A SIMPLE RADIATIVE TRANSFER MODEL OF THE HIGH LATITUDE MESOSPHERIC SCATTERING LAYER

by

John R. Hummel

July 15, 1974

The research reported in this document has been supported by The National Aeronautics and Space Administration under Grant No. NGL 39-009-003.
A simple radiative transfer model of the particle layer found at 85 km over the summer poles is presented. The effects of the layer on the global radiative temperature, the polar region temperature, and the greenhouse effect are discussed. The estimated magnitude of the global radiative temperature change is $3.5 \times 10^{-3}K$ to $2.2 \times 10^{-2}K$, depending on the value of the imaginary part of the particle index of refraction. The layer is shown to have a possible secondary influence on the temperature of the polar region while the contribution which the layer makes to the greenhouse effect is shown to be negligible. The imaginary part of the particle index of refraction is shown to be important in determining the attenuation properties of the layer.
A Simple Radiative Transfer Model of
the High Latitude Mesospheric Scattering Layer

by

John R. Hummel

July 15, 1974

The research reported in this document has been supported by The
National Aeronautics and Space Administration under Grant No.
NGL 39-009-003.

Submitted by:

J. J. Olivero, Assistant Professor of Meteorology
Project Supervisor

Approved by:

J. S. Nisbet, Director
Ionosphere Research Laboratory

Ionosphere Research Laboratory
The Pennsylvania State University
University Park, Pennsylvania 16802
ACKNOWLEDGEMENTS

Encouraging discussions with Dr. J. R. Mentzer and Dr. J. J. Olivero of the Ionosphere Research Laboratory were helpful in preparing this report. Also support by The National Aeronautics and Space Administration under Grant No. NGL 39-009-003 is greatly appreciated.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. ATTENUATION CHARACTERISTICS</td>
<td>2</td>
</tr>
<tr>
<td>3. ENERGY CONSIDERATIONS</td>
<td>21</td>
</tr>
<tr>
<td>4. COMPOSITION AND SHAPE OF THE PARTICLES</td>
<td>29</td>
</tr>
<tr>
<td>5. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>31</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Layer Geometry</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Rayleigh cross section vs. wavelength</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Integrated backscatter cross section vs. wavelength</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>Absorption cross section vs. wavelength</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>Layer vertical optical depth vs. wavelength</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>Layer vertical optical depth vs. $n_2$</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Effective layer extinction vs. $n_2$</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>Layer albedo vs. $n_2$</td>
<td>19</td>
</tr>
<tr>
<td>9</td>
<td>Percent difference in solar flux vs. $n_2$ (global average)</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>$\Delta T$ vs. $n_2$ (global average)</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>$\Delta T$ vs. $n_2$ (polar average)</td>
<td>27</td>
</tr>
</tbody>
</table>
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Adopted properties of the particle layer</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>Solar fluxes incident on the top of the atmosphere</td>
<td>22</td>
</tr>
</tbody>
</table>
ABSTRACT

A simple radiative transfer model of the particle layer found at 85 km over the summer poles is presented. The effects of the layer on the global radiative temperature, the polar region temperature, and the greenhouse effect are discussed. The estimated magnitude of the global radiative temperature change is $3.5 \times 10^{-3}$K to $2.2 \times 10^{-2}$K, depending on the value of the imaginary part of the particle index of refraction. The layer is shown to have a possible secondary influence on the temperature of the polar region while the contribution which the layer makes to the greenhouse effect is shown to be negligible. The imaginary part of the particle index of refraction is shown to be important in determining the attenuation properties of the layer.
CHAPTER I

INTRODUCTION

Attention has recently been focused upon a layer of particulate matter found at an altitude of about 85 km. As reported by Donahue et al., (1972), the layer, which is believed to consist of ice crystals, is found over the geographic poles during the local summer months. Noctilucent clouds, also found in the same altitude region, are thought to be extensions of the layer.

Although little is presently known about the layer, it could play a significant role in processes of the region. As shown in a recent paper by Olivero (1974) surface reactions on these upper atmospheric aerosols could affect the local photochemical balance. The radiative transfer properties of the layer could help to explain the mystery of the extreme temperatures of the high latitude mesopause and influence polar climatology in a non-negligible manner.

