A DATA REDUCTION TECHNIQUE
AND ASSOCIATED COMPUTER PROGRAM
FOR OBTAINING VEHICLE ATTITUDES
WITH A SINGLE ONBOARD CAMERA

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A detailed discussion of the application of a previously developed method to determine vehicle flight attitude using a single camera onboard the vehicle is presented with emphasis on the digital computer program format and data reduction techniques. Application requirements include film and Earth-related coordinates of at least two landmarks (or features), location of the flight vehicle with respect to the Earth, and camera characteristics. Included in this report are a detailed discussion of the program input and output format, a computer program listing, a discussion of modifications made to the initial method, a step-by-step basic data reduction procedure, and several example applications. The computer program is written in FORTRAN IV language for the Control Data 6000 series digital computer.
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SUMMARY

A detailed discussion of the application of a previously developed method to determine vehicle flight attitude using a single camera onboard the vehicle is presented with emphasis on the digital computer program format and data reduction techniques. Application requirements include film and Earth-related coordinates of at least two landmarks (or features), location of the flight vehicle with respect to the Earth, and camera characteristics. Included in this report are a detailed discussion of the program input and output format, a computer program listing, a discussion of modifications made to the initial method, a step-by-step basic data reduction procedure, and several example applications. The computer program is written in FORTRAN IV language for the Control Data 6000 series digital computer.

INTRODUCTION

A postflight photogrammetric method was previously devised for determining a continuous history of vehicle flight attitudes (Euler angles) using only film data from a single onboard camera along with a ground track of the vehicle. A discussion of method requirements, assumptions, mathematical relationships, and results is presented in reference 1. The method is based on work presented in references 2 and 3. Results from other applications and comparisons with statistical trajectory reconstruction techniques are included in references 4 to 6. However, detailed discussions of method application or data reduction system capabilities are not included in the reference documents. Numerous inquiries have been received concerning method application, particularly from the standpoint of computer programing. In addition, several modifications to the program which make the method more versatile and streamline its use have been made for application to the Viking Balloon Launched Decelerator Test (BLDT) program data reduction

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The purpose of this paper is to present details concerning method application, including an updated program listing and example applications, and to discuss current capabilities and restrictions of the associated Langley Research Center data reduction system. This paper is intended as a user's guide for future applications of the method.

SYMBOLS

Measurements are presented in both SI units and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

- \( c \) focal length of camera
- \( h \) height above sea level of camera, meters (feet)
- \( I \) point in image space
- \( K \) constant of proportionality between object space coordinate system and image space coordinate system
- \( n \) number of observations
- \( Q \) point in object space
- \( R \) geocentric earth radius, meters (feet)
- \( X, Y, Z \) object space coordinate axes with origin at focal point; also, distance along these axes, meters (feet)
- \( \bar{X}, \bar{Y}, \bar{Z} \) distance from origin of geocentric coordinate axes, meters (feet)
- \( X_E, Y_E, Z_E \) Earth-fixed axis system
- \( X_I, Y_I, Z_I \) image space coordinate axes with origin at focal point
- \( \bar{x}, \bar{y} \) measured image coordinates in image space coordinate system
- \( \bar{x}_p, \bar{y}_p \) coordinates of point of intersection between focal plane and focal axes in image space coordinate system; center of frame
\( \alpha, \beta \) angles of attack in pitch and yaw, respectively, degrees

\( \Delta \sigma \) change in attitude angle, degrees

\( \epsilon \) error in \( \tilde{x} \)

\( \tilde{\epsilon} \) error in \( \tilde{y} \)

\( \eta \) total angle of attack, degrees

\( \Lambda \) longitude of camera measured positive east from Greenwich, degrees

\( \lambda_{1, \mu_{1}, \nu_{1}}, \lambda_{2, \mu_{2}, \nu_{2}}, \lambda_{3, \mu_{3}, \nu_{3}} \) directional cosines of image space coordinate system relative to object space coordinate system

\( \sigma_{1, \sigma_{2}, \sigma_{3}} \) camera azimuth, pitch, and roll Euler angles relative to Earth-fixed axes, degrees

