Photoelectric Observations of Krzeminski's Star,
The Companion of Cen X-3

Larry D. Petro
University of Michigan

Abstract. V magnitudes of Krzeminski's star and their interpretation are presented. We find $M_{\text{primary}} = 20.5 \pm 3.5 \, M_\odot$, $M_x = 2.5 \pm 1.1 \, M_\odot$, $\sin i = 0.985 \pm 0.02$, and $q = 0.12 \pm 0.05$.
I. INTRODUCTION

Centaurus X-3 was the first discovered X-ray source whose intensity is pulsed and eclipsed (Giacconi et al. 1971, Schreier et al. 1972). Though the X-ray properties of the Cen X-3 system have been intensively studied, the optical properties remained unstudied until Krzeminski's (1973) identification of the optical counterpart of Cen X-3. Observations of the X-ray eclipse duration, pulsation frequency variation, and pulse arrival time variation have been used to place limits on the nature of the system and to predict the nature of the optical companion and X-ray source (Schreier et al. 1972, Sofia 1972, Wilson 1972, Osaki 1972, Davidson and Ostriker 1972, and McCluskey and Kondo 1972). It is the purpose of this paper to present an observational determination of several system parameters based on a preliminary interpretation of the light curve of Krzeminski's star.

II. OBSERVATIONS AND REDUCTIONS

The observations presented here were obtained with the Cerro Tololo Inter-American Observatory 1 meter (Yale) telescope and the 61 cm. (Lowell) telescope in the period 25 January 1974 to 12 February 1974. A 1P21 (CTIO no. 57) was used in pulse counting mode on both telescopes with the V filter of the CTIO no. 2 UBV filter set. The standard CTIO photometers were used, photometer no. 2 being used on the 1 meter telescope and photometer no. 1 on the 61 cm. telescope.

Observations at the telescope were made in the order: sky-comparison-Krzeminski's star, being sure to bracket all observations of Krzeminski's star with comparison observations and to bracket all comparison measures by sky measures. All observations were one minute integrations. Observations were made in this sequence until approximately 10 measures each of the sky, Krzeminski's...
star, and the comparison were obtained. The primary comparison observed was
no. 9 of Brucato, Kristian, and Westphal (1972) and the secondary standard was
no. 6 of the same paper.

In the data reduction sky subtraction was effected by taking the mean of
adjacent sky measures and applying this to the bracketed measures of the com-
parison and of Krzeminski's star. The differential magnitude of Krzeminski's
star was then found with respect to the mean of the comparison star measures
bracketing the measure of Krzeminski's star.

Two problems were encountered in these procedures. The first was due to
the crowded field in which Krzeminski's star is located. Observations obtained
with a 20 arc second aperture differed systematically from those obtained with
a 15 arc second aperture. A special sequence of observations was made on two
nights with the 1 meter telescope and combined with a previous night's observations,
when both apertures on the 1 meter telescope had been employed to determine
the aperture effect to be 0.056 ± 0.005 mag. This correction has been applied
to all observations obtained with the larger apertures. The size of this
effect corresponds to the presence of a star of magnitude 16.5 in the larger
apertures. Residuals from the light curves described in the following section
show no statistically significant difference between the two sets of observa-
tions after correction of the larger aperture set.

The second problem was that the sky subtraction method failed for observ-
ations made near moonrise and moonset. To overcome this we interpolated graphi-
cally in the logarithm of the sky brightness to determine the value of the sky
background at the time of each observation. The difference between the logarithmic
graphical procedure and our standard technique was as large as 0.03 magnitude.
Having taken these effects into account the mean and internal error of each of the 40 minute runs was computed. The standard deviation of an observation of unit weight is 0.008 magnitude. Figure 1 presents these means from which it may be seen that the amplitude of the light curve is approximately 0.08 magnitude, in agreement with the initial announcement by Krzeminski (1973) but in disagreement with the value of 0.12 magnitude reported recently by Mauder (1974) and Krzemsinki (1974). However, if the extreme values of the present observations are employed in determining the amplitude, then a value approaching 0.12 magnitude is found.

III. LIGHT CURVE SYNTHESIS AND SYSTEM PARAMETERS

We have determined the mass ratio \( q = \frac{M_x}{M_{\text{primary}}} \) and the inclination, \( i \), of the system by minimizing \( \chi^2 \) for the observations shown in figure 1 and for synthetic light curves over a range of \( (q, \sin i) \) values. These light curves are computed from formulas given by Kopal and Kitamura (1968) for the light changes of close binary systems. The model employed is the standard Roche model wherein the stars are assumed to be of infinite central condensation, rotating synchronously, and to have the rotational and orbital angular momenta parallel. The observed surface brightness is taken to obey von Zeipel's theorem and a linear limb darkening law. We have assumed that the primary star fills its Roche lobe. The effect of this assumption is that the mass ratios and inclinations determined will be minimum values.

