A NUMERICAL CALCULATION OF OUTWARD PROPAGATION
OF SOLAR DISTURBANCES

by

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Final Technical Report

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SUMMARY

In this contract (NAS8-25101), a systematic theoretical investigation of the dynamical behavior of the solar active region has been performed. As a result of these studies, we have concluded that the most appropriate physical mechanism in helping to understand the disturbed solar atmosphere is the propagation of shock waves in a model solar atmosphere. During the course of this contract, we have examined two important cases:

(i) The Downward Propagation and Response of the Chromosphere

In this study, we have examined the responses of the solar chromosphere to an infalling material stream resulting from the "disparition brusque" of a prominence. We found that the solar chromosphere is heated by the shock resulting from the infalling material stream and radiation is enhanced. The enhanced radiation terminates the shock around the height of the temperature minimum in the Harvard-Smithsonian Reference Atmosphere model. This radiation enhancement is identified as Optical (H\textalpha) flares. The detail of this study was submitted to the National Aeronautics and Space Administration, Marshall Space Flight Center, as an Interim Report for this contract, dated March 1972 (UARI Research Report No. 114). A part of this study is also published in Solar Physics, Vol. 30, Page 111-120, 1973.
(ii) The Upward Propagation of Solar Disturbance and Its Responses

After completion of the study of the downward propagation of a shock through the chromosphere (from ~ 2000 Km to the sun's surface), we felt it logical to examine the responses of the solar atmosphere due to an outward propagation shock. Therefore, in this final research report, we shall report the results of this study, since the other results have been documented already. In this study, we have employed the Lax-Wendroff method to solve the set of non-linear partial differential equations, because the method of characteristics used to analyze the downward propagating shock became invalid due to non-homogeneity in the model of the solar atmosphere. It was found that this theoretical model can be used to explain the solar phenomena of surge and spray. A criterion to discriminate the surge and spray was found. The detailed information concerning the density, velocity, and temperature distribution with respect to the height and time is presented. The complete computer program is also included in this report.

Finally, we would like to summarize the publications and research reports resulting from this contract as follows:

1. Refereed Publications


ii. Research Report


TABLE OF CONTENTS

CHAPTER

ACKNOWLEDGEMENT

SUMMARY

I INTRODUCTION 1

II FORMULATION OF THE PROBLEM 4

II-1 Hydrodynamics Model 4

II-2 Initial and Boundary Conditions 7

II-2-1 Initial Conditions 7

II-2-2 Boundary Conditions 9

III NUMERICAL TECHNIQUE 11

III-1 Conservational Form of Equations 12

III-2 Numerical Stability 16

III-2-1 Time Interval 16

III-2-2 Artificial Viscosity 17

III-2-3 Shock Dissipation 19

III-2-4 Minor Modifications on Difference Equations 21

III-3 Computation Procedure 24

IV RESULTS OF COMPUTATION 26

V CONCLUSIONS AND RECOMMENDATIONS 30

REFERENCES

LIST OF FIGURES

APPENDICES
CHAPTER I
INTRODUCTION

During previous studies [1, 2], we found that the optical ($\lambda$) flare can be identified with the response of the solar chromosphere to a shock wave propagating downward through the chromosphere. The shock wave is related to an infalling material stream resulting from the "disparition brusque" of a prominence as suggested by Hyder [3, 4, 5], and Nakagawa and Hyder [6]. In the general impact theory, there are always two waves generated propagating in opposite directions right at the moment of the impact. According to the coordinate system which we adopted here, one of these two waves is propagating downward through the chromosphere to the photosphere, and another one is propagating upward through the transition region to the corona and beyond. The study of a downward propagating shock through the chromosphere has been completed and reported [1, 2]. Therefore, we shall present the results of upward propagating disturbances (either shocks or subsonic disturbances) through a model solar atmosphere in this report.

To calculate the downward propagating shock through the chromosphere, we have used the CCW (Chisnell, Chester, Whitham) [7] approximation which is based on the theory of characteristics. However, it is noted by Bird [8] that this method of approximation has only provided satisfactory accuracy for a shock propagating into a denser medium. Therefore, it seems to us that it is not quite proper to apply the CCW
approximation to calculate the outward propagating disturbances, because the model solar atmosphere is attenuated outward. Thus, we have chosen the Lax-Wendroff method \([9, 10]\) for the present study. This is a numerical method, which has the advantage of taking care of both the sub- and supersonic disturbances. The numerical accuracy can be controlled by using proper numerical techniques, such as by specifying the proper time increment and grid size in the computation processes based on the physical model of the solar atmosphere.

In this study, the evolution of the disturbances, originating at \(\sim 30,000 \text{ Km} \sim 0.0428 \ R_s \ R_s\) being the solar radius \(\sim 7 \times 10^5\) Km, is examined in detail using the method we mentioned in the previous paragraph. These disturbances are identified as pressure pulses with different strengths and durations. From the present results, we have shown that the short duration (few minutes) and moderate strength pressure pulse \((\Delta p \sim 2)\) will result in the phenomenon of "surge," because it shows that a stream velocity of \(\sim 100 - 200 \text{ km/sec}\) can be achieved in this case. It also demonstrated that there is an essential part of the material falling back to the sun's surface which agrees with the observation.

The longer duration (i.e., 20-40 minutes) and stronger strength pressure pulse \((\Delta p \sim 10)\) shows that a stream velocity of the order of \(\sim 1000 \text{ km/sec}\) is achieved and no falling materials can be seen. Thus, we have identified this case as the "spray".

Finally, we should point out, that there is little difference between the adiabatic calculation and the calculation with a Cox-Tucker type
radiation loss as has been shown in the calculation of the downward propagating case [1]. This is because the Cox-Tucker [11] radiative loss is mainly based on the hydrogen-equilibrium radiative-equilibrium estimation that only covers radiation such as the Balmer series. It is worthwhile to examine the radiative effects in further detail.
CHAPTER II

FORMULATION OF THE PROBLEM

II-1 Hydrodynamics Model

In the solar atmosphere of interest in the present study, the gyro-radius is ~ 10 km, and while the scale height is of the order of a thousand kilometers, we can consider that the medium is filled with collision-dominated plasma. Thus, the physical behavior of this plasma can be considered as a continuum fluid. Consequently, the hydrodynamic model was chosen for the present problem.

In dealing with radiative cooling effects in this problem, we have chosen the Cox-Tucker model [11], because the dominating radiation in this part of the solar atmosphere results from bremsstrehung, recombination radiation, and collision-induced line emission. A summary of this radiative loss is shown in Figure 1. For the convenience of the numerical calculation, a simple, analytical closed-form expression is adopted,

\[ Q_R = \chi \rho^2 T^\alpha. \]  \hspace{1cm} (2-1)

The symbol \( Q_R \) is the radiative cooling rate (ergs/cm\(^3\)sec), \( \rho \) is the gas mass density (gm/cm\(^3\)) and \( T \) is the temperature (K). Finally \( \chi \) and \( \alpha \) constants determined by the results given by Cox and Tucker, as shown in Figure 1. The numerical values for these two constants at various temperatures are presented in Table 1.
Table I. Temperature of Radiative Cooling Rate

<table>
<thead>
<tr>
<th>Range of Temperature °K</th>
<th>( \chi )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T \leq 5.0 \times 10^4 )</td>
<td>( \chi = 1.0 \times 10^{10} )</td>
<td>( \alpha = 3.55 )</td>
</tr>
<tr>
<td>( 5.0 \times 10^4 &lt; T \leq 2.5 \times 10^5 )</td>
<td>( \chi = 3.0 \times 10^{26} )</td>
<td>( \alpha = 0 )</td>
</tr>
<tr>
<td>( 2.5 \times 10^5 &lt; T \leq 7.0 \times 10^6 )</td>
<td>( \chi = 1.0 \times 10^{32} )</td>
<td>( \alpha = -1.172 )</td>
</tr>
<tr>
<td>( 7 \times 10^6 &lt; T )</td>
<td>( \chi = 1.0 \times 10^{23} )</td>
<td>( \alpha = 0.288 )</td>
</tr>
</tbody>
</table>

\[ g \text{ives } Q = \chi \rho^2 T^\alpha \text{ (ergs/cm}^3\text{sec)} \]

As we have discussed previously, the hydrodynamic model can be used for the present study. In order to avoid unnecessary complexity while retaining the basic physical process of the problem, we consider the plasma flow guided upward along a vertical magnetic flux tube. The possibility of such a confinement is discussed by Nakagawa and Hyder [12], and it was shown that confinement is possible when the gas pressure within the plasma flow is smaller than the local magnetic pressure, i.e.,

\[ \frac{B^2}{8\pi} > \frac{2\gamma}{\gamma+1} M^2 p , \]  

(2-2)

where \( B \) is the strength of magnetic field induction, \( \gamma \) the ratio of specific heats, \( M \) the shock Mach number and \( p \) the gas pressure. If we consider that the maximum gas pressure in the model of the solar atmosphere is \( \sim 1.5 \times 10^{-1} \) cgs, which corresponds to the gas flow velocity of the order of \( 4 \times 10^2 \) Km/sec, we find that Eq. (2-2) is satisfied for \( B \geq 60 \) G, which is a reasonable value of \( B \) in an active region. Therefore, the magnetic force effect can be ignored in this calculation. The governing equations for the present problem can be written as
Continuity:
\[ \frac{\partial \rho}{\partial t} + \mathbf{\nabla} \cdot (\rho \mathbf{v}) = 0 , \]  
(2-3)

Momentum:
\[ \rho \frac{\partial \mathbf{v}}{\partial t} + \mathbf{\nabla} \cdot (\rho \mathbf{v} \mathbf{v}) = -\mathbf{\nabla} p + \rho \mathbf{g} , \]  
(2-4)

Energy:
\[ \rho \frac{\partial \varepsilon}{\partial t} + (\mathbf{v} \cdot \mathbf{\nabla}) \varepsilon = -\rho \mathbf{v} \cdot \mathbf{\nabla} \varepsilon + \mathbf{\nabla} \cdot (\mathbf{K} \mathbf{v} \mathbf{v}) - Q_R \]  
(2-5)

where \( \mathbf{v} \) is the flow velocity and \( \mathbf{g} \) is the gravitational acceleration along the normal axis from the sun's surface, toward the sun. \( \varepsilon \) is the internal energy of the gas per unit volume, \( K \) is the thermal conductivity and \( Q_R \) is the radiative cooling loss rate given by Eq. (2-1). Finally the equation of state is
\[ p = \rho RT = \rho \frac{kT}{m} , \]  
(2-6)

and
\[ E = C_v T = \frac{p}{\rho(\gamma-1)} , \]  
(2-7)

with \( R, k, m, \) and \( C_v \) being the gas constant, Boltzmann constant, average molecular weight and specific heat at constant volume, respectively.