\( \Phi \) geodetic latitude of camera measured positive north, degrees

\( \Phi' \) geocentric latitude of camera measured positive north, degrees

\( \psi, \theta, \phi \) vehicle yaw, pitch, and roll Euler angles relative to Earth-fixed axes, degrees

Subscripts:

\( i \) for camera \( i = 1 \) and for object space points \( i = 2, \ldots, n + 1 \)

\( o \) initial conditions

\( S \) vehicle or spacecraft

Superscript:

\( T \) denotes transpose of matrix
CAMERA-VEHICLE ORIENTATION METHOD

In order to determine the orientation of a flight vehicle (or spacecraft) with respect to the Earth by using this photogrammetric method, the following information is required: location of at least two landmarks on each frame of the film, Earth-related coordinates of these landmarks, location of the flight vehicle with respect to Earth (as determined by radar, for example), orientation of the camera within the vehicle, camera focal length, and lens distortion characteristics. Atmospheric refraction corrections are not included because these corrections would have been insignificant for previous applications. Digital computer techniques can then be applied to determine the relationship between the two coordinate systems defined by the film frame (image space) and the Earth (object space). For the convenience of the potential user, the mathematics of the relationship (taken directly from ref. 1) are included as appendix A.

Modifications

For the previous applications of the method, two separate digital computer programs were employed to operate on the raw data (image space identification points read from the film) before the final camera or vehicle orientation angles (Euler angles, figs. 1 and 2) were determined. Also a third program was required to operate on the vehicle Euler angle ($\psi, \theta, \phi$) data and produce vehicle angles ($\alpha, \beta, \eta$) relative to the wind. In the modified version, presented in this paper, the three programs have been combined into one without any reduction in data output and with significant improvement in user and computer efficiency. The new version also includes simplified input-output procedures and a multiple job processing capability. Another significant modification, to be discussed in the following section, involves converting unknown surface features or non-permanent objects appearing on the film into usable "landmarks."

Method Application

Application of the method is discussed in terms of procedures by use of the basic data reduction and computer systems currently available at the NASA Langley Research Center and as used to determine test vehicle motions from the Viking Balloon Launched Decelerator Test (BLDT) Program (refs. 4 and 5).

Landmark selection.- The initial, and frequently very difficult, step encountered when using this technique involves identifying distinct features on the film (1) which appear on a sufficient number of frames and (2) the geodetic coordinates of which can be...
Figure 1.- Camera Euler angles.

Figure 2.- Vehicle Euler angles.
determined. For many applications the user has little control over what Earth-related features will be photographed during flight and must instead work with what the film data offer. Many hours can be spent matching a series of features appearing on the film with the corresponding features on a topographical map so that their coordinates can be determined. A feature whose coordinates have been identified is considered a landmark.

Often, because of the flight trajectory and vehicle motions, a landmark may appear only on a relatively small number of frames. On the other hand, features appearing on the film which cannot be identified on a map may be visible for a relatively long period of time. Such nonpermanent features as clouds, ground vehicles, and ships might fit this category if they were slowly moving with respect to the camera frame rate (that is, quasi-stationary). The program has been modified to allow the user the flexibility to process several "trial" coordinates (latitude, longitude, and altitude) for a single feature in one computer run; this procedure allows for a quick iterative solution of its actual location. The user can accomplish this determination of the location by comparing vehicle Euler angle results obtained by using only known landmarks with those obtained by using both known landmarks and features. Solving for the coordinates of the feature readily follows and permits the feature to be used as a known landmark in other frames of the film where actual landmarks are not distinguishable.

In applying this option, the iteration procedure is initiated by first estimating the location of the feature and running a series of latitudes about that estimation. By comparing the resultant Euler angles with the values from the known landmarks, the best latitude is obtained. A series of longitudes is then processed at that latitude; as a result, a best estimate of longitude is obtained. The best estimate of altitude is established in a similar manner. With these new altitude and longitude values, the process can be repeated to define the latitude more accurately and, subsequently, the longitude and altitude parameters. The landmarks defined in this manner may be used to find the coordinates of other quasi-static features during other parts of the data period. Application of this option permits obtaining vehicle orientation data for time periods when no true landmarks are in view of the camera.

