DIFFRACTION BY A PERFECTLY CONDUCTING RECTANGULAR CYLINDER WHICH IS ILLUMINATED BY AN ARRAY OF LINE SOURCES

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**Abstract**

The geometrical theory of diffraction (GTD) is employed to analyze the radiation from a perfectly-conducting rectangular cylinder illuminated by an array of line sources. The excitation of the cylinder by a single electric or magnetic current line source is considered first, and a solution which includes contributions from the geometrical optics rays and all singly- and doubly-diffracted rays is obtained. A new diffraction coefficient valid in the transition regions of the shadow and reflection boundaries is employed to obtain a continuous total field, except for negligible discontinuities in the doubly-diffracted field at its shadow boundaries. Patterns calculated by the GTD method are found to be in excellent agreement with those calculated from an integral equation formulation. Using superposition the solution for array or aperture excitation of the rectangular cylinder is obtained. A computer program for this solution is included.

**Key Words (Suggested by Author(s))**

- Antenna, Spacecraft and Aircraft Antennas
- Applied Electromagnetic Theory
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I. INTRODUCTION

In this report the geometrical theory of diffraction (GTD) \[1\] is used to treat the diffraction by a perfectly-conducting rectangular cylinder which is illuminated by an array of electric or magnetic current line sources. The illumination by a single line source of the array is considered first with the geometry of the problem shown in Fig. 1. The scattered and total fields are calculated at all points exterior to the rectangular cylinder, except for the shaded regions around the source and edges, which are excluded because of the nature of the high-frequency approximation. More will be said about this later. The solution is then generalized to calculate the field of linear arrays of such line sources radiating in the presence of the cylinder.

The scattering by a rectangular cylinder has been considered previously for the special case of plane wave incidence. Mei and VanBladel \[2,3\] formulated the problem in terms of an integral equation which they solved numerically to find the surface currents on the rectangular cylinder. They give radiation patterns, scattering cross sections, and surface currents for both E- and H-polarized incident waves. There are two disadvantages associated with this method: 1) as the frequency increases, difficulties are encountered with the convergence of the solution (this is particularly true when the technique is employed to solve a three-dimensional scattering problem), 2) the solution provides no physical insight into the scattering mechanism. Morse \[4\] also studied this problem using the ordinary GTD to obtain expressions for the diffracted field away from shadow and reflection boundaries. Since the ordinary theory fails at shadow and reflection boundaries, he introduced supplementary solutions there. He employed Oberhettinger's uniform asymptotic solution \[5\] near the boundaries of the incident and reflected fields, and he employed an integral representation of the field near the shadow boundaries of the fields of the diffracted rays. Thus, he did not obtain a compact high-frequency solution to this problem. The difficulties encountered by Morse at these boundaries may be overcome with a new edge diffraction coefficient derived by Kouyoumjian and Pathak \[6,7\]. This diffraction coefficient can be applied in the transition regions adjacent to the shadow and reflection boundaries so that one obtains a total field which is valid and continuous everywhere away from edges and caustics. Their diffraction coefficient is employed in this analysis.

Using Keller's Generalized Fermat's Principle, we include contributions to the total field from the geometrical optics fields (incident and reflected), as well as singly-diffracted fields which appear to emanate from the edges. Doubly-diffracted fields have also been included to describe the interactions between the edges; however, multiply-diffracted rays of higher order have been neglected because, in general, their fields contribute insignificantly, for the problem defined here. The incident, reflected and diffracted rays may be shadowed in the geometrical optics sense, and hence they contribute to the total field only in their respective regions of illumination. Discontinuities in the
Fig. 1. Line source in the presence of the rectangular cylinder.

...field are introduced at shadow boundaries and at the reflection boundaries, but they are systematically removed by employing the new diffraction coefficient for the edge diffracted field mentioned previously. It should be pointed out that there are some residual discontinuities due to uncompensated discontinuities in the field of the doubly-diffracted rays at their shadow boundaries. However, these discontinuities are so small that they are not apparent in the plotted patterns. The patterns calculated from our GTD solution are found to be in excellent agreement with those calculated from numerical solutions.

Due to the high-frequency approximations of the incident and diffracted fields, our solution is restricted so that the following distances are greater than 0.7 wavelength.

1) The distance between the line source and the closest edge of the rectangular cylinder,

2) the distance between the observation point P and the closest edge of the rectangular cylinder,

3) the distances between the edges of the rectangular cylinder,

4) the distance between the observation point P and the closest line source, when calculating the incident or total fields.
A computer program based on the GTD solution for the rectangular cylinder in the presence of an array source has been written and is included in the report. With this program, it is possible to obtain numerical results for both the near- and far-fields of the cylinder under quite general conditions of illumination. Thus, the program is directly relevant to both antenna and scattering problems. As an example of its versatility, one notes that the program may be used to compute the pattern of an array of magnetic line sources mounted directly on the rectangular cylinder, provided the sources are not too close to an edge.

II. METHOD OF SOLUTION

Keller's geometrical theory of diffraction [1] is an extension of geometrical optics in which diffracted rays are introduced by a generalization of Fermat's principle, the excitation of the diffracted field is treated as a local phenomenon, and away from the diffracting surface the behavior of the diffracted field along its ray is the same as that of the geometrical optics field. The basic idea of GTD is that the field of the line source illuminates the rectangular cylinder giving rise to a reflected field and an edge diffracted field, which consists of the fields of singly and multiply-diffracted rays. The total field $U(P)$ at a point $P$ is equal to the sum of the fields on all rays through $P$.

\[ U(P) = \sum_{\text{rays}} U_i(P) \]

which includes the incident field if $P$ is not in the shadow region. The wave function $U(P)$ represents a magnetic field parallel to the edge in case of a magnetic line source, and an electric field parallel to the edge in the case of an electric current line source. The pertinent rays and their associated fields will be discussed briefly in the following paragraphs.

A. Geometrical Optics Rays - Direct and Reflected

Let us consider the field radiated from a line source at $O$ and observed at $P$ as shown in Fig. 2. Fermat's principle predicts only the direct ray $OP$. If a line source is being considered, the field along $OP$ is given by

\[ U^i(P) = C \frac{e^{-jk s_0}}{\sqrt{s_0}}, \]

where $s_0$ is the distance between $O$ and $P$, and $C$ is a conveniently chosen normalization constant. For the configuration shown in Fig. 2, the space surrounding the right-angle wedge may be divided into three regions:
Fig. 2. Line source in the presence of a wedge.

Region I $0 \leq \phi < \pi - \phi'$,
Region II $\pi - \phi' < \phi < \pi + \phi'$,
Region III $\pi + \phi' < \phi < \frac{3\pi}{2}$.

Region III is the shadow region, which is not penetrated by the incident ray; the incident field vanishes here.

We know that there is a field reflected from the surface $A_QE$. To describe this we introduce an additional class of rays which include on their trajectory a point $QR$ of the surface $A_QE$. Applying Fermat's principle, the distance $OQP$ along the ray path is a minimum and the law of reflection results. This simple extension of Fermat's principle which accounts for the reflected ray is so natural that we accept it without question. The field of the reflected ray is readily deduced from image theory as

$$U_r(P) = U_r(AQE) = \pm C e^{-jks'' \tau} s''$$

where the positive sign is for Neumann (hard) boundary condition associated with the magnetic current line source, the negative sign is for the Dirichlet (soft) boundary condition associated with the electric current source line, and $s''$ is the distance between the image $O'$ and the observation point $P$. The reflected field vanishes in regions II and III, which the reflected ray does not penetrate. Let us consider now a further extension of Fermat's principle.
B. Singly-Diffracted Rays

It is well known that the ray incident on the edge \( Q_E \) in Fig. 2 gives rise to diffraction. To account for this, Keller introduced a class of rays which includes the point \( Q_E \) in its trajectory. This completely determines the diffracted ray path in the isotropic, homogeneous medium of this two-dimensional problem, so the law of edge diffraction becomes trivial under these circumstances.

In terms of GTD, the diffracted field at \( P \) for the line source at \( O \) is

\[
U^d(Q_E) = U^i(Q_E) D_s(\phi, \phi') \frac{e^{-jks}}{\sqrt{s}} ,
\]

where \( D_s \) is the scalar diffraction coefficient for the acoustically soft (Dirichlet) boundary condition and \( D_h \) is the scalar diffraction coefficient for the acoustically hard (Neumann) boundary condition. They are deduced from the general dyadic diffraction coefficient \( D(\phi, \phi', \beta_0) \) obtained by Kouyoumjian and Pathak [6,7]. For the special case where the incident ray is perpendicular to a straight edge, the scalar diffraction coefficients are given by

\[
D_s(\phi, \phi') = \frac{-e^{-j\pi/4}}{2\sqrt{2\pi k}} \left\{ \frac{\cot\left(\frac{\pi+(\phi-\phi')}{2n}\right)}{2n} F[kLa^+(\phi-\phi')] + \frac{\cot\left(\frac{\pi-(\phi-\phi')}{2n}\right)}{2n} F[kLa^-(\phi-\phi')] \right\}
\]

\[
\pm \left\{ \frac{\cot\left(\frac{\pi+(\phi+\phi')}{2n}\right)}{2n} F[kLa^+(\phi+\phi')] + \frac{\cot\left(\frac{\pi-(\phi+\phi')}{2n}\right)}{2n} F[kLa^-(\phi+\phi')] \right\}
\]

where \( n \pi \) is the exterior wedge angle, which equals \( 3\pi/2 \) in this case, and

\[
F(x) = 2j|\chi| e^{jx} \int_{|\chi|}^{\infty} e^{-j\tau^2} d\tau
\]

in which

\[
a^\pm(\phi \pm \phi') = 2 \cos^2\left(\frac{2n\pi N^\pm - (\phi \pm \phi')}{2} \right).
\]
are the integers which most nearly satisfy the following equations

\begin{align*}
(8) \quad 2\pi n N^+ &= \pi + (\phi + \phi') \\
(9) \quad 2\pi n N^- &= -\pi + (\phi + \phi')
\end{align*}

and $kL$ is the large parameter in the asymptotic evaluation of the pertinent integrals involved in the derivation of the dyadic diffraction coefficient. The quantity $L$ may be viewed as a distance parameter which depends upon the type of edge illumination; for line source illumination, $L$ is given by

\begin{equation}
L = \frac{s s'}{s + s'}
\end{equation}

For grazing incidence $\phi' = 0$, $n\pi$, $D_s$ is multiplied by a factor of $\frac{1}{2}$; furthermore, if the diffracted ray grazes the surface in the case of a soft boundary, $D_s = 0$ and the diffracted field vanishes, as it should.

