COMPUTER PROGRAM FOR
THIN-WIRE STRUCTURES IN
A HOMOGENEOUS CONDUCTING MEDIUM

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JUNE 1974
A computer program is presented for thin-wire antennas and scatterers in a homogeneous conducting medium. The analysis is performed in the real or complex frequency domain. The program handles insulated and bare wires with finite conductivity and lumped loads. The output data includes the current distribution, impedance, radiation efficiency, gain, absorption cross section, scattering cross section, echo area and the polarization scattering matrix. The program uses sinusoidal bases and Galerkin's method.
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I. INTRODUCTION

Reference 1 presents the electromagnetic theory for a thin-wire structure in a homogeneous conducting medium, and this report presents the corresponding computer program. The program performs a frequency-domain analysis of thin-wire antennas and scatterers. The wire configuration is a generalized polygon assembled from straight wire segments. The program has been tested extensively with simple structures (linear dipoles, V dipoles, coupled dipoles, square loops, octagonal loops, multiturn loops and coupled loops) and complicated configurations including wire-grid models of plates, spheres, cones, aircraft and ships. Although the air-earth or air-water interface is not considered, the program is applicable in many problems involving buried or submerged antennas or targets. It is useful in locating the poles of the admittance or scattering function for wire structures in the complex frequency domain.

A piecewise-sinusoidal expansion is used for the current distribution. The matrix equation \( ZI = V \) is generated by enforcing reaction tests with a set of sinusoidal dipoles located in the interior region of the wire. Since the test dipoles have the same current distribution as the expansion modes, this may be regarded as an application of Galerkin's method. Rumsey's reaction concept was most helpful in this development, and therefore the formulation is known as the "sinusoidal reaction technique."

The current is assumed to vanish at the endpoints (if any) of the wire, and Kirchhoff's current law is enforced everywhere on the structure. The input data specify the frequency, wire radius, wire conductivity, the parameters of the exterior medium, coordinates of points to describe the shape and size of the wire configuration, and a list of the wire segments. If some or all of the wire segments are insulated, the radius and permittivity of the insulating sleeve are indicated.

Coordinates are required for wire endpoints, corners, junctions and terminals. For accuracy, the longest wire segment should not greatly exceed one-quarter wavelength. Longer segments should be subdivided by defining additional current-sampling points. The program automatically defines a set of \( N \) sinusoidal dipole modes on the wire structure and computes the mutual impedance matrix for these modes. The elements in the matrix are generated by numerical integration when appropriate, or from closed-form expressions in terms of exponential integrals. The computer program uses certain approximations which yield a symmetric matrix even when the wire structure has finite conductivity, lumped loads and insulating sleeves.

In antenna problems, the output data includes the current distribution, impedance, radiation efficiency, gain, patterns and near-zone field. In bistatic scattering problems, the output includes the echo
area and the complex elements of the polarization scattering matrix. In backscatter situations, the output includes also the absorption, scattering and extinction cross sections.

If the wire has finite conductivity or dielectric sleeves, it is assumed that the frequency is real. This restriction can readily be removed if the user will specify the surface impedance of the wire and the complex permittivities of the dielectric sleeves and the ambient medium appropriate for complex frequencies.

The user may make a tradeoff between accuracy and computation costs by specifying the input variable INT. A large value increases the accuracy and the cost. For most problems, the recommended value is INT = 4.

The program was run on an IBM 370/165 computer to determine the broadside backscatter for a wire-grid square plate with edge length L. With a five-by-five grid, there are 60 segments, 36 points and 84 simultaneous equations. With INT = 4, calculations were made for L/\lambda = 0.3, 0.4, 0.5, 0.6 and 0.7. The execution time was 100 seconds. This averages to 20 seconds for each value of L/\lambda. The wire structure was perfectly conducting, uninsulated and located in free space. No advantage was taken of the target symmetries.

The next section presents the thin-wire computer program, instructions for the user, typical input and output data and tables of the mutual impedance of sinusoidal dipoles. Appendices list the computer subroutines and explain their functions.

II. THE THIN-WIRE COMPUTER PROGRAM

Fig. 1 is a Fortran listing of the thin-wire computer program. Near the beginning of this program, the DIMENSION statements reserve storage for a wire structure with up to 50 segments, 55 points and 60 dipole modes. Quantities with the same or related dimensions are grouped together on the same line or consecutive lines.

NM denotes the actual number of monopoles (segments), INM is the corresponding dimension, and the dimension for CG, VG and ZLD is twice INM. The second subscript for MD always has a dimension of 4.

N denotes the number of simultaneous linear equations and ICJ is the corresponding dimension. The dimension for C is (ICJ*ICJ + ICJ)/2.

The DO LOOP ending at statement 15 sets ISC(J) = 0 for all the segments. This indicates that the wires are bare or uninsulated. If some or all of the segments are insulated, the user may set ISC(J) = 1 for the appropriate segment numbers J.
Fig. 1a. The thin-wire computer program.
CALL GANTI(IA,IB,INM,IWR,11,12,13,112,JA,JB,MO,N,NO,NM,AM) 0063
2; C, CD, CG, CMM, D, EFF, GAM, GG, CGO, SG0, VG, Y11, Y11, Z1L, ZS)
WRITE(6,3)EFF,GG,ZS) 0064
WRITE(6,3)EFF,GG,ZS) 0065
200 IF (INEAR.LE.O) GO TO 300 0066
CALL GNFLD(IA,IB,INM,11,12,13,MD,N,NO,NM,AM,CG0,SG0,ETA,GAM)
WRITE(6,3)XP,YP,ZP 0067
WRITE(6,6)EX,EY,EZ 0068
300 IF (IAGAIN.LE.0) GO TO 400 0069
INC=0 0070
PH=PHA 0071
TH=THA 0072
CALL GFFLD(IA,IB,INC,INM,IWR,11,12,13,112,MD,N,NO,NM,AM)
WRITE(6,3)PH,TH,GPP,GTT 0073
WRITE(6,6)ACSP,ACST,ECSP,ECST,SCSP,SCST,SPPM,STPM,STTM,TH 0074
400 IF (ISCAT.LE.O) GO TO 600 0075
INC=1 0076
PH=PHI 0077
TH=THI 0078
CALL GFFLD(IA,IB,INC,INM,IWR,11,12,13,112,MD,N,NO,NM,AM)
WRITE(6,3)PH,TH,SPPM,SPTM,STPM,STTM 0079
WRITE(6,6)ACSP,ACST,ECSP,ECST,SCSP,SCST 0080
500 IF (IBISC.LE.O) GO TO 600 0081
INC=2 0082
PH=PHS 0083
TH=THS 0084
CALL GFFLD(IA,IB,INC,INM,IWR,11,12,13,112,MD,N,NO,NM,AM)
WRITE(6,3)PH,TH,SPPM,SPTM,STPM,STTM 0085
WRITE(6,6)ACSP,ACST,ECSP,ECST,SCSP,SCST 0086
600 CONTINUE 0087
800 CALL EXIT 0088
END 0089

Fig. 1b. The thin-wire computer program.
The first READ statement inputs the following parameters for the dielectric insulation:

BM  outer radius in meters
ER2  dielectric constant relative to free space
SIG2  conductivity in mhos per meter
TD2  loss tangent

The program will use SIG2 or TD2 but not both. The user determines which one will be used by assigning the other a negative value. For an uninsulated wire structure, the program will not use any of the data from the first READ statement.

The second READ statement inputs the following parameters for the wire and the exterior medium:

AM  wire radius in meters
CMM  wire conductivity in megamhos per meter
ER3  dielectric constant relative to free space
SIG3  conductivity in mhos per meter
TD3  loss tangent

The parameters ER3, SIG3 and TD3 are those of the homogeneous ambient medium. Again, the program will use SIG3 or TD3 but not both.

The third READ statement inputs the following data:

IBISC  indicator for bistatic scattering calculations
IGAIN  indicator for antenna gain calculations
INEAR  indicator for near-zone field calculations
ISCAT  indicator for backscatter calculations
IWR  indicator for writeout of current distributions
NGEN  indicator for antenna calculations
NM  number of monopoles (segments)
NP  number of points

For each indicator, a positive value means the calculation or writeout is desired while a zero or negative value means it is not desired.

The fourth READ statement inputs the following data:

FMC  frequency in megahertz
PHA,THA far-field angle for antenna gain
PHI,THI incidence angle for plane-wave scattering
PHS,THS scattering angle for bistatic scattering

The above angles are given in degrees, and they denote values of the angular coordinates in the spherical system (r,θ,φ) widely used in antenna and scattering literature.
The fifth READ statement (in the DO LOOP ending with statement 22) inputs the endpoints IA(J) and IB(J) of segment J. Thus, IA and IB are the index numbers of the two points which are joined by segment J.

The sixth READ statement (in the DO LOOP ending with statement 40) inputs the coordinates X(I), Y(I) and Z(I) of point I in meters. The seventh and last READ statement inputs the coordinates XP, YP and ZP (in meters) of the observation point for near-zone field calculations.

Some of the quantities used in the program are defined as follows:

- **FHZ** — frequency in Hertz
- **OMEGA** — angular frequency
- **EP2** — complex permittivity of insulation
- **EP3** — complex permittivity of ambient medium
- **ETA** — intrinsic impedance of ambient medium
- **GAM** — intrinsic propagation constant of ambient medium
- **ZS** — surface impedance of wire

For an uninsulated wire with perfect conductivity, one may specify complex values for ETA and GAM and delete the following input data and calculations: BM, ER2, SIG2, TD2, ER3, SIG3, TD3, FMC, FHZ, OEMGA, EP2 and EP3.

After reading the input data, the program calls subroutine SORT. This subroutine defines a set of dipole modes on the wire structure. N denotes the total number of dipole modes, the number of simultaneous linear equations, and the size of the impedance matrix $Z_{ij}$. Since this matrix is symmetric, only the upper-right triangular portion (including the entire principal diagonal) is calculated and stored in C(K). SORT calculates N, but the user may predict N as follows to reserve adequate storage. If m wire segments intersect at a point, this point is defined as a junction of order m and degree $n = m - 1$. There will be n dipole modes with terminals at this junction. N is determined by summing the degrees of all the junctions. For an example, an endpoint of a dipole is a junction of order $m = 1$ and degree $n = 0$. The vertex of a V dipole is a junction of order 2 and degree 1. NP denotes the number of points on the wire structure, and each of these points is considered to be a junction.

Mode I is a two-segment V dipole with a sinusoidal current distributed over the intersecting segments JA(I) and JB(I). The dipole has endpoints I1(I) and I3(I) and terminals at I2(I). The reference direction for positive current on dipole I is from I1 to I2 to I3.

A wire segment may be shared by as many as four dipole modes, or as few as one. In the output of subroutine SORT, ND(J) denotes the number of dipoles sharing segment J. The extreme values of ND(J) are MAX and MIN. If MIN is less than one, the wire structure has an un-connected segment and the computation is aborted. (An isolated wire
must have at least two segments and three points.) If $N$ exceeds $ICJ$, the dimensions are inadequate and the run is aborted.

$INT$ specifies the number of intervals for calculating the elements in the impedance matrix with Simpson's-rule integration. A large value for $INT$ improves the accuracy at the expense of greater execution time. For most problems a suitable combination of speed and accuracy is obtained with $INT = 4$. A larger value is recommended if one wire passes close to another as in the helix or the multiturn loop. If in doubt, one may set $INT = 0$ to choose the rigorous closed-form impedance expressions in terms of exponential integrals.

The DO LOOP ending with statement 60 sets all the lumped load impedances and generator voltages to zero. If the wire structure has lumped loads, one may insert a READ command after statement 60 to input a list of complex load impedances $ZLD(J)$. For a wire antenna with just one generator, the program inserts a unit voltage generator with $VG(NGEN) = (1.,0.)$. If the antenna or array has several generators, one may insert a READ command after statement 60 to input a list of complex voltages $VG(J)$.

