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15. Abstract  
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MOLECULAR ATTENUATION AND PHASE DISPERSION BETWEEN 40- AND 140-GHZ FOR PATH MODELS FROM DIFFERENT ALTITUDES

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U.S. DEPARTMENT OF COMMERCE
Frederick B. Dent, Secretary

OFFICE OF TELECOMMUNICATIONS
John M. Richardson, Acting Director

MAY 1973
UNITED STATES DEPARTMENT OF COMMERCE

OFFICE OF TELECOMMUNICATIONS

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Assists other government agencies in the use of telecommunications;

Conducts research, engineering, and analysis in the general field of telecommunication sciences to meet government concerns;

Acquires, analyzes, synthesizes, and disseminates information for the efficient use of telecommunications resources.
Preface

The work described in this report was done at the Institute for Telecommunication Sciences as part of a research program on quantitative millimeter wave spectroscopy of atmospheric oxygen. The project was partially supported by the following agencies:

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(b) NOAA - National Environmental Satellite Service under Order No. E-579-71, dated February 16, 1971, and monitored by Mr. J. Alishouse.

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ABSTRACT

Radio wave propagation in the 40 to 140 GHz band through the first hundred kilometers of the atmosphere is strongly influenced by the microwave spectrum of oxygen (O₂-MS). A unified treatment of molecular attenuation and phase dispersion is formulated. Results of molecular physics are translated into frequency, temperature, pressure, and magnetic field dependencies of a complex refractive index. The intensity distribution of the O₂-MS undergoes several changes with increasing altitude: when h < 10 km, all lines, but one at 119 GHz, are merged to a continuum spectrum under the influence of pressure-broadening; when h > 30 km, a line spectrum with isolated Lorentzians is displayed; when h > 40 km, Zeeman-splitting of each line occurs due to the influence of the earth's magnetic field; for h > 60 km, a Voigt profile governs the transition to a Gaussian line shape and eventually the Doppler-broadened line spectrum vanishes. The influence of water vapor is discussed separately.

Attenuation and dispersion rates for path models are evaluated by computer routines. Examples of computer plots are given as a function of altitude for homogeneous, zenith, and tangential path geometries. Molecular resonances of minor atmospheric gases are discussed briefly, as in the noise which originates from the O₂-MS.

Key words: Atmospheric transmissivity; atmospheric millimeter wave propagation; complex refractive index of air; oxygen microwave spectrum.

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1. INTRODUCTION

Much of the future growth of telecommunications will occur by extending the radio spectrum into the millimeter wavelength range (30 to 300 GHz). As shown in a recent review by Thompson et al. (1972), the successful exploitation of this largely unused portion of the spectrum hinges on the accurate and complete account of all propagation effects. The greatest obstacles to the use of such high frequencies are attenuation and phase dispersion caused permanently by the clear air and intermittently by rain and clouds. This report addresses the use of the frequency spectrum 40 to 140 GHz for radio propagation through the clear atmosphere and discusses in detail ultimate limitations due to air molecules of which oxygen is a major species. The microwave spectrum of oxygen, abbreviated \( O_2 \)-MS, dominates the transfer properties of air throughout the specified frequency range, while molecular effects of water vapor influence them to a lesser extent. Roughly forty lines of the \( O_2 \)-MS contribute to the atmospheric spectrum. Each line has individual pressure and temperature dependencies, and the overlapping of lines below 30 kilometers changes the structure of the spectrum drastically with altitude.

Of prime concern to system engineers are signal degradations (amplitude and phase distortions; interferences) and the minimum transmitted pulse length supported by the propagation medium. These concerns lead in Section 2 to the formulation of a complex transfer function \( \tau(\alpha, \phi) \) for a well-mixed, homogeneous atmosphere. Spectroscopic knowledge of the \( O_2 \)-MS is translated into engineering terms of attenuation \( \alpha \), phase-delay \( \phi_0 \), and dispersion \( \Delta\phi \) rates for atmospheric conditions up to altitudes of 100 kilometers. In Section 3 these results are applied to calculate transfer properties of path models through the U.S. Standard Atmosphere (1962). Cumulative attenuation \( A \) and phase
dispersion $\Delta T$ along slanted ray paths through the total (inhomogeneous) atmosphere are evaluated by computer calculations. An example is shown in Figure 1 for one-way zenith path attenuation between 49 and 72 GHz from different initial altitudes. A graphical presentation is the best introduction to atmospheric transmission and shielding properties due to the $O_2$-MS.

As part of spectrum management, a first step was taken to allocate bands for specific applications through an international agreement (ITU, 1971). For the first time several bands have been reserved exclusively for passive systems. A clear understanding of the $O_2$-MS is required to promote the most efficient use of the millimeter wave spectrum which, just as its lower frequency counterpart, is a saturable resource. Reliable knowledge of the transfer function $\tau$ provides the basis for designing systems which take advantage of unique molecular transmission and emission properties associated with the relatively stable dry part of the atmosphere. The $O_2$-MS affords interference resistance and transmission security for broadband radio communications. Optimum system performances can be assured by tradeoff studies between environmental degradations and component limitations.

The following key phrases give examples of telecommunication applications which incorporate $O_2$-MS properties:

(a) Shielding of broadband satellite-to-satellite links from ground interferences (e.g., Crane, 1971).

(b) Secure high-altitude communication in the valleys of the $O_2$-MS with special channel characteristics (balance between transmission and shielding, phase-dispersive pass-band, phase keying, etc.).

(c) Restricted range, variable range, and local distribution broadband communication systems (e.g., Murray, 1971).
(d) Aircraft collision avoidance by means of ≈60 GHz beacons and "threshold" receivers (M. L. Meeks, private communication, 1969).

(e) Restrictions of radar range (Blake, 1972).

(f) Occultation experiments between orbiting satellites for studies of atmospheric structures.

(g) Absolute differential phase measurements between frequency pairs on either side of the O₂-MS over line-of-sight paths for correction of radio distance (MITRE, 1965; Thompson, 1968; see Section 3.1.1).

More applications will almost certainly follow if the basics are better known.

Active systems may eliminate desirable molecular properties (e.g., shielding) when operating at extremely high power densities. A very rough estimate indicates that, for the onset of saturation at an isolated line center, the power density has to exceed $10^{-2} \text{W/cm}^2\text{torr}^2$. The nonlinear power saturation effects, however, are not treated any further in this report.

The following applications are based on frequency-selective emission originating from the O₂-MS:

(a) Remote sensing of atmospheric temperature structures (Meeks and Lilley, 1963; Lenoir, 1968; Wilheit, 1969; Waters, 1970; Westwater, 1970). Ground-based stratospheric temperature soundings (Waters, 1973) and the NIMBUS-5 (launched Dec. 1972) satellite (Sabatini, 1972), both operate radiometers within the O₂-MS band. The conversion from measured radio noise to meteorological quantities requires accurate knowledge of oxygen absorption. This application in particular has fostered theoretical and experimental work on the O₂-MS (see Sect. 3.2.4).
(b) Stratospheric navigation aids such as velocity and altitude indicators (Mardon, 1969).

(c) Horizon sensor for satellite navigation (Guidice, 1971; Grauling, 1972).

(d) Radiometric detection of strong temperature gradients in the atmosphere for the identification of clear air turbulences (Haroules and Brown, 1969).

(e) Earth magnetic field strength measurements from satellites by means of the normal Zeeman effect (Sect. 2.3.3 and 3.1.1).

(f) Line emission (e.g. 119 GHz) used as a tracer in biological research, for example, to study oxygen exchange rates.

Radio astronomy efforts which are searching for millimeter wavelength emissions from extraterrestrial sources are interested in accurately defining the boundaries of the atmospheric $O_2$-MS, which masks their useful frequency bands.

This report shall provide baseline data on unique atmospheric transfer properties for a largely unused portion of the spectrum.
Figure 1. Millimeter-wave attenuation in the 49 to 72 GHz band due to atmospheric oxygen resonances for zenith paths through the U.S. Standard Atmosphere 1962 from different initial heights \( h_1 \) to outer space (see also Section 3.2). Allocated band utilizations are: A(1-5) - active telecommunications; P(1-3) - passive systems only (I.T.U., 1971; Klass, 1971).
2. THEORY

2.1 Atmospheric Transfer Function

Signal analysis of millimeter waves propagating through a well mixed atmosphere is facilitated by treating the medium as a linear system and applying the transfer function concept, (Morgan and Ekdahl, 1966). The central topic is the development of a microwave transfer function for moist air as reliably as can be at the present state of knowledge. To accomplish this, one has to track down macroscopic properties of the propagation medium to their molecular origin and be confronted with a combination of classical and quantum mechanical theories which describe the interaction between radiation and gas molecules. A rigorous test of these theories is still wanting since it can not be provided by the small amount of inaccurate and sometimes conflicting experimental data. The complex transfer function is defined by

\[ T = \frac{E}{E_0} = \exp \left[ -\left( \frac{\alpha}{20 \log_e} + j\phi \right) L \right] = \exp \left[ j \frac{2\pi \nu L \pi}{c} \right] \]  

(1)

where \( E_0 \) is the initial amplitude and \( E \) is the amplitude and phase received after the geometric path length \( L \) for free space propagation; \( j = \sqrt{-1} \). Power attenuation rate \( \alpha \) and phase-delay rate \( \phi \) are the well known propagation parameters

\[ \alpha = (10 \log e) \left( \frac{4\pi \nu}{c} \right) \pi \quad [\text{dB/km}] \]

\[ \phi = (2\pi \nu/c) \pi \quad [\text{rad/km}] \]

(2)

where \( \nu \) is the microwave frequency, \( c \) is the speed of light, and \( 10 \log e \approx 4.34294 \). The dimensionless macroscopic measure of the interaction between radiation and gas molecules is a complex refractive index.
\textbf{Equation (2a)}

\[ n = n' - jn'' , \]

where

\[ n' = n_o + \Delta n(\nu) \] is the Refraction Spectrum and \[ n'' (\nu) \] is the Extinction Spectrum.

Both spectra are interrelated by Kramers-Kronig integral equations, which are obeyed by the shape functions chosen (Eqs. 17, 23, 28, Table 8). This report is concerned with theoretical and analytical evaluations of the refractive index \( \overline{n} \) for air over the frequency band, \( \nu \approx 40 \) to 140 GHz. The environmental conditions to be considered for altitudes \( h = 0 \) to 100 km are:

- **Dry air pressure** \( p(h) \) [see below]
- **Mixing ratio of dry air components** \( r_k = \text{const.} \) [see Table 1]
- **Temperature** \( T(h) \) [330 to 1800K]
- **Water Vapor pressure** \( p_w(T, h) \) [0 to 50 torr\(^*\)]
- **Earth's magnetic field strength** \( H(h \text{ and geo-coord.}) \) [0.2 to 0.7 gauss\(^*\)]

The field strength \( H \) decreases between \( h = 0 \) and 100 km by 4.8 percent.

**Pressure changes versus altitude** as shown (U. S. Std. Atm. 62):

\[
\begin{array}{c|cccccc}
\text{h [km]} & 0 & 100 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 100 \\
p [\text{torr}\text{*}] & 760 & 199 & 41.5 & 9.0 & 2.2 & 600 & 170 & 41 & 7.8 & 0.23 \\
\end{array}
\]

\(*\) The units [torr] and [gauss] do not comply with recommended S. I. units. The conversions are 1 torr \( \equiv 133.322 \) pascal \((N/\text{m}^2)\) and 1 gauss \( \equiv 10^{-4} \) Wb/\text{m}^2.
The solution of equations (1) and (2) leads to the extinction and refraction spectra of oxygen and water vapor being defined by

\[ n' = n_o + \sum_i S_i F'_i + \Delta n_n + \Delta n_w \]

and by

\[ n'' = \sum_i S_i F''_i + n''_n + n''_w \]  \hspace{1cm} (4)

where

- \(n_o\) is the frequency-independent refraction of air (see Sect. 2.2);
- \(i\) is the label for lines of the \(O_2\) Microwave Spectrum (see Sect. 2.3.1);
- \(S_i\) are the individual line strengths (see Sect. 2.3.2 and 3);
- \(F'_i, F''_i\) are the shape functions (real and imaginary parts) for individual lines (see Sect. 2.4);
- \(n''_n\) and \(\Delta n_n\) indicates the nonresonant oxygen spectrum (see Sect. 2.4.2); and
- \(n''_w\) and \(\Delta n_w\) indicates contributions due to water vapor (see Sect. 2.5).

### 2.2 Frequency-Independent Phase Delay

The non-dispersive phase delay of a millimeter wave propagating through the gaseous atmosphere is commonly expressed in \(N\)-units,

\[ N = (n_o - 1) \times 10^6 = \left[ pR^o_d + p_w \cdot R^o_w (T) \right] / T, \hspace{1cm} \text{[ppm]} \]  \hspace{1cm} (5)

The corresponding phase delay in engineering terms is

\[ \phi_o = (2\pi v / c) \cdot n_o, \hspace{1cm} \text{[rad/km]} \]

*Pressure proportionality is indicated in general for all appropriate parameters by a superscript \(\circ\).*
Table 1. Relative Concentration \( r \) of Dry Air Components (near sea level) and Refractivity \( R^0 \) Measured for Different Air Components at \( \nu = 61156 \) MHz, \( p \leq 20 \) torr, \( T = 280^\circ \)K. (Liebe and Welch, 1972).

<table>
<thead>
<tr>
<th>( r ) Component</th>
<th>( r ) ppm by vol</th>
<th>( R^0(280) ) ppm/torr</th>
<th>( R^0/T ) ppm(^{0\text{K}})/torr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Oxygen ( O_2 )</td>
<td>209 460(^{(a)})</td>
<td>0.3427</td>
<td>95.95</td>
</tr>
<tr>
<td>2 Nitrogen ( N_2 )</td>
<td>780 840</td>
<td>0.3771</td>
<td>105.60</td>
</tr>
<tr>
<td>3 Argon ( Ar )</td>
<td>9 340</td>
<td>0.3562(^*)</td>
<td>99.73(^*)</td>
</tr>
<tr>
<td>4 Carbon Dioxide ( CO_2 )</td>
<td>314</td>
<td>0.6356</td>
<td>178.0</td>
</tr>
<tr>
<td>5 Neon ( Ne )</td>
<td>18.2</td>
<td>0.08659</td>
<td>24.25</td>
</tr>
<tr>
<td>6 Helium ( He )</td>
<td>5.2</td>
<td>0.0450</td>
<td>12.6</td>
</tr>
<tr>
<td>7 Methane ( CH_4 )</td>
<td>2</td>
<td>0.5672</td>
<td>158.8</td>
</tr>
<tr>
<td>8 Krypton ( Kr )</td>
<td>1.1</td>
<td>0.5428</td>
<td>152.0</td>
</tr>
<tr>
<td>9 Nitrous Oxide ( N_2O )</td>
<td>0.5</td>
<td>0.7138</td>
<td>199.9</td>
</tr>
<tr>
<td>Dry Air</td>
<td>10(^6)</td>
<td>0.3696</td>
<td>103.54</td>
</tr>
<tr>
<td>Water Vapor ( H_2O )</td>
<td>variable</td>
<td>6.712</td>
<td>1879(280(^0\text{K}))</td>
</tr>
</tbody>
</table>

\(^{(a)}\) Machta and Hughes (1970).

\(^*\) Reference value; all measured \( R^0 \) are relative to the \( R^0 \) (Ar) - value given by Newell and Baird (1965).
The refractivity values for dry air and water vapor are

\[ R_d^o = \sum_k r_k R_k^o = 103.5 \text{ [ppm}^\circ \text{K/torr]} \]

and

\[ R_w^o = 95.5 + 499500/T \text{ [ppm}^\circ \text{K/torr]} \),

which were measured for the air components at 61.156 GHz as listed in Table 1. Similar results have been reported by Boudouris (1963) and Newell and Baird (1965).

2.3 The Microwave Spectrum of Oxygen

The following simplified explanation can be given as to the nature of the \( \text{O}_2 \)-MS. The magnetic dipole moment of the \( \text{O}_2 \) molecule couples to the microwave radiation field which excites quantized changes (molecular resonances) between fine structure levels of the \( \text{O}_2 \) rotational energy states. At these resonances the molecules momentarily store (phase dispersion), absorb (attenuation), and randomly re-radiate (incoherent noise) microwave energy.