Let us adopt the spherical coordinates for the present study, and further assume that the case of spherical symmetry, the thermal conduction is negligible compared with radiation. The Eqs. (2-3) through (2-5) become
\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r} (\rho u) + \frac{2\rho u}{r} = 0 \]  
(2-8)
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = - \frac{1}{\rho} \frac{\partial \rho}{\partial r} - g, \quad (2-9)
\]
\[
\frac{\partial \varepsilon}{\partial t} + u \frac{\partial \varepsilon}{\partial r} = \frac{\rho}{\varepsilon} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) - Q_R, \quad (2-10)
\]

where \( u \) represents the radial velocity and depends on radius \( r \) and time \( t \).

The gravitational acceleration \( g \) is given by

\[
g = \frac{g_s R_s^2}{r^2}, \quad (2-11)
\]

with \( g_s \) being the gravitational acceleration at the surface of the sun, and \( R_s \) being the solar radius.

Eq. (2-8) through (2-10) are set of non-linear time dependent partial differential equations without dispersion coefficients (i.e., viscosity, diffusivity, etc.). To find an analytical solution for this set of equations with various boundary conditions is impossible. However, it is possible to obtain a numerical solution. Some discussion on the existing numerical method will be given in the next chapter.

II-2 Initial and Boundary Conditions

II-2-1 Initial Conditions

Initially, we have assumed the solar atmosphere at the photosphere and chromosphere to be the Harvard-Smithsonian Reference Atmosphere (HSRA) [13]. Beyond its range, the solar atmosphere is assumed to be in a state of hydrostatic equilibrium, which can be calculated from Eq. (2-4), thus

\[
\frac{\partial \rho}{\partial r} = - \rho g = - \rho g_s \frac{R_s^2}{r^2}, \quad (2-12)
\]
where superscript \( o \) denotes the quantities at steady state.

Substituting \( p^o = p^o R^o T^o \) into (2-12), we obtained

\[
\frac{d\rho^o(r)}{dr} = -\rho^o(r) \left[ \frac{g^o R^o_s r^2}{R^o T^o(r)} \frac{1}{r^2} + \frac{1}{T^o(r)} \frac{dT^o(r)}{dr} \right]
\]  

(2-13)

Eq. (2-13) should be numerically integrated for a given temperature profile of the atmosphere. However for an isothermal atmosphere Eq. (2-13) reduces to a simpler form

\[
\frac{1}{\rho^o} \frac{d\rho^o}{dr} = -\frac{g^o S}{R^o T^o} \frac{R^2}{r^2}.
\]  

(2-14)

Integration of this equation yields the solution for the density profile of the hydrostatic atmosphere,

\[
\rho^o = \rho^o_0 \exp \left[ \frac{g^o S R^2}{RT^o} \left( \frac{1}{r} - \frac{1}{r_1} \right) \right],
\]  

(2-15)

where \( \rho^o_0 \) is the reference density at \( r = r_1 \). Steady state pressure distribution is then

\[
p^o = p^o_0 \exp \left[ \frac{g^o S R^2}{RT^o} \left( \frac{1}{r} - \frac{1}{r_1} \right) \right],
\]  

(2-16)

where \( p^o_0 \) is the pressure at \( r = r_1 \).
II-2-2 Boundary Conditions

There are two boundary conditions, one at $r = 0$ and one at $r \to \infty$, to be given. The lower boundary condition will be characterized by disturbances such as density pulse, temperature pulse, velocity pulse, and pressure pulse. To introduce these pulses, we can specify them by prescribing the amplitude and duration of the pulse which depends on the characteristics of the disturbances.

For example, the disturbance is introduced at the lower boundary as a pressure pulse and the velocity is zero at $t = 0$. But, the velocity on the boundary for $t > 0$ will be determined from the continuity equation thus

$$u^n = \frac{\rho^n}{\rho_2} \frac{u^n}{\rho_1}$$ (2-17)

where subscript 1 denotes the boundary and 2 denotes the mesh point next to the boundary, superscript n denotes the time increment, such that $t = n\Delta t \leq \tau$, $\tau$ being the duration of the disturbance. When $t > \tau$, disturbance is gone, and the lower boundary returns to its unperturbed condition, i.e., a hydrostatic equilibrium state.

On the upper boundary we have used the completely absorbed condition, i.e., all the effects due to reflected waves are ignored. The reason for keeping this assumption is because the time for the reflected wave to reach the upward-propagating wave packet is much longer than the time for the upward-propagating wave to reach the upper boundary. Thus, the non-reflective boundary condition for velocity can be expressed as
\[ \phi^n_j = 2\phi^n_{j-1} - \phi^n_{j-2} \] (2-18)

where \( \phi \) represents a physical quantity, the subscript \( j \) denotes the mesh points at upper boundary and superscript \( n \) denotes the time step.
CHAPTER III

NUMERICAL TECHNIQUE

Numerical computation of time dependent inviscid, compressible flow is a formidable task because of the appearance of discontinuities in the flow field. The conventional way to solve hyperbolic types of differential equations is the method of characteristics. In this method, partial differential equations are usually written in characteristic form. Due to the presence of discontinuities in the fluid, these characteristic equations can not be integrated over the entire region of space. Instead, the integral form of the differential equations is used for the discontinuities while differential equations are applied to the remaining region. Although this method is very simple in principle, its application to practical problems is very lengthy and cumbersome. Furthermore, the fact that one cannot know the time and location of such discontinuities in flow field prior to the computation makes application to actual problems almost impractical. Watts and Rosenverg, et.al. [14] solved the transient adiabatic compressible fluid flow in a duct by using the characteristic method in an elegant manner. Their method can be, in principle, extended to solve the present problem. However, the resulting computational procedures will be too complicated to be practical. It may be worthwhile to examine this method in some detail at a later date. Gentry, et al.,[15] used the FLIC method, known as Fluid in Cell, to describe the time dependent equations of motion for the compressible flow of a fluid. This method has
been used to solve a wide variety of problems in compressible fluid flow. Hundhausen, et al, [16] used this method to simulate the flare generated disturbances in the solar wind.

Currently, the most commonly adopted method for solving the compressible fluid flow problem is probably the Lax-Wendroff difference method. Lax-and Wendroff [9, 10] suggested that partial differential equations are first written in divergence free form, and then the difference equations in the divergence free form can be generated from these equations. The mathematical proofs are beyond the present scope of these studies and will not be presented in this report. However, the basic idea of this method is that errors caused by the discretization process tend to smooth the solution. This allows the representation of shocks by smearing discontinuities over several mesh points.

There are many versions of the difference scheme for the conservative form of equations. Some detailed comparisons among these versions are made by Ehmery [17] and Burstein [18, 19]. One version due to Burstein [20], is used for the present study.

III-1 **Conservational Form of Equations**

Divergence-free form of the governing equations will not create or eliminate any flow variables. After some algebraic manipulation (see Appendix A) the governing equations [2-8], [2-9], and [2-10] can be written in Eulerian pseudo-conservational form;
\[
\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial r} (\rho u) - \frac{2 \rho u}{r}, \tag{3-1}
\]

\[
\frac{\partial (\rho u)}{\partial t} = - \frac{\partial}{\partial r} \left[ (\gamma - 1) E - \frac{(\gamma - 3)}{2} \rho u^2 \right] - \rho g - \frac{2 \rho u^2}{r}, \tag{3-2}
\]

\[
\frac{\partial E}{\partial t} = - \frac{\partial}{\partial r} \left[ u (\gamma E - \frac{(\gamma - 1)}{2} \rho u^2) \right] - \rho ug - Q_R - \frac{2}{r} \left[ u (\gamma E - \frac{(\gamma - 1)}{2} \rho u^2) \right], \tag{3-3}
\]

where \( E \) is the total energy per unit volume, given by

\[
E = \frac{p}{\gamma - 1} + \frac{\rho u^2}{2}. \tag{3-4}
\]

Using vector notation, Eqs. (3-1), (3-2) and (3-3) can be put in the form

\[
\frac{\partial \mathbf{u}}{\partial t} = - \frac{\partial \mathbf{F}}{\partial r} + \mathbf{\kappa}, \tag{3-5}
\]

where \( \mathbf{u}, \mathbf{F} \) and \( \mathbf{\kappa} \) are three components vectors;

\[
\mathbf{u} = \begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix}, \tag{3-6}
\]

\[
\mathbf{F} = \begin{bmatrix} \rho u \\ (\gamma - 1)E - \frac{(\gamma - 3)}{2} \rho u^2 \\ u \left( \gamma E - \frac{(\gamma - 1)}{2} \rho u^2 \right) \end{bmatrix}, \tag{3-7}
\]
Using the difference operator, Equation (3-5) is approximated by

\[ \delta_t \mathbf{U} = -\delta_r \mathbf{F} + \mathbf{K}, \quad \text{where} \]

\[ \delta_t \quad \text{and} \quad \delta_r \quad \text{is yet to be discussed. There are several versions of Lax-Wendroff difference schemes which have been extensively used for a wide range of fluid flow problems.} \]