The field of the singly-diffracted ray is discontinuous at the shadow and reflection boundaries in a way which compensates the discontinuities in the geometrical optics fields there. This is readily demonstrated; consider for example the incident and diffracted fields at the shadow boundary, where to simplify the discussion, it is assumed there is no nearby reflection boundary. Let $\pi + \phi' - \epsilon$ be a point close to the shadow boundary, see Fig. 2. In the illuminated region $\epsilon > 0$ and in the shadow region $\epsilon < 0$.

\begin{equation}
U(\phi) = \begin{cases}
-e^{jks_0} & e^{jks'} \\
\frac{e}{\sqrt{s_0}} + \frac{e}{\sqrt{s'}} \frac{D_s(\phi' + \pi - \epsilon, \phi')}{\sqrt{s}} & e > 0 \\
\frac{e^{jks'}}{\sqrt{s'}} \frac{D_s(\phi' + \pi - \epsilon, \phi')}{\sqrt{s}} & e < 0
\end{cases}
\end{equation}

For $\epsilon$ small it follows from Eq. (5) that

\begin{equation}
D_s(\phi' + \pi - \epsilon, \phi') = \frac{-ej\pi/4}{2\sqrt{2\pi k}} \left\{ \cot \frac{\epsilon}{2n} F[kLa^- (\pi - \epsilon)] + \text{smaller terms which are continuous at the shadow boundary} \right\}
\end{equation}
From Eq. (9),

$$N^- = 0$$

Also, as $\varepsilon \to 0$,

$$\cot \left( \frac{\varepsilon}{2n} \right) = \frac{2n}{\varepsilon}$$

(13) $$a^- (\pi - \varepsilon) = \frac{\varepsilon^2}{2}$$

(14) $$\text{sgn} \varepsilon + \text{smaller, continuous terms},$$

(15) $$F[kL a^- (\pi - \varepsilon)] = \sqrt{\pi kL/2} e^{j\pi/4} |\varepsilon|$$

Substituting Eqs. (13) and (15) into Eq. (12) as $\varepsilon \to 0$,

(16) $$D_s (\phi' + \pi - \varepsilon, \phi') = -\frac{1}{2} \sqrt{\frac{s^i}{s + s'}} \text{sgn} \varepsilon + \text{smaller, continuous terms},$$

(17) $$s_0 = s' + s .$$

Upon substituting Eqs. (16) and (17) into Eq. (11), it is seen that the total field is continuous at the shadow boundary. In an analogous manner it can be shown that the total field is continuous at the reflection boundary.

C. Doubly-Diffracted Rays

When one face of the conducting wedge is terminated at $Q_F$ as shown in Fig. 3, a second order diffracted-ray will emanate from the edge $Q_F$. In terms of the GTD, the doubly-diffracted field at $P$ due to the line source at $O$ can be written as

(18) $$U^d (Q_E, Q_F) = U^i (Q_F) \frac{\sqrt{\pi}}{h} D_s (\phi_2, 0) \frac{e^{-jks}}{\sqrt{s}} = \left\{ U^i (Q_E) D_s \left( \frac{3\pi}{2}, \phi' \right) e^{-jkh} \right\} \frac{D_s (\phi_2, 0)}{h^2} \frac{e^{-jks}}{\sqrt{s}} .$$
Since \( D_s(3\pi/2, \phi') = 0 \), the contribution from the doubly-diffracted rays vanishes for the soft boundary according to the above expression. If a higher order approximation for the doubly-diffracted field is employed, then this contribution is non-vanishing, as will be explained later.

The field of the ray singly-diffracted at \( Q_E \) has a shadow boundary \( SB(Q_E) \) as shown in Fig. 3; the singly-diffracted ray does not penetrate the shaded region. It will be shown next that the discontinuity in the field of the singly-diffracted ray at \( SB(Q_E) \) is compensated by the ray doubly-diffracted from \( Q_F \), so that the total diffracted field is continuous at this boundary. Since the field doubly diffracted vanishes in the case of the soft boundary, we only need to treat the hard boundary here.

Consider a point close to \( SB(Q_E) \) so that \( \phi_2 = \pi - \varepsilon \), the total diffracted field at this boundary is

\[
U_{TD} = \begin{cases} 
U_i(Q_E) D_h(\phi_1, \phi') \frac{e^{-jks}}{\sqrt{s_1}} + U_i(Q_F) D_h(\pi - \varepsilon, 0) \frac{e^{-jks}}{\sqrt{s}} , & \varepsilon > 0 \\
U_i(Q_F) \frac{D_h(\pi - \varepsilon, 0)}{2} \frac{e^{-jks}}{\sqrt{s}} , & \varepsilon < 0 
\end{cases}
\]

Fig. 3. Configuration for double-diffraction.
where \( s_1, \phi_1 \) are the coordinates of the ray diffracted from \( Q_E \), and

\[
U^i(Q_F) = U^i(Q_E) D_h\left(\frac{3\pi}{2}, \phi'\right) e^{-jkh}.
\]

When \( \phi'_2 = 0 \) and the singly-diffracted ray grazes the vertical surface, the second and fourth terms in the expression for the diffraction coefficient are the same, except for the \( \pm \) sign of the latter. This is also true for the first and third terms. As a result,

\[
D_h(\pi - \epsilon, 0) = -\frac{e^{-j\pi/4}}{\sqrt{2\pi}} \left\{ \cot\left(\frac{\epsilon}{2}\right) F[kLa^{-}(\pi-\epsilon)] + \text{smaller terms which are continuous at } SB(Q_E) \right\}.
\]

As \( \epsilon \to 0 \), it is seen from Eqs. (12), (13), (14) and (15) that

\[
D_h(\pi - \epsilon, 0) = -\sqrt{\frac{hs}{s+h}} \text{ sgn } \epsilon;
\]

furthermore,

\[
s_1 = h + s.
\]

Substituting Eqs. (22) and (23) into Eq. (19) and making use of Eq. (20), it is seen that the total diffracted field is continuous at the boundary \( SB(Q_E) \).

As we have already noted, in the case of a soft boundary the field of an incident-ray grazing the surface vanishes, the edge-diffracted field is then proportional to the normal derivative of the incident field at the edge. The proportionality factor is a diffraction coefficient \( D' \) given by Karp and Keller [8]. Thus, for the case of Dirichlet problem, the doubly-diffracted field must be replaced by

\[
U^d(Q_E, Q_F) = \frac{\partial U^i(Q_F)}{\partial n} D'(\phi_2, 0) e^{-jks} \frac{e^{-jks}}{\sqrt{5}}
\]

where

\[
D'(\phi_2, 0) = \frac{1}{jk} \frac{\partial}{\partial \phi'} D_s(\phi_2, 0)
\]
The derivative $3U(Q_p)/\partial n$ is taken with respect to the normal to the surface $Q_eQ_f$. This contribution is weak in comparison with that of the singly-diffracted rays; the contribution of the former is of order $(1/k^2)$, whereas that of the latter is of order $(1/\sqrt{R})$. In calculating the field diffracted from the soft cylinder, it was found that the field of the doubly-diffracted rays did not contribute significantly, so the contribution from these rays can be omitted in this case.

Let us now turn to the diffraction by a rectangular cylinder illuminated by a line source. Depending on the location of the line source, the whole domain surrounded by the cylinder will be divided into regions by the various shadow boundaries and the reflection boundaries. Each of these boundaries is labeled to indicate how it originates. For example, referring to Fig. 4, the notation $SB$ means the shadow boundary of the incident geometrical optics field $U_I(P)$, $RB(A-B)$ is the shadow boundary of the geometrical optics field $U'(A-B)$ reflected from the surface $A-B$, $SB(A)$ is the shadow boundary of the singly-diffracted $U^d(A)$, which emanates from the edge $A$, and $SB(A,B)$ is the shadow boundary of the doubly-diffracted field $U^d(A,B)$ which emanates from the edge $B$. The shadow boundary of the reflected field is referred to simply as the reflection boundary.

![Fig. 4. Shadow and reflection boundaries of the GTD fields.](image-url)
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A check (✓) means the field is non-vanishing in the region

**TABLE I**
In what follows we may locate the line source in the upper left hand quadrant of the space surrounding the cylinder, without loss of generality. Three cases will be considered.

1) First, the line source is located directly above the cylinder. The whole domain around the cylinder is divided into 14 regions as shown in Fig. 4. Table I shows the regions covered by each type of ray. To demonstrate the use of the table, let us consider an observation point P in region IX. Examining the column under region IX of Table I, one finds checks(✓) for \( U^d(A) \), \( U^d(B,A) \), \( U^d(A,D) \), and \( U^d(B,C) \). Thus the total field at P is equal to \( U^t(P) = U^d(A) + U^d(B,A) + U^d(A,D) + U^d(B,C) \).

2) The line source is located to the upper left of the cylinder as shown in Fig. 5. In this case there are 18 regions. Table II shows the regions where a particular ray exists and its field makes a non-vanishing contribution.

3) Line source at grazing incidence. Let the line source be located in the plane which contains the A-D side of the rectangular cylinder as shown in Fig. 6. This case is of special interest because the trailing edge D lies in the shadow of the leading edge A. Also, it lies on the shadow boundary of the direct geometrical optics field. The behavior of the field at the boundary DP must be treated separately, see Appendix I. As before, the domain surrounding the cylinder is divided into regions as shown in Fig. 6 and Table III gives the regions covered by the individual rays.

Recall that one or more of the various field components is discontinuous at each boundary shown in Figs. 4, 5 and 6 but that all except the discontinuities in the doubly diffracted fields are compensated; e.g., the discontinuities in the geometrical optics field are compensated by the field of the singly-diffracted rays and the discontinuities in the field of the singly-diffracted rays are compensated by the fields of the doubly-diffracted rays.

As a final step in the analysis, the fields of the individual line sources are superimposed to give the field of a linear array of line sources radiating in the presence of the cylinder, see Fig. 7. A computer program has been written to calculate the incident, total and scattered fields once the linear array is specified. Unlike its earlier definition for geometrical optics, the term incident field used here means the field of the array in the absence of the cylinder, and the scattered field is simply the difference between the total field and this incident field. The versatility of such a program is evident; the scattering from the cylinder for a wide variety of illuminations can be studied, and the radiation from antennas in the presence of the rectangular cylinder also can be studied. As a matter of fact, the program
was written originally so that the linear array of line sources, when
densely packed, closely approximates the field of an aperture antenna of
finite width W. The aperture antenna (more precisely its axis) is
directed toward a point Q on the surface of the rectangular cylinder as
shown in Fig. 7. A description of the aperture radiation in terms of an
array of discrete line sources is discussed in the following paragraphs.

The aperture distribution may be approximated by a discrete array
of line sources which are properly weighted in amplitude and phase. The
width of the aperture denoted by W, is divided into 2M segments; (M =
integer). The line sources are positioned at the ends of these segments,
which introduces 2M + 1 line sources. In approximating a continuous
distribution, the number M is selected so that 2M + 1 > 10 W/\lambda, where
\lambda = free space radiated wavelength.

Three types of line sources are available in this program:

Type I  An electric current line source
Type II  A magnetic current line source
Type III  A magnetic current moment line source.

As described earlier, the electric current line source radiates an
omnidirectional electric field which is parallel to the edge of the
rectangular cylinder, and the magnetic current line source radiates an
omnidirectional magnetic field which is parallel to the edge of the
rectangular cylinder. The magnetic current moment line source consists
of a continuous array of magnetic current moments directed perpendicular
to the line of the array and parallel to the aperture in question. This
line source radiates an electric field parallel to the edge of the
cylinder; however, the field has a pattern, |\cos \theta|, where \theta is shown in
Fig. 7. The strength of these magnetic type line sources is determined
from the equivalent magnetic surface currents in the aperture. \( K_S = \vec{E} \times \hat{n} \), where \( \vec{E} \) is the electric field distribution in the aperture
(assumed known) and \( \hat{n} \) is the outward normal to the aperture.

