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# A DATA REDUCTION TECHNIQUE AND ASSOCIATED COMPUTER PROGRAM FOR OBTAINING VEHICLE ATTITUDES WITH A SINGLE ONBOARD CAMERA

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16. Abstract

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# A DATA REDUCTION TECHNIQUE AND ASSOCIATED COMPUTER PROGRAM FOR OBTAINING VEHICLE ATTITUDES WITH A SINGLE ONBOARD CAMERA

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## SUMMARY

A detailed discussion of the application of a previously developed method to determine vehicle flight attitude using a single camera onboard the vehicle is presented with emphasis on the digital computer program format and data reduction techniques. Application requirements include film and Earth-related coordinates of at least two landmarks (or features), location of the flight vehicle with respect to the Earth, and camera characteristics. Included in this report are a detailed discussion of the program input and output format, a computer program listing, a discussion of modifications made to the initial method, a step-by-step basic data reduction procedure, and several example applications. The computer program is written in FORTRAN IV language for the Control Data 6000 series digital computer.

## INTRODUCTION

A postflight photogrammetric method was previously devised for determining a continuous history of vehicle flight attitudes (Euler angles) using only film data from a single onboard camera along with a ground track of the vehicle. A discussion of method requirements, assumptions, mathematical relationships, and results is presented in reference 1. The method is based on work presented in references 2 and 3. Results from other applications and comparisons with statistical trajectory reconstruction techniques are included in references 4 to 6. However, detailed discussions of method application or data reduction system capabilities are not included in the reference documents. Numerous inquiries have been received concerning method application, particularly from the standpoint of computer programing. In addition, several modifications to the program which make the method more versatile and streamline its use have been made for application to the Viking Balloon Launched Decelerator Test (BLDT) program data reduction

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(refs. 4 and 5). The purpose of this paper is to present details concerning method application, including an updated program listing and example applications, and to discuss current capabilities and restrictions of the associated Langley Research Center data reduction system. This paper is intended as a user's guide for future applications of the method.

## SYMBOLS

Measurements are presented in both SI units and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

| | |
|---|---|
| c | focal length of camera |
| h | height above sea level of camera, meters (feet) |
| I | point in image space |
| K | constant of proportionality between object space coordinate system and image space coordinate system |
| n | number of observations |
| Q | point in object space |
| R | geocentric earth radius, meters (feet) |
| X,Y,Z | object space coordinate axes with origin at focal point; also, distance along these axes, meters (feet) |
| $\overline{X},\overline{Y},\overline{Z}$ | distance from origin of geocentric coordinate axes, meters (feet) |
| $X_E,Y_E,Z_E$ | Earth-fixed axis system |
| $X_I,Y_I,Z_I$ | image space coordinate axes with origin at focal point |
| $\bar{x},\bar{y}$ | measured image coordinates in image space coordinate system |
| $\bar{x}_p,\bar{y}_p$ | coordinates of point of intersection between focal plane and focal axes in image space coordinate system; center of frame |

| | |
|---|---|
| $\alpha,\beta$ | angles of attack in pitch and yaw, respectively, degrees |
| $\Delta\sigma$ | change in attitude angle, degrees |
| $\epsilon$ | error in $\bar{x}$ |
| $\bar{\epsilon}$ | error in $\bar{y}$ |
| $\eta$ | total angle of attack, degrees |
| $\Lambda$ | longitude of camera measured positive east from Greenwich, degrees |
| $\lambda_1,\mu_1,\nu_1$; $\lambda_2,\mu_2,\nu_2$; $\lambda_3,\mu_3,\nu_3$ | directional cosines of image space coordinate system relative to object space coordinate system |
| $\sigma_1,\sigma_2,\sigma_3$ | camera azimuth, pitch, and roll Euler angles relative to Earth-fixed axes, degrees |
| $\Phi$ | geodetic latitude of camera measured positive north, degrees |
| $\Phi'$ | geocentric latitude of camera measured positive north, degrees |
| $\psi,\theta,\phi$ | vehicle yaw, pitch, and roll Euler angles relative to Earth-fixed axes, degrees |

Subscripts:

| | |
|---|---|
| i | for camera $i = 1$ and for object space points $i = 2, \ldots, n + 1$ |
| o | initial conditions |
| S | vehicle or spacecraft |

Superscript:

| | |
|---|---|
| T | denotes transpose of matrix |

## CAMERA-VEHICLE ORIENTATION METHOD

In order to determine the orientation of a flight vehicle (or spacecraft) with respect to the Earth by using this photogrammetric method, the following information is required: location of at least two landmarks on each frame of the film, Earth-related coordinates of these landmarks, location of the flight vehicle with respect to Earth (as determined by radar, for example), orientation of the camera within the vehicle, camera focal length, and lens distortion characteristics. Atmospheric refraction corrections are not included because these corrections would have been insignificant for previous applications. Digital computer techniques can then be applied to determine the relationship between the two coordinate systems defined by the film frame (image space) and the Earth (object space). For the convenience of the potential user, the mathematics of the relationship (taken directly from ref. 1) are included as appendix A.

### Modifications

For the previous applications of the method, two separate digital computer programs were employed to operate on the raw data (image space identification points read from the film) before the final camera or vehicle orientation angles (Euler angles, figs. 1 and 2) were determined. Also a third program was required to operate on the vehicle Euler angle ($\psi, \theta, \phi$) data and produce vehicle angles ($\alpha, \beta, \eta$) relative to the wind. In the modified version, presented in this paper, the three programs have been combined into one without any reduction in data output and with significant improvement in user and computer efficiency. The new version also includes simplified input-output procedures and a multiple job processing capability. Another significant modification, to be discussed in the following section, involves converting unknown surface features or nonpermanent objects appearing on the film into usable "landmarks."

### Method Application

Application of the method is discussed in terms of procedures by use of the basic data reduction and computer systems currently available at the NASA Langley Research Center and as used to determine test vehicle motions from the Viking Balloon Launched Decelerator Test (BLDT) Program (refs. 4 and 5).

Landmark selection.- The initial, and frequently very difficult, step encountered when using this technique involves identifying distinct features on the film (1) which appear on a sufficient number of frames and (2) the geodetic coordinates of which can be

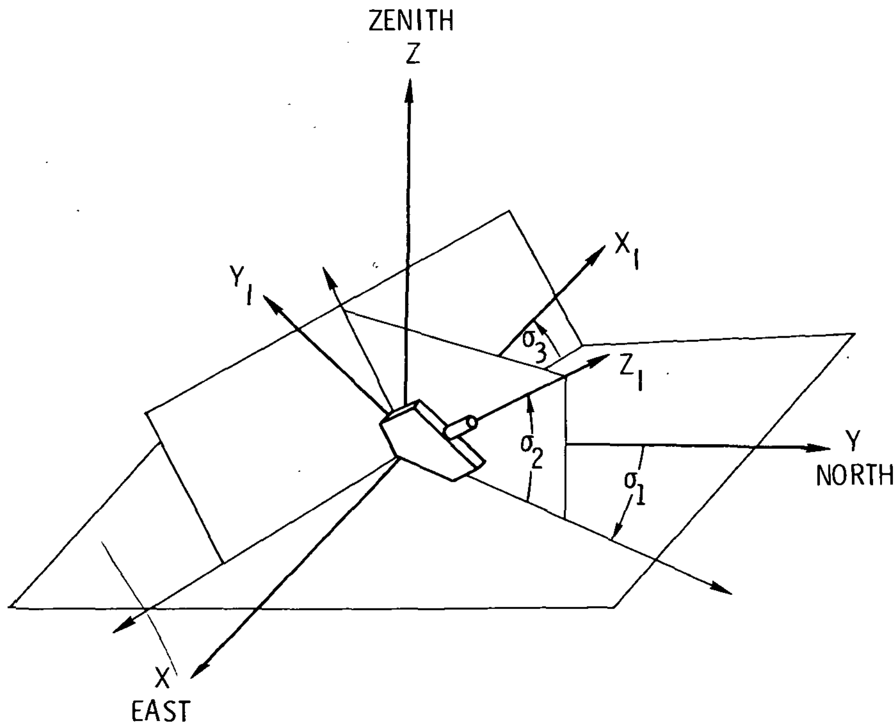



Figure 1.- Camera Euler angles.

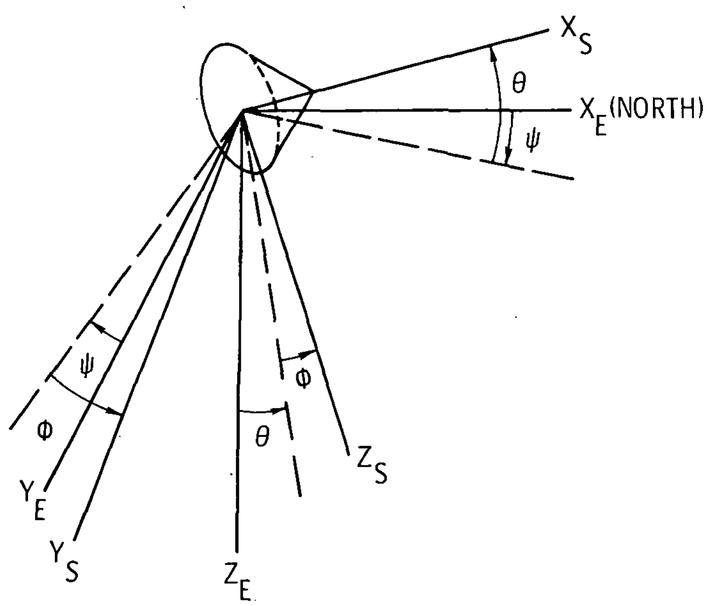



Figure 2.- Vehicle Euler angles.

determined. For many applications the user has little control over what Earth-related features will be photographed during flight and must instead work with what the film data offer. Many hours can be spent matching a series of features appearing on the film with the corresponding features on a topographical map so that their coordinates can be determined. A feature whose coordinates have been identified is considered a landmark.

Often, because of the flight trajectory and vehicle motions, a landmark may appear only on a relatively small number of frames. On the other hand, features appearing on the film which cannot be identified on a map may be visible for a relatively long period of time. Such nonpermanent features as clouds, ground vehicles, and ships might fit this category if they were slowly moving with respect to the camera frame rate (that is, quasi-stationary). The program has been modified to allow the user the flexibility to process several "trial" coordinates (latitude, longitude, and altitude) for a single feature in one computer run; this procedure allows for a quick iterative solution of its actual location. The user can accomplish this determination of the location by comparing vehicle Euler angle results obtained by using only known landmarks with those obtained by using both known landmarks and features. Solving for the coordinates of the feature readily follows and permits the feature to be used as a known landmark in other frames of the film where actual landmarks are not distinguishable.

In applying this option, the iteration procedure is initiated by first estimating the location of the feature and running a series of latitudes about that estimation. By comparing the resultant Euler angles with the values from the known landmarks, the best latitude is obtained. A series of longitudes is then processed at that latitude; as a result, a best estimate of longitude is obtained. The best estimate of altitude is established in a similar manner. With these new altitude and longitude values, the process can be repeated to define the latitude more accurately and, subsequently, the longitude and altitude parameters. The landmarks defined in this manner may be used to find the coordinates of other quasi-static features during other parts of the data period. Application of this option permits obtaining vehicle orientation data for time periods when no true landmarks are in view of the camera.

As previously stated, a minimum of two landmarks is required for each frame. Theoretically, no limits are required on the maximum number but satisfactory results have been obtained by using from two to six landmarks, three landmarks being the preferred number. The current computer program in use at Langley Research Center is set up with a six landmark maximum.

Basic data reduction.- The basic data consist of film (image space) coordinates of the selected landmarks (or features to be converted to landmarks) with respect to the center of the frame. The basic data are punched on digital computer cards.

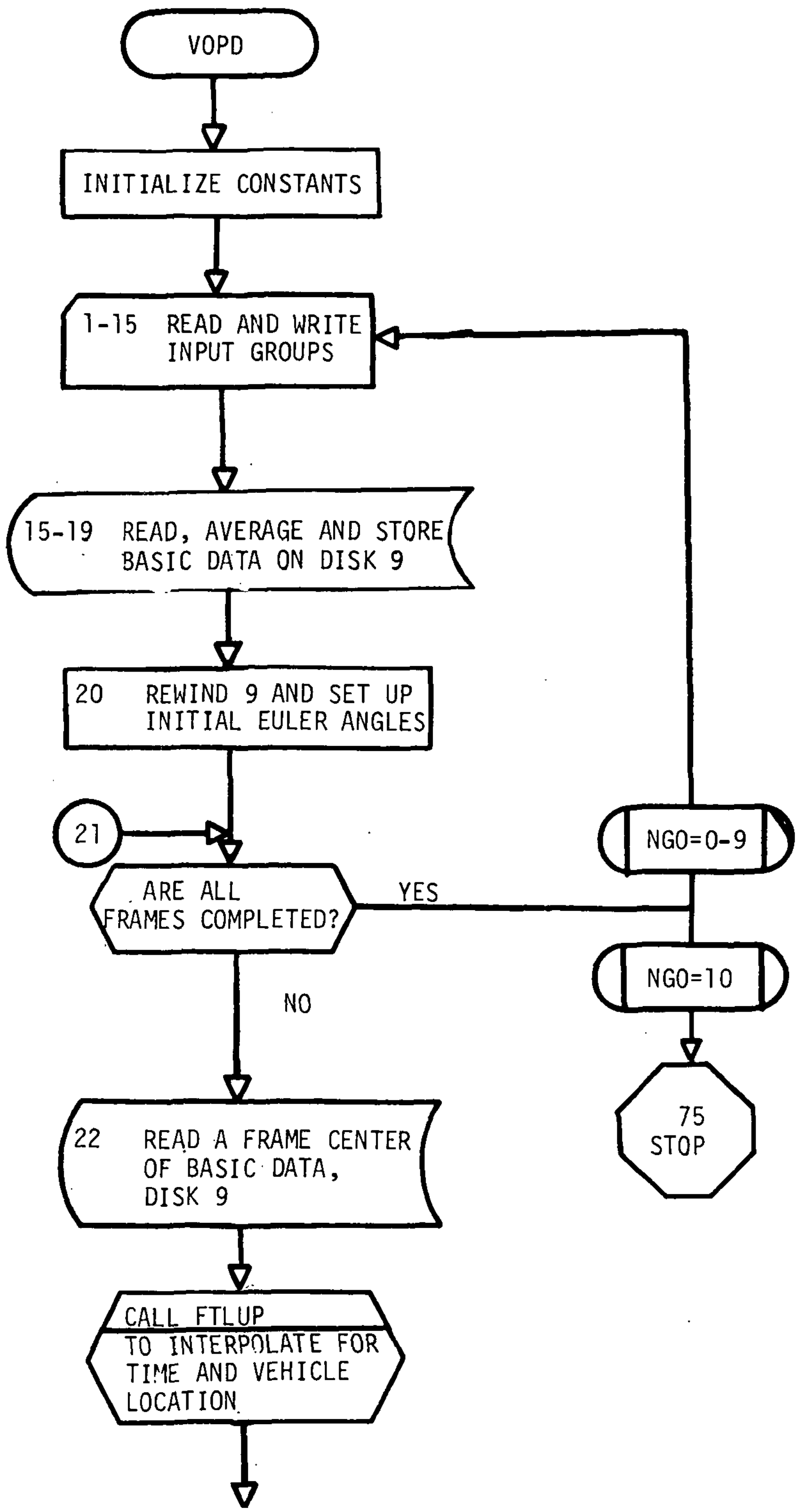
By using the Gerber film reading system employed at the Langley Research Center, the basic data are obtained as follows: First, the processed film from the onboard vehicle camera is inserted into a film reading system capable of providing coordinates for either 16-mm, 35-mm, or 70-mm film. The film reader magnifies a 16-mm frame image to dimensions of approximately 26.7 cm by 37.3 cm (10.5 in. by 14.7 in.) and provides coordinate readings to the nearest 0.00254 cm (0.001 in.) on the enlarged image. The film reading system is coupled with a visual-display electronic digitizer, a card punch, and an electronically operated typer (data typewriter). The card punch produces the basic data deck and the typer provides a record which can be used to determine rapidly whether any gross reading or film reading system errors exist. Then a manually controlled pair of crosshairs located on the film reader are consecutively alined on each corner (or other fiducial point) of the frame. These crosshairs also drive the digitizer. The coordinates and other reference information displayed on the digitizer are next recorded by both the card punch and typer by the user pressing a foot switch. For accuracy purposes, each reading is taken three times. The software is employed to average the three readings for each of the four corners of the frame and subsequently to determine the coordinates of the frame center. In the same manner the film coordinates of each landmark are obtained.