Lax-Wendroff scheme is based on the Taylor series expansion of the vector function \( \mathbf{U}(r, t + \Delta t) \) so as to include the second order term \( \partial^2 \mathbf{U} / \partial t^2 \). A two-step method obtained by Richtmyer [21] is used here. The values at the intermediate points are computed at a time \( t + \Delta t/2 \) using a first-order correct scheme, and then a second-order correct scheme (leap frog) is used to compute the value at time \( t + \Delta t \). The overall scheme has, then a second-order correct differencing scheme. Burstein, et. al. [20] suggested that instead of computing the intermediate value at time \( t + \Delta t/2 \), they compute them at \( t + \Delta t \) and then average the \( \mathbf{F} \) difference at \( t \) and \( t + \Delta t \) so that both the \( \mathbf{U} \) and \( \mathbf{F} \) values are centered at point \( (i, t + \Delta t/2) \). The above differencing method applied to Equation (3-5) yields the following difference approximation:
Intermediate Values

\[
\begin{align*}
\frac{\bar{U}_{i+1/2}^{n+1}}{\bar{U}_i} &= \frac{1}{2} \left( \bar{U}_{i+1}^n + \bar{U}_i^n \right) - \frac{\Delta t}{\Delta x} \left( \bar{F}_{i+1}^n - \bar{F}_i^n \right) + \frac{\Delta t}{2} \left( \bar{K}_{i+1}^n + \bar{K}_i^n \right) \\
\frac{\bar{U}_{i-1/2}^{n+1}}{\bar{U}_i} &= \frac{1}{2} \left( \bar{U}_{i-1}^n + \bar{U}_{i+1}^n \right) - \frac{\Delta t}{\Delta x} \left( \bar{F}_i^n - \bar{F}_{i-1}^n \right) + \frac{\Delta t}{2} \left( \bar{K}_i^n + \bar{K}_{i-1}^n \right) \\
\frac{\bar{U}_i^{n+1}}{\bar{U}_i} &= \frac{1}{2} \left( \bar{U}_{i-1}^n + \bar{U}_{i+1}^n \right) - \frac{\Delta t}{2\Delta x} \left( \bar{F}_{i+1}^n - \bar{F}_{i-1}^n \right) + \Delta t \bar{K}_i^n
\end{align*}
\] (3-10)

where the bar signifies the intermediate flow variables at \((i-1/2)\Delta x, i\Delta x, (i+1/2)\Delta x\) and at \((n+1)\Delta t\). Using these intermediate values of the variables we calculate the final values.

Final Values

\[
\begin{align*}
\bar{U}_i^{n+1} &= \bar{U}_i^n - \frac{\Delta t}{2\Delta x} \left[ \frac{1}{2} \left( \bar{F}_{i+1}^n - \bar{F}_{i-1}^n \right) + \left( \bar{F}_{i+1}^{n+1} - \bar{F}_{i-1}^{n+1} \right) \right] \\
&\quad + \frac{\Delta t}{2} \left( \bar{K}_i^n + \bar{K}_i^{n+1} \right)
\end{align*}
\] (3-11)

where the intermediate values of \(\bar{F}_{i+1}^{n+1}, \bar{F}_{i-1}^{n+1}\) and \(\bar{K}_i^{n+1}\) are calculated by using Eqs. (3-7) and (3-8) with intermediate flow variables obtained in the previous step. Combining Eqs. (3-10) and (3-11), it is easily seen that this is a second-order correct-differencing scheme.
Numerical Stability

Finite difference equations may exhibit rapidly growing and oscillatory solution that cannot resemble the true solutions of partial differential equations. In this case, the difference equation is said to be computationally unstable. The origin of instabilities varies with a particular set of partial differential equations. Many theories and criteria have been developed by many investigators, such as Richtmyer and Morton [22], Hirt [23], and Van Leer [24]. However, it is not possible to have a general theory for higher-order non-linear equations, such as in the present problem. The discussion given here is not rigorous, but presents some heuristic techniques which prove very useful in stabilizing the computations.

Time Interval

As a first approximation, the time interval for successive iterations can be found by applying the stability criterion of Courant, Friedrichs and Lewy,

\[ \Delta t < \frac{\Delta r}{|u| + a}, \]  

(3-12)

where \( a \) is the local sound speed. When a thermal conduction term is included in the governing equations, thermal conduction stability criteria should be considered. Applying the Fourier method proposed by von Neumann concerning the heat conduction equation, one finds the time interval \( \Delta t_c \) to be

\[ \Delta t_c < \frac{\frac{1}{2} p (\Delta r)^2}{K(y-1)T^{7/2}}, \]  

(3-13)
In the case of heat exchange terms becoming arbitrary, namely \( Q_R \), the \( K_3 \) vector in Eq. (3-8) of the present problem, can be varied very rapidly. When such a case occurs, numerical oscillation starts and computation terminates. In order to smear out such oscillation, the time step is chosen according to

\[
|Q_R| \Delta t_Q < \frac{1}{2} \frac{D}{\gamma - 1} .
\]  

(3-14)

For a model that does not include the conduction term, 0.25 \( \Delta t_s \) is sufficiently small to satisfy all stability criteria.

III-2-2 Artificial Viscosity

If the strength of the disturbance is large, a sharp jump in the flow variable occurs in the flow field. We can find this jump condition by applying the Rankine-Hugoniot relations. In numerical simulation, these discontinuities easily cause the onset of instability. Thus, the artificial viscosity is introduced to help offset the instability. The idea of introducing artificial viscosity into shock calculations is due to the work of von Neumann and Richtmyer [25]. The basic requirements for a purely artificial dissipative term are:

(i) all flow variables should have smooth transitions across the discontinuity;

(ii) transitions should have correct jump conditions computable with Rankine-Hugoniot conditions;
(iii) the discontinuity should travel at very nearly the correct speed; and

(iv) the thickness of transition is independent of shock strength, pressure or density of material while the shock is moving [26].

Introducing a pseudo-viscous pressure term in the compression zone, it is shown that all requirements for the artificial viscosity are satisfied. The pseudo-pressure is given by [26]

\[ q = \begin{cases} \left( \rho \lambda^2 \right) \left( \frac{\partial u}{\partial r} \right)^2 & \text{if } \frac{\partial u}{\partial r} < 0 \\ 0 & \text{if } \frac{\partial u}{\partial r} \geq 0 \end{cases} \]  

(3-15)

where \( \lambda \) is a constant having the dimensions of length. Then the total energy, Eq. (3-4), for the compression region, \( \frac{\partial u}{\partial r} < 0 \), is modified to include the pseudo-pressure, such that

\[ E = \frac{P + q + \rho u^2}{\gamma - 1} \]  

(3-16)

It is seen clearly that this correction only affects the compression region and that the continuity equation is inact by this modification. Letting \( \lambda = \beta \Delta r \), the appropriate difference approximation for the altered pressure is then

\[ (p+q)^n_i = \begin{cases} p^n_i + \frac{1}{2} \beta^2 \rho^n_i \left( u^n_{i+1} - u^n_i \right)^2 & \text{if } u^n_{i+1} < u^n_i \\ p^n_i & \text{if } u^n_{i+1} \geq u^n_i \end{cases} \]  

(3-17)

for the intermediate step, and

\[ (p+q)^{n+1}_{i+1/2} = \begin{cases} p^{n+1}_{i+1} + \frac{1}{2} \beta^2 \rho^{n+1}_{i+1/2} \left( u^{n+1}_{i+1/2} - u^{n+1}_{i+1/2} \right) & \text{if } u^{n+1}_{i+1} < \bar{u}^{n+1}_{i+1/2} \\ p^{n+1}_{i+1} & \text{if } u^{n+1}_{i+1} \geq \bar{u}^{n+1}_{i+1/2} \end{cases} \]  

(3-18)
for the final step of computations, where $-$ signifies the intermediate values, and $\beta$ has the value of 1 to 5. The effect of $q$ on the overall picture is carefully tested by several trial runs. There is no significant change in flow variables except at discontinuities, and the transition occurred over 3 or 4 mesh points.

III-2-3 Shock Dissipation

Mechanical energy carried by the shock wave is dissipated into thermal energy of the gas which experiences an irreversible, non-isentropic process as the shock front passes through the gas. For a unit mass of gas, the thermal energy increases in terms of enthalpy which depends on how the post shock gas returns to its pre-shock gas state. Schützmann [27] suggested that the gas expands adiabatically until it comes back to the initial pressure and then cools down until it reaches the initial density. Along this path, the change of enthalpy is given by [28]

$$\Delta h = -\frac{P-P_o}{2} \left( \frac{1}{\rho} + \frac{1}{\rho_0} \right) - \frac{Y}{\gamma-1} \left( \frac{P}{\rho} + \frac{P_0}{\rho_0} \right)$$  \hspace{1cm} (3-19)

where subscript $o$ denotes the pre-shocked gas. Then, for a periodic disturbance propagation, the total enthalpy change of the gas is

$$Q_D = -\frac{P_0}{\omega} \Delta h$$  \hspace{1cm} (3-20)

where $\omega$ is the period of the disturbance. Letting

$$\psi = \frac{P}{P_0}, \quad \sigma = \frac{P}{P_0} \quad \text{and} \quad \xi = \left( \frac{\psi}{\sigma} \right)^{1/2},$$

Eq. (3-20) becomes
\[ Q_d = - \frac{\rho_o a_o^2}{\omega \gamma} \left[ \frac{1}{2} (\psi - 1) \left( \frac{1}{\sigma} + 1 \right) - \frac{\gamma}{\gamma-1} (\xi^2 + 1) \right] , \quad (3-21) \]

where \( \psi \) and \( \sigma \) are functions of shock Mach number; that is,

\[ \psi = \frac{2\gamma}{\gamma+1} \frac{M_s^2}{(\gamma+1)} - \frac{\gamma-1}{\gamma+1} , \quad (3-22) \]

\[ \sigma = \frac{(\gamma+1) M_s^2}{(\gamma-1) M_s^2 + 2} , \]

\[ M_s = \text{shock speed relative to pre-shock gas} \]

\[ \text{sonic speed in pre-shock gas} \quad (3-23) \]

The detailed structure of the shock front must be known in terms of its position and its strength \( M_s \) in order to find the accurate shock dissipation in the gas. However, it is not possible to determine the exact position of the shock front, since the transition occurs over several mesh points. It is not quite clear how to determine the exact shock strength \( M_s \) in this numerical calculation. The pressure difference between the neighboring points does not give the shock strength, because the unperturbed solar atmosphere possesses a density gradient. In order to give an approximate shock strength at position \( i \Delta r \), the following equation is employed:

\[ M_{s,i} = \frac{u_{i-1} - u_i}{a_i} , \quad (3-24) \]

This equation only gives a parameter which is related more to the local gas flow than the shock strength. In other words, Eq. (3-24) is a sufficient condition for a shock, but not a necessary one. It does,
however, provide a mechanism for continuously monitoring the presence of the discontinuity, whenever the discontinuity occurs, in a very simple manner.