The field of the two-dimensional aperture can be adequately repre-
sented in the forward direction by a densely-packed array of type II and
type III line sources, but such an array fails to approximate the field
adequately at aspects behind the aperture. This limitation is particu-
larly troublesome when calculating the total field. To overcome this
difficulty an obliquity factor has been included in the program which
multiplies the pattern of each line source. The obliquity factor is
f(\theta) = \cos^n \theta/2, where n = 0, 1/2, 1, 2. When n = 0, the obliquity factor
is unity so that the array radiates symmetrically with respect to the
axis of its elements. The case n = 2 occurs naturally in the description
of the radiation fields of aperture antennas via the Kirchhoff-Huygen's
approximation (for the forward region). The cases n = 1/2, and n = 1 are
added so that the n which best approximates the measured aperture pattern
may be used. It is evident that the obliquity factor results in a
pattern null in the direction directly behind the aperture at \( \theta = \pi \). In
most practical cases, there is no such null in the backward direction.
Fig. 5. Shadow and reflection boundaries of the GTD fields.
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Fig. 6. Shadow and reflection boundaries of the GTD fields.
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* SEE APPENDIX I

TABLE III
On the other hand, the pattern in the forward region is quite satisfactory when the obliquity factor is included, and as one moves away from the forward region, the pattern drops to a level where the differences between the simulated and measured patterns are unimportant when computing the total field surrounding the rectangular cylinder.

The scattered, and the total fields are computed as a function of \( \theta_s (\frac{-\pi}{2} \leq \theta_s \leq \frac{\pi}{2}) \) at a range \( R_s \). The incident field pattern on the other hand is available for a phase reference at the center of the array (aperture) at 0 in Fig. 7 and computed at a range measured from 0, as well as for a phase reference at Q with a range \( R_s \) (just as for the scattered and the total fields).

The input and output variables for the computer program are described in Appendix II, and the listing of the computer program is given. Also, a sample case is treated in this appendix to illustrate the use of the program.

III. NUMERICAL RESULTS

To assess the accuracy of the GTD solution described in the preceding section, it was applied to several simple examples, where the rectangular cylinder is illuminated by either an electric or magnetic current line source. Far-zone patterns for the total field are calculated by this method and also from numerical solutions of the pertinent integral equation*. The cylinders are square with a side length of 1.6

*The computer programs for the integral equation solutions were provided by Prof. J.H. Richmond of the ElectroScience Laboratory.
wavelength, and in each case the line source illuminates the cylinder from a distance of 0.8 wavelength. These small distances provide a stringent test of our GTD solution; also they give us an opportunity to examine the accuracy of the new scalar diffraction coefficients in a situation where the edges are illuminated by curved wavefronts and where the transition regions are relatively broad. The pattern calculated from the integral equation solution must be considered more accurate for the small dimensions chosen for these examples, since the integral equation method is convergent whereas the GTD solution is an asymptotic approximation.

Patterns for magnetic current line source excitation (hard boundary case) are given in Figs. 8, 9 and 10, where the line source is positioned on the diagonal of the square cylinder, on the centerline directly above the cylinder, and at a point of glancing incidence on one of its surfaces. The agreement between the patterns calculated by the GTD and the integral equation method is remarkable - every detail is the same within the limits of graphical accuracy. The corresponding patterns for electric current line source excitation (soft boundary case) are presented in Figs. 11, 12 and 13. Again there is excellent agreement between the two pattern calculations, except in the vicinity region of forward scatter in Figs. 12 and 13. Note that the level of the patterns is very low in these regions, so that small errors in the solution become significant. We hope to look into the reason for these differences at a later time.

The numerical examples considered here confirm the accuracy and applicability of our GTD solution; this is further demonstrated by an example treated in Appendix II. As the size of the cylinder and the distance between the edges of the cylinder and the source (or sources) increases in terms of a wavelength, one can expect the accuracy of the GTD solution to increase, because it is an asymptotic approximation where \( k = 2\pi/\lambda \) is a large parameter.

In the case of magnetic current line source excitation there is little evidence of shadowing in the forward direction by the small square cylinder; however, there is distinct evidence of shadowing in the case of electric current line source excitation, where the total electric field is parallel to the cylinder and must vanish at its surface. One should expect this.
Fig. 8. Pattern of a magnetic current line source in the presence of a rectangular cylinder.
Fig. 9. Pattern of a magnetic current line source in the presence of a rectangular cylinder.
Fig. 10. Pattern of a magnetic current line source in the presence of a rectangular cylinder.
Fig. 11. Pattern of an electric current line source in the presence of a rectangular cylinder.
Fig. 12. Pattern of an electric current line source in the presence of a rectangular cylinder.
Fig. 13. Pattern of an electric current line source in the presence of a rectangular cylinder.
IV. CONCLUSIONS

The GTD has been applied to calculate the radiation from a perfectly-conducting rectangular cylinder in the presence of a linear array of line sources, which may be of the electric current, magnetic current or magnetic current moment type. When densely packed, these sources may be used to approximate the radiation from an aperture. To insure good accuracy in the fields calculated from the solution described here, the separation of source and field points from the edges of the cylinder and the separation of the edges from each other should be not less than 0.7 wavelength. However, for far-zone pattern calculations, the line sources can be only a few tenth of a wavelength from the nearest edge.

The use of new scalar diffraction coefficients valid in the transition regions makes it possible to calculate continuous patterns in the region surrounding the cylinder away from its edges. Radiation patterns calculated from this solution and from an integral equation solution are found to be in excellent agreement for a number of stringent test cases. This demonstrates the utility and accuracy of the new diffraction coefficients and the overall accuracy of GTD as it has been applied to this problem.
APPENDIX I

THE FIELD AT THE SHADOW BOUNDARY OF A THICK SCREEN FOR GRAZING INCIDENCE

In this appendix we derive an expression for the field near the shadow boundary of a thick, perfectly conducting screen illuminated by a line source at grazing incidence, as shown in Fig. 14. The solution near the shadow boundary in the forward direction is of interest. In the following development we employ Eqs. (5) through (10) in the text, the subscript \( h \) on the hard scalar diffraction coefficient has been omitted, and it is convenient to use the function

\[
(A-1) \quad f(x) = \frac{e^{-jkx}}{\sqrt{x}}
\]

Fig. 14. Shadow boundary of a thick screen for grazing incidence.
Let the strength of the line source be such that the incident field at \( P \) is

\[
U^i(P) = f(\lambda_0) \quad .
\]

The total field at \( P \) is the sum of the incident field plus the field of the ray singly-diffracted from edge 1 and the field of the ray doubly-diffracted from edge 2. In the illuminated region, \( \varepsilon > 0 \),

\[
U(P) = f(\lambda_0) + f(\lambda_1) \frac{\pi}{\lambda_2} \frac{f(\lambda_1)}{D(\phi_1, 0; L_1)} + \frac{1}{2} U^i(2) D(\phi_2, 0, L) f(\lambda_2) .
\]

In the shadow region, \( \varepsilon < 0 \),

\[
U(P) = \frac{1}{2} U^i(2) D(\phi_2, 0, L) f(\lambda_2) .
\]

Here,

\[
\phi_2 = \pi - \varepsilon \quad ,
\]

\[
L_1 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} \quad ,
\]

\[
U^i(2) = f(\lambda_1 + h) + f(\lambda_1) \left[ -\frac{2}{3} e^{\pi/4} \cos \left( \frac{2\pi}{3} \right) F(2kL_1) \right] f(h)
\]

in which

\[
L_1' = \frac{\lambda_1 h}{\lambda_1 + h} \quad .
\]

Thus for \( \varepsilon > 0 \),

\[
U(P) = f(\lambda_0) + f(\lambda_1) \frac{\pi}{\lambda_2} \frac{f(\lambda_1)}{D(\phi_1, 0; L_1)} f(\lambda_1) + \frac{1}{2} f(h) \left[ -\frac{2}{3} e^{\pi/4} \cos \left( \frac{2\pi}{3} \right) F(2kL_1) \right] D(\phi_2, 0, L) f(\lambda_2) \quad .
\]
and for $\varepsilon < 0$,

\[(A-5b) \quad \mathcal{U}(P) = \frac{1}{2} f(\ell_1 + h) \, D(\phi_2, 0, L_2) \, f(\ell_2) + \frac{1}{2} f(\ell_1) f(h),\]

\[\cdot \left[ -\frac{2}{3} e^{-j \pi/4} \cot\left(\frac{2\pi}{3}\right) F(2kL_1') \right] D(\phi_2, 0, L) \, f(\ell_2),\]

where

\[(A-6) \quad L_2 = \frac{(\ell_1 + h) \, \ell_2}{\ell_1 + h + \ell_2}\]

and $L$ is a distance parameter determined by the wavefront curvature of the field incident on edge 2 which has been singly-diffracted from edge 1. Since edge 2 is in the transition region of this field, the curvature of this wavefront is not simply that of a cylindrical wave emanating from edge 1; i.e., $L \neq h \, \ell_2/(h + \ell_2)$. We will determine $L$ by requiring $\mathcal{U}(P)$ to be continuous at the shadow boundary.

As $\varepsilon \to 0$,

\[(A-7) \quad D(\phi_2, 0, L_2) = \frac{-e^{-j \pi/4}}{n \sqrt{2\pi k}} \left(2\pi\right) \sqrt{\frac{\pi kL_2}{2}} |\varepsilon| \, e^{-j \pi/4} - \varepsilon_2 \, \text{sgn} \, \varepsilon.

In a similar manner,

\[(A-8) \quad D(\phi_2, 0, L) = -\sqrt{\varepsilon} \, \text{sgn} \, \varepsilon.

Furthermore, as $\varepsilon \to 0$

\[(A-9) \quad D(\phi_1, 0, L_1) = -\frac{2}{3} e^{-j \pi/4} \cot\left(\frac{2\pi}{3}\right) \frac{F(2kL_1)}{\sqrt{2\pi k}}.

Substituting Eqs. (A-7), (A-8) and (A-9) into Eqs. (A-5a), (A-5b) and requiring $\mathcal{U}(P)$ to be continuous at the shadow boundary $\varepsilon = 0$, we see that
\begin{align}
\text{(A-10)} \quad \frac{e^{jk(\ell_1 + \ell_2)}}{\sqrt{\ell_1 \ell_2}} \left\{ \frac{2}{3} e^{-j \pi/4} \cot \left( \frac{2\pi}{3} \right) \frac{F(2kL_1)}{\sqrt{2\pi k}} \right\} = \\
\frac{e^{-jk(\ell_1 + h + \ell_2)}}{\sqrt{\ell_1 h \ell_2}} \left\{ \frac{2}{3} e^{-j \pi/4} \cot \left( \frac{2\pi}{3} \right) \frac{F(2kL_1)}{\sqrt{2\pi k}} \right\} \sqrt{\bar{L}}
\end{align}

where \( h + \ell_2 = \ell_1 \) at \( \varepsilon = 0 \).