Solutions to problems of atmospheric millimeter wave transmission (Eq. 4) require detailed knowledge of the \( \text{O}_2 \) Microwave Spectrum. Parameters of the \( \text{O}_2 \)-MS which originate from properties of "isolated" molecules are the line center frequencies \( \nu_o \) and the strength values \( S \).

2.3.1 Line Center Frequencies of the \( \text{O}_2 \)-Microwave Spectrum

The \( \text{O}_2 \) fine structure microwave spectrum obeys the selection rule: \( \Delta N = 0, \Delta J = \pm 1 \) (\( N, J \) are the rotational and total angular momentum quantum numbers), and the transition frequencies \( \nu_i \) can be calculated by means of (Welch and Mizushima, 1972)
\[
\nu_0 (N^\pm) = \lambda_0 + \frac{1}{2}\mu_o - 4\mu_1 + (\lambda_1 + \frac{5}{2}\mu_1) (a^2 + a + 2)
\]
\[
\pm \{ f(a) - (2a + 1) [ \beta(a) + \frac{2}{3}\lambda_1 - \mu_1] \} ,
\]

where \( a = N \pm 1, \quad N = 1, 3, \ldots \) (only odd values are allowed),

\[
f(a) = \sqrt{C^2 + a(a + 1)D^2},
\]

\[
C = (2a + 1) [\beta(a) - \frac{1}{2}\mu_o - \frac{1}{2}\mu_1 (a^2 + a + 4)]
\]
\[
- [\lambda_o + \frac{1}{3}\lambda_1 (7a^2 + 7a + 4)] / (2a + 1) ,
\]

\[
D = 2[\lambda_o + \lambda_1 (a^2 + a + 1)] / (2a + 1) ,
\]

\[
\beta(a) = B_0 + 2B_1 (a^2 + a + 1) + B_2 (3a^4 + 6a^3 + 13a^2 + 10a + 4) .
\]

The line designation \( N^\pm \) denotes the transitions \( J = N \pm 1 \rightarrow J' = N. \)

The most recent values of the molecular constants are given by Welch and Mizushima (1972) and they are in megahertz:

**Rotational:**
\[
B_0 = 43100.518 \ (3), \quad B_1 = -0.14492 \ (9),
\]
\[
B_2 = 1.57 \ (11) \cdot 10^{-7}.
\]

**Spin-spin coupling:**
\[
\lambda_o = 59501.342 \ (7),
\]
\[
\lambda_1 = 5.847 \ (3) \cdot 10^{-2}.
\]

**Spin-rotational coupling:**
\[
\mu_o = -252.5865 \ (10),
\]
\[
\mu_1 = -2.464 \ (20) \cdot 10^{-4}.
\]
For all atmospheric transmission problems, it is sufficient to consider \( i = 44 \) individual lines. The known values of \( v_0^i \) are given in Table 2. Most of the center frequencies have been measured with high accuracy, and the few missing ones are predicted by equation (6). These predictions, for example, guided atmospheric emission measurements by Waters (1973) whereby the agreement was excellent (see Table 2).

### 2.3.2 Line Strengths of the O\(_2\)-Microwave Spectrum

For practical applications it is helpful to reduce molecular line shape theory and define a line strength parameter \( S \) (Liebe, 1969a) as a multiplier to a Lorentzian shape function (Sect. 2.4.1). Pressure and temperature dependences of the strength for an individual line of the O\(_2\)-MS are given by

\[
S = S_i^o \frac{p}{r_1} \xi, \quad [\text{Hz}]
\]

where \( r_1 = 0.20946 \) (Table 1), and \( \xi = 0.9976 \), portion* of molecules of mass \(^{16}\text{O}_2\).

The strength parameter \( S_i^o \) was derived from the equation describing the transition probabilities. Assuming for the magnetic field strength \( H = 0 \) (\( H \neq 0 \), see Section 2.3.3) leads to the expression (VanVleck, 1947; Morgan and Ekdahl, 1966; LeFande, 1968; Wilheit, 1969)

\[
S_i^o = \nu_0^i \cdot K \cdot f(N) \cdot f(T), \quad [\text{Hz/torr}]
\]

where \( K = 69.50125 \times 10^{-6} \cdot k(J) \left[ \text{K}^3/\text{torr} \right] \) and \( k(J) \) is given in Table 3 and discussed below.

*The isotopic abundance of \(^{17}\text{O}_2\) and \(^{18}\text{O}_2\) are 0.037 and 0.204 percent respectively.
The intensity functions (Matrix elements) are

\[
f(N^+) = \frac{2N^2 + 3N}{N+1}
\]
\[
f(N^-) = \frac{2N^2 + N - 1}{N}.
\]

The temperature function is in units of [1/ °K^3]

\[
f(T) = \left\{ T^{-3} \exp\left[-2.06858 \cdot N(N+1)/T\right] \cdot \left[1 - (\hbar\nu_i / 2kT)\right] \right\}. 
\]

The error of the approximation \[1 - (\hbar\nu_i / 2kT)\] ≈ 1 is less than 1 percent for all values of interest (see Fig. 3).

The approximation made in calculating the matrix element of the magnetic dipole moment operator was that \(O_2^+\) is a pure Hund's case "b" molecule. The fact that this is not exactly true was accounted for by a correction factor \(k(J)\), which is a mixing coefficient quantifying the purity of a particular state (Zimmerer and Mizushima, 1961; Wilheit, 1969). The \(k(J)\) values given in Table 3 were obtained by diagonalizing the Hamiltonian matrix with the appropriate wave functions. Equation (8) was applied to compute \(S_i^0\) at \(T = 300°K\) up to \(N^\pm = 43\), and the results are listed in Table 3. Figure 2 displays the strength value distribution versus the frequency positions \(\nu_i^0\) for 200° and 300°K.

The temperature dependence of the strength parameter can be expressed relative to the reference temperature, \(T_0 = 300K\), by

\[
S_i^0(T) = S_i^0(300) \cdot \mathcal{J}(N),
\]

with

\[
\mathcal{J}(N) = (300/T)^3 \cdot \exp\left\{-6.89526 \cdot 10^{-3} \cdot N(N+1)((300/T)-1)\right\}. 
\]

The temperature function can be approximated for a small temperature interval \((\Delta T \lesssim 40 °K)\) by

\[
\mathcal{J} \approx (T_0/T)^w(N).
\]
Each N-pair of lines possesses its own temperature dependence $\mathcal{J}(N)$. The values of $\mathcal{J}$ are given in graphical form in Figure 2 for atmospheric temperatures, and numerical values are listed in Table 4 which also shows temperature exponents $w(N)$ of equation (13) for $T_0 \approx 280 \pm 20^\circ K$. The temperature function $\mathcal{J}$ indicates the population of energy states relative to 300$^\circ$K. At low temperatures many of the lines with high $N$-numbers may be neglected.

2.3.3 Zeeman-Splitting of the Lines in the $O_2$ Microwave Spectrum

The $O_2$-MS is greatly complicated by Zeeman-splitting of each fine-structure line into many components under the influence of a permanent magnetic field such as the earth's. The Zeeman effect, which originates from the interaction of the molecular magnetic moment with a static magnetic field of strength $H$, removes the spatial $M$-degeneracy of the $J$ levels, splitting each level into $(2J+1)$ sub-levels ($M$-magnetic quantum number). The complete Zeeman Hamiltonian for $O_2$ was given by Tinkham and Strandberg (1954). In the weak field case ($H < 1$ gauss), however, it is sufficient to retain only the term involving the interaction of the molecular electronic spin moment with the field. Matrix elements of the spin moment, evaluated in the Hund's (case "b") representation, are given by Evenson et al. (1968) and may be used in a perturbation calculation to obtain the Zeeman transition frequencies.

The selection rule on $\Delta M$ (change in magnetic quantum number) for Zeeman transitions is derived from assumptions about the orientation between $H$ and the microwave magnetic field component $\mathcal{E}_\phi$ perpendicular to the direction of propagation. By a judicious choice of the orientation between $H$ and $\mathcal{E}_\phi$, one can selectively excite two different types of Zeeman transitions:
(1) Linearly polarized radiation "induces"

\[ \Delta M = 0, \text{ called } \pi \text{ transitions when } H \text{ and } \varphi \text{ are perpendicular,} \]
\[ \Delta M = \pm 1, \text{ called } \sigma \text{ transitions when } H \text{ and } \varphi \text{ are parallel, and} \]
in both cases \( H \) is perpendicular to the propagation direction.

(2) Circular polarized radiation "induces"

\[ \Delta M = +1 \text{ or } \sigma^+ \text{ transitions for right-handed polarization,} \]
\[ \Delta M = -1 \text{ or } \sigma^- \text{ transitions for left-handed polarization, and} \]
in both cases \( H \) is parallel to the direction of propagation.

The simplest pattern, the normal Zeeman effect, is exhibited by the \( 1^+ \) and \( 1^- \) lines (see Table 5), each splitting into three components whereby only the two \( \sigma \) components shift away from the center (Hill and Gordy, 1953). The anomalous Zeeman effect splits the lines \( N = 3^+, 5^-, \ldots \) into \( 3(2J^* + 1) \) components where \( J^* \) is the smaller of the two \( J \) values involved in the transition.

The frequency of each Zeeman component is determined by

\[ \nu^Z_0 = \nu^0_o + \eta(M, N) \cdot ZH \quad [MHz] \quad (14) \]

The shift is proportional to the external field strength \( H \), and \( \eta \) (Table 5) is less than one. The factor of proportionality for \(^{16}O_2\) microwave lines is \( Z = 2.8026 \quad [MHz/\text{gauss}] \). The earth's magnetic field strength, \( H = 0.2 \) to 0.7 gauss, produces frequency shifts for the Zeeman components of up to 2 MHz.

The line strength of each Zeeman component is given by

\[ S^o_z = \nu^Z_0 \cdot K \cdot f(M, N) \cdot f(T) \quad [Hz/\text{torr}] \quad (15) \]

analogous to equation (8). The intensity function \( f(M, N) \) is listed for the different Zeeman components in Table 5. Summing over polarizations and over all \( M \)-degeneracies of the final state \( J' \) yields \( f(N) \).

Examples for atmospheric conditions are discussed in Section 3.1.1.
Table 2. Observed and Calculated Line Center Frequencies of the \(^{16}\text{O}_2\) Microwave Spectrum, in MHz.

<table>
<thead>
<tr>
<th>Line</th>
<th>Observed</th>
<th>Ref.</th>
<th>Calculated (Eq. 6)^(\text{a}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1(^+)</td>
<td>56264.766(20) a</td>
<td>56264.758</td>
</tr>
<tr>
<td></td>
<td></td>
<td>56264.778(10) b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3(^+)</td>
<td>58446.580(10) c</td>
<td>58446.580</td>
</tr>
<tr>
<td></td>
<td></td>
<td>58446.600(10) b</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5(^+)</td>
<td>59590.978(10) c</td>
<td>59590.979</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60434.776(10) c</td>
<td>60434.778</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60434.788(10) f</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9(^+)</td>
<td>61150.570(10) c</td>
<td>61150.567</td>
</tr>
<tr>
<td></td>
<td></td>
<td>61150.565(5) f</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>11(^+)</td>
<td>61800.169(20) a</td>
<td>61800.167</td>
</tr>
<tr>
<td></td>
<td></td>
<td>61800.155(10) c</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>15(^+)</td>
<td>62411.223(10) c</td>
<td>62411.234</td>
</tr>
<tr>
<td></td>
<td></td>
<td>62997.999(2) d</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>17(^+)</td>
<td>63568.520(10) c</td>
<td>63568.542</td>
</tr>
<tr>
<td></td>
<td></td>
<td>64127.777(20) a</td>
<td>64127.790</td>
</tr>
<tr>
<td></td>
<td>21(^+)</td>
<td>64678.2(2) d</td>
<td>64678.920</td>
</tr>
<tr>
<td></td>
<td></td>
<td>65224.12(20) c</td>
<td>65224.076</td>
</tr>
<tr>
<td>25</td>
<td>25(^+)</td>
<td>65764.744(20) a</td>
<td>65764.760</td>
</tr>
<tr>
<td></td>
<td></td>
<td>66302.06(2)</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>29(^+)</td>
<td>66836.77(30) a</td>
<td>66836.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>67369.51(31) a</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>31(^+)</td>
<td>67900.73(33) a</td>
<td>67900.73</td>
</tr>
<tr>
<td>35</td>
<td>35(^+)</td>
<td>68430.8(35) a</td>
<td>68430.8</td>
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<td></td>
<td></td>
<td>68960.1(37) a</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>37(^+)</td>
<td>69488.7(40) a</td>
<td>69488.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70016.9(41) a</td>
<td></td>
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<td>39</td>
<td>39(^+)</td>
<td>70544.9(44) a</td>
<td>70544.9</td>
</tr>
</tbody>
</table>

\(^{-}\) Uncertainty of the last digits given

\(^*)\) See also Welch and Mizushima (1972)

References:
- a. West and Mizushima (1966)
- b. McKnight and Gordy (1968)
- c. Zimmerer and Mizushima (1961)
- d. Mizushima and Hill (1954)
- e. Waters (1973)
- f. Liebe and Welch (1972)
Table 3. Line Strength $S'_1$ (300) at $T = 300^\circ$K (Eq. 8) and Strength Correction $k(J)$ for the $^{16}\text{O}_2$ Microwave Spectrum.

<table>
<thead>
<tr>
<th></th>
<th>Line $N^+$</th>
<th>$k(J)$</th>
<th>$S'_1$ (300)</th>
<th></th>
<th>Line $N^-$</th>
<th>$k(J)$</th>
<th>$S'_1$ (300)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1$^*$</td>
<td>0.9808</td>
<td>0.348 69</td>
<td>2</td>
<td>1$^-$</td>
<td>1</td>
<td>0.597 25</td>
</tr>
<tr>
<td></td>
<td>3$^+$</td>
<td>0.9941</td>
<td>0.925 05</td>
<td>3</td>
<td>3$^-$</td>
<td>0.9808</td>
<td>0.963 36</td>
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<td>5$^+$</td>
<td>0.9972</td>
<td>1.341 02</td>
<td>5</td>
<td>5$^-$</td>
<td>0.9941</td>
<td>1.348 70</td>
</tr>
<tr>
<td></td>
<td>7$^+$</td>
<td>0.9984</td>
<td>1.562 63</td>
<td>7</td>
<td>7$^-$</td>
<td>0.9972</td>
<td>1.526 31</td>
</tr>
<tr>
<td></td>
<td>9$^+$</td>
<td>0.9989</td>
<td>1.589 94</td>
<td>9</td>
<td>9$^-$</td>
<td>0.9984</td>
<td>1.515 04</td>
</tr>
<tr>
<td>11</td>
<td>11$^+$</td>
<td>0.9992</td>
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<td>11$^-$</td>
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</tr>
<tr>
<td></td>
<td>13$^+$</td>
<td>0.9997</td>
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<td>13$^-$</td>
<td>0.9992</td>
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<td>17</td>
<td>17$^-$</td>
<td>1</td>
<td>0.605 61</td>
</tr>
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<td></td>
<td>19$^+$</td>
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<td></td>
<td>19</td>
<td>19$^-$</td>
<td>1</td>
<td>0.401 19</td>
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<td></td>
<td>21$^+$</td>
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<td>1</td>
<td>0.248 87</td>
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<td>23$^+$</td>
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<td>31$^-$</td>
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<td>33$^-$</td>
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<td>35$^-$</td>
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<td>0.001 564</td>
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<td>37</td>
<td>37$^-$</td>
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<td>43</td>
<td>43$^-$</td>
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<td>0.000 024</td>
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</table>
Figure 2. Line strength parameter for the \(\text{O}_2\)-MS at 200°K and 300°K. Numerical values at 300°K are listed in Table 3.
Table 4. Temperature Dependence of the Line Strength $S^*(300)$ of the $O_2$ Microwave Spectrum