In computation, then, from Eq. (3-21) with Eqs. (3-22) and (3-24), the dissipated energy due to the shock is included in the form

\[
(Q_p)_i^n = \begin{cases} 
\rho_1 a_1^2 / \omega \gamma \left[ \frac{1}{2} (\psi_i^n - 1) \left( \frac{n_i^n}{\sigma_1^n} + 1 \right) - \frac{\gamma}{\gamma - 1} \left( \frac{c_i^n}{\sigma_1^n} \right) \right] & \text{for } \psi_i^n \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]  (3-25)

For a dynamic model, the exact value of \( \omega \) cannot be defined. An estimation of \( \omega \) is made on the ground that the weak shock travels one mesh point within the time \( \Delta t_s \), i.e.,

\[
\Delta t_s = \frac{\Delta r}{|u| + a}.
\]

Since the Lax-Wendroff method is an explicit difference scheme, disturbances travel one mesh point for each full iteration. Thus, \( \omega \) equals to \( \Delta t_s \) approximately. Without the addition of \( Q_p \) in the \( K_3 \) term in equation (3-8), the temperature of the gas just ahead of the shock goes negative sometimes, and the calculation is terminated. With this modification, the solution remained stable and there is no noticeable difference in shock structure.

III-2-4 Minor Modifications on Difference Equations

Due to the exponential decrease of density in a quiet solar atmosphere, the difference scheme needs two minor modifications. For a hydrostatic solar atmosphere, the density variation along the \( r \)-direction is from Eq. (2-15),

\[
\rho_i^n = \rho_1^n \exp (-h_i^n r_1 / \Delta r),
\]  (3-26)
where

\[ h_i^n = \frac{g_i}{RT_i^n} \Delta r \text{ (scale height)} \]  

(3-27)

Then the nearest-neighbor average value appearing in the first term in RHS of Equation (3-10) is larger than the value at the central point, i.e.,

\[ 0.5 (\rho_{i-1}^n + \rho_{i+1}^n) = 0.5 \rho_i^n \left( e^{-h_i^n} + e^{h_i^n} \right) \]

\[ = 0.5 \rho_i^n \left[ 2 + \left( \frac{h_i^n}{2} \right)^2 \right] = \rho_i^n \left( 1 + \frac{(h_i^n)^2}{2} \right) > \rho_i^n \]  

(3-28)

If this difference is not corrected, the density after the intermediate step will become excessively large, and the unchanged vertical pressure cannot support this excess material. Consequently, a downward velocity appears over the entire field. Instead of a simple average, an expression

\[ 0.5 \left( \rho_{i-1}^n + \rho_{i+1}^n \right) + \rho_i^n - 0.5 \left( \rho_{i-1}^n + \rho_{i+1}^n \right) \]

is used for \( \rho_i^n \).

Rearranging this, one gets

\[ 0.5 \left( \rho_{i-1}^n + \rho_{i+1}^n \right) + 0.5 \rho_i^n \left[ 1 - 0.5 \left( e^{-h_i^n} + e^{h_i^n} \right) \right] \]  

(3-29)

If the nearest-neighbor points have the same value as the center, then Eq. (3-29) reduces to the simple neighbor-points average value.

In the process of calculation, the 2nd correction term in Eq. (3-29), is applied for the intermediate step of each iteration for all conservational variables at full mesh points.

Due to the exponential variation of the hydrostatic equilibrium state,
the difference approximation is systematically different from the
derivatives they approximate. That is why the error grows from Eq.
(3-26),
\[
\frac{\partial \rho_i^n}{\partial r} = - \frac{h_i^n}{\Delta r} \rho_i^n
\]  
(3-30)

But the simple centered difference approximation to the first derivative
with respect to \( r \) gives
\[
\delta_r \rho_i^n = \frac{\rho_{i-1}^n - \rho_{i+1}^n}{2\Delta r} = \frac{e^{-h_i^n} - e^{h_i^n}}{2\Delta r} \rho_i^n
\]  
(3-31)

\[
z - \frac{h_i^n}{\Delta r} \left( 1 + \frac{(h_i^n)^2}{6} \right) \rho_i^n < \frac{\partial \rho_i^n}{\partial r}
\]

Unless this discrepancy is corrected, an incorrectly calculated pressure term
in Eq. (3-2) will set the flow field in an upward motion. In order
to avoid this non-physical situation, a correction term \( c_i^n \) is defined such
that
\[
\left( \delta_r + c_i^n \right) \rho_i^n = \frac{\partial \rho_i^n}{\partial r}  
\]  
(3-32)

Combining Equations (3-31) and (3-32), \( c_i^n \) is found to be
\[
c_i^n = - \frac{1}{\Delta r} \left[ h_i^n + 0.5 \left( e^{-h_i^n} - e^{h_i^n} \right) \right]  
\]  
(3-33)

This correction term is included for both steps in each iteration.
III-3 **Computation Procedure**

The actual calculation procedure for the present problem is illustrated in Figure 2.

The steady state temperature $T$ of the atmosphere is assumed to be known and the steady state density is found by using Eq. (2-15), i.e.,

$$
\rho_1^n = \rho_1^n \exp \left[ \frac{g_0 R_0^2}{R T_1^n} \left( \frac{1}{r_i^n} - \frac{1}{r_1} \right) \right].
$$

(3-34)

For a hydrostatic equilibrium state the velocity of the field is zero,

$$
u_1^n = 0
$$

(3-35)

The total energy is then, from Eq. (3-16),

$$
E_i^n = \frac{p_i^n + q_i^n}{\gamma - 1} + \frac{(\rho_i^n u_i^n)^2}{2 \rho_i^n},
$$

(3-36)

where $p_i^n$ and $q_i^n$ are determined by Eqs. (3-17) and (3-18), respectively.

The disturbance at the lower boundary is introduced in terms of a pressure jump. For instance, $p_1^n = 2p_1^n$, $T_1^n = T_1^n$ will give $p_1^n/p_1^n = 2$. This arbitrary disturbance was kept constant for a prescribed time interval $\tau$. After this period of time, the lower boundary returns to its original state.

The time increment $\Delta t$ is found by the CFL (Courant-Fredricks-Levy) condition, and is applied to each mesh point in the flow field.
Using these initial values of conservational variables, the fluxes at time \((n\Delta t)\) are found from Eq. (3-7) and (3-8).

\[
\begin{align*}
F_{1n}^n &= (\rho u)_i^n \\
F_{2n}^n &= (\gamma - 1) E_i^n - \frac{(\gamma - 3)}{2} \rho_i^n (u_i^n)^2 \\
F_{3n}^n &= u_i^n \left( \gamma \, E_i^n - \frac{(\gamma - 1)}{2} \rho_i^n (u_i^n)^2 \right) \\
K_{1n}^n &= \frac{2\rho_i^n u_i^n}{r_i} \\
K_{2n}^n &= -\frac{\rho_i^n u_i^n}{\gamma} \left[ 2 \, \frac{\rho_i^n (u_i^n)^2}{\gamma} \right] \\
K_{3n}^n &= -\rho_i^n u_i^n g_i + (Q_{R_i})^n - \frac{2}{r} \left[ u_i^n \left( \gamma E_i^n - (\gamma - 1) \, \frac{\rho_i^n (u_i^n)^2}{2} \right) \right] + (Q_{D_i})^n
\end{align*}
\]

where

\[
E_i^n \quad \text{is given by Eq. (3-25).}
\]

\[
(Q_{R_i})^n = -\chi \, \rho_i^n (T_i^n) \alpha
\]

and \((Q_{D_i})^n\) is given by Equation (3-25).

Using these fluxes at \(n\Delta t\), the intermediate conservational flow variables at \((n+1)\Delta t\) are found from Eq. (3-10) with Eq. (3-29). Intermediate fluxes at \((n+1)\Delta t\) are found by similar manner as those at \(n\Delta t\) except intermediate flow variables, \(U_i^n\), should be used in Eq. (3-29) and (3-33). The new flow variables at \((n+1)\Delta t\) are then calculated by using Eq. (3-11) with the aid of Eq. (3-33).
CHAPTER IV
RESULTS OF COMPUTATIONS

Numerical results are obtained for given various initial boundary conditions identified as a pressure pulse with different amplitude and duration resulting from possible solar disturbances due to solar activities. All disturbances are placed at the lower boundary which is located at ~30,000 Km (1.043 Rs) above the sun's surface, and all the calculations are carried out to ~ 3 Rs (Rs being the solar radius). The results obtained in this report are the density, temperature and mass flow velocity as a function of height and time for $\Delta p$ (pressure disturbance) equal to 2 $\sim 10$ and $\Delta t$ (duration of the disturbances) equal to 30 sec, 60 sec, 120 sec, 1200 sec, and in some cases, 2400 sec. A detailed discussion of these results will follow.

Figure 3a, b, and c plotted the disturbed density, temperature and velocity due to disturbances of $\Delta p = 2$ and $\Delta t = 120$ sec. It shows that the disturbance has little effect on temperature and density and its influence on mass flow velocity is significant. It appears that this disturbance has created a mass stream shooting out from the upper chromosphere or/lower corona to upper corona with a flow velocity $\sim 50$ Km/sec at $\sim 1R_s$ from the surface (i.e., $\sim 2R_s$ from the center of the sun). Similar plots with different initial strength and duration of disturbances are given in Figures 4a, b, c through 11 a, b, c. These results clearly demonstrate that the characteristics of the disturbances are the essential
parameters of the effects of the disturbed solar atmosphere.

Some general features of the disturbed solar atmosphere can be observed from these results. Namely, the stronger initial disturbance gives a stronger temperature enhancement and its mass flow velocity can reach as high as 1000 Km/sec, and a longer duration of the disturbance will sustain the disturbed solar atmosphere, and there will be material falling back to the sun's surface. For example, we have plotted the velocity versus height for $\Delta p = 6$ and $\Delta t = 30$ sec, 120 sec, 1200 sec, and 2400 sec at $t - 40$ min. after explosion. This shows that the negative velocities (i.e., downward velocity) appeared near the surface of the sun for $\Delta t = 30$ sec, 120 sec and 1200 sec and for $\Delta t = 2400$ sec, mass flow velocity just ejects out from the sun's surface all the way. From this evidence, we may suggest that the surge develops due to a short duration disturbance, because, observations show the material falling back to the sun's surface during a surge.