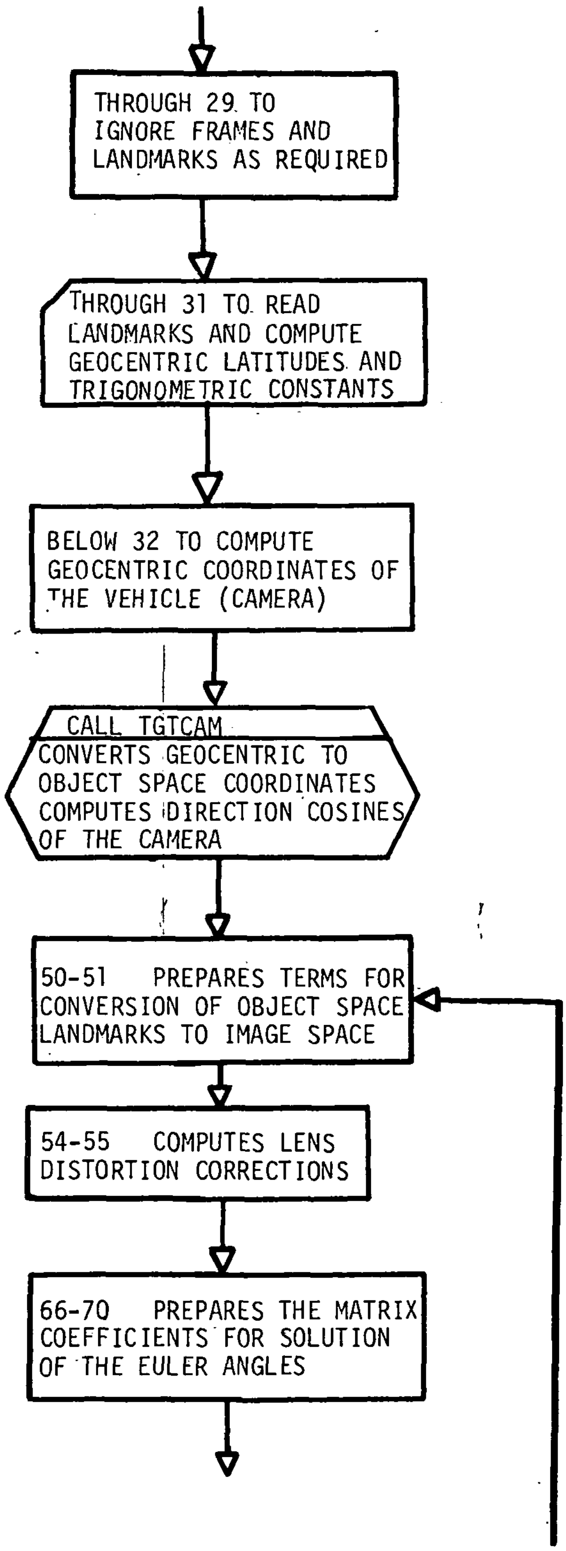
## PROGRAM DESCRIPTION

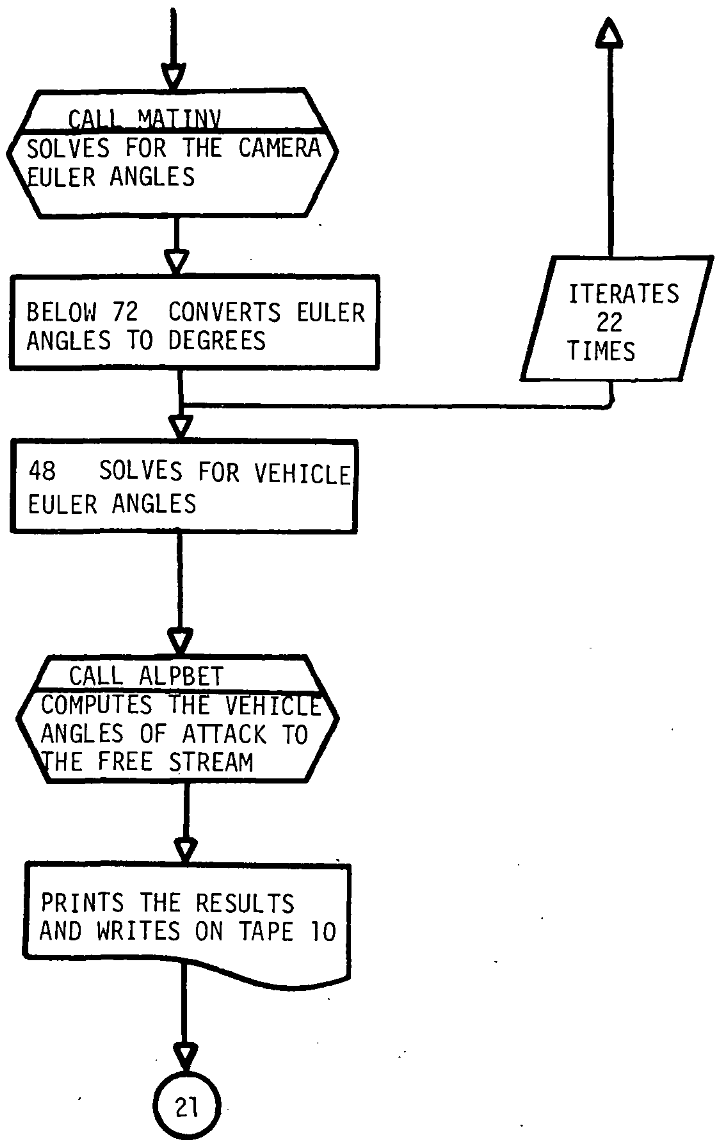
The computer program (Program VOPD; Library No. A4424) used at the Langley Research Center is written in Fortran IV and is set up for a CDC 6600 series computer. For applications utilizing an average of three to four landmarks per frame, computer running time has averaged 0.17 second/frame. The basic data deck contains the film coordinates of the landmarks obtained as previously discussed. The control data include geodetic coordinates of the landmarks, vehicle trajectory data, and camera characteristics.

Program VOPD is the main program. It calls subroutines TGTCAM and ALPBET for auxiliary calculations. A flow diagram for VOPD which indicates the basic calculation procedure follows and a computer listing of the program is given in appendix B. Computer library routines MATINV and FTLUP are called by VOPD and are described in appendices C and D, respectively.

## PROGRAM VOPD FLOW DIAGRAM




THROUGH 29 TO IGNORE FRAMES AND LANDMARKS AS REQUIRED
THROUGH 31 TO READ LANDMARKS AND COMPUTE GEOCENTRIC LATITUDES AND TRIGONOMETRIC CONSTANTS
BELOW 32 TO COMPUTE GEOCENTRIC COORDINATES OF THE VEHICLE (CAMERA)
CALL TGTCAM
CONVERTS GEOCENTRIC TO OBJECT SPACE COORDINATES COMPUTES DIRECTION COSINES OF THE CAMERA
50-51 PREPARES TERMS FOR CONVERSION OF OBJECT SPACE LANDMARKS TO IMAGE SPACE
54-55 COMPUTES LENS DISTORTION CORRECTIONS
66-70 PREPARES THE MATRIX COEFFICIENTS FOR SOLUTION OF THE EULER ANGLES




The following section gives a summary of the basic program operations and is followed by a more in-depth input-output discussion of the control data format and some sample program applications.

## General Operation

The program operates under the following general steps:

(1) The control data and basic data are read

(2) The frame center and each landmark location on the frame is computed and stored on a disk for each frame of the basic data deck

(3) All landmarks are transformed from the geodetic to the geocentric coordinate system and stored in core

(4) Each frame of data is then processed as follows:

(a) The first frame is read from the disk and compared with the control data to determine whether it is to be ignored or whether any landmarks are to be ignored

(b) The object space landmarks are converted to their image space coordinates and lens distortion corrections are computed

(c) The camera Euler angles are solved and compared with the initial estimates from the control deck

(d) Through an iterative process using the new Euler angles as a starting point, the final Euler angles are obtained when a sufficiently small change is detected

(e) These camera angles are then converted to the vehicle-oriented values, and by using the meteorological and trajectory data of the control deck, the vehicle angle-of-attack components are computed and printed

(5) At the completion of all frames of the basic data deck, the program returns to the input group specified by the control deck for further processing.

An initial estimate of camera Euler angles is necessary to start the iteration procedure. For the first frame, the initial estimates should be in or adjacent to the quadrant of the final solution to obtain satisfactory convergence within the 22 iterations provided for in the program. For subsequent frames, initial estimates are taken as the results of the previous frame.

## Program Usage

Basic data card format.- As previously discussed, the basic data consist of film coordinates of the selected landmarks with respect to the center of the frame. The basic data card should include, as a minimum, these data and the information specified in the program control-data deck. For program VOPD, this information includes landmark number and frame number. A separate card is required for each of the three readings of each point, whether it is a fiducial point or landmark. Four separate fiducial points are required for each frame. Thus, a frame with six landmarks would require 30 basic data computer cards.

The basic data card should also contain the information required for identification and review, and also, as an aid for determination of possible reading or system errors.

The format used in the most recent Langley applications is listed in appendix E. Also included in appendix E is a typical sequence of steps employed in reading a frame.

Control data format.- For the convenience of the potential user, program VOPD has been divided into distinctive input and output groups of control data cards. Detailed descriptions in computer terminology of the input for each group are included in appendix F. These descriptions include such information as column location, parameter symbols, parameter description, the number of cards in each group, parameter units, and input format. The following discussion, summarizing the information in each group, is in the order in which each group appears in the deck, and should be reviewed in conjunction with the more detailed information given in appendix F.

Group 1 contains a table of camera lens distortion compared with radial distance from the lens center. These values are used for correcting the landmark image space coordinates and are obtained from camera calibration data.

Group 2 is a title card to annotate each page of output. A "1" must be punched in column 1 to operate the carriage control on the printer and any descriptive information can be punched in columns 2 to 80.

Group 3 consists primarily of data describing the conversion of camera Euler angles to vehicle Euler angles and is strictly a function of the relative orientation between the camera and vehicle axes. To solve for vehicle attitude angles accurately, it is essential to know the exact orientation between the camera and vehicle axes. The numbers of values in the vehicle trajectory table (group 7) and wind table (group 4) are also required in group 3. The conversion data for camera to vehicle Euler angles are the coefficients of the equations defined in the input description (appendix F). Camera and vehicle Euler angles are shown in figures 1 and 2, respectively.

Group 4 is a group of cards each containing an altitude and the corresponding horizontal wind components. The span of altitudes in this group must at least include the altitude span of the vehicle trajectory group (group 7). These data are required only for angle-of-attack and velocity calculations.

Group 5 lists the number of landmarks in the basic data deck, the number of frames of data in the basic data deck, and the number of values in the time-frame table (group 6). The remaining spaces on this first card and all the spaces on the following cards in this group, as many as necessary, comprise a table consisting of the number of landmarks for each frame. These numbers are listed in the order corresponding to the frame sequence in the basic data deck.

Group 6 consists of a set of cards each listing the frame number and an associated time. This group must at least include the frames in the basic data deck. For a constant frame rate camera, only two values are needed for this group.

Group 7 lists the vehicle trajectory data. The origin for this trajectory information is described in group 8. Each card of group 7 contains vehicle flight time, associated vehicle position (X, Y, and Z locations) and velocity, and altitude above mean sea level (MSL). This set of data must encompass the time span of the basic data deck.

Group 8 contains the following data on one card: camera focal length; the origin of the vehicle trajectory coordinate system consisting of geodetic latitude, longitude, and the Earth radius to this origin; and the initial estimates of the camera Euler angles. These initial estimates are used to initiate the iteration on the Euler angles for the first frame of each basic data deck. Subsequent frames employ results from the previous frame to initiate Euler angle iterations.

Group 9 lists the landmarks and their locations. This group includes all the landmarks read from the film using the same landmark identification numbers as used in the basic data deck. Geodetic latitude, longitude, and altitude for each landmark are listed on each card along with the landmark identification number.

Group 10 indicates the number of frames from which specific landmarks are to be ignored. This value is, in effect, the number of group 11 cards. Landmarks are ignored when it is suspected that either their frame coordinates or Earth coordinates have been incorrectly identified.

Group 11 consists of a number of cards as specified in group 10 which lists the frames and landmarks which are to be ignored for these frames. To ignore the entire frame, an option is also included as described in appendix E.

Group 12 defines the following two parameters: First, the input group to which the program returns after processing all frames of the basic data deck. This information allows the user to stack jobs by returning to any input group except 11 and by adding behind the basic data deck only that input group and those following the one specified. Additional data in group 12 consist of the landmark number of any landmarks which are to be ignored in calculations for all frames of the basic data deck and eliminates the necessity for a long group 11 table.

After these groups of cards, the basic data deck is inserted and appears only once in the deck setup. For subsequent processing, the basic data deck is automatically stored on the disk and need not be repeated. The information read from the basic data deck are the landmark number, its coordinates on the frame ($\bar{x}$, $\bar{y}$), and the frame number. The landmark number is zero for the four fiducial (frame corners) points.