This report examines the radiative transfer properties of the layer with regards to global climatic effects. The model developed represents one part of a larger study which will encompass both the chemistry and dynamics of the region. As expected, the particle layer was found to have a negligible effect on the global effective radiative temperature of the Earth, the estimated magnitude of the change ranging from about $3.5 \times 10^{-3} \text{K}$ to $2.2 \times 10^{-2} \text{K}$. The layer is shown to have a possible secondary influence on the temperature of the polar region. Also, the contribution which the layer makes to the greenhouse effect is shown to be negligible. The extent to which the attenuation properties of the layer are dependent upon the imaginary part of the particle index of refraction is also shown. Finally, the composition and the shapes (geometry) of the particles making up the layer are discussed.
CHAPTER II

ATTENUATION CHARACTERISTICS

2.1 Basic Considerations

Radiation interacting with a particle can either be scattered by the particle, at an angle $\theta$ from the direction of propagation, or it can be absorbed by the particle. Here we assume the particles to be spherical and to be evenly distributed within a layer of thickness $\delta z$. The layer is sufficiently tenuous to allow multiple scattering to be neglected.

The radiation incident upon the layer, $I_o$, can be reduced by scattering and absorption within the layer. The radiation directly transmitted by the layer, $I_t$, is the incident radiation less that which has been scattered away and absorbed by the particles.

The scattered radiation can be broken down into forward and backward components. The backward component, $I_{bs}$, represents that radiation reflected back to space. The forward component, $I_{fs}$, represents that passed through the layer but in directions other than the direction of propagation of the incident radiation. At a large enough distance below the layer, $\Delta z >> \delta z$, the total intensity one would measure, $I_t'$, is equal to the sum of that transmitted and that scattered in the forward direction,

$$I_t' = I_t + I_{fs} \quad (1)$$

Calling $I_{abs}$ the intensity of the radiation lost to the particles via absorption we can express the radiation budget as

$$I_o = I_t' + I_{bs} + I_{abs} \quad (2)$$
2.2 **Determination of Backscattering, Absorption, and Extinction Cross Sections**

Let the total energy scattered in all directions by a single particle be equal to the energy of the incident wave falling on the area $\sigma_s$. The total energy absorbed is equal to the energy incident on the area $\sigma_{abs}$. The total energy removed from a pencil beam due to the presence of a particle is then equal to that incident on the area $\sigma_e$ where

\[
\sigma_e = \sigma_s + \sigma_{abs}.
\]

The areas $\sigma_e$, $\sigma_s$, and $\sigma_{abs}$ are, respectively, the cross sections for extinction, scattering, and absorption. Recalling that scattering occurs in both forward and backward directions we can write the scattering cross section as

\[
\sigma_s = \sigma_{bs} + \sigma_{fs}.
\]

Since, for the layer configuration used in this model, the radiation lost is due to backscattering and absorption the effective extinction cross section, $\sigma_e'$, is given by

\[
\sigma_e' = \sigma_{bs} + \sigma_{abs}.
\]

For a scatterer with a complex index of refraction $m$, given by

\[
m = n_1 - i n_2,
\]

the scattering cross section may, in general, be expressed as (Van de Hulst, 1957)

\[
\sigma_s(\lambda) = \frac{1}{K^2} \int \int F(X, \Theta, \Phi) \sin \Theta \, d\Phi d\Theta
\]
where $K$ is the wavenumber

$$ K = \frac{2\pi}{\lambda}, \quad (5) $$

$\lambda$ being the wavelength of the incident radiation, and $F(X, \Theta, \phi)$ is a complicated function, defining the intensity of the scattered radiation from a particle, described by a size parameter $X$ for an arbitrary scattering angle $\Theta$ and azimuth angle $\phi$. The size parameter is given by

$$ X = \frac{2\pi r}{\lambda}, \quad (6) $$

$r$ being the radius of the particle. The general expression for $\sigma_s$ given by (4) can, for various sizes of $X$, be replaced by simpler expressions. For example, when $X \ll 1$, or when the particle is much smaller than the wavelength of the incident radiation the Rayleigh scattering theory is applicable. In this case the scattering cross section is found to be proportional to $\lambda^{-4}$. (See Equation (17).)