Now, we shall calculate the total energy of the disturbance initially introduced into the solar atmosphere. The total energy can be computed from

$$E = \left(\sum V_i\right) \frac{p}{\gamma + 1} + \frac{1}{2} \left(\sum \rho_i u_i^2\right)$$  \hspace{1cm} (4-1)

where on the right hand side, the first term represents the internal energy and the second term represents the kinetic energy of the gas in a volume $\left(\sum V_i\right)$. The results corresponding to various disturbances are shown in Table II.
Table II
Total Energy Per Cross-Section Area for Each Disturbance
(ergs/Km$^2$)

<table>
<thead>
<tr>
<th>$\Delta p$</th>
<th>$\Delta \tau = 30$ sec</th>
<th>$\Delta \tau = 120$ sec</th>
<th>$\Delta \tau = 1,200$ sec</th>
<th>$\Delta \tau = 2,400$ sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$7.31 \times 10^{17}$</td>
<td>$9.42 \times 10^{18}$</td>
<td>$3.80 \times 10^{20}$</td>
<td>$6.72 \times 10^{20}$</td>
</tr>
<tr>
<td>6</td>
<td>$1.98 \times 10^{17}$</td>
<td>$2.63 \times 10^{18}$</td>
<td>$1.38 \times 10^{20}$</td>
<td>$2.76 \times 10^{20}$</td>
</tr>
<tr>
<td>2</td>
<td>$2.60 \times 10^{16}$</td>
<td>$1.48 \times 10^{17}$</td>
<td>$1.8 \times 10^{19}$</td>
<td></td>
</tr>
</tbody>
</table>

If we consider the cross-section area of a disturbance which has a radius
of ~ 500 Km., it will give a total energy of ~ $5.28 \times 10^{26}$ ergs for the
$\Delta p = 10$ and $\Delta \tau = 2400$ disturbance. This may correspond to the total energy
of a class of sub-flare.

From those density profiles, such as Figures 3a, 4a, 5a, 6a, 7a,
8a, 9a, 10a, and 11a, we can estimate the amount of particles which can be
ejected into the corona (or solar wind), and the results are given in Table III.

Table III
Total Number Particles Per Cross Sectional Area
for Each Disturbance (#/Km$^2$)

<table>
<thead>
<tr>
<th>$\Delta p$</th>
<th>$\Delta \tau = 30$ sec</th>
<th>$\Delta \tau = 120$ sec</th>
<th>$\Delta \tau = 1,200$ sec</th>
<th>$\Delta \tau = 2,400$ sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$4.77 \times 10^{26}$</td>
<td>$5.87 \times 10^{27}$</td>
<td>$1.87 \times 10^{29}$</td>
<td>$3.44 \times 10^{29}$</td>
</tr>
<tr>
<td>6</td>
<td>$2.60 \times 10^{26}$</td>
<td>$3.37 \times 10^{27}$</td>
<td>$1.42 \times 10^{29}$</td>
<td>$2.85 \times 10^{29}$</td>
</tr>
<tr>
<td>2</td>
<td>$5.5 \times 10^{25}$</td>
<td>$7.30 \times 10^{26}$</td>
<td>$3.43 \times 10^{28}$</td>
<td></td>
</tr>
</tbody>
</table>
Again, if we consider the cross-section area of the disturbance being ~500 Km in radius, we find that $2.7 \times 10^{35}$ particles can be added to the corona, which is believed by many to be a reasonable number.

From this study, we have shown that the surge and spray can result from disturbances in the solar atmosphere. After the disturbance has been introduced, the corona may settle into a new equilibrium state. Evidence for this has been reported in some of the observations from the ATM/Skylab experiments.
CHAPTER V

CONCLUSION AND RECOMMENDATIONS

In this investigation, we have examined the upward propagating solar disturbances in a model atmosphere. It was found that the characteristics of the disturbances have dominant effects on the disturbed solar atmosphere. We may conclude from this study that the phenomena of surge and spray can be discriminated by the characteristics of the initial disturbance, as we discussed in the previous chapter. Also, the present model can be used to examine the observed X-ray data from the Skylab mission by relating the X-ray emission to the dynamical responses of the solar atmosphere. The initial disturbances introduced in this study can be either subsonic or supersonic without limitations.

The radiation effects on this problem were examined by using the Cox-Tucker radiation loss function. We found that there is no noticeable difference between the adiabatic calculation and the radiative calculation with the Cox-Tucker radiative loss function. This is due to the fact that the radiative loss function given by Cox-Tucker is decreasing as the temperature is increasing. Therefore, it is necessary to calculate the radiative loss energy from the spectral lines in order to have more accurate results. Also, we have ignored the transport effect in the present analysis.

Finally, we shall outline as follows, the steps which should be taken to improve the present analysis:

(1) Include magnetic field in this model calculation.
(2) Include thermal conduction effects.

(3) A detailed radiative hydrodynamic calculation procedure needs to be considered.
REFERENCES


12. Y. Nakagawa and C. L. Hyder, Private Communication.


LIST OF FIGURES

Figure 1  Cox-Tucker Radiative Loss Function

Figure 2  Flow Chart for Calculation

Figure 3a  Disturbed Density (gm/cc) Versus Height (Km) for Δp = 2.0 and Δt = 120 sec.

Figure 3b  Disturbed Temperature (°K) Versus Height (Km) for Δp = 2.0 and Δt = 120 sec.

Figure 3c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 2.0 and Δt = 120 sec.

Figure 4a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 2.0 and Δt = 1200 sec.

Figure 4b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 2.0 and Δt = 1200 sec.

Figure 4c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 2.0 and Δt = 1200 sec.

Figure 5a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 6 and Δt = 30 sec.

Figure 5b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 6 and Δt = 30 sec.

Figure 5c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 6 and Δt = 30 sec.

Figure 6a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 6.0 and Δt = 120 sec.

Figure 6b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 6.0 and Δt = 120 sec.

Figure 6c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 6.0 and Δt = 120 sec.

Figure 7a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for Δp = 6 and Δt = 2400 sec.
Figure 7b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 6$ and $\Delta \tau = 2400$ sec.

Figure 7c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 6$ and $\Delta \tau = 2400$ sec.

Figure 8a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 30$ sec.

Figure 8b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 30$ sec.

Figure 8c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 30$ sec.

Figure 9a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 120$ sec.

Figure 9b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 120$ sec.

Figure 9c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 120$ sec.

Figure 10a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 1200$ sec.

Figure 10b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 1200$ sec.

Figure 10c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 1200$ sec.

Figure 11a  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 2400$ sec.

Figure 11b  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 2400$ sec.

Figure 11c  Disturbed Flow Velocity (Km/sec) Versus Height (Km) for $\Delta p = 10$ and $\Delta \tau = 2400$ sec.

Figure 12  Velocity Profile for $\Delta p = 6.0$ and $\Delta \tau = 30$ sec. $1200$ sec. and $2400$ sec. at time being $40$ min. after the explosion.
Radiative Cooling Rate of an
Optically Thin, High Temperature Plasma

after Cox and Tucker (1969)
FIGURE 2. FLOW DIAGRAM

Start
Read, TSTOP, T

Institute Variables
\( t, \rho^n_i, T^n_i, u^n_i \)

Specify lower boundary \( \rho^*, T^*, u^* \)
\( \Delta t \leq \min \frac{0.25 \Delta t}{|u^n_i| + a^n_i} \)

Calculate old fluxes
\( F^n_i, \bar{F}^n_i \)

Intermediate
Step
Calculate intermediate variables
\( \rho^{n+1}_i, \bar{F}^{n+1}_i, \bar{u}^{n+1}_i \)

Calculate intermediate fluxes
\( \bar{F}^{n+1}_i, \bar{F}^{n+1}_i \)

Final Step
yes

Calculate new variables
\( \rho^{n+1}_i, T^{n+1}_i, u^{n+1}_i \)

except boundaries

Write
\( \rho^{n+1}_i, T^{n+1}_i, u^{n+1}_i \)

Calculate upper boundary variables
using interpolation
\( t - t + \Delta t \)

Lower boundary
\( \rho^{n+1}_1 = \rho^n_1 \)
\( T^{n+1}_1 = T^n_1 \)
\( u^{n+1}_1 = u^{n+1}_1 / \rho^{n+1}_1 \)

no

STOP

TSTOP >
FIG. 3-a
TEMPERATURE

$\Delta T = 120 \text{ sec}$
$\Delta p = 20$

Solar Radius

FIG. 3-b
VELOCITY

$\Delta \tau = 120\ sec$
$\Delta p = 20$

Adiabatic

$\rho^* = 1.6$
$T^* = 1.3$
$\rho^* = 2.0$

FIG. 3 - c
\( \Delta p = 2 \)
\( \Delta \tau = 1200 \text{ sec} \)
TEMPERATURE

\[ \Delta p = 2 \]
\[ \Delta T = 1200 \text{ sec} \]

FIG. 4-b
VELOCITY

$\Delta T = 1200 \text{ sec}$

$\Delta p = 2.0$

FIG. 4-c
DENSITY

$\Delta p = 6$
$\Delta \tau = 30 \text{ sec}$

Solar Radius

FIG. 5-a
TEMPERATURE

FIG. 5-b
VELOCITY

$\Delta T = 30 \text{ sec}$

$\Delta \rho = 6.0$

**FIG.-5-c**
\[ \Delta p = 6.0 \]
\[ \Delta \tau = 120 \text{ sec} \]
TEMPERATURE

FIG. 6-b
VELOCITY

\[ \tau = 120 \text{ sec} \]
\[ p = 6.0 \]

FIG. 6-c
\[ \Delta p = 6.0 \]
\[ \Delta T = 2400 \text{ sec} \]

**FIG. 7-a**

**DENSITY**

\[ 10^{-1} \]
\[ 10^{-2} \]
\[ 10^{-3} \]
\[ 10^{-4} \]
\[ 10^{-5} \]
\[ 10^{-6} \]
\[ 10^{-7} \]

GM/CC

Solar Radius

10  20  40  50
30 min.
TEMPERATURE

FIG. 7-b
VELOCITY

\[ \Delta T = 2400 \text{ sec} \]
\[ \Delta p = 6.0 \]