Generators or lumped loads may be inserted at either end or both ends of segment $J$. First consider a load impedance inserted in the middle of segment $J$. Now slide the load along the segment and let it approach the endpoint $IA(J)$. This load is represented by $ZLD(J)$. Next insert another load in segment $J$ and slide it to approach the endpoint $IB(J)$. This load is designated $ZLD(JJ)$ where $JJ = J + NM$. The same convention is employed for the voltage generators $VG(J)$ and $VG(JJ)$. A generator voltage $VG(J)$ is considered positive if it tends to force a current flow in the direction from $IA(J)$ to $IB(J)$.

Subroutine $SGANT$ calculates the elements of the impedance matrix $Z_{ij}$ and stores them in $C(K)$ where $K = (I-1)*N - (I*I - I)/2 + J$. This subroutine will set $N = 0$ and the run will abort if the wire radius is zero or negative, the shortest segment length is less than the wire diameter, the wire radius is electrically large, or the longest segment is too long.

Subroutine $GANT1$ considers the thin-wire structure as an antenna and solves for the current distribution $CG(J)$, radiation efficiency $EFF$, time-average power input $GG$ and complex power input $Y11$. In the current distribution, $CG(J)$ is the current on segment $J$ as one approaches the endpoint $IA(J)$ and $CG(JJ)$ is the current at the other end $IB(J)$. The reference direction for positive current is from $IA$ to $IB$. Thus, the conventions are the same for the branch currents $CG$ and the branch voltages $VG$.

If the antenna has only one voltage generator with $VG(NGEN) = (1.,0.)$, then $Y11$ is the antenna admittance and $Z11$ is the impedance.
The radiation efficiency EFF is calculated from the time-average power input to the antenna and the time-average power dissipated in the wire and the lumped loads. If the antenna is insulated, the power dissipated in the insulation is neglected. If the wire has perfect conductivity and the loads are purely reactive, the calculated efficiency will be 100 per cent.

The near-field subroutine GNFLD calculates the electric field intensity \((E_X, E_Y, E_Z)\) at the observation point \((X_P, Y_P, Z_P)\). In the calling parameters, \(C_J\) denotes the current distribution on the wire. (The loop currents are stored in \(C_J(I)\) and the branch currents in \(C_G(J)\)). Thus, the currents must be calculated before GNFLD is called. Fig. 1 illustrates the use of GNFLD to calculate the near-zone field in an antenna problem. This subroutine can be called again just above statement 500 to calculate the near-zone scattered field for a wire target. In the calling parameters, \(C_J\) is replaced with \(E_P\) or \(E_T\) to obtain the near-zone field with a phi-polarized or theta-polarized incident plane wave. Reference 1 describes the more sophisticated techniques required when the observation point is extremely close to the wire structure.

The far-field subroutine GFFLD calculates antenna gain if \(INC = 0\), backscattering if \(INC = 1\), and bistatic scattering if \(INC = 2\). If \(INC = 0\), \(PH\) and \(TH\) denote the spherical coordinates \(\phi\) and \(\theta\) of the distant observation point and the output from GFFLD is defined as follows. \(EPPS\) and \(ETTS\) denote the phi-polarized and theta-polarized components of the electric field intensity. For example,

\[
EPPS = r \ e^{Y_P} E_\phi
\]

where \(r\) is the distance from the origin to the observation point. \(GPP\) and \(GTT\) denote the power gains associated with the phi and theta polarizations. Appendix 14 defines \(GPP\) and \(GTT\) more precisely.

If \(INC = 1\), \(PH\) and \(TH\) denote the incidence angles \(\phi_i\) and \(\theta_i\). These are also the spherical coordinates of the distant source. In this backscattering situation, the output data from GFFLD are defined as follows:

- **ACSP, ACST**: absorption cross sections for \(\phi\) and \(\theta\) polarizations
- **ECSP, ECST**: extinction cross sections for \(\phi\) and \(\theta\) polarizations
- **EP, ET**: loop currents induced by \(\phi\) and \(\theta\) polarized waves
- **EPPS**: scattered electric field \(E_{\phi\phi}\)
- **EPTS**: scattered electric field \(E_{\phi\theta}\)
- **ETPS**: scattered electric field \(E_{\theta\phi}\)
- **ETTS**: scattered electric field \(E_{\theta\theta}\)
- **SCSP, SCST**: scattering cross sections for \(\phi\) and \(\theta\) polarizations
- **SPPM**: echo area \(\sigma_{\phi\phi}\)
The echo areas are given in square meters. For the doubly-subscripted quantities such as $E_{\phi\phi}$ and $\sigma_{\phi\phi}$, the first and second subscripts specify the polarizations of the incident and scattered waves, respectively. The complex numbers $E_{PSS}$, $E_{PTS}$, $E_{TPS}$ and $E_{TTS}$ are the elements of the polarization scattering matrix.

If $INC = 2$, $PH$ and $TH$ denote the scattering angles $\phi_s$ and $\theta_s$. These are the spherical coordinates of the distant observer. In this bistatic scattering situation, the only outputs from GFFLD are the polarization scattering matrix and the echo areas.

To obtain antenna patterns, backscattering patterns or bistatic patterns, one may insert DO LOOPS in the program to increment the angles $PH$ and $TH$. The DO LOOP will begin just above the call to GFFLD and terminate just below this call. To obtain the near-zone field distribution along a given probing path, one may insert a DO LOOP beginning just above the call to GNFLD and terminating just below this call.

When the calculations have been completed for one problem, one may GO TO a point just above CALL GANT1 if only the generator voltages are to be changed. One may GO TO a point just below CALL SORT if there is a change in the wire radius or conductivity, the insulation, ambient medium, frequency, load impedances or the coordinates $(X,Y,Z)$. If there is a change in $NM$, $NP$, $IA$ or $IB$, one should GO TO a point above CALL SORT.

Consider an array of three center-fed dipoles, and suppose we desire the $3 \times 3$ admittance matrix for the array. Let each dipole be divided into four segments with segments 1 through 4 on dipole 1, 5 through 8 on dipole 2 and 9 through 12 on dipole 3. The three-port admittance matrix can be obtained by inserting a DO LOOP beginning just above CALL GANT1 and terminating just below this call. GANT1 will be called three times with all the voltages $VG$ set to zero except for a single one-volt generator. On the first, second and third calls, let $NGEN = 3$, 7 and 11 to represent a generator at the center of dipole 1, 2 and 3, respectively. After the first call, set $Y_{11} = CG(3)$, $Y_{12} = CG(7)$ and $Y_{13} = CG(11)$. Set $Y_{22} = CG(7)$ and $Y_{23} = CG(11)$ after the second call and $Y_{33} = CG(11)$ after the third call.

For extremely small antennas, quasi-static or double-precision subroutines are required.

The wire radius must exceed zero, but there is no difficulty with small radii. If the radius exceeds $0.007\lambda$, the thin-wire assumptions are questionable and the accuracy and convergence deteriorate. The length ratio of the longest and shortest segments should not exceed 100. It is
assumed that the wire length exceeds the wire diameter by a factor of at least 30. We are not aware of any lower limit on the segment length, however.

If a wire is bent sharply to form a small acute angle (less than 30 degrees), the thin-wire model is questionable. It is assumed that the wire conductivity greatly exceeds the conductivity of the ambient medium. For insulated wires, the dielectric layer is assumed to be electrically thin.

For each thin-wire problem, calculations should be repeated several times with the wire divided progressively into shorter segments. There is no assurance of accuracy until the output data converge. For a moderately thick wire (with radius \( a = 0.007 \lambda \) or larger), the susceptance may diverge with the delta-gap model. This difficulty may be alleviated or eliminated with the magnetic-frill model and the techniques of Imbriale and Ingerson [2].

Tables 1, 2 and 3 list input and output data for three simple examples of uninsulated wire structures. Each table includes a sketch of the wire configuration with labels to indicate the numbering system for the points and segments. In these examples there are no lumped loads.

In the sinusoidal-reaction formulation, a basic function is the mutual impedance between two sinusoidal filamentary electric dipoles. One dipole is a test source located on the axis of the wire structure, and the other is an expansion mode on the wire surface. In view of the importance of this mutual impedance, short tables are presented next for a few simple cases. The data can be reproduced with the program in Fig. 1 with appropriate input data for uninsulated wires with perfect conductivity and no lumped loads in free space. The data were obtained with the closed-form expressions (\( INT = 0 \)) by writing out the quantities \( C(K) \) just below the call to subroutine SGANT. Double precision was used for these calculations.

Table 4 lists the self impedance of a two-segment sinusoidal V dipole with radius \( a = 0.001 \lambda \). Subroutine SGANT calculates this quantity by setting up one filamentary dipole on the wire axis and another identical dipole on the wire surface. These dipoles lie in parallel planes separated by a distance equal to the wire radius.

In Table 5, dipoles 1 and 2 have terminals at vertices 1 and 2, respectively, and they share the middle segment. Again these dipoles lie in parallel planes separated by a distance equal to the wire radius. For a one-turn planar polygon wire loop, subroutine SGANT would generate the data in Table 4 for the diagonal elements \( Z_{ii} \) and the data in Table 5 for the next elements.
### TABLE 1
**Input and Output Data for Straight Wire**

<table>
<thead>
<tr>
<th>Input Data</th>
<th>2.56</th>
<th>-1.0</th>
<th>0.0005</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>1.00</td>
<td>1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>0.001</td>
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**Output Data**

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### TABLE 2
**Input and Output Data for Square Loop**

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<th>0.0005</th>
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**Output Data**

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<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### TABLE 4

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$h = 0.10\lambda$</th>
<th>$h = 0.15\lambda$</th>
<th>$h = 0.20\lambda$</th>
<th>$h = 0.25\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.59 - j 481</td>
<td>1.4 - j 314</td>
<td>3.1 - j 186</td>
<td>6.1 - j 61</td>
</tr>
<tr>
<td>60°</td>
<td>2.15 - j 547</td>
<td>5.3 - j 337</td>
<td>11.0 - j 177</td>
<td>21.3 - j 21</td>
</tr>
<tr>
<td>90°</td>
<td>4.22 - j 572</td>
<td>10.4 - j 340</td>
<td>21.1 - j 163</td>
<td>40.0 + j 9</td>
</tr>
<tr>
<td>120°</td>
<td>6.31 - j 583</td>
<td>15.3 - j 338</td>
<td>30.9 - j 151</td>
<td>57.7 + j 28</td>
</tr>
<tr>
<td>150°</td>
<td>7.81 - j 587</td>
<td>18.9 - j 335</td>
<td>37.7 - j 144</td>
<td>69.3 + j 39</td>
</tr>
<tr>
<td>180°</td>
<td>8.33 - j 589</td>
<td>20.1 - j 335</td>
<td>39.9 - j 142</td>
<td>73.1 + j 42</td>
</tr>
</tbody>
</table>
Fig. 2. Symmetric center-fed V dipole.

TABLE 5

Mutual Impedance Between Overlapping V Dipoles in Fig. 3
Radius: \( a = 0.001\lambda \)

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>( h = 0.10\lambda )</th>
<th>( h = 0.15\lambda )</th>
<th>( h = 0.20\lambda )</th>
<th>( h = 0.25\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60°</td>
<td>-0.96 + j 338</td>
<td>-2.08 + j 285</td>
<td>-3.45 + j 275</td>
<td>-4.8 + j 298</td>
</tr>
<tr>
<td>90</td>
<td>0.19 + j 322</td>
<td>1.03 + j 276</td>
<td>3.57 + j 271</td>
<td>10.1 + j 297</td>
</tr>
<tr>
<td>120</td>
<td>3.29 + j 336</td>
<td>8.40 + j 290</td>
<td>17.86 + j 285</td>
<td>35.3 + j 309</td>
</tr>
<tr>
<td>150</td>
<td>6.61 + j 346</td>
<td>15.61 + j 299</td>
<td>30.00 + j 291</td>
<td>52.9 + j 309</td>
</tr>
<tr>
<td>180</td>
<td>8.01 + j 349</td>
<td>18.47 + j 301</td>
<td>34.35 + j 292</td>
<td>58.2 + j 308</td>
</tr>
</tbody>
</table>

Fig. 3. Overlapping V dipoles share the middle segment.