When the particle is large compared to the wavelength, $X >> 1$, the general theory according to Mie is employed. In this case $F(X, \Theta, \phi)$ may be expressed as

$$ F(X, \Theta, \phi) = i_2(X, \Theta) \cos^2 \phi + i_1(X, \Theta) \sin^2 \phi \tag{7} $$

where $i_1(X, \Theta)$ and $i_2(X, \Theta)$ are the intensities of the scattered radiation per solid angle perpendicular and parallel, respectively, to the plane of scattering. Combining Equations (4) and (7) we get

$$ \sigma_s(\lambda) = \frac{1}{K^2} \int \int [i_2(X, \Theta) \cos^2 \phi + i_1(X, \Theta) \sin^2 \phi] \sin \Theta \, d\phi \, d\Theta. \quad (8) $$
Assuming the radiation to be linearly polarized and substituting Equation (5) for \( K \), we can express (8) as

\[
\sigma_s(\lambda) = \frac{\lambda^2}{4\pi} \int_0^{2\pi} [i_1(X, \theta) + i_2(X, \theta)] \sin \theta \ d\theta .
\]  

\( (9) \)

The intensity functions \( i_1(X, \theta) \) and \( i_2(X, \theta) \) are complicated expressions involving the Riccati-Bessel functions and their derivatives. When the complete expressions for \( i_1(X, \theta) \) and \( i_2(X, \theta) \) are substituted into (9) the Mie scattering cross section is found to be proportional to \( \lambda^{-2} \).

The absorption cross section is more difficult to represent. In Van de Hulst's (1957) treatment an absorption cross section is developed but the phenomenon represented by the cross section is backscattering plus pure absorption. This definition has been used in aerosol models (Ensor et al., 1971) and is a feature of a popular Mie scattering program (Dave, 1968) but this definition tends to obscure what is meant by absorption by a particle. The absorption properties of a particle are best treated theoretically from a quantum mechanical standpoint or experimentally (e.g., Lin, Baker, and Charlson, 1973). The absorption cross section can be approximated for small \( X \) as (Lin, Baker, and Charlson, 1973)

\[
\sigma_{abs}(\lambda) \approx \frac{2\pi^2 r^3}{\lambda} f(n_1, n_2) .
\]

\( (10) \)

where \( n_1 \) and \( n_2 \) are, respectively, the real and imaginary components of the index of refraction. For values of \( X \leq 10 \), Equation (10) can be approximated as

\[
\sigma_{abs}(\lambda) \approx \frac{2\pi^2 r^3}{\lambda} n_2 f(n_1) .
\]

\( (11) \)

For a real component of index of refraction \( n_1 = 1.33 \), \( f(n_1) \) is assigned the value of 1. Therefore, the absorption cross section is given by
In the present study the imaginary component of the index of refraction was varied from 0 to 0.9.

2.3 Optical Depth Determinations

Consider the layer geometry shown in Figure 1. As shown by Equation (2) the incident radiation is equal to the sum of that transmitted, absorbed, and reflected by the layer. Employing Beer's law the radiation transmitted by the layer of thickness $\delta z$ is given by

$$I_t'(\lambda, z) = I_o(\lambda) \exp[-\tau(\lambda, z)]$$

where $\tau(\lambda, z)$ is the optical depth. The optical depth for vertically incident radiation is given by

$$\tau(\lambda, z) = \int_{z}^{z+\delta z} \sum_{i} \sigma_{ei}(\lambda) n_i(z) \, dz$$

where $\sigma_{ei}(\lambda)$ is the extinction cross section of constituents $i$, given by Equation (3), $n_i(z)$ is the number density of constituent $i$, and the summation is performed over all of the constituents making up the layer. It is assumed here that the layer consists of both particulate matter and the molecular constituents making up the atmosphere. For the high latitude particle layer under consideration the incident radiation is not normal to the layer, therefore, the zenith angle, $\chi$, of the radiation must be considered when determining the amount of transmitted radiation. As $\chi$ increases, the path length of the radiation increases and the amount of transmitted radiation decreases. Therefore, the optical depth becomes
Figure 1. Layer geometry
\[ \tau(\lambda, z, \chi) = \sec \chi \int_z^{z+\delta z} \sum_i \sigma_{ei}(\lambda) n_i(z) \, dz. \]  

