wavelength, the ray theory (or geometric-optics) approach to the propagation problem is valid. In general, the ray equations can be derived from the Fermat's principle, which, in terms of the Hamilton's equations, is expressed as (Landau and Lifshitz, 1959; Whitham, 1961; Yeh and Liu, 1972b)

$$\begin{aligned}\frac{d\vec{r}}{dt} &= \nabla_{\vec{k}} H \\ \frac{d\vec{k}}{dt} &= -\nabla_{\vec{r}} H\end{aligned}\tag{97}$$

where  $\vec{r}$  is a point on the ray path,  $\nabla_{\vec{r}}$  and  $\nabla_{\vec{k}}$  are the gradient operators in  $\vec{r}$ - and  $\vec{k}$ -space respectively, and  $H$  for the acoustic gravity wave system is given by the dispersion relation (70), i.e.

$$H = \omega^4 - \omega^2 c_0^2 k^2 - \omega^2 \omega_a^2 + c_0^2 \omega_b^2 k_h^2 = 0\tag{98}$$

For the nonisothermal atmosphere,  $c_0$  in (98) is the local sound speed and the buoyancy frequency  $\omega_b$  is defined by eq. (9). Substituting (98) into (97), ray equations can be derived. For a spherical coordinate system with the center of the earth as the origin, the equations for  $\vec{r}(r, \theta, \phi)$  and  $\vec{k}(k_r, k_\theta, k_\phi)$  are given by (Georges, 1972)

$$\begin{aligned}\frac{dr}{dt} &= c_0^2 k_r \omega F \\ \frac{d\theta}{dt} &= \frac{1}{r} [c_0^2 k_\theta (\omega^2 - \omega_b^2) F / \omega]\end{aligned}$$

$$\frac{d\phi}{dt} = \frac{1}{r \sin \theta} [c_0^2 k_\phi (\omega^2 - \omega_b^2) F / \omega]$$

$$\frac{dk_r}{dt} = \frac{\omega}{c_0} \frac{\partial c_0}{\partial r} (\omega_a^2 - c_0^2 k^2) F + \frac{gF}{2\omega} \frac{\partial^2 c_0^2}{\partial r^2} (k_\theta^2 + k_\phi^2) + k_\theta \frac{d\theta}{dt} + k_\phi \sin \theta \frac{d\phi}{dt} \quad (99)$$

$$\frac{dk_\theta}{dt} = \frac{1}{r} \left[ \frac{\omega}{c_0} \frac{\partial c_0}{\partial \theta} (\omega_a^2 - c_0^2 k^2) F + \frac{gF}{2\omega} \frac{\partial^2 c_0^2}{\partial \theta \partial r} (k_\theta^2 + k_\phi^2) - k_\theta \frac{dr}{dt} + r k_\phi \cos \theta \frac{d\phi}{dt} \right]$$

$$\frac{dk_\phi}{dt} = \frac{1}{r \sin \theta} \left[ \frac{\omega}{c_0} \frac{\partial c_0}{\partial \phi} (\omega_a^2 - c_0^2 k^2) F + \frac{gF}{2\omega} \frac{\partial^2 c_0^2}{\partial \phi \partial r} (k_\theta^2 + k_\phi^2) - k_\phi \sin \theta \frac{dr}{dt} - r k_\phi \cos \theta \frac{d\theta}{dt} \right]$$

where

$$F = (2\omega^2 - \omega_a^2 - c_0^2 k^2)^{-1} \quad (100)$$

These equations, though complicated as they stand, can be integrated numerically on a digital computer to yield the ray  $\vec{r}(t)$  and the wave vector  $\vec{k}(t)$  for a given atmospheric model. When there is time- and spatial- varying horizontal neutral wind, the equations can still be used with the replacement of Doppler shifted frequency  $\omega'$  for  $\omega$  where (see discussion in section C.4)

$$\omega' = \omega - \vec{k} \cdot \vec{v}_0 \quad (101)$$

An additional equation for frequency is sometimes useful (Jones, 1969)

$$\frac{d\omega}{dt} = \vec{k} \cdot \frac{\partial \vec{v}_0}{\partial t} - \frac{\omega'}{c_0} \frac{\partial c_0}{\partial t} (\omega_a^2 - c_0^2 k^2) F \quad (102)$$

This equation allows integration of the Doppler shift in frequency when the atmosphere changes slowly with respect to time.

These ray equations are valid for three dimensional variations of the atmosphere. In many applications, the horizontal variations can be neglected and the atmosphere can be assumed to be stratified horizontally. For this case, a simplified ray tracing procedure based on Snell's law is applicable. The atmosphere is divided into horizontal slabs of constant temperatures. Snell's law requires the horizontal wave vector  $\vec{k}_h$  to be constant across the slab boundaries. For a given frequency  $\omega$  and horizontal wave vector  $\vec{k}_h$  in a given slab,  $k_z$  can then be computed from the dispersion relation. Then the group velocity  $\vec{v}_g$  is computed using (34) and (41). This gives the position and the direction of the ray in the slab. The computation is then repeated for the next slab and so on. Cowling,

et al. (1971) applied this technique to trace rays of internal gravity waves in a model atmosphere with stratified neutral winds. Fig. 11 shows some typical rays.

In the discussion above, the atmosphere is taken as lossless. For a lossy atmosphere, the ray equations have to be modified (Jones, 1970). In the following section, this problem will be discussed using a different approach.

One point worth noting is the refraction by the earth's gravitational field in a spherical geometry. It has been shown [Francis, 1972] that the resulting ray-path curvature nearly matches the earth curvature for gravity waves but not acoustic waves. This interesting result justifies the use, at least approximately, of plane geometries pertaining to terrestrial environment in discussing propagation of gravity waves.

## 2. Coupled Equation Formulation

Closely related to the ray theory of wave propagation is the WKB solution. The derivation of this approximate solution, and its extension to higher orders can be done conveniently by following the so-called coupled equation formulation. Let us consider a lossy, nonisothermal, stratified atmosphere. As discussed in Sections C.7 and C.8, the loss mechanisms in the upper atmosphere to acoustic-gravity waves are thermal conduction, ion drag, and viscosity. We first consider the combined effect of only the first two mechanisms.

In a nonisothermal atmosphere, the equilibrium pressure is given by eq. (3) with a variable scale height  $H(z)$ . To discuss the wave motion in such an atmosphere, the starting equations again are eqs. (20)-(22). Neglecting Coriolis effect and viscosity and assuming plane wave solution of the form

$$\vec{e}(z) \exp[-i(\omega t - k_x x - k_y y)]$$

the following set of equations can be derived for the height variation function  $\vec{e}(z)$  (Clark et al., 1970)

$$\frac{d\vec{e}(z)}{dz} + i \tilde{A}(z) \cdot \vec{e}(z) = 0 \quad (103)$$

where by definition the vector  $\vec{e}$  has four components given by

$$\begin{aligned} e_1 &= (gH\rho_0)^{1/2} v_z' & e_2 &= p' / (gH\rho_0)^{1/2} \\ e_3 &= (gH\rho_0)^{1/2} \kappa T' & e_4(z) &= \kappa \left[ \frac{dT'}{dz} + \frac{1}{2} \frac{d \ln T_0}{dz} T' \right] \end{aligned} \quad (104)$$

The elements of the matrix  $\tilde{\tilde{A}}$  are given by

$$\begin{aligned}
 A_{11} &= i(1/2H + \nu k_x \cos I \sin I / \Omega_1) \\
 A_{12} &= -\omega/gH + k_x^2/\Omega_1 + k_y^2/\Omega \\
 A_{13} &= \omega/\kappa T_0 gH \\
 A_{21} &= -\omega\Omega/\Omega_1 \\
 A_{22} &= -i(1/2H - \nu k_x \cos I \sin I / \Omega_1) \\
 A_{23} &= i/\kappa T_0 H \\
 A_{33} &= -i/2H \\
 A_{34} &= igH\rho_0 \\
 A_{41} &= i\left(g + \frac{c_0^2}{\gamma-1} \frac{d \ln H}{dz}\right) \\
 A_{42} &= -\omega \\
 A_{43} &= i(k_x^2 + k_y^2)/gH\rho_0 + \omega\gamma/(\gamma-1)\kappa T_0 \\
 A_{44} &= i/2H \\
 A_{14} &= A_{24} = A_{31} = A_{32} = 0
 \end{aligned} \tag{105}$$

where

$$\Omega = \omega + i\nu, \quad \Omega_1 = \omega + \nu \sin^2 I \tag{106}$$

In (104) and (105),  $T_0(z)$  is the equilibrium temperature of the atmosphere;  $\kappa$ , the heat conductivity;  $\nu = \nu_{in} \rho_{i0} / \rho_0$ , the neutral-ion collisional frequency for ion drag; and  $I$ , the geomagnetic dip angle.

Eq. (103) is a set of coupled equations with variable coefficients. Standard procedures may be followed to solve it (Clemmow and Heading, 1954; Budden and Clemmow, Inoue and Horowitz, 1966; Volland, 1967; Yeh and Liu, 1972b). Introduce a transformation matrix  $\tilde{\tilde{B}}$  and a new vector  $\vec{c}$  such that

$$\vec{e}(z) = \tilde{\mathbf{B}}(z) \cdot \vec{c}(z) \quad (107)$$

Substituting (107) into (103) and assuming  $\tilde{\mathbf{B}}$  is nonsingular, we obtain

$$\frac{d\vec{c}}{dz} + i(\tilde{\mathbf{B}}^{-1} \cdot \tilde{\mathbf{A}} \cdot \tilde{\mathbf{B}}) \cdot \vec{c} = -\tilde{\mathbf{B}}^{-1} \cdot \frac{d\tilde{\mathbf{B}}}{dz} \cdot \vec{c} \quad (108)$$

where  $\tilde{\mathbf{B}}^{-1}$  is the inverse of  $\tilde{\mathbf{B}}$ . We now require that the matrix  $\tilde{\mathbf{B}}^{-1} \cdot \tilde{\mathbf{A}} \cdot \tilde{\mathbf{B}}$  to be diagonal

$$\tilde{\mathbf{B}}^{-1} \cdot \tilde{\mathbf{A}} \cdot \tilde{\mathbf{B}} = \begin{pmatrix} k_{z1} & 0 & 0 & 0 \\ 0 & k_{z2} & 0 & 0 \\ 0 & 0 & k_{z3} & 0 \\ 0 & 0 & 0 & k_{z4} \end{pmatrix} = \tilde{\mathbf{\Lambda}} \quad (109)$$

where the  $k_{zi}$ 's are the roots of the equation

$$\det [\tilde{\mathbf{A}} + k_z \tilde{\mathbf{I}}] = 0 \quad (110)$$

For distinct roots of equation (110), the matrix  $\tilde{\mathbf{B}}$  is constructed by taking its  $i$ th column as the  $i$ th eigenvector  $\vec{y}_i$  of the eigenvalue equation

$$[\tilde{\mathbf{A}} + k_{zi} \tilde{\mathbf{I}}] \cdot \vec{y}_i = 0 \quad (111)$$

The normalization of  $\vec{y}_i$  is still arbitrary. It has been shown (Inoue and Horowitz, 1966; Volland, 1969b) that the optimal choice of the normalization for  $\vec{y}_i$  is the one that makes the matrix  $\tilde{\mathbf{B}}^{-1} \cdot \frac{d\tilde{\mathbf{B}}}{dz}$  to have zero diagonal elements. This way, the matrix  $\tilde{\mathbf{B}}$  is determined uniquely.

Expanding eq. (110), we obtain

$$a_4 k_z^4 + a_3 k_z^3 + a_2 k_z^2 + a_1 k_z + a_0 = 0 \quad (112)$$

where

$$a_4 = ikT_0 / \omega g H \rho_0$$

$$a_3 = 2i(k_x v \sin I \cos I / \Omega_1) a_4$$

$$\begin{aligned}
a_2 &= (\kappa T_0/\omega)A_{43} + (1/2H^2 - k_X^2 v^2 \sin^2 I \cos^2 I / \Omega_1^2 - A_{12} A_{21})a_4 \\
a_1 &= 2i\{\kappa T_0 A_{43} k_X v \sin I \cos I / \omega \Omega_1 - ia_3/8H^2 - [\gamma/(\gamma-1)] \frac{d \ln H}{dz}\} \\
a_0 &= (1/4H^2 - k_X^2 v^2 \sin^2 I \cos^2 I / \Omega_1^2 - A_{12} A_{21})a_4/4H^2 \\
&\quad + \kappa T_0 A_{43} / 4H^2 \omega - \kappa T_0 A_{43} A_{12} A_{21} / \omega - 1/H^2 \\
&\quad - \omega A_{21} / gH - g A_{12} / \omega H + [\gamma/(\gamma-1)] (k_X v \sin I \cos I / \Omega_1 - 1/H - g A_{12} / \omega) \\
&\quad \cdot \frac{d \ln H}{dz}
\end{aligned} \tag{113}$$

The solutions of (112) corresponds to four characteristic modes: up- and down-going acoustic-gravity waves, and up- and down-going thermal conduction waves. The general solution is quite complicated. For some special cases, analytical expression can be obtained (Clark et al., 1970). With the  $k_{zi}$ 's, the eigenvectors  $y_i$ 's can be found from (111) and the transformation matrix  $\tilde{B}$  is determined. The next step is the solution of the equation (108). Formally, the solution can be written as

$$\vec{c}(z) = \vec{c}_0(z) - \tilde{C}^{-1}(z) \cdot \int_{z_0}^z \tilde{C}(\xi) \cdot \tilde{B}^{-1} \cdot \frac{d\tilde{B}}{d\xi} \cdot \vec{c}(\xi) d\xi \tag{114}$$

where

$$\begin{aligned}
c_{0i}(z) &= A_i \exp \left[ -i \int_{z_0}^z k_{zi}(\xi) d\xi \right] \\
C_{ij}(z) &= \delta_{ij} \exp \left[ i \int_{z_0}^z k_{zi}(\xi) d\xi \right]
\end{aligned} \tag{115}$$

$z_0$  is a reference level,  $\delta_{ij}$  is the Kroneker delta. Eq. (114) is an integral equation. Based on this equation, an iterative procedure can be devised to obtain the solution numerically. Once  $\vec{c}(z)$  is solved, the wave parameter  $\vec{e}(z)$  can be obtained from (107).

Following the same procedure, the effect of viscosity can be studied. This results in two more pairs of characteristic waves: up- and down-going ordinary and extra-ordinary viscosity waves (Volland, 1969a).

### 3. The WKB Solution

The WKB solution of the problem is obtained from (114) by neglecting the second term on the right-hand side. In this approximation the solution is simply

$$\vec{c}(z) = \vec{c}_0(z) \quad (116)$$

and from (107)

$$\vec{e}(z) = \tilde{B}(z) \cdot \vec{c}_0(z) \quad (117)$$

The WKB solutions are just the individual characteristic waves. They propagate independently of each other as long as the matrix  $\tilde{B}^{-1} \cdot \frac{d\tilde{B}}{dz}$  is small. This matrix is called the coupling matrix since its elements indicate the coupling and reflection among the characteristic waves. Wherever the coupling is strong, the WKB solution fails to be valid. Such regions exist in the neighborhood of those values of  $z$  at which the matrix  $\tilde{B}$  becomes singular. These points are called "coupling points" or "turning points". The coupling or reflection of waves occur in the neighborhood of these points.

As an example, let us consider a lossless, nonisothermal atmosphere. For this case, (103) simplifies to

$$\frac{d}{dz} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} + \begin{pmatrix} -(2-\gamma)/2\gamma H & (i\omega/\gamma)(1-k_h^2 c_0^2/\omega^2) \\ i\omega(1-\omega_b^2/\omega^2)/gH & (2-\gamma)/2\gamma H \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = 0 \quad (118)$$

where  $c_0$  is given by (6) and  $\omega_b$  is given by (9).

Following the procedure outlined in the previous section, we first find the  $k_{zi}$ 's for the characteristic waves,

$$k_z^\pm = \pm \sqrt{\frac{\omega^2}{c_0^2} \left(1 - \frac{\omega^2}{\omega_a^2}\right) - k_h^2 \left(1 - \frac{\omega^2}{\omega_b^2}\right) + (2-2\gamma-\gamma\dot{H}) \left(\frac{d \ln H}{dz}\right)^2} \quad (119)$$

The plus and minus signs indicate down- and up-going acoustic-gravity waves respectively. The over dot on  $H$  signifies a height derivative. Comparing (119) and (28), we can note the contribution due to the variation of the scale height with respect to  $z$ . The transformation matrix is found from (111) and (119) to be

$$\tilde{B} = \begin{pmatrix} R^+ E^+ & R^- E^- \\ E^+ & E^- \end{pmatrix} \quad (120)$$

where

$$R^\pm = - \frac{(2-\gamma)/2\gamma H + i k_z^\pm}{i\omega(1-\omega_b^2/\omega^2)/gH} \quad (121)$$

$$E^\pm = \exp\left[\pm \int_{z_0}^z [dR^\pm/d\tau]/(R^- - R^+) d\tau\right] \quad (122)$$

The coupling matrix is given by

$$\tilde{B}^{-1} \cdot \frac{d\tilde{B}}{dz} = \begin{pmatrix} 0 & \frac{dR^-/dz}{R^+ - R^-} \exp \int_{z_0}^z \frac{[d(R^+ + R^-)/d\tau]}{R^+ - R^-} d\tau \\ \frac{dR^+/dz}{R^- - R^+} \exp \int_{z_0}^z \frac{[d(R^+ + R^-)/d\tau]}{R^- - R^+} d\tau & 0 \end{pmatrix} \quad (123)$$

The characteristic waves can be obtained from (115), (117) and (119).

Examining the coupling matrix, we note that the matrix becomes singular at points where  $R^+ = R^-$ . This corresponds to  $k_z^+ = k_z^-$ , or

$$\frac{\omega^2}{c_0^2} \left(1 - \frac{\omega_a^2}{\omega^2}\right) - k_h^2 \left(1 - \frac{\omega_b^2}{\omega^2}\right) + (2-2\gamma - \gamma H) \left(\frac{dl_n H}{dz}\right)^2 = 0 \quad (124)$$

In the neighborhood of these points, WKB solutions cease to be valid. Reflection occurs. It is of interest to note that by choosing different wave parameters to formulate the problem, one ends up with different matrix  $\tilde{A}$  and different eigenvalues  $k_z$  (Hines, 1965). This seems to imply that the turning points and hence the reflection level will be different for different formulations. To resolve this apparent paradox, one has to remember that the WKB solutions are invalid not only right at the turning point, but also in the neighborhood of it. Reflection or coupling of waves occurs in this whole range of heights. Some of these points are discussed in detail in Einaudi and Hines (1970).

In the WKB solution, the vertical phase speed can be defined as

$$v_{pz} = \omega/k_z \quad (125)$$

This definition is valid only in the range where the WKB solution is valid. When this is used in the interpretation of phase velocity measurements in the atmosphere, care must be taken when it is in the neighborhood of the turning point (Tolstoy, 1972).

#### 4. Attenuation by Loss Processes

From the results of WKB solution, the attenuation of the acoustic-gravity waves in a realistic atmosphere can be studied in a straight-forward manner. For a given atmosphere model, the eigenvalue  $k_z$  for the acoustic-gravity wave can be computed by including the effects of thermal conduction, viscosity, ion drag and background neutral wind. The amplitude of the wave for the velocity perturbation for example can be obtained as

$$\exp \int_{z_0}^z [1/2H - |\text{Im}k_z|] dz$$

Using this procedure, Volland (1969b) has studied the attenuation of the acoustic-gravity waves in the thermosphere with viscosity neglected.

A more accurate solution of the problem is the so-called full wave solution. The procedure is outlined as follows. The atmosphere is divided into thin layers. In each layer, the background atmospheric parameters are taken as constants and equal to the local values of the continuous atmosphere at the center of the slab. The characteristic waves are found within the layer. There can be as many as eight characteristic waves when both thermal conduction and viscosity are taken into account. At the interface between adjacent layers, interfacial conditions must be used to match the characteristic waves in the two layers. These conditions include the continuity of the velocity, pressure, temperature, heat flux, etc. (Volland, 1969b, Klostermeyer, 1972a,b). The computation is carried out throughout the thermosphere. Various authors have carried out the full wave solution for propagation of acoustic-gravity waves for different atmospheric models. Midgley and Liemohn (1966) computed the waves for the lower atmosphere below 200 km. Hines and Reddy (1967) studied the effect of wind on the acoustic-gravity waves. Volland (1969b) has shown that for gravity waves propagation in the thermosphere, the WKB solution is a sufficient approximation for the

full wave computation. Clark et al. (1970) computed the effects due to both thermal conduction and ion drag.

Perhaps the most elaborate full wave computations of acoustic-gravity waves are those made by Klostermeyer (1972a, b, c). Using the set of hydrodynamic equations, including all three dissipative processes, he computed the phase and the amplitude of the up-going gravity wave. Fig. 12 and Fig. 13 show some of his computed results. On the figures, cases where only one of the dissipative processes is present are also plotted to compare with the full solution. The model atmosphere is the 1966 U.S. Standard atmosphere, the exospheric temperature is taken as 1200°K. The geomagnetic inclination is equal to 67°. It is seen that the viscosity and thermal conduction influence the wave in much the same way. Their effects on the attenuation of the wave amplitude are of the same order. The effect of ion drag depends on frequency as expected. All three loss mechanisms affect the wave in some way and should be taken into account simultaneously to compute the phase and amplitude of the wave. Under the combined influence of viscosity, thermal conduction and ion drag,  $\text{Re}k_z$  tends to zero at great heights and  $\text{Im}k_z$  is approximately equal to  $1/2H$  above the  $F_2$ -peak, indicating decaying of the wave amplitude.