Program output is listed by groups in appendix G and contains such information as parameter description, symbols, and units.

## EXAMPLE APPLICATIONS

For a better understanding of program application, several example cases taken from BLDT applications are presented with the corresponding program input and output format (for one frame only) presented in appendix H. Example case inputs are divided into the previously discussed input groups (appendix F) and may be referred to for clarification. For all cases, input group 12 provides instructions to the program to be implemented upon completion of the existing case. Program output consisting of a single frame for all cases is included collectively after the program input for all cases.

Case 1.- This example represents an application where 24 frames are processed (only one frame is presented) and where no question exists concerning landmark location. All 12 input groups, as defined in appendix F, and the basic data deck are included. Three landmarks, identified as landmarks 65, 66, and 67, are used and their coordinates are listed in input group 9. As indicated in group 10, no landmarks are to be ignored. Group 12 provides instructions to the program upon completion of case 1.

Case 2.- Case 2 is a repeat of case 1 except that one landmark (landmark 65) was ignored because there was some question about its exact location on the film. The inputs for all groups, except group 12, are the same as those for case 1.

By comparing the outputs of cases 1 and 2, a difference of about $1^{\circ}$ can be seen for $\psi$ and $\phi$ with little change in $\theta$. By ignoring landmark 65, the residuals are slightly less for case 2 than for case 1. This result indicates that the results from case 2 may be more accurate than the results from case 1 and that the coordinates of landmark 65 may be in error.

Case 3.- Case 3 is similar to case 2 except that the latitude of landmark 66 was changed from $32^{\circ}50'$ to $32^{\circ}50.3'$. (See group 9.) The inputs for all groups, except groups 9, 10, and 12, are the same as those for cases 2 and 1. Case 3 illustrates a typical iteration to define the object space (Earth) coordinates of a previously unknown landmark to convert a feature on the film to a usable landmark. The results of this iteration show a slight improvement in residuals over case 2 and about a $1^{\circ}$ change in $\psi$ and $\theta$.

Case 4.- This case illustrates the combined approach of completely ignoring frame 55 and ignoring landmark 69 for several frames (frames 60 and 65) as instructed through input group 11. This case is independent of the previous three and requires a different set of input data. For this case, the output for frames 60 and 65 are presented. A computer listing of the general camera orientation method program (VOPD) is shown in appendix H.

## CONCLUDING REMARKS

Details of application of a previously devised photogrammetric method to determine a time history of vehicle flight attitudes have been included in this paper. Emphasis has been placed on the techniques involved in reducing the raw photographic data to computer inputs in Fortran IV language and on the computer techniques and programs involved in obtaining vehicle flight attitude results. Also discussed are the major program modifications which allow faster data reduction and permit the user to determine the Earth-related coordinates of unknown or nonpermanent features appearing on the film.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., May 3, 1974.

## APPENDIX A

## PHOTOGRAMMETRIC DETERMINATION OF CAMERA ORIENTATION

In order to describe adequately how the camera orientation can be determined, it is necessary to define the coordinate systems in which observations are made. The coordinate system in which the position of observed points are known and in which the camera is oriented is called the object space. The coordinate system composed of the camera focal plane and focal axis and in which image coordinates are measured is the image space. (See ref. 3.) The relationship between a point in the object space and image space is

$$\left.\begin{aligned} \bar{x} - \bar{x}_p &= c\left(\frac{\lambda_1 X + \mu_1 Y + \nu_1 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z}\right) \\ \bar{y} - \bar{y}_p &= c\left(\frac{\lambda_2 X + \mu_2 Y + \nu_2 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z}\right) \end{aligned}\right\} \tag{A1}$$

The relationship in equations (A1) can be derived with the aid of figure 3. Let $Q(X,Y,Z)$ be a point in the object space, then the image of $Q$ will be $I(X_I,Y_I,Z_I)$ in the image space. Since the origins of the two coordinate systems coincide for all practical purposes, the following transformation describes the coordinates of $Q$ relative to the image space

$$\begin{bmatrix} X_I \\ Y_I \\ Z_I \end{bmatrix} = K \begin{bmatrix} \lambda_1 \mu_1 \nu_1 \\ \lambda_2 \mu_2 \nu_2 \\ \lambda_3 \mu_3 \nu_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{A2}$$

where $K$ is a constant of contraction. In equations (A2) dividing the first and second equations by the third equation removes the constant $K$ and gives

$$\left.\begin{aligned} \frac{X_I}{Z_I} &= \frac{\lambda_1 X + \mu_1 Y + \nu_1 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z} \\ \frac{Y_I}{Z_I} &= \frac{\lambda_2 X + \mu_2 Y + \nu_2 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z} \end{aligned}\right\} \tag{A3}$$

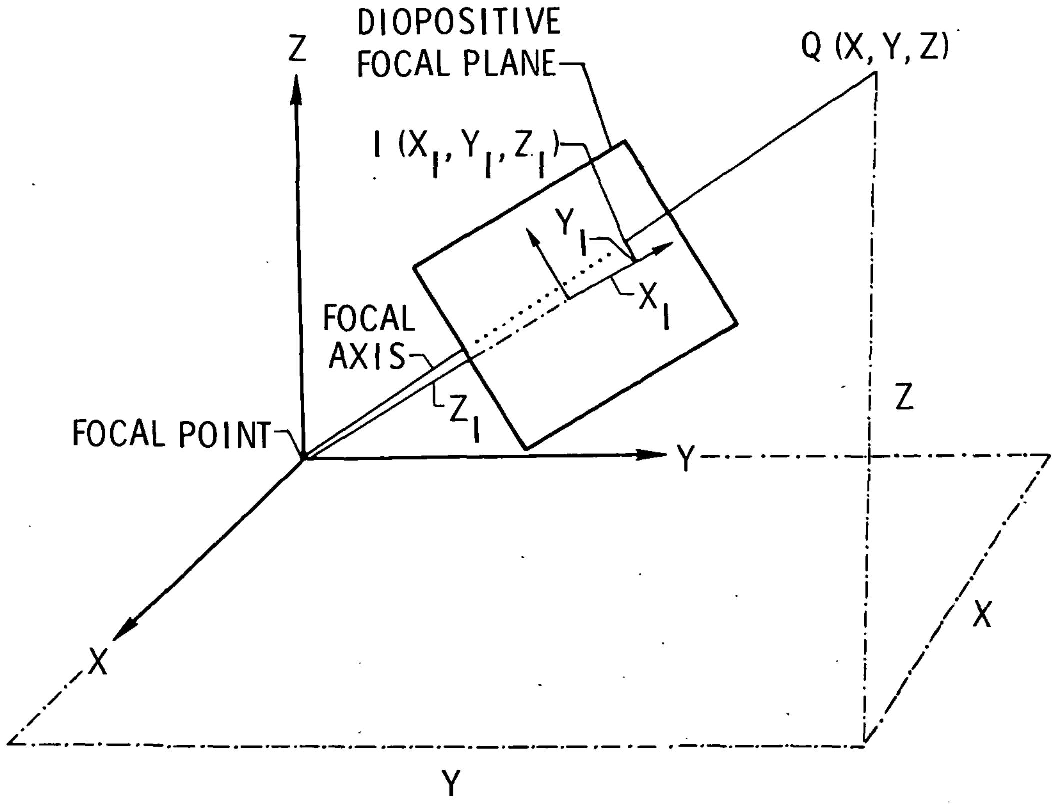



Figure 3.- Axis systems relating image space and object space.

Letting

$$\bar{x} - \bar{x}_p = X_I$$
$$\bar{y} - \bar{y}_p = Y_I$$
$$c = Z_I$$

and multiplying both sides of equations (A3) by c gives

$$\left.\begin{aligned}\bar{x} - \bar{x}_p &= c\left(\frac{\lambda_1 X + \mu_1 Y + \nu_1 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z}\right)\\ \bar{y} - \bar{y}_p &= c\left(\frac{\lambda_2 X + \mu_2 Y + \nu_2 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z}\right)\end{aligned}\right\} \qquad \text{(A4)}$$

Equations (A4) are identical to equations (A1).

For each known point in the object space there exist two equations (A1) relating the object space to the image space. The directional cosines $\lambda_1$, $\mu_1$, $\nu_1$, . . ., of the image space axes relative to the object space axes can be expressed in terms of three angles $\sigma_1$, $\sigma_2$, and $\sigma_3$. Consequently, these angles can be used to describe the orientation of the camera coordinate system relative to the object space coordinate system. To obtain the directional cosines, consider three successive rotations through the angles $\sigma_1$, $\sigma_2$, and $\sigma_3$. By using figure 3 and imposing a constant of contraction for the camera, the following transformation is obtained:

$$\begin{bmatrix} \bar{x} - \bar{x}_p \\ \bar{y} - \bar{y}_p \\ c \end{bmatrix} = K \begin{bmatrix} -\cos\sigma_1 \cos\sigma_3 - \sin\sigma_1 \sin\sigma_2 \sin\sigma_3 & \sin\sigma_1 \cos\sigma_3 - \cos\sigma_1 \sin\sigma_2 \sin\sigma_3 & \cos\sigma_2 \sin\sigma_3 \\ \cos\sigma_1 \sin\sigma_3 - \sin\sigma_1 \sin\sigma_2 \cos\sigma_3 & -\sin\sigma_1 \sin\sigma_3 - \cos\sigma_1 \sin\sigma_2 \cos\sigma_3 & \cos\sigma_2 \cos\sigma_3 \\ \sin\sigma_1 \cos\sigma_2 & \cos\sigma_1 \cos\sigma_2 & \sin\sigma_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = K \begin{bmatrix} \lambda_1 \mu_1 \nu_1 \\ \lambda_2 \mu_2 \nu_2 \\ \lambda_3 \mu_3 \nu_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

This equation is equivalent to equation (A2).

## The Least-Squares Solution

The solution of equations (A1) contains six parameters which under theoretical conditions are constants. These parameters are $\bar{x}_p$, $\bar{y}_p$, c, $\sigma_1$, $\sigma_2$, and $\sigma_3$ of which $\bar{x}_p$, $\bar{y}_p$, and c are measured independently for this experiment and are not unknowns in the solution. A solution of equations (A1) for $\sigma_1$, $\sigma_2$, and $\sigma_3$ can be found with two properly chosen observations. Associated with $\bar{x}$ and $\bar{y}$ in equations (A1) are errors $\epsilon$ and $\bar{\epsilon}$. Since these errors exist, a computational method is needed which yields the best possible results with all the information available. The method of least squares which is described subsequently uses a minimum error criterion and has been used in the data reduction for this investigation.

In general, equations (A1) with the associated errors can be written as

$$\left.\begin{aligned} \bar{x}_i &= F(\sigma_j) + \epsilon_i \qquad && (j = 1, 2, 3) \\ \bar{y}_i &= \bar{F}(\sigma_j) + \bar{\epsilon}_i \qquad && (i = 1, \ldots, n) \end{aligned}\right\} \tag{A5}$$

where F and $\bar{F}$ are nonlinear functions of $\sigma_j$ and in order to find a solution they must be linearized. Expanding equations (A5) in a Taylor's series about a nominal set $\sigma_{j,o}$ and dropping the higher order terms results in the following linear approximations:

$$\left.\begin{aligned} \Delta\bar{x}_i &= \bar{x}_i - \bar{x}_{i,o} = b_{i1}(\sigma_1 - \sigma_{1,o}) + b_{i2}(\sigma_2 - \sigma_{2,o}) + b_{i3}(\sigma_3 - \sigma_{3,o}) + \epsilon_i \\ \Delta\bar{y}_i &= \bar{y}_i - \bar{y}_{i,o} = \bar{b}_{i1}(\sigma_1 - \sigma_{1,o}) + \bar{b}_{i2}(\sigma_2 - \sigma_{2,o}) + \bar{b}_{i3}(\sigma_3 - \sigma_{3,o}) + \bar{\epsilon}_i \end{aligned}\right\} \tag{A6}$$

where

$$b_{i1} = \left.\frac{\partial F_i}{\partial \sigma_1}\right|_{\sigma_{1,o}\sigma_{2,o}\sigma_{3,o}} \qquad \bar{b}_{i1} = \left.\frac{\partial \bar{F}_i}{\partial \sigma_1}\right|_{\sigma_{1,o}\sigma_{2,o}\sigma_{3,o}}$$

$$b_{i2} = \left.\frac{\partial F_i}{\partial \sigma_2}\right|_{\sigma_{1,o}\sigma_{2,o}\sigma_{3,o}} \qquad \bar{b}_{i2} = \left.\frac{\partial \bar{F}_i}{\partial \sigma_2}\right|_{\sigma_{1,o}\sigma_{2,o}\sigma_{3,o}}$$

$$b_{i3} = \left.\frac{\partial F_i}{\partial \sigma_3}\right|_{\sigma_{1,o}\sigma_{2,o}\sigma_{3,o}} \qquad \bar{b}_{i3} = \left.\frac{\partial \bar{F}_i}{\partial \sigma_3}\right|_{\sigma_{1,o}\sigma_{2,o}\sigma_{3,o}}$$

Letting

$$\Delta\sigma_j = \left(\sigma_j - \sigma_{j,o}\right)$$

equations (A6) can be put in the following form:

$$\left.\begin{aligned} \Delta\bar{x}_i &= \sum_{j=1}^{3} b_{ij}\,\Delta\sigma_j + \epsilon_i \qquad (i = 1, \ldots, n) \\ \Delta\bar{y}_i &= \sum_{j=1}^{3} \bar{b}_{ij}\,\Delta\sigma_j + \bar{\epsilon}_i \qquad (i = 1, \ldots, n) \end{aligned}\right\} \tag{A7}$$

For further considerations the linear equations (A7) corresponding to the ith observation are expressed in matrix notation

$$v_i = \begin{bmatrix} \Delta\bar{x}_i \\ \Delta\bar{y}_i \end{bmatrix} \qquad B_i = \begin{bmatrix} b_{i1} & b_{i2} & b_{i3} \\ \bar{b}_{i1} & \bar{b}_{i2} & \bar{b}_{i3} \end{bmatrix} \qquad \Delta\sigma = \begin{bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\sigma_3 \end{bmatrix} \qquad e_i = \begin{bmatrix} \epsilon_i \\ \bar{\epsilon}_i \end{bmatrix}$$

where

$$v_i = B_i\,\Delta\sigma + e_i \tag{A8}$$

Then for n observations there are n matrix equations of the form of equation (A8) which may be written

$$\bar{V} = \bar{B}\,\Delta\sigma + \bar{e} \tag{A9}$$

where

$$\bar{V} = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_n \end{bmatrix} \qquad \bar{e} = \begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{bmatrix}$$

The problem may be restated: given $\bar{V}$ and $\bar{B}$ find the best estimate $\widehat{\Delta\sigma}$ for $\Delta\sigma$.