Tables 6, 7, and 8 list the mutual impedance for other configurations. In all these tables, the data apply to two-segment center-fed sinusoidal dipoles with identical segment lengths \( h \).
TABLE 6
Mutual Impedance Between Overlapping V Dipoles in Fig. 4
Radius: \( a = 0.001\lambda \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( h = 0.10\lambda )</th>
<th>( h = 0.15\lambda )</th>
<th>( h = 0.20\lambda )</th>
<th>( h = 0.25\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>6.74 - j 314</td>
<td>16.24 - j 167</td>
<td>32.17 - j 56</td>
<td>58.7 + j 49.6</td>
</tr>
<tr>
<td>60</td>
<td>3.16 - j 291</td>
<td>7.68 - j 169</td>
<td>15.47 - j 76</td>
<td>28.8 + j 14.2</td>
</tr>
<tr>
<td>90</td>
<td>0.06 - j 278</td>
<td>0.31 - j 172</td>
<td>1.15 - j 92</td>
<td>3.5 - j 12.2</td>
</tr>
<tr>
<td>120</td>
<td>-1.01 - j 256</td>
<td>-2.39 - j 168</td>
<td>-4.47 - j 101</td>
<td>-7.6 - j 35.5</td>
</tr>
<tr>
<td>150</td>
<td>-0.48 - j 207</td>
<td>-1.20 - j 146</td>
<td>-2.40 - j 98</td>
<td>-4.5 - j 50.7</td>
</tr>
</tbody>
</table>

Fig. 4. Overlapping V dipoles share the bottom segment in a planar Y configuration.
TABLE 7
Mutual Impedance Between the Coplanar-Skew Linear Dipoles in Fig. 5
Displacement: d = λ

<table>
<thead>
<tr>
<th>θ</th>
<th>h = 0.10λ</th>
<th>h = 0.15λ</th>
<th>h = 0.20λ</th>
<th>h = 0.25λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.337 + j 1.952</td>
<td>0.880 + j 4.759</td>
<td>1.932 + j 9.547</td>
<td>4.011 + j 17.7</td>
</tr>
<tr>
<td>15</td>
<td>0.322 + j 1.884</td>
<td>0.831 + j 4.585</td>
<td>1.799 + j 9.180</td>
<td>3.671 + j 17.0</td>
</tr>
<tr>
<td>30</td>
<td>0.281 + j 1.684</td>
<td>0.700 + j 4.082</td>
<td>1.448 + j 8.128</td>
<td>2.800 + j 15.0</td>
</tr>
<tr>
<td>45</td>
<td>0.220 + j 1.369</td>
<td>0.521 + j 3.301</td>
<td>1.000 + j 6.519</td>
<td>1.745 + j 11.9</td>
</tr>
<tr>
<td>60</td>
<td>0.149 + j 0.964</td>
<td>0.333 + j 2.310</td>
<td>0.579 + j 4.524</td>
<td>0.860 + j 8.1</td>
</tr>
<tr>
<td>75</td>
<td>0.075 + j 0.497</td>
<td>0.159 + j 1.187</td>
<td>0.252 + j 2.308</td>
<td>0.305 + j 4.1</td>
</tr>
<tr>
<td>90</td>
<td>0.0 + j 0.0</td>
<td>0.0 + j 0.0</td>
<td>0.0 + j 0.0</td>
<td>0.0 + j 0.0</td>
</tr>
</tbody>
</table>

Fig. 5. Center-fed coplanar-skew linear dipoles.
TABLE 8
Mutual Impedance Between the Nonplanar-Skew Linear Dipoles in Fig. 6
Displacement: $d = \lambda$

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$h = 0.10\lambda$</th>
<th>$h = 0.15\lambda$</th>
<th>$h = 0.20\lambda$</th>
<th>$h = 0.25\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>$0.337 + j 1.952$</td>
<td>$0.880 + j 4.759$</td>
<td>$1.932 + j 9.547$</td>
<td>$4.011 + j 17.74$</td>
</tr>
<tr>
<td>15</td>
<td>$0.326 + j 1.886$</td>
<td>$0.850 + j 4.596$</td>
<td>$1.867 + j 9.222$</td>
<td>$3.877 + j 17.14$</td>
</tr>
<tr>
<td>30</td>
<td>$0.292 + j 1.691$</td>
<td>$0.762 + j 4.121$</td>
<td>$1.675 + j 8.269$</td>
<td>$3.482 + j 15.37$</td>
</tr>
<tr>
<td>45</td>
<td>$0.238 + j 1.380$</td>
<td>$0.622 + j 3.365$</td>
<td>$1.369 + j 6.752$</td>
<td>$2.850 + j 12.55$</td>
</tr>
<tr>
<td>60</td>
<td>$0.169 + j 0.976$</td>
<td>$0.440 + j 2.380$</td>
<td>$0.969 + j 4.775$</td>
<td>$2.020 + j 8.88$</td>
</tr>
<tr>
<td>75</td>
<td>$0.087 + j 0.505$</td>
<td>$0.228 + j 1.232$</td>
<td>$0.502 + j 2.472$</td>
<td>$1.047 + j 4.60$</td>
</tr>
<tr>
<td>90</td>
<td>$0.0 + j 0.0$</td>
<td>$0.0 + j 0.0$</td>
<td>$0.0 + j 0.0$</td>
<td>$0.0 + j 0.0$</td>
</tr>
</tbody>
</table>

Fig. 6. Center-fed nonplanar-skew linear dipoles.
III. SUMMARY

This report presents the sinusoidal-reaction computer program for thin-wire antennas and scatterers, instructions for the user, typical input and output data and mutual-impedance tables for sinusoidal dipoles. Appendices list the computer subroutines and explain their functions.
REFERENCES


APPENDIX 1. Subroutine SORT

Subroutine SORT, listed in Fig. 7, defines a set of dipole mode currents on the wire structure. The input data IA, IB, NM, NP, ICJ and INM have been defined already. The output data are defined as follows:

- \( N \) : total number of dipole modes
- \( I1(I) \) : endpoint of dipole I
- \( I2(I) \) : terminal point of dipole I
- \( I3(I) \) : endpoint of dipole I
- \( JA(I) \) : first segment of dipole I
- \( JB(I) \) : second segment of dipole I
- \( MD(J,K) \) : list of dipoles sharing segment J
- \( ND(J) \) : total number of dipoles sharing segment J
- \( MAX, MIN \) : extreme values of ND(J)

At completion of the DO LOOP ending with statement 20, NJK denotes the number of segments intersecting at point K, and JSP is a list of these segments. In the DO LOOP ending with statement 22, the computer sets up the appropriate number MOD of dipoles modes with terminals at point K.

APPENDIX 2. Subroutine SGANT

Subroutine SGANT, listed in Fig. 8, calculates the mutual impedances \( Z_{ij} \) and stores them in \( C(K) \). The input data for SGANT have been defined already. The output data are defined as follows:

- \( C(K) \) : open-circuit impedance matrix
- \( CGD(J) \) : \( \cosh \gamma d \) for segment J
- \( SGD(J) \) : \( \sinh \gamma d \) for segment J
- \( D(J) \) : length of segment J
- \( ZS \) : surface impedance of the wire

The surface impedance is calculated just above statement 12. \( B01 \) denotes \( J_0/J_1 \) where \( J_0 \) and \( J_1 \) are the Bessel functions of order zero and one with complex argument ZARG. It is assumed that all the wire segments have the same radius, conductivity and surface impedance.

In the DO LOOP ending with statement 20, SGANT calculates the segment lengths \( D(J) \). DMIN and DMAX denote the lengths of the shortest and longest segments. If the wire radius or the segment lengths are clearly beyond the range of thin-wire theory, \( N \) is set to zero at statement 25 followed by RETURN to the main program to abort the calculation.

At statement 30, the program selects a segment K, and a few statements below this it selects another segment L. K is a segment of test dipole I, and L is a segment of expansion mode J. The mutual impedance between segments K and L is obtained by calling subroutine GGS or GGMM.
SUBROUTINE SORT(JA,JB,MD,ND,NM,JP,N,MAX,MIN)

DIMENSION JSP(20)
DIMENSION I/I/I(1),I2(I),I3(I),JA(I),JB(I)
DIMENSION IA(I),IB(I),ND(I),MD(I)

I=0
DO 24 K=1,NP
  NJK=0
  DO 20 J=1,ND
    IND=(I/I)/J-K)*I/I(J)-K)
    IF(IND.NE.0)GO TO 20
    NJK=NJK+1
    JSP(NJK)=J
    MOD=NJK-1
  IF(MOD.LE.0)GO TO 24
  DO 22 IMD=1,MOD
    J=JSP(IMD)
    JA(I)=JSP(IMD)
    JA(I)=JA(I)
    JB(I)=JSP(IPD)
    JB(I)=JB(I)
    I(1)=I/1(JA(I))
    IF(I(1).EQ.K)I/I(I)=IB(JA(I))
    I(1)=K
    I(1)=I/1(JB(I))
    IF(I/1(JB(I)).EQ.K)I/I(I)=IB(JB(I))
  CONTINUE
  22 CONTINUE
  24 CONTINUE

N=1
DO 30 J=1,ND
  ND(J)=0
  DO 30 K=1,4
  MD(I,J,K)=0
  I/I=I/I
  IF(N.GT.1CJ)I/I=I/1
  DO 40 I=1,11
    J=JA(I)
    DO 38 L=1,2
      ND(J)=ND(J)+1
      K=1
      M=0
  38 J=JB(I)
  40 CONTINUE
  MIN=100
  MAX=0
  DO 46 J=1,ND
    ND(J)=ND(J)
    IF(ND(J).GT.MAX)MAX=ND(J)
  46 IF(MIN.LT.MAX)MIN=ND(J)
  RETURN
END

Fig. 7. Subroutine SORT
SUBROUTINE SGANT(IA, IB, INM, INT, ISC, I1, I2, I3, JA, JB, MD, N, ND, NM, NP
2, AM, BM, C, CGD, CGM, D, EP2, EP3, ETA, ETA, GAM, SGD, SGD, X, Y, Z, ZLD, ZS)

COMPLEX ZG, ZH, ZS, EGD, GD, GOS, GOS, SGD, GT, B01
COMPLEX P11, P12, P21, P22, Q11, Q12, Q21, Q22, EP2, EP3, ETA, GAM, GOS

COMPLEX EPSILA, CEA, BETA, ZARG

DIMENSION X(I1), Y(I1), Z(I1), X(I1), Y(I1), Z(I1), D(I1), IA(I1), IB(I1), MD(INM, 4)

DATA E0, TP, U0, B.854E-12, 6.28318, 1.2566E-6/

2 FORMAT(3X, 'AM = ', E10.3, 3X, 'DMIN = ', E10.3)

EP=EP3
ICC=(N+N+N)/2
DO 10 I=1, ICC
C(I)=1.0, 0.0
IF (CMM .GE. 0.) GO TO 12

OMEGA=TP*EHZ
EPSILA=CMPLX(E0, -CMM*1.E6/OMEGA)

CWEA = 1.0, 1.0) * EPSILA

BETA = EPSILA * SQRT(U0) * EPSILA

ZARG = EPSILA

CALL CBES(ZARG, B01)
ZS = BETA / CWEA

DMIN = 1.0
DMAX = 0.0
DO 20 J=1, NM

K=IA(J)
L=IB(J)
D(J) = SQRT((X(K)-X(L))**2+(Y(K)-Y(L))**2+(Z(K)-Z(L))**2)

IF (D(J) .LT. DMIN) DMIN = D(J)

EGD = CEPLX(GAM*D(J))

CGD(J) = (EGD*1.0/EGD)**2

SGD(J) = (EGD*1.0/EGD)**2

IF (DMIN .LT. AM) GO TO 25
IF (CGD(J) .GT. TO) GO TO 25
IF (SGD(J) .GT. TO) GO TO 25
IF (AM .GT. TO) GO TO 30

N=0
WRITE (6, 2) AM, DMAX, DMIN
RETURN

DO 30 K=1, NM

NDK=ND(K)

KA=IA(K)

KB=IB(K)

DK=ND(K)

CGDS=CGD(K)

SGDS=SGD(K)

DO 200 L=1, NM

NDL=ND(L)

LA=IA(L)

LB=IB(L)

DL=ND(L)

SGDT=SGD(L)

NIL=D

DO 200 I=1, NDK

1=HD(K, I)

NM=(I-1)*N-(I-1)/2

FI=1.