The validity of the multilayer analysis has been justified by Pierce (1966a) for an inviscid atmosphere without background wind. Recently, Hines (1973) discussed this problem again for more general atmospheric models. He pointed out that in the coupled equation approach the coefficients of the equations in general contain the background parameters as well as their height derivatives. In the standard multilayer analysis, these parameters vary stepwise with height, assuming values equal to those of the continuous atmosphere at the center of each layer, resulting in infinite, delta function, height derivatives. He suggested that, to ensure the validity of the multilayer analysis, the set of wave parameters  $e_i$ 's should be chosen such that the coefficient matrix  $\tilde{A}$  (see eq. (102)) is devoid of height derivatives of the background parameters. The specially selected  $e_i$ 's will constitute a set of wave parameters that remain continuous across the interfaces between the layers. The approach

adopted by Clark et al. (1970) and Klostermeyer (1972, a,b,c) differs from the standard multilayer analysis in that they assume both the background parameters and their height derivatives to be step functions of height. The justification for such a procedure is given by Zurmuhl (1965) in the limit of vanishing layer thickness.

##### 5. Guided Acoustic-Gravity Modes:

So far, the discussion in this chapter has been concentrated on the free acoustic-gravity waves in a realistic atmosphere for which the ground effect is neglected. As discussed in sec. C.2, in the presence of the solid ground, surface waves known as Lamb waves can exist in the atmosphere which has energy concentrated near the surface of the earth. Also discussed in sec. C.5 is the fact that a temperature discontinuity in the atmosphere can support surface waves propagating horizontally with energy concentrated near the temperature discontinuity. These are two examples of guided acoustic-gravity modes. In a realistic atmosphere, many more guided modes can exist. Press and Harkrider (1962), Pfeffer and Zarichny (1963) and Harkrider and Wells (1968) computed the properties of the fully ducted modes in a lossless atmosphere. Friedman (1966) and Tolstoy and Pan (1970) computed the imperfectly ducted modes. They have indicated that in the absence of dissipation, the structure of the spectrum of modes depends on the upper boundary condition (whether it is a free surface, rigid surface or radiation condition). Francis (1973) has shown that the presence of strong upper-atmospheric dissipation can resolve this ambiguity. The reason is that strong dissipation prohibits reflection at the upper boundary. Therefore, any reasonable upper boundary condition that requires the down-going wave at the top of the atmosphere to be small should yield the correct solution.

The solution for the guided modes in a realistic atmosphere can be carried out in quite the same way as described in the previous section for full wave calculation. The additional requirements are the boundary conditions. The atmosphere is again divided into thin layers. For a given  $\omega$  and horizontal wave number  $k_h$ , the eigenvalues  $k_z$ 's are computed for each characteristic mode. In the first layer next to the lower boundary at the surface of the earth, the characteristic waves must be chosen to satisfy the boundary condition. New eigenvalue

and characteristic waves are computed for the next layer. The waves are matched at the interface according to the interfacial conditions. The computation is repeated until we reach the upper boundary. At this point, the down-going wave must have zero amplitude. If this amplitude is nonzero, the value of  $k_x$  must be adjusted so as to eliminate this down-going wave. Each value of  $k_x$  corresponds to one guided mode.

Using this technique, Francis (1973<sup>a</sup>) studied the various guided modes for acoustic-gravity waves in a model atmosphere with realistic dissipations. The model is basically the CIRA [1965] mean atmosphere. Fig. 14 shows some of the computed modes. In general, these curves fall into several categories: a fundamental mode (F), a series of gravity modes ( $G_i$ ), a series of acoustic modes ( $A_i$ ), and a series of lower atmosphere modes ( $L_i$ ). Fig. 15 shows the horizontal attenuation distance  $|I_m k_h|^{-1}$  for the various modes. Several points of interest can be seen from the computed results. The long period gravity modes are all relatively nondispersive. Their time-averaged power is concentrated around the steep temperature gradient at the base of the thermosphere. The ducting mechanism for them is basically the same as for the surface wave along temperature discontinuity discussed earlier. The attenuation distance for the gravity mode is always smaller than that for the other modes. The two lower atmosphere modes are essentially a generalized Lamb mode ( $L_0$ ) and a mesosphericly ducted mode ( $L_1$ ). The attenuation distance for some of these modes can be very long up to many times the circumference of the earth. The short period acoustic modes are primarily ducted in the lower and upper sound channels of the atmosphere (Pfeffer and Zarichny, 1963). The attenuation distances vary as functions of period. In general, they are greater than that for gravity modes. More detailed discussion on these guided modes can be found in the reference (Francis, 1973<sup>a</sup>).

## F. INTERACTION OF INTERNAL WAVES WITH THE IONOSPHERE

In the past few chapters we have discussed possible wave motions that can be sustained in the atmosphere. The ionosphere is ignored almost entirely except perhaps in its effect on wave damping through ion drag. The justification for such an approach is based on the fact that the upper atmosphere is only weakly ionized, being less than one per cent ionized even at the ionospheric peak. Therefore, the ionospheric perturbations are expected to have very little influence on neutral perturbations. Such an observation effectively decouples the problem into two parts: (i) the problem of acoustic gravity wave propagation in which the only effect the ionosphere may have on the wave is damping through ion drag; (ii) the response of the ionosphere to the wave computed in (i). In all previous chapters our concern was exclusively that of part (i). The concern of the present chapter is part (ii). Such an approach has been adopted by many investigators [e.g. Clark et al., 1971; Klostermeyer, 1972a].

The ionization density in the ionosphere obeys the continuity equation [Ratcliffe, 1960; Rishbeth and Garriott, 1969]

$$\partial N / \partial t = Q - L - M \quad (126)$$

where  $N$  is the electron density,  $Q$  the rate of production per unit volume,  $L$  the rate of loss per unit volume due to chemical processes and  $M$  the transport term. A careful analysis of gravity wave induced perturbations in  $Q$ ,  $L$  and  $M$  have been carried out by Hooke [1968]. The conclusion is <sup>that</sup> the perturbations in  $Q$  are unimportant except when the solar ionization rays are nearly in alignment with the wave front. Further, perturbations in  $L$  that may be important in the lower F region are found unimportant in the F2 region (see Section 3). Therefore, as far as F2 region is concerned the most important gravity wave induced perturbation occurs in  $M$ . The transport term is composed of two terms, the diffusion term  $M_d$  and the wave induced term  $M_w$ . Since the ion gyrofrequency is much higher than both the ion neutral collisional frequency and the wave frequency, the charged particles

can move only along the magnetic field lines. Let the  $\zeta$ -axis be parallel to the magnetic field, then the diffusion term takes the form [e.g. Rishbeth and Garriott, 1969]

$$M_d = \text{div } N \vec{V}_d = D_a \frac{\partial^2 N}{\partial \zeta^2} + \left( \frac{\partial D_a}{\partial \zeta} + G \right) \frac{\partial N}{\partial \zeta} + \frac{\partial G}{\partial \zeta} N \quad (127)$$

where  $D_a$  is the ambipolar diffusion coefficient and  $G$  depends on gravity, temperature gradients and ion-neutral collision frequency. For an expression of  $G$  the reader is referred to Clark et al. [1971]. The wave induced motion term is given by

$$M_w = \text{div } N \vec{v}_i \quad (128)$$

Under steady state conditions the ionization velocity  $\vec{v}_i$  in response to neutral atmospheric motion is [MacLeod, 1966]

$$\vec{v}_i = \frac{1}{1+R^2} (R^2 \vec{v}' + R \vec{v}' \times \hat{B}_0 + \vec{v}' \cdot \hat{B}_0 \hat{B}_0) \quad (129)$$

where  $R = \nu_{in}/\omega_i$  is the ratio of ion-neutral collisional frequency and the ion gyrofrequency. In the F2 region  $R$  is very small, (129) immediately reduces to (54). Under this approximation (126) reduces to a parabolic partial differential equation of the second order with only one spatial dimension, that along the magnetic field lines. Such an equation can be solved numerically in the manner done by many investigators [e.g. Bailey et al., 1969; Cho and Yeh, 1970]. Some of these computations have been carried out by Clark et al. [1971]. In the following we discuss various wave induced ionospheric effects.

#### 1. Wave-associated dynamic effect on the ionosphere.

The wave-associated dynamic effect on the ionosphere comes from  $M_w$  given by (128). If the perturbation is small we may assume  $N = N_0(z) + N'(r)$  with  $|N'| \ll |N_0|$ . Then the linearized equation for  $N'$  becomes

$$\partial N' / \partial t + N_0 \text{div } \vec{v}_i + \vec{v}_i \cdot \hat{z} \partial N_0 / \partial z = 0 \quad (130)$$

For plane waves with dependence  $\exp i(\vec{k} \cdot \vec{r} - \omega t)$ , (130) can be used to solve for  $N'$ , yielding

$$N' = (1/\omega) \vec{v}' \cdot \hat{B}_0 \hat{B}_0 \cdot (\vec{k} - i\hat{z}\partial/\partial z) N_0 \quad (131)$$

where (54) has been substituted for  $\vec{v}'_i$ . The properties of (131) have been thoroughly studied by Hooke [1970]. The ionospheric response is shown to be highly anisotropic. For simplicity let us specialize to the ionospheric peak at which  $\partial N_0/\partial z$  vanishes and then (131) reduces to

$$N'/N_0 = (\vec{v}'/v_p) \cdot \hat{B}_0 \hat{B}_0 \cdot \hat{k} \quad (132)$$

where  $\hat{k} = \vec{k}/k$  and  $v_p = \omega/k =$  phase velocity. Now let  $\psi$  be the azimuthal direction of propagation measured eastward from the magnetic north,  $\theta$  be the dip angle of the wave vector from the horizontal and  $I$  be the magnetic dip angle. Further, the waves are assumed to satisfy the Boussinesq approximation so that  $\vec{v}'$  and  $\vec{k}$  are mutually perpendicular. Under these conditions we can obtain, from (132), ionospheric responses at the peak fairly easily. In the following are listed several special cases:

(i)  $\psi = 0$ , waves propagating northward along the magnetic meridian

$$N'/N_0 = (v'/v_p) \sin 2(\theta - I) \quad (133a)$$

(ii)  $\psi = \pm\pi/2$ , waves propagating either eastward or westward

$$N'/N_0 = -(v'/v_p) \sin 2\theta \sin^2 I \quad (133b)$$

(iii)  $\psi = \pi$ , waves propagating southward along the magnetic meridian

$$N'/N_0 = (v'/v_p) \sin 2(\theta + I) \quad (133c)$$

Eq. (133c) has been used by Sterling et al. [1971] to explain the possible observational biases of waves of different periods at locations of different magnetic dip.

It is interesting to note that for certain waves for which either  $\vec{v}' \cdot \hat{B}_0 = 0$  or  $\hat{B}_0 \cdot \hat{k} = 0$ , the ionospheric density at the peak does not change. Again by assuming Boussinesq approximation the conditions for the absence of ionospheric response at the peak can be found. The results are shown in Fig. 16. The directional and frequency filtering by ionospheric response processes is obvious.

Another point worth mentioning is the possibility that the ionosphere may be in relative motion with respect to the neutral atmosphere. Such a relative motion may be created, for example, by the neutral winds and the electric field. If the ionosphere drifts with a velocity very nearly equal to the phase velocity of the internal gravity wave, the ionosphere is expected to be perturbed resonantly. This resonance is called the spatial resonance by Whitehead [1971]. The presence of relative horizontal motion can be taken into account by replacing  $\omega$  in (131) by  $\omega' = \omega - \vec{k} \cdot \vec{v}_{i0}$  where  $\omega$  is the wave frequency in the frame of neutral atmosphere and  $\vec{v}_{i0}$  is the horizontal velocity of the ionosphere relative to the neutral atmosphere. When there is spatial resonance the Doppler shifted frequency  $\omega'$  vanishes. Consequently, according to (131) the ionization perturbation becomes infinite. In practice, however, other effects such as diffusion and nonlinear effects are expected to limit the growth in  $N'$  to a large but finite value.

In some experimental investigations the electron density integrated along a radio ray path from the satellite to the ground observer is measured. This integrated electron density will also be perturbed because of ionospheric response to acoustic gravity waves. By using (131) the perturbed integrated density along an oblique path is

$$I'_{ob} = \int N' dr = (v'_B / \omega \cos \chi) (\hat{r} \times \hat{B}_0) \times \hat{z} \cdot \vec{k} \int N_0(z) e^{i\vec{k} \cdot \vec{r}} dr \quad (134)$$

where  $v'_B = \hat{B}_0 \cdot \vec{v}'$ ,  $\chi$  is the zenith angle of the ray,  $\hat{r}$  is a unit vector along the ray from receiver to satellite over which the integration is carried out, and  $\hat{z}$  is a unit vertical vector. If the background ionosphere is an  $\alpha$ -Chapman model given by

$$N_0(z) = N_m \exp \frac{1}{2} \left[ 1 - \frac{z-z_m}{H} - \exp - \frac{z-z_m}{H} \right]$$

then (134) can be integrated to produce [Georges and Hooke, 1970; Yeh, 1972]

$$|I'/I_0| = (v'_B / v_p \cos \chi) [(\hat{r} \times \hat{B}_0) \times \hat{z} \cdot \vec{k}] / \sqrt{\cosh(\pi k H \cos \eta / \cos \chi)} \quad (135)$$

where  $\theta$  is the angle between  $r$  and  $k$ . In (135) both  $I'$  and  $I$  are referred to the equivalent vertical integration, i.e.  $I' = I'_{ob} \cos \theta$ . A very thorough treatment on electron content perturbation has been carried out by Davis [1973].

## 2. Wave-associated transport effect on the ionosphere

In the upper ionospheric F2 region the diffusion may be very rapid. Its effect on ionospheric response to acoustic-gravity waves can be examined by including the diffusion term approximately in (130) to produce

$$\partial N' / \partial t - D_a \partial^2 N' / \partial z^2 + \text{div } N_0 \vec{v}_i = 0 \quad (136)$$

For plane waves, (136) can be solved to result in

$$N' = (\omega + i D_a k_B^2)^{-1} \vec{v}' \cdot \hat{B}_0 \hat{B}_0 \cdot (\vec{k} - i \hat{z} \partial / \partial z) N_0 \quad (137)$$

where  $k_B = \vec{k} \cdot \hat{B}_0$ . Inspection of (137) shows that the effect of diffusion is to introduce a phase shift from that due to dynamic effect alone [Cunnold, 1967]. Taking  $D_a = 2.1 \times 10^6 \text{ m}^2/\text{s}$ ,  $k_B = 3 \times 10^{-5} \text{ m}^{-1}$  we obtain  $D_a k_B^2 \approx 2 \times 10^{-3} \text{ sec}^{-1}$ . The diffusion term can be ignored only for those waves with  $\omega \gg 10 D_a k_B^2$  which corresponds to waves with periods less than about 5 minutes. Therefore in this case, the diffusion effects are expected to be significant in influencing ionospheric response for waves with periods larger than about 5 minutes in the F2 region.

The importance of diffusion has also been demonstrated by numerical computations. The wave perturbed motion term in the linearized limit is

$$M'_w = \text{div} (N_0 \vec{v}'_i + N' \vec{v}'_{d0} + N_0 \vec{v}'_d) \quad (138)$$

Comparison of the dynamic term  $|N_0 \vec{v}'_i|$  and the diffusion term  $|N' \vec{v}'_{d0}|$  shows that they are of equal order of magnitude for many internal waves investigated [Clark et al., 1971]. Experimentally it has also been found that better agreement can be achieved by including the diffusion term as shown in Fig. 17 [Testud and Francois, 1971].

### 3. Wave-associated photochemical effect on the ionosphere

In the lower F region both photoionization and chemical processes may play important roles in controlling the behavior of the ionosphere. The presence of acoustic-gravity waves will influence the ionization rate through changes in the local neutral density and through changes in the ionization radiation absorption. In an isothermal exponential atmosphere of scale height  $H_i$  for the ionizable constituent the rate of production of ionization per unit volume is given by the classical Chapman formula [Chapman, 1931]

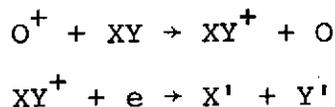
$$q_0 = q_m \exp \left[ 1 - \frac{z-z_m}{H_i} - \exp \left( - \frac{z-z_m}{H_i} \right) \right] \quad (139)$$

When the atmosphere is perturbed by a sinusoidally steady state wave the production function (139) is perturbed. In the linearized limit the perturbed production  $q'$  has been found to satisfy [Hooke, 1968]

$$q'/q_0 = (\rho'/\rho_0) \left[ 1 - \frac{\cos \chi \exp \left[ -(z-z_m)/H_i \right]}{(1-H_i/2H) \cos \chi - iH_i \vec{k} \cdot \hat{r}} \right] \quad (140)$$

where  $\chi$  is the solar zenith angle,  $H$  is the neutral scale height and  $\hat{r}$  is a unit vector pointed from the ionosphere to the sun. Inspection of (140) shows that  $q'$  is expected to be fairly large for those waves that satisfy  $\vec{k} \cdot \hat{r} = 0$ , i.e. waves with wave fronts aligned with the direction of solar radiation. The numerical results of Fig. 18 demonstrate this point clearly.

Perturbations in the neutral atmosphere also cause perturbations in chemical processes. In the lower F region following reactions are believed to take place [Ratcliffe, 1960 p. 385]



Let  $\lambda_1$  be the reaction rate of the charge exchange process given by the top reaction and  $\lambda_2$  the rate of dissociative recombination process given by the second reaction. Then the densities of various ions must satisfy the following conservation equations.

$$\partial n(O^+)/\partial t = q - \lambda_1 n(XY) n(O^+) - \nabla \cdot [n(O^+) \vec{v}(O^+)] \quad (141)$$

$$\begin{aligned} \partial n(XY^+)/\partial t = & \lambda_1 n(XY) n(O^+) - \lambda_2 n(XY^+) n(e) \\ & - \nabla \cdot [n(XY^+) \vec{v}(XY^+)] \end{aligned} \quad (142)$$

Here  $n(\cdot)$  denotes the number density of the particles signified. An equation for electron density  $n(e)$  can be obtained similarly, but this equation is not independent of the two given because of the charge neutrality condition

$$n(e) = n(O^+) + n(XY^+) \quad (143)$$

Now suppose the wave associated perturbations are small and can be linearized. Let the perturbed quantities have time dependence  $\exp -i\omega t$ . Then the linearized equations of (141), (142) and (143) with diffusion effects ignored can be used to solve for the perturbed electron density, yielding

$$\begin{aligned} n'(e) = & [\lambda_2 n_0(e) + \lambda_2 n_0(XY^+) - i\omega]^{-1} [\lambda_1 n_0(XY) - i\omega]^{-1} [\lambda_1 n_0(XY) \\ & + \lambda_2 n_0(XY^+) - i\omega] [q' - \nabla \cdot (n_0(O^+) \vec{v}')] - \lambda_1 \lambda_2 n_0(e) n'(XY) n_0(O^+) \\ & - [\lambda_1 n_0(XY) - i\omega] \nabla \cdot [n_0(XY^+) \vec{v}'] \end{aligned} \quad (144)$$

Even though the formula (144) is long, it is mainly algebraic and can be computed relatively simply. However, because the number of parameters involved is fairly large, many intricate details can be learned only after systematic computations. Many computations have been made by Hooke [1970]. One example is shown in Fig. 18. The effect produced by the alignment of the solar radiation and the wave front is obviously revealed in this figure.

#### 4. Interaction with a realistic ionosphere

In previous several sections we have discussed processes involved in the interaction of internal waves with the ionosphere. These processes, if all included, would make the problem fairly complex. Furthermore, as mentioned in the beginning of Chapter F, the ionospheric response is the second part of a two-part problem,

the first part deals with the propagation of acoustic-gravity waves. Therefore, in order to compute ionization perturbations realistically we must first compute neutral acoustic-gravity wave perturbations realistically. The propagation of acoustic-gravity waves in a realistic atmosphere has been discussed in Chapter E. The problem can be solved by the coupled equation formulation using one of several multilayer approaches [Hines and Reddy, 1967; Volland, 1969b; Clark et al., 1970; Klostermeyer, 1972a]. After obtaining these first-order wave associated perturbations in the neutral atmosphere, the ionospheric response is computed by solving the equation of continuity in its original form (126) or the linearized version [Thome and Rao, 1969; Klostermeyer, 1972a, b]. It should be pointed out that in all these computations in the literature, certain simplifying but not necessarily realistic assumptions have been made. For example in numerical calculations of gravity wave propagation assumptions may be made in relation to neutral background wind, the three loss processes, and the Coriolis force. In this regard the formulation used by Klostermeyer [1972a,b] seems to be most complete. On the other hand the simplifying assumptions in calculating the ionospheric response are related to the diffusion term, the ionization production term and the ionization loss term. Here the formulation used by Clark et al. [1970] seems to be most complete, at least for F region effects. They further show that even if the neutral parameters are of perturbation magnitude, the ionospheric response may be very large especially when the gas parcel orbit and the geomagnetic field are nearly parallel. However, the effects produced by wind induced ionization motion do not seem to have received attention in the literature so far.