The best estimate $\widehat{\Delta\sigma}$ is the value of $\widehat{\Delta\sigma}$ which minimizes the sum of the squares of the residuals $\bar{e}^T\bar{e}$ where

$$\bar{e}^T\bar{e} = \left(\bar{V} - \bar{B}\,\Delta\sigma\right)^T\left(\bar{V} - \bar{B}\,\Delta\sigma\right) \tag{A10}$$

In order to minimize equation (A10), the first variation $\delta$ with respect to $\Delta\sigma$ must vanish; that is,

$$\left.\begin{aligned} \delta\left(\bar{e}^T\bar{e}\right) &= \delta\left[\left(\bar{V} - \bar{B}\,\Delta\sigma\right)^T\left(\bar{V} - \bar{B}\,\Delta\sigma\right)\right] = 0 \\ \delta\left(\bar{e}^T\bar{e}\right) &= -2\left(\bar{V}^T - \Delta\sigma^T\bar{B}^T\right)\bar{B}\delta\,\Delta\sigma = 0 \end{aligned}\right\} \tag{A11}$$

Since $\delta\,\Delta\sigma \neq 0$, equations (A11) can be satisfied if

$$\left(\bar{V}^T - \Delta\sigma^T\bar{B}^T\right)\bar{B} = 0$$

or

$$\bar{B}^T\bar{B}\,\Delta\sigma = \bar{B}^T\bar{V} \tag{A12}$$

Solving for the estimate of $\Delta\sigma$ in equation (A12) gives

$$\widehat{\Delta\sigma} = \left(\bar{B}^T\bar{B}\right)^{-1}\bar{B}^T\bar{V} \tag{A13}$$

A second necessary condition for equation (A10) to be a minimum is that the second variation with respect to $\Delta\sigma$ be positive definite. Upon examination, the second variation is

$$\delta^2\left(\bar{e}^T\bar{e}\right) = 2\delta\ \Delta\sigma^T\bar{B}^T\bar{B}\delta\ \Delta\sigma$$

which is positive definite. Therefore, equation (A13) is a valid expression for $\widehat{\Delta\sigma}$.

Since equation (A13) is based on a linear approximation with nominal $\sigma_{j,o}$, $\widehat{\Delta\sigma}$ can be used to find the best estimates $\hat{\sigma}_j$. With the relationship $\sigma = \sigma_o + \Delta\sigma$, the value of $\widehat{\Delta\sigma}$ which minimized equation (A10) leads to a new nominal $\sigma_{j,o} = \sigma_{j,o} + \widehat{\Delta\sigma}$. This process implies an iterative procedure which continues until $\widehat{\Delta\sigma} \rightarrow 0$ and the value of $\sigma_{j,o}$ that leads to this result is the best estimate of $\hat{\sigma}_j$ for $\sigma_j$.

## Partial Derivatives of Projection Equations

For equations (A5)

$$\bar{x}_i = F\left(\sigma_j\right) + e_i \qquad (j = 1,\ 2,\ 3)$$

$$\bar{y}_i = \bar{F}\left(\sigma_j\right) + e_i \qquad (i = 1,\ .\ .\ .,\ n)$$

The partial derivatives of $F$ and $\bar{F}$ with respect to $\sigma_1$, $\sigma_2$, and $\sigma_3$ are as follows. Let

$$p = \lambda_1 X + \mu_1 Y + \nu_1 Z$$

$$q = \lambda_2 X + \mu_2 Y + \nu_2 Z$$

$$r = \lambda_3 X + \mu_3 Y + \nu_3 Z$$

then

$$\frac{\partial F}{\partial \sigma_1} = \frac{c\left(\frac{\partial p}{\partial \sigma_1} r - \frac{\partial r}{\partial \sigma_1} p\right)}{r^2}$$

$$\frac{\partial F}{\partial \sigma_2} = \frac{c\left(\frac{\partial p}{\partial \sigma_2} r - \frac{\partial r}{\partial \sigma_2} p\right)}{r^2}$$

$$\frac{\partial F}{\partial \sigma_3} = \frac{c\left(\frac{\partial p}{\partial \sigma_3} r - \frac{\partial r}{\partial \sigma_3} p\right)}{r^2}$$

$$\frac{\partial \bar{F}}{\partial \sigma_1} = \frac{c\left(\frac{\partial q}{\partial \sigma_1} r - \frac{\partial r}{\partial \sigma_1} q\right)}{r^2}$$

$$\frac{\partial \bar{F}}{\partial \sigma_2} = \frac{c\left(\frac{\partial q}{\partial \sigma_2} r - \frac{\partial r}{\partial \sigma_2} q\right)}{r^2}$$

$$\frac{\partial \bar{F}}{\partial \sigma_3} = \frac{c\left(\frac{\partial q}{\partial \sigma_3} r - \frac{\partial r}{\partial \sigma_3} q\right)}{r^2}$$

$$\frac{\partial p}{\partial \sigma_1} = \mu_1 X - \lambda_1 Y$$

$$\frac{\partial q}{\partial \sigma_1} = \mu_2 X - \lambda_2 Y$$

$$\frac{\partial r}{\partial \sigma_1} = \mu_3 X - \lambda_3 Y$$

$$\frac{\partial p}{\partial \sigma_2} = -\nu_1\left(X \sin \sigma_1 + Y \cos \sigma_1\right) - \left(\sin \sigma_2 \sin \sigma_3\right)Z$$

$$\frac{\partial q}{\partial \sigma_2} = -\nu_2\left(X \sin \sigma_1 + Y \cos \sigma_1\right) - \left(\sin \sigma_2 \cos \sigma_3\right)Z$$

$$\frac{\partial r}{\partial \sigma_2} = -\nu_3\left(X \sin \sigma_1 + Y \cos \sigma_1\right) + Z \cos \sigma_2$$

$$\frac{\partial p}{\partial \sigma_3} = \lambda_2 X + \mu_2 Y + \nu_2 Z$$

$$\frac{\partial q}{\partial \sigma_3} = -\left(\lambda_1 X + \mu_1 Y + \nu_1 Z\right)$$

$$\frac{\partial r}{\partial \sigma_3} = 0$$

Coordinate Transformation

Object space points and the position of the camera are initially identified in terms of geodetic latitude, longitude, and altitude above sea level. These data are obtained from maps of the photographed area and radar observations of the vehicle trajectory. In order to reference the data relative to the camera as described previously, the Earth-centered geocentric coordinates of both the object space points and the vehicle position are computed by

$$\bar{X}_i = (R_i + h_i)\cos \Phi_i' \cos \Lambda_i$$

$$\bar{Y}_i = (R_i + h_i)\cos \Phi_i' \sin \Lambda_i$$

$$\bar{Z}_i = (R_i + h_i)\sin \Phi_i'$$

and

$$\Phi_i' = \Phi_i - 11'35.6635'' \sin 2\Phi_i + 1.1731'' \sin 4\Phi_i - 0.0025'' \sin 6\Phi_i$$

$$R_i = 6378.388(0.998320047 + 0.001683494 \cos 2\Phi_i - 0.000003549 \cos 4\Phi)$$

where

$i = 1$ for the vehicle position

$i = 2 \ldots n + 1$ for the object space points

In the preceding discussion the $\bar{X}$ axis is in the equatorial plane pointing toward the Greenwich meridian, the $\bar{Y}$ axis $90^\circ$ east in the equatorial plane, and the $\bar{Z}$ axis toward the north pole. A slight error is introduced by adding altitude to the Earth's radius vector but this error is negligible for the accuracy desired of this system.

The following transformation maps the geocentric coordinates into object space coordinates relative to the camera where the X axis points east, the Y axis points north, and the Z axis points toward the zenith.

$$\begin{bmatrix} X_{i+1} \\ Y_{i+1} \\ Z_{i+1} \end{bmatrix} = \begin{bmatrix} -\sin \Lambda_1 & \cos \Lambda_1 & 0 \\ -\cos \Lambda_1 \sin \Phi_1 & -\sin \Lambda_1 \sin \Phi_1 & \cos \Phi_1 \\ \cos \Lambda_1 \cos \Phi_1 & \sin \Lambda_1 \cos \Phi_1 & \sin \Phi_1 \end{bmatrix} \begin{bmatrix} \bar{X}_{i+1} - \bar{X}_1 \\ \bar{Y}_{i+1} - \bar{Y}_1 \\ \bar{Z}_{i+1} - \bar{Z}_1 \end{bmatrix}$$

This transformation is obtained by a positive rotation about the $\bar{Z}$ axis through an angle of $90^\circ + \Lambda_1$ followed by a position rotation about the new X axis through an angle of $90^\circ - \Phi_1$.