IF (KX.EQ.12) GO TO 36
IF (KB.EQ.11) FI = -1.

Fig. 8a. Subroutine SGANT

21
IS=1  
GO TO 40  
36 IF (KA.EQ.13(I)) FI=-1.  
IS=2  
40 DO 200 JJ=1,NDL  
J=MD(J,JJ)  
MMM=MM+J  
IF (I.GT.J) GO TO 200  
FJ=1.  
IF (LB.EQ.12(J)) GO TO 46  
IF (LB.EQ.11(J)) FJ=-1.  
JS=1  
GO TO 50  
46 IF (LA.EQ.13(J)) FJ=-1.  
JS=2  
50 IF (NIL.NE.0) GO TO 168  
NIL=1  
IF (K.EQ.L) GO TO 120  
IND=(LA-KA)*(LB-KA)*(LA-KB)*(LB-KB)  
IF (IND.NE.0) GO TO 80  
C SEGMENTS K AND L SHARE NO POINTS  
CALL GGS(X(KA),Y(KA),Z(KA),X(KB),Y(KB),Z(KB),X(LA),Y(LA),Z(LA))  
2*X(LB),Y(LB),Z(LB),AM,DK,CGDS,SGDS,DL,SGDT,INT,ETA,GAM  
3*P(1,1),P(1,2),P(2,1),P(2,2))  
GO TO 168  
C SEGMENTS K AND L SHARE ONE POINT (THEY INTERSECT)  
80 KG=0  
JL=K  
JC=KA  
KF=-1  
IM=(LB-LA)*(KB-LB)  
IF (IM.NE.0) GO TO 82  
JC=KB  
KF=-1  
IM=LA  
KG=3  
82 LG=3  
JL=LA  
LF=-1  
IF (LB.EQ.JC) GO TO 83  
JL=LB  
LF=1  
LA=0  
83 SGN=KF*LF  
CPSI=((X(JP)-X(JC))*(-X(JM))-X(JC)) + (Y(JP)-Z(JC)) + (Y(JM)-Y(JC))  
= Z(JP)+Z(JC)) + Z(JM)-Z(JC))  
Z(JP)=Z(JM)+Z(JC))/DK*DL  
CALL GGS(X(JP),Y(JP),Z(JP),AM,DK,CGDS,SGDS,DL,SGDT,INT,ETA,GAM  
2*X(LB),Y(LB),Z(LB),AM,DK,CGDS,SGDS,DL,SGDT,INT,ETA,GAM  
3*P(1,1),P(1,2),P(2,1),P(2,2))  
DF1 98 KK=1,2  
KD=IARS(KK-KG)  
DU 98 LL=1,2  
LP=IARS(LL-LG)  
PKP*LP=SGN*Q(KK,LL)  
98 CONTINUE  
GO TO 168  
C K=L (SELF REACTION OF SEGMENT K)  
120 Q11=(.0,.0)  
Q12=(.0,.0)  
IF (CMM.LE.0.) GO TO 150  
GO=GAM*DK  
2G=2H/(SGD**2)  
Q11=Q11-(SGD*CGDS-GO)/2.  

Fig. 8b. Subroutine SGANT
OLZG*(G)-CGDS-SGD) S/2.

ISCK=ISC(K)
P11=(.0,.0)
P12=(.0,.0)
IF(ISCK.EQ.0)GO TO 155
IF(BM.LE.AM)GO TO 155
CALL DSHELL(AM,BM,DK,CGDS,SGDS,EP2,EP,ETA,GAM,P11,P12)
155 Q11=P11+Q11
Q12=P12+Q12
CALL GGMM(*)DK,*.0,DK,AM,CGDS,SGDS,SGDS,1.
2,ETA,GAM,P11,P12,P21,P22)
Q11=P11+Q11
Q12=P12+Q12
P(1,1)=Q11
P(1,2)=Q12
P(2,1)=Q12
P(2,2)=Q11
IF(KA.NE.LA)GO TO 160
GO TO 168
160 P(1,1)=-Q12
P(1,2)=-Q11
P(2,1)=-Q11
P(2,2)=-Q12
168 C(MMM)=C(MMM)+F1*FJ*P(IS,JS)
200 CONTINUE
DO 220 I=1,N
IJ=(I-1)*N-(I*1-1)/2+1
J1=JA(I)
IF(I2(I))EQ.IB(J1))J1=J1+NM
J2=JB(I)
IF(I2(I))EQ.IB(J2))J2=J2+NM
220 C(IJ)=C(IJ)+ZLD(J1)+ZLD(J2)
RETURN
END

Fig. 8c. Subroutine SGANT
In statement 168, this impedance is lumped into \( C(MMM) \). The mutual impedance \( Z_{ij} \) between dipoles \( I \) and \( J \) is the sum of four segment-segment impedances.

In SGANT, segment \( K \) has endpoints \( KA \) and \( KB \), and segment \( L \) has endpoints \( LA \) and \( LB \). It is convenient to think of \( KA \) and \( KB \) as points 1 and 2 on segment \( K \), and \( LA \) and \( LB \) as points 1 and 2 on \( L \). Now we define four segment-segment impedances \( P(IS, JS) \). The first subscript \( IS \) refers to the terminal point on segment \( K \), and the second subscript \( JS \) refers to the terminal point on \( L \). Thus \( IS = 1 \) or \( 2 \) if dipole \( I \) has its terminal point \( 12(I) \) at \( KA \) (point 1) or \( KB \) (point 2), respectively. Similarly, \( JS = 1 \) or \( 2 \) if mode \( J \) has its terminal point \( 12(J) \) at \( LA \) or \( LB \). The impedances \( P(IS, JS) \) are defined with the following reference directions for current flow: from point 1 toward point 2 on each segment. If dipole \( I \) has this same reference direction on segment \( K \), we set \( FI = 1 \); otherwise \( FI = -1 \). Similarly \( FJ = 1 \) or \( -1 \) in accordance with the reference direction for mode \( J \) on segment \( L \). In statement 168, \( P(IS, JS) \) is multiplied by \( FI \) and \( FJ \) before its contribution is added to \( Z_{ij} \).

Subroutine GGMM calculates the impedances \( Q(KK, LL) \) which are like the \( P(IS, JS) \) but have different conventions for reference directions and subscript meaning. The transformation from the \( Q \) impedances to the \( P \) impedances is accomplished in the DO LOOP ending with statement 98.

If the wire has finite conductivity, the appropriate modification is applied to the impedance matrix just above statement 150. (See Eqs. 27 through 29 in Reference 1.) The terms arising from the dielectric shell on an insulated segment are obtained from subroutine DSHELL just above statement 155. Finally, the lumped loads \( ZLD \) are added to the diagonal elements of the impedance matrix in statement 220.

The impedance matrix could be calculated in a different order as follows. Select modes \( I \) and \( J \), calculate \( Z_{IJ} \), and then increment \( I \) or \( J \). Instead, SGANT selects segments \( K \) and \( L \), calculates \( Z_{KL} \), adds \( Z_{KL} \) to all the appropriate elements \( Z_{IJ} \), and then increments \( K \) or \( L \). This minimizes the calls to GGS and GGMM and presumably improves the computational efficiency.

\( K \) is a segment of test dipole \( I \), and \( L \) is a segment of expansion mode \( J \). When the segment numbers \( K \) and \( L \) are equal, SGANT calls GGMM to obtain the mutual impedance between two filamentary electric monopoles. These monopoles are parallel and have the same length. Monopole \( K \) is positioned on the axis of the wire segment, and monopole \( L \) is on the surface of the same wire segment. Thus, the displacement is equal to the wire radius. The two monopoles are side-by-side with no stagger.

When segments \( K \) and \( L \) intersect, SGANT again calls GGMM for the mutual impedance between the two filamentary monopoles. Monopole \( K \) is
situated on the axis of wire segment K, and monopole L is on the surface of wire segment L. The axes of segments K and L define a plane P, and monopole K lies in this plane. Monopole L is parallel with plane P and is displaced from it by a distance equal to the wire radius.

APPENDIX 3. Subroutine CBES

Subroutine CBES, listed in Fig. 9, calculates the quantity \( B_{01} = \frac{J_0(z)}{J_1(z)} \) where \( z \) is complex and \( J_0 \) and \( J_1 \) denote the Bessel functions of order zero and one.

APPENDIX 4. Subroutine DSHELL

Subroutine DSHELL, listed in Fig. 10, calculates the mutual impedance term contributed by the dielectric insulation on the surface of a thin wire. This subroutine uses Eq. 35 of Reference 1.

APPENDIX 5. Subroutine GGS

Subroutine GGS, listed in Fig. 11, calculates the mutual impedance between two filamentary monopoles with sinusoidal current distributions. (The dipole-dipole mutual impedance in Eq. 20 of Reference 1 is the sum of four monopole-monopole mutual impedances.) The endpoints of the axial test monopole s are \((X_A, Y_A, Z_A)\) and \((X_B, Y_B, Z_B)\), and the endpoints of the expansion monopole t are \((X_1, Y_1, Z_1)\) and \((X_2, Y_2, Z_2)\). DS and DT denote the lengths of monopoles s and t, respectively. CAS, CBS and CGS are the direction cosines of monopole s, and CA, CB and CG are the direction cosines of monopole t.

IF INT = 0, GGS calls GGMM for the closed-form impedance calculations. Otherwise GGS calculates the mutual impedance via Simpson's-rule integration with the following number of sample points: \( IP = INT + 1 \). If the monopoles are parallel with small displacement, GGS calls GGMM to avoid the difficulties of numerical integration.

For the fields of the test monopole, GGS uses Eqs. 75 and 76 of Reference 1. The current distribution on the expansion monopole is given by Eq. 74 of Reference 1. With an origin at \((X_1, Y_1, Z_1)\), the coordinate \( T \) measures distance along the expansion monopole. Thus \( T \) is the integration variable.

