MODULATION OF COSMIC-RAY PROTONS AND HELIUM NUCLEI NEAR SOLAR MAXIMUM

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ABSTRACT

Balloon observations of proton and helium spectra in 1970, 1971 and 1972, which extend previous work for the period 1965 to 1969, reveal a factor of ~1.85 deviation from a single valued regression at low rigidities. This deviation decreases with increasing rigidity for both species. The period 1969-1970 is unique because time variations at low and high energies were anti-correlated. When satellite observations are used to extend the balloon observations to energies below 100 MeV/nucleon, the proton spectrum showed a steeper slope in 1970 and 1972 than the characteristic $J = AT$ spectrum observed during 1965-1969. The slope of the helium spectrum became continuously flatter during the same period (1970-1972). Computer generated spectra based on simple two parameter modulation models describe the basic features of the observations if one of the variable parameters is used to characterize the rigidity dependence of the diffusion coefficient. On the other hand, models which do not allow such a variation are not consistent with the observations.

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I. INTRODUCTION

This paper reports new observations of cosmic-ray protons and helium nuclei during 1970, 1971 and 1972 which extend and complement similar observations for the period from 1965 to 1969 (Rygg and Earl, 1971). The new data reported here can be compared directly with the earlier results because they were obtained with the aid of the same balloon-borne hodoscope and because they were reduced using the same computer programs and analysis procedures. To facilitate reference to the earlier report, it will be designated hereafter as Paper I. The interested reader will find there details of the instrumentation and analysis. For the present purposes, the response of the detector can be summarized as follows: Differential spectra between 66 and 254 MeV/nucleon were obtained from measurements of the ionization and range of stopping protons and helium nuclei. The flux of penetrating protons between 254 and 720 MeV was obtained from ionization measurements alone. Integral fluxes of protons above 720 MeV and of helium nuclei above 254 MeV/nucleon also were measured. Luhmann and Earl (1973) have reported on electrons identified by the detector with energies between 15 and 150 MeV. Taken as a whole, the hodoscope results on protons and helium nuclei for the period 1965 to 1972 constitute a consistent set of data suitable for the detailed study of cosmic-ray modulation over a period extending from solar minimum to 3 years past solar maximum.

The solar modulation of cosmic-ray intensity has been studied for more than 40 years (Neher, 1971; Forbush, 1954). Data from sea level detectors, established the existence of temporal variations inversely
correlated with the 11 year solar cycle, but, within the last two decades, direct observations of low energy primaries from balloons and satellites have greatly advanced our understanding of the mechanisms which give rise to solar modulation. In particular, these studies have led to widespread acceptance of the basic idea, embodied in the diffusion convection model of Parker (1958, 1963), that the observed depression of cosmic-ray intensity near earth occurs because galactic particles are swept out of the solar system by the solar wind. However, the same process of scattering by magnetic irregularities that leads to the sweeping effect also leads to adiabatic deceleration, a continuous loss of energy which was first discussed by Laster, Lenchek and Singer (1962) and later incorporated into the diffusion convection picture by Parker (1965). The importance of adiabatic deceleration was emphasized by the finding, in Paper I, that the modulated spectrum of protons displays at low energies a characteristic behavior such that the differential intensity $J$ remains directly proportional to increasing kinetic energy $T$ even though the spectral intensities change by large factors during half a solar cycle. Thus the differential intensity spectrum can be described by $J = AT$ where $A$ is a parameter independent of $T$ but which varies with time. This behavior can be understood only if energy loss plays an important role in shaping the observed spectrum. Another phenomenon which was reviewed by O'Gallagher (1972) and which is closely related to cosmic-ray modulation is the presence of intensity gradients within the solar system.

Figure 1 illustrates how the effects of interest here can be described in terms of the phenomenology that is frequently invoked in
discussions of modulation. Figure 1a shows, in schematic form, regression plots in which intensities at low energies, recorded, for example, by a counter telescope, geiger counter or ionization chamber carried on a balloon or satellite, are plotted against the rate recorded simultaneously by a detector sensitive to much higher energy particles such as a neutron monitor or ground level ionization chamber.

If the depression of cosmic ray intensity depends at all energies upon only one variable, then the relationship between changes at high energies and changes at low energies may or may not be simple, but it is certainly single valued. In this case, points on the regression plot should single cluster on a well defined curve. In actuality, observations over a limited portion of the solar cycle are remarkably consistent with unique regression curves which describe not only the overall long term variations but also short term changes due to Forbush decreases and other transient effects. (See Webber (1967) for an extensive compilation of single-valued regression curves for many components studied over a wide range of energies.) Moreover, attempts to understand the observed form of the regression curves in terms of physical theories characterized by a single parameter have met with some success (Gloeckler and Jokipii, 1967).

An analysis by Simpson (1964) first revealed that the relationship between neutron monitor rates and sunspot numbers is not single valued. Subsequent observations at solar minimum showed that the intensity of low energy cosmic rays measured by geiger counters and ion
chambers on satellites is not simply correlated with neutron monitor rate (Balasubrahmanyan, Hagge, and McDonald, 1968; Kane and Winckler, 1969). On the other hand, the effect was found not to be significant when the rates of neutron monitors with different cut-off rigidities were correlated (Simpson and Wang, 1967). O'Gallagher (1969) showed that the satellite observations could not be accounted for by changes in the spectra of low energy protons and helium alone. He suggested that variations in the intensity of low energy electrons, which also contributed to the rates measured by the satellite detectors, might account for some of the deviation. Recent observations of the electron spectrum (Schmidt 1972; Burger and Swanenburg 1973) and of low energy protons and helium nuclei (Van Hollebeke, Wang and McDonald, 1972) have made it evident that the regression relationship for the current declining phase of solar activity, which is indicated schematically in Figure 1 by the curve DEA, is different from that traced out along ABC during the increasing phase. This deviation from a single-valued relationship is conventionally described as a "hysteresis effect" but the significance of this behavior is simply that the phenomenon of cosmic-ray modulation depends upon more than one variable.