<table>
<thead>
<tr>
<th>LINE $N^\pm$</th>
<th>$180^\circ$K</th>
<th>$200^\circ$K</th>
<th>$220^\circ$K</th>
<th>$240^\circ$K</th>
<th>$260^\circ$K</th>
<th>$280^\circ$K</th>
<th>$300^\circ$K</th>
<th>$320^\circ$K</th>
<th>$340^\circ$K</th>
<th>$w$ $(280^\pm 20^\circ$K)</th>
</tr>
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<td>1</td>
<td>4.5873</td>
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<td>1.9464</td>
<td>1.5329</td>
<td>1.2267</td>
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<td>0.8247</td>
<td>0.6881</td>
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<td>2.4605</td>
<td>1.9131</td>
<td>1.5168</td>
<td>1.2227</td>
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<td>1.4881</td>
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<td>1.0</td>
<td>0.8347</td>
<td>0.7039</td>
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<td>1.0</td>
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<td>0.7962</td>
<td>1.65</td>
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<td>1.0</td>
<td>0.9138</td>
<td>0.8346</td>
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<td>0.7962</td>
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<td>0.9410</td>
<td>0.9796</td>
<td>1.0</td>
<td>1.0055</td>
<td>0.9993</td>
<td>-0.42</td>
</tr>
<tr>
<td>23</td>
<td>0.3661</td>
<td>0.5032</td>
<td>0.6353</td>
<td>0.7542</td>
<td>0.8553</td>
<td>0.9372</td>
<td>1.0</td>
<td>1.0453</td>
<td>1.0750</td>
<td>-1.1</td>
</tr>
<tr>
<td>25</td>
<td>0.2333</td>
<td>0.3590</td>
<td>0.4969</td>
<td>0.6370</td>
<td>0.7709</td>
<td>0.8930</td>
<td>1.0</td>
<td>1.0904</td>
<td>1.1639</td>
<td>-1.8</td>
</tr>
<tr>
<td>27</td>
<td>0.1433</td>
<td>0.2491</td>
<td>0.3809</td>
<td>0.5306</td>
<td>0.6889</td>
<td>0.8476</td>
<td>1.0</td>
<td>1.1413</td>
<td>1.2684</td>
<td>-2.6</td>
</tr>
<tr>
<td>29</td>
<td>0.0849</td>
<td>0.1681</td>
<td>0.2862</td>
<td>0.4359</td>
<td>0.6104</td>
<td>0.8013</td>
<td>1.0</td>
<td>1.1988</td>
<td>1.3913</td>
<td>-3.5</td>
</tr>
<tr>
<td>31</td>
<td>0.0484</td>
<td>0.1104</td>
<td>0.2108</td>
<td>0.3532</td>
<td>0.5363</td>
<td>0.7546</td>
<td>1.0</td>
<td>1.2635</td>
<td>1.5361</td>
<td>-4.4</td>
</tr>
<tr>
<td>33</td>
<td>0.0266</td>
<td>0.0705</td>
<td>0.1522</td>
<td>0.2823</td>
<td>0.4672</td>
<td>0.7078</td>
<td>1.0</td>
<td>1.3363</td>
<td>1.7069</td>
<td>-5.3</td>
</tr>
<tr>
<td>35</td>
<td>0.0141</td>
<td>0.0438</td>
<td>0.1077</td>
<td>0.2226</td>
<td>0.4036</td>
<td>0.6613</td>
<td>1.0</td>
<td>1.4182</td>
<td>1.9091</td>
<td>-6.3</td>
</tr>
<tr>
<td>37</td>
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<td>0.0265</td>
<td>0.0747</td>
<td>0.1730</td>
<td>0.3457</td>
<td>0.6154</td>
<td>1.0</td>
<td>1.5103</td>
<td>2.1491</td>
<td>-7.4</td>
</tr>
<tr>
<td>39</td>
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<td>0.0156</td>
<td>0.0507</td>
<td>0.1327</td>
<td>0.2936</td>
<td>0.5704</td>
<td>1.0</td>
<td>1.6139</td>
<td>2.4351</td>
<td>-8.6</td>
</tr>
<tr>
<td>41</td>
<td>0.0017</td>
<td>0.0089</td>
<td>0.0338</td>
<td>0.1004</td>
<td>0.2472</td>
<td>0.5267</td>
<td>1.0</td>
<td>1.7306</td>
<td>2.7771</td>
<td>-9.8</td>
</tr>
<tr>
<td>43</td>
<td>0.0008</td>
<td>0.0050</td>
<td>0.0221</td>
<td>0.0749</td>
<td>0.2064</td>
<td>0.4844</td>
<td>1.0</td>
<td>1.8622</td>
<td>3.1877</td>
<td>-11</td>
</tr>
</tbody>
</table>
Figure 3. Temperature dependence $\mathcal{J}$ of the line strengths $S_i^0$ (300). Numerical values are listed in Table 4.
Table 5. Frequency Shift \( \eta \) and Intensity Functions \( f(M', N) \) for Zeeman-Components of \( O_2 \) Microwave Spectrum Lines (Eqs. 14, 15).

Initial State: \( N, J, |M| \leq J \)  
Final State: \( N, J', |M'| \leq J' \)  
Selection Rule: \( \Delta M = M' - M \)

<table>
<thead>
<tr>
<th>Zeeman Transitions</th>
<th>Frequency Shift ( \eta ) (M,N)</th>
<th>Intensity Functions ( f(M',N) ) (Matrix Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma^+ ) ( (\Delta M = 1) )</td>
<td>( \frac{M(1 - N) + 1}{N(N + 1)} )</td>
<td>( N^+ ) - Line</td>
</tr>
<tr>
<td>( \pi ) ( (\Delta M = 0) )</td>
<td>( \frac{M(1 - N)}{N(N + 1)} )</td>
<td>( N^+ ) - Line</td>
</tr>
<tr>
<td>( \sigma^- ) ( (\Delta M = -1) )</td>
<td>( \frac{M(1 - N) - 1}{N(N + 1)} )</td>
<td>( N^+ ) - Line</td>
</tr>
</tbody>
</table>

After Hill and Gordy (1954)  
After Lenoir (1968)

Example: \( N = 1 \)

<table>
<thead>
<tr>
<th>( \eta(M,N) )</th>
<th></th>
<th>( f(M',N) )</th>
<th></th>
<th>( f(N) )</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>( \sigma^+ )</td>
<td>( \pi )</td>
<td>( \sigma^- )</td>
<td>( M' )</td>
<td>( \sigma^+ )</td>
</tr>
<tr>
<td>1* Line</td>
<td>+1</td>
<td>0.5</td>
<td>0.5</td>
<td>+1</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>0.375</td>
</tr>
<tr>
<td>( J = 2 \rightarrow J' = 1 )</td>
<td>-1</td>
<td>0.5</td>
<td>0.5</td>
<td>-1</td>
<td>0.125</td>
</tr>
<tr>
<td>1* Line</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>( J = 0 \rightarrow J' = 1 )</td>
<td>-1</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Eq. 8)
2.4 **Line Shapes of the Atmospheric O\textsubscript{2} Microwave Spectrum**

The first complete theory of the atmospheric O\textsubscript{2}-MS was given by Van Vleck (1947). As specific applications based on absorption (emission) properties of the O\textsubscript{2}-MS evolved, the need for more detailed spectroscopic information was developed. Especially the works of Meeks and Lilley (1963), Lenoir (1968), Wilheit (1969), Waters (1970), and Westwater (1970) expanded and refined the theory of the atmospheric O\textsubscript{2}-MS for purposes of remote sensing atmospheric temperature structures.

The molecular theory of the pressure-broadened O\textsubscript{2}-MS was advanced by Dillon and Godfrey (1969, 1972) and Mingelgrin (1972). Both treatments are based on an intermolecular potential model for O\textsubscript{2}-O\textsubscript{2}, O\textsubscript{2}-N\textsubscript{2}, O\textsubscript{2}-Ar interactions, and scattering calculations of the trajectories for rotational and translational motions. Dillon-Godfrey treat the rotational trajectories quantum-mechanically up to N = 7. Mingelgrin solves both trajectories classically up to N = 23, however, the approximate nature of this assumption is expected to yield width parameters which are slightly larger than the true ones (R. Gordon, private communication, 1973).

Measured high-pressure (3 to 40 ktorr) absorption rates of pure O\textsubscript{2} and O\textsubscript{2}-Ar, O\textsubscript{2}-N\textsubscript{2} mixtures over the 48 to 81 GHz range (Mingelgrin et al., 1972) are reproduced by the theoretical calculations. Implications of both theories to the atmospheric O\textsubscript{2}-MS are discussed in Section 2.4.1.

In treating the intensity distribution (line shapes $F'$, $F''$) of the O\textsubscript{2}-MS it is assumed that the microwave power flux density is sufficiently low that nonlinear saturation effects are avoided. As a result, the integrated area of each absorption line is proportional to the number of possible transitions (numbers of molecules)\(^*\) or, on a macroscopic

\(^*\)The number of molecules per cm\(^3\) equals $9.662 \times 10^{18} \cdot \frac{p}{T}$ (ideal gas law).
scale, to pressure. The line strengths $S_1$ (Sect. 2.3.2) have been defined as scale factors to the area $\pi \cdot n'' \cdot \gamma$ (Eqs. 18, 19), which is that of a Lorentzian shape (Fig. 4). While the theory of spectral properties relating to the $O_2$ molecule is well understood, this is not the case for the intensity distribution, especially when considering the full range of conditions specified by equation (3). The atmospheric $O_2$-MS for pressures above 1 torr is broadened by binary collisions (Sect. 2.4.1); for pressure below 1 torr, the gradual transition from collision-to Doppler-broadening is described by a Voigt profile (Sect. 2.4.3).

2.4.1 Pressure Broadening of the $O_2$ Microwave Spectrum

In the simple case of a single, pressure-broadened line, theory predicts the Lorentzian line shape to be valid when $\gamma << \nu_o$ (e.g., Dillon, 1969). The frequency, normalized to a multiple of the width $\gamma$, is

$$z = (\nu_o - \nu) / \gamma$$

yielding the shape factors in the following form:

<table>
<thead>
<tr>
<th>Extinction</th>
<th>Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F'' = 1 / \gamma (1 + z^2)$</td>
<td>$F' = z / \gamma (1 + z^2)$. [1/Hz]</td>
</tr>
</tbody>
</table>

The approximations for the wing regions ($z > 1$) are

$$F'' \approx \gamma / (\nu_o - \nu)^2$$

and

$$F' \approx 1 / (\nu_o - \nu).$$

The peak extinction, the peak dispersion

for $z = 0 (\nu = \nu_o)$ is

$$n''_o = S / \gamma$$

for $z = 1 (\nu = \nu_o \pm \gamma)$ is

$$\Delta n_o = S / 2\gamma$$

(17a)

This yields the two line profiles in the form

$$n'' = n''_o / (1 + z^2)$$

and

$$\Delta n = \Delta n_o 2z / (1 + z^2). [1]$$

(18)
Figure 4 gives a graphical presentation of the equations above. The linewidth $\gamma$ can be expressed for atmospheric air by (Liebe, 1969a)

$$\gamma = \gamma^*(300) \left\{ m_d(300/T)^u \cdot p + m_w(300/T)^v \cdot p_w \right\} [\text{MHz}] \quad (19)$$

Table 6 shows the large body of width parameters $\gamma^*(300)$, which exists for pure oxygen ($m_d = 1$). The other quantities of equation (19) are $m_d$ and $m_w$, the broadening efficiencies for dry air and water vapor, and $u$ and $v$, the associated temperature dependences. The arrangement of $\gamma^*(300)$ versus the quantum number $N$ in Figure 5 seems to support the theoretically predicted $N$-dependence (Dillon and Godfrey, 1972; Mingelgrin, 1972), although the experimental data (some are claimed to be accurate to $\pm 5\%$) are spread over a large range ($\pm 20\%$). The uncertainties of $\gamma$ are one of the largest sources of error in a reliable description of the atmospheric $O_2$-MS (Liebe, 1969b).

The isolated line is a good candidate for quantitative experimental work on the $O_2$-MS. From measured values of $\Delta n$ and/or $n''$ one needs to determine the quantities (Eqs. 18 and 19)

$$\nu_o = S^*(300) \cdot f(T) - \gamma^* - m_d, m_w, u, v \cdot$$

Profiles of $n''$ and $\Delta n$ close to a line center ($|\nu_o - \nu| \approx 10 \text{ MHz}$) and at low pressures ($< 1.5 \text{ torr}$) for different values of $H$ can establish the validity of equations (14) and (15). $O_2$-MS measurements are in progress in our group using dispersion and absorption pressure-scanning spectroscopy (Liebe et al., 1973).

So far, we have measured the $9^+$ line and obtained the following preliminary results (Liebe and Welch, 1972): (20)

(a) $\nu_o(9^+)$—the value is given in Table 2. A pressure-induced shift of $\nu_o$ was not detectable ($< \pm 10 \text{ KHz/ torr}$ for $p \leq 50 \text{ torr}$) which agrees with predictions (Dillon and Godfrey, 1972).
(b) So(300).J(T) - the agreement with calculated values (Tables 3 and 4) was within ±5 percent between 250 and 325°K. More careful work at 300°K yielded agreement within ±1 percent.

(c) F', F'' - the shape factors were verified as being Lorentzian. The pertinent width parameters were

\[ \gamma^\circ(300) = 1.81 \, \text{[MHz/torr]} \pm 2 \, \% \], \[ m_\text{d} = 0.93 \], \[ u = 0.87 \], \[ m_\text{w} = 1.25 \], and \[ v \approx 1 \] (assumed).

(d) The experimental \( \Delta n_0 \) versus \( |\nu_0 - \nu| \) profile at \( H = 0.53 \) [gauss] was in good agreement with "Zeeman" calculations. The responses of \( \Delta n \) and \( n'' \) at \( H = 0.53 \) [gauss] were undistinguishable from those measured for \( H \approx 0 \) for \( p > 1.5 \) torr and \( |\nu_0(9^+) - \nu| > 10 \) MHz.

(e) The comparison between air and oxygen dispersion yielded

\[ \left[ \frac{\Delta n_0(\text{Air})}{\Delta n_0(\text{O}_2)} \right] = 0.225 \] (\( = r_1/m_\text{d} \) according Eqs. 18, 19).

For atmospheric \( \text{O}_2 \)-MS calculations, we have arranged the relevant spectroscopic parameters in Table 7 by order of increasing frequency. The theoretical width values by Dillon-Godfrey (1972) are listed, but they have been scaled up by a factor 1.14 to match our experimental \( \gamma^\circ(9^+) \) value. A computer summation of contributions (Eq. 4) from 44 lines of the \( \text{O}_2 \)-MS for six different pressures revealed the spectra shown in figures 6 through 8. The calculation used the \( S^\circ(300) \) values of Table 7 and \( \gamma_1^\circ = 1.6 \, \text{MHz/torr} \). Pressures chosen are representative for \( h \approx 30 \) to 0 km. We conclude from these results that the atmospheric \( \text{O}_2 \)-MS needs to be discussed separately for three pressure ranges:

1 - Isolated Line Spectrum for \( p < 10 \) torr (Fig. 6).

II - Continuum Spectrum for \( p > 300 \) torr (Fig. 8),

where the envelope is determined by all lines.

I/II - Mixture of I and II (Fig. 7).
In range II, the addition of individual line responses (Beer-Lambert law) may not be a valid procedure since collisions do alter the energy levels to an extent that they can interfere with each other. Mingelgrin (1972) included overlap effects in his calculations, which are compared with the pressure-linear width model (Eq. 19) in Figure 9. A substantial narrowing can be noticed above 100 torr. For standard conditions (760 torr, 298°K) the difference in peak intensity at 60 GHz is as high as +28 percent. Overlap effects between N+ and N− lines have been computed by Dillon (1969) and found to be insignificant for standard conditions. These contradicting predictions need to be clarified through analysis of reliable experimental continuum data.

A necessary condition for overlap in the coalescence of extinction line shapes (Eq. 17) under the influence of pressure-broadening. Table 7 reveals 8 lines where the separation $c$ of the centers is only about 1/5 of the normal spacings. These doublets overlap with increasing pressure first. A second condition comes from pressure-broadening theory, which predicts different sensitivities of the energy levels to mutual interferences. Approximately, one can treat the individual linewidths as being no longer proportional to pressure and formulate

$$\gamma_c (\text{range II}) \approx \gamma_i^p - 5(c, p, N)$$

(21)

For example, the 760 torr attenuation profile by Mingelgrin (Fig. 9) can be fitted approximately by assuming the doublets a linewidth of one-half of the isolated width. Thus the doublets may be responsible for most of the overlap effects of the atmospheric continuum $O_2$-MS. Considering only overlap factors $δ$ for these line pairs would simplify computations of the continuum $O_2$ spectrum.