# APPENDIX B

# PROGRAM LISTING

The program listing follows:

```
      PROGRAM VOPD(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10,TAPE9)
      DIMENSION RRR(25), ISAVE(30), XSAVE(30), YSAVE(30), XRR(25), RR(20
     1), DRR(20), DXY(20)
      DIMENSION XLAM(20), XMU(20), XNU(20), B(2,6), BS(6,2), XN(6,6), EP
     1S(2,20), C(6,1), XMM(20), XNN(20), QQ(20), NG(20), X(20), Y(20), S
     2X(20), SY(20), IDENT(6), CM(20), CN(20), IPIVOT(6), INDEX(6,2)
      DIMENSION PHIBD(100), PHIBM(100), PHIBS(100), XLOBD(100), XLOBM(10
     10)
      DIMENSION XLOBS(100), TXYZT(20,4), TGTTAB(20,3), XYZTAB(3), XX(20)
     1, YY(20), XNTEMP(6,6), XNUNIT(6,6)
      DIMENSION TTHJ(90), TTHJMI(90), TPHJ(90), TPHJMI(90), TRJ(90)
      DIMENSION GEODA(6,3), GEODB(6,3)
      DIMENSION NPT(200), IFRM(100), TIMX(100), XXX(50), YYY(50), ZZZ(50
     1), XFRM(100)
      DIMENSION IOUT(100), IFM(100), JOUT(6,100)
      COMMON /AB/ III,IWV,TX(50),VVX(50),VVY(50),VVZ(50),H(50),ALT(50),V
     1XO(50),VYO(50)
      COMMON /DIRCOS/ XLAM,XMU,XNU
      DATA (RAD=57.29577951),(PI=3.14159265),(FTPKM=3280.8336),(FACTOR=.
     1713815400205),(ERADKM=6378.165),(V=0.0)
      DATA (SFOCAL=.0015),(SLAT=.08333333333333),(SXYZ=200.),(SRJ=200.),
     1(SXPYP=.01),(SXY=0.1),(VFTPKM=.0003048006),(RN=0.)
      TWOPI=2.*PI
      ITGTO=1
      DELF=0.
1     READ (5,76) (XRR(I),RRR(I),I=1,25)
2     READ (5,77)
      IF (EOF,5) 13,200
200   JUMP=0
3     READ (5,78) III,IWV,S1M,S1A,S2M,S2A,S3M,S3A
4     READ (5,79) (ALT(I),VXO(I),VYO(I),I=1,IWV)
5     READ (5,80) NT,NFR,NTFR,(NPT(L),L=1,NFR)
6     READ (5,81) (IFRM(I),TIMX(I),I=1,NTFR)
      DO700I=1,NTFR
700   XFRM(I)=IFRM(I)
7     READ (5,82) (TX(I),XXX(I),YYY(I),ZZZ(I),VVX(I),VVY(I),VVZ(I),H(I),
     1I=1,III)
8     READ (5,82) CC,PHI,THZ,RZ,ALPHAI,XOMEGI,XKAPI
      PHITEM=PHI
      TWOPHI=(2.*PHI)/RAD
      FORPHI=2.*TWOPHI
      SIXPHI=3.*TWOPHI
      SINTPD=SIN(TWOPHI)
      SINFPD=SIN(FORPHI)
      SINSPD=SIN(SIXPHI)
      PHI=PHI-.1932398611111*SINTPD+.0003258611111111*SINFPD+7.222222222
     1222E-07*SINSPD
      PHIZ=PHI
      DPR=57.29577951
      PHIZ=PHIZ/DPR
      THZ=THZ/DPR
```

```
9       DO1100IJK=1,NT                                              A  62
        READ (5,83) ITGT,THJ,THJMI,PHIJ,PHIJMI,RJ                   A  63
        RJ=RJ*1000./FTPKM
        TTHJ(ITGT)=THJ                                              A  65
        TTHJMI(ITGT)=THJMI                                          A  66
        TPHJ(ITGT)=PHIJ                                             A  67
        TPHJMI(ITGT)=PHIJMI                                         A  68
1100    TRJ(ITGT)=RJ
10      READ (5,80) NOUT                                            A  71
        IF (NOUT.LT.1) GO TO 12                                     A  72
11      READ (5,84) (IFM(I),(JOUT(J,I),J=1,6),I=1,NOUT)             A  73
        DO 130I=1,NOUT                                              A  74
        DO 130J=1,6                                                 A  75
        IF (JOUT(J,I).LT.1) JOUT(J,I)=101                           A  76
130     CONTINUE                                                    A  77
12      READ (5,80) NGO,(IOUT(I),I=1,NT)                            A  79
        DO 15 LL=1,NT                                               A  80
        IF (IOUT(LL).LT.1) IOUT(LL)=101                             A  81
15      CONTINUE                                                    A  82
        IF (JUMP.GT.0) GO TO 20                                     A  84
        DO 18 N=1,NFR                                               A  85
        IUP=NPT(N)*3+12                                             A  86
        READ (5,86) (ISAVE(I),XSAVE(I),YSAVE(I),IFRAM,I=1,IUP)      A  87
        YPC=0.                                                      A  88
        XPC=0.                                                      A  89
        DO 16 I=1,12                                                A  90
        XPC=XPC+XSAVE(I)/12.                                        A  91
16      YPC=YPC+YSAVE(I)/12.                                        A  92
        IUP=NPT(N)                                                  A  93
        II=12                                                       A  94
        DO 18 I=1,IUP                                               A  95
        XSAVE(I)=0.                                                 A  96
        YSAVE(I)=0.                                                 A  97
        DO 17 J=1,3                                                 A  98
        II=II+1                                                     A  99
        XSAVE(I)=XSAVE(I)+XSAVE(II)/3.                              A 100
17      YSAVE(I)=YSAVE(I)+YSAVE(II)/3.                              A 101
18      ISAVE(I)=ISAVE(II)                                          A 102
        JUMP=1                                                      A 104
20      REWIND 9                                                    A 105
        READFG=0                                                    A 106
        IFOCFG=1                                                    A 107
        KK=0                                                        A 108
        ALPHAP=ALPHAI                                               A 109
        XOMEGP=XOMEGI                                               A 110
        XKAPP=XKAPI                                                 A 111
21      KK=KK+1                                                     A 112
        IF (KK.LE.NFR) GO TO 22                                     A 113
        REWIND 9                                                    A 114
        END FILE 10                                                 A 115
        WRITE (6,77)                                                A 117
        GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13), NGO                  A 118
22      NJ=NPT(KK)                                                  A 119
        READ (9,87) IFRAM,XPC,YPC                                   A 120
        XFRAM=IFRAM                                                 A 121
        CALL FILUP (XFRAM,TI,1,NTFR,XFRM,TIMX)                      A 122
        CALL FILUP (TI,XYZ,1,III,TX,XXX)                            A 123
        XYZTAB(1)=XYZ                                               A 124
        CALL FILUP (TI,XYZ,1,III,TX,YYY)                            A 125
        XYZTAB(2)=XYZ                                               A 126
        CALL FILUP (TI,XYZ,1,III,TX,ZZZ)                            A 127
        XYZTAB(3)=XYZ                                               A 128
```

```
      IF (NOUT.LT.1) GO TO 25
      DO 24 I=1,NOUT
      IF (IFRAM.NE.IFM(I)) GO TO 24
      IF (JOUT(1,I).NE.100) GO TO 25
      DO 23 L=1,NJ
23    READ (9,87) ITGT,XCO,YCO
      WRITE (6,88) IFRAM
      GO TO 21
24    CONTINUE
25    INJ=0
      NK=0
      WRITE (6,77)
      WRITE (6,89)
      DO 31 JJ=1,NJ
      READ (9,87) ITGT,XCO,YCO
      IF (NOUT.LT.1) GO TO 28
      DO 27 I=1,NOUT
      IF (IFRAM.NE.IFM(I)) GO TO 27
      DO 26 M=1,6
      IF (ITGT.NE.JOUT(M,I)) GO TO 26
      NK=NK+1
      IF ((NJ-NK).LT.2) GO TO 28
      WRITE (6,90) ITGT
      GO TO 31
26    CONTINUE
27    CONTINUE
28    DO 29 I=1,NT
      IF (IOUT(I).NE.ITGT) GO TO 29
      NK=NK+1
      IF ((NJ-NK).LT.2) GO TO 30
      WRITE (6,90) ITGT
      GO TO 31
29    CONTINUE
30    INJ=INJ+1
      X(INJ)=XCO
      Y(INJ)=YCO
      THJ=TTHJ(ITGT)
      THJMI=TTHJMI(ITGT)
      PHIJ=TPHJ(ITGT)
      PHIJMI=TPHJMI(ITGT)
      RJ=TRJ(ITGT)
      IDENT(INJ)=ITGT
      THJ=THJ+THJMI/60.
      GEODA(INJ,1)=THJ
      THJ=THJ+RN*SLAT
      GEODB(INJ,1)=THJ
      PHIJ=PHIJ+PHIJMI/60.
      GEODA(INJ,2)=PHIJ
      PHIJ=PHIJ+RN*SLAT
      GEODB(INJ,2)=PHIJ
      PHIJSA=PHIJ
      GEODA(INJ,3)=RJ
      RJ=RJ+RN*SRJ
      GEODB(INJ,3)=RJ
      RJ=RJ/FTPKM+ERADKM
      PHI=PHIJ
      TWOPHI=(2.*PHI)/RAD
      FORPHI=2.*TWOPHI
      SIXPHI=3.*TWOPHI
      PHI=PHITEM
      SINTPD=SIN(TWOPHI)
      SINFPD=SIN(FORPHI)
      SINSPD=SIN(SIXPHI)
```

```
      PHIJ=PHIJ-.19323986111111*SINTPD+.00003258611111111*SINFPD+7.222222
     1222222E-07*SINSPD
      TGTTAB(INJ,1)=THJ
      TGTTAB(INJ,2)=PHIJ
      TGTTAB(INJ,3)=RJ
31    ITGTF=0
      NJ=INJ
      N=NJ
      CCSAVE=0.
32    IRUN=0
      RUNFLG=1.
      IRUNCO=10
      RNSXYZ=RN*SXYZ
      XYZTAB(1)=XYZTAB(1)+RNSXYZ
      XYZTAB(2)=XYZTAB(2)+RNSXYZ
      XYZTAB(3)=XYZTAB(3)+RNSXYZ
      TXYZT(1,1)=TI
      TXYZT(1,2)=XYZTAB(1)*VFTPKM
      TXYZT(1,3)=XYZTAB(2)*VFTPKM
      TXYZT(1,4)=XYZTAB(3)*VFTPKM
      CALL TGTCAM (01,NJ,RZ,PHIZ,THZ,TXYZT,TGTTAB)
      DO 33 I=1,NJ
      TGTTAB(I,3)=TGTTAB(I,3)*FTPKM/1000.
      WRITE (6,91) IDENT(I),(TGTTAB(I,J),J=1,3),(GEODA(I,J),J=2,3),XLAM(
     1I),XMU(I),XNU(I)
33    TGTTAB(I,3)=TGTTAB(I,3)/FTPKM*1000.
      CCORIG=CC
      CC=CC+RN*SFOCAL
      CCLAST=CC
      IF (CCSAVE.GT.0.) CC=SAVE
      CC1=CC
      ALPSAV=ALPHAP
      OMESAV=XOMEGP
      XKASAV=XKAPP
      WRITE (6,92)
      WRITE (6,93) (XYZTAB(I),I=1,3),XPC,YPC,CC
      XPC=XPC+RN*SXPYP
      YPC=YPC+RN*SXPYP
      XPC=XPC*FACTOR
      YPC=YPC*FACTOR
      XPCSTO=XPC
      YPCSTO=YPC
      XPC1=XPC
      YPC1=YPC
      XPCSAV=XPC
      YPCSAV=YPC
      TWON=2.*N
      DO 34 I=1,N
      XX(I)=X(I)
34    YY(I)=Y(I)
      DO 35 I=1,N
35    X(I)=X(I)+RN*SXY
      DO 36 I=1,N
36    Y(I)=Y(I)+RN*SXY
      WRITE (6,95)
      DO 37 I=1,N
37    WRITE (6,94) IDENT(I),X(I),Y(I)
      DO 38 I=1,N
      X(I)=X(I)*FACTOR
38    Y(I)=Y(I)*FACTOR
      IF (IRUN) 42,42,39
39    GO TO (45,40), IFOCFG
```

```
40    CC=CC+DELF
      XPC=XPCSAV
      YPC=YPCSAV
      XPC1=XPCSAV
      YPC1=YPCSAV
      DO 41 I=1,N
      X(I)=XX(I)
41    Y(I)=YY(I)
42    IRUN=IRUN+1
      IF (IRUNCO-IRUN) 43,45,45
43    IF (RUNFLG) 44,44,32
44    RUNFLG=1.
      CC=CC1
      DELF=-DELF
      IRUN=0
      GO TO 40
45    CONTINUE
      ICASE=1
      ICASE=ICASE+1
      ALPHA=ALPSAV
      XOMEG=OMESAV
      XKAP=XKASAV
      ALPHAR=ALPHA/RAD
      OMEGAR=XOMEG/RAD
      XKAPR=XKAP/RAD
      ILINE=0
      SIGX=1.0
      SIGY=1.0
      NOP1=0
      NOP2=0
      EPOC=50.00
      ICOUNT=0
      ITER=1
      JJ=1
      SUM=0.0
46    CONTINUE
      IF (ICASE-1) 48,48,47
47    CONTINUE
      IF (ITER-22) 50,48,48
48    SIG1=S1M*ALPHA+S1A
      SIG2=S2M*XOMEG+S2A
      SIG3=S3M*XKAP+S3A
      IF (SIG1.LT.0.) SIG1=SIG1+360.
      IF (SIG1.GT.360.) SIG1=SIG1-360.
      IF (SIG2.LT.-180.) SIG2=SIG2+360.
      IF (SIG2.GT.180.) SIG2=SIG2-360.
      IF (SIG3.LT.0.) SIG3=SIG3+360.
      IF (SIG3.GT.360.) SIG3=SIG3-360.
      WRITE (5,96)
      DO 49 I=1,N
49    WRITE (5,97) IDENT(I),X(I),Y(I),RR(I),ORR(I)
      CALL ALPBET (TI,ALFA,BETA,ETA,CVP,SIG2,SIG1,SIG3)
      WRITE (6,98)
      WRITE (6,99) IFRAM,TI,ALPHA,XOMEG,XKAP,XPC,YPC,SIG1,SIG2,SIG3,ALFA
     1,BETA,ETA
      WRITE (10) TI,SIG1,SIG2,SIG3,ALFA,BETA,ETA
50    CONTINUE
      SALPHA=SIN(ALPHAR)
      CALPHA=COS(ALPHAR)
      SOMEGA=SIN(OMEGAR)
      COMEGA=COS(OMEGAR)
      SKAPPA=SIN(XKAPR)
      CKAPPA=COS(XKAPR)
```

```
      AAC=-CALPHA*CKAPPA-SALPHA*SOMEGA*SKAPPA
      BBC=SALPHA*CKAPPA-CALPHA*SOMEGA*SKAPPA
      CCC=COMEGA*SKAPPA
      AACP=CALPHA*SKAPPA-SALPHA*SOMEGA*CKAPPA
      BBCP=-SALPHA*SKAPPA-CALPHA*SOMEGA*CKAPPA
      CCCP=COMEGA*CKAPPA
      DDC=SALPHA*COMEGA
      EEC=CALPHA*COMEGA
      FFC=SOMEGA
      IF (ICASE-1) 52,52,51
51    CONTINUE
      IF (ITER-22) 53,52,52
52    CONTINUE
      WRITE (6,100)
53    CONTINUE
      J=0
54    J=J+1
      XMC=AAC*XLAM(J)+BBC*XMU(J)+CCC*XNU(J)
      XNC=AACP*XLAM(J)+BBCP*XMU(J)+CCCP*XNU(J)
      GC=DDC*XLAM(J)+EEC*XMU(J)+FFC*XNU(J)
      XMM(J)=XMC/GC
      XNN(J)=XNC/GC
      GG(J)=CC/GC
      XX1=SQRT((X(J)-XPCSTO)**2+(Y(J)-YPCSTO)**2)
      RR(J)=XX1
      CALL FTLUP (XX1,DELR,1,25,XRR,RRR)
      DELR=-DELR/1000.
      DRR(J)=DELR
      DISTOR=1.-DELR/XX1
      EPS(1,J)=DISTOR*(X(J)-XPC)-CC*XMM(J)
      EPS(2,J)=DISTOR*(Y(J)-YPC)-CC*XNN(J)
      DXY(J)=SQRT(EPS(1,J)**2+EPS(2,J)**2)
      CM(J)=CC*XMM(J)
      CN(J)=CC*XNN(J)
      IF (ICASE-1) 56,56,55
55    CONTINUE
      IF (ITER-22) 57,56,56
56    CONTINUE
      WRITE (6,101) EPS(1,J),EPS(2,J),IDENT(J)
57    IF (ITER.LE.1) SUM=SUM+EPS(1,J)*EPS(1,J)+EPS(2,J)*EPS(2,J)
      IF (N-J) 58,58,54
58    CONTINUE
      ILINE=ILINE+6+N
      IF (ILINE.GE.45) ILINE=0
      JJ=1
      IF (ITER.LE.1) XXN=N
      DO 62 J=1,N
      IF (J-NG(JJ)) 60,59,60
59    JJ=JJ+1
      GO TO 62
60    IF (ABS(EPS(1,J))-V) 61,61,63
61    IF (ABS(EPS(2,J))-V) 62,62,63
62    CONTINUE
      WRITE (6,102) ((EPS(M,MM),M=1,2),MM=1,N)
      WRITE (6,103)
      GO TO 21
63    DO 64 I=1,3
      C(I,1)=0.0
      DO 64 K=1,3
      XN(I,K)=0.0
64    CONTINUE
      JJ=1
      DO 70 J=1,N
      IF (J-NG(JJ)) 66,65,66
```

```
65     JJ=JJ+1
       GO TO 70
66     FF1=AAC-DDC*XMM(J)
       FF2=BBC-EEC*XMM(J)
       FF3=CCC-FFC*XMM(J)
       FB1=AACP-DDC*XNN(J)
       FB2=BBCP-EEC*XNN(J)
       FB3=CCCP-FFC*XNN(J)
       GG1=XMM(J)*COMEGA+FFC*SKAPPA
       GB1=XNN(J)*COMEGA+FFC*CKAPPA
       HH1=XLAM(J)*SALPHA+XMU(J)*CALPHA
       S1=-FF2*XLAM(J)+FF1*XMU(J)
       S2=FF3*HH1+GG1*XNU(J)
       SB1=-FB2*XLAM(J)+FB1*XMU(J)
       SB2=FB3*HH1+GB1*XNU(J)
       B(1,1)=S1*OO(J)
       B(1,2)=S2*OO(J)
       B(1,3)=-CC*XNN(J)
       B(2,1)=SB1*OO(J)
       B(2,2)=SB2*OO(J)
       B(2,3)=CC*XMM(J)
       DO 67 I=1,3
       BS(I,1)=B(1,I)
       BS(I,2)=B(2,I)
67     CONTINUE
       DO 69 I=1,3
       DO 68 L=1,2
       C(I,1)=C(I,1)+BS(I,L)*(-EPS(L,J))
68     CONTINUE
       DO 69 K=1,3
       DO 69 L=1,2
       XN(I,K)=XN(I,K)+BS(I,L)*B(L,K)
       XNTEMP(I,K)=XN(I,K)+BS(I,L)*B(L,K)
69     CONTINUE
70     CONTINUE
       J1=3
       J2=1
       CALL MATINV (XN(1,1),J1,C(1,1),J2,DETERM,IPIVOT,INDEX,6,ISCALE)
       DO 71 I=1,3
       DO 71 J=1,3
71     XNUNIT(I,J)=0.
       DO 72 I=1,3
       DO 72 K=1,3
       DO 72 J=1,3
72     XNUNIT(I,K)=XN(I,J)*XNTEMP(J,K)+XNUNIT(I,K)
       ALPHAR=ALPHAR+C(1,1)
       OMEGAR=OMEGAR+C(2,1)
       XKAPR=XKAPR+C(3,1)
       IF (XKAPR.LT.0.) XKAPR=XKAPR+TWOPI
       IF (ALPHAR.LT.0.) ALPHAR=ALPHAR+TWOPI
       IF (OMEGAR.LT.-PI) OMEGAR=OMEGAR+TWOPI
       IF (XKAPR.GT.TWOPI) XKAPR=XKAPR-TWOPI
       IF (ALPHAR.GT.TWOPI) ALPHAR=ALPHAR-TWOPI
       IF (OMEGAR.GT.PI) OMEGAR=OMEGAR-TWOPI
       ALPHA=ALPHAR*RAD
       XOMEG=OMEGAR*RAD
       XKAP=XKAPR*RAD
       ITER=ITER+1
       ICOUNT=ICOUNT+1
       IF (ICOUNT-21) 46,46,73
73     SUM=0.
       DO 74 J=1,N
```

```
74    SUM=SUM+EPS(1,J)**2+EPS(2,J)**2
      SUM=SQRT(SUM/TWON)
      WRITE (6,104) SUM
      ALPHAP=ALPHA
      XOMEGP=XOMEG
      XKAPP=XKAP
      IF (ALPHAP.LT.0..OR.ALPHAP.GT.TWOPI) ALPHAP=ALPHAI
      IF (XKAPP.LT.0..OR.XKAPP.GT.TWOPI) XKAPP=XKAPI
      IF (XOMEGP.LT.-PI.OR.XOMEGP.GT.PI) XOMEGP=XOMEGI
      GO TO (21,40), IFOCFG
13    WRITE (6,105)
      STOP
76    FORMAT (2F10.3)
77    FORMAT (80H
     1                                                                                )
78    FORMAT (2I5,6F5.0)
79    FORMAT (3F10.1)
80    FORMAT (16I5)
81    FORMAT (1I5,1F10.1)
82    FORMAT (8F10.1)
83    FORMAT (1I10,5F10.2)
84    FORMAT (7I5)
85    FORMAT (1I5/(7I5))
86    FORMAT (8X,1I2,30X,1F7.3,3X,1F7.3,6X,1I4)
87    FORMAT (1I5,2E13.7)
88    FORMAT (////6X,*FRAME*,1I4,*IGNORED BY USERS DIRECTION*)
89    FORMAT (/1X,*L-MARK LONGITUDE  LAT-GUCN  RAD ER KM  LAT-GODT ALTIT
     1UDE-FT     XLAM         XMU          XNU*)
90    FORMAT (1I5,2X,*IGNORED BY USERS DIRECTION*)
91    FORMAT (1I5,2X,2F10.5,1F10.3,1F10.5,1F11.3,3F10.7)
92    FORMAT(1H0,*VEHICLE      X              Y               Z              XPC          YPC
     1      CC*)
93    FORMAT (7X,3F10.2,3F10.5)
94    FORMAT (1I4,2F15.8)
95    FORMAT (1H0,32HL-MARK  XCO FINAL       YCO FINAL)
96    FORMAT (1H0,*L-MARK*,8X,1HX,14X,1HY,11X,6HR DIST,9X,7HDELTA R)
97    FORMAT (1H ,I3,4F15.5)
98    FORMAT(1H0,* FRAME  TIME     SIG1     SIG2     SIG3     XP       YP
     1PSI         THETA     PHI      ALPHA    BETA     ETA*)
99    FORMAT (1I5,4F8.3,2F7.3,2F10.3,1F8.3,3F8.3,1F9.3)
100   FORMAT (1H0,47HRESIDUALS  RX(I)              RY(I)             L-MARK)
101   FORMAT (2X,2F16.8,1I10)
102   FORMAT (//2E21.8)
103   FORMAT (///11H END OF JOB)
104   FORMAT (1H0,8X,*STANDARD DEVIATION OF RESIDUALS*,1F13.5)
105   FORMAT (//,1X,*END OF CASE*)
      END
```

```
      SUBROUTINE TGTCAM (NC,NJ,RZ,PHIZ,THZ,TXYZT,TGTTAB)
      DIMENSION A(3,3), XYZTAB(3,1), XSYSZS(3,1), TGTTAB(20,3), B(3,3),
     1DXYZJ(3,1), DXYZJP(3,1), AMV(3,1), AMVB(3,1), DELJPT(100,3), TXYZT
     2(20,4)
      DIMENSION ATAB(20), AMUTAB(20), VTAB(20)
      COMMON /DIRCOS/ ATAB,AMUTAB,VTAB
      DPR=57.29577951
      COSPZ=COS(PHIZ)
      COSTHZ=COS(THZ)
      SINPZ=SIN(PHIZ)
      SINTHZ=SIN(THZ)
      XZ=RZ*COSPZ*COSTHZ
```

```
      YZ=RZ*COSPZ*SINTHZ
      ZZ=RZ*SINPZ
      XSYSZS(1,1)=XZ
      XSYSZS(2,1)=YZ
      XSYSZS(3,1)=ZZ
      A(1,1)=-SINTHZ
      A(1,2)=-SINPZ*COSTHZ
      A(1,3)=+COSPZ*COSTHZ
      A(2,1)=COSTHZ
      A(2,2)=-SINPZ*SINTHZ
      A(2,3)=SINTHZ*COSPZ
      A(3,1)=0.
      A(3,2)=COSPZ
      A(3,3)=SINPZ
      INC=0
1     INC=INC+1
      TI=TXYZT(INC,1)
      XYZTAB(1,1)=TXYZT(INC,2)
      XYZTAB(2,1)=TXYZT(INC,3)
      XYZTAB(3,1)=TXYZT(INC,4)
      TGTFLG=0.
      INJ=0
2     INJ=INJ+1
      THJD=TGTTAB(INJ,1)
      PHIJD=TGTTAB(INJ,2)
      RJ=TGTTAB(INJ,3)
      THJ=THJD/DPR
      PHIJ=PHIJD/DPR
      IF (TGTFLG) 3,3,5
3     CONTINUE
      TGTFLG=1.
      DO 4 I=1,3
      DO 4 J=1,3
4     XSYSZS(I,1)=A(I,J)*XYZTAB(J,1)+XSYSZS(I,1)
      XS=XSYSZS(1,1)
      YS=XSYSZS(2,1)
      ZS=XSYSZS(3,1)
      TEMP=XS**2+YS**2
      TEMP=SQRT(TEMP)
      AL=ATAN2(ZS,TEMP)
      ALPT=AL*DPR
      T=ATAN2(YS,XS)
      TPT=T*DPR
      TEMP=XS**2+YS**2+ZS**2
      R=SQRT(TEMP)
      SINT=SIN(T)
      COST=COS(T)
      SINAL=SIN(AL)
      COSAL=COS(AL)
      B(1,1)=-SINT
      B(1,2)=COST
      B(1,3)=0.
      B(2,1)=-SINAL*COST
      B(2,2)=-SINAL*SINT
      B(2,3)=COSAL
      B(3,1)=COSAL*COST
      B(3,2)=SINT*COSAL
      B(3,3)=SINAL
5     CONTINUE
      COSPJ=COS(PHIJ)
      COSTHJ=COS(THJ)
```

```
      SINPJ=SIN(PHIJ)
      SINTHJ=SIN(THJ)
      XJ=RJ*COSPJ*COSTHJ
      YJ=RJ*COSPJ*SINTHJ
      ZJ=RJ*SINPJ
      DELXJ=XJ-XS
      DELYJ=YJ-YS
      DELZJ=ZJ-ZS
      DELJPT(INJ,1)=DELXJ
      DELJPT(INJ,2)=DELYJ
      DELJPT(INJ,3)=DELZJ
      TEMP=DELXJ**2+DELYJ**2+DELZJ**2
      DELRJ=SQRT(TEMP)
      AMV(1,1)=DELXJ/DELRJ
      AMV(2,1)=DELYJ/DELRJ
      AMV(3,1)=DELZJ/DELRJ
      DO 6 I=1,3
      AMVB(I,1)=0.
      DO 6 J=1,3
6     AMVB(I,1)=B(I,J)*AMV(J,1)+AMVB(I,1)
      DXYZJ(1,1)=DELXJ
      DXYZJ(2,1)=DELYJ
      DXYZJ(3,1)=DELZJ
      DO 7 I=1,3
      DXYZJP(I,1)=0.
      DO 7 J=1,3
7     DXYZJP(I,1)=B(I,J)*DXYZJ(J,1)+DXYZJP(I,1)
      ATAB(INJ)=AMVB(1,1)
      AMUTAB(INJ)=AMVB(2,1)
      VTAB(INJ)=AMVB(3,1)
      IF (NJ-INJ) 8,8,2
8     CONTINUE
      DJPTFG=0.
      DJPTFG=1.
      IF (DJPTFG) 9,9,11
9     WRITE (6,14)
      DO 10 INJ=1,NJ
10    WRITE (6,13) (DELJPT(INJ,K),K=1,3)
11    CONTINUE
      IF (NC-INC) 12,12,1
12    RETURN
C
13    FORMAT (1H0,5X,7E15.7)
14    FORMAT (1H0,8X,9HDELTA X J,6X,9HDELTA Y J,6X,9HDELTA Z J)
      END
```

```
      SUBROUTINE ALPBET (TIME,ALPHAD,BETAD,ETAD,CVP,THETA,PSI,PHI)
      COMMON /AB/ III,IWV,TX(50),VVX(50),VVY(50),VVZ(50),H(50),ALT(50),V
     1XO(50),VYO(50)
      DR=.0174532925
      RD=57.29577951
      CALL FTLUP (TIME,VX1,1,III,TX,VVX)
      CALL FTLUP (TIME,VY1,1,III,TX,VVY)
      CALL FTLUP (TIME,VZ1,1,III,TX,VVZ)
      CALL FTLUP (TIME,Z,1,III,TX,H)
      CALL FTLUP (Z,VXO,1,IWV,ALT,VXO)
      CALL FTLUP (Z,VYO,1,IWV,ALT,VYO)
      VZO=0.
      PSIR=PSI*DR
      PHIR=PHI*DR
```

```
      THETAR=THETA*DR
      X1=SIN(PSIR)
      X2=COS(PSIR)
      X3=SIN(PHIR)
      X4=COS(PHIR)
      X5=SIN(THETAR)
      X6=COS(THETAR)
      B1=X6*X2*VX1+X6*X1*VY1-X5*VZ1
      B2=(X2*X5*X3-X1*X4)*VX1+(X2*X4+X1*X5*X3)*VY1+X6*X3*VZ1
      B3=(X2*X5*X4+X1*X3)*VX1+(X1*X5*X4-X2*X3)*VY1+X6*X4*VZ1
      B4=X6*X2*VX0+X6*X1*VY0-X5*VZ0
      B5=(X2*X5*X3-X1*X4)*VX0+(X2*X4+X1*X5*X3)*VY0+X6*X3*VZ0
      B6=(X2*X5*X4+X1*X3)*VX0+(X1*X5*X4-X2*X3)*VY0+X6*X4*VZ0
      UP=B1-B4
      VP=B2-B5
      WP=B3-B6
      ALPHA=ATAN2(WP,UP)
      ETA=ATAN2((SQRT(VP**2+WP**2)),UP)
      BETA=ATAN2(VP,UP)
      ALPHAD=ALPHA*RD
      BETAD=BETA*RD
      ETAD=ETA*RD
      V2=UP*UP+VP*VP+WP*WP
      CVP=SQRT(V2)
      RETURN
      END
```

