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D. A. Throckmorton and D. R. Stone

June 1974

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LANGLEY RESEARCH CENTER, HAMPTON, VIRGINIA 23665
**Title and Subtitle**
Model Wall and Recovery Temperature Effects on Experimental Heat Transfer Data Analysis

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**Abstract**
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**Key Words**
- Fluid Mechanics
- Boundary layers
- Heat transfer

**Distribution Statement**
Unclassified - Unlimited

**Security Classification**
- Unclassified (of this report)
- Unclassified (of this page)

**Price**
$3.00

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Summary

Basic analytical procedures are used to illustrate, both qualitatively and quantitatively, the relative impact upon heat transfer data analysis of certain factors which may affect the accuracy of experimental heat transfer data. Inaccurate knowledge of adiabatic wall conditions results in a corresponding inaccuracy in the measured heat transfer coefficient. The magnitude of the resulting error is extreme for data obtained at wall temperatures approaching the adiabatic condition. High model wall temperatures and wall temperature gradients affect the level and distribution of heat transfer to an experimental model. The significance of each of these factors is examined and its impact upon heat transfer data analysis is assessed.
INTRODUCTION

Development of the phase-change coating technique (reference 1) has provided a valuable tool for obtaining quantitative measurements of the heat transfer to bodies in hypersonic wind tunnels. The technique offers economies of both time and money when compared to thermocouple techniques, and also yields measurement of highly detailed heating distributions not possible with previous methods. These advantages have led to widespread use of the phase-change technique in both basic fluid mechanics research and configurational heating studies. However, the procedures required in utilizing this technique result in data obtained over long test time intervals and, therefore, at model wall temperature levels and gradients not normally encountered in thin-skin testing. In thin-skin testing, data are obtained simultaneously at all points on the model, at times near test initiation such that model wall temperatures are nearly uniform - i.e. temperature gradients are generally negligible. During a phase-change coating test, however, data at various model locations are obtained at different times, with time intervals sufficiently large to allow significant temperature gradients to exist on the model when the data are obtained.

Increased model wall temperature results in an increased sensitivity of phase-change heat transfer coefficients to adiabatic wall temperature, as compared to thin-skin data. Analysis of data obtained on models with wall temperatures approaching adiabatic conditions, and on models with significant surface temperature gradients demands an understanding of the effects of wall temperature on the heat transfer process, to assure accurate interpretation of that data. The significance of each of these factors (\(T_w\), \(T_{aw}\), \(T_w\) gradients) is examined and their impact upon heat transfer data analysis is assessed. The investigation utilized both experimental phase-change coating data and "exact" numerical solutions to the laminar boundary layer equations to observe the nature of the wall temperature effects on the heat transfer to a flat plate, a hemisphere-cylinder, and to the windward centerline of a representative space shuttle orbiter configuration.

NOMENCLATURE

\(c_p\) model material specific heat
\(h\) heat transfer coefficient
\(k\) model material thermal conductivity
\(M\) Mach number
\(q\) heating rate
\(Pr\) Prandtl number
\(r\) laminar recovery factor
ANALYSIS AND RESULTS

Sensitivity of Heat Transfer Coefficient to Adiabatic Wall Temperature

Experimental heating data, obtained using either the thin-skin calorimeter or phase-change coating technique, are usually expressed in the form of the aerodynamic heat transfer coefficient (h). This parameter is defined by Newton's Law of Cooling as the proportionality constant relating the local heat transfer rate (q) and the forcing function of the
heat transfer process; i.e., the difference between the local adiabatic wall temperature (Taw) and the local wall temperature (Tw).

\[ \dot{q} = h (T_{aw} - T_w) \]  

(1)

For the analysis of experimental data, expressions for the aerodynamic heat transfer coefficient are derived from the equation governing the one-dimensional, transient conduction of heat into a solid, with application of appropriate boundary and initial conditions. Both the phase-change and thin-skin calorimeter techniques assume a step heat input, usually obtained by rapid injection of an isothermal model into the airstream. For the thin-skin technique, heat transfer coefficient is based upon heat conduction into a finite solid of known thermal properties:

\[ h = \rho c_p \frac{\lambda}{\alpha} \frac{\partial T_w}{\partial t} \frac{1}{T_{aw} - T_w} \]  