An empirical width model was introduced by Meeks and Lilley (1963) to reconcile atmospheric attenuation rates between the line
spectrum (I) and the continuum spectrum (II). Zenith attenuation data (see Table 10) have been fitted with this model whereby over the range of the continuum spectrum, $\gamma$ was reduced to $1/2$ (Carter et al., 1968) or even to $1/3$ (Reber, 1972). This will be treated further in Sections 3.1 and 2.

2.4.2 Nonresonant $O_2$-Spectrum

In a classical picture the molecular dipole is unable to follow completely the oscillating field. Thus, after every collision there is a preferred reorientation relative to the field. Formally, collisions induce transitions between identical energy levels under the selection rule, $\Delta J = 0$ and $\Delta N = 0$. As a consequence, increasing pressure ($p > 100$ torr) adds to the $O_2$-MS a non-resonant contribution which is centered at $v = 0$ Hz and can be described by a Debye-type polarization (VanVleck, 1947; Zhevakin and Naumov, 1967) which is

$$n'' = S_o F'' \quad \text{and} \quad \Delta n_n = S_o F'$$

The Debye shape factors are

$$F'' = 2 v \gamma_o / (v^2 + \gamma^2) \quad \text{and} \quad F' = [\gamma^2_o / (v^2 + \gamma^2_o)] - 1.$$  \hspace{1cm} (23)

The $O_2$ dispersion spectrum at 760 torr was calculated ($v = 0$ to 80 GHz; $N \leq 25$) by Mingelgrin (1972), and from the result, which is shown in Figure 22, we can deduce approximate values of strength and width

$$S_o \approx 5.2 \cdot 10^{-10} (300 / T)^2 \cdot r_1 \cdot p \quad \text{[1/torr]}, \quad (24)$$

$$\gamma_o \approx 1.8 (300 / T) \cdot p \quad \text{[MHz/torr]}. \quad (25)$$

The example for standard conditions at sea level (760 torr, 300 °K) shows that at microwave frequencies the nonresonant part is small when
compared with the much larger resonance part and it can very well be
neglected in the remainder of this report:

\[\nu \text{ [GHz]} \begin{array}{c|cccc} 
1.37(y_o) & 40 & 90 & 140 \\
\hline 
\alpha_n \text{ [dB/km]} & .021 & .041 & .041 & .042 \\
\Delta \phi_n \text{ [rad/km]} & -.001 & -.070 & -.157 & -.244 \\
\end{array} \]

2.4.3 Transition to Doppler Broadening (Voigt Profile)

As the pressure decreases with altitude (Eq. 3), we have to
consider another broadening mechanism - the Doppler effect. The
transition frequency of a non-colliding molecule moving with velocity
\( c_m \) in respect to the radiation field is \( \nu_o [1 + (c_m / c)] \). The velocity
distribution of gas molecules in thermal equilibrium is Maxwellian.
The resulting Gaussian intensity distribution has a linewidth \( \gamma_D \) which
depends on temperature. The pressure-independent width \( \gamma_D \) makes the
peak intensity proportional to pressure, thus forcing the atmospheric
\( \text{O}_2 \) line spectrum to vanish. The equation for \( \gamma_D \) (see Table 8) yields
the following values for \( ^{16}\text{O}_2 \):

\[
\begin{array}{c|cc}
T[^{\circ}\text{K}] & \gamma_D \text{ [kHz]} & \\
\hline 
180 & 59.1 & 42.5 \\
290 & 74.9 & 53.9 \\
\end{array}
\]

using

\[
\gamma_D = 6.34 \times 10^{-8} \cdot \nu_o \cdot \sqrt{T} \quad \text{[Hz]} \tag{25}
\]

The Gaussian shape of a Doppler-broadened line is for extinction (\( F'' \)) a little broader at the center but decreases much more rapidly
in the wings when compared to a Lorentzian. A normalization factor,
\( \sqrt{\pi \ln 2} \) is introduced for the Gaussian (area = \( \sigma \sqrt{\pi \ln 2} \)) in order to match the Lorentzian (area = \( \sigma \Pi \)) integrated strength (Table 8).

A convolution of the two shape functions leads to a Voigt profile whereby it is assumed that the free motion of molecules is unaffected by collisions ("pressure" and "temperature" are statistically independent) which is not exactly true. The reduced mathematical expressions for the Voigt function are given in Table 8. The Voigt parameters

\[
y = \frac{\gamma_p}{\gamma_D} \quad \text{and} \quad x = \frac{\Delta \nu}{\gamma_D}
\]

(26)
govern the extinction profile \( u(x, y) = n''/n''_o \) and the dispersion profile \( v(x, y) = \Delta n/\Delta n_o \) as shown in Figure 10. For \( y < 0.1 \) the shape is practically Gaussian, and for \( y > 10 \) the shape is Lorentzian. As the pressure goes to zero (\( y < 0.1 \)), the rate for the Lorentzian values \( n''_o \) and \( \Delta n_o \) to approach zero becomes \( 0.678 S^o p/\gamma_D \) for the max. extinction \( n''_o \) and \( 0.610 S^o p/\gamma_D \) for the max. dispersion \( \Delta n_o \) (see Figs. 10 and 11).

Figure 11 also gives the corresponding frequency loci of \( \Delta n_o (p) \) and \( y(p) \). In principle, the Voigt profile has to be applied to each Zeeman component in the presence of a magnetic field \( H \).

2.5 Attenuation and Dispersion Due to Water Vapor

Two rotational lines of \( H_2O \) are centered on either side of the 40 to 140 GHz band. The low-frequency wing of the complete \( H_2O \) rotational spectrum and additional polarization mechanisms also contribute to atmospheric millimeter wave transmissivity. We can separate the resonance contributions from the remainder (q) and define:

\[
\begin{align*}
n''_w(v) = \sum_{i=1}^{2} S_i F''_i + q''_w \\
\Delta n_w = \sum_{i=1}^{2} S_i F'_i + q'_w
\end{align*}
\]

(27)
The resonant contributions by the two $\text{H}_2\text{O}$ lines adjacent to the frequency band under consideration can be evaluated using the following spectroscopic parameters (Liebe, 1969a)

<table>
<thead>
<tr>
<th>$\nu_o$ [GHz]</th>
<th>22.235 15</th>
<th>183.310 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$ [Hz]</td>
<td>$13.9(\frac{300}{T})^{3.5} \cdot \exp\left[2.14\left(1 - \frac{300}{T}\right)\right] \cdot P_w$</td>
<td>$322(\frac{300}{T})^{3.5} \cdot \exp\left[0.653\left(1 - \frac{300}{T}\right)\right] \cdot P_w$</td>
</tr>
<tr>
<td>$\gamma$ [MHz/torr]</td>
<td>$18.0\left(\frac{300}{T}\right)^{0.6} \cdot P + \left(\frac{300}{T}\right) \cdot P_w$</td>
<td>$19\left(\frac{300}{T}\right)^{0.6} \cdot P + \left(\frac{300}{T}\right) \cdot P_w$</td>
</tr>
</tbody>
</table>

The Gross shape was found to give a better fit of experimental data in the wings of strongly foreign-gas-broadened lines,

$$F_G' = 2\nu_o (\nu_o^2 - \nu^2)/[(\nu_o^2 - \nu^2)^2 + (2\nu\gamma)^2] \quad [1/\text{Hz}]$$

$$F_G'' = 4\nu^2\gamma/[(\nu_o^2 - \nu^2)^2 + (2\nu\gamma)^2] \quad [1/\text{Hz}]$$

An example for $P_w = 10$ torr under standard conditions, $P = 760$ torr and $300^\circ\text{K}$, yields the following resonance contributions (Eqs. 2, 27, 28)

<table>
<thead>
<tr>
<th>$\nu$ [GHz]</th>
<th>22($\nu_o$)</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>183($\nu_o$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_w$ [dB/km]</td>
<td>.194</td>
<td>.016</td>
<td>.009</td>
<td>.010</td>
<td>.017</td>
<td>.038</td>
<td>.106</td>
<td>.106</td>
</tr>
<tr>
<td>$\Delta{\phi}_w$ [rad/km]</td>
<td>.001</td>
<td>.026</td>
<td>.047</td>
<td>.071</td>
<td>.103</td>
<td>.153</td>
<td>.245</td>
<td>.017</td>
</tr>
</tbody>
</table>

These contributions are by far not enough to account for millimeter wave $\text{H}_2\text{O}$ attenuation. Laboratory measurements at two frequencies produced results shown in Figure 12. Experimental uncertainties are on the order of 10 percent. A tentative fit of the data was
accomplished by (Liebe et al., 1973)

\[
\alpha(q'_w) \approx \left\{ 3 p_w \left( \frac{\nu}{61} \right) \left( \frac{300}{T} \right)^{2.4} + 0.8 p_w^2 \left( \frac{\nu}{61} \right)^{2.6} \left( \frac{300}{T} \right)^{10} \right\} \cdot 10^{-3}, \text{ [dB/km]}
\]

where the frequency \( \nu \) is in gigahertz.

The term linear with \( p_w \) indicates roughly the contribution expected from the residual water vapor attenuation due to the low frequency wing of the rotational spectrum. VanVleck (1947) estimated for this term the following amount (Liebe, 1969a)

\[
\alpha \approx 1.9 \cdot 10^{-9} \cdot \nu^2 \cdot p_w^2 / T^{2.5}, \text{ [dB/km]}
\]

which yields at 61 GHz and 300 K the value 4.5 \( \cdot 10^{-3} \) [dB/km torr].

The \( p_w^2 \) term, however, points to pressure-induced polarization effects such as dimerization. Noteworthy are the temperature dependence. An Arrhenius plot, \( \log \alpha \) versus \( T^{-1} \), yields a "binding energy" of 4.5 [kcal/mole] \( \pm 10\% \). The \( p_w^2 \) term gives rise to abnormal high attenuation when saturation (\( p_S \)) is approached. A similar \( p_w^2 \) response was measured for the dispersion \( q'_w = n' (\nu_1) - n' (\nu_1 / 2) \) for \( \nu_1 \) at 61.2 GHz (\( T = 280 \) and 325 K). The sketchy experimental evidence of unaccounted, pressure-induced polarization in water vapor certainly warrants a more careful study of \( \alpha(q''_w) \) and \( q'_w \) over a wider range of millimeter wave frequencies and close to saturation pressures \( p_S(T) \). Such experiments can shed light on the fact that the theory of the rotational H_2O spectrum does yield atmospheric attenuation values in the wings of the two microwave spectral lines which are too low when compared with measured data.
Figure 4. Normalized ($S = \gamma$) Lorentzian shape of an isolated pressure-broadened line for dispersion ($F'$) and extinction ($F''$). The normalized frequency $z$ indicates multiples of the width $\gamma$. 
Table 6. Reported width parameters $\gamma^0(300)$ and their temperature dependency $u$ for self-pressure broadened lines of the $O_2$ Microwave Spectrum.

<table>
<thead>
<tr>
<th>$N\pm$</th>
<th>Theory</th>
<th>$\gamma^0(300)$ in MHz/torr</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td></td>
<td>(f)</td>
<td>(g)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(h)</td>
<td>(i)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(j)</td>
<td>(k)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(l)</td>
<td>(m)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(n)</td>
<td>(o)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(p)</td>
<td>(q)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(r)</td>
<td>(s)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(t)</td>
<td>(u)</td>
<td></td>
</tr>
<tr>
<td>1+</td>
<td>2.37</td>
<td>2.015</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.870</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3+</td>
<td>2.35</td>
<td>1.682</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.680</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5+</td>
<td>2.05</td>
<td>1.611</td>
<td></td>
</tr>
<tr>
<td>7+</td>
<td>2.10</td>
<td>1.592</td>
<td></td>
</tr>
<tr>
<td>9+</td>
<td>1.95</td>
<td>(1.59)</td>
<td></td>
</tr>
<tr>
<td>11+</td>
<td>1.95</td>
<td>(1.59)</td>
<td></td>
</tr>
<tr>
<td>13+</td>
<td>1.91</td>
<td>(1.59)</td>
<td></td>
</tr>
<tr>
<td>15+</td>
<td>1.71</td>
<td>(1.59)</td>
<td></td>
</tr>
<tr>
<td>17+</td>
<td>1.73</td>
<td>(1.59)</td>
<td></td>
</tr>
<tr>
<td>19+</td>
<td>1.58</td>
<td>(1.59)</td>
<td></td>
</tr>
<tr>
<td>21+</td>
<td>1.60</td>
<td>(1.59)</td>
<td></td>
</tr>
<tr>
<td>23+</td>
<td>1.52</td>
<td>(1.59)</td>
<td></td>
</tr>
</tbody>
</table>

References:
(a) Mingelgrin (1972)
(b) Dillon (1969)
(c) Liebe & Welch (1972)
(d) Reber (1972)
(e) Carter et al. (1968)
(f) Stafford & Tolbert (1963)
(g) Schulze & Tolbert (1963)
(h) Zimmerer & Mizushima (1961)
(i) Artmann & Gordon (1954)
(k) Hill & Gordy (1954)
(l) Anderson et al. (1952)
Figure 5. Reported line width parameter $\gamma^0(300)$ for the self-pressure broadened $\text{O}_2$ Microwave Spectrum (see Table 6) versus the quantum number $N$ for end-over-end rotational angular momentum.
Table 7. Spectroscopic parameters of the $O_2$ microwave spectrum used for computations of this report.

\[
\begin{array}{cccccc}
\text{i} & \nu_O & \lambda & \nu & S(300) & \gamma(300) & \text{Remarks} \\
\hline
\text{0} & 0 & 1 \text{ to } 25 & 5.2 \times 10^{-10} & 1.8 & \text{Nonreson. Spectrum} \\
\hline
1 & 48.942 & 4 & 43 & .000 024 & 1.81 \\
 & 49.451 & 4 & 41 & .000 073 & 1.81 \\
 & 49.961 & 8 & 39 & .000 216 & 1.81 \\
 & 50.473 & 6 & 37 & .000 598 & 1.81 \\
5 & 50.987 & 3 & 35 & .001 564 & 1.81 \\
 & 51.503 & 02 & 33 & .003 860 & 1.81 \\
 & 52.021 & 17 & 31 & .008 985 & 1.81 \\
 & 52.542 & 23 & 29 & .019 707 & 1.81 \\
 & 53.066 & 80 & 27 & .040 717 & 1.81 \\
10 & 53.595 & 68 & 25 & .079 186 & 1.81 \\
 & 54.129 & 96 & 23 & .144 84 & 1.81 \\
 & 54.671 & 145 (20) & 21 & .248 87 & 1.81 \\
 & 55.221 & 372 (20) & 19 & .401 19 & 1.81 \\
 & 55.783 & 819 (20) & 17 & .605 61 & 1.81 \\
15 & 56.264 & 778 (10) & 17 & .348 69 & 2.29 Doublet \\
 & 56.363 & 393 (20) & 15 & .853 87 & 1.81 \\
 & 56.968 & 160 (20) & 13 & 1.120 4 & 1.81 \\
 & 57.612 & 49 & 11 & 1.359 5 & 1.81 \\
 & 58.323 & 685 (10) & 9 & 1.513 0 & 1.81 Doublet \\
20 & 58.446 & 580 (10) & 7 & .925 05 & 1.91 \\
 & 59.163 & 215 (10) & 5 & 1.528 3 & 1.81 \\
 & 59.590 & 978 (10) & 3 & 1.341 0 & 1.83 \\
 & 60.306 & 044 (10) & 1 & .925 05 & 1.91 \\
 & 60.434 & 776 (10) & 9 & 1.562 6 & 1.81 \\
 & 61.150 & 555 (5) & 7 & 1.589 9 & 1.81 \\
 & 61.800 & 169 (10) & 5 & 1.458 8 & 1.81 \\
 & 62.411 & 223 (10) & 3 & 1.227 2 & 1.81 Doublet \\
 & 62.486 & 255 (10) & 1 & .963 36 & 1.91 \\
30 & 62.998 & 00 & 15 & .795 401 & 1.81 \\
 & 63.568 & 520 (10) & 13 & .689 76 & 1.81 \\
 & 64.127 & 777 (20) & 11 & .465 60 & 1.81 \\
 & 64.678 & 92 & 9 & .294 21 & 1.81 \\
 & 65.224 & 120 (20) & 7 & .174 37 & 1.81 \\
 & 65.764 & 744 (20) & 5 & .097 074 & 1.81 \\
35 & 66.302 & 06 & 3 & .050 820 & 1.81 \\
 & 66.836 & 77 & 1 & .025 041 & 1.81 \\
 & 67.369 & 51 & 1 & .011 621 & 1.81 \\
 & 67.900 & 73 & 9 & .005 083 & 1.81 \\
 & 68.430 & 8 & 7 & .002 096 & 1.81 \\
 & 68.960 & 1 & 5 & .000 815 & 1.81 \\
40 & 69.488 & 7 & 3 & .000 299 & 1.81 \\
 & 70.016 & 9 & 1 & .000 104 & 1.81 \\
 & 70.544 & 9 & 9 & .000 034 & 1.81 \\
44 & 118.750 & 343 (10) & 7 & .597 25 & 2.13 Isolated Line \\
\end{array}
\]
D Doublet (Table 7)

Figure 6

Figure 7

Figure 8

Figure 6 - 8. Computer plots of the pressure-broadened O₂ Microwave Spectrum for six different pressures (10 to 760 torr) corresponding to altitudes between h ≈ 30 and 0 km. The model of linear addition of individual Lorentzians was used (T = 300°K, γᵢ° = 1.6 MHz/torr).