As previously stated, a minimum of two landmarks is required for each frame. Theoretically, no limits are required on the maximum number but satisfactory results have been obtained by using from two to six landmarks, three landmarks being the preferred number. The current computer program in use at Langley Research Center is set up with a six landmark maximum.
Basic data reduction.- The basic data consist of film (image space) coordinates of the selected landmarks (or features to be converted to landmarks) with respect to the center of the frame. The basic data are punched on digital computer cards.

By using the Gerber film reading system employed at the Langley Research Center, the basic data are obtained as follows: First, the processed film from the onboard vehicle camera is inserted into a film reading system capable of providing coordinates for either 16-mm, 35-mm, or 70-mm film. The film reader magnifies a 16-mm frame image to dimensions of approximately 26.7 cm by 37.3 cm (10.5 in. by 14.7 in.) and provides coordinate readings to the nearest 0.00254 cm (0.001 in.) on the enlarged image. The film reading system is coupled with a visual-display electronic digitizer, a card punch, and an electronically operated typer (data typewriter). The card punch produces the basic data deck and the typer provides a record which can be used to determine rapidly whether any gross reading or film reading system errors exist. Then a manually controlled pair of crosshairs located on the film reader are consecutively aligned on each corner (or other fiducial point) of the frame. These crosshairs also drive the digitizer. The coordinates and other reference information displayed on the digitizer are next recorded by both the card punch and typer by the user pressing a foot switch. For accuracy purposes, each reading is taken three times. The software is employed to average the three readings for each of the four corners of the frame and subsequently to determine the coordinates of the frame center. In the same manner the film coordinates of each landmark are obtained.

PROGRAM DESCRIPTION

The computer program (Program VOPD; Library No. A4424) used at the Langley Research Center is written in Fortran IV and is set up for a CDC 6600 series computer. For applications utilizing an average of three to four landmarks per frame, computer running time has averaged 0.17 second/frame. The basic data deck contains the film coordinates of the landmarks obtained as previously discussed. The control data include geodetic coordinates of the landmarks, vehicle trajectory data, and camera characteristics.

Program VOPD is the main program. It calls subroutines TGTCAM and ALPBET for auxiliary calculations. A flow diagram for VOPD which indicates the basic calculation procedure follows and a computer listing of the program is given in appendix B. Computer library routines MATINV and FTLUP are called by VOPD and are described in appendices C and D, respectively.
PROGRAM VOPD FLOW DIAGRAM

1. Initialize constants.
2. Read and write input groups.
3. Read, average and store basic data on disk 9.
4. Rewind 9 and set up initial Euler angles.
5. Are all frames completed? If yes, go to step 6. If no, go to step 2.
6. Read a frame center of basic data, disk 9.
7. Call FILUP to interpolate for time and vehicle location.
8. End program.
THROUGH 29 TO
IGNORE FRAMES AND
LANDMARKS AS REQUIRED

THROUGH 31 TO READ
LANDMARKS AND COMPUTE
GEOCENTRIC LATITUDES AND
TRIGONOMETRIC CONSTANTS

BELOW 32 TO COMPUTE
GEOCENTRIC COORDINATES OF
THE VEHICLE (CAMERA)

CALL TGTCAM
CONVERTS GEOCENTRIC TO
OBJECT SPACE COORDINATES
COMPUTES DIRECTION COSINES
OF THE CAMERA

50-51 PREPARES TERMS FOR
CONVERSION OF OBJECT SPACE
LANDMARKS TO IMAGE SPACE

54-55 COMPUTES LENS
DISTORTION CORRECTIONS

66-70 PREPARES THE MATRIX
COEFFICIENTS FOR SOLUTION
OF THE EULER ANGLES
The following section gives a summary of the basic program operations and is followed by a more in-depth input-output discussion of the control data format and some sample program applications.

General Operation

The program operates under the following general steps:

(1) The control data and basic data are read

(2) The frame center and each landmark location on the frame is computed and stored on a disk for each frame of the basic data deck
(3) All landmarks are transformed from the geodetic to the geocentric coordinate system and stored in core

(4) Each frame of data is then processed as follows:

(a) The first frame is read from the disk and compared with the control data to determine whether it is to be ignored or whether any landmarks are to be ignored

(b) The object space landmarks are converted to their image space coordinates and lens distortion corrections are computed

(c) The camera Euler angles are solved and compared with the initial estimates from the control deck

(d) Through an iterative process using the new Euler angles as a starting point, the final Euler angles are obtained when a sufficiently small change is detected

(e) These camera angles are then converted to the vehicle-oriented values, and by using the meteorological and trajectory data of the control deck, the vehicle angle-of-attack components are computed and printed

(5) At the completion of all frames of the basic data deck, the program returns to the input group specified by the control deck for further processing.