Fig. 19 shows the numerically computed isoionic contours by solving the continuity equation (solid lines) as compared with the results where the only effect included is the wave-associated dynamics in the directed velocity (dotted line).

## G. EXPERIMENTAL OBSERVATIONS IN THE UPPER ATMOSPHERE AND IONOSPHERE

The experimental evidence for the existence of acoustic-gravity waves in the upper atmosphere is rather indirect and in many cases a correct and unambiguous interpretation of the experimental data may not be possible. This is because the wave process deals with space-time fields. In order to completely describe it one has to monitor all perturbed quantities with sufficient resolution in space and in time. On a geophysical scale such a monitoring program will be very difficult and costly. Some of these problems are discussed by Lindzen [1969].

Historically the evidence came first from observations of traveling ionospheric waves (called TIDs). As early as the 1920's communication engineers were already troubled by the presence of fading in ionospherically reflected signals [Mimno, 1937]. Further experimentation showed at least partly the fading was caused by the focusing and defocusing of radio waves by ionospheric ripples [Pierce and Mimno, 1940]. Munro [1950, 1958] made a systematic study of these ripples and obtained direction and velocity of travel. Martyn [1950] proposed to explain these traveling disturbances in terms of cellular atmospheric waves. Martyn's theory has since been modified and developed by Hines [1960]. Hines proposed acoustic-gravity waves as the driving force of traveling disturbances. This proposal seems to be most satisfactory and is accepted by most investigators in the field at present. However, it should be cautioned that acoustic-gravity waves are not the only waves that can be sustained in the upper atmosphere. The neutral atmosphere can also support tides with specific periods [Siebert, 1961; Dikii, 1965] while there can exist a variety of hydromagnetic and plasma waves in the ionosphere [e.g. Ratcliffe, 1959; Ginzburg, 1970; Yeh and Liu, 1972a, b]. There are periodic reports of experimental data that cannot be explained by acoustic-gravity waves [Heisler and Whitehead, 1961; Greenhow and Neufeld, 1961; Witt, 1962; Theon et al., 1967; Herron, 1973]; some of these may be real, but some may be complicated by experimental uncertainties. In some cases a deeper examination of the experimental data has found no inconsistencies with the acoustic-gravity wave hypotheses [Francis, 1973c]. But it seems that most of the observations can be explained by

the acoustic-gravity wave theory [Hines, 1960; Gershman and Grigor'ev, 1968; Vasseur et al., 1972]. The strongest support seems to be ionospheric observations following nuclear detonations in the atmosphere [Hines, 1967; Row, 1967].

## 1. Survey of Experimental Techniques

There have been many techniques used to detect traveling waves in the lower thermosphere and ionosphere. It is impossible to discuss the merits of each of these techniques in this article. In Table 2 and Table 3 we merely list some references where these techniques as well as experimental results are discussed. Table 2 lists those experimental techniques which essentially respond to wave fields in the neutral atmosphere; Table 3 lists those experimental techniques which are essentially measurements on the ionization.

Before closing this section we show in Fig. 20 a comparison between the observed ionization contours and the synthesized ionization contours by assuming that the smoothed background ionosphere is perturbed by an internal gravity wave [Thome and Rao, 1969]. It is obvious that good agreement has been obtained.

## 2. Generation by Impulse Sources

Probably the most stringent test of the acoustic-gravity theory is to subject its theoretical predictions to an experimental verification from a known source. In the atmosphere the experiment that comes closest to this requirement is probably that associated with the atmospheric nuclear detonations. Many observations have shown major ionospheric responses following nuclear explosions in the atmosphere [Dieminger and Kohl, 1962; Obayashi, 1962; Webb and Daniels, 1964; Wickersham, 1966; Kannellakos, 1967; Albee and Kanelkakos, 1968]. Attempts have been made to explain the surface observations in terms of guided modes [See review by Donn and Shaw, 1967]. Most of these surface observations show a period of the order of several minutes. However, in the ionosphere oscillations of tens of minutes to hours have been observed. Further, the oscillation period seems to depend linearly on the distance of the observer from the ground zero. A well-known example is depicted in Fig. 21 which shows oscillations

Table 2. Experimental techniques which essentially respond to wave fields in the neutral atmosphere.

Experimental Technique	Reference
Long enduring visual meteor trails	Liller and Whipple [1954]
Meteor radar	Manning et al. [1954], Greenhow and Neuffeld [1959], Revah and Spizzichino [1964], Revah [1969]
Chemical luminescent trails	Kochanski [1964]
Noctilucent cloud waves	Witt [1962]
In situ measurements	Newton et al. [1969, Dyson et al. [1970]
FW/CW radar observations	Gossard et al. [1970]

Table 3. Experimental techniques which are essentially measurements on the ionization.

Experimental Technique	Reference
Virtual height vs. time data	Munro [1950, 1958], Toman [1955], Gossard [1967, 1969]
Ionosonde observations	Heisler [1958], Baker and Gledhill [1965], Klostermeyer [1969].
HF backscatter sounding	Valverde [1958, Hunsucker and Tveten [1967]
Direction of arrival data	Bramley [1953], Detert [1965]
Oblique incidence Doppler	Chan and Villard [1962]
Vertical incidence Doppler	Davies [1962], Georges 1968]
Electron content from a low orbiting satellite	Rao and Yeh [1968], Rao et al. [1969], Liszka [1971]
Electron content from a geostationary satellite	Davis and deRosa [1969], Cowling et al [1970], Schodel [1972]
Incoherent scatter sounding	Thome [1964], Testud and Vasseur [1969], Evans et al. [1970]

in the ionospheric critical frequency  $f_0F_2$  at five observatories following the Soviet nuclear detonation of October 30, 1961 at Novaya Zemlya. An explanation of the oscillation period dependence on distance based on acoustic gravity wave theory has been made by Hines [1967] and Row [1967]. As shown in Chapter D section 2, the asymptotic response in a lossless isothermal atmosphere to a stationary impulse source can be considered as the Green's function. For large time, the Green's function for a fixed observer essentially consists of three terms, one oscillating at the acoustic cutoff frequency  $\omega_a$ , one at the buoyancy frequency  $\omega_b$  and one at the characteristic frequency  $\omega_c = \omega_b z/R$  where  $z$  is the height of disturbance above the explosion and  $R$  is the slant range from the location of the explosion to the location of the disturbance (see equation (83)). Numerical computations [see Fig. 10, or Liu and Yeh, 1972] indicate that, of the three waves generated, the buoyancy wave will decay fastest, the next is the acoustic wave and the gravity wave has the slowest decay. Therefore, as time progresses the atmospheric response can be described as essentially emission of three waves: acoustic wave which arrives at the observer first, the buoyancy wave which is short living and weak, and the gravity wave which is long living. Asymptotically then, the wave with frequency  $\omega_c$  will be the only one remaining. We note that since  $\omega_c = \omega_b z/R \cong \omega_b z/d$  where  $d$  is the ground distance of the observer from the explosive source, the oscillation period  $T = 2\pi/\omega_c$  is theoretically expected to be directly proportional to  $d$ , as the variation of ionospheric height  $z$  is very small. The behavior  $T \propto d$  is exactly the behavior observed experimentally as shown in Fig. 21. Hence the correctness of the theory has received a strong support. The use of plane geometries in the theory to explain gravity wave observations on a spherical earth can be justified by noting that the earth curvature is nearly compensated by the refraction of the gravity wave ray caused by changing direction of the earth's gravity [Francis, 1972].

We note that a wave with  $\omega = \omega_c$  satisfies the asymptotic relation (30) whose dispersion relation applies to a Boussinesq fluid with  $\omega^2 \ll \omega_b^2$ . In this limit the horizontal group velocity and the horizontal phase velocity are equal.

Other examples of impulsive sources are earthquakes [Leonard and Barnes, 1965; Davies and Baker, 1965; Row, 1967] and possibly impact created by great meteors.

### 3. Generation of Magnetic Storm Related Events

Numerous experiments have suggested the connection between magnetic activities in the auroral zone and the occurrence of traveling disturbances at middle and low latitude ionosphere [e.g. Valverde, 1958; Chan and Villard, 1962; Bowman, 1965; Georges, 1968; Thome, 1968]. There is evidence to indicate that the source of these disturbances is in the evening sector of the auroral oval [Davis, 1971]. The observed disturbances are usually called the large scale disturbances according to the classification scheme of Georges [1968]. These large scale disturbances have periods varying from 30 minutes to several hours, some as high as ten hours [Alcayde et al., 1972]. The horizontal wave fronts may extend several thousand kilometers in the east-west direction and travel southward in the northern hemisphere with horizontal velocities in the range 400 to 700 m/s. Many of these disturbances travel about a quarter of the way around the globe. In many cases the shape of the wave train is preserved even after traveling a distance of one thousand kilometers. An example is shown in Fig. 22. Incoherent scatter observations support the gravity wave interpretation of these waves [Thome, 1968; Testud and Vasseur, 1969]. As discussed in Chapter E there may exist guided acoustic-gravity modes in addition to free modes because of the temperature structure in the thermosphere. It is therefore still necessary to decide whether these large scale traveling waves are excited by free or guided acoustic-gravity waves. Some investigators have ascribed their observations to free waves [Georges, 1968; Testud and Vasseur, 1969], while there are also others in favor of <sup>a</sup>guided wave interpretation [Friedman, 1966; Francis, 1973a]. Recent comprehensive computations of Francis [1973a] suggest that all observed properties of large scale traveling disturbances can be explained by the guided wave theory. These properties include the near sonic velocity of travel, an attenuation distance of the order 1/8 terrestrial circumference, and the tilt of the phase front. As discussed in Chapter E the upper atmosphere can be roughly described as a warm thermosphere on top of a cool lower atmosphere. The guided mode in such an atmosphere has energy confined mainly near the temperature discontinuity (see Fig. 6). Since activities

associated with auroral events are taking place near this temperature discontinuity there should be no difficulty for these activities to deposit the energy in the guided wave field and then to be propagated away.

Suggestions have been made that acoustic-gravity waves may be generated by a supersonic displacement of auroral arcs through Lorentz force [Wilson, 1969] or the Joule heating associated with the sudden surge of auroral electrojet currents [Davis and da Rosa, 1969]. Recent theoretical work [Chimonas and Hines, 1970b; Chimonas and Peltier, 1970] has shown that such waves can be generated by either of the two mechanisms given above. But it should be mentioned that for theoretical convenience an isothermal atmosphere was assumed by these authors. This assumption effectively removes guided waves from consideration.

#### 4. Response to Seismic Waves

At the end of section 2 we mentioned that certain earthquakes may act as an impulsive source for acoustic gravity waves. The wave so generated has the characteristic frequency of oscillation  $\omega_c$ . However, earthquakes also excite seismic waves which can in turn generate atmospheric waves through continuous coupling in a manner discussed in section 4 of Chapter D. Since the Rayleigh wave travels at a velocity much faster than the atmospheric sound velocity the atmospheric wave is launched at a very steep angle. The excited wave usually belongs to the acoustic branch. Numerically, a Rayleigh wave of period 25 seconds, speed 3.9 km/s, and peak-to-peak vertical ground displacement 5 mm will give rise to a vertical air parcel velocity of 30 m/s at 150 km height. Such a velocity will produce effects that are susceptible to experimental measurements.

The experimental evidence to support the excitation of acoustic waves by seismic waves is shown in Fig. 23 [Yuen et al., 1969]. A three-component very-long-period seismograph system located at Oahu, Hawaii recorded data following an earthquake at Hachinoke, Japan. On the figure the  $R_1$  window marks the time interval during which only the Rayleigh component of the seismic wave was important. Simultaneously an HF Doppler recorder at Honolulu recorded ionospheric motions at a 300 km height. As shown in Fig. 23 a good correlation

↪

is obtained if the two records are displaced in time. The time difference agrees very well with the propagation time required for both the Rayleigh wave and the acoustic wave [Yuen et al., 1969].

##### 5. Properties of Medium-Scale Traveling Disturbances

By far the most persistent traveling ionospheric disturbances are those of medium scale. Most of these disturbances appear as a train of quasi-periodic oscillations with a period centered at about 20 to 30 minutes, a horizontal scale of 100 to 200 km and a speed of about 100 to 200 m/s. Extensive observations have been made by Munro [1958], Tveten [1961], Chan and Villard [1962], Davies and Jones [1971] and many others. The observed wave front has a negative tilt in the direction of propagation of about  $-40^{\circ} \pm 10^{\circ}$ . Many reported observations seem to satisfy the dispersion relation for a Boussinesq fluid [Chang 1972; Davies and Jones, 1971]. This experimental evidence suggests that some of these waves are generated by sources with a wide frequency spectrum, such as an impulsive source. It also suggests that the gravity waves responsible for excitation of traveling disturbances are free waves. Several investigators have earlier proposed that these gravity waves are fully or imperfectly ducted [Wickersham, 1966; Friedman, 1966]. However, recent calculations by Francis [1973b] have shown that based on the observed velocities the ducted mode hypothesis is untenable, in support of the free wave hypothesis. The results of Munro [1958] and others show that these medium-scale traveling disturbances often change their shape or may even disappear over a distance of 100 to 200 km. Such a behavior cannot be explained by the wave dispersion alone. It is possible that several waves from different sources may give rise to a very complex interference effect in these cases.

Unlike the large scale traveling disturbances which usually travel equatorward away from the auroral zone, the medium-scale traveling disturbances may travel in any direction. The peculiar daytime directional data are very interesting. Both the southern hemisphere data [Munro, 1958] and the northern hemisphere data [Davies and Jones, 1971] show that the disturbances on the average travel equatorward in the local winter and poleward in the local

summer. The seasonal switch in direction is through the westward direction during equinoxes in the northern hemisphere and through the eastward direction in the southern hemisphere. There does not appear to have any satisfactory explanation for this directional behavior.

Recently several investigators have carried out spectral analysis of various data. When done this way there often exist several fairly sharp lines showing the quasi-periodic nature of the medium-scale traveling disturbances. Superimposed on these sharp lines is the background spectrum which is usually above noise and is statistically meaningful. It represents the presence of random fluctuations of the gravity wave field [Georges, 1968]. Tolstoy and Montes [1971] have analyzed some Doppler sounding records and showed the existence of a break in spectral slope near the buoyancy frequency  $\omega_b$ . Several studies of spectrally analyzed electron content data have also been carried out [Schödel, 1972; Titheridge, 1971<sup>a</sup>]. An average cutoff in spectrum near  $\omega_b$  is commonly observed. The spectral cutoff near  $\omega_b$  is expected on theoretical grounds since waves near  $\omega_b$  are severely damped through both viscosity and thermal conduction processes (see Fig. 8). However, on each individual data basis the observed cutoff may at times be so high that a straightforward interpretation as a cutoff at  $\omega_b$  would lead to an unacceptably low thermospheric temperature of 180°K. It is hypothesized by Davies et al. [1973] and Gupta et al. [1973] that the neutral wind may have Doppler shifted the cutoff frequency upward in these cases. In all these observations there also occur a low frequency cutoff at about 60 to 100 minutes. Again the viscosity and thermal conduction seem to be responsible for filtering out these low frequency (or long period) waves although not as sharply as the high frequency cutoff near  $\omega_b$ . In addition to the low frequency cutoff and the high frequency cutoff there also may occur a bite-out in the spectrum. Setty et al. [1973] have made a study of this phenomenon by using  $f_0F_2$  data at stations of several dip angles. They have shown that this bite-out is related to the gravity wave-ionosphere interaction processes discussed in Chapter F. Since most of their waves propagate in the southward direction (azimuthal angle 180°), certain waves will not be able to excite ionospheric perturbations. These waves have a period that depends on the dip angle as shown in Fig. 16.

Even though the medium-scale traveling disturbances occur almost daily their excitation mechanism and the nature of their sources are still unknown. Most investigators turn their attention to potentially important meteorological causes such as mountain waves, weather fronts, instabilities or distortions of jet streams, and severe storms. Goe [1971] has shown a correlation between the occurrence of traveling disturbances and the existence of a large horizontal wind shear in the jet stream. Recently surface data seem to indicate the possibility that gravity waves with an average period of 3 hours and a horizontal trace speed near 100 knots may be a precursor to the convective storms and may even act to initiate and to reintensify these storms [Uccellini, 1973]. It is not known whether these surface storm-related gravity waves have appreciable amplitude in the 30 minute period region (since in Uccellini's analysis a bandpass filter was used) or whether it is possible to generate higher frequency waves through nonlinear effects. In any case, to establish a causal relationship one is required to trace the group rays of acoustic-gravity waves. Such a ray tracing is found to be rather sensitive to the atmosphere model and very sensitive to the wind model [Cowling et al., 1970]. This means that a fairly accurate atmosphere and wind model for heights up to several hundred kilometers are very desirable. Currently these wind models are extremely gross.

## 6. Effect of Background Winds

Satellite drag studies have revealed the dynamic state of the thermosphere. One of the associated phenomenon is the diurnal wind [See, for example, the review by Rishbeth, 1972]. The effect of winds on the propagation of acoustic gravity waves has been discussed in C.4 and C.6. Experimentally there are observations which indicate the importance of winds. For example, a wave observed by Davies and Jones [1972a] exhibited properties of gravity waves, yet its period near 7 min is less than the buoyancy period of 9.5 min. After examining various possibilities they concluded that their observational results were consistent with our current understanding of the thermospheric properties and the acoustic gravity theory if a horizontal wind blowing in the direction of horizontal phase propagation with a speed of 70 m/s was assumed. Of course the neutral wind was also introduced to explain the observed spectral cutoff when it was

higher than  $\omega_p$  as discussed in the previous section. As another example, the zonal wind pattern in the height range 80 to 110 km has been measured by Revah [1969] by using a meteoric radar. The existence of propagating internal gravity wave was identified. In order to explain their behavior as a function of height he proposed the partial reflection mechanism by temperature and wind discontinuities.

As discussed in C.4 the strong thermospheric winds may act as a directional filter which would permit waves propagating in a certain direction to reach the ionospheric height [Cowling et al., 1971]. Using the theoretically computed time dependent wind model of Cho and Yeh [1970], internal gravity waves of period 30 min. and horizontal phase speed 150 m/s are traced from the ground to a height of 300 km. The time required for these waves to reach the 300 km height is shown as contours in hours in Fig. 24 [Yeh et al., 1972]. Observational data points of Munro [1958] are also plotted on the figure. As seen from the figure, most of observed waves appear near contours of minimum travel time. It should be pointed out that a study of this nature requires an accurate wind profile throughout the atmosphere. Since the winds are known to be highly variable, any conclusion drawn from such model studies must be treated with caution.

## 7. Short Period Disturbances

In addition to long period disturbances that are interpreted as gravity waves there are also observations of short period disturbances belonging to the acoustic branch. Disturbances with a period from 10 seconds to 50 seconds have on occasion been observed [Shrestha, 1967; Rao et al., 1969]. These infrasonic disturbances are interpreted as acoustic waves but because of rarity no systematic study has been made on them. For longer period waves from 2 minutes to about 5 minutes many observations have been made. A connection between the ionospheric appearance of these acoustic waves and the severe weather activities has been established [Georges, 1968; Baker and Davies, 1969]. Backward ray tracing indicates that these waves are coming from thunderstorm cells of an unknown excitation mechanism. The cloud tops are generally greater than 12 km height. The horizontal distance of travel is not large as these waves are observed only near (within 250 km) these cells. The spectral results show two

peaks, one at 4.5 min and one at about 3.5 min. A comprehensive review on this subject has been prepared by Davies and Jones [1972b]. Its relation to infrasound pressure waves recorded on the ground surface is reviewed by Georges [1973].

In the auroral zone the infrasound has also been detected and related to moving auroral electrojets [Wilson, 1969; Liszka and Westin, 1972]. Other sources of acoustic waves are earthquakes [Davies and Baker, 1965] and ground level explosion [Barry et al., 1966]. A fairly complete bibliography on infrasonic waves has been compiled by Thomas et al. [1971].

#### 8. Implication for Thermospheric Dynamics

At present the totality of experimental evidence supports strongly the existence of acoustic-gravity waves in the upper atmosphere, although we must also realize that the atmosphere and ionosphere can support other waves. As these acoustic-gravity waves propagate away from the source they carry with them both momentum and energy. Through interaction with the background atmosphere the momentum and energy may be transferred. The interaction is especially strong near the critical layer at which the background wind velocity matches the horizontal phase velocity [Booker and Bretherton, 1967]. It has been speculated that the interesting phenomenon of quasi-biennial oscillation in the tropical stratosphere may be caused by such a process [Lindzen, 1968; Lindzen and Holton, 1968]. However, the interaction is not restricted to the critical layer; the wave damping processes may also transfer momentum and energy from the wave to the background atmosphere. Theoretical investigations have shown [Lindzen, 1971] that the semi-diurnal tides may contribute significantly to the heating of the upper atmosphere, although the desired second heat source from incoherent scatter results has a diurnal behavior different from that predicted theoretically [See also Volland and Mayr, 1972].