FIG. 7-c
\[ \Delta p = 10 \]
\[ \Delta \tau = 30 \text{ sec} \]
TEMPERATURE

$10^7$

$10^6$

$10^5$

Solar Radius

FIG. 8-b
Delta T = 30 sec
Delta p = 10
DENSITY

\[ \Delta p = 10 \]
\[ \Delta \tau = 1200 \text{ sec} \]

FIG. 9-a
\[ \Delta T = 120 \text{ sec} \]
\[ \Delta p = 10 \]
FIG. 10-a

DENSITY

$\Delta p = 10$
$\Delta T = 1200$

$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
$10^{-8}$

Solar Radius

GM/cc

0.043 0.49 0.95 1.40 1.85 2.30

FIG. 10-a
TEMPERATURE

FIG. 10-b
\[ \Delta \tau = 1200 \text{ sec} \]
\[ \Delta p = 10 \]

**FIG. 10-c**

**VELOCITY**

- Solar Radius vs. KM/SEC
- Graph showing velocity changes over solar radius with labeled points (8, 16, 24, 32, 40 min.)
DENSITY

\[ \Delta p = 10 \]
\[ \Delta \tau = 2400 \text{ sec} \]
TEMPERATURE

Solar Radius

FIG. II-b
\[ \Delta \tau = 2400 \]
\[ \Delta p = 10 \]
$p = 6.0$

$\tau = 40 \text{ min.}$
APPENDIX A

Derivation of Conserved Form of Equations

The governing equations for the present problem written in spherical coordinates are

Continuity;
\[ \frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial r} (\rho u) - \frac{2\rho u}{r} . \] (A-1)

Momentum;
\[ \frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial r} - \frac{1}{\rho} \frac{\partial P}{\partial r} - g . \] (A-2)

Energy;
\[ \rho \frac{\partial \varepsilon}{\partial t} = -\rho u \frac{\partial \varepsilon}{\partial r} + \frac{P}{\rho} \left( \frac{\partial P}{\partial r} + u \frac{\partial \rho}{\partial r} \right) - Q_R, \] but by virtue of Equation (A-1), this can be written
\[ \rho \frac{\partial \varepsilon}{\partial t} = -\rho u \frac{\partial \varepsilon}{\partial r} - p \frac{\partial u}{\partial r} - p \frac{2u}{r} - Q_R \] (A-3)

Continuity equation (A-1) is already in conservational form. Multiplying \( \rho \) to equation (A-2) and rearranging with the aid of equation (A-1), we have momentum conservational form of equations,
\[ \frac{\partial (\rho u)}{\partial t} = \frac{\partial}{\partial r} \left( \rho u^2 + p \right) - \rho g - \frac{2\rho u^2}{r} , \] (A-4)

noting however \( p = (\gamma-1)E - \frac{(\gamma-1)}{2} \rho u^2 \) by definition, equation (A-4) becomes
\[ \frac{\partial (\rho u)}{\partial t} = -\frac{\partial}{\partial r} \left[ \left( \gamma-1 \right)E - \frac{(\gamma-3)}{2} \rho u^2 \right] - \rho g - \frac{2\rho u^2}{r} \] (A-5)

Energy conservational form is found by adding four equations, i.e., \( \varepsilon \times \) eqn. (A-1), \( \frac{pu}{2} \times \) eqn. (A-2), \( \frac{u}{2} \times \) eqn. (A-4) and eqn. (A-3).
Collecting $\partial/\partial t$ terms, we have after some simplification,

$$\frac{\partial}{\partial t} \left( \rho e + \frac{\rho u^2}{2} \right) = \frac{\partial E}{\partial t} \quad \text{(A-6)}$$

Collecting $\partial/\partial r$ terms and other remaining terms, we get, after some simplifications,

$$- \frac{\partial}{\partial r} \left[ u(p + \rho e + \frac{\rho u^2}{2}) \right] - u \rho g - \frac{2u}{r} \left[ \rho e + \frac{\rho u^2}{2} + p \right] = Q_R - \rho gu$$

$$= - \frac{\partial}{\partial r} \left[ u (\gamma e - \frac{(\gamma-1)}{2} \rho u^2) \right] - \rho gu - Q_R - \frac{2}{r} \left[ u(rE - \frac{(\gamma-1)}{2} \rho u^2) \right] \quad \text{(A-7)}$$

Thus, energy conservational form of equation is

$$\frac{\partial E}{\partial t} = - \frac{\partial}{\partial r} \left[ u(\gamma e - \frac{(\gamma-1)}{2} \rho u^2) \right] - \rho gu - Q_R - \frac{2}{r} \left[ u(\gamma E - \frac{(\gamma-1)}{2} \rho u^2) \right] \quad \text{(A-8)}$$
APPENDIX B

COMPUTER LISTING

1O ONE DIMENSIONAL MOVING SHOCK THROUGH THE SOLAR ATMOSPHERE

39 95 INVESTIGATED BY UTILIZING FINITE DIFFERENCE TECHNIQUE

4o BASED ON THE LAH-MEYERDOFF CONSERVATIONAL LAW

59 THE FINAL DIFFERENCE SCHEME IS BASED ON RUBIN-BURSTEIN'S METHOD.

72 PARAMETER IMAX = 40

82 PARAMETER JMAX = 5

92 REAL K1, K2, K3

10 REAL K1H, K2H, K3H


12 COMMON/BLOC1/IM1, IM2

13 COMMON/BLOC2/GAM1, GAM2, GAM3, GSR5, C1, R

14 COMMON/BLOC3/RX(IMAX), X(IMAX), PHI(IMAX), RXS0(IMAX)

15 COMMON/BLOC4/F1(IMAX), F2(IMAX), F3(IMAX), K2(IMAX), K3(IMAX)

16 1S K1(IMAX)

17 COMMON/BLOC5/UI(IMAX), U2(IMAX), U3(IMAX)

18 COMMON/BLOC6/UI(IMAX), TE(IMAX)

19 COMMON/BLOC7/UI(IMAX), U2D, U3D, U1D

20 COMMON/BLOC8/TEC(IMAX), U1C(IMAX)

21 COMMON/BLOC9/UIP(IMAX), U3P(IMAX), TEP(IMAX), IMAX)

22 COMMON/BLOC10/CX(IMAX), CX(IMAX)

23 COMMON/BLOC11/UA1(IMAX), U2A(IMAX), U3A(IMAX), U1B(IMAX)

24 COMMON/BLOC12/FLA(IMAX), F2A(IMAX), F3A(IMAX), FIB(IMAX)

25 COMMON/BLOC12/F3A(IMAX)

26 COMMON/BLOC11/U1A(IMAX), U2A(IMAX), U3A(IMAX)

27 COMMON/BLOC11/IMAX)

28 COMMON/BLOC10/IMAX)

29 COMMON/BLOC11/IMAX)

30 COMMON/BLOC12/IMAX)

31 COMMON/BLOC11/IMAX)

32 COMMON/BLOC12/IMAX)

33 COMMON/BLOC12/IMAX)

34 COMMON/BLOC12/IMAX)

35 COMMON/BLOC12/IMAX)

36 COMMON/BLOC13/U1H(IMAX), U2H(IMAX), U3H(IMAX), TEH(IMAX)

37 COMMON/BLOC14/K1H(IMAX), K2H(IMAX), K3H(IMAX)

38 COMMON/BLOC15/U3H(IMAX), U2H(IMAX), U3H(IMAX)

39 COMMON/BLOC16/DT

40 COMMON/BLOC26/C(IMAX)

41 COMMON/BLOC27/E1(IMAX)

42 COMMON/BLOC28/K1A(IMAX), K1B(IMAX), K2A(IMAX), K2B(IMAX), K3A(IMAX)

43 COMMON/BLOC33/U3C(IMAX), UUC(IMAX)

44 COMMON/BLOC42/QR(IMAX)

45 COMMON/BLOC43/QPC(IMAX)

46 COMMON/BLOC5C/UT

47 GAM = SPECIFIC HEAT RATIO

48 RS = RADIUS OF THE SUN

49 HP = HEIGHT OF THE PROMINENCE

50 HC = HEIGHT OF THE CHROMOSPHERE

51 GS = GRAVITATIONAL ACCELERATION AT THE SURFACE OF THE SUN

52 DX = SPATIAL INCREMENT

53 U1 = VELOCITY

54 U2 = MOMENTUM DENSITY

55 UU = VELOCITY

56 XR = SPATIAL COORDINATE MEASURED UPWARD

57 ZS = SPATIAL COORDINATE MEASURED DOWNWARD

58 SOT = SHOCK DURATION TIME AT THE LOWER BOUNDARY.

59 READ (5,1) ISTOP
58. READ (5,66) (X(I),TE(I),U1(I),I=1,IMAX)
59. IM1=IMAX-1
60. IM2=IMAX-2
61. GAM=5.*3.*
62. GAM1=GAM-1.*
63. GAM3=3.*GAM
64. RS=+.695E+6
65. HP=2.*E5
66. HC=3.*E4
67. GS=.274
68. R=+.83E-2
69. DTP=480.*
70. SDT=6.*Q
71. C1=GAM1/R
72. GSRS=GS*RS**2
73. U10=4.*14713E8
74. C
75. C  INITIALIZATION
76. C
77. RX(I)=PS+HC
78. T=0.*
79. TP=0.*
80. DO 20 I=1,1
81. RX(I)=X(I)+RS
82. RXSQ(I)=RX(I)**2
83. 20 CONTINUE
84. DO 77 I=2,IMAX
85. X(I)=X(I-1)+2.*CUW.
86. RX(I)=X(I)+RS
87. RXSQ(I)=RX(I)**2
88. TE(I)=1.53E6
89. UI(I)=UI0*EXP((GSRS/(R*TE(I)))*(1./RX(I)-1./RX(1)))
90. 77 CONTINUE
91. DO 60 I=1,IMAX
92. CON=2.*E-12
93. U1(I)=CON*U1(I)
94. UU(I)=0.*
95. U2(I)=U1(I)+UU(I)
96. PH1(I)=GSRS/RX(I)
97. U3(I)=U1(I)*TE(I)/C1+5.*UU(I)**2
98. C(I)=SORT(GAM*R*TE(I))
99. EI(I)=U1(I)*TE(I)/C1
100. UIC(I)=U1(I)
101. U3C(I)=U3(I)
102. TEC(I)=TE(I)
103. 60 CONTINUE
104. DO 111 I=2,IMAX
105. HX(I)=GSRS/(R*TE(I))**(X(I+1)-X(I))/RXSQ(I)
106. CK(I)= (HX(I)**5.*EXP(-HX(I))-EXP(HX(I)))/(X(I+1)-X(I))
107. 111 CONTINUE
108. WRITE (6,100) T
109. WRITE (6,105) (X(I),I=1,IMAX)
110. WRITE (6,110) (U1(I),I=1,IMAX)
111. WRITE (6,115) (U1(I),I=1,IMAX)
112. WRITE (6,120) (UU(I),I=1,IMAX)
113. WRITE (6,125) (UU(I),I=1,IMAX)
114. WRITE (6,130) (UU(I),I=1,IMAX)
115. WRITE (6,135)
C BACKGROUND CORONA RADIATION IS DETERMINED BY TUCKER'S EQN.
C RADIATION EFFECTS ARE INTRODUCED BY INCLUDING RADIATION EQN
C OBTAINED BY FITTING OF TUCKER'S RESULTS.