An initial estimate of camera Euler angles is necessary to start the iteration procedure. For the first frame, the initial estimates should be in or adjacent to the quadrant of the final solution to obtain satisfactory convergence within the 22 iterations provided for in the program. For subsequent frames, initial estimates are taken as the results of the previous frame.

Program Usage

Basic data card format.- As previously discussed, the basic data consist of film coordinates of the selected landmarks with respect to the center of the frame. The basic data card should include, as a minimum, these data and the information specified in the program control-data deck. For program VOPD, this information includes landmark number and frame number. A separate card is required for each of the three readings of each point, whether it is a fiducial point or landmark. Four separate fiducial points are required for each frame. Thus, a frame with six landmarks would require 30 basic data computer cards.

The basic data card should also contain the information required for identification and review, and also, as an aid for determination of possible reading or system errors.
The format used in the most recent Langley applications is listed in appendix E. Also included in appendix E is a typical sequence of steps employed in reading a frame.

Control data format.- For the convenience of the potential user, program VOPD has been divided into distinctive input and output groups of control data cards. Detailed descriptions in computer terminology of the input for each group are included in appendix F. These descriptions include such information as column location, parameter symbols, parameter description, the number of cards in each group, parameter units, and input format. The following discussion, summarizing the information in each group, is in the order in which each group appears in the deck, and should be reviewed in conjunction with the more detailed information given in appendix F.

Group 1 contains a table of camera lens distortion compared with radial distance from the lens center. These values are used for correcting the landmark image space coordinates and are obtained from camera calibration data.

Group 2 is a title card to annotate each page of output. A "1" must be punched in column 1 to operate the carriage control on the printer and any descriptive information can be punched in columns 2 to 80.

Group 3 consists primarily of data describing the conversion of camera Euler angles to vehicle Euler angles and is strictly a function of the relative orientation between the camera and vehicle axes. To solve for vehicle attitude angles accurately, it is essential to know the exact orientation between the camera and vehicle axes. The numbers of values in the vehicle trajectory table (group 7) and wind table (group 4) are also required in group 3. The conversion data for camera to vehicle Euler angles are the coefficients of the equations defined in the input description (appendix F). Camera and vehicle Euler angles are shown in figures 1 and 2, respectively.

Group 4 is a group of cards each containing an altitude and the corresponding horizontal wind components. The span of altitudes in this group must at least include the altitude span of the vehicle trajectory group (group 7). These data are required only for angle-of-attack and velocity calculations.

Group 5 lists the number of landmarks in the basic data deck, the number of frames of data in the basic data deck, and the number of values in the time-frame table (group 6). The remaining spaces on this first card and all the spaces on the following cards in this group, as many as necessary, comprise a table consisting of the number of landmarks for each frame. These numbers are listed in the order corresponding to the frame sequence in the basic data deck.

Group 6 consists of a set of cards each listing the frame number and an associated time. This group must at least include the frames in the basic data deck. For a constant frame rate camera, only two values are needed for this group.
Group 7 lists the vehicle trajectory data. The origin for this trajectory information is described in group 8. Each card of group 7 contains vehicle flight time, associated vehicle position (X, Y, and Z locations) and velocity, and altitude above mean sea level (MSL). This set of data must encompass the time span of the basic data deck.

Group 8 contains the following data on one card: camera focal length; the origin of the vehicle trajectory coordinate system consisting of geodetic latitude, longitude, and the Earth radius to this origin; and the initial estimates of the camera Euler angles. These initial estimates are used to initiate the iteration on the Euler angles for the first frame of each basic data deck. Subsequent frames employ results from the previous frame to initiate Euler angle iterations.

Group 9 lists the landmarks and their locations. This group includes all the landmarks read from the film using the same landmark identification numbers as used in the basic data deck. Geodetic latitude, longitude, and altitude for each landmark are listed on each card along with the landmark identification number.

