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# On the Theory of Large Amplitude Alfvén Waves**

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Large amplitude Alfvén waves have been shown to constitute an important component of the MHD turbulence that has been detected by spacecraft magnetometer and plasma probes (v.,e.g., Belcher and Davis, 1971 and Daily, 1973).

The theory of this wave mode has also been studied (v. Barnes and Hollweg, 1974 and references contained there). Two features of the Alfvén mode have been particularly useful in analyzing experimental data. The first is the relationship between the fluctuation magnetic field $\Delta \underline{B}$ and the fluctuation fluid velocity $\underline{v}$. This relationship, which defines the Alfvén mode, is

$$\underline{v} = \pm \frac{\Delta \underline{B}}{(4\pi\rho)^{1/2}} \tag{1}$$

The second feature was used extensively by Daily (1973) to determine the propagation direction of interplanetary Alfvén waves. The technique, briefly described, is to construct a variance matrix of the fluctuating components of the interplanetary field $\underline{B}$. One then examines this variance matrix to determine whether or not it contains a direction of minimum variance. This direction, when it can be found, defines the propagation direction (to within a sign) of a planar Alfvén wave. (The reader is referred to Daily (1973) for a more comprehensive discussion of

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the method). This method only defines the propagation direction of plane waves. It is not clear that one should expect Alfvén waves observed at 1 AU to be predominately planar.

The full MHD equations that describe the propagation of nonplanar, large amplitude, Alfvén waves in an expanding solar wind which contains a spiral mean magnetic field are nonlinear. One might expect the solutions of these equations to include significant mode coupling which is not included in linear treatments (v., e.g., Völk, Morfill, Alpers and Lee, 1974 ) in which Alfvén waves formed near the sun refract into plane waves as they propagate toward and beyond 1 AU.

Another feature of observed interplanetary Alfvén waves is that they are often not even approximately periodic (v. Fig. 1 in Belcher and Davis 1971). Although such disturbances can be described in terms of a Fourier decomposition, it would be useful to be able to discuss the propagation properties of these nonlinear wave packets as a whole, without resort to a Fourier analysis with its attendant problems of determining the importance of mode coupling. It is also not necessary to assume that the solutions are plane waves (cf. Barnes and Hollweg, 1974).

Wave packet solutions can be directly obtained from the MHD equations. In the discussion below we derive solutions of the MHD equations which describe the propagation of nonplanar, large amplitude Alfvén waves in an infinite, homogeneous plasma, with no background flow. Extension of these ideas to the more complicated geometry of a radially expanding solar wind containing a background spiral field is deferred to a later paper (Barish, Goldstein, and Klimas, in preparation).

We begin with the usual MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0 \tag{2}$$

$$\rho\left(\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla\right) \underline{v} = - \nabla \cdot \underline{\underline{P}} + \frac{1}{4\pi} (\nabla \times \underline{B}) \times \underline{B} \tag{3}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) \tag{4}$$

where $\rho$ and $\underline{\underline{P}}$ are the particle mass density and fluid pressure tensor, respectively. We restrict ourselves to the MHD mode in which $\rho$ and $\underline{\underline{P}}$ have no fluctuating components.

Because we are interested in Alfvèn waves, we substitute equation (1) into (2) - (4) and find that $B^2$ is constant for the Alfvèn mode. It is also straightforward to show that if one operates on equation (3) with $\partial/\partial t$, that the set of equations (1) - (4) reduces to

$$\frac{\partial^2}{\partial t^2} \Delta \underline{B} = \frac{1}{4\pi\rho} (\underline{B}_o \underline{B}_o : \nabla\nabla) \Delta \underline{B} \tag{5}$$

where $\underline{B}_o$ and $\Delta\underline{B}$ are the mean and fluctuating magnetic fields, respectively. The interpretation of this wave equation is clear. Non-planar, arbitrary amplitude Alfvèn waves propagate along $\underline{B}_o$ with a group velocity $\underline{V}_A = \underline{B}_o/(4\pi\rho)^{1/2}$. The mode is incompressible ($\rho$ and P are constant) and the magnitude of $\underline{B}$ is conserved.

One can construct explicit solutions to these equations. If we use the spherical coordinate system defined in the figure 1, then the magnitude of $\Delta\underline{B}$ is given by

$$\Delta B = - 2B_o \cos\theta ; \; \pi/2 \leq \theta \leq \pi \tag{6}$$

The equation for $\theta$ and $\phi$ found from the divergence condition is

$$-2 \sin 2\theta \frac{\partial\theta}{\partial z} + 2 \cos 2\theta \sin\phi \frac{\partial\theta}{\partial y} + 2 \cos 2\theta \cos\phi \frac{\partial\theta}{\partial x}$$
$$= \sin 2\theta \left[\sin\phi \frac{\partial\phi}{\partial x} - \cos\phi \frac{\partial\phi}{\partial y}\right] \qquad (7)$$

where $\theta$ and $\phi$ are functions of (x,y,z). One can see that the polarization and spatial extent of these waves can be quite general. In particular, one can look for solutions which are plane polarized ( $\phi = 0$ throughout the disturbance). In this case the characteristic equations which follow from equation (7) can be solved for a variety of boundary conditions on $\theta$ . A typical solution is, qualitatively, a single wave crest of arbitrary steepness and amplitude, which decays to the background field, $\underline{B}_o$, in the direction parallel to $\underline{B}_o$, but does not decay in a plane which intersects $\underline{B}_o$.

