AN ANALYSIS OF FRACTURE TRACE PATTERNS IN AREAS OF FLAT-LYING SEDIMENTARY ROCKS FOR THE DETECTION OF BURIED GEOLOGIC STRUCTURE

(MELVIN H. PODWYSOCKI)

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
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ABSTRACT 

Two study areas in a cratonic platform underlain by flat-lying sedimentary rocks were analyzed to determine if a quantitative relationship exists between fracture trace patterns and their frequency distributions and subsurface structural closures which might contain petroleum. Fracture trace lengths and frequency (number of fracture traces per unit area) were analyzed by trend surface analysis and length frequency distributions also were compared to a standard Gaussian distribution. Composite rose diagrams of fracture traces were analyzed using a multivariate analysis method which grouped or "clustered" the rose diagrams and their respective areas on the basis of the behavior of the rays of the rose diagram. 

Analysis indicates that the lengths of fracture traces are log-normally distributed according to the mapping technique used in this paper. Deviations from log-normality may be associated with both reef (passive) structures whose "closure" is caused by differential compaction of sediments over the reefs and with basement uplift (active) anticlinal structures. The primary control of fracture trace frequency and log-mean lengths is associated with variations in surficial lithology. This variation may be extracted using trend surfaces and the residuals may be analyzed. Fracture trace frequency appeared higher on the flanks of active structures and lower around passive reef structures. Fracture trace log-mean lengths were shorter over several types of structures, perhaps due to increased fracturing and subsequent erosion. 

Analysis of rose diagrams using a multivariate technique indicated lithology as the primary control for the lower grouping levels. Groupings at higher levels indicated that areas overlying active structures may be isolated from their neighbors by this technique while passive structures showed no differences which could be isolated.
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AN ANALYSIS OF FRACTURE TRACE PATTERNS IN AREAS OF FLAT-LYING SEDIMENTARY ROCKS FOR THE DETECTION OF BURIED GEOLOGIC STRUCTURE

INTRODUCTION

Although linear features on the earth’s surface had long been mapped solely on topographic and geologic criteria (Hobbs, 1911; Brock, 1957), more of these subtle features became apparent as aerial photographic coverage became available (Rich, 1928). Since then, airphoto linears have been applied to a wide range of topics such as groundwater studies (Lattman and Parizek, 1964; Siddiqui and Parizek, 1971), mineralization (Keim, 1962; Kutina, 1969) and engineering studies (Parizek and Voight, 1970; Parizek, 1971; Alpay, 1973; Benedict and Thompson, 1973). Linears observable on various scales of aerial photographs and topographic maps have been utilized extensively in regional tectonic studies (Plafker, 1964; Gol’braikh et al., 1968a; Gold et al., 1974). Although several investigators claim that analysis of airphoto linears will allow exploration for geologic structures which may bear petroleum (Permyakov, 1949, 1954; Blanchet, 1957; Mollard, 1957), few exploration techniques have been divulged due to their proprietary nature. This paper will discuss some parameters which can be extracted from an airphoto linear study for the purposes of exploration for several types of oil and gas traps.

NOMENCLATURE

The terms "airphoto linears" or "linears" were used above in order to circumvent the variety of names and non-systematic nomenclature for these topographic and photographic expressions. Barton (1933) used the term "topographic lines" and Gross (1951) used "topographic linears." Although their maps showed they did limit the size of the observed features, no comment was made concerning the distribution of their individual lengths. Only recently has attention been paid to the scale of observations and size of the features (Nemec, 1970; Gold et al., 1974) and until the advent of satellite imagery, there was no convenient format for direct observations of the large features.

Blanchet (1957) categorized his observations on linears observed on aerial photographs as "micro- and macrofractures," dividing the two categories at 2.5 miles (4 km). He claimed, but offered no proof, that microfractures (0.5 - 2.5 miles (0.8 - 4 km)) in length are intrinsic to the sediments themselves whereas macrofractures (greater than 2.5 miles (4 km)) are related to deep seated basement features. Because similar orientations prevailed in different parts of the world, he claimed that the fractures were related to a worldwide tectonic pattern.
Mollard (1957) used the term "lineament" to classify aerial photographic linears. His classification allowed the use of both continuous and discontinuous features ranging from 0.2 - 5 miles (0.3 - 8 km) in length. He too considered them related to global tectonics.

Gol’braikh et al. (1968a,b) use the term "megajoint," which they adopted because of its relationship in hierarchy to other scales of jointing (i.e. micro- and macrojointing) and to the analytical techniques which could be applied regardless of scale. The term megajoint is based on the scale of maps or aerial photographs used and the minimum length (1 cm) which they believe can be precisely measured to determine the bearing of a megajoint. Their published works indicate a range of 1 to 6 km with a peak around 3 km (Mirkin, 1973, pers. comm.). Unfortunately, this scheme is dependent upon the scale of maps or photographs used. Other Russian terms used to describe the same phenomena are "lineamental jointing," "rectilinear elements of topography and stream networks" (Gol’braikh et al., 1968a) and "lineaments" (Shul’ts, 1969).

Lattman (1958) subdivides airphoto linears into "lineaments" and fracture traces," based on their length. He defines fracture traces as naturally occurring linear features observed on aerial photographs as alignments of stream segments, topographic features and soil and vegetational tonals which are expressed continuously for less than one mile in length. He relates them either to small faults or zones of joint concentration which are usually vertical or nearly vertical in cross-section (Lattman and Matzke, 1961). Excluded from the definition are bedding planes, compositional layering, and foliations. Lineaments are defined as consisting of the same morphological landscape elements as fracture traces, except that they are expressed discontinuously in the landscape and are greater than one mile (1.6 km) and up to several tens or hundreds of miles (km) in length. They may consist of zones of increased fracture trace concentrations, transgressing structural, temporal and physiographic provinces and because of their great lengths, they are thought to be recurrent effects associated with basement faults or zones of tectonic adjustment between major crustal blocks (Wise, 1968; Gold et al., 1973, 1974). A plot of aerial photographic linears combining both of Lattman's categories indicates a bimodal distribution, with a minimum occurring at about the one mile length (Lattman, 1969, pers. comm.). Mirkin (1973, pers. comm.) indicates a similar bimodal distribution with his break occurring at the 3-4 km interval.

The present study will use the terminology of Lattman (1958) and will examine whether his definitions agree with observations made during this study.
MEASUREMENT PARAMETERS

Griffiths (1967) characterizes the measurable properties of an object by the following mathematical equation:

\[ P = f \text{(material, size, shape, orientation, packing)} \]  \hspace{1cm} (1)

Size (length) and orientation (bearing) are the most readily measured properties of fracture traces. Shape can be variously defined. Griffiths (1967) characterizes the shape of quartz grains or pebbles as the ratio of their long, intermediate and short axes. In this sense, the ratio of fracture trace width to its length might be a measurable parameter. However, measurement of fracture trace widths is a highly subjective study because of possible erosional and seasonal vegetal enhancement, and until more is known of their character with depth, no consistent classification can be attempted. In addition, since fracture traces are defined as lines, their width can be defined as infinitely small and unmeasurable. A radius of curvature can also be defined as a shape parameter, however, the scarcity of these features would preclude their use as a commonly measured and quantified parameter (Gol'braikh et al., 1968a). The possible significance of these curved and arcuate features has often been overlooked (Podwysocki and Gold, 1974); they may represent the surface expression of periclinal structures, listric faults and intrusive bodies.

The two remaining factors which can be studied are materials and packing. In this study, materials will refer to surficial geologic materials (formations) present in the mapping area. Packing (density or number of fracture traces per unit area) will be one of the parameters calculated as a result of this investigation.

STUDY AREAS

Two study areas were chosen representing different types of "structural traps" for the accumulation of petroleum. Both are located in the relatively stable cratonic platform areas of the central USA. A study area in south-central Kansas was chosen because it was regarded as typical of vertical uplift controlled by basement faulting. The other area was located in west Texas and is underlain by a series of reef structures with overlying sediments draping over them (differential compaction). No basement tectonic control is evident in the latter area.

The Kansas study area, covering approximately 150 square miles (270 sq. km), occupies the southern portion of Pratt and the northern part of Barber Counties (Figure 1). It overlies a portion of the southward plunging nose of the Pratt Anticline, a southerly extension of the Central Kansas Uplift (Merriam, 1963).
Figure 1. Schematic Geologic Map Showing Anticlinal Structures and If Productive, the Name of the Associated Oil Fields. The Outlines of the Fields Are Based on the Lowest Structure Contour Which Indicates Closure, and Structural Closure is the Difference Between the Structural Crest and the Lowest Contour of Structural Closure.
Although deformation occurred as early as Cambrian time (Williams, 1968), the major pulse is Mid-Pennsylvanian (Merriam, 1963), and produced an unconformity between pre-Mississippian and late Pennsylvanian rocks. Structural and structural-stratigraphic traps suitable for the entrapment of petroleum were created by "crenulations" of 2-3 km diameter on the Pratt Anticline. A northeast trending fault underlines the Coats Oil Field, cutting the Precambrian basement (Cole, 1962). No documentation exists for this fault in higher stratigraphic horizons (Williams, 1968). Figure 1 contains a schematic representation of the oil fields in the area and the amount of structural closure as determined by structure contours on top of the Late Pennsylvanian Lansing Group (Williams, 1968). According to cross-sections by Curtis (1956), minor reactivation of some of the structures may have occurred as late as Permian time. Average depth to the top of the producing horizons is approximately 3500 feet (1065 m) below the surface and depth to basement averages about 5200 feet (1585 m).

Figure 1 also contains a surface geologic map. Glacial outwash gravels, sands, silts and some clays of the Pleistocene Kansan and Illinoian Stages predominate. The Illinoian materials are found on the upland surfaces in the northern portions of the study areas whereas Kansan materials are usually found in the southern part and the major stream valleys of the central portion (Layton and Berry, 1973). Thickness of these deposits reaches a maximum of 200 feet (61 m) in the northern part of the study area and gradually tapers to a zero edge where the Permian rocks of the Whitehorse Group crop out in the southern extremity of the study area. The latter consist of reddish-brown siltstones, shales and sandstones with lesser amounts of gypsum, salt, anhydrite and limestone (Layton and Berry, 1973).