Let the coordinate \( s \) measure distance along the test monopole with origin at \((X_A, Y_A, Z_A)\). From any point \( T \) on monopole \( t \), construct a line to the test monopole such that the line is perpendicular to the test monopole. SZ denotes the \( s \) coordinate of the intersection of this line with the test monopole. The length of the line is the radial coordinate \( \rho \), and RS denotes \( \rho^2 \). \( R_1 \) and \( R_2 \) are the distances from \((X_A, Y_A, Z_A)\) and \((X_B, Y_B, Z_B)\) to the point \( T \). \( C_1 \) is the current at \( T \) for the mode with terminals at \((X_1, Y_1, Z_1)\), and \( C_2 \) is the current at \( T \) for the other mode with terminals at \((X_2, Y_2, Z_2)\). \( C \) denotes the Simpson's-rule weighting coefficient.
SUBROUTINE CBES(Z,B01)
COMPLEX ARG,CC,CS,EX
COMPLEX B01,Z,TERMJ,TERMN,MZ24,JN(2)
DATA PI/3.14159/
IF(CABS(Z).GE.12.0) GO TO 10
FACTOR=0.0
TERMN=(0.,0.)
MZ24=-0.25*Z*Z
TERMJ=(1.0,0.0)
DO 1 NP=1,2
   N=NP-1
   JN(NP)=TERMJ
   M=0
   M=M+1
   TERMJ=TERMJ*MZ24/FLOAT(M*(N+M))
   JN(NP)=JN(NP)+TERMJ
1  IF(NP.NE.1) GO TO 3
   FACTOR=FACTOR+1.0/FLOAT(M)
   TERMN=TERMN+TERMJ*FACTOR
3  ERROR=CABS(TERMJ)
   IF(ERROR.GT.1.0E-10) GO TO 2
   B01=JN(1)/JN(2)
RETURN
10  Y=AIMAG(Z)
   IF(ABS(Y).GT.20.)GO TO 20
   ARG=(0.0,1.0)*Z
   EX=CEXP(ARG)
   CC=EX+1./EX
   CS=(0.,-1.)*(EX-1./EX)
   B01=(CS+CC)/(CS-CC)
RETURN
20  B01=(0.,-1.)
   IF(Y.LT.0.)B01=(0.,1.)
RETURN
END

Fig. 9. Subroutine CBES
SUBROUTINE DSHELL(AM, BM, DK, CGDS, SGDS, EP2, EP, ETA, GAM, P11, P12)

COMPLEX CGDS, SGDS, EP2, EP, ETA, GAM, P11, P12, GO, CST

DATA PI/3.14159/

GO = GAM * DK


P11 = -CST * (GD + SGDS * CGDS)

P12 = CST * (GD + CGDS + SGDS)

RETURN
END

Fig. 10. Subroutine DSHELL.
SUBROUTINE GGS(XA,YA,ZA,XB,YB,ZB,X1,Y1,Z1,X2,Y2,Z2,AM)
Z*DS,CDS,SGDS,DT,SGDT,INT,ETA,GAM,P11,P12,P21,P22)
COMPLEX P11,P12,P21,P22,EJA,EJB,EJ1,EJ2,ETA,GAM,C1,C2,CST
COMPLEX EGD,CDS,SGDS,SGDT,ER1,ER2,ET1,ET2
DATA FP/12.56637/
CA=(X2-X1)/DT
CB=(Y2-Y1)/DT
CG=(Z2-Z1)/DT
CAS=(XB-XA)/DS
CBS=(YB-YA)/DS
CGS=(ZB-ZA)/DS
CC=CA*CAS+CB*CBS+CG*CGS
IF(ABS(CC).GT..997)GO TO 200
SZ=(X1-XA)*CAS+(Y1-YA)*CBS+(Z1-ZA)*CGS
IF(INT.LE.0)GO TO 300
INS=2*INT/2
IF(INS.GT.2)INS=2
IP=INS+1
DELT=DT/INS
T=0
DSZ=CC*DELT
P11=(0.,0.)
P12=(0.,0.)
P21=(0.,0.)
P22=(0.,0.)
AMS=AM*AM
SGN=-1.
DO 100 IN=1,IP
   ZZ1=SZ
   ZZ2=SZ*DS
   XXZ=X1+T*CA-XA-SZ*CAS
   YYZ=Y1+T*C B-YA-SZ*CBS
   ZZZ=Z1+T*CG-ZA-SZ*CGS
   RS=XXZ**2+YYZ**2+ZZZ**2
   R1=SQT(RS+ZZ1**2)
   EJA=CEXP(-GAM*R1)
   EJ1=EJA/R1
   EJ2=SQRT(RS+ZZ2**2)
   EJB=CEXP(-GAM*R2)
   EJ2=EJB/R2
   ER1=EJA*SGDS+ZZ1*EJ1*CGDS-ZZ2*EJ2
   ER2=EJB*SGDS+ZZ2*EJ2*CGDS-ZZ1*EJ1
   FAC=.0
   IF(RS.GT.AMS.FAC=-(CA*XXZ+C B*YYZ+CG*ZZZ)/RS
   ET1=CC*(EJ2-EJ1)*CGDS)+FAC*ER1
   ET2=CC*(EJ1-EJ2)*CGDS)+FAC*ER2
   C=3.*SGN
   IF(IN.EQ.1.OR.IN.EQ.IP)C=1.
   EGD=CEXP(GAM*(DT-T))
   C1=C*(EGD-1./EGD)**2
   EGD=CEXP(GAM*T)
   C2=C*(EGD-1./EGD)**2
   P11=P11+ET1*C1
   P12=P12+ET1*C2
   P21=P21+ET2*C1
   P22=P22+ET2*C2
   T=T+DELT
   SIZ=SZ*DS
100 SGN=-SGN
   CST=ETA*DELT/(3.*FP*SGDS*SGDT)
P11=CST*P11
P12=CST*P12

Fig. 11a. Subroutine GGS

20
P21 = CST * P21
P22 = CST * P22
RETURN

200 S1 = (X1 - XA) * CAS + (Y1 - YA) * CBS + (Z1 - ZA) * CGS
RH1 = SQRT((X1 - XA - S1 * CAS) ** 2 + (Y1 - YA - S1 * CBS) ** 2 + (Z1 - ZA - S1 * CGS) ** 2)
S2 = S1 + DT * CC
RH2 = SQRT((X2 - XA - S2 * CAS) ** 2 + (Y2 - YA - S2 * CBS) ** 2 + (Z2 - ZA - S2 * CGS) ** 2)
DDD = (RH1 + RH2) / 2.
IF (DDD GT 20. AND. INT. GT 0) GO TO 20
IF (DDD LT AM) DDD = AM
CALL GGMM(.O, DS, S2, S2, DDD, CGO, SGDS, SGDT, 1, ETA, GAM, P11, P12, P21, P22)
RETURN

300 SS = SQRT(1. - CC * CC)
CAD = (CGS * CB - CBS * CG) / SS
CBD = (CAS * CG - CBS * CA) / SS
CGO = (CBS * CA - CAS * CB) / SS
DK = (X1 - XA) * CAD + (Y1 - YA) * CBD + (Z1 - ZA) * CGD
DK = ABS(DK)
IF (DK LT AM) DK = AM
XZ = XA + S1 * CAS
YZ = YA + S1 * CBS
ZZ = ZA + S1 * CGS
XP1 = X1 - DK * CAD
YP1 = Y1 - DK * CBD
ZP1 = Z1 - DK * CGD
CAP = CBS + CGD - CBS * CGD
CBP = CGD + CAS - CAS * CGD
CGP = CAS * CBD - CBS * CAD
P1 = CAP * (XP1 - XZ) + CBP * (YP1 - YZ) + CGP * (ZP1 - ZZ)
T1 = P1 / SS
S1 = T1 + CC * S2
CALL GGMM(51, 51, DS, T1, T1 + DT, DK, CGO, SGDS, SGDT, CC, ETA, GAM
Z, P11, P12, P21, P22)
RETURN
END

Fig. 11b. Subroutine GGS
Below statement 300, GGS performs some analytic geometry in preparation for calling GGMM. The remaining part of this Appendix concerns this last part of subroutine GGS.

Let \( \hat{s} \) denote a unit vector in the direction from \((X_A, Y_A, Z_A)\) toward \((X_B, Y_B, Z_B)\). Also let \( \hat{t} \) denote a unit vector from \((X_I, Y_I, Z_I)\) toward \((X_2, Y_2, Z_2)\). Then \( \hat{s} \cdot \hat{t} = \cos \theta = \frac{CC}{R} \) where \( \theta \) is the angle formed by the axes of the two monopoles. Let monopole \( s \) lie in one plane \( P_s \) and monopole \( t \) lie in another parallel plane \( P_t \). \( \text{CAD}, \text{CBD} \) and \( \text{CGD} \) are the direction cosines of the unit vector \( \hat{d} = \frac{\hat{t} \times \hat{s}}{\sin \theta} \) which is perpendicular to both planes. To obtain the distance \( DK \) between the two planes, we construct a vector \( R_{31} \) from \((X_A, Y_A, Z_A)\) to \((X_I, Y_I, Z_I)\) and take \( DK = R_{31} \cdot \hat{d} \).

Construct a line from \((X_I, Y_I, Z_I)\) to the test monopole, such that the line is perpendicular to the test monopole. \( S_Z \) denotes the \( s \) coordinate of the intersection of this line with the test monopole, and the cartesian coordinates of this intersection are \( X_Z, Y_Z \) and \( Z_Z \). The direction cosines of \( \hat{s} \times \hat{d} \) are \( \text{CAP}, \text{CBP} \) and \( \text{CGP} \).

From the point \((X_I, Y_I, Z_I)\) in plane \( P_t \), construct a perpendicular line to the point \((X_P1, Y_P1, Z_P1)\) in the plane \( P_s \). This line is parallel with \( \hat{d} \) and has length \( DK \). Let \( R \) represent a vector from \((X_Z, Y_Z, Z_Z)\) to \((X_P1, Y_P1, Z_P1)\). \( P_1 \) denotes \( R \cdot (\hat{s} \times \hat{d}) \). \( S_1 \) and \( T_1 \) are defined in the next Appendix.

APPENDIX 6. Subroutine GGMM

Subroutine GGMM calculates the mutual impedance between two filamentary monopoles with sinusoidal current distributions. The dipole-dipole mutual impedance in Eq. 20 of Reference 1 is the sum of four monopole-monopole mutual impedances. The monopole impedances are calculated by GGS with Simpson's rule or by GGMM with closed-form expressions in terms of exponential integrals.

To explain the input data for GGMM, reference is made to Fig. 12. Subroutine GGMM is listed in Fig. 13. If the monopoles are parallel, let the \( z \) axis be parallel with both monopoles. The coordinate origin may be selected arbitrarily. \( S_1 \) and \( S_2 \) denote the \( z \) coordinates of the endpoints of the test monopole, \( T_1 \) and \( T_2 \) are the \( z \) coordinates of the endpoints of the expansion monopole, and \( D \) is the perpendicular distance (displacement) between the monopoles. The mutual impedance of parallel monopoles is calculated in the last part of GGMM below statement 110.