(2)

where \( \rho \), \( c_p \), and \( \lambda \) are model material density, specific heat and thickness, respectively. The measured quantities are the wall temperature (T_w) and its time-rate-of-change (\( \frac{\partial T}{\partial t} \)). The corresponding equation used to reduce phase-change data is based upon heat conduction into a semi-infinite solid of known thermal properties:

\[ h = \sqrt{\rho c_p k} \sqrt{t_{pc}} \]  

where \( 1 - e^{\theta^2} \) \( \text{erfc} \) (θ) = \( \frac{T_{pc} - T_i}{T_{aw} - T_i} \)  

(3)

The measured quantity is the time (t_{pc}) required for the model surface temperature to increase from some initial value (T_i) to a known coating phase-change temperature (T_{pc}). (Reference 1.)

The adiabatic wall temperature is rarely measured in thin-skin tests, and the phase-change technique is incapable of indicating this temperature. Computed heat transfer coefficients are extremely sensitive to excursions of an assumed value of the adiabatic wall temperature. This is illustrated in Figure 1 where the ratio of heat transfer coefficient to that value computed assuming T_{aw} = T_t is presented as a function of T_{aw}/T_t for constant values of phase-change temperature ratio and model initial temperature ratios. The initial model-to-stream temperature ratios indicated are representative of conditions for a room temperature model and current hypersonic wind tunnels. (The phase-change coating technique is presently routinely used in many hypersonic air facilities which operate at initial temperature ratios of ~0.35, and has been considered for use in facilities which operate near T_i/T_t = 0.65). (Reference 2.) The sensitivity of thin-skin calorimeter data to adiabatic wall temperature is indicated by the
lower curve on each figure (corresponding to the phase-change temperature equal to the initial temperature). The remaining curves indicate the sensitivity of phase-change data, with increasing values of the phase-change temperature. Analysis of the curves clearly indicates the increased sensitivity of phase-change derived data, as opposed to thin-skin data, to the accuracy of the adiabatic wall temperature estimate. In addition, comparison of the plots for the range of $T_f/T_t$ illustrates the magnification of this sensitivity with increasing initial temperature.

The phase-change temperature is constrained by the test time required to effect the coating phase-change; this time must be long enough for accurate measurement, yet short as compared to the thermal diffusion time of the model. The shaded areas of the figure approximate those regions of interest for practical test operations.

Local adiabatic wall temperatures can be adequately estimated for simple shapes by use of "exact" numerical computation techniques; however, for complex geometries, such "exact" numerical solutions are presently beyond the "state-of-the-art." Consequently, it has become common practice to base experimental data on a nominal adiabatic wall temperature ratio $(T_{aw}/T_t)$ assumed constant over an entire configuration. The use of a nominal value of $T_{aw}/T_t = 1.0$ in the data reduction results in data which are in error as indicated in Figure 1. Attempts to reduce this error by assuming a compromise ratio of 0.95 or 0.90 diminish the maximum potential error; however, the functional relationship of heat transfer coefficient to deviations of the assumed adiabatic wall temperature from the actual value is unchanged. It is therefore necessary to assess the impact of inaccuracies in adiabatic wall temperature estimation procedures on heat transfer coefficients derived from experimental data.

The value of the adiabatic wall-to-total temperature ratio may be expressed for an ideal gas, as a function of recovery factor ($r$) and boundary layer edge Mach number ($M_e$) in the form:

$$\frac{T_{aw}}{T_t} = \frac{1 + \frac{r}{2} - \frac{1}{2} M_e^2}{1 + \frac{r}{2} - \frac{1}{2} M_e^2}$$ (4)