Group 10 indicates the number of frames from which specific landmarks are to be ignored. This value is, in effect, the number of group 11 cards. Landmarks are ignored when it is suspected that either their frame coordinates or Earth coordinates have been incorrectly identified.

Group 11 consists of a number of cards as specified in group 10 which lists the frames and landmarks which are to be ignored for these frames. To ignore the entire frame, an option is also included as described in appendix E.

Group 12 defines the following two parameters: First, the input group to which the program returns after processing all frames of the basic data deck. This information allows the user to stack jobs by returning to any input group except 11 and by adding behind the basic data deck only that input group and those following the one specified. Additional data in group 12 consist of the landmark number of any landmarks which are to be ignored in calculations for all frames of the basic data deck and eliminates the necessity for a long group 11 table.

After these groups of cards, the basic data deck is inserted and appears only once in the deck setup. For subsequent processing, the basic data deck is automatically stored on the disk and need not be repeated. The information read from the basic data deck are the landmark number, its coordinates on the frame (x, y), and the frame number. The landmark number is zero for the four fiducial (frame corners) points.

Program output is listed by groups in appendix G and contains such information as parameter description, symbols, and units.
EXAMPLE APPLICATIONS

For a better understanding of program application, several example cases taken from BLDT applications are presented with the corresponding program input and output format (for one frame only) presented in appendix H. Example case inputs are divided into the previously discussed input groups (appendix F) and may be referred to for clarification. For all cases, input group 12 provides instructions to the program to be implemented upon completion of the existing case. Program output consisting of a single frame for all cases is included collectively after the program input for all cases.

Case 1.- This example represents an application where 24 frames are processed (only one frame is presented) and where no question exists concerning landmark location. All 12 input groups, as defined in appendix F, and the basic data deck are included. Three landmarks, identified as landmarks 65, 66, and 67, are used and their coordinates are listed in input group 9. As indicated in group 10, no landmarks are to be ignored. Group 12 provides instructions to the program upon completion of case 1.

Case 2.- Case 2 is a repeat of case 1 except that one landmark (landmark 65) was ignored because there was some question about its exact location on the film. The inputs for all groups, except group 12, are the same as those for case 1.

By comparing the outputs of cases 1 and 2, a difference of about $1^\circ$ can be seen for $\psi$ and $\phi$ with little change in $\theta$. By ignoring landmark 65, the residuals are slightly less for case 2 than for case 1. This result indicates that the results from case 2 may be more accurate than the results from case 1 and that the coordinates of landmark 65 may be in error.

Case 3.- Case 3 is similar to case 2 except that the latitude of landmark 66 was changed from $32^\circ50'$ to $32^\circ50.3'$. (See group 9.) The inputs for all groups, except groups 9, 10, and 12, are the same as those for cases 2 and 1. Case 3 illustrates a typical iteration to define the object space (Earth) coordinates of a previously unknown landmark to convert a feature on the film to a usable landmark. The results of this iteration show a slight improvement in residuals over case 2 and about a $1^\circ$ change in $\psi$ and $\theta$.

Case 4.- This case illustrates the combined approach of completely ignoring frame 55 and ignoring landmark 69 for several frames (frames 60 and 65) as instructed through input group 11. This case is independent of the previous three and requires a different set of input data. For this case, the output for frames 60 and 65 are presented. A computer listing of the general camera orientation method program (VOPD) is shown in appendix H.
CONCLUDING REMARKS

Details of application of a previously devised photogrammetric method to determine a time history of vehicle flight attitudes have been included in this paper. Emphasis has been placed on the techniques involved in reducing the raw photographic data to computer inputs in Fortran IV language and on the computer techniques and programs involved in obtaining vehicle flight attitude results. Also discussed are the major program modifications which allow faster data reduction and permit the user to determine the Earth-related coordinates of unknown or nonpermanent features appearing on the film.