From which
\begin{equation}
\text{(A-11)} \quad \bar{L} = \left( \frac{h \ell_2}{h + \ell_2} \right) m
\end{equation}

with
\begin{equation}
\text{(A-12)} \quad m = \left[ \frac{F(2kL_1)}{F(2kL_1')} \right]^2
\end{equation}
APPENDIX II
DESCRIPTION OF THE COMPUTER PROGRAM

A. Input Variables

N : is the number of sources in the array which approximates the aperture distribution. Here N = 2M+1, where M is an integer (. . N is an odd integer), and M has been introduced earlier in section II. The DIMENSION cards at the beginning of the program must be dimensioned as N or larger.

TYPE : is a reference parameter. TYPE is set equal to 1.0 when sources of type I (see section II) are used. TYPE is likewise set equal to 2.0 for type II, and is set equal to 3.0 for type III sources, respectively.

AL : is the aperture width (= W of Fig. 7).

AM(I): is the magnitude of the Ith source in the array which approximates a given aperture distribution.

AP(I): is the phase of the Ith line source in the array (which approximates a given aperture distribution), in RADIANS.

X : is the point of incidence on the 2-D box and corresponds to X shown in Fig. 7.

XL : is the length of the box (corresponding to XL of Fig. 7).

H : is the height of the box (it corresponds to h in Fig. 7).

RI : is the incident range (corresponding to R_i in Fig. 7).

RS : is the scattered range (corresponding to R_s in Fig. 7).

THI : is the angle of incidence (corresponding to e_i in Fig. 7).

XLAMDA: is the transmitted wavelength.

Note that the variables AL, XL, H, X, RI and RS have the same units as XLAMDA.

B. Output Variables

THS : is the angle of scattering (corresponding to e_s in Fig. 7) in degrees.

ATAL : magnitude of the total field.
DBTAL: magnitude of the total field in dB.

DBSAS: magnitude of the scattered field in dB.

DBGA : incident field with phase center at Q (Fig. 7) as a function of THS, in dB.

PHASE: phase of the total field in degrees.

TH : angular variable corresponding to (Fig. 7), in degrees

DBHA : incident field with phase center at 0 (Fig. 7) in dB, as a function of TH. Note that DBHA is an output variable in the Subroutine TEST.

C. Instructions for Representing Aperture Field Distribution by a 2-D Line Source Array.

When dealing with the input variables AM(I) and AP(I) for Ith source in the planar array used to approximate a given aperture distribution, the ordering of the array elements is done as follows:

Let N = 11, where N is the number of line sources approximating a given aperture distribution over the aperture width W. The ordering arrangement for these sources is indicated in Fig. 15. The source at the center of the aperture is the one for which I = 1. I = 2,3,4,5 and 6 for sources to the right of the one designated by I = 1 (as one views the 2-D box from the aperture center). Similarly, I = 7,8,9,10 and 11 for sources to the left of the source at the center (designated by I = 1).

Let the aperture distribution (assumed known) be represented by the quantity $F = |F|e^{i\psi}$ over the aperture. $|F|$ represents the magnitude of

![2-D APERTURE OF WIDTH W](image)

![2-D BOX](image)

Fig. 15. The ordering arrangement of the line sources used to approximate a given aperture distribution.
the field distribution over the width \( W \), and \( \psi \) represents the phase of the field distribution over the width \( W \). Hypothetical plots of \(|F|\) and \( \psi \) over the aperture are indicated below:

Fig. 16. Field distribution over the aperture.

Fig. 16 clearly indicates the amplitudes and phases of the sources designated by \( I = 1, 5 \) and 9. For example, the magnitude of the line source strength corresponding to \( I = 9 \) is given by \( AM(9) \), and its phase is given by \( AP(9) \). Similarly, one can obtain the amplitudes and phases of all the other line sources. Note that the aperture is divided into \( 2M \) segments, where \( M = 5 \). Hence, the number of sources, \( N = 2M+1 = 11 \).
D. Instructions for Using the 'Obliquity Factor'

The fields radiated by apertures are non-symmetrical on either side of the aperture, in most practical cases. The fields radiated by the 2-D line source array discussed above are symmetrical on either side of the planar 2-D array. Thus, an obliquity factor of the type \( \cos^n \theta/2 \) (please refer to the discussion in section II) is included for computing the field radiated by each source in the array. The obliquity factor is different for each \( n \), where \( n = 0, \frac{1}{2}, 1, 2 \). A function statement \( \text{FB(SX)} \) computes this obliquity factor for a given value of \( n \). Specifically, the statement concerning \( \text{FB(SX)} \) reads:

\[
\text{FB(SX)} = \text{ABS(COS(SX/2.0))}^{**2.0}
\]

and corresponds to an obliquity factor with \( n = 2.0 \). If any other value of \( n \) is desired, the appropriate value must be punched into a new card which replaces the previous one. Note that the value of \( n \) directly follows the ** symbol in the statement.

The obliquity factor \( \cos^n \theta/2 \) is plotted as a function of \( \theta \) for different values of \( n \) (\( n = \frac{1}{2}, 1 \) and 2) in Fig. 17. When the pattern of an isotropic source is multiplied by \( \cos^n \theta/2 \), it is evident from the resultant pattern that the obliquity factor serves to control the level of the radiation pattern primarily in the range \( \pi/3 < \theta < 5\pi/3 \). The case \( n = 0 \) corresponds to the isotropic case.

E. Instructions for Computing the Incident Field

Two incident field patterns are computed, one is for a phase reference at the center of the aperture, and the other is for a phase reference at \( Q \) (see Fig. 7). In the former case, the radiation pattern in dB is designated by \( \text{DBHA} \), and is obtained as a function of \( \theta \) (or \( \text{TH} \) as defined in the computer program). In the latter case, the radiation pattern in dB is designated by \( \text{DBGA} \) and is obtained as a function of \( \theta_s \) (or \( \text{THS} \) as defined in the computer program). \( \text{DBGA} \) is computed at a distance equal to \( R_s \) from \( Q \). \( \text{DBHA} \) has been programmed for a range of \( R_i + R_s \) from 0 (center of the aperture as shown in Fig. 7); however, if the user wishes to change the present range for \( \text{DBHA} \), only one card in the program deck needs modification. A subroutine designated TEST computes \( \text{DBHA} \) at a range of \( R_i + R_s \) from 0; the call statement for this subroutine is

\[
\text{CALL TEST}(N, \text{AL}, \text{RS}+\text{RI}, \text{A}, \text{TYPE})
\]

If a different value of the range is desired, one must replace \( \text{RS}+\text{RI} \) in the call statement above by a number which equals the desired value for the range. Note that the new range should have the same unit as those of \( \lambda \) (corresponding to \( \text{XLAMDA} \) in the computer program).
Fig. 17. Patterns of the obliquity factor for different values of n.

F. Sample Programs

In this section, we present a sample case which serves to illustrate the use of our computer program. The example selected involves an array of three magnetic line sources of unit strength which illuminate a rectangular cylinder, as in Fig. 18. We utilize the computer program for calculating the incident field of the array, the field scattered by the 2-D box (rectangular cylinder) and the total field (incident + scattered) surrounding the 2-D box.
Fig. 18. An array of three line sources in the presence of a square cylinder.

The ordering of the array elements is shown in Fig. 18, where the source at the center is labeled source #1. Note that source #1 corresponds to $I = 1$, and sources #2 and #3 correspond to $I = 2$, and $I = 3$, respectively. For this particular problem, $AM(1)$, $AM(2)$ and $AM(3)$ are each equal to $1.0$, and $AP(1)$, $AP(2)$ and $AP(3)$ are each equal to $0.0$, because the line sources are of unit strength and zero phase. An obliquity factor corresponding to $n = 2$ (i.e., $\cos^2 \theta/2$) has been incorporated into the program for the incident field pattern, and the incident field pattern corresponding to DBHA (phase reference at source #1) is plotted in Fig. 19. Also included in Fig. 19 is the incident field pattern without the obliquity factor ($n = 0$ case) for the sake of comparison. The pattern of the scattered field designated by DBSAS, and computed for values of $\theta_s$ (or THS) which lie in the range $-180^\circ < \theta_s < 180^\circ$, is plotted in Fig. 20. The scattered field obtained by our method is compared against that obtained from a numerical solution to the integral equation for this problem given by J. H. Richmond; these results agree perfectly. Finally, the total field (incident plus scattered, each being phase referenced at $Q$) is also obtained, and is designated by DBTAL. DBTAL is computed as a function of $\theta_s$ (THS in the program) and the results are indicated in Fig. 21 by a dashed curve. The solid curve is added for the sake of comparison; it corresponds to the total field when the incident field has no obliquity factor in it.
Fig. 19. Patterns of an array of three magnetic line sources of equal strength, with and without the obliquity factor.
Fig. 20. Pattern of the field scattered by a square cylinder which is illuminated by an array of three magnetic line sources.
Fig. 21. Pattern of an array of three magnetic line sources in the presence of a square cylinder.
PROGRAM LSOS30X(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C      PROGRAM LSOS30X IS PHENIX AUGUST 1972
C      N THE NUMBER OF LINE SOURCES OR ELEMENTS IN THE ARRAY
C      TYPE 1.0 IS ELECTRIC CURRENT LINE SOURCE
C      TYPE 2.0 IS MAGNETIC CURRENT LINE SOURCE
C      TYPE 3.1 IS MAGNETIC CURRENT MOMENT LINE SOURCE
C      AL APERTURE WIDTH
C      AM(I) MAGNITUDE OF THE(I)TH ELEMENT
C      AP(I) PHASE OF THE(I)TH ELEMENT
C      A(I) FIELD STRENGTH OF THE(I)TH ELEMENT
C      U(I) SPACING BETWEEN(I+1)TH AND(I)TH ELEMENTS
C      XLAMCA TRANSMITTED WAVELENGTH
C      X THE POINT OF INCIDENCE
C      XL THE LENGTH OF THE BOX
C      H THE HEIGHT OF THE BOX
C      RI INCIDENCE RANGE
C      KS SCATTERING RANGE
C      THI ANGLE OF INCIDENCE 0 TO 65
C      TSH ANGLE OF SCATTERING -65 TO 65
C      XP,YP OBSERVING POINT
C      XS(I),YS(I) SOURCE POINT OF THE(I)TH ELEMENT
C      DS DIFFRACTION COEFFICIENT FOR SOFT BOUNDARY
C      DH DIFFRACTION COEFFICIENT FOR HARD BOUNDARY
C      GA(I) DIRECT INCIDENT FIELD DUE TO(I)TH ELEMENT
C      RA(I) REFLECTED FIELD DUE TO(I)TH ELEMENT
C      DAR1(I) FIRST ORDER DIFFRACTED FIELD FROM THE RIGHT EDGE DUE TO(I)TH ELEMENT
C      DAR2(I) 2ND ORDER DIFFRACTED FIELD FROM THE RIGHT EDGE DUE TO(I)TH ELEMENT
C      DAR3(I) 3RD ORDER DIFFRACTED FIELD FROM THE RIGHT EDGE DUE TO(I)TH ELEMENT
C      DAL1(I) FIRST ORDER DIFFRACTED FIELD FROM THE LEFT EDGE DUE TO(I)TH ELEMENT
C      DAL2(I) 2ND ORDER DIFFRACTED FIELD FROM THE LEFT EDGE DUE TO(I)TH ELEMENT
C      DAL3(I) 3RD ORDER DIFFRACTED FIELD FROM THE LEFT EDGE DUE TO(I)TH ELEMENT
C      TA(I) THE FIELD AT THE OBSERVING POINT DUE TO(I)TH ELEMENT
C      TOTA TOTAL FIELD AT OBSERVING POINT DUE TO ALL THE ELEMENTS
C      DBITAL MAGNITUDE OF THE TOTAL FIELD EXPRESSED IN DB
C      DBASAS MAGNITUDE OF THE SCATTERED FIELD EXPRESSED IN DB
C      DBHA DIRECT INCIDENT FIELD EXPRESSED IN DB
C      PHASE PHASE OF THE TOTAL FIELD EXPRESSED IN DEGREES
C      ATAL AMPLITUDE OF TOTAL FIELD AND ALSO OF INCIDENT FIELD IN SUBROUTINE TEST
C
000003   DIMENSION DAL(100)
000003   DIMENSION CAL(51),CAL2(51),DAL3(51),TA(51)
000440   PIL2=0.5*PI
000442   PIL4=2.5*PI
000444   WLNCH=M=1.0/XLAMDA
000445   N=1.0
C
000446   IF(TYPE=V.2.*C) R=-1.0
COC452   XL=XL/XLAMDA
000453   H=W/HXLAMDA
000455   THI=THI+TT
C
000457   L=181
C   L IS 1.GT.THE NUMBER OF POINTS,NPTS, TO BE PLotted
000460   THS=181.0
000462   GO TO 5998
C
* 9999 CONTINUE
000462   THS=THS+TTDEG
COC464   ATAL=0.
000464   CBTAI=0.
000465   DBGA=0.
000466   CBSAS=0.
000467   PHASE=0.
000471   PRINT 4, THS,ATAL,CBTAL,DBGA,CBSAS,PHASE
000510   IF ( ATAL.EQ.0.0) GO TO 5
C
* 9998 CONTINUE
0005111  I=1
000512   THS=THS-2.0
000514   IF(THS.LT.-180.0) GO TO 1
000516   THS=THS+TT
000517   XS(I)=X-R1*SIN(THI)
000525   YS(I)=R1*COS(THI)
000532   XP=X+R5*SIN(THS)
000536   YP=R5*COS(THS)
000541   TE=ATAN2(YP,XP)
000544   TB=ATAN2(YP,XP-XL)
000552   20 CONTINUE
000552   TC=ATAN2(YS(I),XL-XS(I))
000557   TD=ATAN2(YS(I),-XS(I))
* 000564   BOND=0.01*TT
*
C