(15)

As stated previously, the layer has been found to extend over the summer geographic poles. For this work, it is assumed that the layer extends from 75° latitude to the poles. Furthermore, it is assumed that the number density of the particulate matter is uniform throughout the layer. With these assumptions and approximating \( \sec \chi \approx \sec \chi \), (15) can be written as

\[ \tau(\lambda, z) = \sec \chi \int_z^{z+\delta z} \sum_i \sigma_{ei}(\lambda) n_i(z) \, dz \]  

(16)

where \( \chi \) is the mean zenith angle for the layer. Table 1 lists the properties adopted for the layer.

The adopted atmospheric model consists of gaseous constituents, notably O, O₂, and N₂, and the particles making up the layer. It is assumed that for wavelengths above about 0.2 μ the gaseous (molecular) constituents interact with solar radiation only by scattering. (Note that at these high zenith angles little radiation at wavelengths below 0.2 μ reaches the mesosphere.) The scattering cross section for these constituents, describable by the Rayleigh scattering theory is (Van de Hulst, 1957)

\[ \sigma_{ray}(\lambda) = \frac{8\pi}{3} \left[ \frac{m_a^2 - 1}{\lambda^2 n_m} \right]^2 \frac{6 + 3\delta}{6 - 7\delta} \]  

(17)

where \( m_a \) is the index of refraction of the atmosphere, assumed real, \( n_m \) is the total number density of the molecular constituents, and \( \delta \) is the
Table 1. Adopted Properties of the Particle Layer

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude, $z$</td>
<td>85 km</td>
</tr>
<tr>
<td>Thickness, $\delta z$</td>
<td>3 km</td>
</tr>
<tr>
<td>Spatial</td>
<td>Uniform from $75^\circ$ latitude to the poles</td>
</tr>
<tr>
<td>Number density of molecular constituents, $n_m$</td>
<td>$1.532 \times 10^{14} \text{cm}^{-3}$ (CIRA, 1965)</td>
</tr>
<tr>
<td>Aerosol number density, $n_L$</td>
<td>$33 \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Aerosol particle radius (monodispersed), $r$</td>
<td>$0.13\mu$</td>
</tr>
<tr>
<td>Mean zenith angle, $\bar{\chi}$</td>
<td>$79.7^\circ$</td>
</tr>
<tr>
<td>Range of particle index of refraction, $m$</td>
<td>$1.33 - 0i$ to $1.33 - 0.9i$</td>
</tr>
</tbody>
</table>
depolarization factor, assumed negligible. The index of refraction is, in general, given by (Bean, 1962)

\[ m_a = 1 + \frac{7.76 \times 10^{-5} P}{T} + \frac{7.72 \times 10^{-5} e}{T} + \frac{0.375 e}{T^2} \quad (18) \]

where \( P \) is the pressure in millibars, \( T \) is the temperature in degrees Kelvin, and \( e \) is the partial pressure of water vapor in millibars. Assuming negligible amounts of water vapor we can approximate (18) as

\[ m_a \approx 1 + \frac{7.76 \times 10^{-5} P}{T} \quad (19) \]

For this work \( m_a = 1, m_a - 1 = 1.66 \times 10^{-9} \). In Figure 2 the Rayleigh cross section is plotted against wavelength. The optical depth for the molecular constituents may be expressed as

\[ \tau_m(\lambda, z) = \sec \chi \frac{H}{n_{om}} \sigma_{ray}(\lambda) \quad (20) \]

where \( H \) is the mean scale height of the atmosphere and \( n_{om} \) is the number density of the molecular constituents at the base of the layer. The optical depth expressed by (20) is that for a column of air extending from the base of the aerosol layer to infinity.