The Texas study area, covering approximately 180 square miles (324 sq. km), is situated in the northwestern portion of Nolan and southwestern part of Fisher Counties (Figure 2). It lies on the eastern shelf of the Midland Basin, the site of the Pennsylvanian and pre-Pennsylvanian Concho Arch and Platform (Hope, 1956). Two major unconformities exist with a hiatus from Late Ordovician through the Mississippian and another from Triassic through the Cretaceous ages (Hope, 1956; Shamburger, 1967). During Pennsylvanian time the area was the site of extensive reef-building, caused by repetitive advances and retreats of the seas across this shallow platform area (Van Siclen, 1958). Subsequent deposition commonly covered the reefs with fine-grained clastic sediments, eventually draping over them, due to differential compaction, to create "structural highs" (Conselman, 1959). In addition, stratigraphic traps associated with the updip pinchout of fore-reef detritus are common. No documentation exists for faulting in the study area (Hope, 1956). Depth to "Canyon Reef" production horizons averages about 6000 feet (1830 m).
Figure 2. Schematic Geologic Map Showing Known Reef "Structures" and the Name of the Associated Productive Oil Fields. The Outlines of the Fields Are Based on the Lowest Structural Contour Which Indicates Closure, and Structural Closure is the Difference Between the Structural Crest and the Lowest Contour of Structural Closure.
Figure 2 contains a schematic representation of reef production with a minimum "structural closure" indicated for each reef and excludes stratigraphic traps such as the "Canyon Sands."

The surface geology is also portrayed in Figure 2. The Permian Whitehorse Group consists of sandstone, siltstone and shale redbeds with some interspersed gypsum beds. It crops out in the northern and extreme eastern portion of the study area. The Triassic Dockum Group crops out sporadically in the northern and eastern parts of the study area because of its cover by the Cretaceous and Tertiary units and its erosion during a later hiatus (Conselman, 1959; Shamburger, 1967). This unit consists mainly of red and tan conglomerates, sandstones and shales. Because of its small extent and similarity in lithology to the Permian, the two units have been grouped together. Dips on the Permo-Triassic and older subsurface units is about 0.5 - 1 degree to the west. The Cretaceous Trinity Group occupies the extreme southern part of the study area and consists of medium to coarse-grained quartz sands up to 80 feet thick which vary in color (Shamburger, 1967). Directly above is the Fredricksburg Group, consisting of thin to thick bedded arenaceous and fossiliferous limestones. Maximum thickness approaches 200 feet (61 m) in the Edwards Plateau directly to the south of the area, but it is considerably thinner locally. Karst features such as broad, shallow, poorly defined sinkholes are also found. Although these limestones underlie most of the central and western portion of the area, they are masked by a relatively thin cover of Tertiary Ogallala deposits. The Cretaceous units have been consolidated into one map unit because of the small lateral extent of the Trinity Group. Regional dips on these units usually do not exceed 1 degree to the southeast. The Pliocene Ogallala Formation consists of caliche, sands, gravels and some light colored clays; it forms a thin mantle over the central and western parts of the field area and, where exposed in cross-section in limestone quarries, it does not exceed 8 feet in thickness.

MAPPING METHOD

U.S. Department of Agriculture aerial photographs at a scale of 1:20,000 taken in the early 1960's were used as a basis for mapping the fracture traces. A pocket stereoscope was used in areas of moderate relief (up to 150 feet (46 m)), and for low relief areas (5 - 20 feet (1.5 - 6 m)), individual photographs were viewed at low oblique angles while the photos were rotated to view all possible "look directions." Mapping was done in flightlines, spending about 1/2 hour per stereo pair. Trainer (1967) showed that 84-89% of the fracture traces could be found in the first 20 minutes of observation.

As a check to determine if this operator was consistent in the selection of fracture traces, parts of the sidelap between adjacent flightlines were mapped.
and compared. A minimum of 83% of fracture traces were mapped consistently between several pairs of flightlines.

As a test of variations in the recognition of fracture traces, several experienced operators were compared to determine if the same general trends were mapped amongst the operators. Four sets of airphoto stereo pairs representing different types of topography in the study were mapped by two additional operators. Friedman Two-Way Analysis of Variance (Siegal, 1956) indicated that each operator mapped a different number of fracture traces on the four examples, based on a 0.05 level of rejection. However, the relative ranking by the operators of fracture trace direction indicated that in three out of four cases, there was no reason to reject the hypothesis that the operators were choosing the same directions. Thus, even though absolute numbers of fracture traces varied between operators, the same patterns of orientation and the same relative magnitude remained when the data were plotted in rose diagram plots. Gol'braikh et al. (1968a) achieves the same end by converting the absolute number or length of megajoints to percent rose diagrams in order to eliminate variation due to different operators and to more clearly discern the signal pattern.

Fracture traces were mapped directly onto aerial photographs by marking their endpoints with a soft colored pencil. In order to minimize planimetric errors, the fractures were mapped only within a three inch radius of the photograph centerpoint. These data were then transferred using a Saltzman projector to standard U.S. Geological Survey 1:24,000 scale topographic maps which were used as a base map. Figures 3 and 4 represent the fracture maps of the two areas. The grid on the left and top margins will be discussed later.

Cultural features such as pipelines and fencerows were usually readily distinguishable on aerial photographs. Subsequent field examinations verified and eliminated these features. Difficulty was encountered in differentiating some cultural features from fracture traces, notably relict plow patterns. This manifested itself in two fashions: 1) Plow patterns which paralleled some fracture traces would most likely cause the operator to overlook these fractures. This would eliminate north-south and east-west oriented fractures in the Kansas area. In west Texas, due to the orientation of the cultural pattern, those fractures oriented within several degrees of N12W and N78E could be easily overlooked. Conversely, old plowing practices did not heed the "lay of the land," and plowing was done normal to local slope. Those plow furrows normal to the slope would enhance and concentrate runoff in this direction, creating a series of parallel first and second order stream channels. Contour plowing practices alleviated this problem, however, they may have additionally obscured some of the original fracture pattern. Figures 5 and 6 are obvious examples of some
Figure 3. Fracture Trace Map of Kansas Study Area. The Upper and Left Margins Correspond to the Same Margins of Figure 1 and All Later Maps Using the Same Base.
Figure 4. Fracture Trace Map of Texas Study Area. The Upper and Left Margins Correspond to the Same Margins of Figure 2 and All Later Maps Using the Same Base.
Figure 5. Vertical Aerial Photograph of a Portion of Pratt County, Kansas, Taken in 1950. Note the Finishing Passes in the Plow Pattern (A), the Fracture Trace (B) and Areal Extent of Exposed Carbonate-Rich "B" Soil Horizon at C. Compare with Figure 6.

Figure 6. Vertical Aerial Photograph of a Portion of Pratt County, Kansas, Taken in 1963. Finishing Plow Patterns (A) Might be Mistaken for Fracture Traces. Fracture Trace (B in Figure 5) Has Been Obliterated by Land Contouring. Poor Agricultural Practices Have Caused Erosion and Exposed More Carbonate-Rich "B" Horizon (C).
of these phenomena. Many others exist where a decision concerning their origin is more difficult. 2) Plow patterns with their characteristic finishing passes through the field diagonals create linear patterns which later show through as relict patterns through a newer plowing pattern. Because most of these lines pass through field or section corners, they were regarded with suspicion and their significance downgraded. Gol'braikh et al. (1968a) noted similar problems in the USSR.

For the purpose of this study and to eliminate some of the subjectivity of the mapping, only continuous features were mapped as individual fracture traces. Thus, if a linear feature of 4 cm length on an aerial photograph appeared to have a break in its length, dividing it into two individual fractures, it would be mapped as such.

DATA HANDLING

Due to the large amount of information obtained, a computer-based data handling system was devised. A cartesian coordinate system was established with its origin in the upper left corner of each of the map areas. The X axis was chosen as latitudinal and positive to the right and the Y axis meridional and positive downward. The beginning and end points of each fracture trace could now be referenced with respect to this system which is illustrated in Figures 3 and 4. The map data were digitized onto standard 80 column Hollerith computer cards and preliminary treatment performed by a FORTRAN IV program TRANSFORM. Program listings and additional detail are described in Podwysocki (1974). The punched card output of this program contained the beginning, end and midpoints of each fracture trace as well as its length in millimeters on the map, azimuth and several other parameters which were then used in additional computer programs. Subsequent programs utilized the established cartesian base, dividing the map area into various grid cell sizes, and summarized the data in several fashions.

These programs were designed so that not only could data be summarized within a grid cell specified by the user, but the increment by which this grid cell was moved across the map could be specified. Thus, 1) the whole map could be treated as a single grid cell and all information would be summarized within that one cell, 2) the map could be subdivided into a series of smaller cells with the summaries taking place in those individual cells or 3) the map could be subdivided as in 2 above and the summary cell size could be incremented at a value less than the grid cell size, creating a "running average" or smoothing effect (see Podwysocki, 1974). Gol'braikh et al. (1968a) used the latter technique to look for changes in the number of megajoints and their orientation which might be associated with the presence of structural complications (i.e. structural closures, faults).
ANALYSIS OF FRACTURE TRACE LENGTHS

Treating the whole map of each area as a single grid cell, and classifying the fracture traces into 0.05 mile (0.08 km) class intervals, produced the results shown in Figures 7 and 8. VECLEN, the computer program for this classification, which is described in Appendix A, summarizes a fracture trace by its length if its midpoint falls within a grid cell. In both study areas the distributions of fracture trace lengths are highly skewed towards the shorter lengths. Gol'braikh and Mirkin (1973, pers. comm.) showed similar results for their studies of the Vilyuisk Syncline and the Preverkhoyansk Downwarp. Although no conscientious effort was made by the operator to discriminate against linear features greater than one mile (1.6 km) in length, it should be noted that all but a few fracture traces mapped were less than the maximum defined length of one mile as defined by Lattman (1958).

Because of the marked similarity between the observed distribution of fracture trace lengths and plots of sediment grain size distribution from sieve analysis, a variation of Krumbein's Phi scale transformation (1938) was applied to the data as follows:

\[ z = \log_2 x + 6 \]  

where \( x \) is the original length of the fracture trace in miles, \( z \) is the transformed value of the fracture trace length and 6 is a constant added to each value so that all resultant values in this work would be positive. Repeated analysis using the same techniques listed above produced the results illustrated in Figures 9 and 10. The histograms look like Gaussian distributions, however, the summary statistics in the figures do not bear this out. The following discussion of fracture trace lengths will utilize the transformed data.

It was thought that mixing of geologically different populations might cause the deviations from log-normality in the transformed data. The study areas were divided into quarters and each analyzed independently. Results indicated that only some areas showed normal distributions. It was noted that the log-mean fracture length was different for each of the 4 quarters of each of the two study areas.