ANY ONE OF THE 11 IF STATEMENTS ENDING IN GO TO 9999 MAY CAUSE THE
C
CURRENT ODD NUMBERED THIS ANGLE IN DEG TO BE SKIPPED
C

JR=(XP*YS(I)+XSI(I)+YPY)/(YS(I)+YP).
RO(I)=SQRT((XP-XSI(I))*(XP-XP(I))+(YP-YS(I))*(YP+YS(I)))/XLAMDA
OE(I)=PI
IF(ABS(THS+THI)*GT.BCND) OB(I)=FA(R5,R1,R0(I)*XLAMDA)
K1=WLNUM
*SQRT((XR-XSI(I))*(XR-XSI(I))+(YS(I))*(YS(I))
K2=WLNCRM
*SQRT((XP-XK)*(XP-XK)+(YP+YP))
K3=WLNCR
*SQRT((XL-XS(I))*(XL-XS(I))+(YS(I))*(-YS(I)))
K4=WLNCRM
*SQRT((XP-XL)*(XP-XL)+(YP+YP))
K5=WLNCRM
*SQRT((XP-XL)*(XP-XL)+(YP+H))
R6=WLVJR
*SQRT((XSI(I))**(-XSI(I)))+(-YS(I))*(-YS(I)))
W7=WLNCRM
*SQRT((XP*XP)+(YP*YP))
R8=WLNUM
*SQRT((XP*XP)+(YP+H))
THK=ATAN2(YP,XP-XK)
K9=WLNUM
*SQRT((XSI(I))*XSI(I))+(YS(I)+H)*(YS(I)+H)
PHP=(PI-ATAN2(YS(I),XS(I)))*TTDEG
PH1=(PI-ATAN2(YF,XP-XL))*TTDEG
PH2=(PI-ATAN2(YF,-H-YP))*TTDEG
PH3=270.0-PH1
THP=(ATAN2(YS(I),XS(I)))*TTDEG
TH1=(PI-ATAN2(YF,-XP))*TTDEG
TH2=PI-ATAN2(-XP,-H-YP)*TTDEG
TH4=ATAN2(-XSI(I),YS(I)+H)

C

IF(XSI(I),LT.0.0,AND.Abs((THI+THS)/TT-180.0L),LE.1.0) GO TO 9999
IF(XSI(I),GT.0.0,AND.Abs((THI-THP-180.0L),EQ.0.0) GO TO 9999
IF(XSI(I),LT.0.0,AND.Abs((TH2+TH4)/TT-180.0L),EQ.0.0) GO TO 9999
IF(Abs(Phl-PhP-180.0L),EQ.0.0) GO TO 9999

C

CA=G(R3,K4)
CALL DFKC=(DA,1.5,CA,PH1-PHP,1.)
CALL DFKCF(DA,1.5,QA,PH1-PHP,1.)
DK1=DH(DA,0B)

**
IF (TYPE\(=\) 3\(\times\)) DAL2(1) = DAL2(1) * COS(0.5 * PI + TH1 - THP * TT)
0.04139

IF (TYPE\(=\) 3\(\times\)) DAL8(1) = DAL8(1) * COS(TH1 - TH4)
0.04147

IF (TYPE\(=\) 3\(\times\)) DB2(1) = DB2(1) * COS(TH1 - TH4)
0.04163

IF (TYPE\(=\) 3\(\times\)) DLR(1) = DLR(1) * COS(0.5 * PI - TH1 - THP * TT)
0.04177

IF (TYPE\(=\) 3\(\times\)) DLR(1) = DLR(1) * COS(0.5 * PI + TH1 - THP * TT)
0.04186

**

IF (YP\. GT\. 0\. 0) GO TO 3333
0.04205

**

IF (XS(1). GE\. 0\. 0) AND\(\) PH1\.-\(\) PHP\. EQ\. 1\(8\) 0\. 0 OR \(\) TH1\.-\(\) THP\. EQ\. 1\(8\) 0\. 0)
1 GO TO 9999
0.04246

**

IF (XS(1). GE\. 0\. 0) AND\(\) PH1\.-\(\) PHP\. EQ\. 1\(8\) 0\. 0 AND\(\) TH1\.-\(\) THP\. GE\. 1\(8\) 0\. 0) GA(1) =
1 CMPLX(0.0, 0.0)
0.04264

**

IF (XS(1). LT\. 0\. 0) AND\(\) PH1\.-\(\) PHP\. EQ\. 1\(8\) 0\. 0 OR \(\) TH2\.-\(\) TH4\. TT\. EQ\. 1\(8\) 0\. 0)
1 GO TO 9999
0.04312

**

IF (XS(1). LT\. 0\. 0) AND\(\) PH1\.-\(\) PHP\. GT\. 1\(8\) 0\. 0 AND\(\) (TH2\.-\(\) TH4\. TT\. GT\. 1\(8\) 0\. 0)
1 GA(1) = CMPLX(0.0, 0.0)
0.04313

IF (PH1\.-\(\) GT\. 2\(7\) 0\. 0) AND\(\) PH1\.-\(\) LT\. 3\(6\) 0\. 0) CAR(1) = CMPLX(0.0, 0.0)
0.04356

IF (TH1\.-\(\) GT\. 2\(7\) 0\. 0) AND\(\) TH1\.-\(\) LT\. 3\(6\) 0\. 0) DALL(1) = CMPLX(0.0, 0.0)
0.04375

IF (PH2\.-\(\) GE\. 2\(7\) 0\. 0) AND\(\) PH2\.-\(\) LT\. 3\(6\) 0\. 0) CAR2(1) = CMPLX(0.0, 0.0)
0.04414

IF (TH2\.-\(\) GE\. 2\(7\) 0\. 0) AND\(\) TH2\.-\(\) LT\. 3\(6\) 0\. 0) CAL(1) = CMPLX(0.0, 0.0)
0.04433

IF (TH2\.-\(\) LT\. 0\. 0) OR\(\) TH2\.-\(\) LT\. 3\(6\) 0\. 0) DAL(1) = CMPLX(0.0, 0.0)
0.04452

IF (PH3\.-\(\) LT\. 0\. 0) OR\(\) PH3\.-\(\) LT\. 3\(6\) 0\. 0) CAR3(1) = CMPLX(0.0, 0.0)
0.04463

IF (TH2\.-\(\) GE\. 2\(7\) 0\. 0) AND\(\) TH2\.-\(\) LT\. 3\(6\) 0\. 0) DAL(1) = CMPLX(0.0, 0.0)
0.04474

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A11(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04513

**

IF (YP\. EQ\. -H\. (R\. YR\. FC\. 0\. 0) GO TO 9999
0.04557

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
1 RA(1) = RA(1) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04566

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04612

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04645

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04664

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04674

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04723

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04723

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04734

**

IF (XS(1). LT\. 0\. 0) AND\(\) XP\.-\(\) LT\. 0\. 0) RA(1) = A1(I) * R * (SQR(T((XS(1) + XP) *
1) (XS(1) + XP) + (YS(1) - VP) * (YS(1) - VP))) / XLAMDA)
0.04742
TA(I) = TA(I) + ADCA(I)  
TA(I) = TA(I) +ocrats(I)  
TA(I) = TA(I) + ADDS(I)  
TA(I) = TA(I) + ADDS(I)  
GO TO 4444  
CONTINUE
05626 M=1+1
05633 K(M)=SQRT((XP-YS(M))*5/(YP-YS(M)))*X/LAMDA
05644 UB(M)=FAU(I)*FLOAT(I),RO(1)*X/LAMDA,RO(M)*X/LAMDA
05655 IF(I.GT.11)OM=M-FAU(I)*FLOAT(I),RO(1)*X/LAMDA;
100(M)*X/LAMDA)
05672 IF(UB(I).GT.PLO2)UB(M)=UB(I)-FAU(I)*FLOAT(I),RO(1)*X/LAMDA;
100(M)*X/LAMDA)
05707 IF(CH(I).GT.PLO2)AND1.GT.NN)
100(M)=UB(I)+FAU(I)*FLOAT(I),RO(1)*X/LAMDA,RO(M)*X/LAMDA)
05734 IF(11.GT.NN)GO TO 21
05740 I=I+1
05741 GO TO 20
05741 11 CONTINUE
05751 S_ATL=COMPLX(0.0,0.0)
05744 R_ATL=COMPLX(0.0,0.0)
05746 T_ATL=COMPLX(0.0,0.0)
05751 UU 3 K=1,N
05752 K_ATL=K_ATL+GG(X(K))
05759 SATL=S_ATL+SAS(K)
05766 3 TOTAL=TOTAL+TA(K)
05776 ATAL=CAHST(TOTAL)
05780 DB_ATL=20.0*ALG10(ATAL)
06005 DBGA=20.0*ALG10(GA)
06005 DBSA=20.0*ALG10(SA)
06015 WR=REAL(TOTAL)
06017 WI=IMAG(TOTAL)
06020 PHASE=ATAN2(WI,WR)
06023 THS=THS+TTOEG
06025 PHASE=PHASE+TTOEG
06027 WRITE(6,4)THS,TOTAL,UBAL,UBGA,UBASA,PHASE
06036 4 FORMAT(5X,*THS=*,F7.2,5X,*ATAL=*,E12.4,5X,*DBTAL=*,E12.4,5X,
1*UBGA=*,F12.4,5X,*UBASA=*,E12.4,5X*PHASE=*,E12.4)