C

DO 62 I=1,IMAX
   IF (TE(I)*GT.5*E4) GO TO 11
      C2=1.6E10
      C3=3.59
   GO TO 79
   IF (TE(I)*GT.2.5E5) GO TO 12
      C2=3.6E26
      C3=0
   GO TO 79
   IF (TE(I)*GT.7.6E6) GO TO 13
      C2=1.5E32
      C3=-1.172
   GO TO 79
   C2=1.6E23
   C3=0.288
   79 QRC(I)=C2* (UI(1)*
   2I
   *(TE(I)+C3)
   CONTINUE

C PLOTTING OF THE INITIAL VALUES OF THE VARIABLES...

C

CALL PLOT (U,UI,UU,TE)
   TP=TP+DTP
   U11=U1(1)
   U21=U2(1)
   U31=U3(1)
   TE1=TE(I)

C UPWARD PROPAGATING SHOCK SIMULATION IN TERMS OF DENSITY,
C ENERGY, TEMPERATURE GRADIENT
C REFLECTED SHOCK MACH NUMBERS ARE ASSUMED TO BE KNOWN
C AT THE LOWER BOUNDARY, THEN THE JUMP CONDITIONS ARE FOUND BY
C RANKINE-HUGONIOT RELATIONS,

C

U10=U1(1)*2
   UU0=0.
   U20=U10*UU0
   TE0=TE(I)
   U30=U10* (TE0/C1**S*UU0**2)

C

K1=1
   U1(1)=U10
   UU(1)=UU0
   U2(1)=U20
   U3(1)=U30
174* \( \text{TE}(1) = \text{TE} \)
175* \( C(1) = \sqrt{\text{GAM} \times \text{TE}(1)} \)
176* \( E(1) = U(1) \times \text{TE}(1)/C \)
177* 70 \text{ CONTINUE}
178* C PREDICTOR STEP
179* C CALL DTIME \((UU, C)\)
180* IF \((DT \times LT > 0.01)\) \text{ GO TO 200}
181* IF \((T < 3.0)\) \(DT = 0.01\)
182* SDD = SDD + 30.
183* IF \((\text{T} \times \text{GT} \times \text{SDD} \times \text{AND} + \text{T} \times \text{LT} \times \text{SDD})\) \(DT = 1.0\)
184* CALL FLUX1 \((U1, U2, U3, \text{TE}1, UU, C)\)
185* CALL EQN1 \((U1, U2, U3, F1, F2, F3, K2, K3, DT, CX, HX, K1)\)
186* C CALL FLUX2 \((U1, U1B, U2A, U2B, U3A, U3B, U1H, U2H, U3H, \text{TE}1, UU, C)\)
187* I = I + DT
188* C CORRECTOR STEP
189* C CALL EQN2 \((U1, U2, U3, F1, F2, F3, K2, K3, F1A, F2A, F3A, K1, K1H, E1, K3B)\)
190* T = T + DT
191* C REFLECTED SHOCK SIMULATION IN TERM OF THE KNOWN MACH NUMBER
192* C CALL BOUND
193* IF \((\text{KW} \times \text{GT} \times \text{C})\) \text{ GO TO 555}
194* \text{ GO TO 666}
195* 555 \text{ CONTINUE}
196* WRITE \((6, 10) T\)
197* WRITE \((6, 125) U1(I)\)
198* WRITE \((6, 116) (U1(I), I = 1, IMAX)\)
199* WRITE \((6, 125) U3\)
200* WRITE \((6, 116) (U3(I), I = 1, IMAX)\)
201* WRITE \((6, 125) \text{TE}\)
202* WRITE \((6, 116) (\text{TE}(I), I = 1, IMAX)\)
203* WRITE \((6, 125) EI\)
204* WRITE \((6, 116) (EI(I), I = 1, IMAX)\)
205* DO 404 \(I = 1, IMAX\)
206* 404 \text{ CONTINUE}
207* KW = 3
208* 666 \text{ CONTINUE}
209* KW = KW + 1
210* C PLOTTING OF THE PERTURBED VARIABLES
211* C IF \((\text{T} \times \text{GT} \times \text{TP})\) \text{ GO TO 150}
212* \text{ GO TO 160}
213* 150 \text{ CONTINUE}
214* \text{ CALL PLOT } (I, U1, UU, TE)
215* TP = TP + \text{DT}
216* 160 \text{ IF } (\text{T} \times \text{GT} \times \text{TSTOP}) \text{ GO TO 200}
217* \text{ GO TO 70}
700 CONTINUE
    K=I
DO 190 I=1,IMAX
X(K)=X(I)
K=K+1
190 CONTINUE

THE FOLLOWING IS TO RECORD THE DATAS ON THE TAPE:

DO 210 J=1,JMAX
DO 210 I=1,IMAX
IF (I.EQ.IMAX) X(I)=1.E37
IF (I.EQ.IMAX) UIP(J,I)=1.E37
IF (I.EQ.IMAX AND J.EQ.JMAX) X(I)=9.E37
IF (I.EQ.IMAX AND J.EQ.JMAX) UIP(J,I)=9.E37
WRITE (9) X(I),UIP(J,I)
210 CONTINUE
DO 300 J=1,JMAX
DO 300 I=1,IMAX
IF (I.EQ.IMAX) X(I)=1.E37
IF (I.EQ.IMAX) UUP(J,I)=1.E37
IF (I.EQ.IMAX AND J.EQ.JMAX) X(I)=9.E37
IF (I.EQ.IMAX AND J.EQ.JMAX) UUP(J,I)=9.E37
WRITE (9) X(I),UUP(J,I)
300 CONTINUE
DO 400 J=1,JMAX
DO 400 I=1,IMAX
IF (I.EQ.IMAX) X(I)=1.E37
IF (I.EQ.IMAX) TEP(J,I)=1.E37
IF (I.EQ.IMAX AND J.EQ.JMAX) X(I)=9.E37
IF (I.EQ.IMAX AND J.EQ.JMAX) TEP(J,I)=9.E37
WRITE (9) X(I),TEP(J,I)
400 CONTINUE

STOP

FORMAT (E15.4)
FORMAT (2F15.4,E15.6)
FORMAT (1H1)
FORMAT (1X,9E11.4)
FORMAT (1H1)

END

NO OF COMPIILATION: NO DIAGNOSTICS.
SUBROUTINE FLUXI (U1, U2, U3, TE, UUIC)
PARAMETER IMAX=4
REAL K1, K2, K3
COMMON/BLOC1/IM1, IM2
COMMON/BLOC2/GAM1, GAM2, GAM3, GSRS, CI, R
COMMON/BLOC3/PHI(MAX), RX(MAX), RXS0(MAX)
COMMON/BLOC4/F1(MAX), F2(MAX), F3(MAX), K2(MAX), K3(MAX)
COMMON/BLOC8/TEC(MAX), U1C(IMAX)
COMMON/BLOC16/DT
COMMON/BLOC42/QR(IMAX), F2(IMAX), F3(IMAX)
COMMON/RLOCB/TEC(IMAX), IUC(IMAX)
COMMON/BLOC3/DT
COMMON/RLOC42/QR(IMAX)
DIMENSION UI(IMAX), U2(IMAX), U3(IMAX)
DIMENSION UUIC(IMAX), IC(IMAX), QD(IMAX), DM(IMAX)
DIMENSION UD(IMAX)

C2 AND C3 ARE CONSTANTS USED IN TUCKER'S RADIATION EQN.
CO=CONVERSION FACTOR
AC=3.
RADIATION EFFECTS ARE INTRODUCED BY INCLUDING RADIATION EQN.
OBTAINED BY FITTING OF TUCKER'S RESULTS.
DO 6 I=1, IMAX
6 IF (TE(I)> 5. E4) GO TO 11
C2 = 1. E10
C3 = 3. 55
GO TO 77
IF (TE(I) > 2. 5E6) GO TO 12
C2 = 3. E26
C3 = 0
GO TO 77
IF (TE(I) > 7. E6) GO TO 13
C2 = 1. E32
C3 = -1. 172
GO TO 77
C2 = 1. E23
C3 = 0. 28
QRI = C2*(U1(I)*C2)*(TE(I)*C3)*CO.
CONTINUE
DO 88 I=2, IM1
88 CONTINUE
QRI = -OR(I) + 5*OR(I+1) + 5*OR(I-1) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(MAX) + 5*OR(M
59. C
60. DO 50 I=2,1M1
61. DM(I)=UU(I-1)/C(I-1)-UU(I)/C(I)
62. IF (DM(I) GT 1.0) GO TO 80
63. QD(I)=0.
64. GO TO 50
65. 80 CONTINUE
66. SM=DM(1)
67. PAI=(2.*GAM/(GAM+1.))*(SM+2)-(GAM)/(GAM+1))
68. SIG=(GAM+1.)*(SM+2)/(GAM+SM+2+2.)
69. ZETA=SQRT(PAI/SIG)
70. QD(I)=U1(I-1)*C(I-1)*ZETA*(PAI-1.)/(1./SIG+1.)
71. I=(GAM/GAM1.)*(ZETA+2+1.)*7/(DT*GAM)
72. QD(I)=ABS(QD(I))
73. 50 CONTINUE
74. QD(IMAX)=O.
75. C
76. DO 10 I=1,IMAX
77. F1(I)=U2(I)
78. F2(I)=GAM1*U3(I)+(GAM3*U2(I)*2/U1(I))
79. 1+5*(AC*2)+U1(I)+(UD(I)**2)
80. F3(I)=(U2(I)/U1(I))+(GAM1*U3(I)-5*GAM1*U2(I)**2/U1(I)
81. 1+5*(AC*2)+U1(I)+(UD(I)**2)
82. K1(I)=-2.*U2(I)/RX(I)
83. K2(I)=U1(I)*GSRS/RXSQ(I)
84. 1+2.*U2(I)**2/RX(I)*U1(I))
85. K3(I)=-U2(I)*GSRS/RXSQ(I)
86. 2+2.*U2(I)**2/(GAM3*U3(I)-GAM1*U2(I)**2/U1(I)
87. 3+5*(AC*2)+U1(I)+(UD(I)**2))/(RX(I)*U1(I))
88. 4+9D(I)
89. 10 CONTINUE
90. RETURN
92. END

ID OF Compilation: NO DIAGNOSTICS.