In the acoustic-gravity wave spectrum the calculations deduced from radiosonde observations up to a height of 4 km show that there is appreciable leakage of energy into the upper atmosphere [Gossard, 1962]. The calculated spectral window extends from 0.14 mHz to 1.6 mHz (or 10 min to 2 hrs in period) with a maximum leakage from 12 min to 20 min. Surface measurements indicate that the vertical energy flux during quiet days is of the order of 0.02 joules/m<sup>2</sup>-s [Gossard, 1962]. A 90% of this total flux is in the range of 15 min.

to 2 hrs. with a maximum at 26 min. During storms and the passage of a frontal system the vertical flux may increase to  $0.1 \text{ joules/m}^2\text{-s}$ . For a few times in a year it may be as large as  $1 \text{ joule/m}^2\text{-s}$ . Even for quiet days, the flux represents a heat source of  $2 \times 10^{-7} \text{ joules/m}^3\text{-s}$  if we assume that the wave energy is dissipated uniformly over a 100 km thickness atmosphere. The solar EUV heat source [Volland, 1969c] has a maximum  $4.5 \times 10^{-10} \text{ joules/m}^3\text{-s}$  in the upper atmosphere. A comparison of these two numbers would indicate the potential of gravity waves as a heat source in the upper atmosphere. Of course only a very small portion of the leaked acoustic gravity waves will reach the F region heights. Hines [1965b] estimated that 50 per cent of the time an energy flux of  $10^{-4} \text{ joules/m}^2\text{-s}$  will reach F region heights. Based on actual measurements many observed traveling ionospheric disturbances have been interpreted as supplying a heat input of  $10^{-10} \text{ joules/m}^3\text{-s}$  to the F region atmosphere [Testud, 1970; Yeh, 1972]. Depending on the length such a heat source is acting, the thermospheric temperature may increase by 40 to  $80^\circ\text{K}$  [Klostermeyer, 1973].

The wave associated transport of heat is not restricted to the vertical direction alone; there may also exist horizontal transport. Satellite drag data have shown an increase in exospheric temperature following magnetic activity. The <sup>temperature</sup> increase in middle and low latitudes comes only a few hours after the storm, a time too short to be explained by molecular diffusion. Gold [private communication to Hines, 1965b] suggested that the spread of heating from auroral region to lower latitudes was caused by wave dissipation. Calculations by Klostermeyer [1973] tend to support Gold's suggestion.

We mentioned earlier that acoustic gravity waves carry not only energy but also momentum. As these waves are being dissipated they deposit a part of their momentum in the background flow [Hines, 1972]. The average vertical flow of horizontal momentum associated with the wave is  $\text{Re} \rho_0 v_h' v_z'^*$  where the complex notation is used. The expression is especially simple for a low frequency wave in the Boussinesq fluid in which  $v_z' = v_h' \omega / \omega_b = v_h' T_b / T$ . Making this assumption, the vertical flux of horizontal momentum reduces to  $\rho_0 v_h'^2 T_b / 2T$ . In an inviscid atmosphere this vertical flux is constant as expected. The presence of dissipative processes will make this flux decay with

height. For estimation purposes let the wave be dissipated completely over a height range  $h$ , then this wave flux must be transferred to the background flow in the amount  $\rho_0 h \dot{v}_0$  where  $\dot{v}_0$  is the time rate of change of the horizontal flow. Equating these two fluxes produces the formula [Hines, 1972]

$$\dot{v}_0 = v_h'^2 T_b / 2Th \quad (145)$$

Numerically Hines took  $v_h' = 100$  m/s,  $T_b = 5$  min,  $T = 200$  min and  $h = 6.5$  km. The use of (145) yields  $\dot{v}_0 = 70$  m/s/hr, a value that points out the potential importance of this process in generating winds in the upper atmosphere. Indeed Hines suggested that the dissipation of semidiurnal tide might be the mechanism responsible for the observed super-rotation of the thermosphere.

The existence of <sup>a</sup>/random acoustic gravity field could alter atmospheric properties in many respects. The expected changes come about through wave induced processes which affect (i) transport coefficients, (ii) chemical reaction coefficients, and (iii) increased damping of other acoustic gravity waves. Hoult [1968] has derived an expression for the diffusion coefficient due to vertically propagating random acoustic waves in an isothermal atmosphere. The enhanced diffusion may alter the atmospheric composition above the height at which the diffusive separation takes place. This implies a change of chemical reaction coefficients. King [1966, 1967] has hypothesized that the storm related random acoustic waves was responsible for the enhanced mixing in order to explain the decrease in observed ionization following magnetic activities. The presence of random fields will also give rise to attenuation of coherent acoustic gravity waves. This attenuation comes about because of randomization of the coherent wave through scattering. This effect has been considered by Liu [1970] and Wenzel and Keller [1971].

## H. CONCLUSION

In the past chapters we have reviewed the recent theoretical work on the propagation and generation of acoustic gravity waves and the experimental results that support the existence of such waves in the upper atmosphere. Because of the presence of the ground and the large temperature gradient in the lower thermosphere, the atmosphere is capable of supporting guided modes as well as free waves. The free waves have a continuous spectrum, a real horizontal and vertical wave number, and a wave energy independent of height in the inviscid part of the atmosphere. On the other hand, the guided waves have a complicated discrete spectrum and a complex horizontal and vertical wave number implying attenuation by loss processes and energy leakage to the thermosphere. The wave energy of such guided modes is confined either near the ground or near the height of large temperature gradient. At present our understanding of propagation effects of these waves is fairly complete. However, in order to predict theoretically the outcome of a wave in the thermosphere it is necessary to prescribe completely the atmosphere en route. This includes not only temperature, density and composition but also the three dimensional wind vector.

Several processes are involved in the interaction between acoustic gravity waves and the ionosphere. The physics of the interaction processes seems to be well understood. But a quantitative prediction still requires numerical computation. Most of the experimental data comes from observations of traveling disturbances in the ionosphere. Experimental evidence indicates that these disturbances are excited by the propagating acoustic gravity waves.

Even though we understand the propagation effects and the processes involved in the interaction with the ionosphere, we still do not know precisely the mechanism responsible for the creation of acoustic gravity waves. Theoretical studies have been carried out by assuming the atmosphere to be either isothermal or inviscid or both. The isothermal assumption removes the possibility of exciting guided modes from consideration, while an assumed ad hoc upper

boundary condition such as that necessary in an inviscid atmosphere is both ambiguous and unrealistic. Experimental observations indicate that certain thunderstorm cells are responsible for generating acoustic waves and certain auroral events are followed by large-scale traveling disturbances. But the precise physical mechanism is still not known. As for the medium-scale waves, which by far are the most frequently observed waves, we do not even have hard evidence to relate them to some causal events or mechanisms.

In all previous discussions in this paper we have assumed implicitly that the wave associated perturbations are small enough so that the linearization procedure is permitted. One distinct feature of the linear theory is that it predicts an exponential height growth for the perturbed pressure, density and velocity. Such growth can go on until it is limited by either dissipation or nonlinear effects. Observations in the stratosphere show the existence of many nonlinear waves [Gossard et al., 1970; see also many articles in Radio Science, vol. 4, No. 12, 1969]. In the upper atmosphere Baker and Gledhill [1965] analyzed ionograms that appear to show wave breaking. Such breaking wave irregularities have also been observed by in situ measurements [McClure and Hanson, 1973]. The electron content observations occasionally show isolated irregularities [Titheridge, 1971<sup>b</sup>] that look suspiciously similar to solitary waves. Further experimentation is needed to identify the nature of such irregularities. In the equatorial region frequent nighttime large oscillations in electron content during equinoctial months have been observed [Yeboah-Amankwah and Koster, 1972]. Irregularities embedded in these oscillations tend to scatter radio waves in a transequatorial path [Röttger, 1972]. However, these large nighttime oscillations in electron content have also been observed in middle latitudes but seasonally with a summer maximum [Titheridge, 1969]. It is not known whether the middle latitude oscillations and the equatorial oscillations are excited by acoustic gravity waves, but some of their properties do not differ much from those of medium-scale traveling disturbances.

The theory of nonlinear waves is old but it has received a new impetus of late. In the atmosphere many investigations have

been carried out. Einaudi [1969] studied the formation of shock as an acoustic wave propagated vertically upward by using a singular perturbation technique. The possibility of creating unstable regions in the atmosphere by internal gravity waves was investigated by Breeding [1971]. Using a wave-wave interaction approach, Yeh and Liu [1970] showed the possibility of trapping an acoustic gravity wave through resonant interaction with a sinusoidal wind shear. In the presence of dissipative loss Juren and Stenflo [1973] worked out the condition for a resonant trio of waves to interact. They used their theory to explain observations of Litzka and Westin [1972] on the generation of 2 Hz infrasound by moving auroral electrojet. In the linear theory the propagation properties are governed by the dispersion relation such as eq. (28). When the wave has a finite amplitude, a nonlinear dispersion relation is obtained [Cho, 1972; Cho et al., 1973]. Stability conditions can be investigated by using this nonlinear dispersion relation. The upper atmosphere is also capable of supporting long waves [Cho, 1972], such waves were initially investigated in connection with water waves. It should be mentioned that there is a large class of nonlinear waves. Experimental identification and theoretical interpretation of observed nonlinear waves will be a major task.

The acoustic gravity wave spectrum is an important part of the dynamic spectrum in the upper atmosphere. Its understanding is crucial to a complete understanding of thermospheric dynamics.

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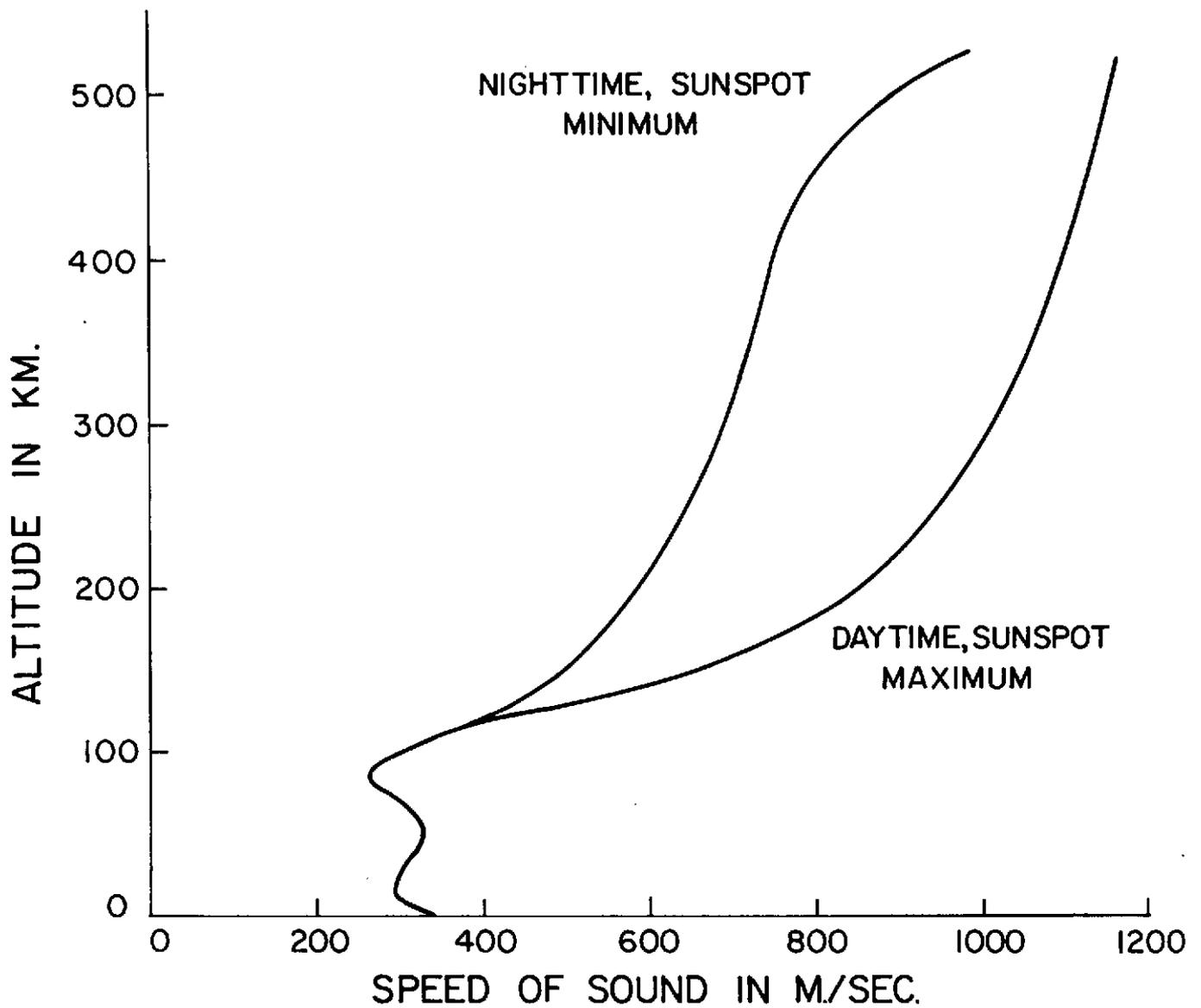


Fig. 1 The vertical structure of the speed of sound. For heights below 100 km the U.S. Standard Atmosphere 1962 is used. For heights above 100 km the CIRA 1965 models for nighttime, sunspot minimum and for daytime, sunspot maximum conditions are used.

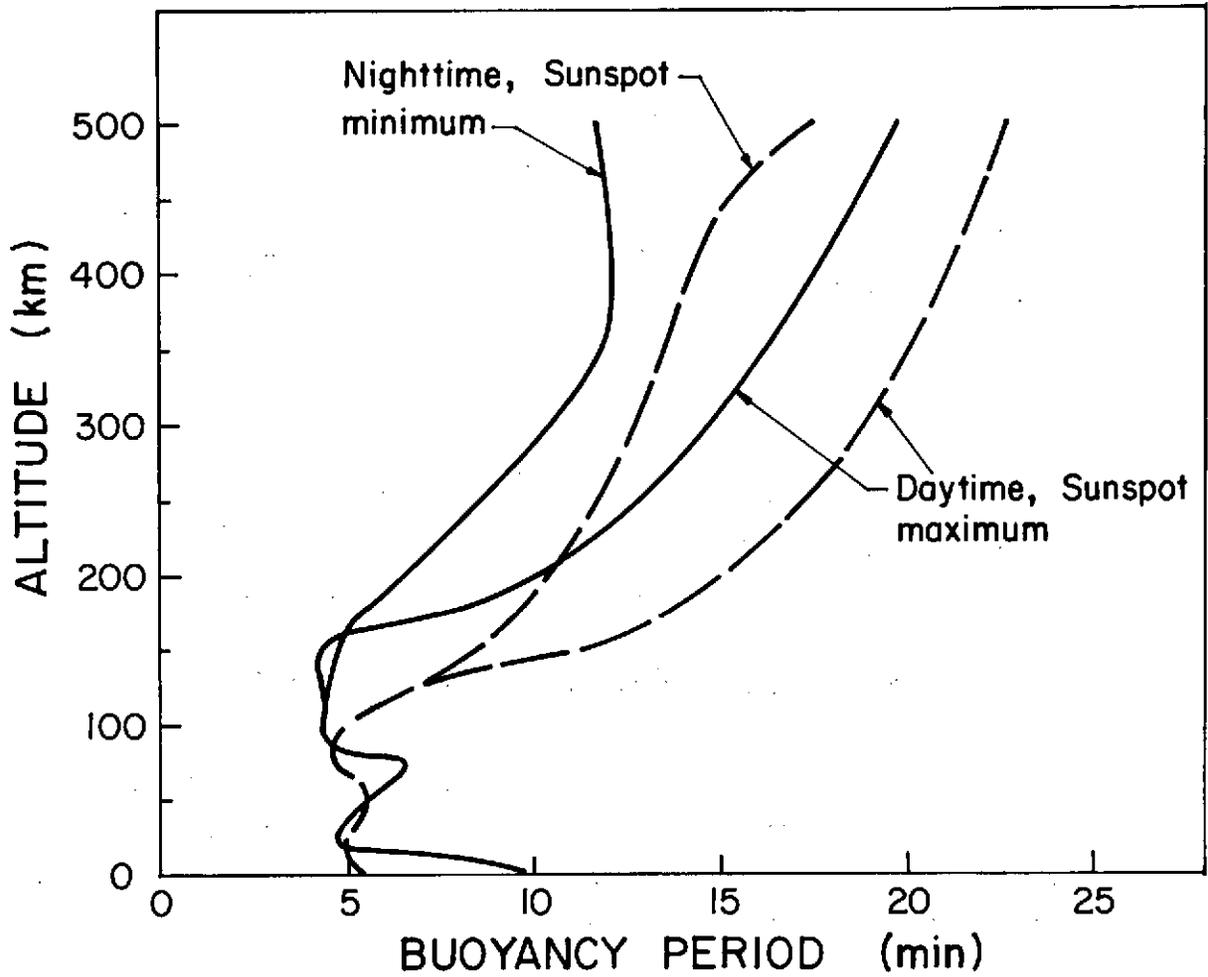


Fig. 2 The vertical structure of the buoyancy period computed by using equation (9) which applies to a nonisothermal atmosphere (solid lines) and equation (10) which applies only to an isothermal atmosphere (dashed lines). The atmosphere is the same as that used in Fig. 1.

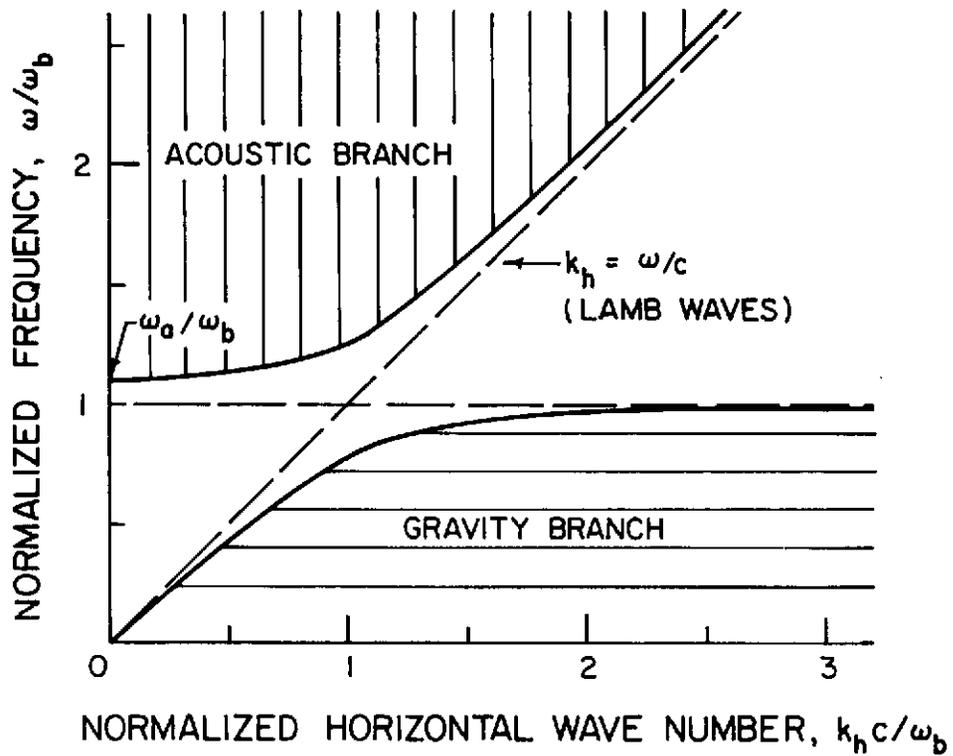


Fig. 3 Regions of propagation of the gravity branch and the acoustic branch in an isothermal atmosphere. The bounding curves are given by  $k_z=0$ . The regions correspond to finite real  $k_z$  values are shown hatched.