To isolate those areas which were anomalous, the study areas were again quartered, producing a 1/16th unit of the total map area and the analysis performed on each unit. In addition, the summary unit cell was incremented by 1/2 cell intervals in both the X and Y directions, creating a running average as described earlier. The summaries produced cells which were approximately 3.5 by 3.9 miles (5.6 by 6.2 km) in the west Texas area and 3.6 by 2.9 miles
Figure 7. Plot of the Distribution of Fracture Trace Frequency versus Linear Length for the Kansas Study Area and the Test for Normality of the Distribution.
Figure 8. Plot of the Distribution of Fracture Trace Frequency versus Linear Length for the Texas Study Area and the Test for Normality of the Distribution
TEST OF FRACTURE TRACE DISTRIBUTION TO LOG-NORMALITY BY CHI SQUARE
KANSAS STUDY AREA: WHOLE AREA TREATED AS ONE CELL
MILEAGE CONVERTED TO LOG SCALE: Z=(I/LOGIO(I)+LOGIO(X)+6

|| ROW | COLUMN | 1 | 0 < X < 920; 0 < Y < 800 |
|---|---|---|---|---|---|---|
| **LOWER LIMIT** | **UPPER LIMIT** | **CLASS LIMIT** | **EXPECTED FREQUENCY** | **CHI SQUARE CONTRIB.** | **OBSERVED FREQUENCY** |
| 10 | 2.26 | 2.50 | 2.1 | 0.02 | 2.0 | > |
| 11 | 2.56 | 2.75 | 6.62 | 6.62 | 0.0 | > |
| 12 | 2.76 | 3.00 | 20.20 | 20.20 | 0.0 | > |
| 13 | 3.08 | 3.25 | 49.75 | 5.64 | 33.0 | >XX |
| 14 | 3.28 | 3.50 | 69.00 | 1.99 | 113.0 | >XXXXXXXXXXXXXXXXXXXXXXXXX |
| 15 | 3.50 | 3.75 | 159.33 | 21.51 | 215.0 | >XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX |
| 16 | 3.76 | 4.00 | 207.22 | 0.66 | 219.0 | >XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX |
| 17 | 4.00 | 4.25 | 217.09 | 0.36 | 209.0 | >XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX |
| 18 | 4.26 | 4.50 | 185.22 | 9.49 | 155.0 | >XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX |
| 19 | 4.50 | 4.75 | 112.76 | 4.62 | 103.0 | >XXXXXXXXXXXXXXXXXXXXXXXXX |
| 20 | 4.76 | 5.00 | 70.68 | 0.08 | 73.0 | >XXXXXXXXXXXXXXXXX |
| 21 | 5.00 | 5.25 | 31.75 | 0.05 | 33.0 | >XXX |
| 22 | 5.26 | 5.50 | 11.51 | 3.56 | 10.0 | >XX |
| 23 | 5.50 | 5.75 | 3.37 | 0.12 | 4.0 | > |
| 24 | 5.76 | 6.00 | 0.99 | 6.04(14.27) | 3.0 | > |
| 25 | 6.00 | 6.25 | 0.16 | 4.60 | 1.0 | > |
| 26 | 6.25 | 6.50 | 0.03 | 0.03 | 0.0 | > |
| 27 | 6.50 | 6.75 | 0.00 | 252.63 | 1.0 | > |
| **TOTALS** | | | 1193.00 | | 321.12 | 1193.00 |

DEGREES OF FREEDOM (NON-FOLDED) = 151; CHI-SQUARE PROBABILITY = 0.2249E-58
DEGREES OF FREEDOM (FOLDED) = 121; CHI-SQUARE PROBABILITY = 0.5680E-10

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STATISTICAL MOMENTS

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<td>0.289</td>
<td>0.537</td>
<td>0.531</td>
<td>3.513</td>
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Figure 9. Plot of Fracture Trace Frequency versus Log-Length for the Kansas Study Area and the Test for Log-Normality of the Distribution. Numbers Within Parentheses Represent Values When Distribution Tails Were Folded So That Expected Frequency > 0.95.
TEST OF FRACTURE TRACE DISTRIBUTION TO LOG-NORMALITY BY CHI SQUARE

TEXAS STUDY AREA: WHOLE AREA TREATED AS ONE CELL

MILEAGE CONVERTED TO LOG SCALE; \( Z = \frac{1}{\log_{10}(2)} \cdot \log_{10}(X) \cdot 6 \)

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<th>( 0 &lt; Y &lt; 1050 )</th>
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<td>7.02</td>
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<td>2</td>
<td>( 0.50 ) - ( 1.00 )</td>
<td>16.65</td>
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<td>2</td>
<td>( 1.00 ) - ( 2.00 )</td>
<td>42.24</td>
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<tr>
<td>13</td>
<td>3</td>
<td>( 2.00 ) - ( 3.00 )</td>
<td>81.57</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>( 3.00 ) - ( 4.00 )</td>
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<td>( 4.00 ) - ( 5.00 )</td>
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<td>( 5.00 ) - ( 6.00 )</td>
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<tr>
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<td>4</td>
<td>( 6.00 ) - ( 7.00 )</td>
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<td>( 7.00 ) - ( 8.00 )</td>
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<td>4</td>
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<td>5</td>
<td>( 9.00 ) - ( 10.00 )</td>
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<td>5</td>
<td>( 10.00 ) - ( 11.00 )</td>
<td>52.96</td>
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<td>( 11.00 ) - ( 12.00 )</td>
<td>24.61</td>
</tr>
<tr>
<td>23</td>
<td>5</td>
<td>( 12.00 ) - ( 13.00 )</td>
<td>9.74</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>( 13.00 ) - ( 14.00 )</td>
<td>3.29</td>
</tr>
<tr>
<td>25</td>
<td>6</td>
<td>( 14.00 ) - ( 15.00 )</td>
<td>28.29</td>
</tr>
<tr>
<td>26</td>
<td>6</td>
<td>( 15.00 ) - ( 16.00 )</td>
<td>0.24</td>
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</table>

TOTALS: 1482.93 1483.00 EACH \( *X *= 5.00 \) VECTOR(S)

DEGREES OF FREEDOM (NON-FOLDED) = 15; CHI SQUARE PROBABILITY = 0.4196E-11

DEGREES OF FREEDOM (FOLDED) = 14; CHI SQUARE PROBABILITY = 0.2375E-11

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STATISTICAL MOMENTS

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<th>ROOT MEAN</th>
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Figure 10. Plot of Fracture Trace Frequency versus Log-Length for the Texas Study Area and the Test for Log-Normality of the Distribution. Numbers Within Parentheses Represent Values When Distribution Tails Were Folded So That Expected Frequency > 0.95.
(5.8 by 4.6 km) in the Kansas area for a total of 64 cells (8 by 8 in each area). Summary statistics such as the log-mean fracture trace length, standard deviation, skewness ($\sqrt{\beta_1}$) and kurtosis ($\beta_2$) of each cell's frequency distribution as well as the number of fracture traces for each unit cell were produced for use in additional analyses.

The summary statistics produced in the above mentioned compilations of the data were analyzed using linear regression analysis. Due to the paucity of fracture traces in the southernmost tier of cells (less than 5 in each) in the Texas study area, these cells were eliminated from the analysis.

The significance test for correlations between the statistical moments for the Kansas data (Table 1) indicates a significant correlation for 1) log-mean fracture trace length and skewness, 2) number of fracture traces per unit cell and standard deviation and 3) skewness versus kurtosis. Figure 11 represents the plot of the standard deviation versus number of fracture traces per unit cell. The plot indicates low standard deviations associated with cells containing few fracture traces (lower left part of diagram). Because the reliability of the statistical moments for such small sample sizes is highly questionable, the offending samples (all cells containing less than 45 samples), which occurred along the eastern and southern margins of the map area, and were due to incomplete mapping coverage, were eliminated from consideration in further tests. Repeated regression analysis on the data exclusive of the mentioned marginal cells indicated no significant correlation between two of the three previously determined associations. However, it should be noted that a significant correlation did remain between the skewness and kurtosis measures; Figure 12, based on the original analysis of 64 samples, serves to illustrate the results. A small group of samples located near the right margin of the plot contains kurtosis values which are highly leptokurtic* (8-10). These cells contain several very long fractures (greater than the accepted length for a fracture trace) that were inadvertently included, and will be discussed later in the log-normality analysis. A removal of these four anomalous cells and repeated regression analysis indicated no significant correlation between the two moments. Removal of these correlations, or attributing them to some sampling inconsistencies, indicates the samples are homogeneous, that is, several discrete and very distinct populations do not exist in the data.

* More peaked than normal
Table 1

Results of Linear Regression Analysis
On Log-Mean Fracture Trace Moments

Kansas Data - 64 Samples

<table>
<thead>
<tr>
<th></th>
<th>Log-Mean Length</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>NS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>$S^*$</td>
<td>NS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>NS</td>
<td>NS</td>
<td>$S^{**}$</td>
<td></td>
</tr>
<tr>
<td>No. of Fracture Traces per Unit Cell</td>
<td>NS</td>
<td>$S^{**}$</td>
<td>NS</td>
<td>NS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Log-Mean Length</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
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<tbody>
<tr>
<td>Standard Deviation</td>
<td>$S^{**}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>$S^{**}$</td>
<td>NS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>NS</td>
<td>$S^*$</td>
<td>NS</td>
<td></td>
</tr>
<tr>
<td>No. of Fracture Traces per Unit Cell</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
<td>$S^{**}$</td>
</tr>
</tbody>
</table>

NS = non significant
$S^* = $ significant at 0.05 level
$S^{**} = $ significant at 0.01 level
Figure 11. Regression Analysis Plot of Standard Deviation versus the Number of Fracture Traces Per Unit Cell for the Kansas Data. Numbers Within the Plot Indicate the Number of Data Points Located in that Position.
Figure 12. Regression Analysis Plot of Skewness versus Kurtosis for the Kansas Data.
The regression analyses on the Texas data are given in Table 2. Significant correlations exist between:

1) log-mean fracture trace length and standard deviation (Figure 13), which indicates increasing standard deviation with increasing mean fracture trace length;

2) log-mean fracture trace length and skewness (Figure 14), which illustrates an increasing positive skewness (mode displaced towards smaller values with respect to the mean of the distribution) with increasing fracture trace length;

3) standard deviation and kurtosis, (Figure 15); and

4) kurtosis and the number of fracture traces per unit cell (Figure 16).

Figures 13-15 can be interpreted together to indicate one of two possible causes. If the assumption is made that the samples were taken from a single homogeneous population, then the sampling technique indicates a bias. Conversely, the population may not be homogeneous, and the tests may indicate the sampling of two or more discrete and distinct populations of fracture traces. The second of the two hypotheses will be proven and more clearly illustrated by the use of trend surface analysis which will be discussed later.

The last significant correlation occurs between kurtosis and the number of fracture traces per unit cell (Figure 16). These high values are associated with large sample populations and are anomalous, perhaps suggesting some mixing of several populations of fracture traces.

Tests were performed using the Chi Square, skewness and kurtosis criteria (Griffiths and Ondrick, 1968), comparing the observed against a hypothetical Gaussian distribution. Deviations of each of the criteria were ranked, assigning values to those populations which significantly differed from normality at the 0.05 and 0.01 levels. Rankings were assigned as illustrated in Table 3.