As an alternative to the regression analysis, the intensities can be plotted as functions of time. In this case, the hysteresis effect appears, in Figure 1b, as a phase lag between two periodic variations. This lag could be due to an actual temporal delay analogous to the phase difference between voltage and current in a reactive component.
Here the time derivative of solar activity could be identified, somewhat artifically, as a second variable associated with deviations from a unique regression relationship. Evidently the large derivatives of short term fluctuations would have to be averaged out in the definition of this variable. On the other hand, the relationship could be (phenomonologically) analogous to that between B and H in a magnetic material where the phase lag is independent of the frequency of sinusoidal excitation and depends only upon the shape of the hysteresis curve. In this case, the second variable would be characterized by the phase of the sinusoid.

The behavior can also be described in terms of energy spectra. To illustrate this point of view, Figure 1c shows spectra for each of the lettered points A - E in Figure 1a and 1b. Here, as is exemplified by curves B and E which coincide at high energies, hysteresis appears as a depression of the spectrum observed at low energies during the increasing phase of solar activity from that observed during the decreasing phase.

Schmidt (1972) has noted that the intensity of low energy electrons decreased from 1969 to 1970 at a time that neutron monitor rates were increasing. An objective of this paper is to demonstrate that low energy protons exhibited a similar behavior and to show that the portion CD of the regression curve traced out during this period was a well defined region of negative slope. The significance of this behavior is not evident but any valid theory of solar modulation must explain
the fact that the hysteresis loop is closed at solar maximum by a finite segment of negative slope rather than at a cusp like that indicated at point A in Figure 1a. It is worth noting that, for magnetic materials, regions of negative slope in the hysteresis curve do not occur because they correspond to an energetically unstable condition. The spectra presented in Figure 1c for times C and D exhibit a crossover at point X somewhere between the high energy and low energy regions. The observations reported in the following section show that significant spectral variations take place on the short time scale associated with the interval between two flights in the same month for both 1969 and 1970. These spectral variations are characterized by a cross-over similar to that described above.

The presentation of new observations and the discussion that follow are structured according to the ideas discussed above and illustrated by Figure 1. In Section II, proton and helium spectra observed in 1970, 1971 and 1972 are presented in the same format employed in Paper I. Section III contains a simplified treatment of "hysteresis" which considers models characterized by only two variable parameters. The modulation parameter \( \eta \), introduced by Silberberg (1966), is adopted as one parameter. The second parameter is taken to be either the rigidity dependence of the diffusion coefficient or the size of the modulation region. Variations in either of these lead to changes in the low energy spectrum with \( \eta \) held constant, but only the former case shows good quantitative agreement with observations.
II. RESULTS

The series of balloon flights described in Paper I, which began in 1965, has been extended with two flights in 1970, two in 1971 and one in 1972. Detailed information is summarized for these five flights in Table 1. Integral fluxes of penetrating protons and helium nuclei are presented in Table 2. Differential spectra for protons and helium nuclei are given in Table 3. These results are presented graphically in Figures 2-8 where new data are all designated by open symbols and where data reported in Paper I, included for comparison, are represented by solid symbols.

The analysis of proton measurements during Flights 1293 and 1330 is intricate, because solar particles were present. Flight 1293 is a relatively simple case because solar protons were present only late in the flight. Consequently, it is believed that the data reported here, which refer to the eight hours at ceiling prior to the solar onset, are free of contamination. In contrast, Flight 1330, which was launched during the active period preceding the great solar event of 1972 August 7, recorded not only an abrupt onset of particles from a flare but also a persistent flux attributed to another flare which occurred on the back side of the sun three days before the flight. The former contamination was easily eliminated by restricting the analysis to data recorded during the five hours at ceiling before the influx. To estimate the latter persistent contribution, the steep solar proton spectrum recorded between 20 and 80 MeV by the Goddard Space Flight Center experiment on IMP V was extrapolated upward to the energies of concern using a power law dependence on energy. Unfortunately, simultaneous data are not available
because the satellite entered the radiation belts just as the balloon reached ceiling, but the flux observed on IMP varied by only 25% during the 12 hours preceding the loss of data. Consequently, the extrapolated solar proton spectrum derived from the six hours preceding the flight, which is shown as a solid line in Figure 2, is thought to be a valid approximation to that present during the first five hours of Flight 1330. In any case, the solar contribution was negligible above 170 MeV. In the interval 102-134 MeV (Range 4), the estimated solar intensity was \(0.6 \pm 0.3\) protons/\(\text{m}^2\) sec sterad while that in 141-167 MeV (Range 5) was \(0.2 \pm 0.1\) protons/\(\text{m}^2\) sec sterad. These corrections were subtracted from the 1972 spectrum, plotted as open circles in Figure 2, but the corresponding entries in Table 3 include the solar contribution. In view of these corrections, the spectra graphed in Figures 2-8 can all be taken as representative of the modulated intensity of galactic cosmic rays.