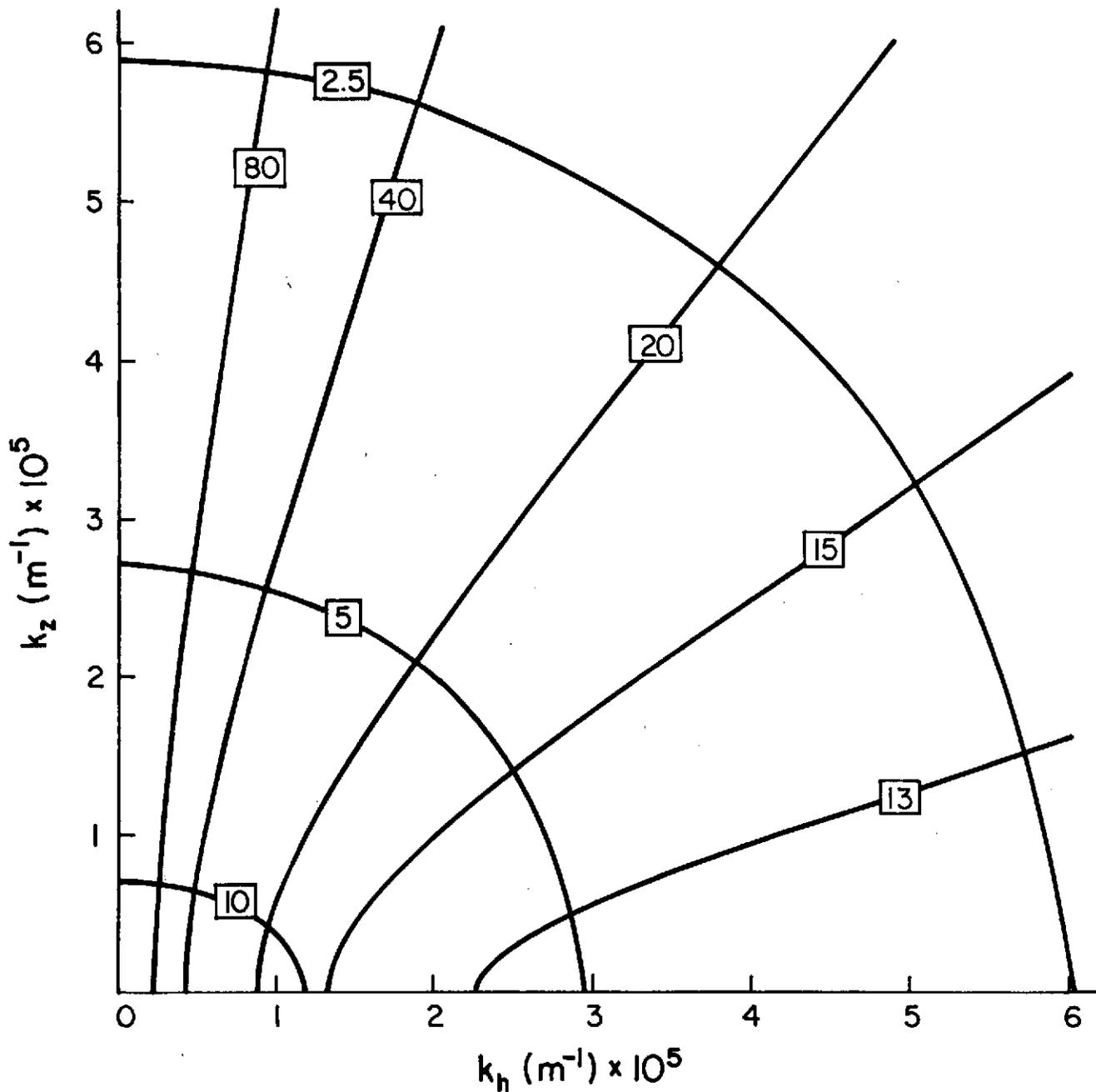
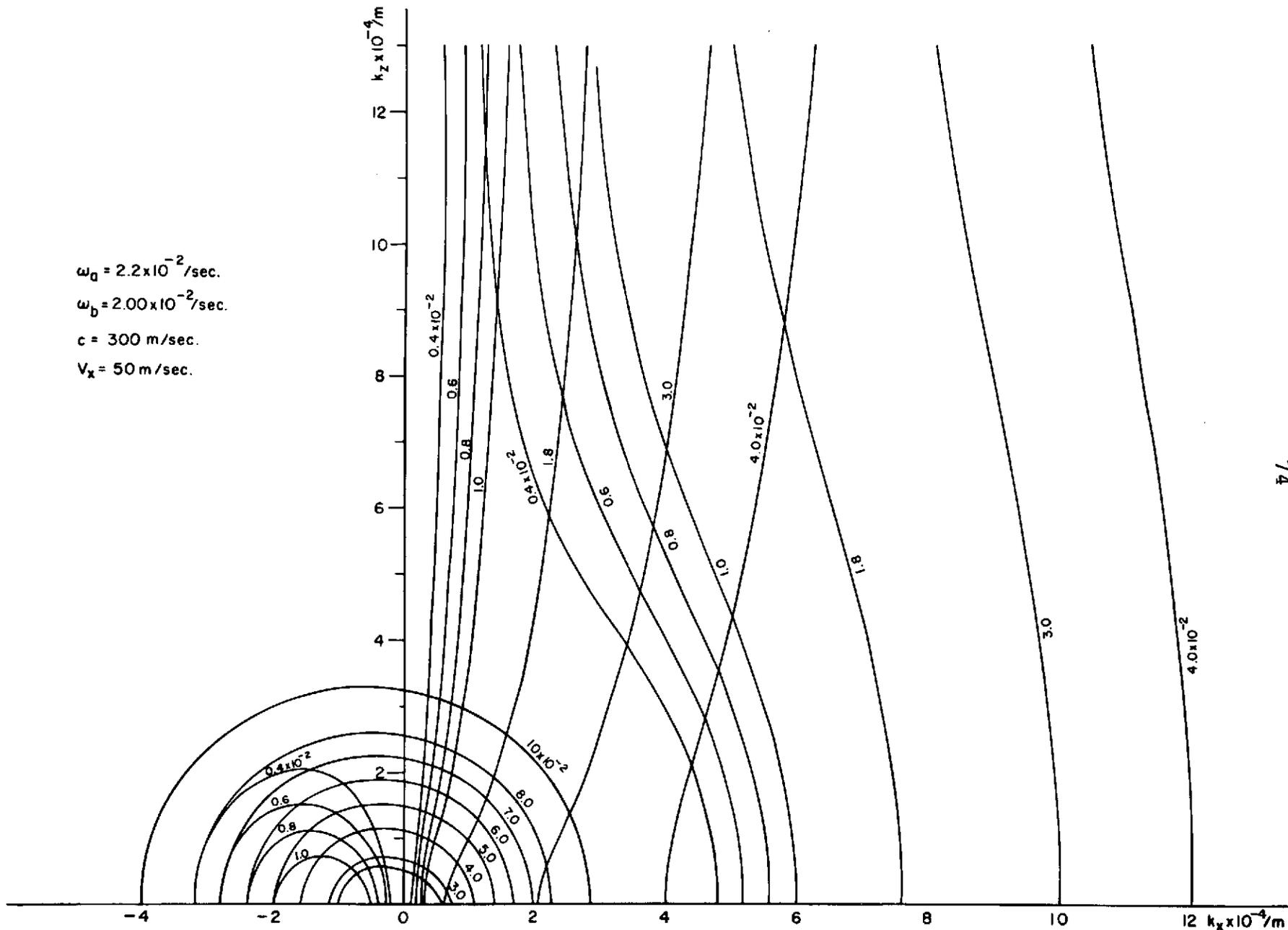


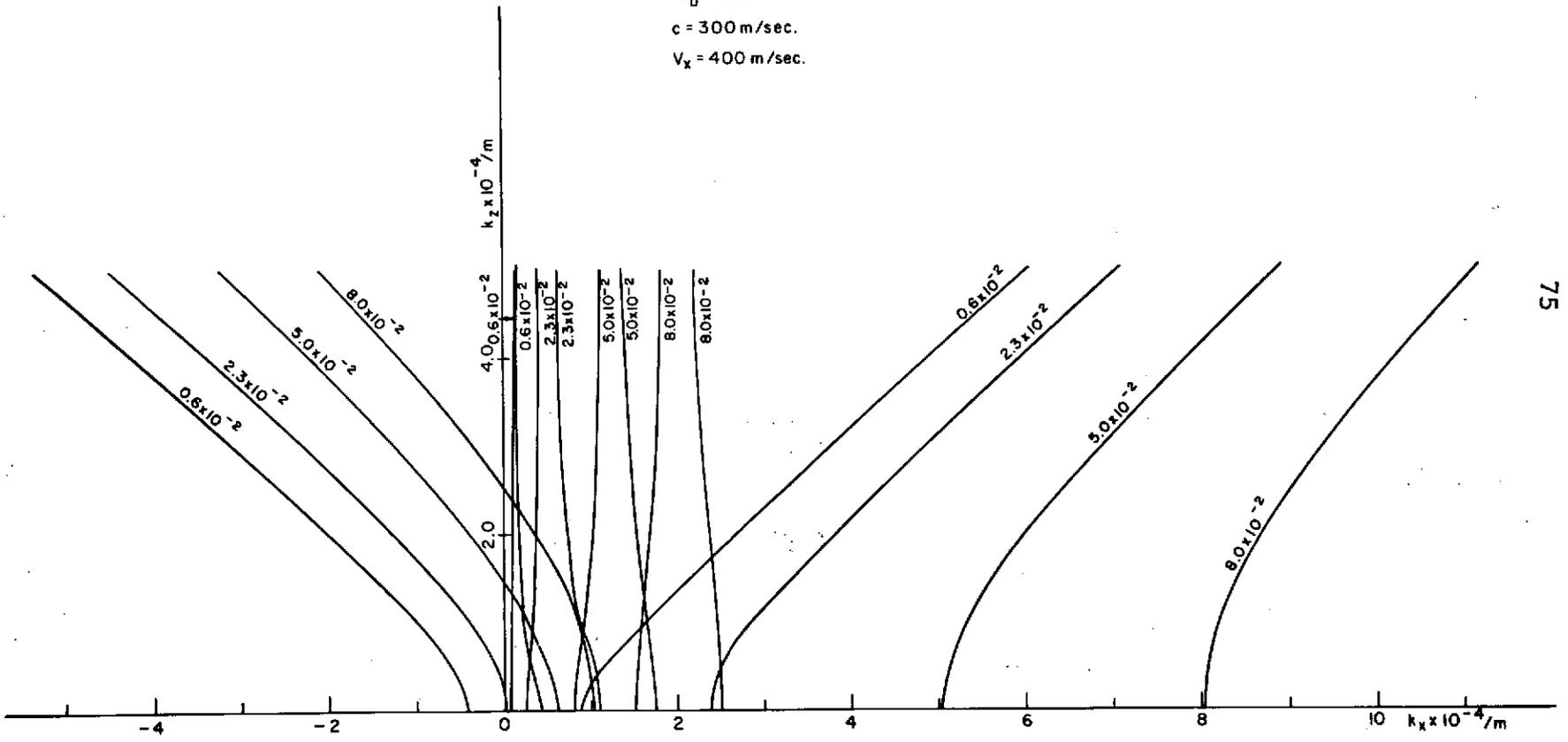
Fig. 4 Dispersion surfaces in a model atmosphere [CIRA, 1965] in which  $\gamma = 1.40$ ,  $g = 9.2050 \text{ m/s}^2$ ,  $H = 37.371 \text{ km}$ , whence  $T_a = 11.38 \text{ minutes}$ ,  $T_b = 12.48 \text{ minutes}$ . These values correspond to a height of 200 km. The periods shown in boxes are given in minutes. [After Hines, 1960]



$\omega_0 = 2.2 \times 10^{-2} / \text{sec.}$   
 $\omega_b = 2.00 \times 10^{-2} / \text{sec.}$   
 $c = 300 \text{ m/sec.}$   
 $V_x = 50 \text{ m/sec.}$

Fig. 5 Dispersion surfaces in an atmosphere with background wind.  
 (a)  $V_x = 50 \text{ m/sec.}$

$\omega_0 = 2.22 \times 10^{-2} / \text{sec.}$   
 $\omega_b = 2.00 \times 10^{-2} / \text{sec.}$   
 $c = 300 \text{ m/sec.}$   
 $V_x = 400 \text{ m/sec.}$



(b)  $V_x = 400 \text{ m/sec.}$

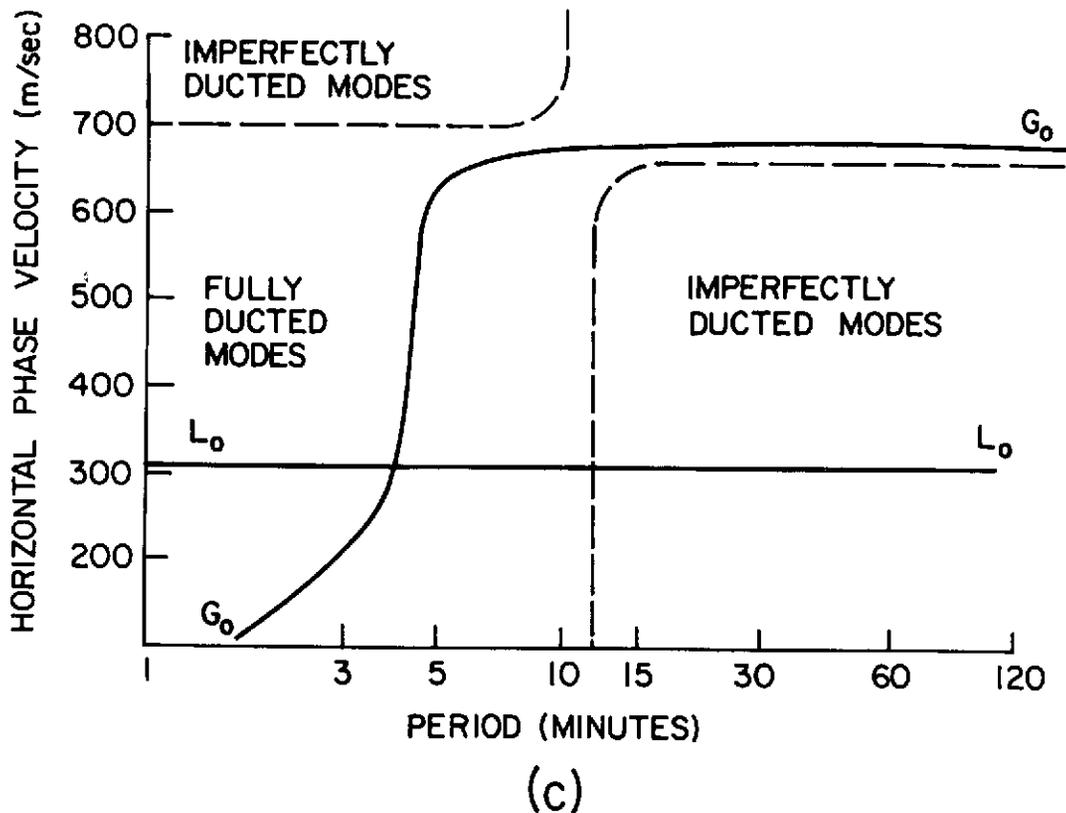
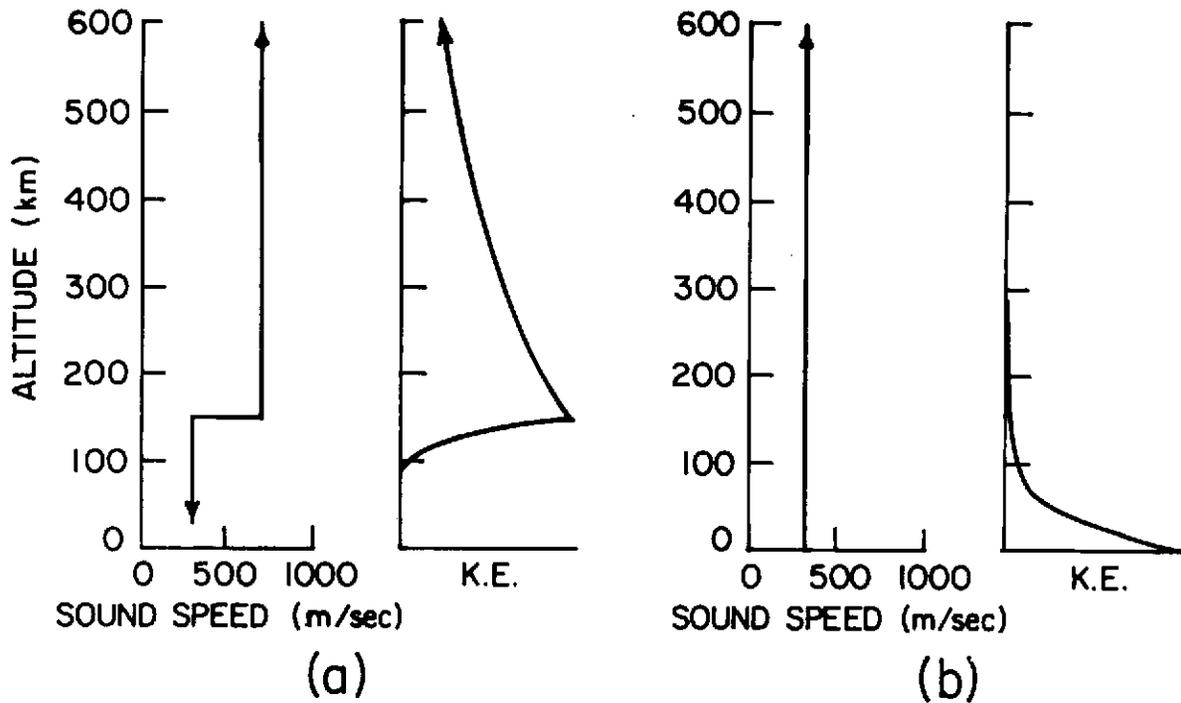


Fig. 6 (a) The kinetic energy profile for a long period boundary wave in a model atmosphere. The upper half space has a sound speed 700 m/sec and the lower half space has a sound speed 310 m/sec. (b) The kinetic energy profile of a Lamb mode in an isothermal atmosphere supported by the solid ground. (c) The dispersion curves of the modes of (a) and (b). [From Francis, 1973<sup>a</sup>]

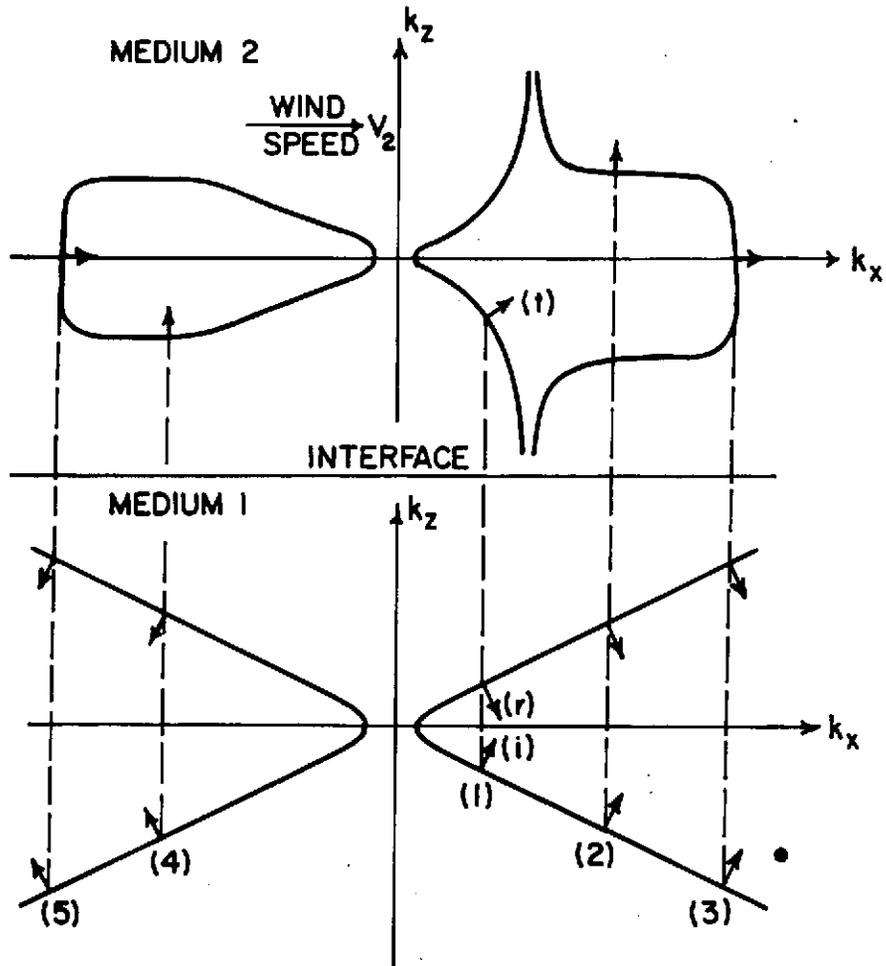


Fig. 7 The use of dispersion surfaces to illustrate the reflection and transmission at a vortex sheet. Medium 1 is stationary and medium 2 is in motion. To demonstrate several possible cases incident gravity waves with wave numbers labeled 1 through 6 are chosen. Total reflection occurs for those labeled 3 and 5. Wave amplification occurs for the wave labeled 2 since (53) is satisfied for this wave. [After McKenzie, 1972].