# APPENDIX C

## LANGLEY LIBRARY SUBROUTINE MATINV

Language: FORTRAN

Purpose: MATINV solves the matrix equation $AX = B$, where A is a square coefficient matrix and B is a matrix of constant vectors. The solution to a set of simultaneous equations, the matrix inverse, and the determinant may be obtained. If the user does not want the inverse, use SIMEQ for savings in time and storage. For the determinant only, use DETEV.

Use: CALL MATINV(A,N,B,M,DETERM,IPIVOT,INDEX,NMAX,ISCALE)

| | |
|---|---|
| A | A two-dimensional array of the coefficients. On return to the calling program, $A^{-1}$ is stored in A |
| N | The order of A, $1 \leqq N \leqq NMAX$ |
| B | A two-dimensional array of the constant vectors B. On return to the calling program, X is stored in B |
| M | The number of column vectors in B. The expression $M = 0$ signals that the subroutine is used solely for inversion; however, in the CALL statement an entry corresponding to B must still be present |
| DETERM | Gives the value of the determinant by the formula $DET(A) = \left(10^{100}\right)^{ISCALE}(DETERM)$ |
| IPIVOT | A one-dimensional array of temporary storage used by the routine |
| INDEX | A two-dimensional array of temporary storage used by the routine |
| NMAX | The maximum order of A as stated in the DIMENSION statement of the calling program |
| ISCALE | A scale factor computed by the subroutine to keep the results of computation within the floating-point word size of the computer |

Restrictions: Arrays A, B, IPIVOT, and INDEX have variable dimensions in the subroutine. The maximum size of these arrays must be specified in a DIMENSION statement of the calling program as A(NMAX,NMAX), B(NMAX,M), IPIVOT(NMAX), and INDEX(NMAX,2). The original matrices A and B are destroyed. They must be saved by the user if there is further need for them. The determinant is set to zero for a singular matrix.

Method: Jordan's method is used to reduce a matrix A to the identity matrix I through a succession of elementary transformations $l_n$, $l_{n-1}$, . . ., $l_1$. A = I. If these transformations are simultaneously applied to I and to a matrix B of constant vectors, the results are $A^{-1}$ and X where AX = B. Each transformation is selected so that the largest element is used in the pivotal position. (See ref. (a).)

Accuracy: Total pivotal strategy is used to minimize the rounding errors; however, the accuracy of the final results depends upon how well-conditioned the original matrix is.

Reference: (a) Fox, L.: An Introduction to Numerical Linear Algebra. Oxford Univ. Press, 1965.

Storage: $542_8$ locations.

Subroutine date: August 1, 1968.

## APPENDIX D

## LANGLEY LIBRARY SUBROUTINE FTLUP

Language: FORTRAN

Purpose: Computes $y = F(x)$ from a table of values using first- or second-order interpolation. An option to give $y$ a constant value for any $x$ is also provided.

Use: CALL FTLUP(X, Y, M, N, VARI, VARD)

X The name of the independent variable $x$.

Y The name of the dependent variable $y = F(x)$.

M The order of interpolation (an integer)

M = 0 for $y$ a constant. VARD(I) corresponds to VARI(I) for I = 1, 2, . . ., N. For M = 0 or $N \leqq 1$, $y$ = F(VARI(1)) for any value of $x$. The program extrapolates.

M = 1 or 2. First or second order if VARI is strictly increasing (not equal).

M = -1 or -2. First or second order if VARI is strictly decreasing (not equal).

N The number of points in the table (an integer).

VARI The name of a one-dimensional array which contains the N values of the independent variable.

VARD The name of a one-dimensional array which contains the N values of the dependent variable.

Restrictions: All the numbers must be floating point. The values of the independent variable $x$ in the table must be strictly increasing or strictly decreasing. The following arrays must be dimensioned by the calling program as indicated: VARI(N), VARD(N).

Accuracy: A function of the order of interpolation used.

References: (a) Nielsen, Kaj L.: Methods in Numerical Analysis. The Macmillan Co., c.1956, pp. 87-91.

(b) Milne, William Edmund: Numerical Calculus. Princeton Univ. Press, c.1949, pp. 69-73.

Storage: $430_8$ locations.