The spectra corresponding to a regression curve of positive slope "nest" without crossing as is indicated in Figure 1c. This behavior is evident in Figure 2 where spectra for 1972 (dashed curve representing Flight 1330) and 1971 (dotted curve representing Flights 1308 and 1316 combined) are compared with the spectrum observed in 1970 (heavy solid curve representing Flight 1297) which was more deeply modulated at low energies than any other recorded during the whole series of flights. These three spectra, which correspond, respectively, to curves A, E and D in Figure 1c, embody a progression toward larger fluxes opposite to the decreasing progression shown for 1965 to 1969 by Figure 11 of Paper I.
In Paper I, spectra of the form $J = AT$ which fit the balloon data above 100 MeV also gave excellent agreement at low energies with satellite data reported by the University of Chicago. In contrast, the dotted and dashed straight lines in Figure 2, which describe spectra with $A = 0.004$ and 0.0105 particles/cm$^2$ sec. sterad (MeV)$^2$ respectively and which fit the balloon data, lie well above the comparable satellite data points for 1971 and 1972 (Garcia-Munoz, et al., 1973, J.A. Simpson, private communication). This difference cannot be attributed to a time variation, because in both 1971 and 1972, the average Deep River neutron monitor rate (Steljes, 1970), over the duration of the satellite measurements was within 1.4% of the neutron rate on the days of the flights. Furthermore, the difference is not of instrumental origin, because the GSFC data for July 1972 (Van Hollebeke, et al. 1973) (crosses in Figure 2) confirm the intensity level obtained by the Chicago group and because the hodoscope results in 1969 were in good agreement with those obtained by the same satellite instrument that recorded the 1971 and 1972 Chicago spectra (See Paper I and Figure 3.) Thus, we conclude that the proton spectrum has a steeper positive slope during the decreasing phase of solar activity than it does during the increasing phase when the $J = AT$ form applies. More specifically, the thin solid lines in Figure 2 which represent spectra with $J \propto T^{1.4 \pm 0.1}$ provide good fits to the 1971 and 1972 proton data.

To emphasize the spectral behavior associated with a regression relationship of negative slope and to exhibit the crossover phenomenon illustrated in the introduction by curves C and D of Figure 1c, Figure 3 shows proton spectra recorded during Flights 1293 and 1297 made in 1970 and Flights 1273 and 1274 made in 1969. In Paper I, proton data from the latter flights were combined to form the spectrum reported for 1969, but these spectra are plotted separately here. To avoid overlap, spectra for different
years are displaced by one decade. The two 1970 flights took place late in the recovery phase of a Forbush decrease that began on July 1. Over the 10 day interval between them, the Deep River neutron rate increased by 1.7%. At the same time, a significant decrease was observed in the differential spectrum of protons below 260 MeV while the flux at 260 MeV remained the same within errors. This decrease at low energies manifests the anticorrelation with intensities at high energies and exhibits the crossover behavior discussed in the introduction. The crossover energy is at or slightly above the highest differential energy interval covered by our instrument. It is evident in 1969 that Flights 1273 and 1274 exhibited the same pattern as 1293 and 1297 except that the 0.8% increase in neutron rate over 7 days was not associated with a Forbush decrease and the point at which the two spectra crossed was at a higher energy. As was discussed in Paper I and as was mentioned above, the relationship \( J = AT \) which characterizes the behavior of modulated cosmic rays was confirmed in 1969 at energies below those accessible to balloons by satellite observations plotted as crosses in Figure 3 (Hsieh, Mason and Simpson, 1971). Comparable satellite data for 1970 have not been published.

Because the proton spectra accurately maintained the \( J = AT \) form throughout the period, from 1965 to 1969, the parameter A had special significance because it was independent of energy. Although the spectra observed in 1971 and 1972 cannot be described over a wide range of energies by a unique value of A, the deviations from \( J = AT \) within the relatively narrow range of energies over which the balloon experiment recorded a positive slope are unimportant compared to statistical uncertainties. Consequently, to facilitate
comparison with the previous work, it is appropriate to characterize the spectrum within this region by an empirical parameter \( A \) computed as if \( J = AT \) from the flux measured between the air cutoff at \( \sim 100 \) MeV and the change in spectral slope at 150 to 250 MeV. In Figure 4, where this parameter \( A \) is plotted against Deep River neutron rate, four points derived from the spectra in Figure 3 define a regression relationship of negative slope (dotted curve) which corresponds to the segment CD in Figure 1a. Because of its unique character, this inverse relationship will be discussed before the conventional hysteresis effect is considered. The latter effect appears, as in Figure 1a, as the separation between segment ABC, defined in Figure 4 by the data for 1965-1968 from Paper I (solid line), and segment DEA, defined by the new 1970, 1971 and 1972 points (dashed line).

The inverse relationship is characterized by a persistent, well defined correlation rather than, for example, by random transitions between parallel regression curves of positive slope. In Figure 4, the straight segment which connects the two values of \( A \) measured in 1969 (solid points) lines up on the dotted curve with the corresponding segment defined by the 1970 measurements (open points). This identity in both slope and magnitude between short term variations separated by one year is to be expected if a unique relationship exists, but it would be unlikely to occur accidentally. The observations of Lockwood, Lezniak and Webber (1972), which refer to integral flux above 60 MeV per nucleon, do not display the inverse relationship emphasized here. However, the very weak dependence
upon neutron rate exhibited by this flux between June 1969 and June 1971 is to be expected when a few low energy particles following a steep inverse relationship are mixed together with higher energy particles having a positive correlation. Thus, the presence of an inverse relationship at low energy is consistent with the fact that two cycles of short term intensity variations were observed by these authors to track on a single regression line characterizing this weak dependence. The behavior of low energy electrons (Schmidt, 1972; Burger and Swanenburg, 1973) is also consistent with an inverse relationship. These facts single out the period from June 1969 to June 1971 as a unique episode of the modulation cycle. There is no obvious explanation for the behavior of modulated cosmic rays during this period. However, one point deserves special emphasis: Not only did the low energy intensity respond to interplanetary conditions in a direction opposite to that seen at high energies but also it was extraordinarily sensitive to these conditions as is indicated by the steep slope of the inverse relationship in Figure 4 and by the fact that the large changes which were documented in Figure 3 took place in a few days.