For skew monopoles, let the test monopole \( s \) lie in the \( xy \) plane and the expansion monopole \( t \) in the plane \( z = D \). \( D \) is the perpendicular distance between the parallel planes.) If the monopoles are viewed along a line of sight parallel with the \( z \) axis as in Fig. 12, the extended axes of the two monopoles will appear to intersect at a point on the \( xy \) plane. Let \( s \) measure the distance along the axis of
Fig. 12. Coordinates for parallel and skew monopoles in subroutine GGMM.
SUBROUTINE GGMM(S1,S2,T1,T2,D,CGO,SGD1,SGD2,CPSI,ETA,GAM  
2,P11,P12,P21,P22)  
DOUBLE PRECISION R1,R2,DPO,SIS,TS1,TS2,ST1,ST2,CD,BD,CPSS,SK 
2,TL1,TL2,TD1,TD2,SO1,DP51,DU2,2D  
COMPLEX CGO,SGD1,SGD2,SGD1,SGD2,ETA,GAM,P11,P12,P21,P22  
COMPLEX CST,EB,EC,EK,EL,EKL,EGZI,ES1,ES2,ET1,ET2,EXPA,EXPB  
COMPLEX E(2,2),F(2,2)  
COMPLEX EGZ(2,2),GM(2),GP(2)  
DATA PI/3.14159/  
DSQ=D*D  
SGI=SG1  
IF(S2.LT.SI)SGO1=-SGDI  
SGD1=SGD2  
IF(T2.LT.T1)SGDT=-SGD2  
IF(ABS(CPSI).GT.997)GO TO 110  
ES1=CEXP(GAM*S1)  
ES2=CEXP(GAM*S2)  
ET1=CEXP(GAM*T1)  
ET2=CEXP(GAM*T2)  
DD=D  
DPSI=CPSI  
TD1=T1  
TD2=T2  
CPSS=DPSI*DPSI  
CD=DD/DSORT(1.DO-CPSS)  
C=CD  
BD=CD*DPSI  
B=BD  
EB=CEXP(GAM*CMPLX(.O,8))  
EC=CEXP(GAM*CMPLX(.O,9))  
DO 10 K=1,2  
10 E(K,L)=(.O,.O)  
TS1=TD1*TD1  
TS2=TD2*TD2  
DPO=DD*DD  
SI=S1  
DO 100 I=1,2  
F1=(-I)**I  
SI=SI  
SIS=SI*SD1  
ST1=2.*SIS*TD1*DPSI  
ST2=2.*SIS*TD2*DPSI  
R1=DSORT(DPO+SIS+TS1-ST1)  
R2=DSORT(DPO+SIS+TS2-ST2)  
EK=EB  
DO 50 K=1,2  
FK=(-1)**K  
SK=FK*SD1  
EL=EC  
DO 40 L=1,2  
FL=(-1)**L  
EKL=EK*EL  
XX=FK*BD+FL*CD  
TL1=FL*TD1  
TL2=FL*TD2  
RR1=R1+SK+TL1  
RR2=R2+SK+TL2  
CALL EXPJ(GAM*CMPLX(RR1,-XX),GAM*CMPLX(RR2,-XX),EXPA)  
CALL EXPJ(GAM*CMPLX(RR1,XX),GAM*CMPLX(RR2,XX),EXPB)  
E(K,L)=E(K,L)+F1*(EXPA*EKL+EXPB*EKLI)  
40 EL=1./EC

Fig. 13a. Subroutine GGMM
EK=1./*EB
ZD=SD1*PSI
ZC=ZD
EGZI=CEXP(GAM*ZC)
R1=R1+ZD-TD1
R2=R2+ZD-TD2
CALL EXPJ(GAM*R1,GAM*R2,EXPB)
R1=R1-ZD+TD1
R2=R2-ZD+TD2
CALL EXPJ(GAM*R1,GAM*R2,EXPA)
F(I,1)=2. *SGDS*EXPB*EGZI
F(I,2)=2. *SGDS*EXPB*EGZI
100 SI=S2
CST=ETA/(16. *PI*SGDS*SGDT)
P11=CST*F(1,1)+E(2,2)*ES2-E(1,2)/ES2)*ET2
A1=-F(1,2)-E(2,1)*ES2+E(1,1)/ES2)/ET2
P12=CST*((-F(1,1)-E(2,2)*ES2+E(1,2)/ES2)*ET1
B1=F(1,2)+E(2,1)*ES2-E(1,1)/ES2)/ET1
P21=CST*((-F(2,1)-E(2,2)*ES1+E(1,1)/ES1)*ET2
C1=F(2,2)+E(2,1)*ES1-E(1,1)/ES1)/ET2
P22=CST*((F(2,1)-E(2,2)*ES1+E(1,1)/ES1)*ET1
D1=-F(2,2)-E(2,2)*ES1-E(1,1)/ES1)/ET1
RETURN
IF(CPSI.LT.0.)GO TO 120
TA=T1
TB=T2
GO TO 130
120 TA=T1
TB=T2
SGDT=-SGDT
130 SI=S1
DO 150 I=1,2
TJ=TA
DO 140 J=1,2
ZIJ=ZIJ-S1
R=SORT(DSO+ZIJ*ZIJ)
W=R+ZIJ
IF(ZIJ.GT.0.)W=DSQ/(R-ZIJ)
V=R-ZIJ
IF(J.EQ.1.)V=W
IF(J.EQ.1.)V=W
EGZ(I,J)=CEXP(GAM*ZIJ)
140 TJ=TB
CALL EXPJ(GAM*V1,GAM*V,GPI)
CALL EXPJ(GAM*W1,GAM*W,GM(1))
150 SI=S2
CST=ETA/(8. *PI*SGDS*SGDT)
P11=CST*(GM(1)*EGZ(1,2)+GP(1)/EGZ(1,2))
2-EGDS*(GM(1)*EGZ(1,2)+GP(1)/EGZ(1,2))
0111
P12=CST*(-GM(2)*EGZ(2,1)-GP(2)/EGZ(2,2))
2+EGDS*(GM(1)*EGZ(2,1)+GP(1)/EGZ(2,1))
0112
P21=CST*(GM(1)*EGZ(1,2)+GP(1)/EGZ(1,2))
2-EGDS*(GM(2)*EGZ(2,1)+GP(2)/EGZ(2,2))
0113
P22=CST*(-GM(1)*EGZ(2,1)-GP(1)/EGZ(1,1))
2+EGDS*(GM(2)*EGZ(2,1)+GP(2)/EGZ(2,1))
RETURN
END

Fig. 13b. Subroutine GGMM
the test monopole with origin at the apparent intersection. S1 and S2
denote the s coordinates of the endpoints of the test monopole. Simi-
larly, let t measure distance along the axis of the expansion monopole
with origin at the apparent intersection. T1 and T2 denote the t co-
ordinates of the endpoints of the expansion monopole. Let \( \hat{s} \) and \( \hat{t} \) be
unit vectors parallel with the positive s and t axes, respectively.

Then \( \text{CPSI} = \hat{s} \cdot \hat{t} = \cos \psi \). The monopole lengths are \( d_s \) and \( d_t \), and
the remaining input data are defined as follows:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CGDS</td>
<td>( \cosh \gamma d_s )</td>
</tr>
<tr>
<td>SGD1</td>
<td>( \sinh \gamma d_s )</td>
</tr>
<tr>
<td>SGD2</td>
<td>( \sinh \gamma d_t )</td>
</tr>
</tbody>
</table>

GGMM calls EXPJ for the exponential integrals.

The output data from GGMM are the impedances \( P_{11}, P_{12}, P_{21}, \) and
\( P_{22} \). In defining these impedances, the reference direction is from
S1 to S2 for the current on monopole s, and from T1 to T2 for the
current on monopole t. In the impedance \( P_{ij} \), the first subscript is
1 or 2 if the test dipole has terminals at S1 or S2 on monopole s.
The second subscript is 1 or 2 if the expansion dipole has terminals
at T1 or T2 on monopole t. The endpoint coordinates S1, S2, T1 and T2
may be positive or negative. The monopole lengths \( d_s \) and \( d_t \) are
assumed positive in defining the input data CGDS, SGD1 and SGD2.

For parallel monopoles, \( \text{CPSI} = 1 \) or \(-1\). S1, S2, T1 and T2 are
cartesian coordinates for parallel monopoles and spherical coordinates
for skew monopoles. For skew monopoles, the radial coordinates S1, S2,
T1 and T2 tend to infinity as the angle \( \psi \) tends to zero or \( \pi \). Therefore,
if the monopoles are within 4.5° of being parallel, they are approxi-
mated by parallel dipoles.

APPENDIX 7. Subroutine EXPJ

Subroutine EXPJ, listed in Fig. 14, evaluates the exponential
integral defined as follows:

\[
W_{12} = \int_{V_1}^{V_2} e^{-v} \, \frac{dv}{v} = E_1(V_1) - E_1(V_2) + j 2\pi
\]

where the integration path is the straight line from \( V_1 \) to \( V_2 \) on the
complex v plane and

\[
E_1(z) = \int_{z}^{\infty} e^{-t} \, \frac{dt}{t}
\]
SUBROUTINE EXPJ(V1, V2, W1, W2)

COMPLEX EC, E15, S, T, UC, VC, V1, V2, W1, W2

DIMENSION V(21), W(21), D(16), E(16)

DATA V/ 0.22284667E00, 0.20.11889321E01, 0.29927363E01, 0.57751436E01, 0.1BB74674E01, 0.15982874E02, 0.93307812E-01, 0.49269174E00, 0.12155954E01, 0.22699495E01, 0.36676227E01, 0.54253366E01, 0.75659162E01, 0.1012022RE02, 0.13130282E02, 0.16654408E02, 0.20776479E02, 0.25623894E02, 0.31407519E02, 0.38530683E02, 0.48026086E02/

DATA W/ 0.45896460E00, 0.41700083E00, 0.11337338E00, 0.10399719E01, 0.89549791E01, 0.74411568E02, 0.16021761E03, 0.23862195E03, 0.50094687E03, 0.12254778E02, 0.10161976E02, 0.47219591E02, 0.21069574E02, 0.22046490E01, 0.89728244E02/

DATA D/ 0.22495842E02, 0.74411568E02, -0.41431576E03, -0.78754339E02, -0.16021761E03, -0.23862195E03, -0.50094687E03, -0.12254778E02, -0.10161976E02, -0.47219591E02, -0.21069574E02, 0.22046490E01, 0.89728244E02/

DATA E/ 0.21103107E02, -0.37959780E03, -0.97489220E02, 0.12900672E03, -0.12910931E03, -0.55705574E03, 0.1352480£E02, 0.17949528E02, -0.32981014E00, 0.31028836E02, 0.22236961E02, 0.39124892E02, 0.81636799E01/

Z = Vt DO 100 JIM = I, 2

X = REAL( Z )

Y = AIMAG(Z)

E15 = (0., 0.)

AB = CABS(Z)

IF (AB.EQ.0.) GO TO 90

IF (X.GE.0. AND. AB.GT.0.) GO TO 80

YA = ABS(Y)

IF (X.LE.0. AND. YA.GT.0.) GO TO 80

IF (YA-X.GE.17.5 OR. YA.GE.8.5 OR. X+YA.GE.5.5 OR. X.GE.3.) GO TO 20

IF (X.LE.-10.) GO TO 40

N = 6.+3.*AB

E15 = I./(N-I.)-Z/N_2

N = N-1

E15 = I./(N-I.)-Z-E15/N

IF (N.GE.3) GO TO 15

E15 = Z.*E15-CMPLX(0.577216+ALOG(AB), ATAN2(Y, X))

GO TO 90

20 J1 = 1

J2 = 6

GO TO 31

30 J1 = 7

J2 = 21

S = (0., 0.)

