A COMPUTATIONAL PROCEDURE TO ANALYZE METAL MATRIX LAMINATES WITH NONLINEAR LAMINATION RESIDUAL STRAINS

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TECHNICAL PAPER proposed for presentation at the Composite Reliability Conference sponsored by the American Society for Testing and Materials Las Vegas, Nevada, April 15-16, 1974
A COMPUTATIONAL PROCEDURE TO ANALYZE METAL MATRIX LAMINATES WITH NONLINEAR LAMINATION RESIDUAL STRAINS

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ABSTRACT

An approximate computational procedure is described for the analysis of angleplied laminates with residual nonlinear strains. The procedure consists of a combination of linear composite mechanics and incremental linear laminate theory. The procedure accounts for initial nonlinear strains, unloading, and in-situ matrix orthotropic nonlinear behavior. The results obtained in applying the procedure to boron/aluminum angleplied laminates show that this is a convenient means to accurately predict the initial tangent properties of angleplied laminates in which the matrix has been strained nonlinearly by the lamination residual stresses. The procedure predicted initial tangent properties results which were in good agreement with measured data obtained from boron/aluminum angleplied laminates.

KEY WORDS: Fiber composites, boron/aluminum composites, nonlinear analysis, approximate computational procedure, nonlinear residual strains, initial properties, computer program.

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Determining the reliability of structural components fabricated from fiber composites requires use of various deterministic mathematical models. These models, for example, may be for relating stresses to applied forces, stress intensities at the tips of cracks to nominal stresses in the component, buckling resistance to applied force, vibration response to excitation sources, etc. Deterministic models of the cases just mentioned require initial tangent and strain dependent stress-strain relationships. Experimental data \(^{(1)}\) indicate that the presence of residual stress has significant influence on the initial tangent properties of boron/aluminum angleplied laminates. Analysis of experimental data shows that the lamination residual stresses (the fabrication process induces thermal strains) may be of sufficiently high magnitude to strain the aluminum matrix nonlinearly in certain ply orientation configurations. When the matrix is strained nonlinearly, the stress-strain relationships of the laminate become load-path dependent. Though nonlinear response of composite behavior has received some attention \(^{(2-6)}\), the lamination residual strain nonlinearity facet, including its effects on subsequent loading, has not been examined. It is the purpose of this investigation to describe a computational procedure for predicting initial tangent stress-strain relationships of angleplied laminates in which the matrix has been strained nonlinearly by the residual stress.

\(^{(1)}\) The numbers in the parenthesis refer to list of references appended to this paper.
BACKGROUND AND DESCRIPTION OF PROCEDURE

The background leading to the procedure, its theoretical basis, and the computational procedure evolved are described in this section.

Background

The computational procedure described herein evolved from the analysis of test data from boron/aluminum angleplied laminates \(^{(1,7)}\). These laminates had various ply configurations and were loaded at various angles to their material axis of symmetry. Application of linear laminate analysis to these laminates predicted results for the initial tangent modulus (modulus of elasticity) which were considerably higher than the corresponding measured values. This observation suggested that the lamination residual stresses are present and of sufficient magnitude to strain the matrix nonlinearly in some plies. Further analysis of these data indicated that the residual stress may strain the matrix at considerably different nonlinear strain magnitudes along the fiber direction, transverse to it, and in intralaminar shear. It was concluded, therefore, that a computational procedure should allow for orthotropic nonlinear behavior of the in-situ matrix.

A computational procedure using the finite element method \(^{(8-10)}\) is a logical approach for such analyses. However, this method consumes large computational times in nonlinear analyses. Therefore, an alternate approximate method was pursued. In addition, this alternate method could be readily implemented through the use of a presently available computer code \(^{(11)}\) with the addition of three equations to be described subsequently.
Theoretical Basis

The computational procedure evolved is an approximate analysis. It consists of linear composite mechanics and a piece-wise linear laminate analysis to handle the nonlinear response of the in-situ matrix. Tangent properties at the current cumulative matrix strain level are used as inputs to compute the strains due to the next load increment. The equations of force equilibrium are satisfied at the micromechanics, macro-mechanics, and laminate levels.