Regardless of its interpretation, the inverse regression relationship along segment CD constitutes a transition between the segment ABC which specifies the regression relationship which applies during the increasing phase of solar activity and segment DEA which applies during the decreasing phase. In the discussion that follows, the hysteresis between these two regions of positive slope will be characterized by the ratio, at a given
neutron rate, of the low energy intensity on segment ABC to that on segment DE. This hysteresis ratio is remarkably constant over the range of Deep River neutron rates from 6200 to 6800. In Figure 4, only the values of A from Flight 1297 (Figure 3) and from the 1971 flights (1308 and 1316) lie on segment DE while that recorded during Flight 1330, which lies very close to those obtained in 1965, shows that the hysteresis loop had closed in July 1972. In spite of this limited coverage by balloons, the relationship along DE is well documented by the satellite data of Van Hollebeke et al. (1972) which embody at lower energies the same qualitative picture presented in Figure 4.

The segments ABC and DE in Figure 4 lie parallel and separated by a factor $1.85 \pm 0.1$. (Note the logarithmic scale on the ordinate.) This ratio is 35% smaller than the factor of 2.6 reported by Van Hollebeke et al. (1972), but an energy dependence of exactly this amount would result from the fact, illustrated in Figure 2, that the proton spectrum has a steeper positive slope along DE than it does along ABC.

Obviously, the hysteresis ratio decreases with increasing proton energy, for it must ultimately be unity for the particles whose secondary neutrons are counted. This tendency is illustrated in Figure 5 where the flux of penetrating protons is plotted vs. neutron rate for two energy groups, fast protons, $E > 720$, and slow protons, $260 < E < 720$ MeV. The hysteresis ratios for slow and fast protons are $1.5 \pm 0.08$ and $1.07 \pm 0.07$, respectively, Note that the intensities recorded in 1972 lie on the solid curve for 1965-1969. Consequently, these points were not included in determining the dashed 1970-71 curves which yield the above ratios.
The results on helium extend and complement the proton data presented above. The spectra presented in Figure 6 are in good agreement with satellite results obtained by the Chicago group. In particular, the 1972 spectrum connects smoothly with the flat helium spectrum reported by Garcia-Munoz et al. (1973). The spectrum in 1970 confirms the finding of Paper I that the helium spectrum approaches at solar maximum the characteristic behavior $J = AT$. However, note that the 1970 spectrum lies below the 1969 spectrum above 160 MeV. The 1969 and 1970 proton spectra indicated in Figure 3 exhibit a similar effect in which the greatest depression appears between 200-300 MeV.

In the light of these observations of a power law spectrum of helium at solar maximum ($J \propto T^{1.0}$) and near solar minimum ($J \propto T^{0.2}$), the spectrum measured in 1971 is significant because it is also a power law of intermediate slope ($J \propto T^{0.8}$). This behavior was also seen in 1965 ($J \propto T^{0.5}$) and in 1967 ($J \propto T^{0.75}$) when the spectrum was nearly identical to that seen in 1971. From the present point of view, the flat slope of the 1972 spectrum may be regarded as only an extreme case in a continuous sequence of possible slopes rather than as an anomaly. It is certainly worth noting and it may be significant that the modulated spectrum of deuterium displayed, between 1967 and 1969 exactly the same sort of steepening that was noted here for helium (Hsieh, Mason and Simpson, 1971, Figure 5).

The hysteresis effect for helium in the region of positive spectral slope is illustrated in Figure 7 which shows regression plots vs. neutron rate of the helium intensity within energy windows from 100 to 150 MeV/
nucleon and from 200 to 250 MeV/nucleon. The hysteresis ratio of 1.25±0.1, obtained as before from the displacement between the solid and dashed curves in Figure 7, is the same for both windows but it is significantly smaller than the ratio of 1.85 obtained for protons at comparable energies. At 60 MeV/nucleon, Van Hollebeke, et al. (1972) obtained a hysteresis ratio of 2 for helium. The fact that this number is larger than that obtained here at higher energies is similar to the situation described above for protons. It can be described as a relative steepening of the helium spectrum, analogous to that of protons, during the decreasing phase of solar activity.

Penetrating helium with energy greater than 260 MeV/nucleon displays very little hysteresis. The hysteresis ratio for this component is 1.05±0.05 as can be seen in the regression plot shown in Figure 8.
III. Two Parameter Models for Hysteresis

In many of the traditional treatments of solar modulation it has been customary to characterize the changes in observed spectra by a single time variable parameter (e.g. Silberberg, 1966, Gloeckler and Jokipii 1967, Ormes, Webber 1968, Gleeson and Axford 1968b, Hsieh, 1970) which has often been referred to as the "modulation parameter" $\eta$. As was discussed in the introduction, the existence of "hysteresis" effects in the modulated spectra demonstrates that such a formulation is inadequate and more than one variable parameter is required. In an effort to isolate the physical processes involved we have investigated the effect on calculated modulated spectra produced by separately varying the other basic physical parameters of the modulation region one at a time while holding the traditional parameter $\eta$ constant. In this way we have limited consideration to cases involving no more than two time variable parameters in a very simple model.

In the interest of simplicity we have deliberately avoided introducing any more parameters than are absolutely necessary in a diffusion convection model (Parker 1963, 1965; Jokipii 1971) and have considered only four basic quantities. These are: (a) the solar wind velocity $V$, (b) the effective depth of the modulating region $R$, (c) the diffusion coefficient $K_0(P_0)$ at some reference value of magnetic rigidity $P_0$ and (d) the exponent $n$ of the magnetic rigidity in an assumed power law dependence given by

$$K(P, B) = \frac{\beta}{\beta_0} K_0 \left( \frac{P}{P_0} \right)^n$$  \hspace{1cm} (1)

where $\beta$ is the particle velocity with respect to the velocity of light and $\beta_0$ its value at the reference rigidity (evaluated for protons).
Although electron modulation studies have indicated that the dependence in (1) above cannot extend to the lowest rigidities and in a complete model one would need at least one additional parameter, the simple form shown is sufficient for the range of rigidities considered here.