YS = Y*Y

DO 32 I = J1, J2

XI = V(I)*X

CF = H(I)/(XI*Y*YS)

32 S = S+CMPLX(XI*CF, -YA*CF)

GO TO 54

40 T3 = X*Y*YS

T4 = T3*Y*YS

T5 = X*T3-YA*T4

T6 = X*T4+YA*T3

Fig. 14a. Subroutine EXPJ
UC=CMPLX(D(11)+D(12)*X+D(13)*X*T3+T5-E(12)*YA-E(13)*T4),
    E(11)+E(12)*X+E(13)*X*T3+T6+D(12)*YA+D(13)*T4)
VC=CMPLX(D(14)+D(15)*X+D(16)*X*T3+T5-E(15)*YA-E(16)*T4),
    E(14)+E(15)*X+E(16)*X*T3+T6+D(15)*YA+D(16)*T4)
GO TO 52
50 T3=X*X-Y*Y
    T4=2.*X*YA
    T5=X*T3-YA*T4
    T7=X*T5-YA*T6
    T8=X*T6+YA*T5
    T9=X*T7-YA*T8
    T10=X*T8+YA*T7
    UC=CMPLX(D(1)+D(2)*X+D(3)*X*T3+D(4)*X*T5+D(5)*T7+T9-(E(2)*YA+E(3)*T4)
    2*E(4)*T6+D(5)*T8+D(6)*T9+D(7)*T10+D(8)*T11+D(9)*T12+D(10)*T13+D(11)*T14+D(12)*T15)
    VC=CMPLX(D(6)+D(7)*X+D(8)*X*T3+D(9)*X*T5+D(10)*T7+T9-(E(7)*YA+E(8)*T4)
    2*E(9)*T6+D(10)*T8+D(11)*T9+D(12)*T10+D(13)*T11+D(14)*T12+D(15)*T13+D(16)*T14)
52 EC=UC/VC
    S=EC/CMPLX(X,YA)
    EX=EXP(-X)
    T=X*CMPLX(COS(YA),SIN(YA))
    E15=S*T
56 IF (Y.LT.0.) E15=CONJG(E15)
    GO TO 90
80 E15=.409319/(Z+.02304)+.421831/(Z+.02666)+.147126/(Z+.56788)+
    .230335/(Z+.90035)+.00401E-2/(Z+.18215)+.158654E-4/(Z+12.734)+
    .317031E-7/(Z+.19.3957)
   E15=E15*CEXP(-Z)
90 IF (J1M.EQ.1) W12=E15
100 Z=V2
    TH=ATAN2(AIMAG(Z),REAL(Z))-ATAN2(AIMAG(V2),REAL(V2))
    2*TH=ATAN2(AIMAG(V1),REAL(V1))
    AB=ABS(TH)
    IF (AB.LT.1.) TH=0
    IF (TH.GT.1.) TH=6.2831853
    IF (TH.LT.-1.) TH=-6.2831853
   W12=W12-E15+CMPLX(.0,TH)
RETURN
END

Fig. 14b. Subroutine EXPJ
The exponential integral $E_i(z)$ is defined in Reference 3. To generate $W_{12}$, subroutine EXPJ calculates $E_i(V_1)$, subtracts $E_i(V_2)$ and adds $j2n\pi$. The term $j2n\pi$ is determined by the requirement that $W_{12}$ vanish in the limit as $V_1$ approaches $V_2$. The integer $n$ may assume values of $-1$, $0$ or $+1$. If the integration path does not cross the negative real axis in the $v$ plane, $n$ is zero. The term $j2n\pi$ is calculated below statement 100.

APPENDIX 8. Subroutine GANTI

Subroutine GANTI, listed in Fig. 15, considers the wire structure as an antenna. In the input data, $V_G(J)$ is the voltage of a generator at point $IA(J)$ of segment $J$. $V_G(JJ)$ is the voltage of a generator at point $IB(J)$ of segment $J$. The DO LOOP ending with statement 50 uses the delta-gap model to determine the excitation voltages $C_I(I)$ for all the dipole modes. These are also stored temporarily in $CG(I)$. Then subroutine SQROT is called to obtain a solution of the simultaneous linear equations. SQROT stores the solution (the loop currents) in $C_I(I)$.

In the DO LOOP ending at statement 80, the complex power input is calculated and stored in $Y_{11}$. $GG$ denotes the time-average power input and is the real part of $Y_{11}$. If the antenna has only one voltage generator (with unit voltage and zero phase angle), then $Y_{11}$ also denotes the antenna admittance and $Z_{11}$ is the antenna impedance at that port.

Subroutine RITE is called to make the transformation from the loop currents $C_I(I)$ to the branch currents $CG(J)$. If $IWR$ is a positive integer, RITE will write out the list of branch currents.

Finally, GANTI calculates the radiation efficiency $EFF$. $PIN$ denotes the time-average power input. Subroutine GDISS is called to obtain the time-average power dissipated. $DISS$ is the total power dissipated in the lumped loads and the imperfectly-conducting wire. $PRAD$ is the time-average power radiated, defined by the difference between $PIN$ and $DISS$. If the antenna has perfect conductivity and purely reactive loads, the radiation efficiency is considered to be 100 percent.

APPENDIX 9. Subroutine SQROT

Subroutine SQROT is listed in Fig. 16. This subroutine considers the matrix equation $ZI = V$ which represents a system of simultaneous linear equations. If the square matrix $Z$ is symmetric, SQROT is useful for obtaining the solution $I$ with $V$ given. $NEQ$ denotes the number of simultaneous equations and the size of the matrix $Z$.

On entry to SQROT, $S$ is the excitation column $V$. On exit, the solution $I$ is stored in $S$. Let $Z(I,J)$ denote the symmetric square
SUBROUTINE GANTI (IA, IB, INM, IWR, I1, I2, I3, I12, I13, JA, JB, MD, N, ND, NM, AM)
C, CJ, CG, CMM, D, EFF, GAM, GG, CDO, SGD, VG, Y11, Z11, ZLD, ZS)
COMPLEX C(I), CJ(I), CGD(I), SGD(I), VG(I), ZLD(I), Y11(I), Z11(I), ZS, GAM, CG(I)
DIMENSION D(I), IA(I), IB(I), JA(I), JB(I)
DIMENSION I1(I), I2(I), I3(I), MD(INM, 4), ND(I)
2 FORMAT(1X,115,8F10.2)
5 FORMAT(1HO)
DO 50 I=1,N
CJ(I)=(.0,.0)
K=JA(I)
DO 40 KK=1,2
KA=IA(K)
KB=IB(K)
JJ=K
FI=I.
IF(KB.EQ.I2(I))GO TO 36
IF(KB.EQ.I1(I))FI=-1.
CJ(I)=CJ(I)+FI*VG(JJ)
GO TO 40
36 IF(KA.EQ.I3(I))FI=-1.
JJ=K+NM
CJ(I)=CJ(I)+FI*VG(JJ)
40 K=JB(I)
50 CONTINUE
DO 55 I=1,N
55 CG(I)=CJ(I)
CALL SQROT(C, CJ, O, I12, N)
I12=2
IF(I12.EQ.0) GO TO 80
Y11=Y11+CJ(I)*CONJG(CG(I))
CALL RITE(IA, IB, INM, IWR, I1, I2, I3, MD, ND, NM, CJ, CG)
GG=REAL(Y11)
Z11=1./Y11
PIN=GG
CALL G0ISS(AM, CG, CMM, D, DISS, GAM, NM, SGD, ZLD, ZS)
PRAD=PIN-DISS
EFF=100.*PRAD/PIN
RETURN
END

Fig. 15. Subroutine GANTI
SUBROUTINE SQROT(C,SS,IWR,II2,NEQ)

  COMPLEX C(1),SS(1)
  FORMAT(1X,11S15.1,F10.3,1F15.7,1F10.0,2F15.6)
  N=NEQ
  IF(II2.EQ.2)GO TO 20
  C(1)=CSQRT(C(1))
  DO 4 K=2,N
  C(K)=C(K)/C(1)
  DO 10 I=2,N
    IMO=I-1
    IPO=I+1
    II=(I-1)*N-(I*I-I)/2
    IE=IMO+I
    DO 5 L=IMO,IE
      LI=(L-1)*N-(L*L-L)/2+I
      C(LI)=C(LI)/C(II)
      IF(IPO.GT.N)GO TO 10
      DO 8 J=IPO,N
        IJ=I+J
        DO 6 M=IMO,IE
          MI=MO+M
          MJ=MI+J
          DO 6 C(IJ)=C(IJ)-C(MJ)*C(MI)
        6 CONTINUE
      5 C(II)=CSQRT(C(II))
      IF(II2.EQ.2)GO TO 20
      C(1)=CSQRT(C(1))
      DO 4 K=2,N
  10 CONTINUE
  20 S(I)=S(I)/C(1)
  DO 30 I=2,N
    IMO=I-1
    DO 25 L=IMO,IE
      LI=(L-1)*N-(L*L-L)/2+I
      S(I)=S(I)-C(LI)*S(L)
      II=(I-1)*N-(I*I-I)/2+I
    25 S(I)=S(I)/C(II)
    NN=((N+1)*N)/2
    S(N)=S(N)/C(N)
  30 CONTINUE
  NMO=N-1
  DO 40 I=1,NMO
    K=N-I
    KPO=K+1
    KD=(K-1)*N-(K*K-K)/2
    DO 35 L=KPO,N
      KL=KD+L
      35 S(K)=S(K)-C(KL)*S(L)
      KK=KD+K
    DO 40 S(K)=S(K)/C(KK)
  40 IF(IWR.LE.0)GO TO 100
  CNOR=0
  DO 50 I=1,N
    SA=CAABS(S(I))
    50 CONTINUE
  IF(SA.GT.CNOR)CNOR=SA
  IF(CNOR.LE.0)CNOR=1.
  DO 60 I=1,N
    SS=S(I)
    SA=CAABS(SS)
    SNOR=SA/CNOR
    PH=0
    IF(SA.GT.0.)PH=5.29578*ATAN2(AIMAG(SS),REAL(SS))
  60 WRITE(6,2)I,SNOR,SA,PH,SS
  100 RETURN
END

Fig. 16. Subroutine SQROT
matrix. On entry to SQROT, the upper-right triangular portion of $Z(I,J)$ is stored by rows in $C(K)$ with

$$K = (I - 1) \cdot \text{NEQ} - (I^2 - I) / 2 + J$$

If $I12 = 1$, SQROT will transform the symmetric matrix into the auxiliary matrix (implicit inverse), store the result in $C(K)$ and use the auxiliary matrix to solve the simultaneous equations. If $I12 = 2$, this indicates that $C(K)$ already contains the auxiliary matrix.

The transformation from the symmetric matrix to the auxiliary matrix is programmed above statement 10, and the solution of the simultaneous equations is programmed in statements 20 to 40. If IWR is positive, the program below statement 40 will write the solution.

SQROT uses the square root method described in Reference 4. The original symmetric matrix $Z$ and the upper triangular auxiliary matrix $A$ are related by

$$Z = A' \cdot A$$

where $A'$ is the transpose of $A$.

In the thin-wire application, SQROT must be called with $I12 = 1$ before it is called with $I12 = 2$. With a large matrix, the execution time in SQROT is much smaller with $I12 = 2$ than with $I12 = 1$.

APPENDIX 10. Subroutine RITE

Subroutine RITE is listed in Fig. 17. Given the list of loop currents $CJ(I)$, this subroutine generates a list of branch currents $CG(J)$. $CG(J)$ and $CG(JJ)$ denote the currents at $IA(J)$ and $IB(J)$, respectively, on the wire segment $J$, where $JJ = J + NM$. If IWR is a positive integer, the program below statement 110 writes a list of the branch currents. The symbols in this list are defined as follows:

- $K$ the segment number
- $ACJ$ normalized current magnitude at $IA(K)$
- $BCJ$ normalized current magnitude at $IB(K)$
- $PA$ phase of current at $IA(K)$
- $PB$ phase of current at $IB(K)$
- $CJA$ complex current at $IA(K)$
- $CJB$ complex current at $IB(K)$

The phase angles $PA$ and $PB$ are in degrees. Even if IWR is negative, RITE generates the branch-current list for use in subroutine GDISS.
SUBROUTINE RITE(IA,IB,INM,IR,W,I1,I2,I3,MD,ND,NM,CJ,CG)

COMPLEX CJ(1),CG(1),CJA,CJB

DIMENSION IA(1),IB(I1),I1(I1),I2(I1),I3(I1),MD(INM,4),ND(I1)

5 FORMAT(15,I5,10,E10.3,10,E10.0,15,E15.6)

AMAX=0

DO 100 K=1,INM
  KA=IA(K)
  KB=IB(K)
  CJA=1.0,0.0)
  CJB=1.0,0.0)
  NDK=ND(K)
  DO 40 II=1,NDK
    I=MD(K,II)
    FI=I.
    IF (KB.EQ.12(I)) GO TO 36
    IF (KB.EQ.I1) FI=-I.
    CJA=CJA+FI*CJ(I)
  GO TO 40
36 IF (KA.EQ.13(I)) FI=-I.
    CJB=CJB+FI*CJ(I)
40 CONTINUE
CG(K)=CJA
  KK=K+NM
  CG(KK)=CJB
  ACJ=CA(BS(CJA)
  BCJ=CA(BS(CJB)
  IF (ACJ.GT.AMAX)AMAX=ACJ
  IF (BCJ.GT.AMAX)AMAX=BCJ

100 CONTINUE
IF (IR.GT.0) GO TO 110
RETURN
110 IF (AMAX.LE.0.1)AMAX=1.

DO 200 K=1,INM
  CJA=CG(K)
  KK=K+NM
  CJB=CG(KK)
  ACJ=CA(BS(CJA)/AMAX
  BCJ=CA(BS(CJB)/AMAX
  PA=57.29578*ATAN2(AIMAG(CJA),REAL(CJA))
  PB=57.29578*ATAN2(AIMAG(CJB),REAL(CJB))

200 WRITE(6,2)K,ACJ,BCJ,PA,PB,CJA,CJB

WRITE(6,5)
RETURN
END

Fig. 17. Subroutine RITE
APPENDIX 11. Subroutine GDISS

Subroutine GDISS is listed in Fig. 18. This subroutine uses Eq. 50 of Reference 1 to calculate the time-average power dissipated in the imperfectly conducting wire. This is accomplished in the DO LOOP terminating at statement 100. The power dissipated in the lumped loads is calculated in the DO LOOP terminating with statement 140. DISS denotes the time-average power dissipated in the wire and the loads.