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APPENDIX A

PHOTOGRAMMETRIC DETERMINATION OF CAMERA ORIENTATION

In order to describe adequately how the camera orientation can be determined, it is necessary to define the coordinate systems in which observations are made. The coordinate system in which the position of observed points are known and in which the camera is oriented is called the object space. The coordinate system composed of the camera focal plane and focal axis and in which image coordinates are measured is the image space. (See ref. 3.) The relationship between a point in the object space and image space is

\[ x - x_p = c \left( \frac{\lambda_1X + \mu_1Y + \nu_1Z}{\lambda_3X + \mu_3Y + \nu_3Z} \right) \]  

\[ y - y_p = c \left( \frac{\lambda_2X + \mu_2Y + \nu_2Z}{\lambda_3X + \mu_3Y + \nu_3Z} \right) \]  

The relationship in equations (A1) can be derived with the aid of figure 3. Let \( Q(X,Y,Z) \) be a point in the object space, then the image of \( Q \) will be \( I(X_I,Y_I,Z_I) \) in the image space. Since the origins of the two coordinate systems coincide for all practical purposes, the following transformation describes the coordinates of \( Q \) relative to the image space

\[
\begin{bmatrix}
X_I \\
Y_I \\
Z_I
\end{bmatrix}
= K
\begin{bmatrix}
\lambda_1\mu_1\nu_1 & X \\
\lambda_2\mu_2\nu_2 & Y \\
\lambda_3\mu_3\nu_3 & Z
\end{bmatrix}
\]  

where \( K \) is a constant of contraction. In equations (A2) dividing the first and second equations by the third equation removes the constant \( K \) and gives

\[
\begin{align*}
\frac{X_I}{Z_I} &= \frac{\lambda_1X + \mu_1Y + \nu_1Z}{\lambda_3X + \mu_3Y + \nu_3Z} \\
\frac{Y_I}{Z_I} &= \frac{\lambda_2X + \mu_2Y + \nu_2Z}{\lambda_3X + \mu_3Y + \nu_3Z}
\end{align*}
\]  

(A3)
Figure 3.- Axis systems relating image space and object space.

Letting

\[ \bar{x} - \bar{x}_p = X_I \]
\[ \bar{y} - \bar{y}_p = Y_I \]
\[ c = Z_I \]

and multiplying both sides of equations (A3) by \( c \) gives

\[ \begin{align*}
\bar{x} - \bar{x}_p &= c \left( \frac{\lambda_1 X + \mu_1 Y + \nu_1 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z} \right) \\
\bar{y} - \bar{y}_p &= c \left( \frac{\lambda_2 X + \mu_2 Y + \nu_2 Z}{\lambda_3 X + \mu_3 Y + \nu_3 Z} \right)
\end{align*} \]  

(Equation A4)

Equations (A4) are identical to equations (A1).

For each known point in the object space there exist two equations (A1) relating the object space to the image space. The directional cosines \( \lambda_1, \mu_1, \nu_1, \ldots \) of the
image space axes relative to the object space axes can be expressed in terms of three angles $\sigma_1$, $\sigma_2$, and $\sigma_3$. Consequently, these angles can be used to describe the orientation of the camera coordinate system relative to the object space coordinate system. To obtain the directional cosines, consider three successive rotations through the angles $\sigma_1$, $\sigma_2$, and $\sigma_3$. By using figure 3 and imposing a constant of contraction for the camera, the following transformation is obtained:

\[
\begin{bmatrix}
\bar{x} - \bar{x}_p \\
\bar{y} - \bar{y}_p \\
c
\end{bmatrix} = K
\begin{bmatrix}
\cos \sigma_1 \cos \sigma_3 - \sin \sigma_1 \sin \sigma_2 \sin \sigma_3 & \sin \sigma_1 \cos \sigma_3 - \cos \sigma_1 \sin \sigma_2 \sin \sigma_3 & \cos \sigma_2 \sin \sigma_3 \\
\sin \sigma_1 \cos \sigma_3 - \sin \sigma_2 \cos \sigma_3 & \cos \sigma_1 \cos \sigma_3 - \sin \sigma_1 \sin \sigma_2 \cos \sigma_3 & \cos \sigma_2 \cos \sigma_3 \\
\sin \sigma_1 \cos \sigma_2 & \cos \sigma_1 \cos \sigma_2 & \sin \sigma_2
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = K
\begin{bmatrix}
\lambda_1 \mu_1 \nu_1 \\
\lambda_2 \mu_2 \nu_2 \\
\lambda_3 \mu_3 \nu_3
\end{bmatrix}
\]

This equation is equivalent to equation (A2).