The definition of the traditional modulating parameter $\eta$ is given by

$$\eta(t) = \int_{\text{lau}}^{R} \frac{Vdr}{K_0}$$

(2)

For our simple model we retain $\eta$, defined at $P_o = 10$ GV to be one of the two time variable parameters. At this value of $P_o$ hysteresis effects are small and $\eta$ characterizes the modulation at all higher rigidities. For our choice of the second variable parameter we have limited consideration to two cases in which three of the four physical quantities listed above are varied. (The three quantities involved in the definition of $\eta$ are not independent if $\eta$ is held constant.) First of all we will consider the effect of variations in $n$ holding $R$, $V$, and $K_o$ (and therefore $n$) constant. The physical basis for this choice is that $n$ can be related to the power spectrum of interplanetary magnetic field fluctuations (Jokipii 1967, 1971) which has been observed to vary on a short time scale (Jokipii and Coleman 1968, Sari and Ness, 1969). Secondly we will consider the effect of a variation in $R$ holding $n$ and $\eta$ constant. In this case holding $\eta$ constant while varying $R$ requires that either $V$ or $K_o$ be varied simultaneously. We have accomplished this by allowing $K_o$ to vary while holding $V = 400$ km/sec. The physical basis for choosing $R$ as an independently variable parameter is that adiabatic deceleration effects depend on the length of time particles spend in the expanding solar wind which in turn
depends on $R$. We have not investigated the effect of a separate variation in $V$ since observations over a solar cycle indicate very little, if any, systematic variation which could account for the large observed solar modulation.

Some recent models (Van Hollebeke, et al. 1972, Burger and Swanenburg, 1973; Bedijn, Burger and Swanenburg, 1973) have considered the variation of several parameters in addition to those we consider here, for example, parameters which characterize the radial dependence of the diffusion coefficient and its rigidity dependence and the rigidity dependence of the effective size of the modulating region. Any one or all of such variables may change with time in such a way as to produce hysteresis and models based on these parameters can be made to fit the observations. In these studies no attempt was made to isolate the effect of separate variation of individual parameters and furthermore there is no direct physical basis for distinguishing between such models since they all fit the observations.

For comparison with the calculated spectra, the basic features of the observed hysteresis effect are illustrated in Figure 9 which presents the differential spectra observed in our balloon data for the years 1966, 1969 and 1971. In plotting these spectra the characteristic response energy for the Deep River Neutron intensity has been taken to be approximately 10 GeV. The integral flux point for "fast protons" $>720$ MeV has been plotted by fitting the exponent of a power law spectrum between .7 and 10 GeV to the observed flux under the further assumption that the exponent for $E > 10$ GeV is -2.5. The years 1966 and 1971 were chosen since the neutron intensities for flights made at these times are different
by only \( \approx 1\% \), therefore the spectra at and above 10 GeV are essentially the same (i.e. \( \eta \) is the same). On the other hand for energies <10 GeV the 1971 spectrum shows that the intensities have not recovered fully from the low value at solar maximum in 1969. The 1969 spectrum is shown for reference corresponding to a time when the neutron intensity was more than 9% lower than in 1966 and 1971. Effectively the spectra for 1966 and 1971 are an empirical realization of curves B and E respectively in Fig. 1c discussed in the introduction. To a very good approximation, all three spectra at low energies are consistent with the \( J = AT \) behavior characteristic of the period 1965-1969 discussed in Paper I. As we have discussed with respect to Figure 2, the extension of the 1971 spectrum to lower energies based on satellite data from the University of Chicago group is more consistent with a steeper spectrum \( (J \propto T^{1.4}) \) over the range from 50-250 MeV. However as we have noted in Section II, the parameter \( A \) continues to be a useful quantity to characterize the level of the modulated spectrum between 100 MeV and \( \approx \) 250 MeV above the range of satellite observations. In particular, the value of 1.85 for the ratio of the values of \( A \) corresponding to the lines through the low energy 1966 and 1971 spectra corresponds to the same ratio indicated in Figure 4 for the two long segments of the regression loop. In comparing these observations with the calculated spectra, it is this feature, namely a gradual convergence of two spectra separated by a factor of 1.85 at low energies to the same spectra at high energies (\( \approx 10 \) GeV), that we wish to reproduce quantitatively.

For our calculations we have used computer solutions to a Fokker-Planck equation describing the modulation in terms of diffusion-convection with energy loss effects included (Parker 1965) which has been developed
further by others (Gleeson and Axford 1968a, Fisk and Axford 1969). The computer techniques have also been described in detail (Fisk 1971) and need not be discussed in detail here. It should be borne in mind however that the solutions have been obtained under the assumptions of spherical symmetry and quasi-stationary conditions. The procedure was as follows. In each case, a reference spectrum was calculated by modulating an assumed interstellar spectrum using a value of \( \eta \) which gives a good fit to the 1966 spectrum in Figure 9. For an interstellar power law in total energy we have used \( R = 3 \) a.u., \( V = 400 \) km/sec and \( k_o (10^{6} \text{ GeV}) = 3 \times 10^{22} \) cm\(^2\)/sec and \( \eta = 1 \).