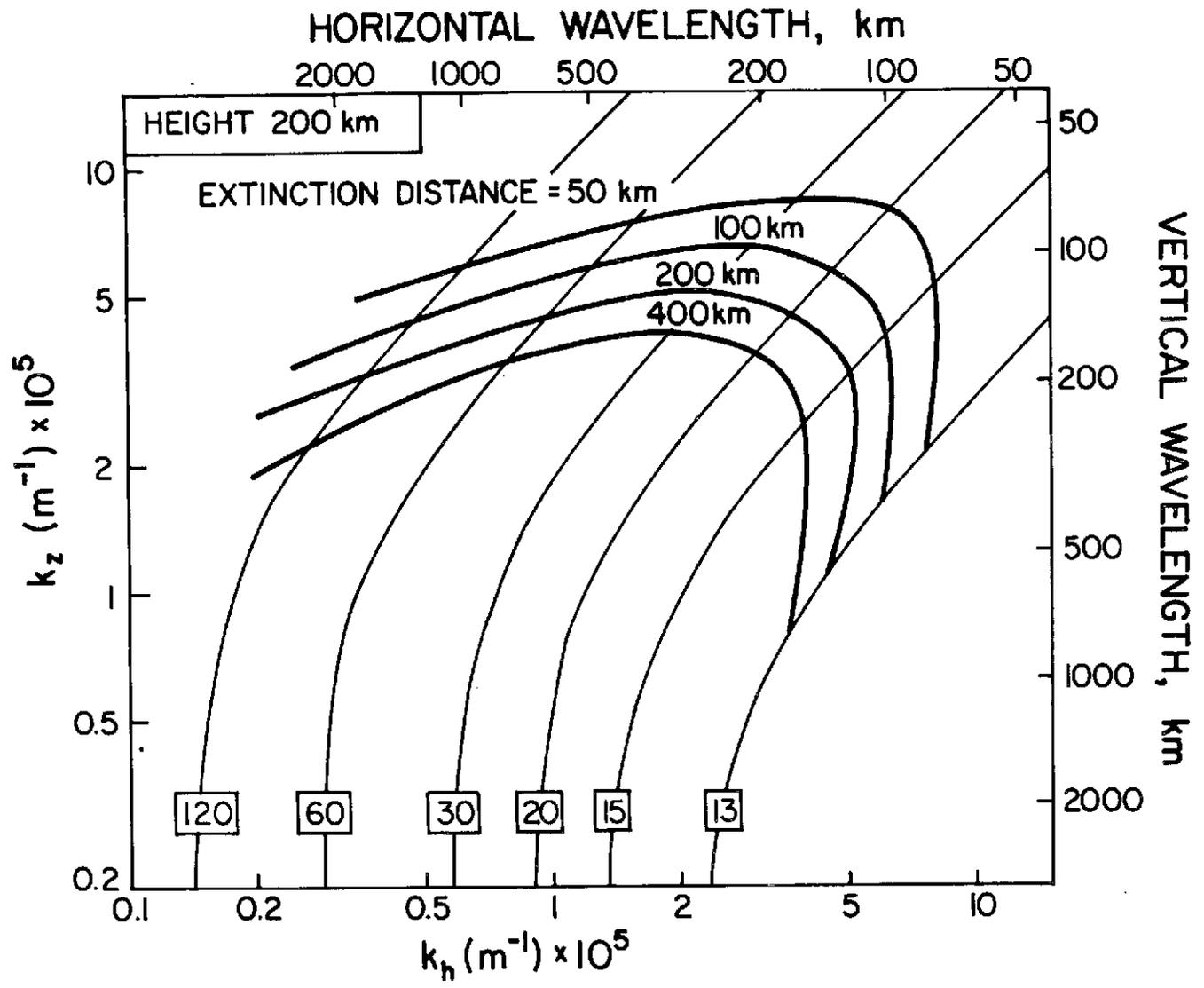


Fig. 8 Contours of extinction distance owing to viscons damping for the average CIRA [1965] atmosphere.

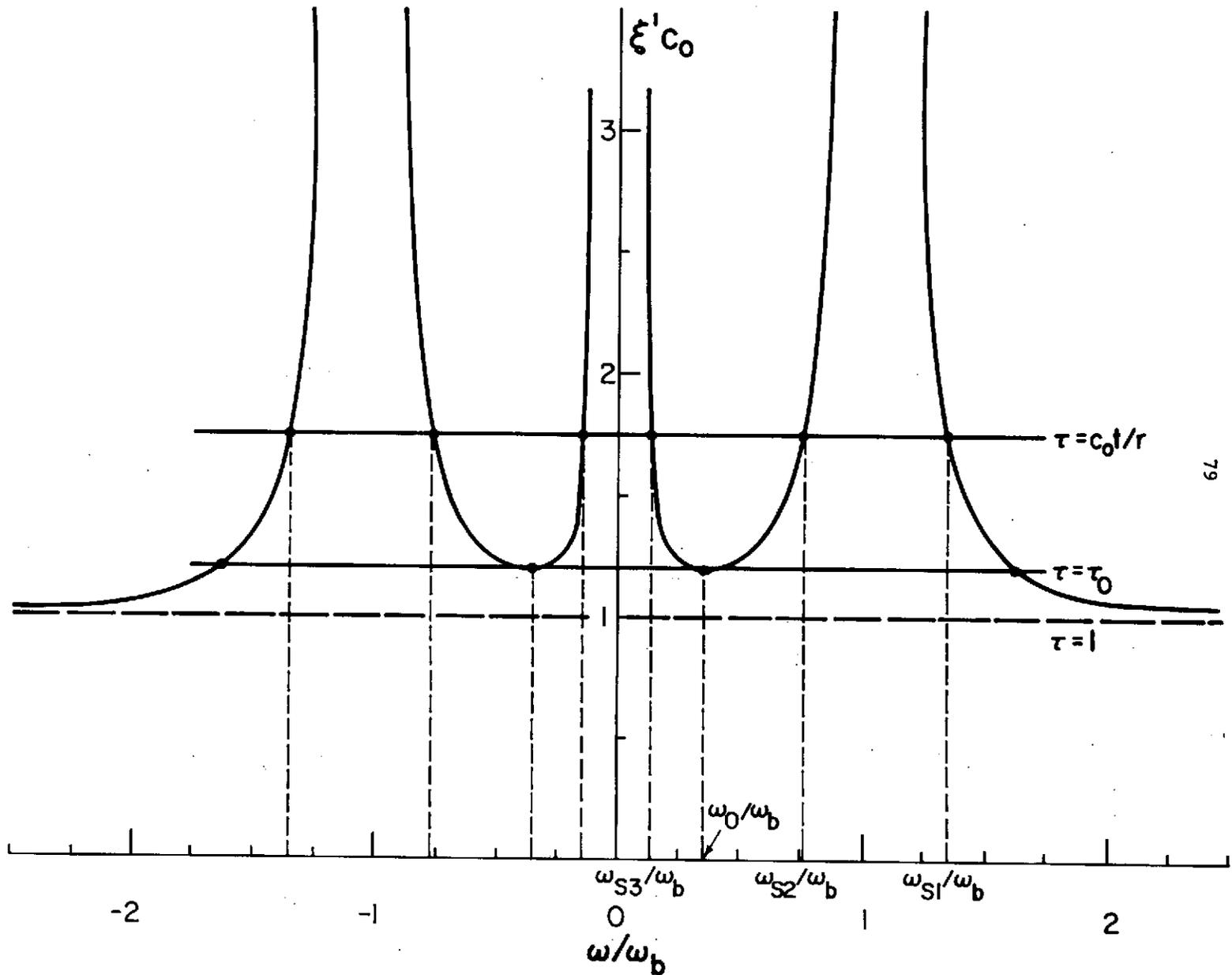


Fig. 9 Plot showing  $\xi'c_0$ , as function of  $\omega/\omega_b$ . Saddle points are obtained for  $\tau > 1$  when the horizontal line intersects the curves [From Liu and Yeh, 1971].

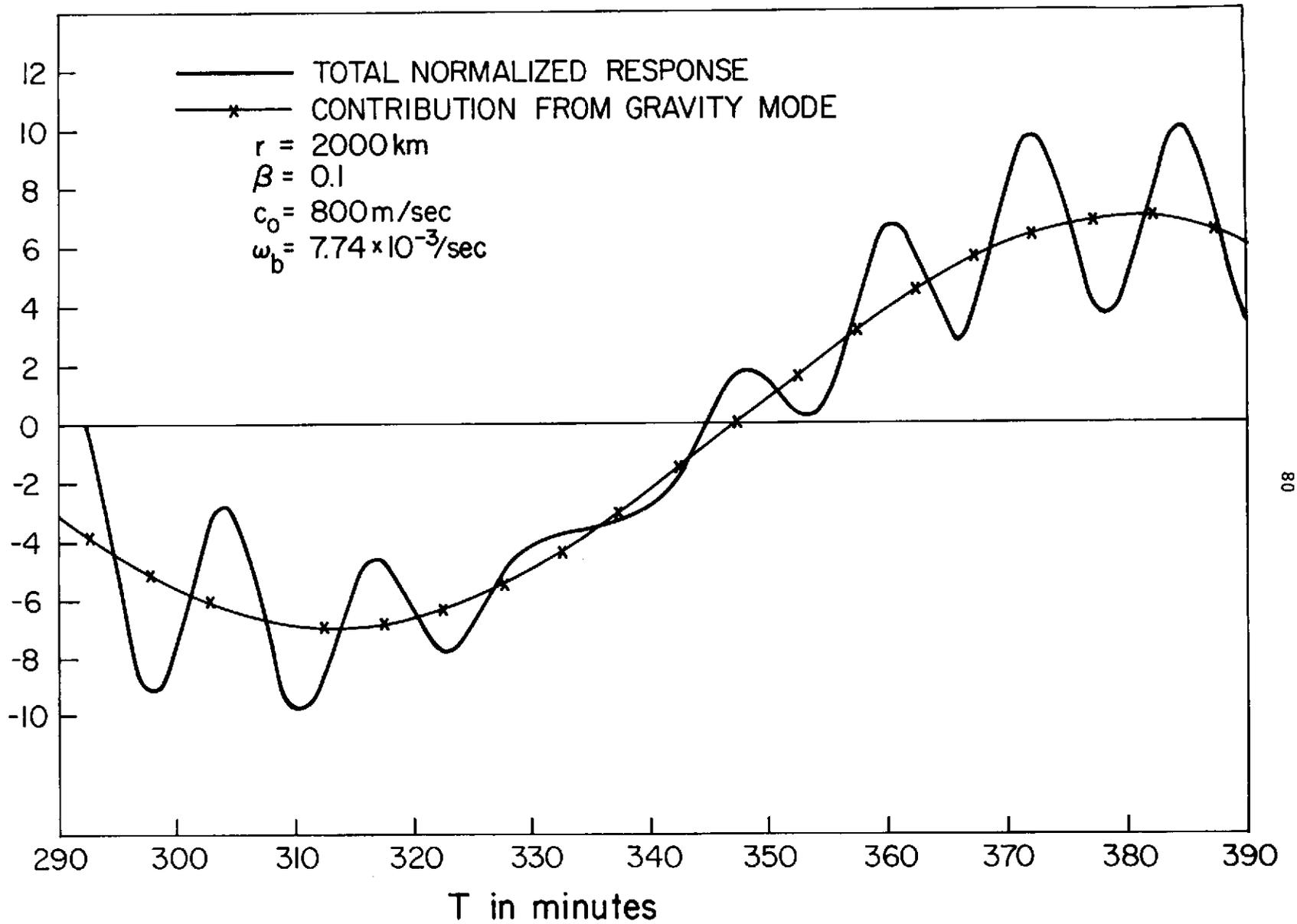


Fig. 10 The large time response of the atmosphere to an impulsive point source showing essentially the internal gravity wave modulated by the acoustic wave [From Liu and Yeh, 1971].

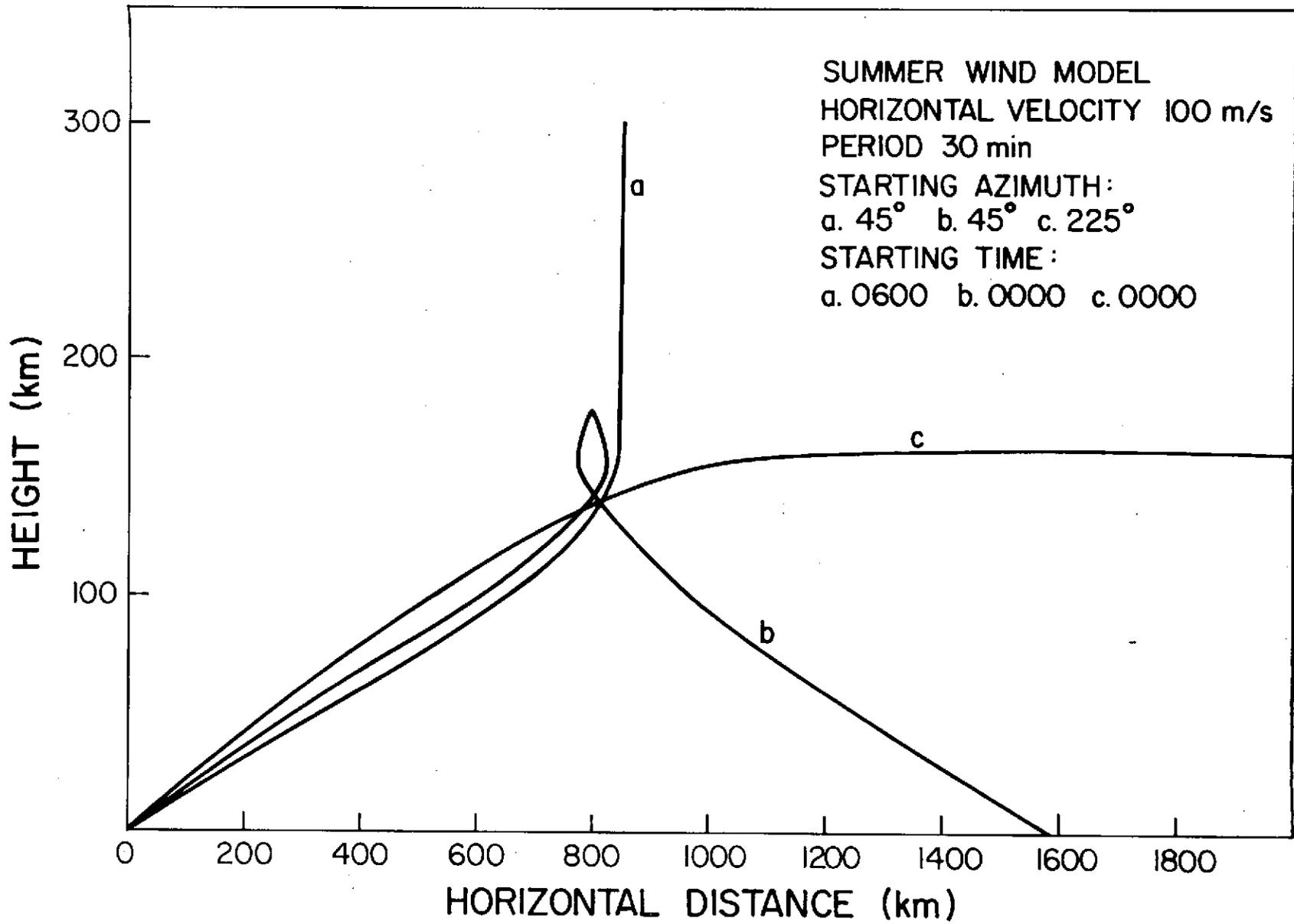


Fig. 11 Plot showing three possible outcomes of a group ray due to horizontal winds. Curve a shows penetration, b reflection and c asymptotic trapping [From Cowling et al., 1971].

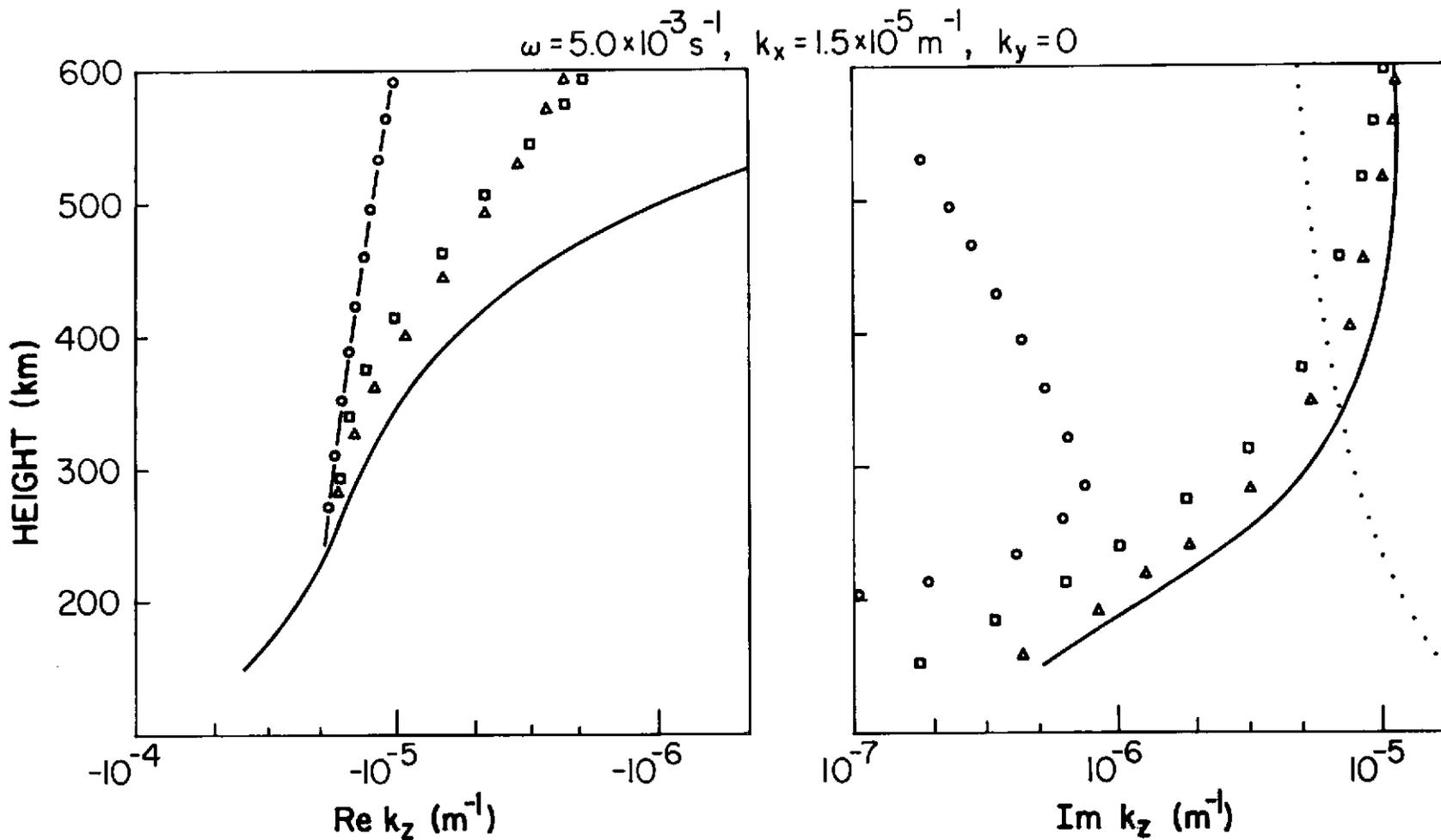


Fig. 12 Vertical profiles of the real part and the imaginary part of the vertical component of the propagation vector. Computations were carried out by including viscosity only (squares) or thermal conductivity only (triangles) or ion drag only (circles) or all three dissipative processes (solid line) or none of the dissipative processes (dashed line). The dotted line shows the height dependence of the term  $1/2H$  [From Klostermeyer, 1972c].

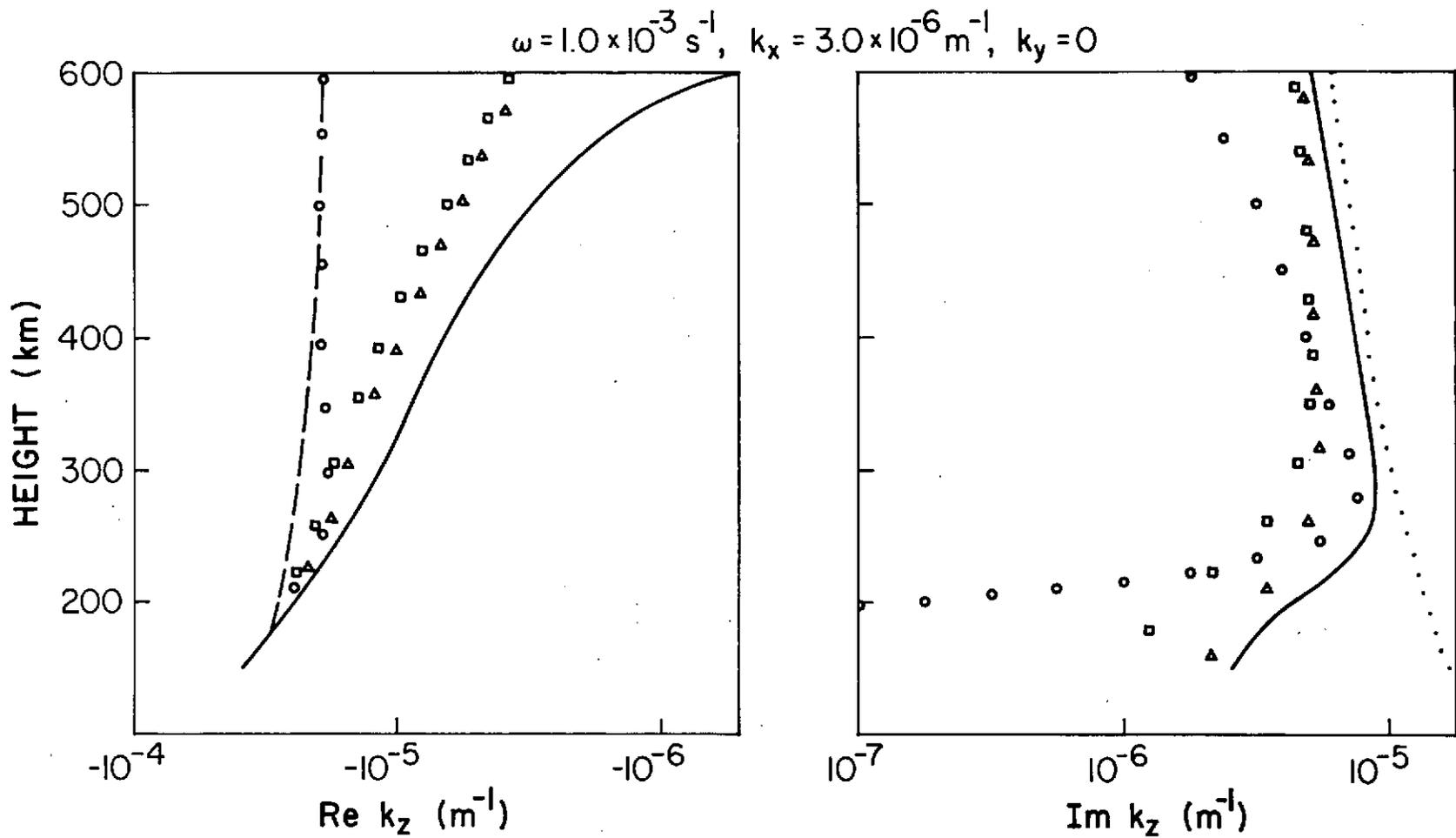


Fig. 13 Same as Fig. 12 for a different wave [From Klostermeyer, 1972c].