* C L-1 STROKES THE COMPUTED POINTS IN REVERSE ORDER IN THE ARRAY, THAT IS,
* THIS FROM -179 TC +179 DEG FOR CARTESIAN PLOTTING
* 2
06046 5 L=L-1
06050 THSTFD(L)=THS
06052 AMPFD(L)=ATAL
006053  DBTFD(L)=DBTAL
006055  PHTFD(L)=PHASE
006056  G0 TU 9958

* 006057  1 CONTINUE
C FINO MAX AMPTFD
006057  DO 6956 J=2,180
006064  IF (AMPTFD(J)>EQ.0.) AMPTFD(J)=(AMPTFD(J-1)+AMPTFD(J+1))/2.
006070  I F (AMPTFD(J)<E A.MAX) GO TO 6956
006073  AMAX=AMPTFD(J)
006074  TMAX=THSTFD(J)
006076  6956 CONTINUE

* C NORMALIZE AMPTFD
CC6100  DC 6957 J=1,180
006102  AMPTFD(J)=AMPTFD(J)/AMAX
CC6104  6957 CONTINUE
006106  PRINT 5553,AMAX,TMAX
006115  5553 FORMT(// MAX AMPTFD=*F9.5* FOR THSTFD=*F3* DEG*)
* 006115  PRINT 29,(THSTFD(J),AMPTFD(J),DBTFD(J),PHTFD(J), J=1,180)
006135  29 FORMT(//2*THS*5X*AMPTFD10*7X*DBTFD*8X*PHASE*/(F5.3(4X10.5)))

* 006135  THSTFD(181)=-180.
006136  THSTFD(192)=90.
006140  AMPTFD(181)=0.
CC6140  AMPTFD(192)=-1.
006142  PHTFD(181)=-180.
006142  PHTFD(182)=90.
006144  CALL PLTTFD(THSTFD,PHTFD,180)
006146  CALL PLTTFD(THSTFD,AMPTFD,180)

* *
C REORDER THSTFD VS DBTFD IN THE ARRAY, FROM 1 TO 359 DEG, FOR POLAR PLOT
* 006151  KK=90
006152  DO 1113 J=1,90
006154  KK=KK+1
006156  TEMP=THSTFD(KK)
006157  THSTFD(KK)=THSTFD(J)+360
006162   THSTFD(J) = TEMP
006164   TEMP = CBTFD(KK)
006165   DBTFD(KK) = DBTFD(J)
006167   DBTFD(J) = TEMP
006170   1113 CONTINUE

** IN LSCSBOX

CC6172   FIELD = 5HOTAL
006173   DMAX = DBTFD(1)
006174   IMAX = THSTFD(1)
006176   DO 6958 J = 2, 180
006200   IF (DBTFD(J) .EQ. 0.) DBTFD(J) = (DBTFD(J-1) + DBTFD(J+1)) / 2.
006204   IF (CBTFD(J) .LE. DMAX) GO TO 6958
006207   DMAX = DBTFD(J)
006210   Tmax = THSTFD(J)
006212   6958 CONTINUE

* C NORMALIZE DBTFD
006214   DO 6959 J = 1, 180
006216   DBTFD(J) = (DBTFD(J) - DMAX) / DMAX
006220   IF (CBTFD(J) .LT. -40.) CBTFD(J) = -40.
006224   6959 CONTINUE
006226   PRINT 5554, DMAX, Tmax
006236   5554 FORMAT(//, * MAX DBTFD = * F9.5* FOR THSTFD = * F3* DEG*)
006236   PRINT 5555, (THSTFD(J), CBTFD(J), J = 1, 180)
006252   5555 FORMAT(//, * THSTFD = 7X* DBTFD NOR*/ (2X F3, 8X F10.5))

* 006252   CALL POLPLOT(THSTFD, CBTFD, 180, 7.5, 8.1, N, TYPE, AL, XMADA, XLH, HR, RS
       1, THI, DMAX, FIELD, SOURCE)
006275   PRINT 8888
006301   8888 FORMAT(///)

* 006301   CALL TEST(N, AL, RS, HR, AT, TYPE, SOURCE)
006337   GO TO 6789
006310   1112 CONTINUE
006310   CALL CALPLOT(J, 0., 559)
006313   STOP
006315   END
SUBROUTINE DFRCP(D,XX,Y,B,XLMCA)
   C
   C THIS ROUTINE IS TO COMPUTE THE DIFFRACTION COEFFICIENT
   C
   C COMPLEX F1J,F2J,G,R,T1,D1,T2,D2,D1,F1,F2,
   C
   COMMON/PICCNST/PI,TWOP1,STP,SQRTPI,TT,TT0EG,P102,P1D4
   C
   00C010 *  
   00C010  BR=D*TT
   00C010  ARL=(PI+BR)/(2.0*XX)
   00C010  ARG2=(PI-BR)/(2.0*XX)
   00C020  CX1=COS(ARG1)
   00C022  CX2=CCS(ARG2)
   00C024  SX1=SAN(ARG1)
   00C026  SX2=SAN(ARG2)
   00C030  X1=(BR+PI)/(2.0*XX*PI)
   00C037  N1=X1
   00C041  E1=X1-N1
   00C043  IF(E1.GT.0.5) N1=N1+1
   00C050  IF(E1.LT.-0.5) N1=N1-1
   00C054  FN1=FLOAT(N1)
   00C055  X2=(BR-PI)/(2.0*XX*PI)
   00C061  N2=X2
   00C063  E2=X2-N2
   00C065  IF(E2.GT.0.5) N2=N2+1
   00C072  IF(E2.LT.-0.5) N2=N2-1
   00C076  FN2=FLOAT(N2)
   00C077  A1=1.0+CO8(8-8+2.0*XY*PI)*FN1)
   00C013  A2=1.0+C08(8-8+2.0*XY*PI*FN2)
   00C017  SAI=SQRT(A1)
   00C019  SA2=SQRT(A2)
   00C033  XX=(SQRT(TWOPI*YY))*SA1
   00C014  YY=(SQRT(TWOPI*YY))*SA2
   00C016  XX=XX*XX
   00C018  YY=YY*YY
   00C160  PC=SQRT(P102)
   00C154  CALL CS(C1,S1,XXS)
   00C166  CC1=0.5-C1
   00C170  SS1=0.5-S1
   00C172  F1J=PC*COMPLEX(SS1,CC1)
   00C202  CALL CS(C2,S2,YYS)
   00C205  CC2=0.5-C2
GOJ2V1
SS2=U*S-S2
00711
F2J=PI*CMPLX(55,CC2)
0LC21
P=-PI/4
0C0222
U=CMPLX(0,0,P)
0C0225
R=CEXP(U)
0C0227
S=(U+0.25)*K*SQRT(XLMDA)/(PI*XN)
0C0234
T1=S+2*D*CEXP(CMPLX(0,0,XSS))
0C0270
D1=T1*FLJ*CX1*(XX/SX1)
0C0332
T2=S+2*D*CEXP(CMPLX(0,0,YXS))
0C0316
D2=T2*F2J*CX2*(YY/SX2)
0C0330
D=D1+D2
0GC340
RETURN
0C0341
END

SUBROUTINE KKDC(D,XN,XL,PHIP,PHI,XLMDA,T)
C  C  -  KARKET  -  KELLER  DIFFRACTION  COEFFICIENT
C  XL  -  DISTANCE  PARAMETER  IN  ANY  UNIT
C  XLMDA - WAVELENGTH  IN  THE  SAME  UNIT
C  PHIP - ANGLE  OF  INCIDENCE  IN  DEGREES
C  PHI  -  ANGLE  OF  OBSERVATION  IN  DEGREES
C  XN  -  WEDGE  ANGLE  = (2.*XN)*PI
C  T  -  D/D(PHIP)  OR  D/D(PHI)  = 1  OR  0
0C012
INTEGER T
0C012
COMPLEX D,D1,D2
0C012
BETAP=PHI+PHIP
0C013
BETAM=PHI-PHIP
0C015
CALL CC(D1,XN,XL,BETAP,XLMDA, 1 ,T)
0C022
CALL CO(D2,XN,XL,BETAP,XLMDA,-1 ,T)
0C033
D=D2-D1
0C043
RETURN
0C044
END
SUBRoutines CD(D,XN,XL,BETA,XLMCA,T1,T2)
C T1 = BETA OR BM 1 OR -1
C T2 = CD(PHIS) OR CD(PHIO) 1 OR 0
000012 INTEGER T1,T2
000015 COMPLEX D,CP,CM,DP,DM,V,P,CCP,CMC,C,F1,F2
000017 P(X)=CEXP(COMPLEX(0,X))

000030 COMMON/PICONST/PI,TWCPI,STP,SCRTP,T,T,TD,EDG,PI02,PI04
*
000031 XK=TWOP/I*XLMDA
000032 C=P(-PI04)/(4.*((XN**2)*SRT(TWOPI*XK)))
000033 BR=BE TA**TT
000034 TP=(PI+BR)/(2.*XN)
000035 ATP=ABS(TP)
000036 IF(ATP .LT. 0.01)GO TO 10
000037 XNP=(BR+PI)/(2.*XN*PI)
000038 NP=XNP
000039 E1=XKF-NP
000041 IF(E1 .GT. 0.5)NP=NP+1
000042 IF(E1 .LT. -0.5)NP=NP-1
000043 AP=1+COS(-BR+2.*XN*NP*PI)
000044 XKAP=XK**XL*AP
000045 CP=CEXP(COMPLEX(0,-PI04))/(4.*((XN**2)*SRT(TWCP**I*XK)*((PI+BR)/
1(2.*XN))**2))
000046 CCP=CP V(XKAP)
000048 GO TO 11
000050 10 A1=2.*XK**XL*(XN**2)*((ATP**2)
000051 CALL CS(C1,S1,1)
000053 F1=CMPLX(C,4.*XK**XL*(XN**2)+2.*SRTPI*P(A1)*((2.*XK**XL*(XN**2)
1)*1.5)*ATP*P(-PIC4**I-SRT(2.)*CMPLX(C1,-S1))
000056 CCP=CP F1
000058 GO TO 11
000061 TM=(PI-BR)/(2.*XN)
000063 ATM=ABS(TM)
000065 IF(ATM .LT. 0.01)GO TO 20
000067 XNM=(BR-PI)/(2.*XN*PI)
000069 NM=XNM
000071 E2=XN**NM
000072 IF(E2 .GT. 0.5)NM=NM+1
000073 IF(E2 .LT. -0.5)NM=NM-1
000075 AM=1+COS(-BR+2.*XN**NM*PI)
000077 XKAM=XK**XL*AM
000079 CM=CEXP(COMPLEX(0,-PI04))/(4.*((XN**2)*SRT(TWOPI*XK)*((PI-BR)/
1)))
000080 20 70000000
000081 70100000
000082 70200000
000083 70300000
000084 70400000
000085 70500000
1.2*21
00421       GO 10 21
00525       AL=2.*XX*XL*(XN**2)*(ATP**2)
00536       CALL CS(C1,S1,A1)
00546       F2=CMPLX((J,4.*XX*XL*(XN**2)+2.*SQRTPI *P(A1)*((2.*XX*XL*(XN**2))
1)**L.5)*ATM*(PI-P104 )-SQRT(2.*CMPLX(C1,-S1))
00557       CCM=C2F2
00551       IF(T1 <EQ. 0 )GO TO 1
00552       DP=CCP
00553       DM=-CCM
00554       IF(T2 =EQ. 0 )DP=-CCP
00555       IF(T2 =EQ. 0 )DM=CCM
00556       GO TO 2
00557       DP=CCP
00558       DM=-CCM
00560       U=DP+OM
00561       RETURN
00562       END