Since the model we are using is an idealized one, the values assigned to the model parameters are for illustration only and are not meant to be taken literally. Rather they exhibit the effect of changes in particular parameters for a range of variation which might correspond to modulation over a solar cycle. For instance, note that the quantity \( R \) is simply a parameter representing the effective size of the modulation region assuming a constant diffusion coefficient \( K \). If \( K \) increases with increasing heliocentric range the actual distance to the modulation boundary could be substantially greater than the value of the parameter \( R \). Therefore the value of \( R = 3 \) a.u. used for illustration in the reference spectrum corresponding to near minimum solar modulation should not be interpreted as being in conflict with Pioneer 10 measurements showing no boundary to beyond 5 a.u. in 1974 (which is also a time when there was substantially more modulation than in 1966 which our reference spectrum approximates).
Our analysis can be reduced to two cases in which the parameters $n$ and $R$ are varied as described below, keeping the value of $n(10 \text{ GV})$ and thus the high energy spectrum constant at all times.

Case A. The effect of variation in $n$. Calculated spectra for the reference spectrum ($n = 1$) and values of $n = 1.1$, $1.2$ and $1.3$ are shown in Figure 10. Any pair of these calculated spectra can be seen to be qualitatively consistent with curves B and E of Fig. 1c so that variation in $n$ does produce a hysteresis effect. Comparison of the corresponding spectra for 1966 and 1971 in Fig. 9 with these calculated spectra shows that a value of $\Delta n \gtrsim 0.15$ is most consistent with the observed effect. The calculated curves for the reference spectrum (dotted line) with $n = 1.0$ and a spectrum with $n = 1.15$ (dashed line) are included in Figure 9 to illustrate this agreement. The observed spectra maintain the characteristic $J = AT$ form to somewhat higher energies than the smooth calculated curves but the overall agreement is good. The value for $\Delta n$ obtained here by comparison of observed proton spectra is close to the value of $\sim 0.2$ derived for essentially the same parameter by VanHollebeke et al. (1972) from the hysteresis loops for $60 \text{ MeV}$ protons and helium with respect to the Deep River Neutron Monitor Intensity.

Thus we see that a systematic small increase in $n$ between the epoch of increasing solar activity and that of decreasing solar activity could produce the observed hysteresis. This would correspond physically to a steeper magnetic field power spectrum in time of increasing solar activity than in periods of decreasing activity. Clearly the calculated spectra are very sensitive to small changes in $n$ which may explain why the corresponding change required to account for the observations has not been detected.
It should be noted that all models which result in a change in the effective average diffusion coefficient as a function of rigidity are equivalent to the case discussed. For example if the effective size of the modulating region is a function of magnetic rigidity and time separately, curves identical to those in Fig. 10 can be derived. However there is no basis for attributing any more physical significance to such a model than to the case discussed here.

Case B. The effect of variations in R. The effect of differing energy losses in regions of greatly differing sizes is illustrated in Fig. 11. Here the same reference spectra as in Fig. 10 with \( R = 3 \) a.u. is compared with the modulated spectrum in a modulating region 10 times as large with \( R = 30 \) a.u. (upper set of curves) with the modulation at 10 GV \((n)\) and value of \( n \) maintained constant \((n = 1)\) for both spectra. Also shown in Fig. 11 (lower curves) are calculated spectra for \( R = 3 \) and 30 a.u. obtained by modulating an interstellar spectrum which is a power law in kinetic energies and steeply rising towards lower energies. These latter spectra require considerably more modulation to be consistent with the observed 1966 spectrum.

The effect of variation of \( R \) can be summarized as follows:

1. Spectral differences are produced in modulating regions differing only in their size. The effect is most significant in the case of an interstellar spectrum which is steep at low energies.

2. Physically the effect manifests itself as a depletion of particle intensity in the larger region at higher energy and an enhancement at lower energies and thus is consistent with the effect of more pronounced energy losses. However this dependence on energy is
qualitatively unlike the observed hysteresis between the 1966 and 1971 spectra since these spectra which coincide at high energies do not cross at lower energies.

3. The magnitude of the effect is insufficient to account for observed long term hysteresis even for the case of the steep interstellar spectrum.

Thus it is evident that the use of R as the second variable modulation parameter does not explain what is conventionally regarded as the hysteresis effect in any straight forward manner. However the behavior emphasized by point (2) above does produce an effect quite similar to that observed during the period of negative slope in the regression plots (segment CD in the regression plots). Compare for instance, the calculated curves in Figure 11 with the observed spectra in Figure 2. This emphasizes that this mechanisms or some other phenomena which affects the relative importance of energy loss processes should be considered seriously as part of the general problem of understanding the time behavior of modulated spectra.

It should be noted that in case B we are simply comparing the modulated solutions with two strongly differing radial dependences of \( K_o \) (i.e. \( K_o(r) = 3 \times 10^{22} \text{ cm}^2/\text{sec} \) for \( r < 3 \text{ a.u.} \) and \( K_o(r) = \infty \), for \( r > 3 \text{ a.u.} \) versus \( K_o(r) = 4.5 \times 10^{23} \text{ cm}^2/\text{sec} \) for \( r < 30 \text{ a.u.} \) and \( K_o(r) = \infty \) for \( r > 30 \text{ a.u.} \)). Therefore our result shows that variations in the radial dependence of the magnitude of \( K \) while maintaining fixed dependence on magnetic rigidity (constant \( n \)) will not account for hysteresis. Note in particular that a change in the magnitude of \( K \) in the inner solar system which propagates outward with the solar wind velocity can be
though of as a radial dependence of $K_0(r)$ which is changing in time and therefore can be reduced to the idealized example of our case B. It follows then that such a mechanisms cannot by itself be responsible for hysteresis. It is necessary to postulate some variation in the rigidity dependence of $K$.