If a criterion value was non-significant, it was assigned a zero value. The rankings of the three criteria for each cell were then summed to create an index value characteristic of the population distribution in each cell. High ranking values indicate strong deviations from log-normality as illustrated in Figures 17 and 18.
Figure 13. Regression Analysis Plot of Log-Mean Fracture Trace Length versus Standard Deviation for the Texas Data.
Figure 14. Regression Analysis Plot of Log-Mean Fracture Trace Length versus Skewness for the Texas Data.
<table>
<thead>
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<th>Standard Deviation of Log-Mean</th>
<th>Kurtosis</th>
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<tbody>
<tr>
<td>0.475</td>
<td>7.366</td>
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<tr>
<td>0.503</td>
<td>7.241</td>
</tr>
<tr>
<td>0.536</td>
<td>7.116</td>
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<tr>
<td>0.566</td>
<td>6.991</td>
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<tr>
<td>0.596</td>
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<td>0.627</td>
<td>6.741</td>
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<tr>
<td>0.657</td>
<td>6.616</td>
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<tr>
<td>0.687</td>
<td>6.491</td>
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<tr>
<td>0.718</td>
<td>6.366</td>
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<tr>
<td>0.748</td>
<td>6.241</td>
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Figure 15. Regression Analysis Plot of Standard Deviation versus Kurtosis for the Texas Data.
Figure 16. Regression Analysis Plot of Kurtosis versus Number of Fracture Traces per Unit Cell for the Texas Data.
Figure 17. Results of Test for the Distribution of Fracture Trace Lengths to Log-Normality. Rank-values are associated with the centerpoint of each grid cell. Refer to Table 3 and text for key to rank values.
Texas Study Area

Figure 18. Results of Test for Distribution of Fracture Trace Lengths to Log-Normality. Rank-values Are Associated with the Center-point of Each Grid Cell. Refer to Table 3 and Text for Key to Rank Values.
Table 3

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<th>Rank</th>
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<td></td>
<td>0.01</td>
<td>4</td>
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<td>Skewness</td>
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<tr>
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<td></td>
<td>0.01</td>
<td>2</td>
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</table>

In many cases, significant deviations from log-normality occur over known structures. Analysis of fracture trace log-lengths in the Kansas study area indicates a consistent positive skewing (mode displaced towards smaller values with respect to the mean) in cells which rank four or higher; their respective kurtosis values are leptokurtic. Several factors may account for these variations. First, structural control may exist, possibly causing development of shorter fractures over structures due to enhancement of surface factors such as erosion along the fractures. Secondly, control may be due to changes in lithology. Analysis of fracture trace lengths does indicate a lithologic control and will be discussed shortly. Thus, mixing of two surface rock types within a grid cell may cause this type of discrepancy. However, it should be pointed out that similar mixing also takes place in the two Pleistocene aged formations in the eastern part of the study area, and these types of deviations do not exist in this area. Very high deviations in the southern portion of the Kansas area (ranked 7 and 8) may be due to the proximity to Permian outcrops and/or the influence of two fractures greater than one mile (1.6 km) in length that were inadvertently mapped (see Figure 7). Because these features were larger by a factor of two over all other fracture traces in the area, they cause highly significant deviations from log-normality. These same fractures were responsible for the high correlation between the skewness and kurtosis in the regression analysis plot (see Figure 12). Thirdly, biases due to operator fatigue or cultural land practices may occur. These hopefully were minimized with field checking, rest periods during mapping and cross checking with photographic coverage of earlier dates to eliminate these possible errors.

Significant deviations from log-normality (ranked four or higher) also occur over some of the known reefs in the west Texas study area. Skewness and kurtosis behave similarly to the anomalies in the Kansas area. The same three arguments stated in the previous paragraph may be employed. Cultural effects have been minimized by reference to earlier photographic coverage.
Because lithology shows little control in the northern part of the area where cells transgress lithologic boundaries, it is probably not a controlling factor in the anomalous eastern portion of the study area. The high value (a rank of 6 in row 4, column 3) in the central portion of the map area is due to the presence of a lineament. The fracture trace length distribution for this area is unlike those over the reefs; it is skewed positive and is nearly normal in its kurtosis. In some cases, the anomalous ranks do not directly overlie the structure, but lie on its flanks. Harris et al., (1960); Gol’braikh et al., (1968a) and Saunders (1969) indicate that increased fracture density may occur along the flanks of a structure, however, no mention has been made of changes in fracture length.

Further reduction of the grid cell size produced many cells with too small a population, and thus reliable statistics were not possible. Analysis of fracture trace length distributions in individual 10 degree azimuth classes in each grid cell also proved fruitless because of the small number of fracture traces in each cell.

ANALYSIS OF FRACTURE TRACE FREQUENCY

Trainer and Ellison (1967) define frequency as the number of fracture traces, irrespective of their length, which fall within a unit area under consideration. Trend surface analysis (O’Leary et al., 1966) was applied to the fracture trace frequency values generated by the VECLEN program for the 1/16th unit areas discussed above. This technique attempts to fit surfaces which represent polynomial equations of increasing order to map data. Increasing polynomial order represents increasing complexity of the surface, which thus more closely approximates the given data. It can be used in some instances to extract different components responsible for variations which may be present in the data. In most cases, first through sixth order surfaces were fitted to the data. Analysis of variance was applied to the output statistics of this technique to determine which surfaces were a significant improvement over their lower order neighbors (Krumbein and Graybill, 1965); the probability level used was based on $P = 0.005$. Only selected surfaces which achieved the prescribed level of significance and their residual plots will be discussed. Tables 4 and 5 summarize the data for each study area.

Figure 19 illustrates the second order surface for the Kansas data and accounts for 82% of the variations. It shows that fracture trace frequency is highest in the southeastern part of the map area near the Permian outcrops, and decreases northward toward the younger Pleistocene deposits and towards the map peripheries, where coverage is incomplete or control is lacking. This suggests that lithology may be a controlling factor for one of several reasons. 1) The
Table 4

Analysis of Variance of Trend Surfaces Data for Fracture Trace Frequency Kansas Study Area

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<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
<th>Cumulative Percent Variation Explained</th>
<th>Percent Variation Improvement for Each Surface</th>
</tr>
</thead>
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<td>.005-.001</td>
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<td>.10-.25</td>
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Table 5

Analysis of Variance of Trend Surface Data for Fracture Trace Frequency Texas Study Area

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<th>Surface Order</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
<th>Cumulative Percent Variation Explained</th>
<th>Percent Variation Improvement for Each Surface</th>
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<td>&lt;.001</td>
<td>67.5</td>
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</table>
Figure 19. Second Order Trend Surface for Fracture Trace Frequency
unconsolidated Pleistocene sediments may have a masking effect, subduing the number of fractures propagated to the surface; 2) the younger sediments may have been subjected to lower stress levels, fewer periods of deformation and a shorter time for the propagation of the fractures; or 3) different rock types may have different mechanical properties. Because Pleistocene unconsolidated deposits do thicken northward, the first two factors are probably the most significant.

Analysis of the residuals* map (Figure 20) indicates a large positive residual (greater than the calculated model) in the southeast part of the map, underlain by outcropping Permian rocks, and may be explained by several factors. Harris et al. (1960) noted changes in jointing frequency due to contrasting lithologies over the Goose Egg Dome in Wyoming. Not only did they find a progressive decrease in frequency from siliceous limestone, calcareous quartz sandstone, soft sandstones to ductile shales, but also that fracture frequency was inversely proportional to strata thickness. In his study on joints in the Great Scar limestones in England, Doughty (1968) recorded changes between differing limestone types of similar age. Huntington (1969) found changes in fracture trace frequency due to contrasting lithology and suggests that observations be confined to like rock types. DeSitter (1964) recognized lithology and strata thickness, amongst others, as controls of rock fracturing intensity. Another factor which should be considered is the possible masking of the fractures due to the strong contrast in mechanical properties of the consolidated Permian deposits as opposed to the unconsolidated Pleistocene materials, which could act as a filter, either totally obliterating or subduing some fracture traces.

Another positive residual is associated with a series of structural closures in the vicinity of the town of Coats (Figure 20). Although the anomaly overlies two different map units, the mechanical contrast between these two unconsolidated Pleistocene deposits should be minimal. Excluding possible operator bias, the residual might reflect the subsurface Pennsylvanian structures. Residuals along the map peripheries are discounted due to lack of control. Gol'braikh et al. (1968a), Saunders (1969) and Dranovskii (1970) have suggested that the number of airphoto linears per unit area (frequency) is an indicator of structural culmination. Moreover, Dranovskii (1970) further states that in box-like uplifts, maximum fracturing occurs on the fold limbs, while in ridge-like uplifts it develops on the crest of the structure.

The second order surface for the Texas data accounts for 67% of the variation and is illustrated in Figure 21. It shows fewer fracture traces over the

* For any given observed data point on the map: residual = observed - expected value calculated for the coordinates of the observed data points.
Figure 20. Map of Second Order Trend Surface Residuals for Fracture Trace Frequency
TEXAS STUDY AREA

Figure 21. Second Order Trend Surface for Fracture Trace Frequency
Tertiary deposits and the immediately underlying Cretaceous limestones, whereas more fracture traces occur over the Permo-Triassic rocks. In conformity with the previously stated conclusions of Harris et al. (1960) and other workers, the same reasons may explain the lower frequency over the Cretaceous-Tertiary rocks. The Ogallala Formation forms a thin blanket, not exceeding 6 - 8 feet in thickness over the study area. In addition, inspection of several quarries in the Fredricksburg Group limestones revealed a large population of curved joint surfaces which usually terminated at bedding planes. These are non-systematic joints that are not associated with quarrying operations. The paucity of vertical systematic joints suggests that most stresses may have been taken up and diffused in the non-systematic joints, thereby precluding the formation of wide zones of weakness suitable for the development of fracture traces. Because the fourth order residual map more clearly illustrates the results, a discussion of the second order map residual is unnecessary.

Figure 22 shows the results of the fourth order fit and answers 89% of the variation. The model contours tend to parallel the north-south flightlines, which suggests an operator bias due to changes in acuity during mapping, however, higher frequencies again occur over the Permo-Triassic rocks. This inter-flightline variation was the predominating signal in the residual plot of the second order surface. It is therefore suggested that mapping of fracture traces either be done on a suitable scaled mosaic or that individual photographs or pairs should be picked randomly from the total available set so that this type of variation might be distributed more evenly.

Figure 23 contains the residuals map based on the fourth order surface. Although some alignment parallel to the north-south flightlines does occur, most has been removed by this surface. The large positive anomaly in the northern portion of the area is associated with the "saddle" in the trend surface (Figure 16) and is anomalous. The strong negative anomalies in the eastern portion of the map area appear to be associated with the flanks of three of the reef structures. This observation is further enhanced by the fact that they occur along several flightlines, thereby indicating a consistency between flightlines after removal of the inter-flightline variation.