The macro- and micro-residual strains are obtained using linear composite mechanics and average matrix thermal and mechanical properties. These average properties are used as inputs in the micromechanics level to generate the ply properties. The ply properties and the temperature difference between processing and use-temperatures are used as inputs in the laminate analysis level to compute the residual strains and stresses in the plies. The maximum residual strains in the matrix are computed using the ply strains and the strain magnification factors. The average residual strains in the matrix are computed using the following equations.

Along the fiber direction

\[ \varepsilon_{m11R} = \varepsilon_{l11R} - \Delta T \alpha_{l11} \]  \hspace{1cm} (1)

Transverse to the fiber direction

\[ \varepsilon_{m22R} = 2\beta_{22} (\varepsilon_{l22} - \Delta T \alpha_{l22}) / \gamma \]  \hspace{1cm} (2)
Intralaminar shear

\[ \varepsilon_{m12R} = 2\beta_{12} \varepsilon_{l12} / \Delta T \]  

Equation (3)

The notation in equations (1), (2), and (3) is as follows: \( \varepsilon \) denotes strain; \( \Delta T \) is the difference in temperature between processing and use-temperature; \( \alpha \) is the thermal coefficient of expansion, and \( \beta \) is the matrix magnification factor. The subscript \( m \) denotes matrix, \( R \) residual, and \( l \) ply. The numerical subscript 1 denotes direction parallel to the ply fibers and 2 normal to them.

Equations (2) and (3) were obtained by assuming that the transverse and intralaminar shear strains in the matrix decrease from their maximum values as a cosine function. Determining the integrated average of this cosine variation of strain yields the desired result.

The suitability of approximating the strain distribution in the matrix assuming a cosine variation was compared with the corresponding distribution using a two dimensional second order finite element analysis. The results are shown in figure 1. As can be seen in this figure, the areas under the two curves are almost the same. Therefore, the integrated average of the cosine distribution for the average matrix strain is a reasonable approximation.

The average residual strains in the matrix in the various plies are used as inputs in the micromechanics level to compute the ply tangent properties with initial strains. The ply tangent properties and a small load increment are used as inputs in the laminate analysis level. The laminate analysis yields the initial tangent composite properties.
Computational Procedure

The computational procedure is in the form of a computer program. In the present investigation, the computer program was generated by modifying an available computer program \((11,12)\). Briefly, this program consists of a collection functional modules to carry out composite micromechanics, macromechanics, and laminate analysis. The inputs to this computer program are constituent material properties, and composite geometry. The program allows the in-situ matrix to be orthotropic.

In the modified version, the temperature dependent thermal and mechanical properties of the matrix are read in the program in the form of tables. The fiber properties are read as in reference 11. A qualitative flow chart of the resulting computer program is shown in figure 2. Note the temperature difference is read in. Note also the provision to check the positive definite condition of the array of the stress-strain relations of the aluminum matrix. Violation of this condition leads to predictions of negative moduli of elasticity for both the ply and the composite.

The last block in the flow chart, figure 2, is enclosed by an interrupted line. This is to note that the procedure can be readily extended due to continue the nonlinear analysis to mechanical and/or additional thermal loads.
APPLICATION OF COMPUTATIONAL PROCEDURE

The computational procedure described previously and shown in the qualitative flow chart, figure 2, was used to predict initial tangent properties of boron/aluminum composites. In this section, the computed properties are compared with the measured data reported in references 1 and 7. The constituent materials for these composites were 4-mil diameter boron fiber and 6061-0 aluminum alloy matrix. The fiber volume ratio was about 0.50. The laminates were made by hot pressing at 970°F.

Input Data and Test Specimens

The input data required for the computational procedure consisted of tabular data of matrix thermal properties and stress-strain data due to mechanical load. The thermal data of the matrix was obtained from reference 13 and are shown in table 1.

The temperature difference used in the residual strain computations in this investigation was 700°F. The reason for this is that the matrix starts supporting stress of greater than 1000 psi between 800°F and 700°F. See table 1.

The mechanical load stress-strain data were obtained from axial tension and from torsion tests on thin tubes made from 6061-0 aluminum. The tube test section was 2 inches inside diameter, 10 inches long, and .060 inch in wall thickness. The tube was machined to the test section dimensions from commercial tube stock allowing 1 inch at each end for gripping. The tubes were instrumented at their midlength with strain gage rosettes. The tubes were loaded in a multiaxial testing machine under tension and
torsion loads. Load and strain data were recorded at regular load intervals. These data were reduced to engineering stress-strain data using a strain-gage data-reduction program. Tensile load data suitable for input into the computational procedure are shown in table 2. The corresponding shear data are shown in table 3.