The calculated results for Case A and Case B are summarized and compared directly with the observations in Figure 12. What is plotted is the "hysteresis ratio" as a function of particle rigidity for three selected pairs of calculated spectra. Curve a) is the hysteresis ratio expected at earth for protons having an interstellar spectrum which is a power law in kinetic energy in a modulating region of $R = 30$ a.u. relative to that in a region of $R = 3$ a.u. where the dependence of $K$ on magnetic rigidity is the same throughout both regions. Curves b) and c) are the hysteresis ratios at earth for protons and helium nuclei respectively in a modulation region with $n = 1.0$ relative to one with $n = 1.15$ for assumed interstellar spectra which are power laws in total energy per nucleon and $R = 3$ a.u. for both regions. All three curves are near unity at 10 GV and increase slightly down to 2 GV. Below 2 GV curve a) diverges from the other curves and yields a hysteresis ratio less than unity below 0.7 GV. Curves b) and c) on the other hand continue to increase below 2 GV eventually diverging/below 0.7 GV as first the helium curve and then the proton curve approach a constant ratio at low rigidities. This constant ratio corresponds to the near parallel displacement at low energies of the spectra calculated for different values of $n$ shown in Figure 10. It is a further manifestation of the $J = AT$ behavior reported in Paper I. Note in particular that the proton and helium curves split such that
the constant value at low rigidity for helium is less than that for protons.

The data points plotted in Figure 12 for comparison with the calculated curves are taken from the ratio of the spectra observed in 1966 to that in 1971 for protons (Figure 9) and for helium as well. Clearly curve a) is qualitatively inconsistent with the observations as was noted above in that the direction of the hysteresis effect at low energies is opposite to that observed. If the balloon observation alone are considered the data are consistent with the calculated curves b) and c) except that the leveling of the ratio takes place at a higher rigidity than calculated for both protons and helium corresponding to a constant value of 1.85 for protons (open circles connected by dashed line) and 1.25 for helium nuclei (open squares connected by dashed line). On the other hand the extension of the proton spectrum in 1971 (and 1972) to lower energies based on satellite data as shown in Figure 2 indicates a steeper spectrum than was observed in 1966. This extension to lower energies is shown by the dotted line in Figure 12. Also shown are the values of 2.6 for protons and 2.0 helium from the work of Van Hollebeke et al. (1973) (solid and open bands). Barring some systematic differences between the balloon and satellite observations the lower energy observations indicate that the hysteresis ratio continues to increase at lower energies for both protons and helium nuclei. Whether the hysteresis ratio is constant or varying with energy below ~100 MeV, all the observations show a splitting in the behavior of protons and helium nuclei with the ratio for helium being less than that for protons. This is certainly qualitatively consistent with the calculated curves.
We have pointed out that all models whose effect is to produce a variation with time in the effective rigidity dependence of K reduce to our case A and that no physical insight beyond that implicit in case A can be drawn from the agreement of these models with the observation of modulation of the nucleonic component near the earth. We have also pointed out that a sudden change in the magnitude of K (not its rigidity dependence) which propagates outward in the modulating region is equivalent to case B which does not agree with the observed effect. Therefore the only way to produce "hysteresis" under the assumptions of time independent diffusion and spherical symmetry is to postulate some variation for the rigidity dependence of K. At present no observational evidence or theoretical basis (other than hysteresis) exists for or against such models.

Finally it should be noted that some of the observed behavior might be related to a breakdown in the basic assumptions noted above. Some work has been reported by Fisk et al. (1973) concerning the effect of a non-spherically symmetric modulating region and O'Gallagher (1973) has shown that time-dependent diffusion effects could account for some of the phase lags associated with hysteresis.
IV. SUMMARY AND CONCLUSIONS

Balloon observations during the epoch of solar maximum have led to the following results:

1. The long term modulation of protons and helium nuclei between 100 and 250 MeV/nucleon exhibits a hysteresis effect slightly smaller than that observed near 60 MeV/nucleon on satellites.

2. The regression relationship observed between the proton intensity in the interval 100-250 MeV and the Deep River neutron intensity between the Summer of 1969 and the Summer of 1970 indicates that their relative behavior can be characterized by a segment of negative slope around solar maximum.

3. Proton spectra in 1971 and 1972 based upon a combination of balloon and satellite data can be represented below ≈ 200 MeV/nucleon by a power law of the form $J = T^{1.4 \pm 0.1}$. The positive slope of these spectra, recorded during the decreasing phase of solar activity, is steeper than that of the $J = AT$ behavior reported in Paper I for the increasing phase.

4. The helium spectra became less steep from year to year from 1969 ($J = T^{1.0}$) through 1972 ($J = T^{0.2}$) over the energy range from 30-250 MeV/nucleon.

5. Hysteresis loops for protons and helium nuclei at all energies investigated here had closed in the summer of 1972.

An investigation of the behavior of modulated spectra computed with only two variable parameters in an effort to describe hysteresis as simply as possible has lead to the following conclusions.
1. The long term "hysteresis effect" in cosmic ray modulation is well described by a model involving only two variable parameters; a) the "traditional" modulating parameter \( n \) (defined here at a high rigidity \( P_0 = 10 \text{ GV} \)) and b) the exponent \( n \) in an assumed power law dependence of the effective diffusion co-efficient \( K \) on magnetic rigidity.

2. The modulated spectra are highly sensitive to variations in \( n \) with the observations of a fairly pronounced "hysteresis" being well described by a change in \( n \) from \( n = 1.0 \) to \( n = 1.15 \).

3. A model involving two parameters a) \( n \) as defined above and b) the size of the modulating region \( R \) does not fit the observed long term hysteresis behavior.

4. A model depending on \( n \) and \( R \) does produce some spectral changes by changing the relative importance of adiabatic deceleration. Although this behavior is not consistent with the long-term hysteresis it does produce a "cross-over" in the low energy modulated spectrum which resembles that observed during the time period corresponding to the segment of negative slope.