For analysis of atmospheric conditions see Figures 14 through 17.
Figure 9. Attenuation of oxygen between 54 and 66 GHz at four different pressures.
Solid lines: Linear addition of Lorentzians (see Figs. 6-8) using Mingelgrin's width parameters (Table 6).
Dashed lines: Scattering calculation by Mingelgrin (1972).
### Table 8. Shape Function for an Isolated Spectral Line

<table>
<thead>
<tr>
<th>Line Profile</th>
<th>Broadening Mechanism</th>
<th>Refraction Spectrum</th>
<th>Extinction Spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>LORENTZIAN</td>
<td>Pressure $p$ (Collisions)</td>
<td>$F' = \left( \frac{z}{1 + z^2} \right) \frac{1}{\gamma}$</td>
<td>$F'' = \left( \frac{1}{1 + z^2} \right) \frac{1}{\gamma}$</td>
</tr>
<tr>
<td>(See Fig. 4)</td>
<td>$\gamma = \gamma^0 p$</td>
<td>&quot;Frequency&quot; $z = (\nu - \nu_0)/\gamma$</td>
<td></td>
</tr>
<tr>
<td>VOIGT</td>
<td>Convolution between Gaussian and Lorentzian</td>
<td>$v(x, y) = \int_{-\infty}^{\infty} \frac{e^{-\frac{(x')^2}{2}}}{(x - x')^2 + y^2} dx'$</td>
<td>$u(x, y) = \int_{-\infty}^{\infty} \frac{e^{-\frac{(x')^2}{2}}}{(x - x')^2 + y^2} dx'$</td>
</tr>
<tr>
<td>(See Fig. 10, $y \neq 0$)</td>
<td>&quot;Frequency&quot; $x = (\nu - \nu_0)/\gamma_D$</td>
<td>&quot;Pressure&quot; $y = \gamma/\gamma_D$</td>
<td></td>
</tr>
<tr>
<td>GAUSSIAN</td>
<td>Doppler Effect due to $T$ (Velocity distribution). Doppler width, $\gamma_D = (\nu_0 / c)\sqrt{2kT/M}$</td>
<td>$F' = \frac{4\sqrt{\ln 2}}{\gamma_D} \exp[-ln2x^2] \int_0^{x\sqrt{\ln 2}} \exp(x'^2) dx'$</td>
<td>$F'' = \frac{\sqrt{\pi \ln 2}}{\gamma_D} \exp(-ln 2x^2)$</td>
</tr>
<tr>
<td>(See Fig. 10, $y = 0$)</td>
<td>$M$ - molecular mass</td>
<td>Dawson's Function$^a$</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Miller and Gordon (1931).
Figure 10. Normalized Voigt profiles for dispersion \( v(x, y) \) and extinction \( u(x, y) \) (Faddeyeva and Terent'ev, 1961). The mathematical functions are given in Table 8.

"Frequency" \( x = (v_0 - v) / \gamma_D \), solid-line curves.

"Pressure" \( y = \gamma_0 p / \gamma_D \), dashed-line curves.
Figure 11. Frequency loci of the maximum dispersion $|\Delta n_o|$ and line half-width ($\gamma$) for a Voigt profile. Also shown is the normalized amount of $|\Delta n_o|$. The ordinate is in normalized pressure units ($y$).
Figure 12. Pure water vapor attenuation as a function of the vapor pressure for two different temperatures at 61.16 GHz, and at 30.58 GHz.
3. COMPUTER ANALYSIS OF ATMOSPHERIC PATH MODELS

3.1 Homogeneous Path Transmissivity at Various Altitudes

Before applying theoretical results of Section 2 to actual atmospheric conditions, it seems appropriate to establish their validity. A serious test of the theory is possible by comparison with experimental results of sufficient accuracy which in turn can only be obtained under controlled laboratory conditions.

The $O_2$ Line Spectrum

One experiment yielded quantitative attenuation data for eight isolated lines around 60 GHz (Stafford and Tolbert, 1963) and the single line at 119 GHz (Schulze and Tolbert, 1963). The conversion of peak attenuation $\alpha_0$ and linewidth $\gamma^o$ to a strength parameter (Eq. 11) is accomplished by means of (Eqs. 2, 18)

$$S^o = 5493 \alpha_0 \gamma^o/v_o \quad [\text{Hz/torr}] \quad (30)$$

where $v_o$ is in megahertz (from Table 7). This is illustrated by the tabulation below

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\alpha_0$ (Air)</th>
<th>$\gamma^o$</th>
<th>T</th>
<th>$S^o$</th>
<th>$S^o_{\text{Th}}$</th>
<th>$(S^o/S^o_{\text{Th}}) - 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$[\text{dB/km}]$</td>
<td>$[\text{MHz/torr}]$</td>
<td>$[^\circ\text{K}]$</td>
<td>$[\text{Hz/torr}]$</td>
<td>$[\text{Hz/torr}]$</td>
<td>$[%]$</td>
</tr>
<tr>
<td>9$^+$</td>
<td>$7.58$ (±12.2%)</td>
<td>1.39 1.76</td>
<td>310</td>
<td>.993</td>
<td>1.406</td>
<td>-29 ? -10</td>
</tr>
<tr>
<td>3$^+$</td>
<td>4.73</td>
<td>1.74 1.86</td>
<td>&quot;</td>
<td>.774</td>
<td>.846</td>
<td>- 9 - 2</td>
</tr>
<tr>
<td>7$^-$</td>
<td>6.50</td>
<td>1.43 1.76</td>
<td>&quot;</td>
<td>.864</td>
<td>1.407</td>
<td>-38 ? -24 ?</td>
</tr>
<tr>
<td>5$^+$</td>
<td>5.94</td>
<td>1.61 1.78</td>
<td>&quot;</td>
<td>.880</td>
<td>1.230</td>
<td>-29 ? -21 ?</td>
</tr>
<tr>
<td>5$^-$</td>
<td>8.58 (±5%)</td>
<td>1.86 1.78</td>
<td>&quot;</td>
<td>1.46</td>
<td>1.237</td>
<td>+18 ? +12</td>
</tr>
<tr>
<td>7$^+$</td>
<td>8.63</td>
<td>1.62 1.76</td>
<td>&quot;</td>
<td>1.27</td>
<td>1.441</td>
<td>-12 - 4</td>
</tr>
<tr>
<td>9$^+$</td>
<td>8.85 (1.97)</td>
<td>1.67 1.76</td>
<td>&quot;</td>
<td>1.33</td>
<td>1.475</td>
<td>-11 - 5</td>
</tr>
<tr>
<td>11$^+$</td>
<td>7.70 (1.97)</td>
<td>1.46 1.76</td>
<td>&quot;</td>
<td>1.00</td>
<td>1.366</td>
<td>-27 ? -11</td>
</tr>
<tr>
<td>1$^-$</td>
<td>6.50 (1.60)</td>
<td>1.99 2.07</td>
<td>300</td>
<td>.598</td>
<td>.597</td>
<td>0 ± 3</td>
</tr>
</tbody>
</table>

9$^+$ (Eq. 20) | 1.81 | 300 | 1.59 | 1.5899 | 0
The difference between $S^*$ and $S^*_{\text{Th}}$ is for five lines larger than the combined experimental uncertainties. The fit improves when the linewidth parameters of Table 7 are applied. The discrepancies are probably due to insufficient control of experimental conditions, incorrect assumptions in reducing the data to 310°K ($u=w=2$), and baseline ($\alpha=0$) problems. This, together with our experimental results for the $9^+$ line (Eq. 20), leads us to believe that the theoretical line strengths are valid as given by equations (7), (11), (12), and Table 7.

The other important parameter of the isolated $O_2$ line spectrum is the correct width. Ab initio calculations of $\gamma^*(300)$ by Mingelgrin (1972) and Dillon-Godfrey (1972) are sensitive to the intermolecular potential parameters which are obtained from data unrelated to the $O_2$-MS. Results from both theories differ by about 20 percent as do the experimental values (Table 6, Fig. 5). Confidence in the measured $\gamma^*(9^+)$ parameter (Eq. 20) and support by other experiments of an $N$-dependence for the width (Fig. 5), made us use the 1.81 MHz/torr value to "calibrate" the Dillon-Godfrey widths.

The atmospheric transfer function versus frequency above an altitude of about 25 kilometers resembles a string of individual $O_2$-MS lines. The analysis of the $O_2$ line spectrum seems to be in fairly good shape. Uncertainties are connected with the width parameters $\gamma^*, m_d, u, m_w$, and $v$ (Eq. 19) for other lines than the $9^+$. Further experimental work with the dispersion spectrometer (Liebe et al., 1973) will hopefully provide the missing information. A guide to treatments of the line spectrum in this report with respect to practical applications can be found in the overview on the next page.
**Guide to Treatments of the Atmospheric Oxygen Line Spectrum (h > 25 km)**

<table>
<thead>
<tr>
<th>Altitude</th>
<th>Line Broadening</th>
<th>Line Shape</th>
<th>&gt; 25 km</th>
<th>&gt; 40 km</th>
<th>&gt; 60 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>Pressure p</td>
<td>Lorentzian</td>
<td>Eq.</td>
<td>Table</td>
<td>Fig.</td>
</tr>
<tr>
<td></td>
<td>Magnetic Field H</td>
<td>Lorentzian-Voigt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Temperature T</td>
<td>Voigt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transfer Fct.</td>
<td>$\tau$</td>
<td>1, 2</td>
<td>6, 17</td>
<td>1, 2</td>
<td>18, 19</td>
</tr>
<tr>
<td>Refract. Index</td>
<td>$n$</td>
<td>4, 18</td>
<td>4</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>Line Centers</td>
<td>$v_o$</td>
<td>6</td>
<td>2, 7</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>Strengths</td>
<td>$S^*$</td>
<td>7, 11</td>
<td>3, 7</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>Temper. Fct.</td>
<td>$\varphi$</td>
<td>12, 13</td>
<td>4, 3</td>
<td>12, 13</td>
<td>12, 13</td>
</tr>
<tr>
<td>Shape Fct.</td>
<td>$\psi$</td>
<td>17</td>
<td>4</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Widths</td>
<td>$\gamma$</td>
<td>19</td>
<td>6, 7</td>
<td>5, 13</td>
<td>19</td>
</tr>
</tbody>
</table>
The complete $O_2$-MS and detailed insight into its transfer properties require a good deal of theory. Therefore, it is important to test theoretical results by as many reliable experiments as possible.

Interesting properties of isolated lines are their markedly different temperature dependences. The peak attenuation $a_o$ varies approximately as (Eqs. 2, 13, and 18)

$$a_o \sim (T_o / T)^{w-u} .$$

Looking at Table 4 and assuming $u=0.9$, we find that $a_0$ of the two $N=15$ lines is almost independent of temperature and thus a good candidate as an atmospheric pressure sensor (from a satellite). One can also pick lines with inversely linear ($N=11$) and square-law ($N=3$) dependences as well as linear ($N=15$), square-law ($N=23$), and higher order temperature dependences. The high rotational lines are very sensitive functions of temperature (Waters, 1973). For example, a 1 percent ($+1.8^\circ K$) raise at 280$^\circ$K produces for $N=27$ lines a 3.5 percent increase in $a_0$, and at 220$^\circ$K the increase is even 5.1 percent.

The $O_2$ Continuum Spectrum

One early experiment (Artman and Gordon, 1954) provided attenuation data ($\pm 10$ percent) at 25 frequencies between 52 and 61 GHz under controlled laboratory conditions ($p=760, 380, 190$ torr; $T=300^\circ$K). Several workers (e.g., Meeks and Lilley, 1963; Westwater, 1970) used these data, different line shape functions, and theoretical line strengths to deduce the pressure-broadening line parameters. It was found that the isolated width parameter (assumed equal for all lines) needs to be reduced by 30 to 50 percent (see Eq. 12) to fit more or less the continuum spectrum, and in all cases it was stated clearly that future laboratory measurements are needed to

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remove the encountered uncertainties. Since such experiments have not been reported up to now, we have had to rely upon measurements made in the atmosphere.

Results of field measurements contain information not only about the $O_2$-MS but also about the structure of the atmosphere. Many of the uncertainties in present knowledge of the transfer function cannot be distinguished from uncertainties in pressure and temperature distributions which makes field data an unsatisfactory alternative to laboratory measurements.

Once the $O_2$-MS is completely understood it is an easy task to use spectroscopic properties as a probe of (dry) atmospheric conditions. Right now, however, we have to content with an empirical linewidth model introduced by Meeks and Lilley (1963) which determines the individual width from ($\gamma_i$ applied to all lines except the 1$^\text{st}$)

$$\gamma_i(p) \approx G(h) \left(\frac{p}{760}\right) \left(\frac{300}{T}\right)^u$$

and

$$\gamma_i(h) \approx \sqrt{\gamma_i^2(p) + \gamma_D^2}$$

Carter et al. (1968) and Reber (1972) measured oxygen attenuation of the atmosphere by the zenith angle technique using the sun as a source. Their ground level results are given in Table 10. Carter et al. accomplished a ± 8. 8 percent fit (squared deviations) to their data which include airplane measurements (taken between $h = 2.6$ and 12.2 km) by the following scheme for $G(h)$:

<table>
<thead>
<tr>
<th>$h$ [km]</th>
<th>$G(h)$ [MHz]</th>
<th>$\gamma_i^o(300)$ [MHz/torr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 8</td>
<td>640</td>
<td>0.85 &gt;180 0.84</td>
</tr>
<tr>
<td>8 to 25</td>
<td>640 to 1357, linear with $h$</td>
<td>1357</td>
</tr>
<tr>
<td>&gt; 25</td>
<td>1357</td>
<td>1.79</td>
</tr>
</tbody>
</table>

Carter et al. (1968) and Reber (1972) measured oxygen attenuation of the atmosphere by the zenith angle technique using the sun as a source. Their ground level results are given in Table 10. Carter et al. accomplished a ± 8.8 percent fit (squared deviations) to their data which include airplane measurements (taken between $h = 2.6$ and 12.2 km) by the following scheme for $G(h)$:
With these values, equation (32) becomes the "CMR-Model" which is used for the analysis described in the remainder of the report.

Another approximation, equation (33), is introduced to take care of the transition to Doppler-broadening (Section 2.4.3). The Lorentzian shape is assumed for the vanishing $O_2$-MS, and the transition to a Gaussian shape (Voigt profile) is not done. At $h = 50$ km this approximation was found to be indistinguishable over the strongest 90 percent of a line from the more correct Voigt profile (Waters, 1970). Equation (33) is sufficient in the context of analyzing cumulative effects through the atmosphere (Sect. 3.2), but for a calculation of the line profile over a homogeneous path above $h = 60$ km the Voigt profile has to be applied.

3.1.1 Analysis of Attenuation and Dispersion for Dry Air Between 49 and 72 GHz.

The analysis includes all lines with rotational quantum numbers $N = 1$ to 43 (Table 7), which omits lines with strengths less than about $2 \cdot 10^{-5}$ of that of the strongest line. This cutoff is sufficient even for computations of transmission over long paths. The assumptions made for the computer analysis of homogeneous attenuation and dispersion rates (Eq. 2) were:

(a) The U. S. Standard Atmosphere (1962) models the distribution of pressure $p$ and temperature $T$ versus altitude $h$.