In summary, the predominating portion of the variation in frequency of fracture traces is associated with differences in lithology. Lesser amounts of the variation are due to operator variability due to changes in perceptibility during fracture trace mapping. Another variation which may occur is associated with "structural closures." Basement uplift structures are accentuated by positive (high) fracture trace frequencies along their flanks, whereas "passive" structures such as reefs, may be associated with negative (low) fracture trace frequencies in these study areas.
Figure 22. Fourth Order Trend Surface for Fracture Trace Frequency
Figure 23. Map of Fourth Order Trend Surface Residuals for Fracture Trace Frequency
ANALYSIS OF LOG-MEAN FRACTURE TRACE LENGTHS

Log-mean fracture trace lengths generated for the 1/16th unit areas by the computer program VECLEN were also analyzed using trend surface analysis. Significance of improvement in the information level of each surface was tested as described earlier. Tables 6 and 7 summarize the results. Only the highest order surface showing the prescribed level of significance (less than 0.005) will be discussed.

Figure 24 illustrates the fifth order surface for the Kansas study area and accounts for 80% of the variation. The model shows longer fracture traces in the northern part of the study area, becoming progressively shorter towards the Permian outcrop area. This may be interpreted as a masking effect of the glacial overburden, causing the operator to overlook shorter fracture traces due to their less pronounced nature or their complete obliteration by the overburden. The model also shows a parallelism between some of the contours and geologic formation boundaries, as exemplified by the 4.1 contour, which further reinforces the lithologic control hypothesis. The parallelism of contours and their steep gradient in the western part of the area is due to lack of control in this area.

The corresponding residuals map (Figure 25) indicates a broad positive residual trending northwest in the central part of the area, and parallels the boundaries between the two mapped Pleistocene units. The positive bands in the northeast and southwest sectors also may be associated with the formational boundaries. A negative residual is present in the west-central portion of the map and coincides with the increased fracture trace frequency derived from the second order residual (Figure 20). These two factors may be inter-related; increased deformation may cause more intense fracturing (higher frequency) and because of surficial processes, greater erosion generates more linear first and second order streams, which manifest themselves as fracture traces.

The fifth order surface for the west Texas study area (Figure 26) indicates longer fracture traces over the Cretaceous-Tertiary deposits, with shorter fracture traces occurring in the Permo-Triassic rocks. The observed differences may be due to masking effects as discussed earlier for the Kansas area, however, Trainer and Ellison (1967) found that longer fractures traces occurred in the limestone units of the Shenandoah Valley. They suggested that this might be due to solution and coalescence of joint planes and zones of weakness, a process which has operated in this area as evidenced by the development of karst features.

Figure 27 illustrates the residuals map associated with the fifth order surface. No consistent pattern is found with respect to the reef structures. The dominant features include a negative residual trending northwest in the central part of
### Table 6

Analysis of Variance of Trend Surface Data for Fracture Trace Log-Mean Length Kansas Study Area

<table>
<thead>
<tr>
<th>Surface Order</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Squares</th>
<th>F</th>
<th>Significance</th>
<th>Cumulative Percent Variation Explained</th>
<th>Percent Variation Explained By Each Surface</th>
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### Table 7

Analysis of Variance of Trend Surface Data for Fracture Trace Log-Mean Length Texas Study Area

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<th>Significance</th>
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40
Figure 24. Fifth Order Trend Surface for Log-Mean Fracture Trace Length
Figure 25. Map of Fifth Order Trend Surface Residuals for Log-Mean Fracture Trace Length
TEXAS STUDY AREA

Figure 26. Fifth Order Trend Surface for Log-Mean Fracture Trace Length
Figure 27. Map of Fifth Order Trend Surface Residuals for Log-Mean Fracture Trace Length
the map area. Its extension across several flightlines tends to bear out the reality of this feature. Examination of an aerial photographic mosaic reveals a lineament passing through the area in this direction which is most likely related to this anomaly. The same argument previously discussed concerning increased structural deformation, which produces a greater number of shorter fractures, may be invoked. Again, some variation is noted between flightlines and some of the positive anomalies in the south.

In summary, log-mean fracture trace lengths are predominantly controlled by the type of sedimentary material. Variations between flightlines may occur, but their effect is not as pronounced as in fracture trace frequency. Fracture traces may be shorter over basement uplift structures. This effect may not occur over passive structures, such as those formed by a draping of sediments over bedrock highs.

ANALYSIS OF ROSE DIAGRAMS

Several formats are available for displaying directional data. Three dimensional data, such as attitude of joint planes, can be efficiently portrayed on stereographic (Wulff) or equal area (Schmidt) nets. Statistical analysis of these data are cumbersome, but has been discussed by several works (Chayes, 1949; Fisher, 1953; Pincus, 1953). Two dimensional data, such as the strike of fracture traces, may be displayed as histograms or as rose diagrams (Podwysocki, 1974). Both these formats can be conveniently tested and may be generated by computers and will be used in this paper because of their suitability for visual comparison.

Several methods can be utilized to summarize the data for rose diagrams. Trainer and Ellison (1967) use the terms "frequency" and "density." Frequency as described earlier, refers to the number of fracture traces, irrespective of their length, while density refers to the total length of fracture traces. Each of these respective techniques has its disadvantages. Summarization using frequency eliminates a bias due to length. Thus, a fracture trace of 0.25 mile (0.4 km) is given as much weight as one 0.75 mile (1.2 km) long. However, due to the mapping technique employed in this paper, which breaks up fracture traces into components based on their continuous exposure, the shorter fracture traces are favored. Thus, density was chosen as the analysis criterion in this work in order to minimize this bias. In addition, the need for a standard unit to facilitate comparison has been noted elsewhere. Gol'braikh et al. (1968) suggest conversion of the units into percent values prior to plotting, so that the size of all roses will be standardized.
Joints were measured in several bedrock exposures in the west Texas study area. Their orientation frequencies were compared to the density of rose diagrams of fracture traces measured in grid cells approximately three miles square surrounding each of these localities. Data in 10 degree azimuth classes were analyzed using AZMAP and ROSE, computer programs written by Podwysocki (1974). In order to compare statistically the two dissimilar units of measurement, both sets of data were converted to percentages. A total of 69 systematic joints were measured in Cretaceous limestones of the Fredricksburg Group, exposed in a quarry near the central-western edge of the map area. Nearly all systematic joints were vertical, eliminating the need to use three-dimensional displays and making the measurements suitable for comparison with the fracture trace distribution. As discussed earlier, there also were many non-systematic joints. Permian exposures in the extreme central-eastern part were measured and consisted of a roadcut in a gypsiferous sandstone and a railroad cut in a massive sandstone. A total of 59 joints were measured and combined from these two adjacent cuts. All joints in these cuts were within 5 degrees of vertical. Figure 28 contains a graphical comparison of the two sets of patterns.

Neither set of rose diagrams show a good visual fit; a Chi Square test comparing the fracture trace and joint orientations for each locality indicates that the patterns were not similar based on a 0.01 level of rejection. Neither could it be influenced that much by population size, because Gol’braikh et al. (1968a) indicated that 40 - 50 joint measurements were required to achieve statistical reliability. It should be noted that while there is conformity in direction in the Permian rocks there is a consistent angular displacement between the two patterns in the Cretaceous rocks. The former set may reflect fracture traces that are occupied by zones of joints sub-parallel to the direction of the fracture trace (Lattman, 1969, pers. comm.); the latter may represent a displacement of the second order joints from the direction of maximum shear stress. This phenomenon has been documented by Renner (1969) and may relate to a hierarchichal structural framework as postulated by Moody and Hill (1956) and discussed by Nemec (1970) and Gold et al. (1973). Because of the dissimilarity in the Cretaceous patterns and partial agreement in the Permian patterns, it might also be suggested that either the rocks have behaved differently when subjected to the same stresses or that the older units were subjected to an additional period of stress not experienced by the younger units.

These results are partly contrary to those of Lattman and Nickelsen (1958), Hough (1959), Boyer and McQueen (1964) and Alpay (1973), who generally found good agreement between fracture trace and joint directions in their investigations in sedimentary rocks dipping less than 5 degrees. Matzke (1961), Lattman and Matzke (1961, 1971) and Trainer and Ellison (1967), however, reported that fracture traces and joint directions do not totally
Figure 28. Comparison of Joint and Fracture Trace Rose Diagrams, Texas Study Area
coincide. Their observations were made in more deformed rocks (i.e. the Appalachian fold belt). Lattman and Matzke (1961) suggest that joint patterns in relatively stable cratonic areas are paralleled by fracture traces whereas, local structure in highly deformed materials impress their own local joint sets which may deviate from the regional trends.

Although orientation directions coincide for the Permian rocks, the length of the rays (degree of preferred orientation) is greater for the joints. This is probably due to the big difference in the size scale of the areas sampled (9 square miles (23 sq. km) versus 2 outcrops 1/2 mile apart).

ANALYSIS OF FRACTURE TRACE PATTERNS

Pattern recognition of preferred orientation in fracture analysis tends to be more difficult due to the large amount of data and its multivariate nature. Several approaches have been used to enhance patterns. Haman (1961, 1964) isolated and plotted all macrofractures (lineaments) and mesofractures (fracture traces) which fell within narrow azimuth ranges and used them in a qualitative fashion to discern faulting and to locate changes in regime of individual tectonic blocks. Maffi and Marchesini (1964) describe the use of optical and computer processing techniques to filter and isolate individual trends. Gol'braikh et al. (1968a) also isolated regional structures by plotting their megajoint densities for narrow azimuth ranges, and showed the applicability of Permyakov's (1949) "rule of the parallelogram" to determine regional trends by analysis of the rose diagram modes. Little has been published on a method for the comparison of several rose diagrams. Chudinskii (Mirkin, 1973, pers. comm.) suggests that rose diagrams of small subsets of the total area should be compared against the grand rose diagram for the whole territory. A variation of the Chi Square criterion could then be used to compare the subset against the composite rose diagram. Those which proved to vary significantly from the composite diagram were zones of "tectonic complications." Lattman (1969, pers. comm.) suggested a similar technique, but instead of comparing a subset against the composite rose diagram, the subset was compared against all of its adjacent neighbors. Significant variations between neighboring diagrams would then indicate structural complexities.

The fracture trace data compiled by the TRANSFORM program was processed by AZMAP (Podwysocki, 1974), which classified the fracture traces into direction categories within each unit cell (1/16th of the total study area). As described previously in the analysis of fracture trace frequency and lengths, a 1/2 cell sliding average increment also was used. An azimuth class interval of 10 degrees was utilized during the classification.
Several classification techniques could be used in AZMAP. The first, entitled "Part" analyzed only that portion of the fracture trace length which lies within the cell. "Mid" considered the whole fracture trace within the cell if its midpoint fell within the cell. A comparison of the two techniques showed that there was no significant difference if the results of the two classification techniques were compared against each other for each of the 49* grid cells of each area, using the Chi Square test and a rejection level of 0.05.

Punched card output of the summary length of fracture traces per azimuth class per grid cell were processed by a computer program ROSE (Podwysocki, 1974), which produced rose diagrams (see Figures 29 and 30). The punched card output from AZMAP also was utilized in a multivariate analysis computer program CLUS (Rubin and Friedman, 1967). Each rose diagram consisted of 18 variables or measurements (the sum total length of fracture traces within each of the 10 degree azimuth classes). A total of 49 grid cells (objects) were generated by AZMAP for each study area and these were treated as 49 samples.