Error condition: If the VARI values are not in order, the subroutine will print TABLE BELOW OUT OF ORDER FOR FTLUP AT POSITION xxx TABLE IS STORED IN LOCATION xxxxxx (absolute). It then prints the contents of VARI and VARD, and STOPS the program.

Subroutine date: September 12, 1969.

## APPENDIX E

## BASIC DATA

### Computer Card Format

A typical basic data computer card format is as follows:

| Column | |
|---|---|
| 1 | shows whether the data are for a landmark or fiducial point and, if the latter, which fiducial system is being used: 4 is used for landmarks; 1 for frame corner fiducial points; and 0 for film sprocket hole fiducial points |
| 2 to 5 | vehicle or test identification numbers |
| 7 | onboard camera identification number |
| 8 | blank |
| 9 to 10 | landmark number for landmark readings; 00 for fiducial point readings |
| 11 | film reading system identification number |
| 12 | blank |
| 13 to 14 | 1, 2, or 3; indicates which of the required three readings (for averaging purposes) of each fiducial point or landmark is on the card |
| 15 to 19 | accumulated point number, including fiducials |
| 20 to 40 | blank |
| 41 to 47 | $\bar{x}$-value (in inches $\times 10^{-3}$) of the landmark or fiducial point on the frame, measured horizontally on the projected image, with origin at right edge of image or right sprocket holes of film |
| 48 to 50 | blank |

Column

| | |
|---|---|
| 51 to 57 | $\bar{y}$-value (in inches × $10^{-3}$) of the landmark or fiducial point on the frame, measured vertically on the projected image, with origin at bottom edge of projected image or bottom sprocket holes of film |
| 58 to 63 | blank |
| 64 to 67 | frame number based on a convenient zero reference frame |

Only the data in columns 9 to 10 and after column 40 serve as inputs to the computer program. The remaining data are used for identification and gross reading or system error analyses only and may be altered at the user's discretion. For the system employed at Langley Research Center, the data through column 11 are input manually on dials located on the digitizer. The remaining data are input automatically and displayed on the digitizer. All data are recorded by the typer as well as on the punched computer cards.

## Data Reading

The following discussion is related to the procedures and system currently employed at the NASA Langley Research Center utilizing the Gerber film reading system and focuses on getting the basic data into a form usable by the computer.

First a frame reference time must be established and the landmarks and features to be used must be identified. Next, sketches showing the position of the landmarks (and features) in relation to other distinct features (for example, mountain ranges, rivers, surface discolorations) on the film should be prepared. These sketches are useful in determining the general landmark location on the projected film image.

A typical sequence in reading a frame of film, after the film has been installed in the film reader, is enumerated. These instructions are for the Gerber film reading system at the Langley Research Center and may be modified for other systems.

(1) Set the frame counter to the proper value

(2) Establish the image space axes system ($\bar{x}$, $\bar{y}$) origin (for example, frame corner)

(3) Focus the frame (do not change the focus until the next frame)

(4) Assure that the $\bar{x}$ and $\bar{y}$ values increase in the desired direction by moving the crosshairs

(5) Aline the crosshairs on the first fiducial point (for example, frame corner, film sprocket hole)

(6) Assure that all dials on the digitizer are properly set and reset the $\bar{x}$ and $\bar{y}$ values on the digitizer to zero

(7) Assure the readiness of the typer and card punch

(8) Punch the fiducials (three readings for each of the four fiducials)

(9) Adjust the digitizer dials as necessary for landmark readings and punch the landmarks (three readings each)

(10) Check the printing from the typer for general correctness

(11) Advance to the next frame to be read and repeat these steps.

Finally, the cards should be interpreted and listed. A review of this listing should reveal any errors or blank cards which should be removed prior to processing.

# APPENDIX F

## CONTROL DATA AND BASIC DATA INPUT DESCRIPTION

### Control Data

For user's convenience, program VOPD has been divided into distinctive input and output groups. Following is a detailed description in computer terminology of the input for each group of control data.

| Column | Parameter | Description | Units |
|---|---|---|---|
| Input Group 1 | Format (2F10.3) | 25 cards | |
| 1 to 10 | XRR | Radial distance from frame center | mm |
| 11 to 20 | RRR | Lens radial distortion | $\mu$m |
| Input Group 2 | Format (80H) | 1 card | |
| 1 | 1 | Punch a 1 in column 1 | |
| 2 to 80 | TITLE | Descriptive title, printed at top of each output page | |
| Input Group 3 | Format (2I5, 6F5.1) | 1 card | |
| 1 to 5 | III | Number of data values in Input Group 7 | |
| 6 to 10 | IWV | Number of data values in Input Group 4 | |
| 11 to 15 | S1M | Multiplying factor to convert camera yaw angle $(\sigma_1)$ to vehicle yaw angle $(\psi)$ | |
| 16 to 20 | S1A | Addition factor to convert camera yaw angle $(\sigma_1)$ to vehicle yaw angle $(\psi)$ PSI = S1M * SIG1 + S1A, S1M = +1 or -1 | deg |

| Column | Parameter | Description | Units |
|---|---|---|---|
| 21 to 25 | S2M | Multiplying factor to convert camera pitch angle $(\sigma_2)$ to vehicle pitch angle $(\theta)$ | |
| 26 to 30 | S2A | Addition factor to convert camera pitch angle $(\sigma_2)$ to vehicle pitch angle $(\theta)$ THETA = S2M * SIG2 + S2A, S2M = +1 or -1 | deg |
| 31 to 35 | S3M | Multiplying factor to convert camera roll angle $(\sigma_3)$ to vehicle roll angle $(\phi)$ | |
| 36 to 40 | S3A | Addition factor to convert camera roll angle $(\sigma_3)$ to vehicle roll angle $(\phi)$ PHI = S3M * SIG3 + S3A, S3M = +1 or -1 | deg |
| Input Group 4 | Format (3F10.1) | IWV cards | |
| 1 to 10 | ALT | Altitude table, mean sea level | ft |
| 11 to 20 | VXO | North-South wind velocity table, + from South | fps |
| 21 to 30 | VYO | East-West wind velocity table, + from West | fps |
| Input Group 5 | Format (16I5) | (NFR + 3)/16 cards | |
| 1 to 5 | NT | Number of landmarks read on this film, also number of Input Group 9 data values | |

| Column | Parameter | Description | Units |
|---|---|---|---|
| 6 to 10 | NFR | Number of frames read from this film | |
| 11 to 15 | NTFR | Number of Input Group 6 data values | |
| 16 to 20 (etc.) | NPT(I) | Number of landmarks in each frame, etc., I = 1 to NFR | |
| Input Group 6 | Format (1I5, 1F10.1) | NTFR cards | |
| 1 to 5 | IFRM | Frame number, does not have to correspond to frames read, but must encompass those of the basic data deck | |
| 6 to 15 | TIMX | Flight time of this frame | sec |
| Input Group 7 | Format (8F10.1) | III cards | |
| 1 to 10 | TX | Time table | sec |
| 11 to 20 | XXX | X-location of the vehicle from origin, + East | ft |
| 21 to 30 | YYY | Y-location of the vehicle from origin, + North | ft |
| 31 to 40 | ZZZ | Z-location of the vehicle from origin, + zenith | ft |
| 41 to 50 | VVX | Vehicle velocity, + toward North | fps |
| 51 to 60 | VVY | Vehicle velocity, + toward East | fps |

| Column | Parameter | Description | Units |
|---|---|---|---|
| 61 to 70 | VVZ | Vehicle velocity, + toward Earth | fps |
| 71 to 80 | H | Altitude of vehicle, mean sea level | ft |
| Input Group 8 | Format (7F10.0) | 1 card | |
| 1 to 10 | CC | Camera focal length | mm |
| 11 to 20 | PHI | Latitude of the origin of Group 7 coordinate system, geodetic | deg |
| 21 to 30 | THZ | Longitude of the origin of Group 7 coordinate system, geodetic | deg |
| 31 to 40 | RZ | Distance from center of Earth to origin of Group 7 coordinate system | $10^3$ ft |
| 41 to 50 | SIG1 | Initial estimate of camera Euler yaw angle | deg |
| 51 to 60 | SIG2 | Initial estimate of camera Euler pitch angle | deg |
| 61 to 70 | SIG3 | Initial estimate of camera Euler roll angle | deg |
| Input Group 9 | Format (1I10, 5F10.1) | NT cards | |
| 1 to 10 | ITGT | Landmark ID number | |
| 11 to 20 | THJ | Longitude of this landmark, degree part only | deg |

| Column | Parameter | Description | Units |
|---|---|---|---|
| 21 to 30 | THJMI | Longitude of this landmark, minute part only | min |
| 31 to 40 | PHIJ | Geodetic latitude of this landmark, degree part only | deg |
| 41 to 50 | PHIJMI | Geodetic latitude of this landmark, minute part only | min |
| 51 to 60 | RJ | Landmark altitude, mean sea level | ft |
| Input Group 10 | Format (1I5) | 1 card | |
| 1 to 5 | NOUT | Number of data values in Input Group 11 | |
| Input Group 11 | Format (7I5) | NOUT cards | |
| 1 to 5 | IFM | Frame number as read from film | |
| 6 to 10 (etc.) | JOUT(I) | Landmarks to be ignored for this frame only, I = 1 to 6 for as many as required | |

If JOUT(I) = 100 this entire frame is ignored; do not remove its data from the basic data deck.

| Column | Parameter | Description | Units |
|---|---|---|---|
| Input Group 12 | Format (16I5) | NT/16 cards | |
| 1 to 5 | NGO | Input group number to which program returns after processing all the frames of this job = 13 to stop further calculations | |
| 6 to 10 (etc.) | IOUT | Landmarks to be ignored in computations for all frames | |

### Basic Data

The basic data deck as punched from the film reader is input after the control data deck. It consists of three cards for each of four fiducial frame corners plus three cards for each of the NPT landmarks for a total of 12 + 3 * NPT cards. Following is a list of the essential information from the basic data deck required for program operation. For an explanation of inputs for all columns of the basic data cards see appendix E.

| Column | Description | Units |
|---|---|---|
| 9 to 10 | Landmark number | |
| 41 to 47 | $\bar{x}$-value of landmark on the film | in. |
| 51 to 57 | $\bar{y}$-value of landmark on the film | in. |
| 64 to 67 | Frame number | |

Only one copy of this basic data deck is required. For additional calculations with these data only these input groups, including and following the one specified by NGO, must be inserted behind this basic data deck.

## APPENDIX G

## PROGRAM VOPD OUTPUT DESCRIPTION

The output of this program is listed one page to a film frame and includes the following groups of data:

| Parameter | Description | Units |
|---|---|---|
| The title card is printed at the top of the page. | | |
| Output Group 1 | NPT lines | |
| L-MARK | Landmark number | |
| LONGITUDE | Landmark longitude | deg |
| LAT-GOCN | Landmark geocentric latitude | deg |
| RAD-ER-KFT | Earth center to landmark distance | $10^3$ ft |
| LAT-GODT | Landmark geodetic latitude | deg |
| ALTITUDE-FT | Landmark altitude, mean sea level | ft |
| XLAM, XMU, XNU | Direction cosines of image space coordinate system relative to object space coordinate system (see appendix A) | |
| Output Group 2 | 1 line | |
| X | X-location of vehicle, same as Input Group 8 XXX | ft |
| Y | Y-location of vehicle, same as Input Group 8 YYY | ft |
| Z | Z-location of vehicle, same as Input Group 8 ZZZ | ft |

| Parameter | Description | Units |
|---|---|---|
| XPC | X-location of film frame center, average of fiducials | in. |
| YPC | Y-location of film frame center, average of fiducials | in. |
| CC | Camera focal length | mm |
| Output Group 3 | NPT lines | |
| XCO FINAL | X-location of landmark in image space (on film) | in. |
| YCO FINAL | Y-location of landmark in image space (on film) | in. |
| Output Group 4 | NPT lines | |
| X | X-location of landmark in object space | in. |
| Y | Y-location of landmark in object space | in. |
| R DIST | Landmark location from frame center, image space | in. |
| DELTA R | Radial distortion of landmark due to camera lens | in. |
| Output Group 5 | 1 line | |
| FRAME | Frame number | |
| TIME | Flight time | sec |
| SIG1 | Camera Euler angle, yaw | deg |
| SIG2 | Camera Euler angle, pitch | deg |
| SIG3 | Camera Euler angle, roll | deg |
| XP | X-location of frame center in object space | in. |
| YP | Y-location of frame center in object space | in. |

| Parameter | Description | Units |
|---|---|---|
| PSI | Vehicle Euler angle, yaw | deg |
| THETA | Vehicle Euler angle, pitch | deg |
| PHI | Vehicle Euler angle, roll | deg |
| ALPHA | Vehicle pitch angle of attack | deg |
| BETA | Vehicle yaw angle of attack | deg |
| ETA | Vehicle total angle of attack | deg |
| Output Group 6 | NPT lines | |
| RX(I) | Error in X-coordinate from frame center | in. |
| RY(I) | Error in Y-coordinate from frame center | in. |
| L-MARK | Landmark number | |
| Output Group 7 | 1 line | |

The standard deviation of the residuals is listed here.

# APPENDIX H

## EXAMPLE DATA

Example input and output data for several sample cases are presented in this appendix.

### Input

```
COLUMN    11111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Sample case 1

```
0.          0.
.5          5.
1.          10.
1.5         15.
2.          20.
2.5         30.
3.          40.
3.25        45.
3.5         55.
3.75        65.
4.          70.
4.2         75.
4.4         76.5                                                    GROUP 1
4.6         77.5
4.8         77.5
5.          76.5
5.2         76.
5.4         75.
5.6         70.
5.8         65.
6.2         60.
6.4         50.
6.6         40.
6.8         30.
7.          22.5
```

```
COLUMN    11111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890

1                                   VIKING  BLDT  AV-4   FOWARD  MILLIKEN  CAMERA       GROUP 2

   4   3 1.0   0.0   1.0   0.0   1.0 -210.                                              GROUP 3

130000.   32.       -90.
140000.   -4.       -144.                                                               GROUP 4
150000.   -2.       -160.