We have employed the NLTE, line blanketed, plane parallel, hydrostatic model atmospheres of Mihalas (1972) to compute the gravity darkening coefficients \( \tau_0 \) and \( \tau_1 \), the values of which depend upon the spectral type of the primary.
The following spectral types for Krzeminski's star have been suggested:
09-09.5 V, B0 I-III, and B0 II by Krzeminski (1974) based on the UBV colors of the star; 09 III-V (Vidal et al. 1974); 09.5-B0.5 Ib by Rickard (1974) from spectra; and 06.5 III-V (Osmer, Hiltner, and Whelan 1974) from spectra.

We have explored this range of spectral types by employing the gravity darkening coefficients appropriate to the endpoints 06.5 III-V and 09.5 III-V. The values of $(\tau_0, \tau_1)$ are $(0.295, -0.45)$ and $(0.460, -0.45)$ respectively. It is found that the gravity darkening coefficients are essentially identical for luminosity classes III and V and that the spectral types near B0 Ib have gravity darkening coefficients within this range. The limb darkening coefficient has been taken to be 0.6 (Grygar 1965).

The best fit to all 41 observations for the above range of gravity darkening coefficients is shown as line segment a in figure 2. The small q endpoint is the best fit 09.5 III-V solution and the large q endpoint is the best fit 06.5 III-V solution. The error ellipses on these endpoints are the formal errors from the $\chi^2$ solution. The 06.5 III-V and 09.5 III-V solutions represent the observations equally well and thus we cannot distinguish between the published spectral types on the basis of the light curve analysis alone.

Consideration of the residuals from the best fitting synthetic light curve shows that the light curve provides a good representation of the observations since there are no systematic deviations. However, the internal errors do not provide a good estimate of the deviation of the observations from the best fit light curve. This can be interpreted as evidence that the primary is irregularly varying in brightness with a dispersion of approximately 0.011 magnitude.

To test the sensitivity of the light curve parameters q and sin i to this variability, we fit the 37 observations obtained by deleting four discrepant
observations near the primary and secondary minima, since the light curve
parameters have a maximum sensitivity to observations at the extrema. The
results of the fitting procedure for these observations are shown as line
segment b in figure 2. These results lie outside the formal 1σ solution
region based on all 41 observations, but not greatly so.

Thus, the present observations can be represented by the model developed
by Kopal and Kitamura (1968) with a mass ratio near 0.123 and sin i near 0.984,
the exact values depending upon the spectral type of the primary and the inter-
pretation of the observations which are far from the mean light curve. These
two ambiguities lead to a 24 percent uncertainty in q and 2 percent uncertainty
in sin i.

These values of q and sin i are minimum values in that we have assumed
that the Roche lobe is filled. Davidson and Ostriker (1973) have suggested
that early spectral type members of close binaries may rotate at less than the
synchronous velocity in which case it is possible for a star to be in dynamical
equilibrium at sizes larger than the Roche lobe. The size and mass ratio of
a star principally affect the mean amplitude of the light variation, whereas
the relative depth of the two minima is principally determined by the inclin-
ation of the system. We therefore believe that our lack of knowledge about
whether the Roche lobe is filled (or overfilled if the tidal lobe model is
appropriate) should not affect our estimation of the uncertainty in sin i.

To estimate the uncertainty in q due to our lack of knowledge of the
size of Krzeminski's star we note that the lowest order equations for the
light changes of a close binary (Kopal 1959, p. 412) can be used to find the
following useful interpolation formula:

\[ \Delta \ell = \sin^2 \iota \, q \, r^3 \left( 1 - r_0 \right) \frac{(15 + x)}{(3 - x)} \]
where the amplitude, $\Delta l$, is given in terms of the radius of the star, $r$ (in units of the separation); the limb darkening coefficient, $x$; and previously defined quantities. Since, near $q = 0.1$ the tidal lobe radius is 7 percent greater than the Roche lobe radius (Davidson and Ostriker 1973) $q$ may be smaller by as much as 21 percent. The approximate $q$ limits for the tidal lobe model for solution tracks $a$ and $b$ are shown in Figure 2 as line segments $c$ and $d$ respectively. Underfilling of the Roche lobe seems less likely since the spectroscopic observations of Osmer, Hiltner, and Whelan (1974) constrain $q$ from being much greater than 0.1.

We have estimated the total uncertainty of $q$ to be 45 percent and of $\sin i$ to be 2 percent. It is expected that further observation and study of the system will greatly reduce the present uncertainty.

IV. DISCUSSION

The most important result which follows from the present study is an observational determination of the masses of Krzeminski's star and of Cen X-3 and of their separation. From the $4.8 \times 10^8$ X-ray pulsation mass function and the projected size of the Cen X-3 orbit the following relations for the primary mass, $M$, and the separation of the components, $a$, are found:

\[
2) \quad M = 15.5 \left( 1 + q \right)^2 / \sin^3 i
\]
\[
3) \quad a = 17.1 \left( 1 + q \right) / \sin i
\]

where the mass and separation are in solar units. It can be seen that the primary mass and the separation are principally sensitive to $\sin i$.

Equations 2) and 3); the equivalent radius, $R$, of the Roche lobe (Kopal 1959)
in units of the component separation; and the radial velocity semi-amplitude, \( K \), of Krzeminski's star in terms of the X-ray source velocity yield the results shown in Table 1.