Solutions of the compressible laminar boundary layer equations indicate that recovery factor is a function of Prandtl number, pressure gradient, and also boundary layer edge Mach number. For zero pressure gradient flows with low edge Mach number ($M_e < 2$), recovery factor ($r$) is closely approximated by square root of the Prandtl number ($Pr$) (Reference 3). The work of Wortman and Mills (Reference 4), however, indicates that for accelerating laminar boundary layers, recovery factor is highly dependent upon the pressure gradient parameter, $\beta$, decreasing monotonically to an asymptote $r \to Pr$ as $\beta \to \infty$; a weaker dependence upon edge Mach number is indicated. Stone, et.al (Reference 2) illustrated the combined effect which edge Mach
number and pressure gradient variations exhibit on recovery factor in high Mach number (~20) flows in Helium. The impact of this effect on lower Mach number hypersonic air flows is less pronounced, as edge Mach number and pressure gradient parameter are normally much lower; yet their significance should be ascertained for any specific flow in question.

In the absence of flow field surveys, estimation of the local boundary layer edge Mach number presents a further obstacle to accurate determination of adiabatic wall temperature. Two methods for estimating boundary layer edge conditions are the tangent wedge/cone, and the normal-shock expansion approximations. The tangent wedge/cone technique models the inviscid flow as that occurring on a wedge or cone surface with half angle equal to the local flow deflection angle ($\delta$); which approximation method (wedge or cone) is more accurate depends upon the geometry of the flow to be modeled. The variation of adiabatic wall temperature with flow deflection angle, as defined by the tangent cone and wedge approximations, is illustrated in Figure 2 for a range of recovery factors typical of hypersonic air flows over real configurations. Adiabatic wall temperature level is shown to be equally sensitive to both recovery factor and flow deflection angle.

Another approach to approximation of local boundary layer edge conditions involves use of a measured or analytically determined surface pressure distribution coupled with a local entropy assumption to define the desired quantities. Possibly, the most common model of this type assumes a Newtonian pressure distribution with edge conditions resulting from an isentropic expansion of the flow from a stagnation point behind a normal shock. This constant entropy assumption, although commonly used in boundary layer computation, may not be valid for particular flows of interest.

Figure 3 presents adiabatic wall temperature distributions on the windward center line of a space shuttle delta wing orbiter configuration at $30^\circ$ angle of attack as computed by the tangent cone and the normal shock expansion techniques. A constant value of recovery factor ($r = 0.84$) was used for both calculation methods in order to emphasize the sensitivity to edge Mach number estimation techniques alone; edge Mach number and pressure gradient were low so that the previously discussed effects of these parameters upon recovery factor were small. It is important to note that although the maximum deviation between the adiabatic wall temperatures computed by these methods is only about 5 percent, the heat transfer results which are dependent upon these values may differ by 5 to 25 percent as a function of model initial temperature ($T_i/T_t$) and phase-change temperature ($T_{pc}/T_t$) as illustrated in Figure 1.

Wall Temperature Effects

A basic assumption used in derivation of the expressions for aerodynamic heat transfer coefficient, for both the thin-skin and phase-change techniques, is that the model experiences a step input in heat transfer coefficient to a value which is constant with time. For thin-skin testing, this is a normally valid assumption as test time is short and the model
remains essentially isothermal over the duration of the transient test. Phase-change testing, however, may violate this assumption as the data are obtained over relatively long test intervals and, therefore, at model wall temperature levels and distributions not normally encountered in thin-skin testing.

Chapman and Rubesin (Reference 5) indicated that for laminar boundary layer flows with variable surface temperature, local boundary layer properties (and, therefore, heat transfer) depend not only on the local temperature potential, but on the entire surface temperature distribution upstream of the point in question. The effects of temperature level and distribution on the value of heat transfer coefficient are due to what they termed "the inappropriateness of the conventional heat transfer coefficient when applied to flows with variable surface temperature." This "inappropriateness" is most apparent as wall temperature approaches the local adiabatic wall condition. For variable wall temperatures (Tw) and wall temperature level of the order of local adiabatic wall temperature (Tw = Taw), heat transfer coefficient may reverse sign and even become infinite. This anomalous behavior is illustrated in Figure 4 for a flat plate at zero angle of attack with surface temperature distribution as indicated. These curves result from "exact" solutions to the laminar boundary layer equations (Reference 6); adiabatic wall temperature was calculated from Equation (4) with \( r = 0.84 \).