Multivariate techniques have been shown by Dahlberg and Griffiths (1967) to be an effective method for determining the relationships between objects with interacting properties. The Rubin and Friedman program is appropriate for determining the relationships between samples because the procedures allow classification on the basis of a number of groups determined by the user. A determination of the optimum grouping is made on the basis of several computer generated criteria for each classification.

The inverse of the Wilk's lambda criterion, \( \log \left( \frac{\text{max} |T|}{\text{max} |W|} \right) \), is used as an informal indicator of the best number of groups (Friedman and Rubin, 1967), where:

- \( W \) is the pooled within-group matrix of the cross products of deviations,
- \( T \) is the matrix of cross products of deviations for the total sample,
- \( B \) is the matrix of between-group cross products of deviations of groups from the grand means weighted by group size (Cooley and Lohnes, 1962),

and

\[ T = B + W. \]

*A total of 15 cells occupying the easternmost and southernmost areas was eliminated due to the low fracture trace frequency caused by incomplete photo coverage.
Figure 29. Rose Diagram Plot of Fracture Trace Patterns for the Kansas Study Area
Figure 30. Rose Diagram Plot of Fracture Trace Patterns for the Texas Study Area
The best partition may also be determined by use of the total generalized distance, the Mahalanobis $D^2$ criterion, where $D^2$ is defined as the sum of the distances between multivariate means of all possible pairs of groups, in terms of standardized measurements.

Using principle components, a plot of the eigenvalues of the total correlation matrix indicates a gradual decrease in the amount of variation explained by each additional component (Figure 31). It was arbitrarily decided to choose the 8 component level as the cutoff. A total of 85% of the variation is explained by the 8 components in the Kansas data and 82% is explained in the west Texas data.

The two sets of data were processed by the program CLUS, using 2 through 11 groups. Figure 32 illustrates the plot of the two criteria using the log $(\max |T|/|W|)^*$ algorithm for the Kansas data using 8 components. An inflection at the three group level in both criteria is interpreted as significant. The six group level also indicates a major inflection of the $D^2*$ criterion. An additional run on the data using six components produced exactly the same classification for the 6 group level, but showed a more marked increase in the value of both criteria. Figure 33 illustrates the three group classification, which in a crude fashion, tends to outline the geology. Group 2 mainly occupies the northern part of the area of exposed Illinoian deposits, group three occupies the area underlain by the Kansan and Permian deposits and group 1 covers areas occupied by a mixture of groups 2 and 3. The six group level (Figure 34) contains some isolated members of groups 3 and 5 within the central part of the map. These overlie the Coats Anticline, which has a structural closure of approximately 250 feet (76 m) and may thus have affected the overlying fracture pattern. Examination of the rose diagram patterns in Figure 29 reveals a pronounced enhancement of the northeast ray directly over the structure (row 3, column 3), which may be associated with the northeast trending fault in the basement rocks underlying this structure (Cole, 1962).

Figure 35 illustrates the plot of the two indicator criteria for the west Texas data. No pronounced peaks were noted, although a change in slope for both criteria occurs at the two and seven group level. The two group classification (Figure 36) seems to be related to geologic materials exposed on the surface. Group 1 tends to overlie areas of Permo-Triassic rocks whereas group 2 occupies areas of Cretaceous-Tertiary deposits. The 7 group level (Figure 37) shows no obvious relation to any of the reef structures. The classification is again partly related to lithology; groups 1, 2, and 3 overlie Cretaceous-Tertiary deposits, whereas groups 4, 5, and 7 overlie the transition between the two

*Based on the grouping of log $(\max |T|/|W|)$. 

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Figure 31. Plot of Eigenvalues versus Principal Components for Both Study Areas
Figure 32. Plot of CLUS Classification Criteria for the Kansas Data Using 8 Components, Log \( \left( \frac{|T|}{|W|} \right) \) Maximized.
Figure 33. Results of the 3 Group Classification of Rose Diagrams
Figure 34. Results of the 6 Group Classification of Rose Diagrams
Figure 35. Plot of CLUS Classification Criteria for the Texas Data Using 8 Components, Log ($\frac{|T|}{|W|}$) Maximized
Figure 36. Results of the 2 Group Classification of Rose Diagrams
TEXAS STUDY AREA

Figure 37. Results of the 7 Group Classification of Rose Diagrams
major map units. Group 6 occupies mainly Permo-Triassic rocks. The mis-
classification of the cells in the southwestern part is probably due to the small
sample size in this area due to incomplete coverage, producing rose diagrams
without any preferred rays.

In summary, classification of the rose diagrams using a multivariate classifi-
cation scheme produces groupings which are predominantly controlled by
surface lithologic factors if classification is limited to a small number of groups.
Active structures (i.e., basement uplift anticlines) may be recognizable because
their fracture patterns may differ from their immediate neighbors and may be
isolated by classifications at higher group levels. Passive (reef) "structures"
do not create fracture patterns which can readily be isolated from their
surrounding neighbors by this technique.

CONCLUSIONS

Detailed quantitative analysis of fracture trace patterns can be routinely per-
formed using repetitive techniques and computer algorithms. Cultural
features can affect the ability to map fracture traces. Fracture trace lengths
tend to be log-normally distributed. Deviations from log-normality tend to be
associated with structural closures in both study areas, suggesting that fracture
pattern may be disturbed over the structures.

Trend surface analysis may allow extraction of several levels of information
that may be present in a set of data. Examination of fracture traces by trend
surface analysis indicates that lithology mainly controlled the frequency and
log-mean fracture trace length. Frequency was also affected by an operator
bias, which caused alignment of some of the model contours with flightline
paths in at least one of the study areas. Higher order surfaces extracted the
majority of these variations. Residuals in the frequency analysis isolated
areas of increased fracture frequency in the Kansas area that appeared to be
associated with either bedrock exposures or with structural culminations. In
the west Texas area, strong negative residuals appear to be related to reef
structures. The increase in frequency in the active structure (anticline) and
the scarcity in the passive structure (reef) suggests either different mechanisms
for propagation of fractures through these two types of structural discontinuities
or different stress fields produced above the structures. Analysis of residuals
for log-mean fracture trace length indicates that in at least one instance,
fracture traces may be shorter over active structures in the Kansas study area.
The west Texas residuals map shows some alignment parallel to flightline
paths; however, a strong negative anomaly (shorter lengths) may be associated
with a through-going lineament in the area. Both areas show a shortening of
fracture traces in areas underlain by tectonic structures (anticlines, lineaments),
possibly due to increased fracturing and subsequent erosion of the fractures.
Fracture traces and joints measured in an area underlain by Permian rocks (sandstones) coincide in orientation, but there may be large differences in the length of the frequency rays. In an area of Cretaceous rocks (limestone), the apparent displacement in orientation between joints and fracture traces may represent a possible second order shear relationship between the fracture trace and jointing directions.

Analysis of rose diagrams using a multivariate statistical approach shows that the basic source of variation is due to differences in surface lithologies, and that a lesser amount may be due to deformational effects in an active structure, thus changing the fracture pattern. For example, in the Kansas area, an anticlinal structure, with a normal fault at depth, was isolated from its surrounding neighbors, whereas in the west Texas area, the predominant effect was lithology even in the larger group classifications.

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FUTURE WORK

Similar areas should be studied to determine if a valid exploration technique has been developed. Additional work also should be carried out to determine if lithologic control may be extracted from fracture trace orientations summarized as rose diagrams. Conversion to percent rose diagrams may achieve this end.
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APPENDIX A

SOURCE LISTING OF FORTRAN IV COMPUTER PROGRAM "VECLEN"
VECTOR LENGTH PROGRAM

THE PROGRAM WAS WRITTEN BY MELVIN PODWYSSOCKI OF THE GEOSCIENCES
DEPT., THE PENNSYLVANIA STATE UNIVERSITY, APRIL, 1972 FOR THE VLN00095
IBM 360/67 COMPUTER, AND WAS MODIFIED IN APRIL, 1974, FOR USE VLN00095
ON OTHER COMPUTERS HAVING THE EQUIVALENT OF 120K BYTES STORAGE.VLN00095

PROGRAM SUMMARIZES VECTOR DATA AS FREQUENCY-LENGTH DISTRIBUTIONS.VLN00095
OPERATOR SPECIFIES CLASS LENGTH, SIZE OF MAXIMUM CLASS AND DETER=VLN00105
MINES THE DIMENSIONS OF THE AREA (GRID CELL) IN WHICH THE DATA VLN00115
ARE SUMMARIZED. A VECTORS IS COUNTED IN A GRID CELL IF ITS MID- VLN00125
POINT FALLS IN THE CELL. NO CONSIDERATION IS GIVEN TO VECTOR AZI=VLN00135
MUTH. PROVISION IS MADE FOR A LOG BASE 2 TRANSFORMATION IF DESI- VLN00145
RED TO ATTEMPT NORMALIZATION OF THE DATA. A TEST FOR NORMALITY ISVLN00155
MADE BY COMPARING A THEORETICAL DISTRIBUTION USING THE CALCULATIOVLN00165
MEAN AND STANDARD DEVIATION OF THE OBSERVED POPULATION AGAINST VLN00175
THE DISTRIBUTION OF THE OBSERVED POPULATION UTILIZING THE CHI-VLN00185
SQUARE CRITERION. EXAMPLES OF PROGRAM OUTPUT AND APPLICATIONS AREVLN00195
GIVEN IN. PODWYSSOCKI, M.P., 1974, "ANALYSIS OF FRACURE TRAJECT VLN00205
TIONS OF BURIED GEOLOGIC STRUCTURE"; NASA - GODDARD SPACE FLIGHT VLN00215
CENTER DOCUMENT X-523-74-200. DATA ARE READ FROM CARDS GENERATED VLN00235
BY VECTOR TRANSFORM PROGRAM (SEE PODWYSSOCKI, M.P., 1974, "FORTRAN VLN00245
IV PROGRAMS FOR SUMMARIZATION AND ANALYSIS OF FRACURE TRAJECT VLN00255
LINEAMENT PATTERNS"; NASA - GSFC DOCUMENT X-664-74-31). VLN00265
CONTROL & TITLE CARDS ARE READ FROM CARD READER WHILE DATA CARDS VLN00275
MAY BE READ FROM ANY UNIT DECLARED BY 'ITAPE' ON CONTROL CARD 4. VLN00285

ALL NUMERIC INPUT DATA IS RIGHT JUSTIFIED: "I" INDICATES INTEGER VLN00305
FORMAT, "F" INDICATES FLOATING POINT FORMAT, "A" INDICATES CHAR- VLN00315
ACTER FORMAT. "*" PRECEEDING NUMBERS INDICATES COLUMNS USED FOR VLN00325
EACH PARAMETER. TO SPECIFY NONUSE OF AN OPTION, PUNCH 0 VLN00325

**********CONTROL CARDS 1 THRU 3--------TITLE CARDS

TITLE WILL BE PRINTED AT THE TOP OF EACH GRID CELL SUMMARIZED
(20A4,#1-80), NOTE: 3 CARDS MUST BE USED; IF ALL 3 ARE NOT USEDVLN00375
, BLANK CARDS MUST BE INSERTED IN THEIR PLACE.