The Least-Squares Solution

The solution of equations (A1) contains six parameters which under theoretical conditions are constants. These parameters are $\bar{x}_p$, $\bar{y}_p$, $c$, $\sigma_1$, $\sigma_2$, and $\sigma_3$ of which $\bar{x}_p$, $\bar{y}_p$, and $c$ are measured independently for this experiment and are not unknowns in the solution. A solution of equations (A1) for $\sigma_1$, $\sigma_2$, and $\sigma_3$ can be found with two properly chosen observations. Associated with $\bar{x}$ and $\bar{y}$ in equations (A1) are errors $\epsilon$ and $\bar{\epsilon}$. Since these errors exist, a computational method is needed which yields the best possible results with all the information available. The method of least squares which is described subsequently uses a minimum error criterion and has been used in the data reduction for this investigation.

In general, equations (A1) with the associated errors can be written as

\[
\begin{align*}
\bar{x}_i &= F(\sigma_j) + \epsilon_i \\
\bar{y}_i &= \bar{F}(\sigma_j) + \bar{\epsilon}_i
\end{align*}
\]

where $F$ and $\bar{F}$ are nonlinear functions of $\sigma_j$ and in order to find a solution they must be linearized. Expanding equations (A5) in a Taylor's series about a nominal set $\sigma_{j,0}$ and dropping the higher order terms results in the following linear approximations:

\[
\begin{align*}
\Delta \bar{x}_i &= \bar{x}_i - \bar{x}_{i,0} = b_{i1}(\sigma_1 - \sigma_{1,0}) + b_{i2}(\sigma_2 - \sigma_{2,0}) + b_{i3}(\sigma_3 - \sigma_{3,0}) + \epsilon_i \\
\Delta \bar{y}_i &= \bar{y}_i - \bar{y}_{i,0} = \bar{b}_{i1}(\sigma_1 - \sigma_{1,0}) + \bar{b}_{i2}(\sigma_2 - \sigma_{2,0}) + \bar{b}_{i3}(\sigma_3 - \sigma_{3,0}) + \bar{\epsilon}_i
\end{align*}
\]
where

\[
\begin{align*}
 b_{11} &= \frac{\partial F_i}{\partial \sigma_1} \bigg|_{\sigma_1, \sigma_2, \sigma_3, o} \\
 b_{12} &= \frac{\partial F_i}{\partial \sigma_2} \bigg|_{\sigma_1, \sigma_2, \sigma_3, o} \\
 b_{13} &= \frac{\partial F_i}{\partial \sigma_3} \bigg|_{\sigma_1, \sigma_2, \sigma_3, o} \\
 \tilde{b}_{11} &= \frac{\partial \tilde{F}_i}{\partial \sigma_1} \bigg|_{\sigma_1, \sigma_2, \sigma_3, o} \\
 \tilde{b}_{12} &= \frac{\partial \tilde{F}_i}{\partial \sigma_2} \bigg|_{\sigma_1, \sigma_2, \sigma_3, o} \\
 \tilde{b}_{13} &= \frac{\partial \tilde{F}_i}{\partial \sigma_3} \bigg|_{\sigma_1, \sigma_2, \sigma_3, o}
\end{align*}
\]

Letting

\[
\Delta \sigma_j = (\sigma_j - \sigma_j, o)
\]

equations (A6) can be put in the following form:

\[
\begin{align*}
 \Delta \hat{x}_i &= \sum_{j=1}^{3} b_{ij} \Delta \sigma_j + \epsilon_i \\
 (i &= 1, \ldots, n) \\
 \Delta \hat{y}_i &= \sum_{j=1}^{3} \tilde{b}_{ij} \Delta \sigma_j + \tilde{\epsilon}_i \\
 (i &= 1, \ldots, n)
\end{align*}
\]  

(A7)