For the aerosol components the optical depth is given by

\[ \tau_L(\lambda, z) = \sec \chi \sigma_e(\lambda) n_L \delta z \quad (21) \]

where \( n_L \) is the average number density of the particles in the layer. From Equation (3) we can write (21) as

\[ \tau_L(\lambda, z) = \sec \chi \left[ \sigma_{abs}(\lambda) + \sigma_{bs}(\lambda) \right] n_L \delta z \quad (22) \]

where \( \sigma_{bs}(\lambda) \) is the integrated backward scattering cross section referenced to the layer orientation as shown in Figure 1. For this orientation \( \sigma_{fs}(\lambda) \)
Figure 2. Rayleigh cross section vs. wavelength
In Figures 3 and 4 are, respectively, the backscattering and absorption cross sections plotted against wavelength for various values of \( n_2 \). It is evident that as the imaginary part of the index of refraction, \( n_2 \), increases the absorption cross section begins to dominate. This result has been confirmed by other workers (Kattawar and Plass, 1967; Plass, 1966; and Ensor et al., 1971). As one might expect, the particle layer optical depth was found to be greater than that of the molecular constituents above the 85 km level, the ratio \( \tau_L/\tau_m \) being on the order of \( 10^2 \) to \( 10^5 \).

In Figure 5 the layer vertical optical depth is plotted against wavelength for various values of \( n_2 \). In Figure 6, the layer vertical optical depth is plotted against \( n_2 \) for a wavelength of 0.55\( \mu \). It is again worth noting that the backscatter cross sections used in determining (22) are integrated over all backscatter angles. The results reported by Donahue et al., (1972) were obtained from the OGO-6 photometer which measured airglow emissions of sodium (5893 \( \AA \)) and the atomic oxygen green line (5577 \( \AA \)), (Donahue et al., 1973). Their vertical optical depth values of \( 4 \times 10^{-5} \) to \( 7 \times 10^{-5} \) were determined for a scattering angle of 80°, particle radius of 0.13\( \mu \), and real index of refraction of 1.33 (Guenther, personal communication). Using these conditions and a wavelength of 0.5577\( \mu \), the vertical optical depth from (22) is \( 1.5 \times 10^{-4} \).

From (22) and knowing that \( \tau_L >> \tau_n \) we can write (13) as

\[
I_t(z, \lambda) = I_0(\lambda) \exp \left\{ - \sec \chi \left[ \sigma_{\text{abs}}(\lambda) + \sigma_{\text{bs}}(\lambda) \right] n_L \delta z \right\}
\]

(23)

By dividing both sides of (23) by \( I_0(\lambda) \) one gets an expression for the transmittance of the layer, \( T_L \), the fraction of the incident radiation...
Figure 3. Integrated backscatter cross section vs. wavelength
Figure 4. Absorption cross section vs. wavelength
Figure 5. Layer vertical optical depth vs. wavelength
Figure 6. Layer vertical optical depth vs. \( n_2 \)

\[ \begin{align*}
\lambda &= 0.55 \mu \\
r &= 0.13 \mu \\
n_i &= 1.33
\end{align*} \]
that is transmitted by the layer,

\[ T_r(z, \lambda) = \frac{I'_L(z, \lambda)}{I_o(\lambda)} \approx \exp \{ - \sec \chi [ \sigma_{abs}(\lambda) + \sigma_{bs}(\lambda) ] n_L \delta z \}. \]  

(24)

In Figure 7, \( 1 - T_r(z, \lambda) \), the effective layer extinction, is plotted against \( n_2 \) for \( \lambda = 0.55 \mu \). The first feature that is evident is that the layer is essentially transparent to the incoming radiation. For longer wavelengths, \( \lambda > 1.0 \mu \), the transparency is more striking. The other feature is that as \( n_2 \) increases \( T_r \) decreases, due to the increasing importance of absorption within the layer.

One may also determine the layer albedo, \( A_r \), the fraction of incident radiation reflected back to space,

\[ A_r(z, \lambda) = \frac{I_{bs}(z, \lambda)}{I_o(\lambda)}, \]

where \( I_{bs}(z, \lambda) \) is the reflected radiation, given by

\[ I_{bs}(z, \lambda) = I_o(1 - \exp \{ - \sec \chi \sigma_{bs}(\lambda) n_L \delta z \}). \]  

(25)

Employing the approximation that \( e^{-x} = 1 - x \) we can express the albedo as

\[ A_r(z, \lambda) = \sec \chi \sigma_{bs}(\lambda) n_L \delta z. \]  

(26)