**********CONTROL CARD 4--------OPTIONS CARD

XINC=INCREMENT OF X-AXIS TRAVERSE IN MM. (14.,#1-4) VLN00405
YINC=INCREMENT OF Y-AXIS TRAVERSE IN MM. (14.,#5-8) VLN00415
XSTART=STARTING POINT FOR X-AXIS TRAVERSE IN MM. (14.,#9-12) VLN00425
YSTART=STARTING POINT FOR Y-AXIS TRAVERSE IN MM. (14.,#13-16) VLN00435
XSTOP=END OF X-AXIS TRAVERSE IN MM. (14.,#17-20) VLN00445
YSTOP=END OF Y-AXIS TRAVERSE IN MM. (14.,#21-24) VLN00455

NOTE: PROGRAM SUCCESSIVELY SCANS DATA IN MAP GRID CELLS 'XCELL' VLN00465
BY 'YCELL' IN SIZE, INCREMENTING BY 'XINC' UNTIL 'XMAX' > VLN00475
'YMAX'. WHEN 'YINC' IS INCREMENTED, PROGRAM TERMINATES WHEN VLN00485
'YMAX' > 'YSTOP'. NONE OF THE ABOVE 6 VALUES CAN BE NEGATIVE. VLN00495
AMPSCALE=MAP SCALE ENTERED AS MILES/MM. (F5.4,#25-29) VLN00505
NOTE: WHEN VECTORS ARE MEASURED ON A 1:24000 SCALE MAP AND OUT-VLN00525
PUT IS DESIRED IN MILES,*AMPSCALE*.0149 (I.E. 1 MM=.0149 MILES) VLN00525
DHINC=NUMERICAL VALUE OF EACH 'X' INCREMENT OF FREQUENCY-LENGTH VLN00535
HISTOGRAM (I.E., EACH X'= 2 VECTORS) (F5.2,#30-34) VLN00545
SCINC=NUMBER CLASS INTERVAL; DEPENDENT ON DATA TREATMENT VLN00555
(SH. 'TRAN' BELOW). (F5.2,#35-39) VLN00565
SLMAX=UPPER CLASS LIMIT OF LAST FREQUENCY-LENGTH CLASS (SEE VLN00575
'TRAN' BELOW). (F5.2,#40-44) VLN00585
NMST=PUNCH 1 FOR FREQUENCY-LENGTH HISTOGRAM IN PRINTED OUTPUT VLN00595
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C
NPUChC--PUNCH 1 IF FREQUENCY-LENGTH DISTRIBUTION IS DESIRED
C ON CARDS (I..4G)
C NSTAT--PUNCH 1 IF FREQUENCY MOMENTS (I.E. MEAN, STANDARD DEVIATION)
C, SKEWNESS, ETC.) ARE DESIRED ON CARDS (I..4G)
C XCELL=CELL SIZE (IN MM.) IN X DIRECTION (I..4G-51)
C YCELL=CELL SIZE (IN MM.) IN Y DIRECTION (I..4G-52-55)
C NFOLD--PUNCH 1 TO FOLD TAILS OF FREQUENCY-LENGTH DISTRIBUTION TO
C SATISFY REQUIREMENTS FOR CHI SQUARE TEST. TAILS ARE
C EXPECTED FREQUENCY OF A CLASS IS < 0.95 (I..4G-56)
C ITAPE2=LOGICAL UNIT FOR READING DATA CARDS GENERATED BY "TRANS-
C FORM" PROGRAM (I..4G-57-58)
C NTRAN--SELECTS DATA TREATMENT (I..4G-59-60)
C PUNCH I IF DATA IS TO BE TREATED IN A LINEAR FASHION (I.E.
C INTERVAL IN MILES AS SPECIFIED IN 'AMPSCL'). NOTE: 'NFOLD' CAN
C NOT BE 1 IF THIS OPTION IS CHOSEN
C PUNCH -1 IF DATA IS TO BE CONVERTED TO LOG BASE 2 BY THE FOR-
C MULA: Z = (1/LOGIO(2))*LOGIO(X)+6, WHERE X = VECTOR
C LENGTH IN MILES. NOTE: FOLD OPTION MAY BE USED.
C 'SCMAX' AND 'SCINC' ARE GOVERNED BY 'NTRAN'. FREQUENCY-LENGTH CLASSES BEGIN WITH A MINIMUM VALUE OF 0 AND INCREMENT BY 'SCINC'
C UNTIL 'SCMAX' IS REACHED. IF A LINEAR SCALE IS USED, 'SCMAX' IS
C > THE LARGEST VECTOR LENGTH IN MILES. IF DATA IS TRANSFORMED TO LOGARITHMS, 'SCMAX' IS DETERMINED BY THE CONVERSION OF 'SCINC'
C CHOSEN APPROPRIATELY FOR EACH CASE.
C
******* DATA CARDS ------
C VECTOR DATA INPUT FROM VECTOR TRANSFORM PROGRAM
C
DIMENSION TITLE(60),VECLEN(200),XMID(200),YMID(200),Z(200),SECVLN00895
IIN(40),SCMAX(40),FNUM(40),ZI(40),AREA(40),DIFF(40),FREQX(40),CHISVLN00905
20(40),D(40),F(40),FD(40),FD(40),FD(40),FD(40)
DIMENSION FFreqX(40), FNUM(40),CHISQ(40)
C READ CONTROL CARDS
C
READ (READ,5) (TITLE(L),L=1,60)
READ (READ,10) XINC,YINC,XSTART,YSTART,XSTOP,YSTOP,AMPSCL,DHINC,
LSCINC,SCMAX,NHIST,NPUNCH,NSTAT,XCELL,YCELL,NFOLD,ITAPE2,NTRAN
IF(XINC.LT.0.OR.YINC.LT.0.OR.XSTART.LT.0.OR.YSTART.LT.0.OR.XSTOP.
LT.0.OR.YSTOP.LT.0.OR.XCELL.LT.0.OR.YCELL.LT.0.OR.NFOLD.EQ.1.AND
2,NTRAN.GT.0) BOMB=1
11 IF(BOMB) 18,18,13
13 WRITE(IPRINT,15) XINC,YINC,XSTART,YSTART,XSTOP,YSTOP,XCELL,YCELL
1,1,NFOLD,NTRAN
GO TO 800
C
READ DATA CARDS FROM LOGICAL UNIT 'ITAPE2'
C
18 GO 25 I=1,5000
READ(ITAPE2,20,END=30) VECLEN(I),XMID(I),YMID(I)
IF(I.EQ.0000) 25,25,22
22 WRITE(IPRINT,24)
GO TO 800
25 NUM =1

A-3
30 DO 35 M=1,NUM
31 Z(M)=VECLEN(M)*AMPSCL
32 IP(NTRAN)=32,13,35
33 CONTINUE
34 IF(NCLASS=40)39,39,36
35 CONTINUE
36 WRITE(IPRINT,38)SCLMAX,SCINC
37 CONTINUE
38 DO 70 N=1,NCLASS
39 SCMI(N)=0.
40 SCMAX(N)=SCMIN(N)+SCINC
41 CONTINUE
42 WRITE(IPRINT,38)SCLMAX,SCINC
43 IF(NCLASS=40)39,39,36
44 WRITE(IPRINT,38)SCLMAX,SCINC
45 CONTINUE
46 CONTINUE
47 CONTINUE
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227 CONTINUE
228 CON
150 CONTINUE
  NC=(SCMAX-0.)/SCINC+0.5
C ELIMINATION OF LOWER EMPTY CLASSES
C DO 161 JK=1,NCLASS
  IF(FNUM(JK)) 161,161,162
161 CONTINUE
162 JKL=JK
C ELIMINATION OF EMPTY UPPER CLASSES
C DO 163 JK=1,NCLASS
  KH=(NCLASS-JK)+1
  IF(FNUM(KH)) 163,163,164
163 CONTINUE
164 JKH=KH
C CALCULATE STATISTICAL MOMENTS FOR EACH GRID CELL
MAXCLS=1
DO 166 M=JKL,JKH
  IF(FNUM(M)) 166,166,165
165 MAXCLS=M
166 CONTINUE
  CLSMCP=(SCMIN(MAXCLS)+SCMAX(MAXCLS))/2
DO 170 WS=JKL,JKH
  D(MS)=((SCMIN(MS)+SCMAX(MS))/2-CLSMCP)/SCIINC
170 CONTINUE
  SD=0.
  SD2=0.
  SD3=0.
  SD4=0.
  SD5=0.
DO 175 I=JKL,JKH
  FD(I)=FNUM(I)*D(I)
  SD=SD+FD(I)
  SD2=SD2+FD2(I)
  SD3=SD3+FD3(I)
  SD4=SD4+FD4(I)
  SD5=SD5+FD5(I)
175 CONTINUE
  GCK=SD-(4.*SD3)+6.*SD2)-4.*SD)+TFNUM
  AMOM1=SD/TFNUM
  AMOM2=SC2/TFNUM
  AMOM3=SC3/TFNUM
  AMOM4=SC4/TFNUM
  AMOM5=AMOM1*SCINC
  TMC1=AMOM1*SCINC
  TMC2=SCINC**2*AMOM2-(AMOM1**2)
  TMC3=SCINC**3*(AMOM3-3.*AMOM2*AMOM1+2.*SCINC**2)
  PT4A=AMOM4-(4.*AMOM3*AMOM1)+6.*SCINC**2*AMOM2)
  PT4B=PT4A-3.*AMOM1**3)
  TMC4=SCINC**4*PT4B
  XBAR=CLSMDP+TMC1
  VAR=TMC2
  STDV=SQRT(VAR)
  RTBI=(TMC3/(SQRT(VAR))**3))
C CALCULATE CHI SQUARE CONTRIBUTION & EXPECTED FREQUENCIES FOR EACH C CLASS ASSUMING A NORMAL DISTRIBUTION WITH A XBAR & STDV OF THE C OBSERVED POPULATION