Additional input data were boron fiber material properties, composite geometry, and the correlation coefficients described in reference 11. The fiber properties used were: modulus $= 57 \times 10^6$ psi, Poisson's ratio $= .2$, and thermal coefficient of expansion $= 2.8 \times 10^{-6}$ in/in/°F.

Results and Comparisons

As was mentioned previously, the computational procedure described herein was used to predict initial tangent properties for test specimens as described in reference 1.

A summary of the calculated lamination residual strains in the plies and the corresponding average residual strains in the matrix are shown in table 4. Compared to the corresponding strains in tables 2 and 3, it is seen that the average residual strains in the matrix in some plies are of sufficient magnitude to strain the matrix well into the nonlinear range.

The matrix properties selected by the computational procedure to be used in subsequent calculations are summarized in table 5. The orthotropic nonlinear behavior of the in situ matrix is clearly illustrated by the results in this table since $E_{m22} \neq E_{m11}$ for the majority of the cases shown.

Initial tangent modulus and Poisson's ratio values predicted by the computational procedure with and without residual strains are compared with
measured data for several composites in Table 6. As can be seen in this table, the predicted moduli from the "with residual strain" case are in good agreement with the measured data. The predicted moduli from the "no residual strain" case are considerably higher than the measured data. In some cases, the predicted values exceed the measured ones by as much as 35 percent.

The comparisons for the Poisson's ratio values, however, are not good for some composites for the "with residual strain" case. The difficulty in comparing Poisson's ratio values arises from the fact that in certain cases there is a transverse strain gradient through the thickness of the laminate. The predicted value is an average value while the measured value is a local one, i.e., that on the material surface.

To illustrate the aforementioned point, the following calculations were performed. For the (0±30)₅ and the (0±45)₅ laminates, the Poisson's ratios were computed for laminates consisting of all 0-plies, all ±30-plies, or all ±45 plies. In these computations, the presence of residual strains was incorporated by using aluminum matrix properties for each ply from corresponding plies in Table 5. The results from these calculations are compared below with measured values.

<table>
<thead>
<tr>
<th>Laminate</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>predicted</td>
</tr>
<tr>
<td>(0±30)₅</td>
<td>measured</td>
</tr>
<tr>
<td>(+30)₅</td>
<td>predicted</td>
</tr>
</tbody>
</table>
Laminate | Poisson's Ratio
--- | ---
0 | predicted 0.27
(0_{+45})_S | measured 0.30
(0_{-45})_S | predicted 0.99

As can be seen from these comparisons, the measured Poisson's ratio is between the two predicted values but close to that for the all 0-plies as would be expected. To eliminate the transverse strain gradient, it is suggested that the following types of test specimens be used for measuring Poisson's ratio values in laminates with nonlinear residual stresses: (1) wide, flat test specimens to minimize or eliminate free-edge effects, (2) thin tube specimens, or (3) specimens subjected to biaxial stress fields.

Predicted initial tangent properties "with" and "without" residual strains are summarized in table 7 for the composites investigated. As can be seen in this table, the differences are substantial, especially for the shear modulus.

DISCUSSION AND IMPLICATIONS

The computational procedure described previously was found to be a convenient tool to compute initial tangent properties in composites whose in-situ matrix had been strained nonlinearly by residual strains. In all the cases investigated, one load increment was sufficient to determine the initial tangent properties. The computer CPU time per case was less than five seconds in the UNIVAC 1106.
In the cases examined in the present investigation, no ply unloading occurred in any of the composites. Had the unloading occurred, then two or more load increments would have been required to establish the initial tangent properties. The actual computing time per load increment is about one to two seconds.

The procedure described herein has two important distinctions when compared with other available approaches. These are: (1) the approximate treatment of the interfiber matrix strain variation and (2) the accounting of orthotropic nonlinear behavior of the in-situ matrix. The main advantages of this approach are: (1) computer running time economy, (2) ease of input data preparation, (3) measured nonlinear stress-strain data for the ply under combined load is not required, and (4) the three-dimensional tangent properties for both ply and composite are computed routinely as a part of the analysis.