One can in principle find solutions to equation (7) which are spatially confined in three dimensions. The technique is to leave $\phi$ as an arbitrary function of space and prescribe the spatial dependence of $\theta$. For example, a spatially confined wave packet can be described by

$$\theta(x,y,z) = \frac{\pi}{2} \left[1 + \frac{\epsilon}{2} \exp - (x^2 + y^2 + z^2)\right] \qquad (8)$$

where $0 < \epsilon < 1$ parametrises the amplitude of the perturbation. Substituting (8) into (7) yields a complicated partial differential equation for $\phi(x,y,z)$. While it does not appear possible to solve analytically for $\phi$ (x,y,z), a solution can, in principle, be obtained numerically.

## Conclusions

Large amplitude Alfvénic disturbances of arbitrary spatial shape and polarization are described by the MHD equations, without resort to the usual assumption of planarity. Such disturbances propagate along the mean field with a group velocity, $\underline{V}_A = \underline{B}_o (4\pi\rho)^{-1/2}$. Their Alfvénic nature is characterized by constancy of $|\underline{B}|$ and the relation (1) between $\underline{v}$ and $\Delta\underline{B}$. However, because of their nonplanar nature, the direction of propagation of these disturbances cannot, in general, be determined by looking for minima in a variance matrix constructed from observed field fluctuations. When such minima exist (Daily, 1973), one is observing that subset of interplanetary Alfvén waves that is essentially planar. An extension of this work that describes the propagation of these MHD waves in a model solar wind is in preparation.

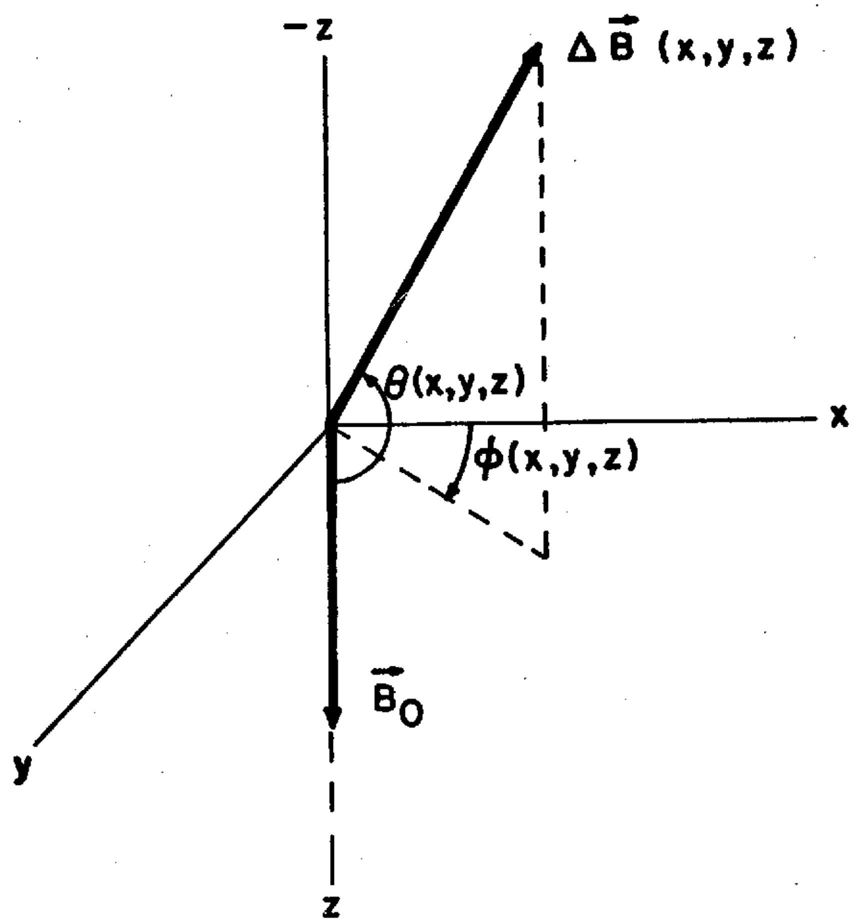
## References


1) Barnes and J. Hollweg, 1974 preprint.

2) J. W. Belcher and L. Davis, Jr., 1971, J. of Geophys. Res., 76, 3534, (see figure 1, p. 3537).

3) W. D. Daily, 1973, J. of Geophys. Res., 78, 2043.

4) H. J. Völk, F. Morfill, W. Alpers and M. A. Lee, 1974, Astrophys. & Space Science, in press.




A spherical coordinate system in which $\theta$ (x, y, z) and $\phi$ (x, y, z) define the direction of $\Delta \underline{B}$ with respect to $\underline{B}_o$. The magnitude of $\Delta \underline{B}$ follows from the constancy of $\underline{B}_o$, and is given by equation (6) in the text.