NDF=(JKH-JKL)-2
DO 250 J=JKL,JKH
ZI(J)=(SCMAX(J)-XBAR)/STDV
200 CONTINUE
TDIFF=0.
TFREX=0.
TCISHO=0.
IJK=0
IKC=JKL-1
AREA(IKC)=0.
DO 250 I=JKL,JKH
IF(ZI(I))210,220,220
210 AVT=-ZI(I)
GO TO 230
220 AVT=ZI(I)
GO TO 230
230 AREA(I)=1.+AVT*((A1+AVT*(A2+AVT*(A3+AVT*(A4+AVT*A5)))))
AREA(I)=0.5/(AREA(I)**2)
IF(ZI(I))240,250,250
240 DIFF(I)=AREA(I)-AREA(IIV)
GO TO 260
250 IJK=IJK+1
IF (IJK-1) 260,260,270
260 DIFF(I)=1.-AREA(I)-AREA(IIV)
GO TO 280
270 DIFF(I)=AREA(IIV)-AREA(I)
280 TDIFF=TDIFF+DIFF(I)
FREQEX(I)=DIFF(I)/FNUM
TFREQX=TFREQX+FREQEX(I)
TCISHO=((FNUM(I)-FREQEX(I))**2)/FREQEX(I)
TCISHO=TCISHO+CHISO(I)
290 CONTINUE
CHIPR=PRICH(TCHISQ,NDF)
NFLAG=0
MFLAG=0
IF(NFOLD)495.495. 300
C FOLD LOWER TAIL OF DISTRIBUTION IF REQUIRED
DO 302 MP=JKL,JKH
FCISHO(MP)=CHISO(MP)
FFNUM(MP)=FNUM(MP)
302 CONTINUE
DO 310 LL=JKL,MAXCLS
IF(TFREQX(LL)-0.95)305.31C3,10
305 FFREQX(LL+1)=FFREQX(LL)+TFREQX(LL+1)
FFNUM(LL+1)=FFNUM(LL)+FFNUM(LL+1)
FCISHO(LL)=0.
JKL=LL+1
NFLAG=1
310 CONTINUE
JKL=(JKH-MAXCLS)+1
C FOLD UPPER TAIL OF DISTRIBUTION IF REQUIRED

A-6
REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR

C
DO 320 LHI=1,JKL
KHH= (JKH-LHI+1)
IF(FFRQX(KHH)-0.95) 315,320
315 FFRQX(KHH-1)=FFRQX(KHH-1)+FFRQX(KHH)
FFNUM(KHH-1)=FFNUM(KHH-1)+FFNUM(KHH)
FFCHSQ(KHH)=0.
JKHI=KHH-1
MFLAG=1
CONTINUE
IF(NFLAG) 325,325,330
325 J2=JKL
GO TO 335
330 J2=JKL
335 IF(NFLAG) 340,340,345
340 J3=JKH
GO TO 350
345 J3=JKH
350 TFFRXQ=0.
TFCHSQ=0.
DO 355 JI=J2,J3
TFFRXQ=TFFRXQ+FFRQX(JI)
TFCHSQ=TFCHSQ+FCHISQ(JI)
CONTINUE
355 NFDF=(J3-J2)-2
FCHPFE=PRBCHI(TFCHSQ,NFDF)
OUTPUT
495 WRITE(IPRINT,500) (TITLE(L),L=1,60)
NR=NMIN+YINC/ YINC
NCOL=NMIN+XINC/XINC
WRITE(IPRINT,510) NR,NCOL,XMIN,XMAX,YMIN,YMAX
WRITE(IPRINT,520)
NXERR=0
DO 630 IJ=JKL,JKH
WRITE(IPRINT,530) IJ,SCMIN(IJ),SCMAX(IJ),FREQEX(IJ),FCHISO(IJ),
IF(NFOLDEQ.1.AND.NFLAG.EQ.1.AND.J2.EQ.IJ) WRITE(IPRINT,532)
IFFRX(IJ),FCHISO(IJ),FFNUM(IJ)
IF(NFOLDEQ.1.AND.NFLAG.EQ.1.AND.J3.EQ.IJ) WRITE(IPRINT,532)
IFFRX(IJ),FCHISO(IJ),FFNUM(IJ)
IF(NPUNCH-1) 540,540,560
WRITE(IPUNCH,550)NR,NCOL,IJ,SCMIN(IJ),SCMAX(IJ),
FNUN(IJ)
540 WRITE(IPUNCH,550)NR,NCOL,IJ,SCMIN(IJ),SCMAX(IJ),
560 IF(NHIST-1) 630,630,630
570 NUmx= FFNUM(IJ)/DHINC
IF(NUMX) 600,580,600
580 WRITE(IPRINT,590)
GO TO 630
620 WRITE(IPRINT,625) (IHX,IXUKA=1,NUMX)
630 CONTINUE
WRITE(IPRINT,640) TFFRXQ,TCHISO,TFNUM,DHINC
IF(NFOLDEQ.1.AND.NFLAG.EQ.1.OR.MFLAG.EQ.1) WRITE(IPRINT,645)
TFCHEQ
IF(NXERR.EQ.1) WRITE(IPRINT,650)
WRITE(IPRINT,652) NF,CHIPRB
IF(NFOLDEQ.1.AND.NFLAG.EQ.1.OR.MFLAG.EQ.1) WRITE(IPRINT,654)
INFDF,FCHPRB
WRITE(*,655) MAXCLS,CLSMDP,FNUM(MAXCLS)
WRITE(*,660) XBAR,STDV,RTBH,RTB2
WRITE(*,665) NROW,NCOL,XMIN,XMAX,YMIN,YMAX,
WRITE(*,670) GCK,SDS
700 CONTINUE
5 FORMAT(20A4)
10 FORMAT(6I4,F5.4)  I FORMAT(1014.F5.4)
15 FORMAT(6I4,F5.4)
20 FORMAT(25,F6.I)
24 FORMAT(1*,ENUMBER OF FREQUENCY-LENGTH CLASSES EXCEEDS PROGRAM LIMIT OF 40 WHERE:
SCLMAX/SCINC=NCLASS/,
= 1,T69,F6.2
500 FORMAT(3(T30,2CA4//))
510 FORMAT(*0*,#COLUMN=I49T30'(o'I5 < X < I51°; Y < I51°
520 FORMAT(2X,'LOWER*.T42.ICHI'.o/T9.'CLASS'.o/ 'EXPECTED*.T41.'CLASS'.o/ 'OBSERVED'.t/)
530 FORMAT(3I5o2FI0.2.FI2.2)
532 FORMAT(°0O'T70'GaNE OR MORE FREQUENCY CLASSES EXCEED HISTOGRAM LIMITS')
550 FORMAT(°0O'T70'CEGREES OF FREEDOM (NON-FOLDED) =',E10.4)
625 FORMAT(°0O'T70'CHI SQUARE PROBABILITY =',E10.4)
652 FORMAT(°0O'T70'GIVEN A VALUE OF CHI-SQUARE AND ITS DEGREES OF FREEDOM,
FUNCTION CHI PROBCHI COMPUTES THE PROBABILITY OF A GREATER VALUE OF CHI-SQUARE.
FUNCTION CHI PROBCHI COMPUTES THE PROBABILITY OF A GREATER VALUE OF CHI-SQUARE.
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INTEGER TEST

ALL REAL*8 ARGUMENTS CHANGED TO DOUBLE PRECISION BY M.PODWYSOCKI.

DOUBLE PRECISION DSQRT, DEXP, ARG, SCHISQ, XPVL

DOUBLE PRECISION Q, R, S, T, V, V9, PRCB, S2PI, Z005, APPROX

DATA S2PI/2.5066282000/

Q(ARG)=(DEXP(-ARG*ARG*0.5)/2.5066282D0)*(T*(0.3183646D0+T*(-1.2216315D0+T*(-0.6673645D0+T(-0.095.05909D0*0T))))*XPL=2.37623596DO0 PRBCHI=0.0

IF(CHISO.LT.0.0) RETURN

IF(IDF.LE.0) RETURN

100 SCH(SQ=CHISQ S=1.0 V=IDF V9=2.0/FLOAT(9*IDF) U=-SCHISO*0.5 SCHISO=DSQRT(SCHISO)

IF (CABS(U).LT.1.74) GO TO 110

PROB=0.0

GO TO 240

C CHECK FOR DEGREES OF FREEDOM GREATER THAN 100 OR GREATER THAN 30

110 IF (IDF.GT.100) GO TO 200

IF (IDF.GT.30) GO TO 170

C DEGREES OF FREEDOM LESS THAN OR EQUAL TO 30

PROB=0.0

TEST=MOD(IDF,2)

IF (TEST.NE.0) GO TO 140

C EVEN DEGREES OF FREEDOM ** LESS THAN OR EQUAL TO 30 ** FORMULA

26*4.5, PAGE 941

IRANGE=(IDF-2)/2

IF (IRANGE.EQ.0) GO TO 130

do 120 I=1,IRANGE

IR=I+1

S=S*IR

120 PROB=PRCB+SCHISQ**IR/S

GO TO 230

C ODD DEGREES OF FREEDOM ** LESS THAN OR EQUAL TO 29 ** FORMULA

C 26*4.4, PAGE 941

140 IRANGE=(IDF-1)/2

IF (IRANGE.EQ.0) GO TO 160

do 150 I=1,IRANGE

IR=I+1

S=S*IR

150 PROB=PRCB+SCHISQ**IR/S

GO TO 230

T=1.0/(1.0+0.23314090D0*CHISQ)

PROB=2.0*(Q(SCHISQ)+2.0*(DEXP(U)/S2PI)*PRB)

GO TO 230
REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR.

C ************ GREATER THAN 30 DEGREES OF FREEDOM ************
C AN APPROXIMATE VALUE OF CHISO IS FIRST COMPUTED THEN COMPARED WITH
C THE GIVEN CHISO. IF THE APPROX. VALUE IS GREATER THAN THE GIVEN
C VALUE, Q(CHISO, IDF) IS RETURNED AS .995.
C ************
C FOR GREATER THAN 30 AND LESS THAN OR EQUAL TO 100 DEGREES OF FREEDOM,
C THE APPROX. VALUE OF CHISO AT THE .995 LEVEL IS COMPUTED BY FORMULA
C 26.4.17, PAGE 941. THE SIGN OF X(P) IN THE FORMULA WAS CHANGED
C FROM + TO - TO ALLOW COMPUTATION OF CHISO AT THE .995 LEVEL RATHER
C THAN THE .005 LEVEL AS IS THE CASE WHEN THE SIGN IS +.
C
170 APROX=((-XPL+DSQRT(V+V-1.0))**2)*0.5
   IF (APROX.LE.CHISO) GO TO 180
   GO TO 210
180 V=2.0*CHISO-DQRT(2.0*CHISO)-DQRT(2.0*V-1.0)
190 T=1.0/(1.0+0.2316419D0+V)
   PROB*Q(V)
   GO TO 230
C GREATER THAN 100 DEGREES OF FREEDOM. THE APPROX. VALUE OF CHISO
C IS COMPUTED BY FORMULA 26.4.16, PAGE 941. THE SIGN OF X(P) WAS
C CHANGED FOR THE SAME REASON AS ABOVE.
C
200 APROX=((-XPL+DSQRT(V+V-1.0))**2)*0.5
   IF (APROX.LE.CHISO) GO TO 220
210 PROB=0.995
   GO TO 240
220 V=DQRT(2.0*CHISO)-DQRT(2.0*V-1.0)
   GO TO 190
230 IF (PROB.GT.0.995) GO TO 210
240 PRBCHI=PROB
   RETURN
END