CMPLX FUNCTION V(X)
TRANSITION FUNCTION FOR THE INTEGRAL WITH THE INTEGRAND HAVING THE
POLE OF ORDER TWO CLOSE TO THE SADDLE
X = K*ATM(A)
CMPLX P
C
P (X) = CECP(CMPLX(0, X))

CCM=CMPLX(PI,PICNST/PI, TP,STP, SQRTPI,TT,TTD,PI92,PI34
CALL CS(C,S,X)
V=CMPLX(0.0,2.*X)+2.*(X**L.5)*P(X)*SQRTPI *(P(-PI04 )-SQRT(2.*
CMPLX(C,-S))
RETURN
END
SUBROUTINE WANG(PH, PHP, XN, XL, D1, D2)
  COMPLEX  D1, D2, DA, CB, CH, DS, U, V
  DLU(U, V) = SQRT(6.283185301796)*((U+V)
  DLS(U, V) = SQRT(6.283185301796)*((U-V)
  CALL DHRCF(DA, XN, XL, PH-PHP, 1.0)
  CALL DFRCF(CB, XN, XL, PH+PHP, 1.0)
  D1 = UH(DA, DR)
  D2 = DLS(CA, DR)
  RETURN
END

SUBROUTINE CS(C, S, X)
  COMPUTES THE FRESNEL INTEGRALS

  DESCRIPTION OF PARAMETERS
  C       THE RESULTANT VALUE C(X)
  S       THE RESULTANT VALUE S(X)
  X       THE ARGUMENT OF FRESNEL INTEGRALS
          IF X IS NEGATIVE, THE ABSOLUTE VALUE IS USED

  THE ARGUMENT VALUE X REMAINS UNCHANGED

  C(X) = INTEGRAL(COS(T)/SQRT(2*LI*T) SUMMED OVER T FROM 0 TO T) * X
  S(X) = INTEGRAL(SIN(T)/SQRT(2*LI*T) SUMMED OVER T FROM 0 TO T)

  EVALUATION
  USING DIFFERENT APPROXIMATIONS FOR X .LT. 4 AND X .GT. 4

  REFERENCE
  COMPUTATION OF FRESNEL INTEGRALS BY H.GERSMA,
  MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION, VOL. 14,
  1960, NO. 72, P. 380

  Z = ABS(X)
  IF(Z .LE. 1, 1, 2)
  C = SQRT(Z)
  S = Z*C
  Z = (4.4-Z)*(4.4+Z)
  C = C**((1.181-1.00785E-11)*Z+5.244297E-9)*Z+5.451182E-7)*Z
  + 3.273398E-51*Z+1.2041E-3)*Z+1.02544E-2)*Z+1.84965E-1)
  S = S**((1.66768E-10*Z+5.883158E-8)*Z+5.051141E-6)*Z
  + 2.441816E-4)*Z+6.121320E-3)*Z+8.026490E-2)
SUBROUTINE TEST(N,AL,XS,A,TYPE)
C THIS SUBROUTINE COMPUTES THE RADIATION FIELD OF HORN ANTENNA
C
C 00010 DIMENSION X(180),AM(180),AP(180),DG(180),AI(180),ANG(180)
C 00010 DIM X,h,THI,F(185),AMPIF(185),OBIF(185)
C 00010 COMPLEX F,A,D,G,C,D,TAL
C
C 00010 COMMON XLAMDA,XPIN,XLH,RH,RS,THI,DMAX,FIELD,SOURCE
C 00010 COMMON/PLICNST/PI, TP,STP,SQRTPI,TT,TTOEG,PI02,PIJ4
C 00010 XPIN IS POINT OF INCIDENCE WHICH IS X IN MAIN PROGRAM
C
C 00010 F(Z)=EXP(CMPLX(0.,-6.2831853071796*Z))/SQRT(6.2831853071796*Z)
C 00036 FA(SA,SB,SC)=ACOS((SB*SB+SC*SC-SA*SA)/(2.0*SB*SC))
C 00057 FC(AA,SH,SC)=SQRT(1/2.0)*SB*SC*COS(AA)
C 00010 FB(SX)=ABS(COS(SX/2.0))**2.0
C
C FB(SX) IS THE OBLIQUITY FACTOR. TO IGNORE IT, SET FB(SX)=1.
C
C 000120 NN=(N+1)/2
C
C 000123 D=0.0
C 000123 IF(N,GT,1) D=AL/FLAT(N-1)
C
C 000130 TH=0.0
C 000130 CONTINUE
C
C 000131 IF(TH,EQ,90.0) GO TO 15
C 000133 TH=TH+T
C 000134 ANG(1)=TH
C 000136 X(1)=XS
C 000137 DG(1)=A(1)*F(XS)*FA(TH)
C 000153 IF(TYPE,FQ,3.0,AND,TH,LT,PI02 ) DG(1)=DG(1)*COS(TH)
C 000175 IF(TYPE,FC,3.0,AND,TH,GT,PI02 ) DG(1)=DG(1)*COS(PI-TH)
C
C 000221 IF(N,EQ,1) GO TO 11
C
C 000223 I=2
C 000223 CONTINUE
C
C 000224 X(I)=FC(PI*0.5,TH,FLOAT(I-1)*D,XS)
C 000236 IF(I,GT,NN) X(I)=FC(PI02,-TH,FLOAT(I-NN)*D,XS)
C 000251 IF(TH,GT,90.0) X(I)=FC(1.5,PI-TH,FLOAT(I-1)*D,XS)
C 000266 IF(TH,GT,90.0,AND,I,GT,NN) X(I)=FC(TH-PI02,FLOAT(I-NN)*D,XS)
C
C 000311 ANG(I)=TH+FA(FLOAT(I-1)*D,XS,X(I))
C 000323 IF(I,GT,NN) ANG(I)=TH-FA(FLOAT(I-NN)*D,XS,X(I))
C 000336 IF(TH,GT,90.0) ANG(I)=TH-FA(FLOAT(I-1)*D,XS,X(I))
C 000352 IF(TH,GT,90.0,AND,I,GT,NN) ANG(I)=TH+FA(FLOAT(I-NN)*D,XS,X(I))
000376  DG(I)=A(I)+F(X(I))*FB(ANG(I))
000416  IF (TYPE.EQ.3.0.AND.ANG(I).LT.PI/2) DG(I)=DG(I)+COS(ANG(I))
000442  IF (TYPE.EQ.3.0.AND.ANG(I).GT.PI/2) DG(I)=DG(I)+COS(PI-ANG(I))
000470  IF (EC.N) GO TO 11
000472  1=1+1
000473  GO TO 3
000474  11 CONTINUE
000474  TAL=CMPLX(C,O,O,O)
000477  DO 33 K=1,N
000500  COG=UG(K)
000503  ANGL=ANG(K)*TTDEG
000505  ADG=ABS(COG)
000507  TAL=TAL+UG(K)
000516  33 CONTINUE
000523  ATAL=CAH(S,TAL)
000524  IF (TH.EQ.0.0) XNML=ATAL
000532  DBTAL=20.0*ALOG10(ATAL)
000536  WRITE(6,4) TH,ATAL,EDTAL
000547  4 FOKMAT(5X,TH=\#F10.4,5X,\*MAGNITUDE=*,E15.4,5X,\*DBHA=*,E15.4)
000547  L=L+1
000551  *  THIFO(L)=TH
000553  AMPIFO(L)=ATAL
000554  DBIFO(L)=DBTAL
000556  CONTINUE
000556  TH=TH*2.0
000556  15 CONTINUE
000556  IF (TH.LE.30D0) GO TO 2
000556  PRINT 39,(THIFO(J),AMPIOF(J),DBIFO(J),J=1,L)
000534  39 FOKMAT(//7X,TH=\#F7X,MAGNITUDE=7X,DBHA=\*(F5,2(5XF70.5)))
000564  C FIND MAX AMPIFO
000564  AMAX=AMPIOF(1)
000575  Tmax=THIFO(1)
000567  DO 6556 J=2,L
000567  IF (AMPIOF(J).LE.AMAX) GO TO 6596
000556  AMAX=AMPIOF(J)
000562  TMAX=THIFO(J)
000562  6556 CONTINUE
000564  C NORMALIZE AMPIFO
000564  DO 6597 J=1,L
000564  AMPIFO(J)=AMPIFO(J)/AMAX
000567  6597 CONTINUE
000567  PRINT 5555,AMAX,TMAX
000567  5555 FOKMAT(//5X,MAGNITUDE=\*F9.5,FOR THIFO=\*F3\* DEG)
** IN TEST **

**

J061 FIELD=5HIND 0
J062 DMAR=CBIFD(1)
J064 TMAX=THIFD(1)
J066 DD 6598 J=2.
J065 IF(DBIFD(J).LE.DMAX) GO TO 6598
J066 DMAR=CBIFD(J)
J065 TMAX=THIFD(J)
J067 6598 CONTINUE

C normalize DBFA which is DBIFD
J062 DC 6579 J=1.
J063 DBIFD(J)=DBIFD(J)-DMAR
J065 IF(CBIFD(J).LT.-40.) CBIFD(J)=-40.
J067 6599 CONTINUE

J067 PRINT 5556,DMAR,TMAX
J070 FORMAT(/** MAX DBIFD=*.F9.5* FOR THIFD=*F3. DEG*)
J070 PRINT 40, (THIFD(J), AMPTFD(J), DBIFD(J), J=1,L)
J072 CALL POLPLT(THIFD,AMPTFD, 180,7.5,8,1,N,THI, X,AMPTFD,NPTS,1,RS,THI,DMAR,FIELD,SOURCE)

***

J071 THIFD(1+1)=0.
J073 THIFD(1+2)=30.
J075 AMPTFD(1+1)=0.
J075 AMPTFD(1+2)=.1
J077 CALL PLIFD(THIFD,AMPTFD,180)
J071 RETURN
J072 END

SUBROUTINE PLIFD(THIFD,AMPTFD,NPTS)