APPENDIX 12. Subroutine GNFLD

Subroutine GNFLD, listed in Fig. 19, inputs the loop currents CJ(I), calls GNF for the near-zone field of each wire segment, and sums over all the segments to obtain the near-zone field of the wire antenna or the near-zone scattered field of the wire scatterer. EX, EY and EZ denote the cartesian components of this field at the observation point (XP,YP,ZP). This calculated field does not include the incident fields of the magnetic frills or loops associated with generators on the antenna. It also does not include the radiation from the polarization currents in the dielectric insulation.

This subroutine could be simplified and speeded by inputting the branch currents CG(J) instead of the loop currents CJ(I). However, this would increase the storage requirements because the far-field subroutine GFFLD would have to store the branch currents induced by the phi-polarized and theta-polarized incident waves.

APPENDIX 13. Subroutine GNF

Subroutine GNF, listed in Fig. 20, uses Eqs. 75 and 76 of Reference 1 to calculate the near-zone electric field of a sinusoidal electric monopole with endpoints at (XA,YA,ZA) and (XB,YB,ZB). The observation point is at (X,Y,Z). EX1, EY1 and EZ1 are the components of the field generated by the mode with unit current at (XA,YA,ZA). EX2, EY2 and EZ2 denote the field generated by the mode with unit current at (XB,YB,ZB). GNF is similar to GGS, and Appendix 5 defines many of the symbols used in both subroutines.

APPENDIX 14. Subroutine GFFLD

The far-field subroutine GFFLD, listed in Fig. 21, is discussed in section II. In antenna gain calculations with INC = 0, the loop currents CJ(I) are employed by GFFLD to calculate the far-zone field. The field of each segment is obtained by calling GFF, and a summation over all the segments yields the field of the antenna.

In a bistatic scattering situation with INC = 2, the input data include the loop currents EP and ET induced by phi-polarized and theta-polarized incident waves. These currents were calculated by GFFLD in a
SUBROUTINE GDISS(AM, CG, CMM, D, DISS, GAM, NM, SGD, ZLD, ZS)
COMPLEX CG(1), SGD(1), ZLD(1), CJA, CJB, GAM, ZS
DIMENSION D(1)
DATA PI/3.14159/
DISS=.0
IF (CMM.LE.0.) GO TO 120
ALPH=REAL(GAM)
BETA=AIMAG(GAM)
RH=REAL(ZS)/(4.*PI*ALPH)
DO 100 K=1,NM
DK=D(K)
DEN=CABS(SGD(K))**2
EAD=EXP(ALPH*DK)
CAD=(EAD+1./EAD)/2.*
CBD=COS(BETA*DK)
SAD=DK
IF (ALPH.NE.0.) SAD=(EAD-1./EAD)/(2.*ALPH)
SBD=DK
IF (BETA.NE.0.) SBD=SIN(BETA*DK)/BETA
FA=RH*(SAD*CAD-SBD*CBD)/DEN
FB=2.*RH*(CAD*SBD-SAD*CBD)/DEN
CJA=CG(K)
L=K+NM
CJB=CG(L)
100 DISS=DISS+FA*(CABS(CJA)**2+CABS(CJB)**2)
2*FB*(REAL(CJA)*REAL(CJB)+AIMAG(CJA)*AIMAG(CJB))
120 DO 140 J=1,NM
K=J+NM
140 DISS=DISS+REAL(ZLD(J))*(CABS(CG(J))**2)
2*REAL(ZLD(K))*(CABS(CG(K))**2)
RETURN
END

Fig. 18. Subroutine GDISS
SUBROUTINE GNFLD (IA, IB, INM, I1, I2, I3, MD, N, ND, NM, AM, CGD, SGD, ETA, GAM)

COMPLEX EX, EY, EZ, EX1, EY1, EZ1, EX2, EY2, EZ2, ETA, GAM

DIMENSION IA(I), IB(I), I1(I), I2(I), I3(I), D(I), X(1), Y(1), Z(1)

DATA PI, TP/3.14159, 6.28318/

EX=(.0, .0)
EY=(.0, .0)
EZ=(.0, .0)

DO 140 K=1, NM
KA=IA(K)
KB=IB(K)
CALL GF (X(KA), Y(KA), Z(KA), X(KB), Y(KB), Z(KB), XP, YP, ZP, AM, D(K))

NDK=ND(K)

DO 140 II=1, NDK
I=MD(K, II)

IF(KB.EQ.I1(I)) GO TO 136
IF(KB.EQ.I2(I)) I=I-1.
EX=EX+F*I*EX1*CJ(I)
EY=EY+F*I*EY1*CJ(I)
EZ=EZ+F*I*EZ1*CJ(I)
GO TO 140

IF(KA.EQ.I3(I)) I=I-1.
EX=EX+F*I*EX2*CJ(I)
EY=EY+F*I*EY2*CJ(I)
EZ=EZ+F*I*EZ2*CJ(I)

CONTINUE
RETURN
END

Fig. 19. Subroutine GNFLD
Fig. 20. Subroutine GNF
SUBROUTINE GFFLD(IA,IB,INC,INM,IWR,I1,I2,I3,I4,MD,N,ND,NM,AM) 0001

3,ETPS,ETTS,GG,GP,EY,ET,PH,SGD,SCSP,SCST,SPPH,STPH,STPM,STTM,TH 0003
4,XY,Z,ZLD,ZST,E(TA,GAM) 0004
COMPLEX CJ,ET,ET2,ETEPS,ETTS,ETTS,EP,EPTS,EPTS,EPTS,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSES,EPTS,ETPSE

Fig. 21a. Subroutine GFFLD
Fig. 21b. Subroutine GFFLD
previous call for the backscattering situation with INC = 1. Thus, a bistatic call must be preceded by a backscatter call.

EPP(I) and ETT(I) denote the phi-polarized and theta-polarized far-zone fields of dipole mode I with unit terminal current. In a backscattering situation, the excitation voltages EP(I) and ET(I) are obtained by multiplying EPP and ETT by the constant CJI. (See Eqs. 38, 39 and 40 in Reference 1.) Then calls are made to SQROT which stores the solution (the induced loop currents) in EP(I) and ET(I). RITE is called for the branch currents CG(J), and GDISS is called for the time-average power dissipated in the imperfectly conducting wire and the lumped loads. This power is denoted PDIS and TDIS for phi-polarized and theta-polarized incident waves, respectively.

In scattering problems, the incident plane wave has unit electric field intensity at the coordinate origin. GGG denotes the time-average power density of the incident wave at the origin. ACSP and ACST denote the absorption cross sections for the phi and theta polarizations.

PIN and TIN denote the time-average power input to the wire structure, delivered by the equivalent voltage generators VP and VT at the terminals. PIN and TIN apply for the phi and theta polarizations, respectively. The time-average power input is regarded as the sum of the time-average power dissipated (in the wire and the lumped loads) and the time-average power radiated or scattered by the wire. ECSP and ECST denote the extinction cross sections and SCSP and SCST are the scattering cross sections.

The distant field is calculated in the DO LOOP ending with statement 180 for scattering situations, and in the DO LOOP ending with statement 260 for the antenna situation. In these fields, the range dependence is suppressed as in Eq. (1).

The radar cross sections (echo areas) SPPM, SPTM, STPM and STTM are defined as in Eq. 72 of Reference 1 with the incident power density (S_i or GGG) evaluated at the coordinate origin. The user selects the location of the origin when supplying the input data for the coordinates of all the points on the wire.

For an antenna, the following definition is employed for the power gain:

\[ G_p(\theta, \phi) = \lim_{r \to \infty} 4\pi r^2 e^{2\alpha r} S(r, \theta, \phi) / P_i \]

where \( P_i \) (or G in the program) denotes the time-average power input and \( S(r, \theta, \phi) \) is the time-average power density in the radiated field. For an antenna in a lossless medium, \( \alpha \) vanishes and Eq. (6) reduces to the standard definition of power gain. Without the factor \( e^{2\alpha r} \) in Eq. (6), the power gain would vanish for a finite antenna in a conducting medium. GPP and GTT denote the power gains associated with the phi-polarized and theta-polarized components of the field, respectively.
APPENDIX 15. Subroutine GFF

Subroutine GFF, listed in Fig. 22, uses the equations in Appendix 2 of Reference 1 to calculate the far-zone field of a sinusoidal electric monopole. The monopole has endpoints \((X_A,Y_A,Z_A)\) and \((X_B,Y_B,Z_B)\). \(E_{P1}\) and \(E_{T1}\) denote \(E_a\) and \(E_E\) for the mode with unit current at \((X_A,Y_A,Z_A)\). \(E_{P2}\) and \(E_{T2}\) denote the fields for the mode with unit current at \((X_B,Y_B,Z_B)\). The range dependence is suppressed as in Eq. (1). The far field vanishes in the endfire direction where \(G_K = 0\).
SUBROUTINE GFF (XA, YA, ZA, XB, YB, ZB, D, 
2CGD, SGD, CTH, STH, CPH, SPH, 
2GAM, ETA, ET1, ET2, EP1, EP2)
COMPLEX ET1, ET2, EP1, EP2, GAM, ETA
COMPLEX GD, SGD, EGD
COMPLEX EGFA, EGFB, EGGD, ESA, ESB
COMPLEX CST
FP = 12.56637
XAB = XB - XA
YAB = YB - YA
ZAB = ZB - ZA
CA = XAB/D
CB = YAB/D
CG = ZAB/D
G = (CA * CPH + CB * SPH) * STH + CG * CTH
GK = 1.0 - G
ET1 = (0.0, 0.0)
ET2 = (0.0, 0.0)
EP1 = (0.0, 0.0)
EP2 = (0.0, 0.0)
IF (GK .LT. 0.001) GO TO 200
FA = (XA * CPH + YA * SPH) * STH + ZA * CTH
FB = (XB * CPH + YB * SPH) * STH + ZB * CTH
EGFA = CEXP (GAM * FA)
EGFB = CEXP (GAM * FB)
EGGD = CEXP (GAM * GD)
CST = ETA / (CK * SGD * FP)
ESA = CST * EGFA * (EGGD - G * SGD - CGD)
ESB = CST * EGFB * (1.0 / EGGD + G * SGD - CGD)
T = (CA * CPH + CB * SPH) * CTH + CG * STH
PX = -CA * SPH + CB * CPH
ET1 = T * ESA
ET2 = T * ESB
EP1 = P * ESA
EP2 = P * ESB
200 CONTINUE
RETURN
END

Fig. 22. Subroutine GFF