In Figure 8 the layer albedo is plotted against \( n_2 \) for \( \lambda = 0.55 \mu \) and \( \chi = 79.7^\circ \). Again, the importance of \( n_L \) and hence absorption within the layer is evident. The vertical optical depths of Donahue et al., (1972) correspond to albedo values of \( 2.2 \times 10^{-4} \) to \( 3.9 \times 10^{-4} \) when the zenith angle dependence is included. For noctilucent clouds, Sharanov (1960) has calculated apparent albedo values of \( 2.3 \times 10^{-5} \) to \( 4.7 \times 10^{-5} \).
Figure 7. Effective layer extinction vs. $n_2$

- $\lambda = 0.55 \mu$
- $\bar{x} = 79.7^\circ$
- $r = 0.13 \mu$
- $n_1 = 1.33$
Figure 8. Layer albedo vs. $n_2$

- $r = 0.13 \mu$
- $\lambda = 0.55 \mu$
- $\bar{X} = 79.7^\circ$
- $n_1 = 1.33$
Employing a scattering angle of 80° and a wavelength of 0.5577μ the albedo calculated via (26) is $8.4 \times 10^{-4}$. 
CHAPTER III

ENERGY CONSIDERATIONS

3.1 Global

Consider now the balance between the radiant energy absorbed by the Earth and that emitted by the Earth. This may be expressed as

\[(1 - A) S_o = 4 \sigma T^4\]  \hspace{1cm} (27)

where A is the planetary albedo, \(S_o\) the incident solar flux striking the atmosphere, \(\sigma\) the Stefan-Boltzmann constant, and T the mean radiative temperature. It should be realized that the mean radiative temperature is not the surface temperature of the Earth but that which would be obtained from radiometric observations from outside the atmosphere. The solar flux values used in this work were taken from the Handbook of Geophysics and Space Environment (1965) and are listed in Table 2. A planetary albedo of 0.33 was taken from Pollack and Toon (1974).

Let us assume that Equation (27) represents the case where the aerosol layer is absent. Assuming, now, the presence of the layer we can write for the energy balance

\[(1 - A) \bar{S} = 4 \sigma (T')^4\]  \hspace{1cm} (28)

where \(\bar{S}\), the mean flux perturbed by the presence of the aerosol layer, is given by

\[\bar{S} = a S_L + b S_o\]  \hspace{1cm} (29)

where \(a\) is the fractional area of the Earth covered by the particle layer, \(S_L\) is the solar flux transmitted by the layer given by (22), \(b\) is the
Table 2. Solar Fluxes Incident on the Top of the Atmosphere
(Handbook of Geophysics and Space Environments, 1965)

<table>
<thead>
<tr>
<th>$\lambda$ ((\mu))</th>
<th>Flux (watts m(^{-2})) *</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>2.30</td>
</tr>
<tr>
<td>0.28</td>
<td>2.60</td>
</tr>
<tr>
<td>0.30</td>
<td>14.91</td>
</tr>
<tr>
<td>0.32</td>
<td>17.97</td>
</tr>
<tr>
<td>0.34</td>
<td>22.62</td>
</tr>
<tr>
<td>0.36</td>
<td>24.96</td>
</tr>
<tr>
<td>0.38</td>
<td>25.14</td>
</tr>
<tr>
<td>0.40</td>
<td>30.96</td>
</tr>
<tr>
<td>0.44</td>
<td>129.28</td>
</tr>
<tr>
<td>0.50</td>
<td>100.50</td>
</tr>
<tr>
<td>0.55</td>
<td>97.06</td>
</tr>
<tr>
<td>0.65</td>
<td>91.53</td>
</tr>
<tr>
<td>0.65</td>
<td>81.31</td>
</tr>
<tr>
<td>0.70</td>
<td>105.18</td>
</tr>
<tr>
<td>0.80</td>
<td>113.09</td>
</tr>
<tr>
<td>0.90</td>
<td>89.79</td>
</tr>
<tr>
<td>1.02</td>
<td>104.59</td>
</tr>
<tr>
<td>1.25</td>
<td>136.77</td>
</tr>
<tr>
<td>1.60</td>
<td>113.97</td>
</tr>
<tr>
<td>2.50</td>
<td>53.73</td>
</tr>
<tr>
<td>3.50</td>
<td>15.34</td>
</tr>
<tr>
<td>4.25</td>
<td>3.45</td>
</tr>
</tbody>
</table>