The effect of wall temperature on heat transfer coefficient is again illustrated in Figure 5, by solutions to the laminar boundary layer on a hemisphere-cylinder. In the case of uniform wall temperature, a change in the temperature level results in an alteration to the heat transfer coefficient distribution due to the changed relationship between the wall and adiabatic wall temperatures. As the wall temperature is increased, the local heat transfer coefficient decreases. This decrease is negligible for low values of the wall-to-total temperature ratio, but becomes significant as wall temperature approaches the adiabatic condition. In contrast, a negative gradient of surface temperature along the wall, results in an increase in the local heat transfer coefficient. The predicted increase shown in Figure 5 is a result of the temperature gradient effect plus the opposing effect of an increased wall temperature level. (Comparing the heat transfer coefficient distributions for the gradient case and the constant wall temperature \( Tw/T_t = 0.1 \) case, heat transfer coefficients would be expected to decrease for the gradient case as wall temperature has everywhere increased above the \( Tw/T_t = 0.1 \) level). The temperature gradient effect predominates and heat transfer coefficient increases. The wall temperature distribution indicated for this calculation, is typical of that on a hemisphere phase-change model at a specific instant in time during test in a hypersonic wind tunnel.

For the more practical case of a space shuttle orbiter configuration, wall temperature gradient effects are illustrated in Figure 6. Again, the indicated temperature distribution would exist on a phase-change model at a
specific instant during a hypersonic heat transfer test. The predictions result from solution of the laminar boundary layer by application of the axisymmetric analog to the flow on the orbiter lower surface plane of symmetry. The inviscid pressure distribution and streamline divergence information were computed by the method of DeJarnette and Hamilton (Reference 7). The impact of variable wall temperature is less significant for this flow than for that about the hemisphere. This is attributed to the smaller temperature gradient present on the orbiter lower surface aft of the nose region.

Heating Data Impact

Phase-change test derived heating distributions on spheres have been frequently used in an inverse method for determining phase-change model material thermophysical properties (Reference 8). This procedure requires comparison of the experimental heating distribution with a theoretical distribution to obtain that value of the phase-change thermal properties parameter, $p_{C_p k}$, which results in the best correlation. The quality of the measurement is a direct function of the sophistication of the theoretical method utilized. Figure 7 presents experimental phase-change heating data for a hemisphere-cylinder at Mach 20.3 in helium (from Reference 4). The measured heat transfer data were reduced using an adiabatic wall temperature distribution obtained by the non-similar solution technique. Also shown is the theoretical heat transfer distribution for a constant wall temperature equal to the model initial temperature. As can be seen from Figure 7, the use of a constant wall temperature theory with phase-change data may result in a derived thermal property value which is substantially in error. Thermal properties so derived will be accurate only if the theory utilized adequately models the non-isothermal nature of the phase-change test itself.

Conversely, if experimental heating data are to be used to verify theoretical calculation procedures, it is important that the theory accurately model the experiment which produced that data. Data which may exhibit significant wall temperature effects should not be used to verify a theory which lacks the sophistication to account for them.

CONCLUDING REMARKS

Basic analytical procedures have been used to illustrate, both qualitatively and quantitatively, the relative impact upon heat transfer data analyses of certain factors which may affect the accuracy of experimental heat transfer data. It is recognized that the physical principles involved, i.e. wall temperature effects on heat transfer, recovery factor and adiabatic wall temperature computation procedures, have all been previously discussed in detail by other investigators. However, recent widespread adoption of the phase-change coating technique for use in a variety of heat transfer investigations requires a renewed awareness, by the experimentalist, of the possible error sources (and the significance
of each) which exist in regimes common to phase-change testing. The subject material leads to the following comments:

1. Experimental heat transfer coefficient data accuracy (for either thin-skin or phase-change) is directly dependent upon accurate knowledge of the local adiabatic wall temperature. Phase-change coating data exhibits a significantly greater sensitivity to this quantity than does thin-skin calorimeter data. Errors in heat transfer coefficient resulting from inaccurate knowledge of the adiabatic condition may be diminished by testing at decreasing values of the model initial-to-stream total temperature ratio. In the limit, however, accuracy in computed heat transfer coefficients is directly proportional to the accuracy of the adiabatic wall temperature.