Acknowledgements. We are grateful to Dr. Lennard Fisk for providing the computer programs used in this analysis. We also wish to thank Dr. J.A. Simpson for supplying us with the 1971 proton spectrum observed on IMP 5 and Drs. F. McDonald and M. Van Hollebeke for the solar proton data during July 1972. Lawrence Bagg, H. William Leech, Janet Luhmann, Charles Meegan and Lawrence Votta provided support for the balloon operations. John McCarthy assisted in the computer analysis of modulated spectra. This work was supported by NASA Grant NGR 21-002-066.
REFERENCES


### TABLE 1. SUMMARY OF FLIGHT DATA

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* These times refer to the day after launch. All times are Universal Time.
† Portion of flight before arrival of solar particles.
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* Differential fluxes are quoted in units of particles/m² sec. ster. MeV/nucleon.
† Energies are quoted in units of MeV/nucleon.
†† A solar contribution of 0.6±0.3 to Range 4 and 0.2±0.1 to Range 5, included in these values, was subtracted from the spectrum plotted in Figure 2. Solar contamination of Ranges 6 - 10 was negligible.
FIGURE CAPTIONS

Figure 1. Hysteresis in cosmic ray modulation can be represented in three different ways: (a) as a closed loop in the regression plot of low energy flux vs. neutron monitor rate (b) as a phase lag between the solar cycle variations of intensity at widely separated energies and (c) as a difference in the shapes of differential energy spectra recorded in different epochs of the solar cycle.

Figure 2. In 1971 and 1972, the proton spectrum below 200 MeV was a power law $J \propto T^{-1.4}$ with steeper slope than the form $J \propto T$ which applied from 1965 to 1969. Solar protons contribute only a negligible flux to the observed spectrum above 170 MeV in 1972.

Figure 3. Proton energy spectra recorded in 1970 (Flights 1293 and 1297) display at ~400 MeV the crossover designated by X in Figure 1c. This effect occurs for any pair of spectra described by the inverse regression relationship between C and D in Figure 1a. A similar phenomenon was seen at >500 MeV in 1969 (Flights 1273 and 1274. Note that these spectra are displaced upward by a factor of 10 and are described by the right hand ordinate scale.) All four spectra display an abrupt change of slope near 200 MeV.

Figure 4. This plot of the parameter A vs. Deep River neutron rate is an empirical realization of Figure 1a. Note that five data points lie on the dotted line which indicates the inverse regression relationship that applied from June 1969 to June 1971.

Figure 5. For penetrating protons, the hysteresis effect becomes smaller as the proton energy increases.
Figure 6. The spectrum of helium below 150 MeV is a power law whose slope and intercept change with modulation. Helium spectra in 1969 and 1970 also display an abrupt change in slope similar to that shown for protons in Figure 2 and 3.

Figure 7. The hysteresis ratio for helium nuclei stopping in the detector is smaller than that shown in Figure 4 for protons.

Figure 8. For penetrating helium, the hysteresis effect is almost imperceptible.

Figure 9. The spectra observed in 1966 and 1971 are the same at high energies and diverge substantially at lower energies manifesting a strong hysteresis as shown in Figure 1c. The solar maximum spectrum in 1969 is shown for reference and two spectra calculated for diffusion co-efficients proportional to rigidity (dotted curve) and rigidity to the 1.15 power (dashed curve) are compared with the 1966 and 1971 spectra.

Figure 10. Small changes in the exponent n of the assumed power law dependence of diffusion co-efficient on magnetic rigidity P produce large changes in the modulation at low energies.

Figure 11. Changes in the radius of the modulation region produce changes in the resulting spectra due to changing effects of energy loss processes. The effect is most pronounced for an interstellar spectrum U₀ which is a power law in kinetic energy but it is too small and qualitatively dissimilar to the observed hysteresis.

Figure 12. Observed values of the hysteresis ratio for protons and helium are compared with calculated curves. Curves b) and c) correspond to case A for protons and helium respectively and fit the data better than curve a) corresponding to case B for protons. Open data points are from the balloon observations of the present work. The dotted line and bars indicate behavior at lower energies based on satellite observations discussed in the text.
Figure 1
Figure 3
Figure 4

PROTONS $E \leq 250$ MeV

$A = \frac{\text{PROTONS}}{M^2 \cdot \text{sec} \cdot \text{ster} \cdot (\text{MeV})^2}$

- BALLOON 1965-69
- BALLOON 1970-72
- SATELLITE 1965-69

DEEP RIVER NEUTRON RATE
Figure 5

PROTONS/M²-sec-sr

FAST PROTONS
E > 720 MeV

SLOW PROTONS
260 - 720 MeV

DEEP RIVER NEUTRON RATE

1965 - 1969

1970 - 1972

1.07 x

1.5 x
Figure 6
HELIUM$^4$

- 1965 - 69
- 1970 - 72

200 - 250 MeV/N

100 - 150 MeV/N

DEEP RIVER NEUTRON RATE

HELIUM NUCLEI / M$^2$ - sec. - ster. - MeV/nucleon

Figure 7
HELIUM

$E \geq 260 \text{ MeV/NUC.}$

Figure 8
OBSERVED PROTON SPECTRA

PROTONS/M^2 sec - sr - MeV

KINETIC ENERGY (MeV)

1966
1969
1971
Figure 10

Unmodulated Spectrum

Calculated Spectra

$K \propto P^n \beta$

PROTONS/$(M^2 \cdot sec \cdot sr \cdot MeV)$

KINETIC ENERGY (MeV)
Unmodulated Spectrum

R = 30 a.u.

R = 3 a.u.

PROTON KINETIC ENERGY (MeV)

Figure 11
Figure 12

- PROTONS
- HELIUM NUCLEI

**Figure 12**

- Rigidity (BV)
- Hysteresis Ratio