In view of the differences in the predicted and measured data for Poisson's ratio and since the measured values appear to be sensitive to the ply stacking sequence, it might be necessary to measure these values in combined stress fields. This will assure constant transverse strain through the laminate thickness.

Additional experimental data are needed to assess the suitability of this computational procedure in predicting initial values for the elastic constants under combined loading. The computational procedure can handle combined loadings including bending and nonuniform temperature through the thickness.
Since the response of material with nonlinear initial strains becomes load-path dependent, care should be used in selecting measured properties for a particular design. If initial tangent properties are selected, the measured and predicted data should be consistent with the anticipated load path.

In view of the large residual strains in the matrix in some composite configurations, particular care should be taken when designing components from those composites which will be subjected to fatigue in order to avoid failure in a few cycles.

**SUMMARY OF RESULTS**

A convenient computational procedure has been described which can be used to accurately predict the initial tangent properties of metal matrix composites in which the in situ matrix has been strained nonlinearly by the lamination residual strains.

Application of this procedure to various angleplied laminates made from 4-mil diameter boron/6061-0 aluminum alloy composites predicted results which were in good agreement with measured data for the initial tangent modulus. This agreement was possible because the in-situ matrix was considered to respond nonlinearly in an orthotropic manner.

The results showed that predicted values for initial tangent modulus, not accounting for nonlinear residual strains, are higher than measured data by as much as 35 percent for some laminates.


### TABLE 1. - MATRIX PROPERTIES VS TEMPERATURE

6061-0 ALUMINUM ALLOY

<table>
<thead>
<tr>
<th>TEMPERATURE, °F</th>
<th>MODULUS, 10^6 PSI</th>
<th>POISSON'S RATIO</th>
<th>THER COEFF EXP, 10^-6 IN./IN./°F</th>
<th>YIELD STRESS, PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.3</td>
<td>0.310</td>
<td>12.1</td>
<td>8000</td>
</tr>
<tr>
<td>100</td>
<td>11.0</td>
<td>.330</td>
<td>12.8</td>
<td>8000</td>
</tr>
<tr>
<td>200</td>
<td>10.7</td>
<td>.350</td>
<td>13.5</td>
<td>8500</td>
</tr>
<tr>
<td>300</td>
<td>10.4</td>
<td>.345</td>
<td>14.1</td>
<td>8000</td>
</tr>
<tr>
<td>400</td>
<td>10.1</td>
<td>.335</td>
<td>14.7</td>
<td>7000</td>
</tr>
<tr>
<td>500</td>
<td>9.9</td>
<td>.330</td>
<td>15.2</td>
<td>4000</td>
</tr>
<tr>
<td>600</td>
<td>9.8</td>
<td>.330</td>
<td>15.8</td>
<td>3000</td>
</tr>
<tr>
<td>700</td>
<td>9.3</td>
<td>.330</td>
<td>16.5</td>
<td>2000</td>
</tr>
<tr>
<td>800</td>
<td>8.8</td>
<td>.345</td>
<td>17.2</td>
<td>1000</td>
</tr>
<tr>
<td>900</td>
<td>8.0</td>
<td>.350</td>
<td>17.9</td>
<td>0</td>
</tr>
<tr>
<td>1000</td>
<td>7.0</td>
<td>.350</td>
<td>18.6</td>
<td>0</td>
</tr>
</tbody>
</table>

### TABLE 2. - MATRIX STRESS/STRAIN PROPERTIES

6061-0 ALUMINUM ALLOY

<table>
<thead>
<tr>
<th>STRAIN, %</th>
<th>STRESS, PSI</th>
<th>MODULUS, PSI</th>
<th>POISSON'S RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>11.0x10^6</td>
<td>0.366</td>
</tr>
<tr>
<td>.024</td>
<td>2 590</td>
<td>10.5</td>
<td>.336</td>
</tr>
<tr>
<td>.050</td>
<td>5 260</td>
<td>9.8</td>
<td>.334</td>
</tr>
<tr>
<td>.078</td>
<td>7 920</td>
<td>8.2</td>
<td>.332</td>
</tr>
<tr>
<td>.098</td>
<td>9 250</td>
<td>3.6</td>
<td>.392</td>
</tr>
<tr>
<td>.44</td>
<td>10 500</td>
<td>2.7x10^5</td>
<td>.409</td>
</tr>
<tr>
<td>1.2</td>
<td>11 900</td>
<td>8.8x10^4</td>
<td>.400</td>
</tr>
<tr>
<td>1.5</td>
<td>11 900</td>
<td>9.9x10^4</td>
<td>.397</td>
</tr>
<tr>
<td>5.0</td>
<td>11 900</td>
<td>1.0x10^4</td>
<td>.450</td>
</tr>
</tbody>
</table>