2. Wall temperature gradients, which may result from model geometry characteristics and/or long run times, can significantly affect measured heat transfer coefficient distributions. Wall temperature gradients and their resulting effects are minimized by thin-skin testing or, in the case of phase-change testing, utilizing phase-change temperatures which approach the initial condition.

3. If experimental data are to be used to verify theoretical calculation procedures, it is important that the theory accurately model the experiment which produced that data. Data which may exhibit significant wall temperature effects should not be used to verify a theory which lacks the sophistication to account for them.

REFERENCES


Figure 1. - Effect of adiabatic wall temperature assumption on computed heat transfer coefficient.
Figure 2.- Adiabatic wall temperature variation with flow deflection angle. ($M_\infty = 10.0$)
Figure 3.- Delta-wing shuttle orbiter lower surface centerline adiabatic wall temperature distribution. \( \alpha = 30^\circ \).
Figure 4.— Effect of wall temperature on heat transfer coefficient for a flat plate at Mach 10.0 in air. $\alpha = 0^\circ$. 
Figure 5.- Theoretical heat transfer distributions on a hemisphere-cylinder at Mach 10.0 in air.
Figure 6.- Theoretical heat transfer distributions on the windward centerline on a space shuttle orbiter configuration at Mach 10.0 in air. 
$\alpha = 30^\circ$. 
Figure 7.- Effect of wall temperature variation on measured heat transfer on a hemisphere-cylinder at Mach 20.3 in helium.
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D. A. Throckmorton* and D. R. Stone**
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ABSTRACT

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<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$c_p$</td>
<td>model material specific heat</td>
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<tr>
<td>$h$</td>
<td>heat transfer coefficient</td>
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<td>$k$</td>
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<td>$M$</td>
<td>Mach number</td>
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<td>$\dot{q}$</td>
<td>heating rate</td>
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<td>$\rho$</td>
<td>model material density</td>
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Subscripts:
- $aw$: adiabatic wall condition
- $e$: boundary layer edge condition
- $i$: initial condition
- $o$: stagnation point condition
INTRODUCTION

Development of the phase-change coating technique (reference 1) has provided a valuable tool for obtaining quantitative measurements of the heat transfer to bodies in hypersonic wind tunnels. The technique offers economies of both time and money when compared to thermocouple techniques, and also yields measurement of highly detailed heating distributions not possible with previous methods. These advantages have led to widespread use of the phase-change technique in both basic fluid mechanics research and configurational heating studies. However, the procedures required in utilizing this technique result in data obtained over long test time intervals and, therefore, at model wall temperature levels and gradients not normally encountered in thin-skin testing. In thin-skin testing, data are obtained simultaneously at all points on the model, at times near test initiation such that model wall temperatures are nearly uniform - i.e. temperature gradients are generally negligible. During a phase-change coating test, however, data at various model locations are obtained at different times, with time intervals sufficiently large to allow
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ANALYSIS AND RESULTS

Sensitivity of Heat Transfer Coefficient to Adiabatic Wall Temperature

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\dot{q} = h \left( T_{aw} - T_w \right) \tag{1}
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For the analysis of experimental data, expressions for the aerodynamic heat transfer coefficient are derived from the equation governing the one-dimensional, transient conduction of heat into a solid, with application of appropriate boundary and initial conditions. Both the phase-change and thin-skin calorimeter techniques assume a step heat input, usually obtained by rapid injection of an isothermal model into the airstream. For the thin-skin technique, heat transfer coefficient is based upon heat conduction into a finite solid of known thermal properties:

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\]

where \(\rho\), \(c_p\), and \(\lambda\) are model material density, specific heat and thickness, respectively. The measured quantities are the wall temperature \((T_w)\) and its time-rate-of-change \((\partial T/\partial t)\). The corresponding equation used to reduce phase-change data is based upon heat conduction into a semi-infinite solid of known thermal properties:

\[
h = \sqrt{\frac{\rho c_p k}{T_{pc}}} \frac{\theta}{\sqrt{T_{pc}}} \quad \text{where} \quad 1 - e^{-\theta^2} \text{erfc} (\theta) = \frac{T_{pc} - T_i}{T_{aw} - T_i} \tag{3}
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Chapman and Rubesin (reference 5) indicated that for laminar boundary layer flows with variable surface temperature, local boundary layer properties (and, therefore, heat transfer) depend not only on the local temperature potential, but on the entire surface temperature distribution upstream of the point in question. The effects of temperature level and distribution on the value of heat transfer coefficient are due to what they termed "the inappropriateness of the conventional heat transfer coefficient when applied to flows with variable surface
temperature." This "inappropriateness" is most apparent as wall temperature approaches the local adiabatic wall condition. For variable wall temperatures (\(T_w\)) and wall temperature level of the order of local adiabatic wall temperature (\(T_w \approx T_{aw}\)), heat transfer coefficient may reverse sign and even become infinite in magnitude. This anomalous behavior is illustrated in Figure 4 for a flat plate at zero angle of attack with surface temperature distribution as indicated. These curves result from "exact" solutions to the laminar boundary layer equations (Reference 6); adiabatic wall temperature was calculated from Equation (4) with \(r = 0.84\).

The effect of wall temperature on heat transfer coefficient is again illustrated in Figure 5, by solutions to the laminar boundary layer on a hemisphere-cylinder. In the case of uniform wall temperature, a change in the temperature level results in an alteration to the heat transfer coefficient distribution due to the changed relationship between the wall and adiabatic wall temperatures. As the wall temperature is increased, the local heat transfer coefficient decreases. This decrease is negligible for low values of the wall-to-total temperature ratio, but becomes significant as wall temperature approaches the adiabatic condition. In contrast, a negative gradient of surface temperature along the wall, results in an increase in the local heat transfer coefficient. The predicted increase shown in Figure 5 is a result of the temperature gradient effect plus the opposing effect of an increased wall temperature level. (Comparing the heat
transfer coefficient distributions for the gradient case and the constant wall temperature $T_w/T_t = 0.1$ case, heat transfer coefficients would be expected to decrease for the gradient case as wall temperature has everywhere increased above the $T_w/T_t = 0.1$ level.) The temperature gradient effect predominates and heat transfer coefficient increases. The wall temperature distribution indicated for this calculation, is typical of that on a hemisphere phase-change model at a specific instant in time during test in a hypersonic wind tunnel.

For the more practical case of a space shuttle orbiter configuration, wall temperature gradient effects are illustrated in Figure 6. Again, the indicated temperature distribution would exist on a phase-change model at a specific instant during a hypersonic heat transfer test. The predictions result from solution of the laminar boundary layer by application of the axisymmetric analog to the flow on the orbiter lower surface plane of symmetry. The inviscid pressure distribution and streamline divergence information were computed by the method of DeJarnette and Hamilton (Reference 7). The impact of variable wall temperature is less significant for this flow as compared to the hemisphere. This is attributed to the smaller temperature gradient present on the orbiter lower surface aft of the nose region.

**Heating Data Impact**

Phase-change test derived heating distributions on spheres have been frequently used in an inverse method for determining phase-change model material thermophysical properties (Reference 8). This procedure
requires comparison of the experimental heating distribution with a theoretical distribution to obtain that value of the phase-change thermal property: parameter, $\sqrt{\rho c_p k}$, which results in the best correlation. The quality of the measurement is a direct function of the sophistication of the theoretical method utilized. Figure 7 presents experimental phase-change heating data for a hemisphere-cylinder at Mach 20.3 in helium (from Reference 4). The measured heat transfer data were reduced using an adiabatic wall temperature distribution obtained by the non-similar solution technique. Also shown is the theoretical heat transfer distribution for a constant wall temperature equal to the model initial temperature. As can be seen from Figure 7, the use of a constant wall temperature theory with phase-change data may result in a derived thermal property value which is substantially in error. Thermal properties so derived will be accurate only if the theory utilized adequately models the non-isothermal nature of the phase-change test itself.