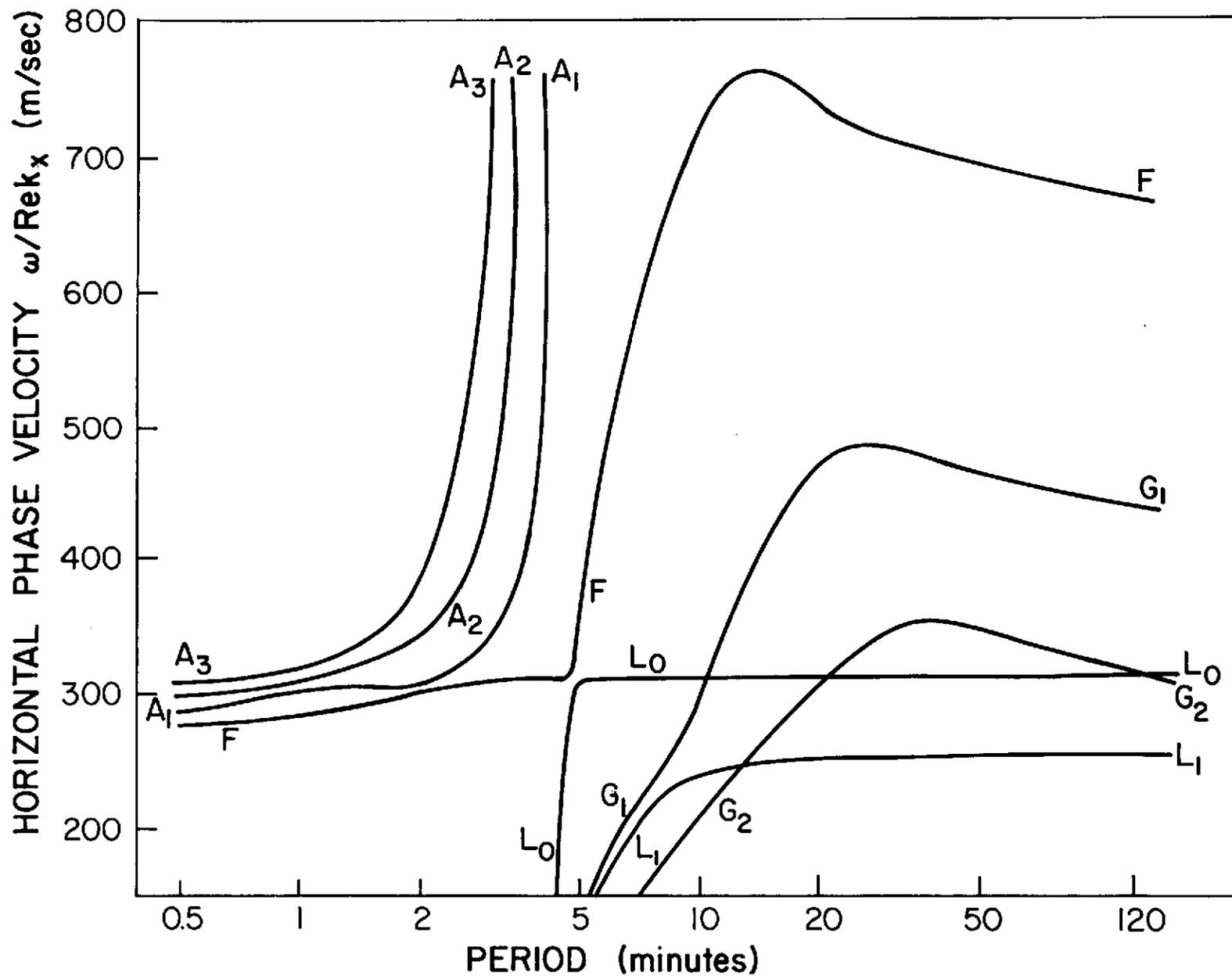


Fig. 14 Plot showing the horizontal phase velocity as a function of period for guided modes in a mean model atmosphere [From Francis, 1973].

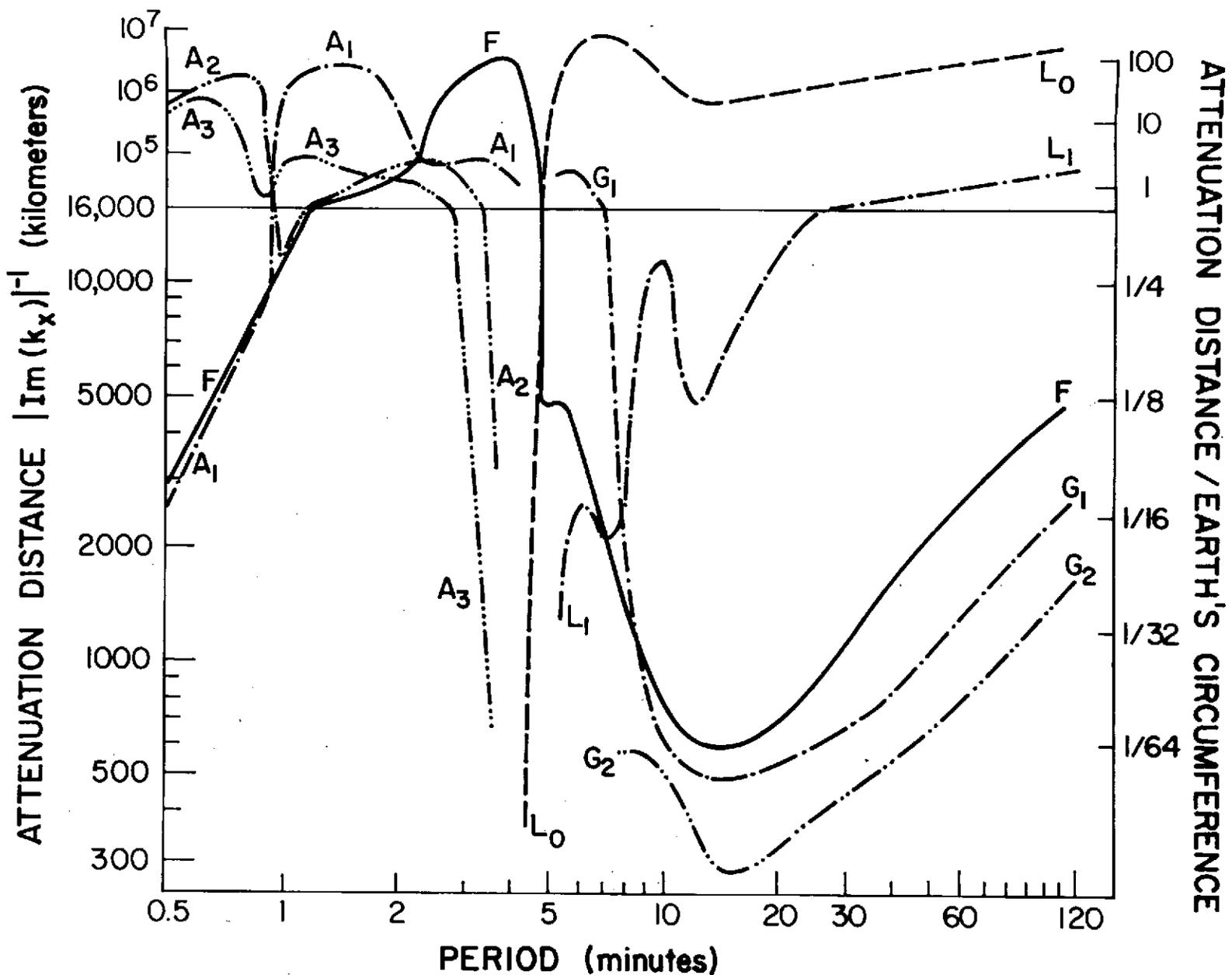


Fig. 15 Plot showing the attenuation distance as a function of period for guided modes in a mean model atmosphere [From Francis, 1973].

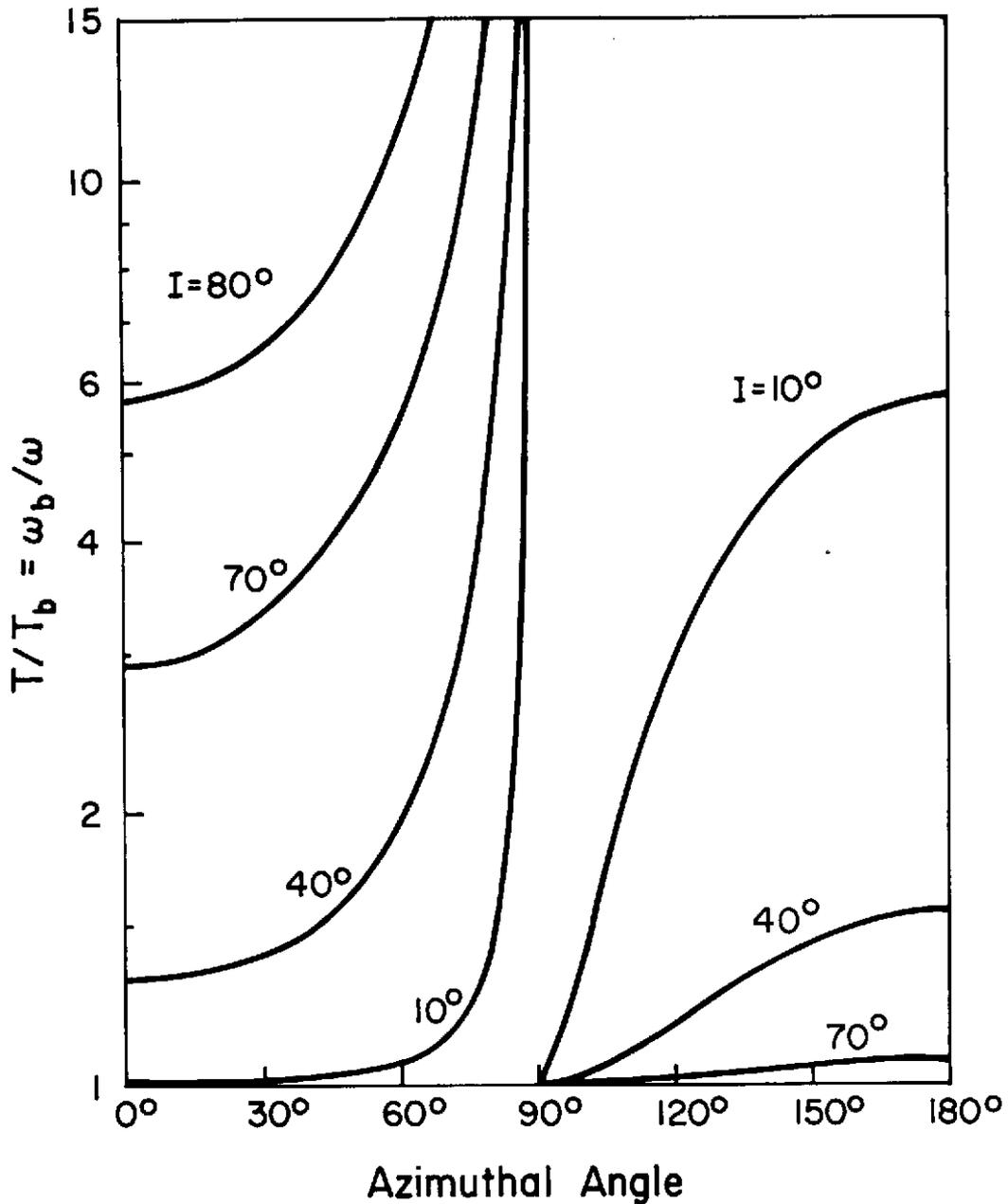


Fig. 16 Plot showing period and azimuthal direction of internal gravity waves under the Boussinesq approximation that will not give rise to any ionospheric response at the peak as predicted by (141). Azimuthal angle is measured eastward from the magnetic north,  $I$  is the magnetic dip angle. The curves are symmetrical about 0 degree azimuth. For curves in the azimuthal range  $-90^\circ$  to  $90^\circ$  the absence of ionospheric response comes about because  $\vec{v}' \cdot \hat{B}_0 = 0$  while those in the range  $90^\circ$  to  $270^\circ$  because  $\vec{k} \cdot B_0 = 0$ .

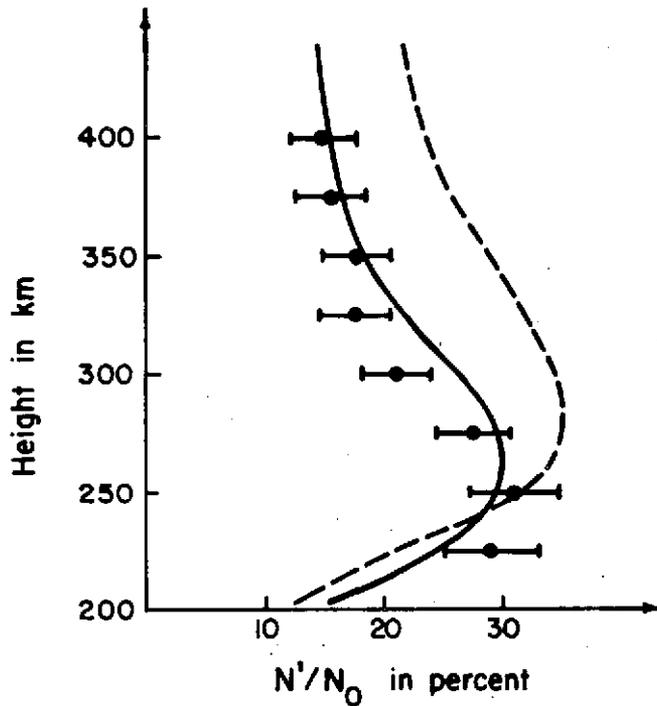


Fig. 17 Comparison of percent ionization density perturbations between those observed experimentally by the incoherent scatter technique on September 13, 1967 (shown with error bars) and those computed theoretically. The dashed line includes only the dynamic effect, while the solid line includes additionally the diffusion effect. [From Testud and Francois, 1971].

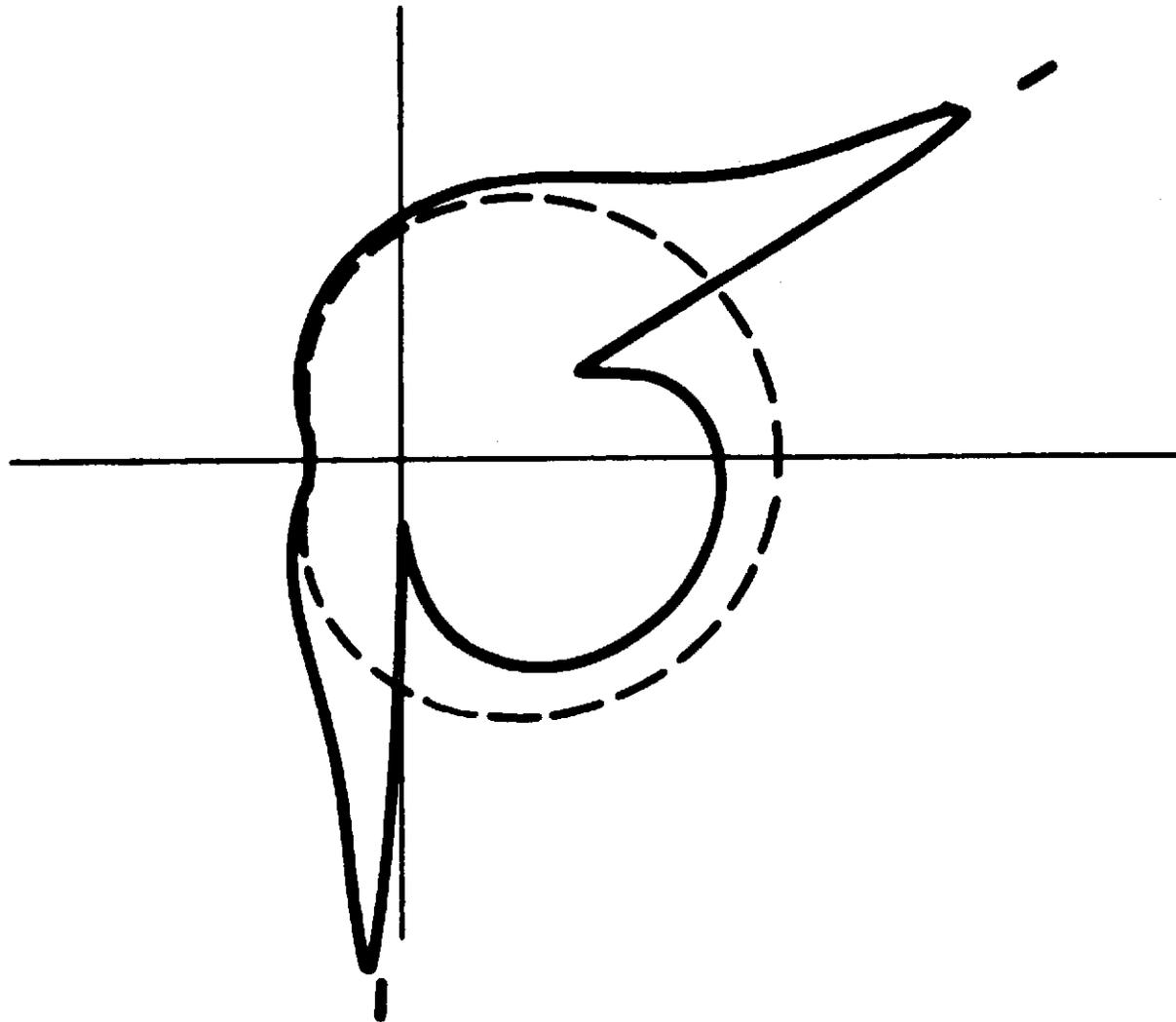


Fig. 18 Relative response of the lower F region to internal gravity waves. Computations have been made for latitude  $35^{\circ}\text{S}$ , magnetic dip- $57^{\circ}$ , height 200 km, solar declination  $23^{\circ}$ , local time 1000. The wave has parameters  $\lambda_h = 200$  km,  $\lambda_z = 200$  km,  $T = 15.9$  min and  $v_p = 209$  m/s. The dashed curve includes only the dynamic effect while the solid curve includes photochemical effects as well. A short line marks the azimuthal direction for which the condition  $\vec{k} \cdot \vec{r} = 0$  is satisfied. [From Hooke, 1970].