SUBROUTINE EQN1 (U1,U2,U3,F1,F2,F3,K2,K3,DT,CX,HX,K1)
PARAMETER IMAX=40
REAL K1,K2,K3
COMMON/BLOC1/IM1,IM2
COMMON/BLOC2/GAM3,GAM1,GAM3,GSRS,CL1,R
COMMON/BLOC3/RX(1,IMAX),C(1,IMAX),PH1(IMAX),RXSQ(IMAX)
COMMON/BLOC4/U1A(IMAX),U2A(IMAX),U3A(IMAX),UIA(IMAX),
1U2B(IMAX),U3B(IMAX)
COMMON/BLOC5/F1A(IMAX),F2A(IMAX),F3A(IMAX),F1R(IMAX),
F2R(IMAX),F3R(IMAX)
COMMON/BLOC6/U1H(IMAX),U2H(IMAX),UHH(IMAX),TH(EIMAX)
COMMON/BLOC7/K1H(IMAX),K2H(IMAX),K3H(IMAX)
COMMON/BLOC8/U3H(IMAX),U4H(IMAX),UUB(IMAX)
DIMENSION HX(IMAX),CX(IMAX)
DIMENSION F1(IMAX),F2(IMAX),F3(IMAX),K2(IMAX),K3(IMAX),K1(IMAX)
DIMENSION U1(IMAX),U2(IMAX),U3(IMAX)
DO 52 I=2, IM1
10 U1A(I)=5*(U1(I)+1)+U1(I)-(DT/X(I+1)-X(I))*F1(I+1)-F1(I)
11 IF2B(IMAX),F3B(IMAX),F1B(IMAX)
20 U2A(I)=5*(U2(I+1)+U2(I)-(DT/X(I+1)-X(I))*F2(I+1)-F2(I)
21 U3A(I)=5*(U3(I+1)+U3(I)-(DT/X(I+1)-X(I))*F3(I+1)-F3(I)
22 U1B(I)=5*(U1(I)+1)+U1(I)-(DT/X(I))+F1(I-I)
23 U2B(I)=5*(U2(I)+1)+U2(I)-(DT/X(I)+F2(I-I)
24 U3B(I)=5*(U3(I)+1)+U3(I)-(DT/X(I)+F3(I-I)
25 U1H(I)=5*(U1(I)+1)+U1(I-I)-(DT/X(I+1)+X(I-I))*F1(I+1)
26 U2H(I)=5*(U2(I)+1)+U2(I-I)-(DT/X(I+1)+X(I-I))*F2(I+1)
27 U3H(I)=5*(U3(I)+1)+U3(I-I)+(DT/X(I+1)+X(I-I))*F3(I+1)
28 UAA(I)=U2A(I)/U1A(I)
29 UAB(I)=U2B(I)/U1B(I)
30 UAH(I)=U2H(I)/U1H(I)
31 THER(I)=C1*(U3H(I)/UH(I)) +5*UHH(I)*D2
32 UHH(I)=U2H(I)/U1H(I)
33 RETURN
34 END

10 OF COMPILATION: NO DIAGNOSTICS.
SUBROUTINE EQN2(U1,U2,U3,F1,F2,F3,K1,K2,K3,F1A,F2A,F3A,K1A,K2A,K3A)

PARAMETER (MAX = 2)

REAL K1,K2,K3
REAL K1A,K2A,K3A

COMMON/BLOC1/IMAX,IMAX1,IMAX2,IMAX3
COMMON/BLOC2/GAM2,GAM1,GAM3,GSRS,C1,IDX
COMMON/BLOC3/RX(IMAX),X(IMAX),PHI(IMAX)
COMMON/BLOC4/Ut(IMAX),U3CC(IMAX)
COMMON/RLOC1/Te(IMAX),Te(IMAX)
COMMON/RLOC2/Ut(IMAX),Te(IMAX)

DIMENSION FI(IMAX),F2(IMAX),F3(IMAX),K2(IMAX),K3(IMAX)

10 CONTINUE
RETURN
END
PROGRAM"Il-talk"
DIMENSION UI(IMAX),U2(IMAX),U3(IMAX)
DIMENSION E1(IMAX)
DIMENSION E1T(IMAX),E1E(IMAX)

DO 48 I=2,IMAX
FIT(I)=EI(I)
DO 50 I=2,IMAX

FIT(I)=(E1T(I)-E1(I))/E1T(I)
IF (E1E(I)-E1T(I)-E1(I)) .LT. C*50) GO TO 47
E1(I)=E1(I)+5*E1T(I)
TE(I)=E1(I)+C1/U1(I)
U3(I)=U1(I)+(TE(I)/C1+5*U(U(I)+2)

CONTINUE

C
DO 47 I=2,IMAX
E1E(I)=(E1T(I)-E1(I))/E1T(I)
IF (E1E(I)-E1T(I)-E1(I)) .LT. C*50) GO TO 47
E1(I)=E1(I)+5*E1T(I)
TE(I)=E1(I)+C1/U1(I)
U3(I)=U1(I)+(TE(I)/C1+5*U(U(I)+2)

CONTINUE

C
RETURN
END

OF COMPILATION: NO DIAGNOSTICS.
SUBROUTINE FIOUN

PARAMETER IMAX=40
COMMON/BLOC1/IM1,IM2
COMMON/BLOC2/GAM,GAM1,GAM3,GSRS,C1,R
COMMON/BLOC5/UI(IMAX),U2(IMAX),U3(IMAX)
COMMON/BLOC6/U2C,TGC,U2O,UUG,U1O
COMMON/BLOC7/UI,U21,UUGI,U31,TE1
COMMON/BLOC8/TEG(IMAX),UIC(IMAX)
COMMON/BLOC9/TEC(IMAX),UIC(IMAX)
COMMON/BLOC33/U3C(IMAX),UUG(IMAX)
COMMON/BLOC30/TE(IMAX)
COMMON/BLOC31/T
SDT=600
IF (T GT SDT) GO TO 101
UI(1)=U10
U2(1)=U2(2)
U2(1)=U2(1)/U1(1)
TE(1)=TEG
E1(1)=U1(1)*TE(1)/CI
U3(1)=U1(1)*TE(1)/CI*S*U2(1)**2
GO TO 151
101 CONTINUE
UI(1)=U11
TE(1)=TE1
U2(1)=0.
U1(1)=U1(1)/U1(1)
E1(1)=U1(1)*TE(1)/CI
U3(1)=U1(1)*TE(1)/CI*S*U2(1)**2
151 CONTINUE
C  IF (U1(1M1), GT, 2O) GO TO 150
UI(IMAX)=UIC(IMAX)
U1(IMAX)=UUC(IMAX)
U2(IMAX)=UI(IMAX)+U1(IMAX)
TE(IMAX)=TEC(IMAX)
U3(IMAX)=U3C(IMAX)
E1(IMAX)=U1(IMAX)*TE(IMAX)/CI
GO TO 105
150 U1(IMAX)=U1(IM1)
U2(IMAX)=U2(IM1)
U3(IMAX)=U3(IM1)
U1(IMAX)=U1(IM1)
*TE(IMAX)=TE(IM1)
E1(IMAX)=E1(IM1)
105 CONTINUE
RETURN
END

D OF COMPI LATION: NO DIAGNOSTICS.
SUBROUTINE PLOT (IFLAG, UI, UU, TE)
PARAMETER IMAX = 4
PARAMETER JMAX = 5
COMMON/BLOC9/UIP(JMAX,IMAX),UUU(JMAX,IMAX),TEP(JMAX,IMAX)
DIMENSION UI(IMAX), UU(IMAX), TE(IMAX)
IF (IFLAG .EQ. 1) GO TO 20
J = 1
K = 1
20 CONTINUE
DO I = 1, IMAX
UIP(J, K) = UI(I)
UUP(J, K) = UU(I)
TEP(J, K) = TE(I)
K = K + 1
10 CONTINUE
J = J + 1
K = 1
RETURN
END

SUBROUTINE DTIME (UU, C)
PARAMETER IMAX = 4
COMMON/BLOC1/IM1, IM2
COMMON/BLOC2/GAM1, GAM1, GAM3, GSRS, C1, R
COMMON/BLOC3/RX(IMAX), X(IMAX), PHI(IMAX), RXSQ(IMAX)
DIMENSION UU(IMAX), C(IMAX)
DIMENSION DDT(IMAX)
DO I = 1, IMAX
DDT(I) = 25 * (X(I+1) - X(I)) / (ABS(UU(I)) + C(I))
10 CONTINUE
HDT = DDT(1)
DO 20 I = 2, IMAX
DO 20 QDT = DDT(I)
20 CONTINUE
IF (HDT * GT, QDT) GO TO 30
DO 30 I = 2, IMAX
HDT = QDT
30 CONTINUE
DT = HDT
RETURN
END

ID OF COMPILATION: NO DIAGNOSTICS.
APPENDIX C

Presentations

During the period of performance of this contract, the following papers were presented.


(2) "Hα Flares: The Response of the Chromosphere to a Downward Shock Wave," Annual Meeting of the Solar Physics Division, American Astronomical Society, University of Maryland, April 4-6, 1972, with S. M. Han and Y. Nakagawa.