Conversely, if experimental heating data are to be used to verify theoretical calculation procedures, it is important that the theory accurately model the experiment which produced that data. Data which may exhibit significant wall temperature effects should not be used to verify a theory which lacks the sophistication to account for them.

CONCLUDING REMARKS

Basic analytical procedures have been used to illustrate, both qualitatively and quantitatively, the relative impact upon heat
transfer data analyses of certain factors which may affect the accuracy of experimental heat transfer data. It is recognized that the physical principles involved, i.e. wall temperature effects on heat transfer, recovery factor and adiabatic wall temperature computation procedures, have all been previously discussed in detail by other investigators. However, recent widespread adoption of the phase-change coating technique for use in a variety of heat transfer investigations requires a renewed awareness, by the experimentalist, of the possible error sources (and the significance of each) which exist when testing in the regimes required by the phase-change technique. The subject material leads to the following comments:

1. Experimental heat transfer coefficient data accuracy (for either thin-skin or phase-change) is directly dependent upon accurate knowledge of the local adiabatic wall temperature. Phase-change coating data exhibits a significantly greater sensitivity to this quantity than does thin-skin calorimeter data. Errors in heat transfer coefficient resulting from inaccurate knowledge of the adiabatic condition may be diminished by testing at decreasing values of the model initial-to-stream total temperature ratio. In the limit, however, accuracy in computed heat transfer coefficients is directly proportional to the accuracy of the adiabatic wall temperature.

2. Wall temperature gradients, which may result from model geometry characteristics and/or long run times, can significantly affect measured heat transfer coefficient distributions. Wall temperature gradients and their resulting effects are minimized by thin-skin
testing or, in the case of phase-change testing, utilizing phase-change temperatures which approach the initial condition.

3. If experimental data are to be used to verify theoretical calculation procedures, it is important that the theory accurately model the experiment which produced that data. Data which may exhibit significant wall temperature effects should not be used to verify a theory which lacks the sophistication to account for them.

REFERENCES


LIST OF FIGURES

Figure 1.- Effect of adiabatic wall temperature assumption on computed heat transfer coefficient.

Figure 2.- Adiabatic wall temperature variation with flow deflection angle. \( M_\infty = 10.0 \)

Figure 3.- Delta-Wing shuttle orbiter lower surface centerline adiabatic wall temperature distribution. \( \alpha = 30^\circ \).

Figure 4.- Effect of wall temperature on heat transfer coefficient for a flat plate at Mach 10.0 in air. \( \alpha = 0^\circ \).

Figure 5.- Theoretical heat transfer distributions on a hemisphere-cylinder at Mach 10.0 in air.

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Figure 7.- Effect of wall temperature variation on measured heat transfer on a hemisphere-cylinder at Mach 20.3 in helium.
Figure 1.- Effect of adiabatic wall temperature assumption on computed heat transfer coefficient.
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\[
\begin{align*}
\frac{T}{T_t} & \quad -10^{-3} \quad 0 \quad 10^{-3} \\
\frac{\dot{q}}{\text{BTU/ft}^2\text{sec}} & \quad 0 \quad 0.4 \quad 0.8 \quad 1.2 \quad 1.2 \\
\frac{h}{\text{BTU/ft}^2\text{sec}^0\text{R}} & \quad 0 \quad 0.4 \quad 0.8 \quad 1.2 \quad 1.2 \\
\end{align*}
\]
Figure 5. - Theoretical heat transfer distributions on a hemisphere-cylinder at Mach 10.0 in air.
Figure 6. - Theoretical heat transfer distributions on the windward centerline on a space shuttle orbiter configuration at Mach 10.0 in air. $\alpha = 30^\circ$. 

CONSTANT $\frac{T_W}{T_t} = 0.31$
Figure 7. - Effect of wall temperature variation on measured heat transfer on a hemisphere cylinder at Mach 20.3 in helium.