(b) The atmosphere is dry, $p_w = 0$.

(c) The sum of 44 $O_2$ lines (Eq. 4) is the only source of attenuation and dispersion over the frequency band considered.

(d) The intensity distribution of each line is a Lorentzian (Eq. (Eq. 17), and the theoretical strength values (Table 7, Eqs. 7, 11, 12) are valid.
(e) The individual widths are the same for each line. They are given by the "CMR-Model" (Eqs. 32, 33) whose approximate (empirical) nature provides at best analytical estimates of transfer properties. The altitude dependence of the width is shown in Figure 13.

Results of the calculation are given in Figures 14 through 19. Only the graphical presentation provides an overview of the structure of the transfer function at different altitudes. Numerical results (computer print-outs) for any given set of conditions are available upon request.

At \( h = 0 \) km, Figure 14 shows a continuum spectrum (II). The envelope is not a sensitive function of the width parameter:

1. \( \gamma^0 = 0.876 \text{ MHz/torr} \) plus arbitrary variations of \( \pm 25 \) percent (CMR-Model);
2. \( \gamma^0 = 1.27 \text{ MHz/torr} \), a value obtained by Westwater from fitting Artman and Gordon's (1954) data. Atmospheric attenuation for sea level conditions was reported at 21 frequencies between 40 and 120 GHz (Thompson, et al., 1972, Fig. 3).

At \( h = 10 \) and 20 km, Figures 15 and 16 show a mixture of continuum (II) and line (I) spectra. The influence of the width becomes more pronounced as the lines gain identity. At \( h = 30 \) km (Fig. 17), we have a pressure-broadened line spectrum.

At \( h = 50 \) km, Figures 18 and 19 show the effects of Zeeman splitting of two individual (9+ and 27-) lines. Figure 18 gives changes in transmissivity for the case of linearly polarized radiation. Each of the 19 \( \pi \) - or 38 \( \sigma \) - components (Eq. 14) is treated as a pressure-broadened line of Lorentzian shape (Eq. 17) with a width determined by equations (32) and (33). The line strengths are calculated using equation (15). The contributions from each Zeeman component are summed up over frequency (Eq. 4). The 9+ line widens under the influence of \( H = 0.53 \) gauss while the peak attenuation is reduced by
20 to 36 percent, respectively, for the \( \pi \) - and \( \sigma \) -envelopes. The \( \sigma \) -envelope splits for circular-polarized radiation depending on the orientation. The two halves are shown in the example for the 27° line in figure 19. The individual Zeeman components are not resolved in both cases. A consequence of the Zeeman effect is that above \( h \approx 40 \) km the atmosphere becomes a medium with polarization-dependent anisotropic transfer properties.

The evaluation of a practical case requires the determination of the type of polarization for the propagating radiation. The next step is to define the orientation between the direction of \( H \) and the microwave field component \( \psi \). Lenoir's (1968) transfer function in tensor form does this in general form for geomagnetic coordinates. Groom (1971) discussed 24 possible "pure" constellations between \( H \) and \( \psi \) for the case of emission from the 1 line, where \( \psi \) takes the role of the radiometer antenna polarization.

The simple Zeeman pattern of the 1 and 1+ lines eases considerably the interpretation of emission (attenuation) measurements in the vicinity of these lines. Such emissions can be picked up from satellite-born radiometers and then converted to mesospheric temperature structures and magnetic field \( (H) \) data on a global basis (Lenoir, 1968; Groom, 1971). At these heights the 1+ line is well isolated from its 1- satellite. The peak attenuation of both lines (Eqs. 2, 7, 18, and Table 7) have for \( T = 300 \) K the values

\[
\alpha_0(1^+) = 351 \text{ and } \alpha_0(1^-) = 136 \text{ dB/km.}
\]

Croom (1971) gives \( \alpha_0(1^+) = 3.8 \) dB/km which is wrong since the overlap-reduced width (Eq. 32) was used. At \( 286 \) K we calculate 1.54 dB/km (Eqs. 2, 11, 12, 18, 19, and Table 7). This error reduces the zenith attenuation, which should be closer to \( A_0(1^-) = 110 \) dB (0.7-1.5-100 km) instead of 266 dB as given by Croom.
Figure 13. Linewidth $\gamma(h)$ (same for all lines of O$_2$-MS) versus altitude $h$ and pressure $p$ as obtained from the CMR-model (Carter et al., 1968).
Figure 14. Horizontal (homogeneous) transmissivity at sea level, $h = 0$ km. Variations are shown due to different linewidth values:

1. $\gamma_1 = 666$ MHz ± 25%,
2. $\gamma_1 = 968$ MHz.
Figure 15. Horizontal (homogeneous) transmissivity at \( h = 10 \text{ km} \) (U.S. Std. Atm. 62). Variations are shown due to different linewidth values: (1) \( \gamma_1 = 255 \text{ MHz} \pm 25\% \).
Figure 16. Horizontal (homogeneous) transmissivity at $h = 20$ km

The linewidth values are: $\gamma_1 = 86.6$ MHz.

(U.S. Std. Atm. 62).
Figure 17. Horizontal (homogeneous) transmissivity at h = 30 km (U.S. Std. Atmos. 62). The linewidth values are $\gamma = 21.2$ MHz.

(1) $H \parallel \vec{\phi}$ to (ii) $H \perp \vec{\phi}$ with...
Figure 18. Effect of Zeeman-splitting upon the horizontal (homogeneous) transmissivity at \( h = 50 \) km (U. S. Std. Atm. 62) in the vicinity of the 9+ line of the \( \text{O}_3 \)-MS for linearly polarized radiation with \( \vec{y} \perp H (\pi) \) or \( \vec{y} \parallel H (\sigma) \).
Figure 19. Effect of Zeeman-splitting upon the horizontal (homogeneous) transmissivity at $h = 50$ km (U.S.Std. Atm. 62) in the vicinity of the $27^-$ line of the $O_2$-$MS$ (after Waters, 1970) for linearly polarized radiation with $\phi \perp H(\pi)$ or for circularly polarized (right-, left-handed) radiation ($\sigma^+$, $\sigma^-$).
The intensities of both lines differ by about 1 to 4, but technology at 56 GHz \((1^+)\) is more advanced. The NIMBUS 5 radiometer, for example is working well in space at 60 GHz (D. Staelin, private communication, 1973).

3.1.2 Phase Dispersion Between 10- and 140- GHz

While attenuation (or emission) is the subject of all papers dealing with applications of the \(\text{O}_2\)-MS, data on phase dispersion are not readily available. In general, the complete transfer function \(\tau(\alpha, \phi)\) (Eq. 1) is required for an evaluation of atmospheric limitations imposed on system performances. The phase term has particular relevance for broadband communication and for various disciplines of microwave measurements. For example, phase dispersion limits the useable, phase-coherent channel bandwidth of a broadband system (Morgan and Ekdahl, 1966); propagation delay and its fluctuations determine the accuracy of clock synchronization via satellite (Levine, 1970), the resolution of long-baseline interferometers (Schaper et al., 1970), the accuracy of radio range measurements (e.g., Thompson, 1968), etc.

The atmosphere delays radio signals causing an apparent increase \(\Delta L\) in the geometric path length \(L\). The propagation time

\[
t = \frac{(L + \Delta L)}{c} \quad [s]
\]

is determined for refractive indices \(n' \geq 1\) by the phase velocity \(c/ n'\) and for \(n'< 1\) by the group velocity \(c n'\).

The case \(n'< 1\) exists for the \(\text{O}_2\)-MS only under special circumstances when the negative resonance dispersion \(\Delta n(v)\) of an isolated line (Fig. 4) is larger than the non-dispersive refraction \(n_0\) (Section 2.2). This case occurs at frequencies up to several megahertz above a line center (Liebe et al., 1973, Fig. 4), but it will not be considered in the following discussion.
We define

\[ \Delta L = \int_L \left( n' - 1 \right) \, \text{d}l = \int_L \left[ \textbf{N} \cdot 10^{-6} + \Delta n(\nu) \right] \, \text{d}l \quad [\text{m}] \tag{35} \]

where the integration is taken along the ray path. For example, the additional electrical length of a zenith path from sea level to outer space is typically \( \Delta L \approx 2 \) meters (Schaper et al., 1970). Extreme values of zenith path length changes due to \( O_2 \)-MS dispersion are only +12 millimeters at 58.3 GHz and -14 mm at 62.6 GHz. The extreme values for \( \Delta L(\nu) \) from different initial altitudes are listed in Table 9. The calculation procedure of these results will be described in Section 3.2.

System engineers express dispersion effects of the homogeneous atmosphere by a phase change in radians per unit distance:

\[ \Delta \phi(\nu) = \left( \frac{2\pi \nu}{c} \right) \Delta n(\nu) \quad [\text{rad/} \text{km}] \tag{36} \]

Calculations of \( \Delta \phi \) over the 49 to 72 GHz range (\( O_2 \)-MS attenuation) have been described in Section 3.1.1, and various examples for \( h = 0 \) to 50 km display the \( O_2 \)-MS dispersion spectrum (Figs. 14 through 18). It is evident that the molecular resonance dispersion extends over a much larger frequency range. The reason lies in the shape function \( F' \) whose wing intensity decreases with \( \nu_o - \nu \) as compared to \( \nu_o - \nu \)^2 for \( F'' \) (Eq. 17 and Figs. 4, 10).

The calculations are extended to cover the frequency range 10 to 140 GHz for \( h = 0 \) and 10 km, and the graphical results are shown in figure 20. The dispersion of the 22 GHz \( H_2O \) line puts a "wrinkle" into the dry air phase response. At \( h = 0 \), we calculate (Eqs. 27, 28, 36):

\[ (\Delta \phi_w)_{\text{max}} = \pm 1.2 \cdot 10^{-3} \text{ rad/km per torr of water vapor at } 19.2 \text{ and } 25.3 \text{ GHz respectively, while the } O_2 \text{-MS dispersion over the same frequency range is } \pm 25 \cdot 10^{-3} \text{ rad/km.} \]
The dispersion $\Delta n(\nu)$ of the $O_2$-MS was first calculated for $h = 0$ by Zhevakin and Naumov (1967). The complete oxygen dispersion spectrum including overlap and nonresonant dispersion (Sect. 2.4) was recently evaluated by Mingelgrin (1972), and his result at $h = 0$ km for $\nu = 0$ to 80 GHz is shown in figure 21. The peak value span (7.87 ppm) of $O_2$ translates for air into 2.11 rad/km, which agrees well with the value of (2) in figure 14. Another dispersion calculation for dry air at $h = 0$ km was performed by Sullivan and Richardson (MITRE, 1965). They predicted a dispersion of 0.4 ppm between 45 and 90 GHz while our calculated value is 0.381 ppm. A broadband system operating on the ground close to the $O_2$-MS attenuation band, e.g., in the 45 or 75 GHz bands, does experience phase dispersion on the order of $\pm 0.03$ rad/km per gigahertz.

Dispersion measurements are difficult in the laboratory because the small effect is heavily masked by instrumental instabilities. A few measurements have been made in our laboratory using a differential microwave refractometer (Thompson and Vetter, 1968) similar to the one described by Liebe et al. (1973). The difference of the refractivity at 26 and at $(26/3)$ GHz was measured at 610 torr and 300°K in dry air to be $0.12 \pm 0.03$ ppm (M. J. Vetter, private communication, 1968). The calculated value for the same conditions is $(0.394 - 0.261) = 0.133$ ppm.

In another experiment we measured the difference in refractivity between 23.6 and $(23.6/2)$ GHz (Liebe, unpublished results, 1969). In dry air from $p = 50$ to 600 torr at 300°K, the following result was obtained $(1.25 \pm 0.3) \cdot 10^{-4} \cdot p$, while the calculated response is $(6.03 - 4.55) \cdot 10^{-4} \cdot p = 1.48 \cdot 10^{-4} \cdot p \ [ppm/torr]$.

These two experiments provide a rough (±25 percent) confirmation of $O_2$-MS dispersion outside the 60 GHz attenuation band where the dispersion is nearly independent of the widths $\gamma$ (Eqs. 19, 21).
Atmospheric refraction limits the accuracy of such systems as electronic distance meters and tracking radars. The measurement of distance is based on determining the propagation time $t$ (Eq. 34), whereas the deviation of the vertical tracking angle is proportional to the gradient $\frac{dn'}{dh}$. These errors can be reduced if pertinent atmospheric parameters of the propagation path are known. The dispersion of the $O_2$-MS offers a means of evaluating the refractive index $n'$ of dry air averaged along an atmospheric path. Systems operating at two (or more) phase-coherent frequencies can extract path conditions from the difference in phase shifts experienced by the signals when travelling the same path (MITRE, 1965; Thompson, 1968; Liebe, 1969b).

The potential of a differential phase measurement is analyzed for signal frequencies in the wings of the $O_2$-MS. Two different signal pairs are chosen for short and long operating ranges in the following example.

<table>
<thead>
<tr>
<th>Master Freq.</th>
<th>Multipl. Factor X</th>
<th>Signal Freq. v</th>
<th>Phase Difference $\phi$</th>
<th>Temperature Sensitivity of $\phi$</th>
<th>Pressure Sensitivity of $\phi$</th>
<th>Operating Range L</th>
</tr>
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<tbody>
<tr>
<td>[GHz]</td>
<td>[GHz]</td>
<td>[rad/km]</td>
<td>[rad/km°K]</td>
<td>[rad/km torr]</td>
<td>[km]</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>55</td>
<td>1.93</td>
<td>$-11.8 \cdot 10^{-3}$</td>
<td>$9.3 \cdot 10^{-4}$</td>
<td>$&lt; 10$</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>66</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.2</td>
<td>5</td>
<td>51.0</td>
<td>0.98</td>
<td>$-4.3 \cdot 10^{-3}$</td>
<td>$4.7 \cdot 10^{-4}$</td>
<td>$&lt; 100$</td>
</tr>
<tr>
<td>10.2</td>
<td>7</td>
<td>71.4</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

$\alpha \approx 40 \text{ dB}$
The phase shifts $\vartheta_{1,2}$ experienced by each signal over a path length $L$ are (Eqs. 35, 36)

$$\vartheta_1 = \left[ 2\pi v_1 (1 + \overline{N} \cdot 10^{-6})/c + \Delta\phi(v_1) \right] \cdot L$$

and

$$\vartheta_2 = \left[ 2\pi v_2 (1 + \overline{N} \cdot 10^{-6})/c - \Delta\phi(v_2) \right] \cdot L,$$

when the signal frequencies are chosen so that $|\Delta\phi(v_1)| \approx |\Delta\phi(v_2)|$ (see Fig. 22) and $\overline{N}$ is the refractivity averaged along the path. The averaged refractivity can be separated into a "drv" and a "wet" term,

$$\overline{N} = \overline{N_d} + \overline{N_w}.$$

Assuming (a) a straight line-of-sight ray path (no reflections or bending), (b) a perfect correlation of time-varying phase changes (mainly due to water vapor density fluctuations), and (c) a common frequency $\overline{\nu} = (\nu_1 + \nu_2)/2$ (e.g., by heterodyning $\nu_1$ and $\nu_2$ and then multiplying each IF by the opposite factor $X$) one can measure at the receiving end the difference

$$\vartheta_1 - \vartheta_2 = \phi L = \left\{ \Delta\phi_1(\overline{\nu}) + |\Delta\phi_2(\overline{\nu})| \right\} \cdot L \quad [\text{rad}]$$

and the sum

$$\vartheta_1 + \vartheta_2 \approx 2\pi (\nu_1 + \nu_2) (1 + \overline{N} \cdot 10^{-6}) L/c \quad [\text{rad}].$$

The $O_2$-MS dispersion $\phi$ can be tabulated in terms of dry air density, $\rho_d = 464 \text{ p}/T [\text{g/m}^3]$. Approximate pressure and temperature sensitivities are listed on the previous page. For the path length $L$, first the uncorrected value can be used with some possible later iterations. Thus the value $\overline{\rho_d}$ found through a measurement of $\phi$ determines the dry term refractivity (Eqs. 5, 39)

$$N_d = 0.223 \overline{\rho_d} \quad [\text{ppm}].$$
Table 9. Extreme values of zenith phase dispersion $\Delta T$ and electrical path length change $\Delta L(\nu)$ with associated frequency $\nu$ and attenuation $A$ for different initial altitudes, $h_1$ (CMR-Model).