**Table 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>0.12 ± 0.05</td>
</tr>
<tr>
<td>( \sin i )</td>
<td>0.985 ± 0.02</td>
</tr>
<tr>
<td>( a )</td>
<td>19.5 ± 1.2 ( R_\odot )</td>
</tr>
<tr>
<td>( R )</td>
<td>12.8 ± 0.8 ( R_\odot )</td>
</tr>
<tr>
<td>( M_{\text{primary}} )</td>
<td>20.5 ± 3.5 ( M_\odot )</td>
</tr>
<tr>
<td>( M_x )</td>
<td>2.5 ± 1.1 ( M_\odot )</td>
</tr>
<tr>
<td>( K )</td>
<td>51 ± 19 km sec(^{-1} )</td>
</tr>
</tbody>
</table>

The quoted uncertainty in each quantity reflects our estimate from the previous section of the Roche lobe filling uncertainty, the spectral type uncertainty, and the observational interpretation uncertainty.

It is interesting to compare these values with stellar masses and radii tabulated as functions of spectral type. The masses of 06.5 III-V models are given as approximately 30 \( M_\odot \) (Stothers 1972) and 45 \( M_\odot \) (Robertson 1973). From equation 2) the value of \( \sin i \) necessary to bring agreement between the model masses and the observed mass can be found. Lines \( f \) and \( g \) in figure 2 show these values of \( \sin i \) for 45 \( M_\odot \) and 30 \( M_\odot \) respectively. Clearly no agreement can exist with the present observations. However, the predicted masses of 09.5 III-V stars are in accord with the mass found here. Effects of mass exchange on close binary members are not well understood and it is therefore unwise to rule out an atmospheric quantity, the spectral type, on the basis of a mass determination. We conclude that Krzeminski's star cannot be an 06.5 star of "normal" mass. Radii of OB stars tabulated by Panagia (1973) show the radii of 09-B0 supergiants to be twice that of Krzeminski's star. We, therefore,
conclude that Krzeminski's star cannot be a normal 09-BO supergiant. Main sequence and giant stars in the range 06.5 - 09.5 are compatible with the Roche lobe diameter determined here.

From the Doppler variation of the $^{48}$X-ray pulsations it is found that the line of sight velocity variation of Cen X-3 is 415 km sec$^{-1}$ (Schreier et al. 1972). The line of sight velocity variation of the optical primary is simply $q$ times this value. Though the observations of Osmer, Hiltner and Whelan (1974) do not place a strong constraint on the radial velocity variation of Krzeminski's star, the radial velocity variation determined from the light curve analysis presented here is consistent with these observations.

The duration of X-ray eclipse places constraints on the size of the occulting object and the inclination of the system. For purposes of comparison we may assume that the size of the occulting object is given by the Roche lobe. The observed eclipse duration then constrains the possible values of $q$ and $\sin i$ to a locus in the $(q, \sin i)$ plane. For the duration of X-ray eclipse we have taken 0.488 (Schreier et al. 1972), excluding the transition period. Curve e in figure 2 shows the result of a calculation in which we have employed the third order expression for the surface radius given by Kopal (1959, p. 131). The mass ratio and inclination found from the light curve do not fall on this locus. A possible resolution of the discrepancy can be made by assuming that the X-ray occulting surface is larger than the Roche lobe by approximately 13 percent while maintaining the photosphere at the Roche lobe. The X-ray occulting surface would necessarily not be in dynamical equilibrium as suggested previously for HD 77581 — 3U 0900-40 by Petro and Hiltner (1974). An alternative explanation follows from the tidal lobe suggestion of Davidson and Ostriker (1973),
namely that the X-ray occulting surface and the photosphere are closely coincident and are both near the tidal lobe limit.

V. CONCLUSION

The lack of firm understanding of the structure of upper main sequence stars and of the evolution of these stars in close binaries precludes the use of the results of this study to discriminate between the widely disparate spectral types suggested for Krzeminski's star. The mass of Krzeminski's star is established as $20.5 \pm 3.5 \, M_\odot$ while that of Cen X-3 is established as $2.5 \pm 1.1 \, M_\odot$. Resolution of the spectral type ambiguity, further light curve observations, and further radial velocity observations should lead to a much better determination of all parameters discussed in this paper.

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REFERENCES


FIGURE CAPTIONS

Figure 1 - Differential V light curve of Krzeminski's star. Also shown is the best fitting synthetic light curve for all 41 observations.

Figure 2 - Constraints on the system Cen X-3 - Krzeminski's star. a is the 06.5 III/V - 09.5 III/V light curve solution track for all 41 observations. b is the same track for 37 of the observations. The error ellipses shown at the endpoints depend only on the internal errors of the observations. Lines c and d are approximate lower limits to the value of q if the tidal lobe model is correct. Locus e gives the (q, sin i) values compatible with X-ray eclipse being caused by material at the Roche lobe. Points to the right of this locus correspond to eclipse being caused by material beyond the Roche lobe. Lines f and g are the approximate values of sin i necessary if the primary mass is to be 45 M\(_\odot\) or 30 M\(_\odot\) respectively.