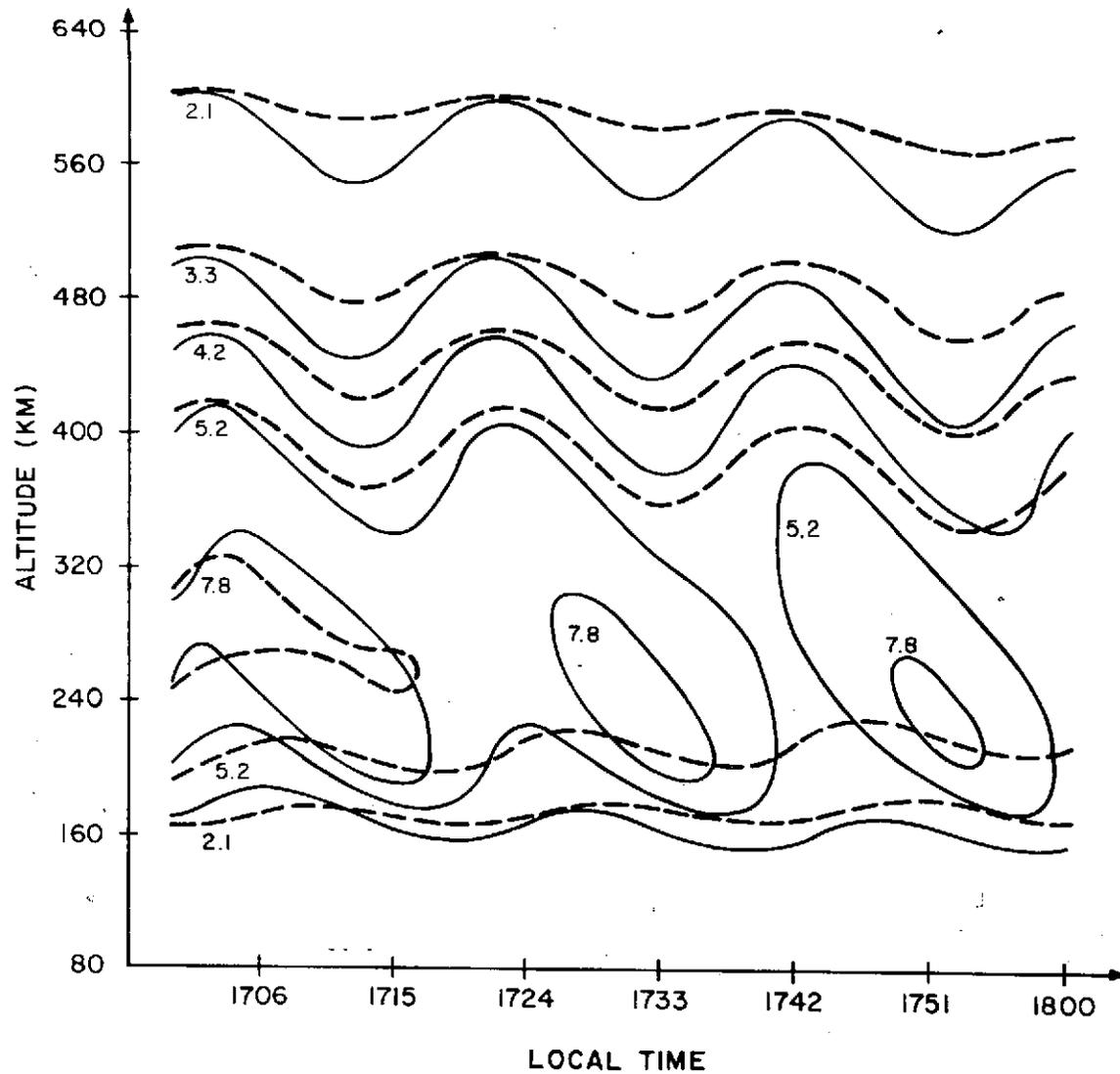


Fig. 19 Isoionic contours in response to an internal gravity wave with  $k_h = 0.02 \text{ km}^{-1}$ ,  $T = 20 \text{ min}$ , propagating eastward. The contours are given in units of  $10^{11} \text{ m}^{-3}$ . Dotted lines are computed by including only the directed velocity, the solid lines by solving continuity equation. [From Clark et al., 1971].

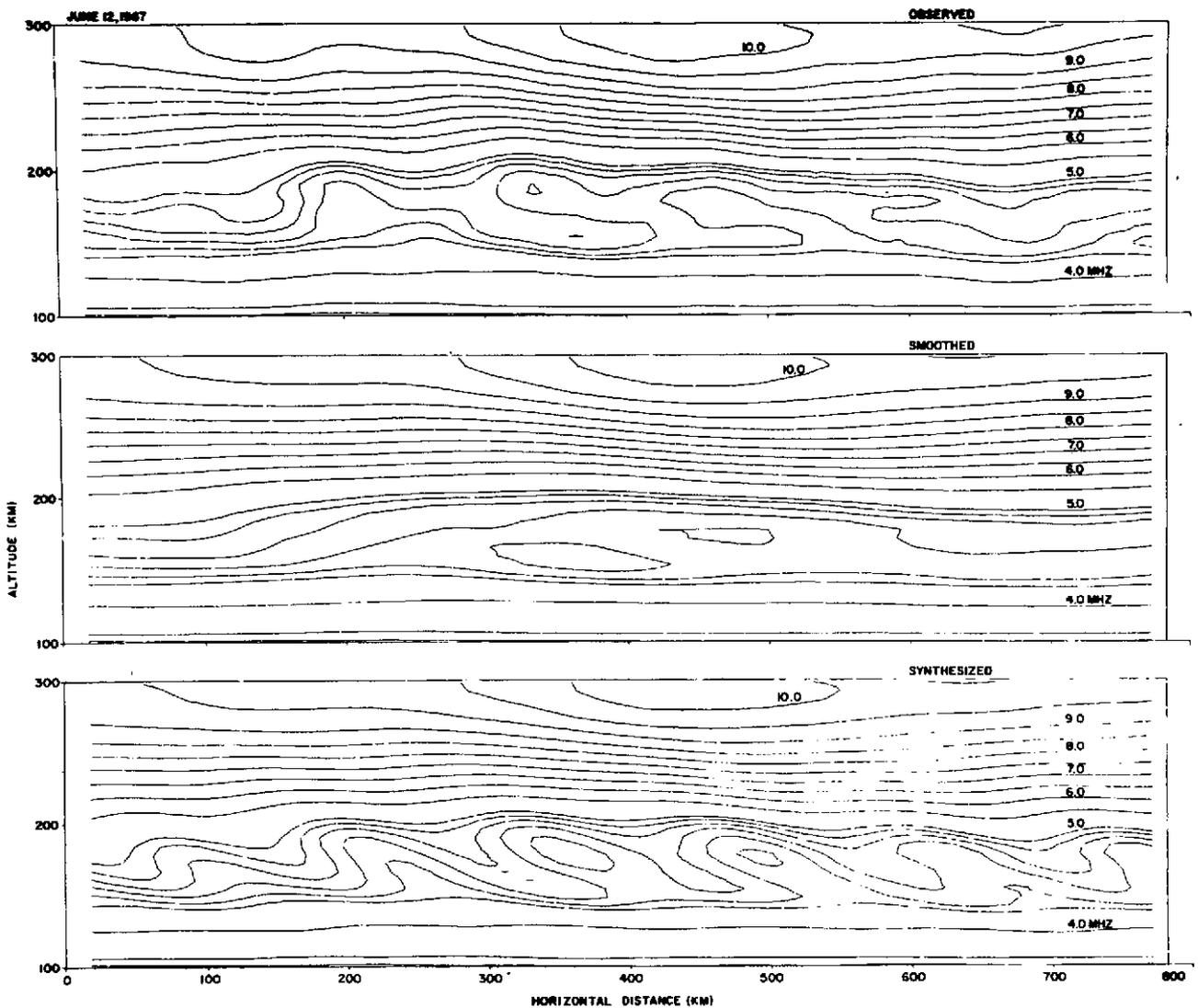
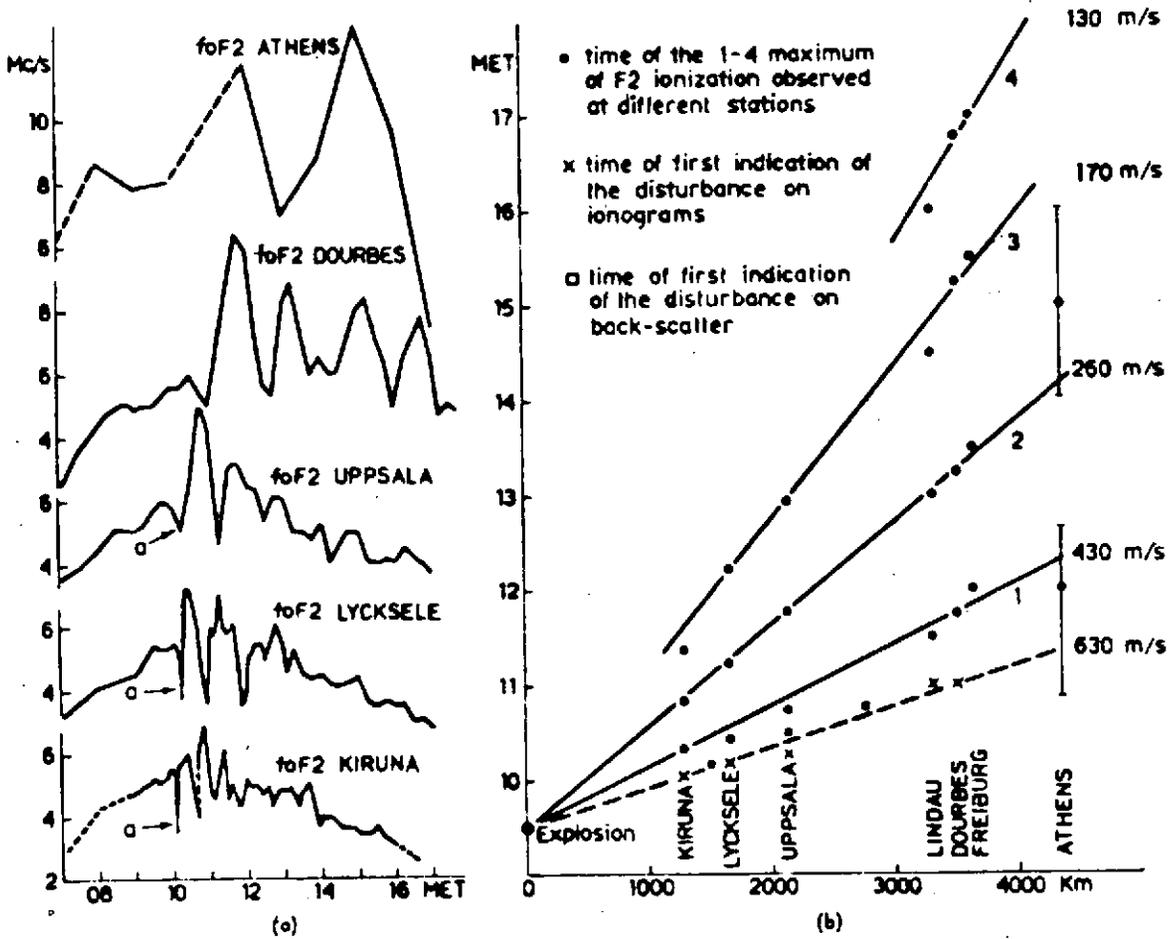


Fig. 20 Comparison of the ionization contours showing observed TID and that synthesized by using the acoustic gravity theory. (a) Contours deduced from incoherent scatter observations at Arecibo on June 12, 1967 showing a TID with a 20 min period. (b) Smoothed ionosphere with the 20 min periodicity removed. (c) Synthesized contours by assuming the smoothed ionosphere is perturbed by an acoustic gravity wave. [From Thome and Rao, 1969].



Time history of  $f_oF_2$  versus distance following Soviet nuclear detonation of October 30, 1961 (from *Stoffregen* [1962] and *Kohl* [1964]).

Fig. 21 Ionospheric response to the Soviet nuclear detonation at Novaya Zemlya on October 30, 1961. [From Row, 1967].

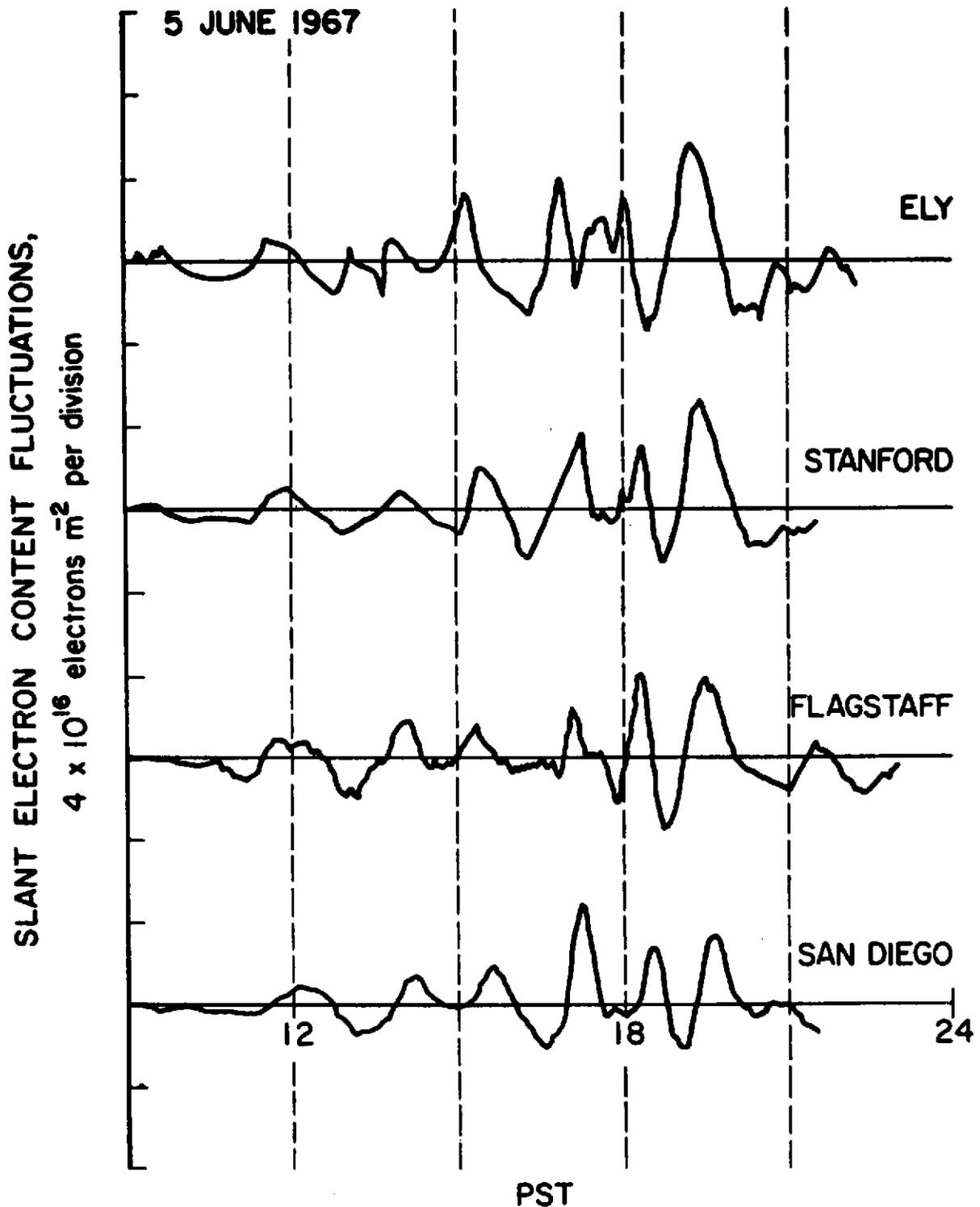
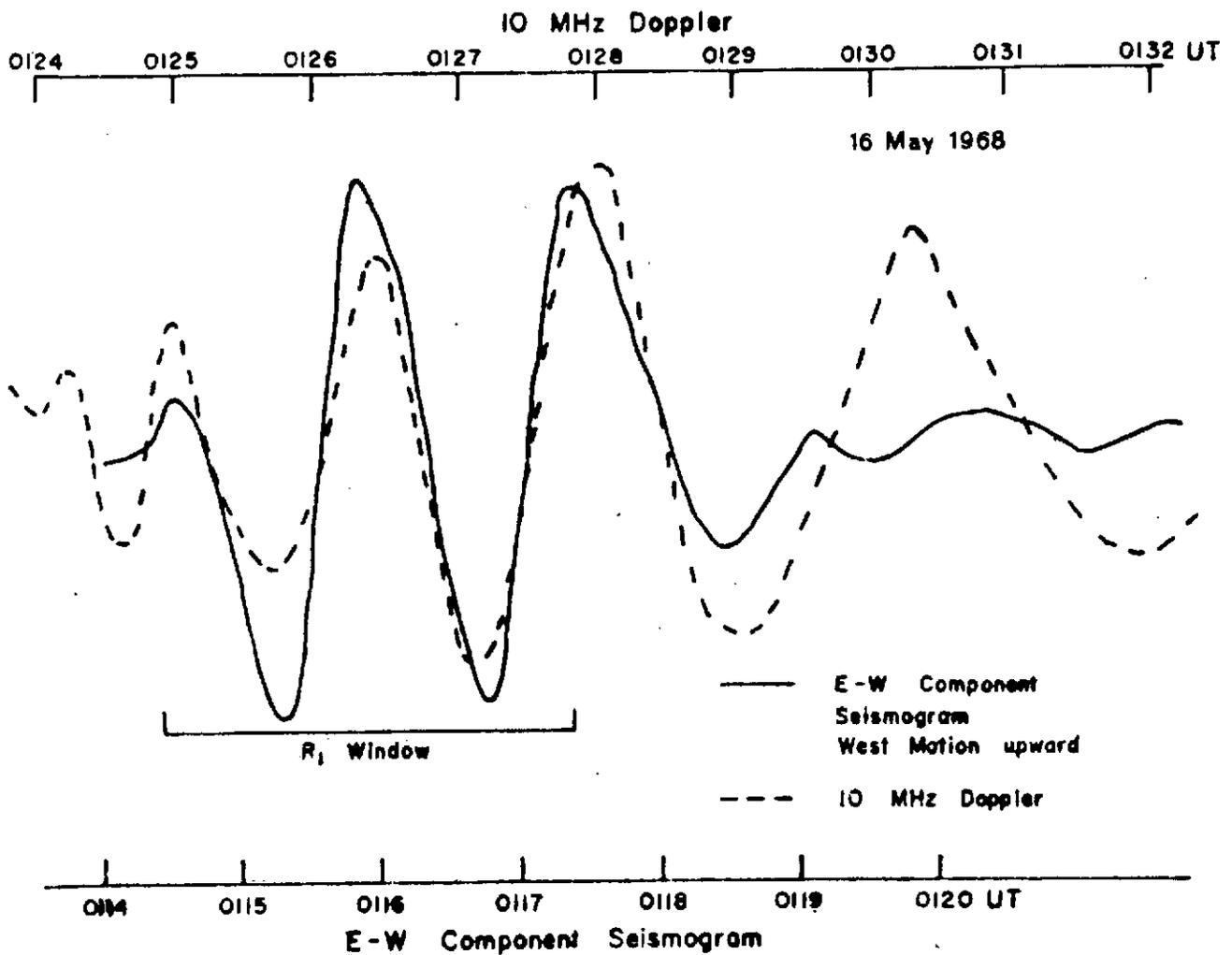


Fig. 22 Large scale traveling disturbances observed at four locations on a magnetically disturbed day. Slant electron content fluctuations are obtained by digital filtering of the Faraday rotation data. The separation between Ely and San Diego is about 700 km in the geomagnetic north-south direction and that between Flagstaff and Stanford is about 900 km in the geomagnetic east-west direction. Note the similarity of wave forms. [From Davis and daRosa, 1969].



### Comparison of Doppler and longer-period seismogram data.

Fig. 23 Comparison of the long period seismogram data obtained at Oaku and the Doppler data recorded at Honolulu of 10 MHz transmissions from WWVH. The time displacement between these two records is 10.5 minutes. [From Yuen et al., 1969].

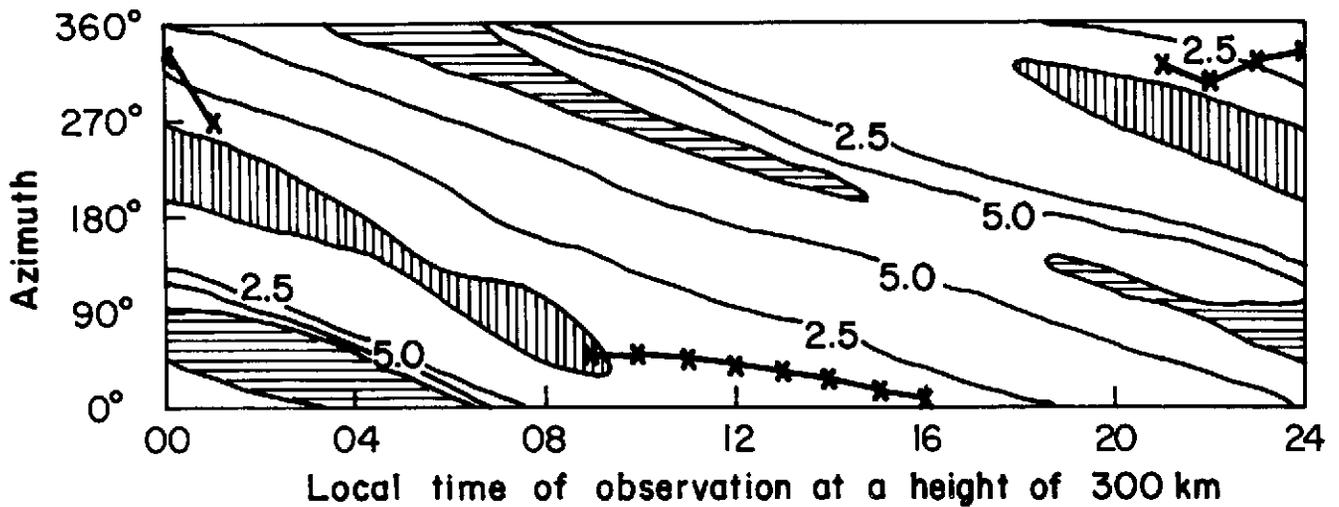


Fig. 24 Contours showing number of hours required for gravity waves to reach a height of 300 km. Shaded regions indicate asymptotic trapping (horizontal hatching) or reflection (vertical hatching). The period of the wave is 30 min. and the horizontal phase speed is 150 m/s. The data points are Munro's [1953] observational values [From Yeh et al., 1972].

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