<table>
<thead>
<tr>
<th>$h_1$ (km)</th>
<th>See Figure</th>
<th>Number of Integrat. Layers</th>
<th>Maximum Values</th>
<th>$\Delta T$</th>
<th>$\Delta L(\nu)$</th>
<th>$\nu$ (MHz)</th>
<th>$A$ (dB)</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>25</td>
<td>151</td>
<td></td>
<td>14.7</td>
<td>12.0</td>
<td>58260</td>
<td>179</td>
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<tr>
<td>5</td>
<td>27</td>
<td>101</td>
<td></td>
<td>11.5</td>
<td>9.4</td>
<td>58280</td>
<td>130</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>81</td>
<td></td>
<td>8.5</td>
<td>6.9</td>
<td>58300</td>
<td>94</td>
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<tr>
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<td>31</td>
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<td></td>
<td>6.1</td>
<td>5.0</td>
<td>58300</td>
<td>56</td>
</tr>
<tr>
<td>20</td>
<td>33</td>
<td>61</td>
<td></td>
<td>4.5</td>
<td>3.7</td>
<td>58320</td>
<td>30</td>
</tr>
<tr>
<td>25</td>
<td>35</td>
<td>56</td>
<td></td>
<td>3.6</td>
<td>2.9</td>
<td>58320</td>
<td>14.3</td>
</tr>
<tr>
<td>30</td>
<td>37</td>
<td>51</td>
<td></td>
<td>3.0</td>
<td>2.5</td>
<td>58320</td>
<td>5.2</td>
</tr>
</tbody>
</table>
Figure 20. Phase dispersion in dry air due to the microwave spectrum of O$_2$ for horizontal paths at $h = 0$ (760 torr, 15°C) and $h = 10$ km (199 torr, -50°C) using the linear model (sum of Lorentzians).
Figure 21. Total dispersion $\Delta n_t$ of the O$_2$ Microwave Spectrum for 760 torr and 298°C as obtained for pure oxygen using scattering calculations (Mingelgrin, 1972). The associated attenuation is shown in Figure 9.
Figure 22. Phase dispersion $\Delta \phi$ and associated attenuation $\alpha$ versus temperature $T$ for two different harmonious frequency pairs in dry air at sea level using the linear model (sum of Lorentzians). $\phi_{1,2}$ - differential dispersion between 51.0/71.4 and 55/66 GHz.
The wet term refractivity $N_w$ originates from the water vapor within the path. With equation (5) and $\rho_w = 288.8 \frac{p_w}{T}$ [g/m$^3$] one obtains

$$N_w \approx 1.73 \cdot 10^3 \cdot \frac{\rho_w}{T} \quad \text{[ppm]} \quad (42)$$

It is possible to solve for the average water vapor density along the path, $\bar{\rho}_w$, using the measured sum of phase shifts (Eqs. 40, 38, 41, 42).

The proposed measurement principle offers the potential to follow air and water vapor density fluctuations separately in real time as they occur as spatial averages along a line-of-sight path. This information is also a wanted correction term (Eqs. 34, 35) for radio distance measurements.

3.2 Slant Path Transmissivity

3.2.1 Calculation Procedure and Critical Comments

Computations of non-turbulent transfer properties through the first 100 kilometers of the atmosphere depend on the exact functional form of $\alpha(h)$ and $\Delta\phi(h)$ [Sect. 2], on the model atmosphere which determines $p(h)$ and $T(h)$ [U. S. Stand. Atm. 1962], on the mixing ratio distribution $r_k(h)$ [Fig. 23], on the ray path geometry [Fig. 24], and on the refractivity distribution $N(h)$ [Eq. 5]. Cumulative transmissivity along a ray path is defined by

$$A = \int_{h_i}^{\infty} \alpha(\lambda) \, d\lambda \quad \text{and} \quad \Delta T = \int_{h_i}^{\infty} \Delta\phi(\lambda) \, d\lambda \quad (43)$$

where $d\lambda$ is the increment of the ray path and $h_i$ is the initial altitude. The atmosphere has only a weak dependence on geographic coordinates and in the first approximation it can be considered as a layered homogeneous medium which does not change with time. The numerical method used for the evaluation of equation (43) consisted of assuming a spherically
stratified atmosphere which was divided into slabs of thickness $\Delta h$. The altitude increments $\Delta h$ were chosen so that each layer was quasi-homogeneous ($\Delta p < 10$ torr and/or $\Delta T < 5^\circ$K across each slab) amounting to a number of $n = 151$ for $h = 0$ to 100 km (see Table 9). Attenuation $\alpha$ and dispersion $\Delta \phi$ were calculated for each slab as described in Section 3.1.1. The clean, dry atmosphere sustains a constant mixing ratio up to $h > 90$ km for all gases which might have a measurable ($r_k > 10^{-6}$) influence upon transfer properties (Fig. 23). An exception is ozone (and possible air pollutants), which is discussed briefly in Section 3.2.3. The correct evaluation of a path trajectory would include refractive ray bending commonly expressed by Snell's law (e.g., Blake, 1972). The effect is most severe for a horizontal starting angle ($\theta = 90^\circ$) defining a tangential path and is zero for a vertical starting angle ($\theta = 0$) defining a zenith path. We assume a straight line slant path with the increment $d\ell(\theta) = \Delta \ell(\theta)$ determined by the algorithm given in figure 24. Finally, a numerical integration based on Simpson's rule was employed to compute the integrals of (43); i.e.,

\[
A \approx \sum_n 0.5 \left( \alpha_n + \alpha_{n+1} \right) \Delta \ell_n(\theta)
\]

\[
\Delta T \approx \sum_n 0.5 \left( \Delta \phi_n + \Delta \phi_{n+1} \right) \Delta \ell_n(\theta)
\]

A word of caution regarding these calculations should be noted at this point. Results obtained by the method described above can only qualify as analytical estimates allowing an overview on altitude-dependent transfer properties of the $O_{2}$-MS. The following key-words underline the approximate nature: (a) empirical linewidth model, (b) straight line ray path, (c) numerical integration, (d) neglect of water vapor, (e) neglect of temporal variations of $p$ and $T$, and (f) a more or less representative model atmosphere. The climate dependence of the
$O_2$-MS is not too strong. The attenuation peaks of $A$, for example, decrease by about 10 percent in a tropical climate and increase by about 20 percent in an arctic climate when compared with results (Figs. 25, 17) for the U. S. Std. Atm. 62 (Reber et al., 1970). Values for $A$ and $\Delta T$ are, however, consistent in themselves, and the spectra could be scaled to results of reliable experiments.

3.2.2 Zenith and Tangential Path Transmissivities Between 49 and 72 GHz at Various Altitudes

A computer program was developed for evaluations of equation (44). The routine was tested for the two extreme (zenith and tangential) cases of a slant path. We present examples of results in the form of computer plots in Figures 25 through 39. The print-outs are too lengthy for reproduction here, but Table 11 gives an example of the format. Such listing can be made available upon request for any given set of variables. The path transmissions between a given initial altitude and outer space are obtained by summing the contributions of 44 $O_2$-MS lines, integrating over height, and finally integrating over frequency. For example, the computation of the results shown in figure 25 require for frequency increments of 20 MHz about $23 \times 10^6$ individual calculations to cover the 49 to 72 GHz band which takes about 5 minutes on a CDC 3800 machine.

The zenith path spectra are labeled by the initial altitude $h_i$ which was varied between $h = 0$ and 30 km in increments of 5 km. The tangential spectra are calculated for a straight-line path through the total atmosphere (satellite-to-satellite) and they are labeled by the height $h_o$, marking the closest approach of the tangent to sea level. Factors that determine tropospheric path loss are not only molecular attenuation but also stratification and path geometry which can introduce
multipath, diffraction, and ground reflection effects. To avoid considering these problems, we limited $h_0 \geq 5$ km. The peak values of a zenith and a tangential path for $h_1 = h_0$ differ by about 1 to 30. The tangential spectra were extended to $h_0 = 50$ and 75 km (Figs. 38, 39) to gain insight into the problem how deep a satellite-satellite link can penetrate the atmosphere. At these altitudes there is still appreciable attenuation at the line centers due to the long path length, but in between are many "clear" channels with bandwidths on the order of 400 MHz.

A few experimental results are available on zenith attenuation from $h = 0$ (Table 10). Carter et al. (1968) also reported some aircraft measurements for different altitudes between $h_1 = 2.6$ and 13.7 km. Their data points for $h_1 = 9.1$ km are shown in figure 29. Figure 25 illustrates the degree of agreement obtained between experiment and the CMR-Model analysis (Eqs. 32, 33) which is on the order of 10 percent. Table 10 gives the numerical values.

Reber (1972) published recently more extensive data for $A$ on both sides of the $O_2$-MS (Table 10). Special care was taken to eliminate water vapor attenuation, being typically on the order of < 1 dB total at zenith. The dry air data were fitted by yet another linewidth model (R-Model) which consists of a two-piece linear pressure dependence for the width $\gamma$ of the 60 GHz line complex excluding the 1° line --

Range 1, $\gamma < 52.7$ MHz,

$$\gamma = 1.88 p \frac{300}{T} \text{ [MHz]}$$

(45)

Range 2, $\gamma \geq 52.7$ MHz ($p_1 = 20.7$ torr at $T = 222^\circ$K),

$$\gamma = 52.7 + (1,88/3) \left( p - p_1 \right) \left( 300/ T \right) \text{ [MHz]}.$$  

We substituted the R-Model (Eq. 45) into the computer routine and, as expected, got a good fit to the Reber data. Both empirical width models (Eqs. 32, 45) are a somewhat arbitrary description of an intensity distribution which deviates from a linear addition of individual shapes.
The experimental zenith data (Table 10) strongly suggest overlap effects in the $O_2$ continuum spectrum (Sect. 2.4.1), especially when compared with $A$-values which have been calculated using the pressure-linear linewidth model (Eq. 19). The experimental values, however, are only wing data and as such not really representative of the $O_2$ continuum spectrum.

Center data cannot be obtained by the radiometric technique since the band between 55 and 65 GHz is practically opaque for any source in space. Reliable experimental information on the pressure dependence of the intensity distribution of the $O_2$ continuum spectrum can be obtained by a controlled laboratory experiment. An analysis of continuum data is based on the accurate knowledge of the $O_2$ line spectrum and overlap effects as shown by the theoretical treatments of Mingelgrin (1972) and Dillon (1969).

The $O_2$-MS offers between 55 and 65 GHz one feature not found at any lower frequency; namely, high attenuation and phase dispersion values related to the fairly stable dry part of the lower atmosphere. This affords shielding of satellite-satellite links against ground interferences. The first 100 km of the atmosphere present a fairly complicated "natural" filter which can be used to advantage in special system applications. The strong frequency dependences of $A(\alpha)$ and $AT(\Delta\phi)$ permit frequency diversity techniques and signal preprocessing such as phase-keying. Figures 25 through 39 demonstrate that the atmosphere, which is opaque close to sea level, becomes transparent at higher altitudes in 16 channels each possessing a bandwidth of several hundred megahertz. One such channel is centered around 58.8 GHz, and for a tangential path geometry the loss due to the $O_2$-MS varies as follows
The curves presented in figures 25 through 39 for cumulative transmission and zenith ray path properties are for one-way radio communication cases. The distinction between the highly-structured line spectrum (I) and the smooth continuum spectrum (II) can clearly be made within each cumulative spectrum.

Aircraft can communicate with one another or with satellites between the $O_2$-MS lines when flying above 10 kilometers while being shielded against the ground. Several atmospheric channels might be combined to increase the bandwidth by means of frequency transposition. The cumulative phase dispersion, $\Delta T$, in the pass band centered at 58.8 GHz varies for a zenith path from $-5.7 \text{ rad per } 350 \text{ MHz}$ at $h = 15 \text{ km}$, to $-4.5 \text{ rad per } 680 \text{ MHz}$ at $h = 30 \text{ km}$.

### Table 1

<table>
<thead>
<tr>
<th>$h_0$ [km]</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
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<th>50</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$ [dB]</td>
<td>5240</td>
<td>2760</td>
<td>835</td>
<td>203</td>
<td>46.5</td>
<td>9.8</td>
<td>.54</td>
<td>$1.5 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

3.2.3. Some Spectroscopic Properties of Ozone and Other Minor Atmospheric Gases.

The mixing ratio for the main constituents of clean, dry air is constant up to $h \gtrsim 90 \text{ km}$ for all $r_k \gtrsim 10^{-5}$ (Fig. 23). Several minor constituents and gaseous pollutants exhibit, in addition to $O_2$ and $H_2O$, spectral lines over the 40 to 140 GHz band (Table 12). These lines are part of rotational spectra due to molecular electric dipole moments.

Linear rotor molecules ($N_2O$, CO) exhibit an almost harmonic spacing of lines, while asymmetric rotors with three different principle moments of inertia ($H_2O$, $SO_2$, $NO_2$, $CH_2O$) display an irregular line pattern. Although the absorption coefficients of electric dipole transitions are considerably larger than those for $O_2$, these spectra are of no importance to transfer properties of the lower ($h < 20 \text{ km}$) atmosphere.
Trace concentrations and strong foreign-gas-broadening essentially "wash-out" these resonances.

Ozone is practically the only molecule which causes some concern. The line shape is altered according to the manner in which the gas is distributed throughout the atmosphere. The increase of the O$_3$-mixing ratio with height produces line profiles of several megahertz width. Waters (1970) calculated 48 O$_3$-lines between 39.426 and 142.172 GHz of which 14 lines are masked by the O$_2$-MS. The 7 strongest O$_3$ lines are listed in Table 12, and the maximum values for the total zenith attenuation indicate that the vicinity of these lines probably should be avoided by ground-to-satellite links. On the other hand, one could "tune-in" on these resonances and use phase or amplitude variations of a transmitted signal as tracer for high-altitude ozone activity.

3.2.4 Atmospheric Noise Due to Molecular Attenuation.

Atmospheric noise reaching an antenna as a function of orientation and frequency is, as derived from radiative transfer theory, an integral over the temperature $T(\ell)$ and the attenuation rate $\alpha(\ell)$ along a ray path (see Sect. 3.2.1). A ground-based antenna "sees" the noise temperature (e.g., Blake, 1972)

$$T_A = \int_0^\infty T(\ell) \alpha(\ell) \exp \left[ -\int_0^\ell \alpha(s) ds \right] d\ell \quad [\circ K].$$

(46)

The tropospheric O$_2$-MS attenuation between about 55 and 65 GHz is so high (opaque atmosphere), that most of the noise originates from the lowest layers. The noise temperature equals approximately the surface temperature since the integrand of (46) is large at $\ell = 0$ and rapidly approaches zero for $\ell > 0$. At the band edges of the O$_2$-MS, the atmosphere becomes semi-transparent, and $T_A$ starts to drop when $A \lesssim 30$ dB (Eq. 43).
Figure 23. Model profiles of atmospheric constituents showing their relative concentrations versus altitude up to h = 90 km (after Waters, 1970). * denotes molecules with resonance spectra below 140 GHz (Tables 7, 12 and Sect. 2.5). Numerical values see Table 1.
\begin{align*}
(r_n + h_n) \cos \theta &= \eta \\
\Delta l_1 &= \sqrt{\eta^2 + (h_{n+1} - h_n)(h_{n+1} + h_n + 2r_o) - \eta} \\
\Delta l_2 &= \sqrt{\eta^2 + (h_{n+2} - h_n)(h_{n+2} + h_n + 2r_o) - (\eta + \Delta l_1)} \\
\Delta l_3 &= \sqrt{\eta^2 + (h_{n+3} - h_n)(h_{n+3} + h_n + 2r_o) - (\eta + \Delta l_1 + \Delta l_2)} \\
\Delta l_4 &= \ldots \text{ etc.}
\end{align*}

Figure 24. Algorithm used for the calculation of the differential slant path length $\Delta l_n$ between two shells of a spherically stratified atmosphere. $\theta$ - starting angle against zenith ($n = 151$ for $h = 0$ to 100 km).
Table 10. Zenith Oxygen Attenuation A from Sea Level.