* Integrated over adjacent wavelength intervals, $\Delta \lambda$, indicated by interval centers
fractional area of the Earth not covered by the layer, and \( T' \) is the mean radiative temperature in the presence of the layer. Knowing that \( b = 1 - a \), we can write the temperature difference, \( \Delta T \), as

\[
\Delta T = T - T' \left( \frac{1 - A}{4 \sigma} \right)^{\frac{1}{4}} \left[ S_0^{\frac{1}{4}} - \overline{S}^{\frac{1}{4}} \right].
\]

(30)

The percent change in solar flux due to the layer,

\[
\frac{\Delta S}{S_0} = \frac{S_o - \overline{S}}{S_o},
\]

is shown in Figure 9 plotted against \( n_2 \), ranging from \( 5.6 \times 10^{-3}\% \) to \( 3.5 \times 10^{-2}\% \). The global temperature difference ranged from \( 3.5 \times 10^{-3} K \) to \( 2.2 \times 10^{-2} K \), again depending on \( n_2 \), shown in Figure 10. The temperature difference averaged globally represents a negligible cooling of the Earth. Again, it is mentioned that the temperature changes reflect changes in the mean radiative temperature of the Earth and not the surface temperature. In order to make surface calculations the energy of the atmospheric circulation, condensation, and evaporation must be considered. As mentioned by Rasool and Schneider (1971) surface temperature changes on the order of 3.5 K, if sustained over a period of several years, could be enough to trigger an ice age. The significance of such particle layers on global climatology is beyond the scope of the present investigation.

B. Polar

The radiative balance of the polar region can also be examined using a form of (27). In this case the radiative energy balance is given by

\[
a_c (1 - A_p) S_o = a_s 4 \sigma T^4
\]

(31)

where \( a_c \) is the fraction of the projected area poleward of \( 75^\circ \) latitude,
Figure 9. Percent difference in solar flux vs. \( n_2 \) (global average)
Figure 10. $\Delta T$ vs. $n_2$ (global average)
A_p is the albedo of the polar region, taken to be 0.75 (Atwater, 1970), and a_s is the fraction of the surface area poleward of 75° latitude. Again, a temperature difference between the layer and non-layer cases can be found, given by

\[ \Delta T = \left[ \frac{a_c (1 - A_p)}{a_s^4 \sigma} \right]^{\frac{1}{4}} \left[ S_o^{\frac{1}{4}} - S_L^{\frac{1}{4}} \right]. \] (32)

In Figure 11, \( \Delta T \) is plotted against \( n_2 \) for a zenith angle of 56.4°, corresponding to the summer solstice, and ranges from 6.09 x 10^{-2}K to 0.533K. The low \( n_2 \) values represent insignificant temperature differences but the upper extreme values are possibly non-negligible.

The layer is a present feature of the earth-atmosphere system and represents a cooling term in the energy budget. If the layer was removed the polar region would, in this steady state model, experience a warming up to a few tenths of a degree, depending on \( n_2 \), due to the added radiation reaching the surface. This warming alone most likely would not be enough to alter the climatology of the region, but as Oort (1974) has pointed out it may have a secondary influence modifying the temperature anamolies brought about by variations of the energy influx from middle latitudes or the Earth's surface.

C. Contributions to the Greenhouse Effect

The radiation emitted by the Earth undergoes attenuation by the layer also. Radiation reflected back to the Earth from the layer provides another heat source for the overall energy budget of the Earth, contributing to the so-called greenhouse effect. As shown by Figure 3, the backscatter cross section is extremely small for longer wavelengths, approaching the
Figure 11. $\Delta T$ vs. $n_2$ (polar average)
Rayleigh regime. For a wavelength of 11.0\mu, \sigma_{bs} = 10^{-15} \text{cm}^2 for 
\mu = 1.33 - 0i, which corresponds to a vertical optical depth of \tau_L \approx 
10^{-8}. Approximating the transmittance as 

\[ T_r(z, \lambda) = 1 - \tau_L \]

it can be seen that the layer is essentially transparent to the Earth's 
outgoing radiation. The heating resulting from the reflected radiation 
from the scattering layer is on the order of $10^{-6}$K averaged globally. 
For the polar region the heating is of the order of $10^{-5}$K.
CHAPTER IV

COMPOSITION AND SHAPE OF THE PARTICLES

The composition of the particles in the layer is crucial to understanding the attenuation properties of the aerosol layer. Unfortunately, little is known about the precise make-up of the particles. Polarization measurements of noctilucent clouds, which are believed to be equatorward extensions of the layer, indicate that noctilucent cloud particles are non-metallic with a real refractive index between 1.2 - 1.6 (Witt, 1960 a, b; Villmann, 1962). As mentioned earlier, the imaginary part of the index of refraction is more difficult to establish.