G0006 DIMENSION THIFD(1),AMPTFD(1)
G0006 TMAJ=1.
G0006 THI=3.
J0011 CALL USHI(0.00,0.0,2.,28.,11HTOTAL FIELD,0.,11)
J0015 CALL CALPLT(2.,1.,0.,-3)
J0022 CALL CALL (0.0,0.,90.,0.,0.,0.,1.,1.,5.,9HAMPLITUDE.,28,9)
J0033 CALL CALL (J,0.,1.,12.,-180.,30., TMAJ,THI,THIFD),DEJS,.28,-10)
J0045 CALL CALL (0.,0.,12.,0.,0.,0.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.)
J0051 CALL CALL (12.,0.,0.,90.,10.,0.,0.,0.,1.,1.,5.,1.,0.,0.,-1)
J0054 CALL CALL (THIFD,AMPTFD,NPTS,1,0,0,0)
J0071 CALL CALL (STRAMP)
J0074 CALL RETURN
J0105 END
SUBROUTINE PLTFDP(THSTFD,PHTFD,NPTS)
DIMENSION THSTFD(1),PHTFD(1)
TMAJ=1.
TMIN=3.
CALL NCTATE(0.,0.,2.,28.,11HTOTAL FIELD,0.,11)
CALL CAL3LT(0.,1.,0.,-3)
CALL AXES(0.,0.,90.,12.,-180.,30.,1.,3.,9HPHASE,DEG.,28.,9)
CALL AXES(0.,C.,0.,12.,-180.,30.,TMAJ,TMIN,10HTHSTFD,DEG.,28.,-10)
CALL AXES(0.,12.,0.,12.,-180.,30.,TMAJ,TMIN,1H.,3.,1)
CALL AXES(12.,0.,0.,90.,12.,-180.,30.,1.,3.,14.,0.,-1)
CALL LINE(THSTFD,PHTFC,NPTS,1,1,0.,0.)
CALL NFRAME
RETURN
END
SUBROUTINE PULPLT(DEGS, DBS, NPTS, DIAM, NOCIR, NPLOTS, N, TYPE, AL,
1XLAMDA, X, XL, H, RI, RS, THI, DMAX, FIELD, SOURCE)

*  DICE DEGS(1), EPS(1), CHTAB(16)
*  DICE ANUM(11), FIELD(2)
0026  DICE CC(400), RHOX(400), RHOY(400), PHI(400)
0026  RAD = DIAM / 2.0
0027  HGT = .21

*  CHTAB(1) = 1H
0031  CHTAB(2) = 1H5
0032  CHTAB(3) = 2H10
0034  CHTAB(4) = 2H15
0035  CHTAB(5) = 2H20
0037  CHTAB(6) = 2H25
0039  CHTAB(7) = 2H30
0041  CHTAB(8) = 2H35

*  CALL NOTATE (0., 0., 0., 1)
0045  X = 10.
0046  P = 10.
0047  AY = 5.0
0048  PY = 5.
0049  K = 1
0050  HGT = .14

*  CONTINUE
0067  1 CONTINUE
0068  DO 5 I = 1, NOCIR
0069  RADF = RAD
0071  CALL CIRCLE(CX, AY, 0., 360., RAD, RADF, 3)
0073  IF (I .EQ. 3) K = 2
0074  CALL NOTATE (CX, AY, HGT, CHTAB(1), 270. + K)
0076  RADO = RADO - DELTR
0078  CX = CX - DELTR
0080  5 CONTINUE

*** COORDINATES FOR CENTER OF CIRCLE ****
0127  CNT HX = PX - RAD
0130  CNTRY = PY

96100000
96200000
96300000
96400000
96500000
96600000
96700000
96800000
96900000
97000000
97100000
97200000
97300000
97400000
97500000
97600000
97700000
97800000
97900000
98000000
98100000
98200000
000132 ANGLE=.174532925
000133 TIC=.1
00435 RPT=RAC+TIC
000136 DELTA=.174532925

*** DRAW TIC MARKS EVERY 10 DEGREES ***

000140 DO 50 I=1,36

*** PAGE COORDINATES ***

000141 TICX1=CNTRX+RPT*COS(ANGLE)
000145 TICY1=CNTRY+RPT*SIN(ANGLE)
000152 TICX2=CNTRX+RPT*COS(ANGLE)
000157 TICY2=CNTRY+RPT*SIN(ANGLE)
000164 CALL CALPLT(TICX1,TICY1,3)
000167 CALL CALPLT(TICX2,TICY2,2)
000172 ANGLE=ANGLE+DELTA

50 CONTINUE

*** DRAW LABELS ***

00202 ANUM(1)=N
00203 ANUM(2)=TYPE
00205 ANUM(3)=AL
00206 ANUM(4)=XVALMA
00210 ANUM(5)=X'
00211 ANUM(6)=X
00213 ANUM(7)=H
00214 ANUM(8)=R
00216 ANUM(9)=R S
00217 ANUM(10)=THI/(.1*DELTA)
00222 ANUM(11)=D MAX

00024 CALL NUTATE(1.140,9.0,.21,14)TRANSMITED W L=,270.,14)
00023 CALL NUMBHR(1.140,6.5,.21,ANUM(4),270.,2)
00026 CALL NUMBHR(1.140,4.3,.21,12HINCID RANGE=,270.,12)
00029 CALL NUMBHR(1.140,2.0,.21,ANUM(13),270.,2)
00025 CALL NUTATE(1.10,5.0,.21,12HINCID POINT=,270.,12)
000254 CALL NUMBHR(1.10,6.7,.21,ANUM(5),270.,2)
000262 CALL NUTATE(1.10,4.3,.21,12HSCATER RANGE=,270.,12)
000266 CALL NUMBHR(1.10,2.0,.21,ANUM(9),270.,-1)
000274 CALL NUTATE(1.10,8.0,.21,1480 UX LENGTH=,270.,11)
000300 CALL NUMBHR(1.10,8.0,.21,ANUM(6),270.,2)
000306 CALL NUTATE(1.10,8.0,.21,12HINCID ANGLE=,270.,12)
000312 CALL NUMBHR(1.10,8.0,.21,ANUM(10),270.,-1)
CALL NOTATE(1.050, .9, 21, 11) HBOX HEIGHT=270.11)
006324 CALL NUMBER(1.050, .9, 21, ANUM(7), 270., -1)
006332 CALL NOTATE(1.050, .9, 21, 12HOMETIZ D3X, 270..12)
006336 CALL NUMBER(1.050, .20, 21, ANUM(11), 270., 3)
006344 CALL NOTATE(1.70, .4, 3, 21, 15HRNE SOURCES, N=270..15)
006350 CALL NUMBER(1.70, 1.6, 21, ANUM(1, 270., -1)
006356 CALL NOTATE(1.40, .8, 21, 20H CURRENT TYPE=270..20)
006362 CALL NUMBER(1.40, 1.2, 21, ANUM(2), 270., -1)
006370 CALL NOTATE(1.40, .8, 21, SOURCE=270..9)
006375 CALL NOTATE(1.10, .9, 21, 11H FIELD=270..11)
006381 CALL NOTATE(1.10, 9.0, 21, FIELD, 270..5)
006386 CALL NOTATE(1.10, 4, 21, 12HAPERT WIDTH=270..12)
006392 CALL NUMBER(1.10, 1.75, 21, ANUM(3), 270., 3)
*** DRAW X AXIS
006400 AX=PX-DIAM-.5
006408 CALL CALPLT(AX, AY, 3)
006416 AX=PX-.5
006424 CALL CALPLT(AX, AY, 2)
*** DRAW ZERO DEGREES AFTER HORIZONTAL PATH
006432 HGT=.01
006436 AX=PX
006444 AY=AY-.05
006452 CALL NOTATE(AX, AY, HGT, 1H0, 270., 1)
006460 AX=AX+.21
006468 AY=AY-.12
*** DEGREE SYMBOL
006476 CALL NOTATE(AX, AY, .07, 1H0, 270., 2)
*** DRAW Y AXIS
006484 AY=5.0
006492 AX=PX-RAD
006500 AY=AY-RAD-.5
006508 CALL CALPLT(AX, AY, 3)
006516 AY=AY+DIAM+1.
006524 CALL CALPLT(AX, AY, 2)
*** DRAW 90 DEGREES
006532 AY=AY+.1
006540 CALL NOTATE(AX, AY, HGT, 2H90, 270., 1)
006548 AX=AX+HGT
006556 AY=AY-.32
006564 CALL NOTATE(AX, AY, .07, 1H0, 270., 1)
*** DRAW 180 DEGREES
006572 AX=PX-CIAM-.3
006580 AY=5.0

006800000
000527  CALL NCTATE (AX,AY,HGT,3H180,270.,3)
000533  AX=AX+.21
000535  AY=AY-.5
000537  CALL NCTATE (AX,AY,.07,1H0,270.,1)
*
***DRAW 270 DEGS
000543  AX=PX-RAD
000545  AY=5.0-RAD-.05
000550  CALL NCTATE (AX,AY,HGT,3H270,270.,3)
000554  AX=AX+HGT
000556  AY=AY-.5
000560  CALL NCTATE (AX,AY,.07,1H0,270.,1)
*
**
*** PLOT POLAR CURVES ***
000564  151 CONTINUE
*
*** SCALE DECIBELS AND CONVERT POLAR COORDINATES TO CART. COORDS.***
000566  DC 500 JJ=1,NPTS
000571  PHI=DEGS(JJ)*.0174532925.
000573  CCB(JJ)=DRS(JJ)+40.
000576  CDB(JJ)=.025*RAD*CDB(JJ)
000581  RHOX(JJ)=CCB(JJ)*COS(PHI)+PX-RAD
000586  RHOY(JJ)=CDB(JJ)*SIN(PHI)+5.0
000590  500 CONTINUE
000616  LIM=NPTS-1
***BEGIN PLOTTING WITH PEN UP***
000625  IPEN=3
000627  DO 100 I=1,LIM
000630  J=I+1
000632  CALL CIRCLE(RHOX(I),RHOY(I),DEGS(I),DEGS(J),CDB(I),CDB(J),IPEN)
000635  IF(1.011J.GT.1)GO TO 110
000636  IPEN=2
000644  110 CONTINUE
000665  100 CONTINUE
*
000670  CALL NFRAME
000671  RETURN
000672  END
SUBMULTIPLF PLTIFUL(THIFO, AMPIFO, NPTS)

CALL MULTM THIFO(1), AMPIFO(1)

CALL MULTM (0.0, 0.2, 28, 14) INCIDENT FIELD, 0..14

CALL MULTM (0.0, 1.0, -3)

CALL AXS(0.0, 0.0, 0.0, 1.0, 5.0, 9) AMPLITUDE, -28, 9

CALL AXS(0.0, 0.0, 12.0, 0.0, 30.0, 1MAJ, TMIX, YTHIFO, DEG, 28, -9)

CALL AXLS (0.0, 1.0, 0.0, -180, 30, 1MAJ, TMIX, IH, 0.0, 1)

CALL AXLS (12.0, 0.0, 90, 0.0, 90, 1.0, 5.0, 1IH, 0.0, -1)

CALL LIN(THIFO, AMPIFO, 180, 1, 0, 0, 0)

CALL NFRAKE

RETURN

END
REFERENCES


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