### TABLE 3. - MATRIX SHEAR STRESS-STRAIN PROPERTIES

6061-0 ALUMINUM ALLOY

<table>
<thead>
<tr>
<th>STRAIN, %</th>
<th>STRESS, PSI</th>
<th>MODULUS, 10^6 PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3.27</td>
</tr>
<tr>
<td>.04</td>
<td>1150</td>
<td>3.44</td>
</tr>
<tr>
<td>.07</td>
<td>2450</td>
<td>3.57</td>
</tr>
<tr>
<td>.11</td>
<td>3760</td>
<td>3.56</td>
</tr>
<tr>
<td>.15</td>
<td>5060</td>
<td>3.32</td>
</tr>
<tr>
<td>.19</td>
<td>6430</td>
<td>2.03</td>
</tr>
<tr>
<td>.33</td>
<td>7810</td>
<td>.867</td>
</tr>
<tr>
<td>.50</td>
<td>8300</td>
<td>.100</td>
</tr>
<tr>
<td>1.00</td>
<td>8300</td>
<td>.010</td>
</tr>
</tbody>
</table>
### Table 4. - Thermal Strains in Plies and in Matrix

Boron/Aluminum Composites; 4-Mil Fiber; 6061-0 Aluminum Alloy;
0.50 Fiber Volume Ratio; ΔT = -700° F

<table>
<thead>
<tr>
<th>Composite Configuration</th>
<th>Ply</th>
<th>Thermal Ply Strains in 10^-3 in./in.</th>
<th>Integrated Avg Matrix Residual Strains in 10^-3 in./in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E_{11}$</td>
<td>$E_{22}$</td>
</tr>
<tr>
<td>[0/8]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-2.48</td>
<td>-8.03</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-2.53</td>
<td>-7.99</td>
</tr>
<tr>
<td>[0₂±25°]₅₀</td>
<td>0</td>
<td>-2.50</td>
<td>-7.90</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-2.86</td>
<td>-7.57</td>
</tr>
<tr>
<td>[0₂±30°]₅₀</td>
<td>0</td>
<td>-2.67</td>
<td>-7.48</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-3.88</td>
<td>-6.28</td>
</tr>
<tr>
<td>[0₂±45°]₅₀</td>
<td>0</td>
<td>-3.22</td>
<td>-6.56</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-4.89</td>
<td>-4.89</td>
</tr>
<tr>
<td>[0₂±90°]₅₀</td>
<td>0</td>
<td>-4.73</td>
<td>-4.73</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-8.4</td>
<td>-6.56</td>
</tr>
</tbody>
</table>

### Table 5. - Summary of Matrix Properties

Selected by Computational Procedure in Presence of Nonlinear Residual Strains; 6061-0 Aluminum Alloy

<table>
<thead>
<tr>
<th>Composite</th>
<th>Ply</th>
<th>Moduli in 10^6 psi</th>
<th>Poisson's Ratio, ν_m12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Only</td>
<td></td>
<td>$E_{m11}$</td>
<td>$E_{m22}$</td>
</tr>
<tr>
<td>[0₂±25°]₅₀</td>
<td>0</td>
<td>11.0</td>
<td>10.6</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>11.0</td>
<td>11.0</td>
</tr>
<tr>
<td>[0₂±30°]₅₀</td>
<td>0</td>
<td>11.0</td>
<td>11.0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>10.6</td>
<td>10.6</td>
</tr>
<tr>
<td>[0₂±45°]₅₀</td>
<td>0</td>
<td>8.4</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>[0₂±90°]₅₀</td>
<td>0</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>
### Table 6. Comparisons of Measured and Predicted Results

**Initial Tangent Elastic Constants; Boron/Aluminum Composites; 4-Mil Diam Fiber;** 6061-0 Aluminum Alloy; 0.50 Fiber Volume Ratio; $\Delta T = -700^\circ F$