Rocket samples of noctilucent cloud particles revealed the presence of nickel and iron in the particles, suggesting a meteoric origin (Witt et al., 1964). Also, in one series of rocket samples the particles were found to have been coated with a volatile substance, at the time of collection, believed to be ice, (Hemenway et al., 1964; Skrivanek and Soberman, 1964). The findings of Donahue et al., (1972) involving the particle layer were based on the assumption of spherical ice particles of radius 0.13μ and real refractive index of 1.33.

The value of 1.33 for the real part of the particle index of refraction was chosen for two reasons. First, use of \( n_1 = 1.33 \) allowed the results from the model to be compared with the results of Donahue et al., (1972). Second, tabular results for the scattering functions of particles with \( n_1 = 1.33 \) are known, thereby permitting comparison of the results in the model.

The shapes of the particles are also poorly understood. Classically, the particles have been assumed spherical in order to simplify the scattering problem. Rocket samples reveal, though, that the particles need not
be spherical (Farlow et al., 1970 and Frank et al., 1970). Their results show that the particles may be rodlike, irregular, or have fluffy, dendritic shapes in clusters.

The shapes of particles would greatly affect the scattering and polarization results and the area-to-mass ratio data for meteorites. As mentioned before, theoretical results of scattering and polarization assume spherical particles for simplicity, the reason being that determining scattering from non-spherical particles is an extremely difficult task. Also, it is extremely difficult to determine particle shape from experimental data (e.g., lidar), which then makes it difficult for one to make estimates of area-to-mass ratio.
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

A simple radiative transfer model has been developed to examine the radiation attenuation properties of the particle layer found at 85 km over the summer geographic poles. As expected, the layer was found to have a negligible effect on the global radiative temperature of the Earth. The layer, though, does have a possible secondary influence on the temperature of the polar region. The contribution to the greenhouse effect was also found to be minimal. In all of these findings the imaginary part of the particle index of refraction was shown to be important in determining the attenuation properties of the layer.

The model assumed a monodispersed distribution for the particles within the aerosol layer. From polarization measurements (Witt, 1960a, b; and Villmann, 1962) and rocket samples (e.g., Hemenway et al., 1964b) of noctilucent clouds it is known that the particles have a size distribution that can be approximated by a Junge distribution

\[ N(d) = A d^{-\rho}, \]

where \( N(d) \) is the number density, \( d \) is the particle diameter, \( A \) is a constant, and \( 3 < \rho < 4 \). The attenuation properties of the layer assuming a polydispersed distribution need to be examined.

The greatest source of uncertainty in the present model lies in characterizing the absorption cross section. Estimates of the imaginary part of the index of refraction, \( n_2 \), must be considered uncertain to an order of magnitude. In addition, the expression adopted for the absorption cross section itself, while probably quite adequate for small values of \( n_2 \) (of order \( 10^{-2} \)), is likely to be uncertain to factors of the order of 2 for the larger values of \( n_2 \) used in the present work. Much still needs to be done to firmly establish the absorption properties of upper atmospheric aerosols.
REFERENCES


A simple radiative transfer model of the particle layer found at 85 km over the summer poles is presented. The effects of the layer on the global radiative temperature, the polar region temperature, and the greenhouse effect are discussed. The estimated magnitude of the global radiative temperature change is $3.5 \times 10^{-3} K$ to $2.2 \times 10^{-2} K$, depending on the value of the imaginary part of the particle index of refraction. The layer is shown to have a possible secondary influence on the temperature of the polar region while the contribution which the layer makes to the greenhouse effect is shown to be negligible. The imaginary part of the particle index of refraction is shown to be important in determining the attenuation properties of the layer.