   3   24    2    2    2    2    2    3    3    3    2    2    3    3    3    3         GROUP 5
   3    3    3    3    3    3    3    3    3    3

   0  48.073                                                                            GROUP 6
2520 126.774

 65.      -26842.93 -1290.53  145928.29 -409.093  -431.207  416.378   145945.5
 70.      -28678.45 -3130.53   143619.3 -314.96   -339.688   499.32    143639.1         GROUP 7
 71.      -29014.85 -3436.18   143114.4 -295.61   -330.916   509.091   143134.7
 75.      -30305.95 -4442.24   140992.7 -211.34   -308.957   543.106   141014.9

 10.4      33.083333 -106.3333 20925.698  198.      -38.      285.                      GROUP 8

         65 -106.     -36.65    32.       50.3      10000.
         66 -106.     -32.7     32.       50.       10000.                              GROUP 9
         67 -166.     -24.5     32.       49.75      4000.
```

```
COLUMN    11111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890

 0                                                                   GROUP 10

 12                                                                  GROUP 12

1122402000410100100                 000003   000003   000730
1122402000410200100                 000002   000004   000730
1122402000410300100                -000004  -000004   000730
1122402000410100101                 014481   000002   000730
1122402000410200101                 014477  -000001   000730
1122402000410300101                 014475  -000003   000730
1122402000410100102                 014472   010454   000730
1122402000410200102                 014476   010455   000730
1122402000410300102                 014474   010457   000730
1122402000410100103                 000008   010458   000730
1122402000410200103                 000002   010457   000730        BASIC
1122402000410300103                 000005   010457   000730        DATA
4122402065410100104                 010178   001655   000730
4122402065410200104                 010174   001651   000730
4122402065410300104                 010174   001649   000730
4122402066410100105                 011147   002976   000730
4122402066410200105                 011137   002977   000730
4122402066410300105                 011127   002986   000730
4122402067410100106                 012869   006237   000730
4122402067410200106                 012867   006242   000730
4122402067410300106                 012864   006246   000730
```

```
COLUMN    11111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Sample case 2

```
    9   65                                                                  GROUP 12
```

Sample case 3

```
        65 -106.     -36.65    32.       50.3      10000.                   GROUP 9
        66 -106.     -32.7     32.       50.3      10000.
        67 -106.     -24.5     32.       49.75      4000.
```

```
    0                                                                       GROUP 10
```

```
    2   65                                                                  GROUP 12
```

Sample case 4

```
1                                  VIKING  BLDT  LOAD  BAR  CAMERA  AV-4    GROUP 2
```

```
    3    2 1.    0.    1.    0.    1.    0.                                 GROUP 3
```

```
120000.   -22.      -70.                                                    GROUP 4
123000.   -28.      -84.
```

```
COLUMN    111111111122222222223333333333444444444455555555556666666666777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
--------------------------------------------------------------------------------
    3    3    2    3    3    3                                                      GROUP 5
--------------------------------------------------------------------------------
   55  -.782                                                                        GROUP 6
  101  0.
--------------------------------------------------------------------------------
 -.8       31160.44  74640.67  120591.62 -34.555  -91.947   -10.208   120767.6
 -.4       31124.71  74627.08  120598.8  -36.199  -90.249    -9.951   120765.1     GROUP 7
 0.        31088.2   74611.38  120607.16 -36.677  -93.154     -.064   120762.6
--------------------------------------------------------------------------------
10.4       33.083333 -106.3333 20925.698  180.     -89.      180.                   GROUP 8
--------------------------------------------------------------------------------
        28 -106.     -14.2     33.       22.5       4300.
        62 -106.     -18.3     33.       15.8       4300.                          GROUP 9
        69 -106.     -20.7     33.       21.7       4300.
--------------------------------------------------------------------------------
    3                                                                               GROUP 10
--------------------------------------------------------------------------------
   55  100
   60   69                                                                          GROUP 11
   65   69
--------------------------------------------------------------------------------
   13                                                                               GROUP 12
--------------------------------------------------------------------------------
```

```
COLUMN    11111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890

1122402000410100001                     -000002  -000003   000055
1122402000410200001                     -000002   000001   000055
1122402000410300001                      000004  -000001   000055
1122402000410100002                      014397  -000002   000055
1122402000410200002                      014393  -000005   000055
1122402000410300002                      014392   000004   000055
1122402000410100003                      014391   010506   000055
1122402000410200003                      014392   010499   000055
1122402000410300003                      014389   010494   000055
1122402000410100004                      000000   010497   000055
1122402000410200004                     -000000   010496   000055
1122402000410300004                      000002   010493   000055
4122402028410100005                      010255   008847   000055
4122402028410200005                      010245   008857   000055
4122402028410300005                      010237   008860   000055
4122402062410100006                      008296   003371   000055
4122402062410200006                      008298   003379   000055
4122402062410300006                      008293   003379   000055
4122402069410100007                      010159   004780   000055
4122402069410200007                      010164   004784   000055
4122402069410300007                      010155   004783   000055
1122402000410100008                     -000002  -000010   000060
1122402000410200008                      000005   000012   000060
1122402000410300008                     -000004   000004   000060
1122402000410100009                      014386   000002   000060
1122402000410200009                      014388   000006   000060
1122402000410300009                      014393   000006   000060
1122402000410100010                      014386   010485   000060
1122402000410200010                      014386   010484   000060
1122402000410300010                      014386   010480   000060
1122402000410100011                     -000003   010482   000060
1122402000410200011                      000001   010488   000060
1122402000410300011                     -000002   010480   000060
```

BASIC
DATA

```
COLUMN    1111111111222222222233333333334444444444555555555566666666667777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

| | | | | |
|---|---|---|---|---|
| 4122402028410100012 | 010287 | 008795 | 000060 | BASIC DATA |
| 4122402028410200012 | 010284 | 008794 | 000060 | |
| 4122402028410300012 | 010281 | 008802 | 000060 | |
| 4122402062410100013 | 008302 | 003388 | 000060 | |
| 4122402062410200013 | 008291 | 003381 | 000060 | |
| 4122402062410300013 | 008282 | 003388 | 000060 | |
| 4122402069410100014 | 010159 | 004787 | 000060 | |
| 4122402069410200014 | 010163 | 004786 | 000060 | |
| 4122402069410300014 | 010156 | 004788 | 000060 | |
| 1122402000410100015 | -000006 | 000000 | 000065 | |
| 1122402000410200015 | -000005 | 000003 | 000065 | |
| 1122402000410300015 | -000007 | 000000 | 000065 | |
| 1122402000410100016 | 014381 | -000004 | 000065 | |
| 1122402000410200016 | 014378 | 000001 | 000065 | |
| 1122402000410300016 | 014377 | 000004 | 000065 | |
| 1122402000410100017 | 014375 | 010487 | 000065 | |
| 1122402000410200017 | 014372 | 010480 | 000065 | |
| 1122402000410300017 | 014374 | 010486 | 000065 | |
| 1122402000410100018 | 000005 | 010484 | 000065 | |
| 1122402000410200018 | -000001 | 010480 | 000065 | |
| 1122402000410300018 | 000001 | 010488 | 000065 | |
| 4122402028410100019 | 010260 | 008810 | 000065 | |
| 4122402028410200019 | 010259 | 008808 | 000065 | |
| 4122402028410300019 | 010264 | 008804 | 000065 | |
| 4122402062410100020 | 008282 | 003363 | 000065 | |
| 4122402062410200020 | 008283 | 003367 | 000065 | |
| 4122402062410300020 | 008281 | 003368 | 000065 | |
| 4122402069410100021 | 010159 | 004763 | 000065 | |
| 4122402069410200021 | 010144 | 004777 | 000065 | |
| 4122402069410300021 | 010138 | 004782 | 000065 | |

Output

Sample case 1

```
                                 VIKING  BLDT  AV-4   FOWARD  MILLIKEN  CAMERA

L-MARK LONGITUDE   LAT-GOCN  RAD ER KM  LAT-GODT ALTITUDE-FT     XLAM        XMU         XNU
  65   -106.61083  32.66227 20935.698  32.83833  10000.000 -.3354096 -.5099642 -.7921091
  66   -106.54500  32.65728 20935.698  32.83333  10000.000 -.2214179 -.5359534 -.8146950
  67   -106.40833  32.65313 20929.698  32.82917   4000.000  .0349096 -.5390263 -.8415652

VEHICLE     X          Y          Z          XPC        YPC       CC
       -28971.56  -3396.84  143179.38   7.23925    5.22825   10.40000

L-MARK  XCO FINAL       YCO FINAL
 65    10.17533300     1.65166670
 66    11.13700000     2.97966670
 67    12.86666700     6.24166670

L-MARK          X              Y            R DIST          DELTA R
 65          7.26332        1.17899        3.30309         -.04712
 66          7.94977        2.12693        3.21206         -.04424
 67          9.18444        4.45540        4.08156         -.07204

 FRAME  TIME    SIG1    SIG2    SIG3     XP      YP       PSI        THETA     PHI      ALPHA    BETA     ETA
 730  70.871 195.270 -38.653 285.837  5.167  3.732    195.270    -38.653  75.837   -4.916  19.770  20.283

RESIDUALS  RX(I)           RY(I)          L-MARK
        -.01101123      .17502027         65
         .14423583     -.05060775         66
        -.11077771     -.12648144         67

          STANDARD DEVIATION OF RESIDUALS       .11718
```

Sample case 2

```
                                        VIKING  BLDT  AV-4   FOWARD  MILLIKEN  CAMERA

L-MARK LONGITUDE  LAT-GOCN  RAD ER KM  LAT-GODT ALTITUDE-FT    XLAM       XMU        XNU
  65  IGNORED BY USERS DIRECTION
  66  -106.54500  32.66227 20935.698  32.83833  10000.000 -.2227328 -.5279743 -.8195323
  67  -106.40833  32.65313 20929.698  32.82917   4000.000  .0349096 -.5390263 -.8415652

VEHICLE     X          Y         Z          XPC        YPC       CC
       -28971.56  -3396.84 143179.38   7.23925    5.22825  10.40000

L-MARK  XCO FINAL      YCO FINAL
 66    11.13700000    2.97966670
 67    12.86666700    6.24166670

L-MARK         X              Y              R DIST        DELTA R
 66         7.94977        2.12693         3.21206        -.04424
 67         9.18444        4.45540         4.08156        -.07204

 FRAME  TIME    SIG1    SIG2    SIG3      XP      YP      PSI        THETA     PHI      ALPHA    BETA     ETA
 730  70.871 194.194 -38.640 284.972  5.167  3.732   194.194   -38.640  74.972  -5.179  20.137  20.693

RESIDUALS  RX(I)          RY(I)          L-MARK
        .04850922      .08970406        66
       -.03905631     -.08823835        67

        STANDARD DEVIATION OF RESIDUALS       .07020
```

Sample case 3

```
                           VIKING  BLDT  AV-4   FOWARD  MILLIKEN  CAMERA

L-MARK LONGITUDE  LAT-GOCN  RAD ER KM  LAT-GODT ALTITUDE-FT    XLAM      XMU       XNU
  65  IGNORED BY USERS DIRECTION
  66  -106.54500  32.65728 20935.698  32.83333  10000.000 -.2214179 -.5359534 -.8146950
  67  -106.40833  32.65313 20929.698  32.82917   4000.000  .0349096 -.5390263 -.8415652

VEHICLE    X          Y          Z          XPC        YPC       CC
       -28971.56  -3396.84 143179.38   7.23925    5.22825  10.40000

L-MARK  XCO FINAL       YCO FINAL
 66     11.13700000     2.97966670
 67     12.86666700     6.24166670

L-MARK         X              Y              R DIST         DELTA R
 66         7.94977        2.12693        3.21206         -.04424
 67         9.18444        4.45540        4.08156         -.07204

 FRAME  TIME    SIG1    SIG2    SIG3      XP      YP       PSI        THETA     PHI     ALPHA    BETA     ETA
 730  70.871 193.258 -38.307 283.630  5.167  3.732   193.258    -38.307  73.630  -5.099  20.826  21.342

RESIDUALS  RX(I)           RY(I)          L-MARK
         .04554002       .08425219       66
        -.03671808      -.08287359       67

        STANDARD DEVIATION OF RESIDUALS       .06593
```

Sample case 4

```
                              VIKING  BLDT  LOAD  BAR  CAMERA  AV-4

L-MARK LONGITUDE  LAT-GOCN  RAD ER KM  LAT-GODT ALTITUDE-FT    XLAM       XMU        XNU
  69  IGNORED BY USERS DIRECTION
  28  -106.23667  33.19748 20929.998  33.37500   4300.000 -.0116687  .2646699 -.9642685
  62  -106.30500  33.08611 20929.998  33.26333   4300.000 -.1877649 -.0732142 -.9794815

VEHICLE    X          Y         Z         XPC       YPC      CC
        31151.24  74637.17 120593.47   7.19333   5.24325  10.40000

L-MARK  XCO FINAL      YCO FINAL
 28    10.28400000    8.79700000
 62     8.29166670    3.38566670

L-MARK        X             Y            R DIST        DELTA R
 28        7.34089       6.27944       3.36187       -.04947
 62        5.91873       2.41674       1.54041       -.01540

 FRAME  TIME    SIG1    SIG2   SIG3     XP      YP       PSI       THETA    PHI      ALPHA   BETA     ETA
  60   -.697 225.133 -86.339 181.954  5.135  3.743   225.133   -86.339 181.954 112.219-156.597 111.912

RESIDUALS  RX(I)          RY(I)        L-MARK
        .00887809      .02759145       28
       -.01100635     -.03052913       62

        STANDARD DEVIATION OF RESIDUALS       .02176
```

VIKING BLDT LOAD BAR CAMERA AV-4

| L-MARK | LCNGITUDE | LAT-GOCN | RAD ER KM | LAT-GODT | ALTITUCE-FT | XLAM | XMU | XNU |
|---|---|---|---|---|---|---|---|---|
| 69 | IGNORED BY USERS DIRECTION | | | | | | | |
| 28 | -106.23667 | 33.19748 | 20929.998 | 33.37500 | 4300.000 | -.0116060 | .2646890 | -.9642640 |
| 62 | -106.30500 | 33.08611 | 20929.998 | 33.26333 | 4300.000 | -.1877016 | -.0731902 | -.9794954 |

| VEHICLE | X | Y | Z | XPC | YPC | CC |
|---|---|---|---|---|---|---|
| | 31143.65 | 74634.28 | 120594.99 | 7.18700 | 5.24242 | 10.40000 |

| L-MARK | XCO FINAL | YCO FINAL |
|---|---|---|
| 28 | 10.26100000 | 8.80733330 |
| 62 | 8.28200000 | 3.36600000 |

| L-MARK | X | Y | R DIST | DELTA R |
|---|---|---|---|---|
| 28 | 7.32447 | 6.28682 | 3.36010 | -.04940 |
| 62 | 5.91183 | 2.40271 | 1.55080 | -.01551 |

| FRAME | TIME | SIG1 | SIG2 | SIG3 | XP | YP | PSI | THETA | PHI | ALPHA | BETA | ETA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | -.612 | 225.465 | -86.386 | 181.407 | 5.130 | 3.742 | 225.465 | -86.386 | 181.407 | 112.175 | -158.836 | 111.930 |

| RESIDUALS | RX(I) | RY(I) | L-MARK |
|---|---|---|---|
| | .01133048 | .03564586 | 28 |
| | -.01404192 | -.03941264 | 62 |

STANDARD DEVIATION OF RESIDUALS .02806

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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