<table>
<thead>
<tr>
<th>Composite Ply Orientation</th>
<th>Loading Angle</th>
<th>Initial Tangent Modulus $10^6$ PSI</th>
<th>Initial Tangent Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Predicted w/ No Residual Strain</td>
<td>Predicted w/ Residual Strain</td>
</tr>
<tr>
<td>$[0_8]$</td>
<td>0</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>$[0_2\pm5]$</td>
<td>0</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td>30</td>
<td>23</td>
</tr>
<tr>
<td>$[0_2\pm15]_S$</td>
<td>0</td>
<td>27</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>-37.5</td>
<td>28</td>
<td>16</td>
</tr>
</tbody>
</table>

*Measured values were taken at about 10% of the composite fracture strain.

### Table 7. Predicted Initial Tangent Elastic Constants

**Boron/Aluminum Composites; 4-Mil Diam Fiber; 6061-0 Aluminum Alloy; 0.50 Fiber Volume Ratio**

[Subscript $x$ denotes property along load direction, $y$ is $90^\circ$ to $x$, $c$ denotes composite property, and $sn$ denotes shear strain-normal strain coupling.]

<table>
<thead>
<tr>
<th>Composite</th>
<th>Loading Angle</th>
<th>No Residual Strain</th>
<th>With Residual Strain $\Delta T = -700^\circ F$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Moduli in $10^6$ PSI</td>
<td>Poisson's Ratio, $\nu_{cxy}$</td>
</tr>
<tr>
<td>$[0_8]$</td>
<td>0</td>
<td>33.9 21.3 7.0</td>
<td>0.25</td>
</tr>
<tr>
<td>$[0_2\pm5]$</td>
<td>0</td>
<td>33.7 21.2 7.0</td>
<td>0.26</td>
</tr>
<tr>
<td>$[0_2\pm15]_S$</td>
<td>0</td>
<td>32.5 20.9 7.5</td>
<td>0.28</td>
</tr>
<tr>
<td>$[0_2\pm30]_S$</td>
<td>-80</td>
<td>20.7 30.9 7.8</td>
<td>0.20</td>
</tr>
<tr>
<td>$[0_2\pm90]_S$</td>
<td>0</td>
<td>29.5 20.4 8.9</td>
<td>0.33</td>
</tr>
<tr>
<td>$[0_2\pm15]_S$</td>
<td>-37.5</td>
<td>26.8 20.8 9.1</td>
<td>0.34</td>
</tr>
<tr>
<td>$[0_2\pm30]_S$</td>
<td>0</td>
<td>27.0 21.5 9.3</td>
<td>0.34</td>
</tr>
<tr>
<td>$[0_2\pm90]_S$</td>
<td>-37.5</td>
<td>24.7 23.2 9.1</td>
<td>0.30</td>
</tr>
</tbody>
</table>

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FLOW CHART OF COMPUTATIONAL PROCEDURE

READ IN TABLES OF
CONSTITUENT MATERIAL
PROPERTIES & COMPOSITE
GEOMETRY

READ IN EFFECTIVE
TEMP CHANGE

COMPUTE AVERAGE
THERMAL PROPERTIES
FROM TABLE

COMPUTE PLY
UNIDIRECTIONAL
PROPERTIES VIA
MICROMECHANICS

COMPUTE COMPOSITE
PROPERTIES VIA
LAMINATE ANALYSIS

COMPUTE PLY
THERMAL STRAINS VIA
LAMINATE ANALYSIS

COMPUTE MAXIMUM
MATRIX RESIDUAL
STRAINS VIA STRAIN
MAGNIFICATION FACTOR

COMPUTE INTEGRATED
AVERAGE OF MATRIX
RESIDUAL STRAINS

DETERMINE MATRIX
PROPERTIES FROM TABLE

CHECK POSITIVE
DEFINITE CONDITION
OF MATRIX ELASTIC
CONSTANT

SET UP A SMALL
LOAD INCREMENT

COMPUTE PLY
PROPERTIES VIA
MICROMECHANICS

COMPUTE COMPOSITE
PROPERTIES VIA
LAMINATE ANALYSIS

DISPLAY COMPOSITE
UNIDIRECTIONAL

PROCEED WITH
NONLINEAR ANALYSIS
DUE TO MECHANICAL
&/OR THERMAL LOADS

THESE ARE THE INITIAL TANGENT COMPOSITE PROPERTIES.

Figure 2.