<table>
<thead>
<tr>
<th>Frequency $\nu$ ± 1 MHz</th>
<th>Measured Values</th>
<th>Calculated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. in Fig. 25</td>
<td>No. of Samples</td>
</tr>
<tr>
<td>[GHz]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.000</td>
<td>W 1957 2</td>
<td>15</td>
</tr>
<tr>
<td>50.200</td>
<td>R &quot;</td>
<td>130</td>
</tr>
<tr>
<td>50.700</td>
<td>R &quot;</td>
<td>120</td>
</tr>
<tr>
<td>51.250</td>
<td>R &quot;</td>
<td>226</td>
</tr>
<tr>
<td>51.750</td>
<td>R &quot; 7</td>
<td>638</td>
</tr>
<tr>
<td>52.220</td>
<td>R &quot;</td>
<td>171</td>
</tr>
<tr>
<td>52.500</td>
<td>SW 1969 6</td>
<td>33</td>
</tr>
<tr>
<td>53.400</td>
<td>CMR 1968 1</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>R 1972</td>
<td>123</td>
</tr>
<tr>
<td>53.500</td>
<td>SW 1969 6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>CMR 1969</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>R 1972</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>R 1972</td>
<td>43</td>
</tr>
<tr>
<td>53.811</td>
<td>CMR 1968 1</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>R 1972</td>
<td>120</td>
</tr>
<tr>
<td>54.352</td>
<td>CMR 1968 1</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>CMR &quot; 1</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>R 1972</td>
<td>44</td>
</tr>
<tr>
<td>54.895</td>
<td>CMR 1968 1</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>R 1972</td>
<td>20</td>
</tr>
<tr>
<td>66.056</td>
<td>R &quot;</td>
<td>122</td>
</tr>
<tr>
<td>66.565</td>
<td>R &quot;</td>
<td>43</td>
</tr>
<tr>
<td>67.016</td>
<td>R &quot;</td>
<td>230</td>
</tr>
<tr>
<td>67.696</td>
<td>R &quot;</td>
<td>144</td>
</tr>
<tr>
<td>68.144</td>
<td>R &quot; 7</td>
<td>402</td>
</tr>
<tr>
<td>68.680</td>
<td>R &quot;</td>
<td>81</td>
</tr>
<tr>
<td>69 to 69</td>
<td>TS 1963 3</td>
<td>-</td>
</tr>
<tr>
<td>69.216</td>
<td>R 1972</td>
<td>154</td>
</tr>
<tr>
<td>69.732</td>
<td>R &quot;</td>
<td>81</td>
</tr>
<tr>
<td>70.0</td>
<td>C 1958 4</td>
<td>-</td>
</tr>
<tr>
<td>70.260</td>
<td>R 1972 7</td>
<td>447</td>
</tr>
<tr>
<td>72.0</td>
<td>H 1964 5</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^a\) W - Whitehurst et al., 1957
R - Reber, 1972
SW - Snider and Westwater, 1969
CMR - Carter et al., 1968

A survey of measured values of clear sky zenith attenuation for the frequency range 15 to 140 GHz has been made by Thompson III (1971; Fig. 9-11).
Table 11. Tangential Attenuation  
Tangential Phase Dispersion  
U. S. Standard Atmosphere 1962  

<table>
<thead>
<tr>
<th>Frequency, MHz</th>
<th>30Km</th>
<th>30Km</th>
<th>20Km</th>
<th>20Km</th>
<th>10Km</th>
<th>10Km</th>
<th>00Km</th>
<th>00Km</th>
</tr>
</thead>
<tbody>
<tr>
<td>49039</td>
<td>3.4978 + 360 rad</td>
<td>4.6254 - 002 dB</td>
<td>1.7364 + 001 rad</td>
<td>9.7549 - 001 dB</td>
<td>6.3603 - 004</td>
<td>1.4701 + 001 Total Attenuation, dB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49029</td>
<td>3.5064 + 000 rad</td>
<td>4.6490 - 002</td>
<td>1.7407 + 001 rad</td>
<td>9.7801 - 001 dB</td>
<td>6.4012 + 001</td>
<td>1.4771 + 001 Attenuation Rate, dB/km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49040</td>
<td>3.5156 + 000 rad</td>
<td>4.6736 - 002</td>
<td>1.7449 + 001 rad</td>
<td>9.8069 - 001 dB</td>
<td>5.4217 + 001</td>
<td>1.5006 + 001 Total Phase Dispersion, rad</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49050</td>
<td>3.5237 + 000 rad</td>
<td>4.6927 - 002</td>
<td>1.7492 + 001 rad</td>
<td>9.8336 - 001 dB</td>
<td>6.4423 + 001</td>
<td>1.5241 + 001 Phase Dispersion Rate, rad/km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49070</td>
<td>3.5323 + 000 rad</td>
<td>4.7152 - 002</td>
<td>1.7535 + 001 rad</td>
<td>9.9215 - 001 dB</td>
<td>6.4630 + 001</td>
<td>1.5476 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49130</td>
<td>3.5411 + 000 rad</td>
<td>4.7379 - 002</td>
<td>1.7579 + 001 rad</td>
<td>9.9593 - 001 dB</td>
<td>6.4837 + 001</td>
<td>1.5711 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49190</td>
<td>3.5516 + 000 rad</td>
<td>4.7608 - 002</td>
<td>1.7621 + 001 rad</td>
<td>1.0017 + 000 dB</td>
<td>5.5064 + 001</td>
<td>1.5946 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49240</td>
<td>3.5617 + 000 rad</td>
<td>4.7840 - 002</td>
<td>1.7665 + 001 rad</td>
<td>1.0066 + 000 dB</td>
<td>5.5299 + 001</td>
<td>1.6180 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49290</td>
<td>3.5719 + 000 rad</td>
<td>4.8073 - 002</td>
<td>1.7709 + 001 rad</td>
<td>1.0114 + 000 dB</td>
<td>5.5533 + 001</td>
<td>1.6404 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49360</td>
<td>3.5823 + 000 rad</td>
<td>4.8310 - 002</td>
<td>1.7754 + 001 rad</td>
<td>1.0163 + 000 dB</td>
<td>5.5767 + 001</td>
<td>1.6628 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49430</td>
<td>3.5934 + 000 rad</td>
<td>4.8564 - 002</td>
<td>1.7799 + 001 rad</td>
<td>1.0213 + 000 dB</td>
<td>5.6001 + 001</td>
<td>1.6852 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49500</td>
<td>3.6043 + 000 rad</td>
<td>4.8826 - 002</td>
<td>1.7840 + 001 rad</td>
<td>1.0263 + 000 dB</td>
<td>5.6235 + 001</td>
<td>1.7077 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49590</td>
<td>3.6152 + 000 rad</td>
<td>4.9092 - 002</td>
<td>1.7885 + 001 rad</td>
<td>1.0313 + 000 dB</td>
<td>5.6469 + 001</td>
<td>1.7302 + 001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49680</td>
<td>3.6264 + 000 rad</td>
<td>4.9374 - 002</td>
<td>1.7923 + 001 rad</td>
<td>1.0363 + 000 dB</td>
<td>5.6703 + 001</td>
<td>1.7527 + 001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total Phase Dispersion, rad
at $h_0$

Minimum Altitude $h_0$

Total Attenuation, dB
at $h_0$

Total Phase Dispersion, rad
at $h_0$

Phase Dispersion Rate, rad/km
at $h_0$

Total Attenuation, dB
at $h_0$

Phase Dispersion Rate, rad/km
at $h_0$
Captions for Figures 25 to 39

---Computer Plots---

a) Figure     

<table>
<thead>
<tr>
<th>h₁ [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
</tr>
<tr>
<td>27</td>
</tr>
<tr>
<td>29</td>
</tr>
<tr>
<td>31</td>
</tr>
<tr>
<td>33</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>37</td>
</tr>
</tbody>
</table>

Total attenuation $A$ and phase dispersion $\Delta T$ due to the O₂-MS between 49 and 72 GHz for a zenith path through the U.S. Standard Atmosphere (1962) from different initial altitudes $h₁$ to outer space. $P₀$ and $T₀$ identify the pressure and temperature conditions at $h₁$ (CMR-Model for $γ₁$ - Eqs. 32, 33).

b) Figure     

<table>
<thead>
<tr>
<th>h₀ [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
</tr>
<tr>
<td>28</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>32</td>
</tr>
<tr>
<td>34</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>38</td>
</tr>
<tr>
<td>39</td>
</tr>
</tbody>
</table>

Total tangential attenuation $A_t$ and dispersion $\Delta T_t$ due to the O₂-MS between 49 and 72 GHz for a straight-line path (see Fig. 24) through the total U.S. Standard Atmosphere (1962) for different minimum altitudes above ground $h₀$ (CMR-Model for $γ₁$ - Eqs. 32, 33).
Figure 25.

Zenith Attenuation $A$, dB

Frequency, GHz

$50, 55, 60, 65, 70$

$1, 2, 3, 4, 5, 6, 7$

Carter et al., 1968

See Table 10

$h_1 = 0$ km

$p_0 = 760$ torr

$T_0 = 15 ^\circ C$

Figure 25.
Tangential Attenuation $A_t$, dB

Frequency $\nu$, GHz

$h_o = 5$ km

Figure 26.
Figure 27.

- $h_1 = 5\text{ km}$
- $p_0 = 405\text{ torr}$
- $T_0 = -17.5\text{ °C}$
Figure 29. 

Carter et al., 1968

$h_1 = 10$ km  
$p_0 = 199$ torr  
$T_0 = -50^\circ$C

Zenith Attenuation $A$, dB

Frequency, GHz

Zenith Dispersion $\Delta T$, rad
Tangential Dispersion

Tangential Attenuation $A_T$, dB

$h_0 = 15 \text{ km}$

Figure 30.
Figure 31.
Figure 32.
Figure 33.

Zenith Attenuation $A$, dB

$A = 20$ km

$p_0 = 41.5$ torr

$T_o = -56.5$ °C

Zenith Dispersion $\Delta T$, dB
Figure 34.
Figure 35.
Figure 37.

- $h_1 = 30\ \text{km}$
- $p_0 = 9.0\ \text{torr}$
- $T_0 = -46.7^\circ\text{C}$
Tangential Dispersion $\Delta T_1$, $\text{rad}$

$\text{Tangential Attenuation } A_1, \text{ dB}$

$h_0 = 50 \text{ km}$

Figure 38.
Figure 39.
Table 12. Stronger Spectral Lines of Minor Atmospheric Gases in the 40 to 140 GHz Frequency Band

<table>
<thead>
<tr>
<th>Molecule (type of rotor)</th>
<th>Center Freq. ( \nu_0 ) ([\text{GHz}])</th>
<th>Max. Absorpt. ( A_0 ) ([\text{dB/km}])</th>
<th>Zenith Absorpt. ( A_0 ) ([\text{dB}])</th>
<th>Ground Level Concentration clean/polluted ([\text{ppm/vol}])</th>
<th>Elec. Dipole Moment ([\text{Debye or } \text{Asm}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>03 (asymmetr.)</td>
<td>96.2288</td>
<td>426 a</td>
<td>.16 *</td>
<td>0 to up to 100</td>
<td>3.336 ( \times 10^{-9} )</td>
</tr>
<tr>
<td></td>
<td>101.7368</td>
<td>803</td>
<td>.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>110.835</td>
<td>1230</td>
<td>.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>124.086</td>
<td>1750</td>
<td>.5</td>
<td>3 to 6</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>125.389</td>
<td>1030</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>136.883</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>142.172</td>
<td>2330</td>
<td>.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*) Calculated by Waters (1970) for U.S. Standard Atmosphere (1962) and daytime ozone distribution (max. 5 \( \times 10^{11} \) molecules/cm³ at \( h = 20\text{km} \)).

<table>
<thead>
<tr>
<th>Molecule (type of rotor)</th>
<th>Center Freq. ( \nu_0 ) ([\text{GHz}])</th>
<th>Max. Absorpt. ( A_0 ) ([\text{dB/km}])</th>
<th>Zenith Absorpt. ( A_0 ) ([\text{dB}])</th>
<th>Ground Level Concentration clean/polluted ([\text{ppm/vol}])</th>
<th>Elec. Dipole Moment ([\text{Debye or } \text{Asm}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂O (linear)</td>
<td>50.2460</td>
<td>20 a</td>
<td></td>
<td>0.5</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>75.3696</td>
<td>65 a</td>
<td></td>
<td>up to 10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100.4197</td>
<td>152 a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>125.6137</td>
<td>282 b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>150.735</td>
<td>480 b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO (linear)</td>
<td>115.2712</td>
<td>104 b</td>
<td></td>
<td>0.01 to 20</td>
<td>0.112</td>
</tr>
<tr>
<td></td>
<td>230.5380</td>
<td>838 b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SO₂ (asymmetr.)</td>
<td>12 lines listed between 53.529 and 131.015 GHz c)</td>
<td></td>
<td>0 to 0.02</td>
<td>up to 20</td>
<td>1.615</td>
</tr>
<tr>
<td>NO₂ (asymmetr.)</td>
<td>10 lines listed between 40.358 and 41.278 GHz c)</td>
<td></td>
<td>0 to 0.01</td>
<td>up to 5</td>
<td>0.316</td>
</tr>
<tr>
<td>CH₂O (asymmetr.)</td>
<td>8 lines listed between 45.063 and 140.839 GHz c)</td>
<td></td>
<td>0 to 0.01</td>
<td>up to 5</td>
<td>2.34</td>
</tr>
<tr>
<td>O₂ (linear, fine structure)</td>
<td>1⁺ (Table 7)</td>
<td>1.56</td>
<td>120 (Fig. 25)</td>
<td>209460 Magnetic Moment</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1⁻</td>
<td>6.06</td>
<td>( \approx 110 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Waters (1970)
b) French (1968)
c) Cord et al. (1968)
Equation (46) was evaluated numerically for $O_2$ and $H_2O$ attenuation, for
elevation angles $\theta = 0^\circ$ to $90^\circ$ and $h = 0$ over the frequency band $0.1$ to
100 GHz, by Blake (1972, Fig. 102). The sensitivity of millimeter wave
receivers is not limited by atmospheric but by detection noise. Uncooled
Schottky-diode mixers, for example, exhibit at 60 GHz presently single
sideband noise temperatures on the order of 1200 $^\circ K$.

In radio communication, data are transmitted through the atmos-
phere in such a way that the system channel parameters are matched to
the propagation characteristics of the medium to maximize received
data; in passive remote sensing, the medium itself provides the data.
Meeks and Lilley (1963) introduced the idea of using $O_2$-MS emission to
remotely sense the temperature structure of the lower ($h>30$ km)
atmosphere from an orbiting satellite. Their work has been extended
by Lenoir (1968) and Croom (1971) and was applied by Westwater (1970)
and Waters (1970, 1973) to ground-based radiometric sensing. All
these treatments take advantage of microwave spectroscopic properties
of $O_2$ and other atmospheric gases and their dependences upon frequency
and meteorological conditions, which has been the subject of this report.

Finding the temperature $T(\ell)$ in equation (46) as a function of an
independent variable [e.g., $p(h)$] from observations of $T_A$ at selected
frequencies requires numerical inversion methods. The reliability
of the inversion depends to a great extent on the accuracy to which
$\alpha(\nu, p, T, H, r_k)$ due to the $O_2$-MS is known. A general descrip-
tion of the atmospheric $O_2$-MS can be accomplished only by theory.
However, existing data are insufficient for a proper test of the available
theory. More laboratory work on the intensity distribution of the atmos-
pheric $O_2$-MS is required under careful control of conditions (Meeks a:
4. CONCLUSIONS

The review of molecular theory for millimeter wave properties of air, its translation into engineering terms (Section 2), the development of analytical schemes to predict propagation characteristics on the basis of meteorological phenomena, and the description of computer routines together with many examples of unique atmospheric transfer properties (Section 3) shall provide a sound basis for the development of future systems operating in the 40 to 140 GHz frequency range.

The main emphasis has been placed on the atmospheric $\text{O}_2$-MS which dominates transfer properties throughout the 40 to 140 GHz band and existing gaps in the knowledge of the spectrum have been identified. It was attempted not to seriously violate real atmospheric situations; however, over the full range of meteorological variables, it remains to be established how reliable the discussed analytical schemes really are. Presently, the correlation between atmospheric attenuation and phase dispersion rates on one hand and meteorological conditions (Eq. 3) on the other hand is not very accurate. The correspondence between both sets of data needs to be improved by inputs from controlled laboratory measurements on isolated line parameters and continuum spectra data. A manageable theory should be brought forward which predicts the continuum spectrum based on properties of the line spectrum, circumventing more or less arbitrary empirical models.

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