For further considerations the linear equations (A7) corresponding to the ith observation are expressed in matrix notation

\[
\begin{align*}
 v_i &= \begin{bmatrix} \Delta \hat{x}_i \\ \Delta \hat{y}_i \end{bmatrix} \\
 B_i &= \begin{bmatrix} b_{i1} & b_{i2} & b_{i3} \\ \tilde{b}_{i1} & \tilde{b}_{i2} & \tilde{b}_{i3} \end{bmatrix} \\
 \Delta \sigma &= \begin{bmatrix} \Delta \sigma_1 \\ \Delta \sigma_2 \\ \Delta \sigma_3 \end{bmatrix} \\
 e_i &= \begin{bmatrix} \epsilon_i \\ \tilde{\epsilon}_i \end{bmatrix}
\end{align*}
\]

where

\[
 v_i = B_i \Delta \sigma + e_i
\]  

(A8)
Then for \( n \) observations there are \( n \) matrix equations of the form of equation (A8) which may be written

\[
\bar{V} = \bar{B} \Delta \sigma + \bar{e} \tag{A9}
\]

where

\[
\begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_n
\end{bmatrix} = \begin{bmatrix}
  B_1 \\
  B_2 \\
  \vdots \\
  B_n
\end{bmatrix} \Delta \sigma + \begin{bmatrix}
  e_1 \\
  e_2 \\
  \vdots \\
  e_n
\end{bmatrix}
\]

The problem may be restated: given \( \bar{V} \) and \( \bar{B} \) find the best estimate \( \hat{\Delta} \sigma \) for \( \Delta \sigma \).

The best estimate \( \hat{\Delta} \sigma \) is the value of \( \hat{\Delta} \sigma \) which minimizes the sum of the squares of the residuals \( \bar{e}^T \bar{e} \) where

\[
\bar{e}^T \bar{e} = (\bar{V} - \bar{B} \Delta \sigma)^T(\bar{V} - \bar{B} \Delta \sigma) \tag{A10}
\]

In order to minimize equation (A10), the first variation \( \delta \) with respect to \( \Delta \sigma \) must vanish; that is,

\[
\begin{align*}
\delta(e^T \bar{e}) &= \delta[(\bar{V} - \bar{B} \Delta \sigma)^T(\bar{V} - \bar{B} \Delta \sigma)] = 0 \\
\delta(e^T \bar{e}) &= -2(\bar{V}^T - \Delta \sigma^T \bar{B}^T)\bar{B} \delta \Delta \sigma = 0
\end{align*} \tag{A11}
\]

Since \( \delta \Delta \sigma \neq 0 \), equations (A11) can be satisfied if

\[
(\bar{V}^T - \Delta \sigma^T \bar{B}^T)\bar{B} = 0
\]

or

\[
\bar{B}^T \bar{B} \Delta \sigma = \bar{B}^T \bar{V} \tag{A12}
\]

Solving for the estimate of \( \Delta \sigma \) in equation (A12) gives

\[
\hat{\Delta} \sigma = (\bar{B}^T \bar{B})^{-1} \bar{B}^T \bar{V} \tag{A13}
\]
A second necessary condition for equation (A10) to be a minimum is that the second variation with respect to $\Delta \sigma$ be positive definite. Upon examination, the second variation is

$$\delta^2(\mathbf{e}\mathbf{e}^T) = 2\delta \Delta \sigma \mathbf{B}^T \mathbf{B} \delta \Delta \sigma$$

which is positive definite. Therefore, equation (A13) is a valid expression for $\hat{\sigma}$. Since equation (A13) is based on a linear approximation with nominal $\sigma_{j,0}$, $\hat{\sigma}$ can be used to find the best estimates $\hat{\sigma}_j$. With the relationship $\sigma = \sigma_0 + \Delta \sigma$, the value of $\hat{\sigma}$ which minimized equation (A10) leads to a new nominal $\sigma_{j,0} = \sigma_{j,0} + \hat{\sigma}$. This process implies an iterative procedure which continues until $\Delta \sigma \to 0$ and the value of $\sigma_{j,0}$ that leads to this result is the best estimate of $\hat{\sigma}_j$ for $\sigma_j$.

Partial Derivatives of Projection Equations

For equations (A5)

$$\hat{x}_i = F(\sigma_i) + e_i \quad (i = 1, 2, 3)$$

$$\hat{y}_i = \bar{F}(\sigma_i) + e_i \quad (i = 1, \ldots, n)$$

The partial derivatives of $F$ and $\bar{F}$ with respect to $\sigma_1$, $\sigma_2$, and $\sigma_3$ are as follows. Let

$$p = \lambda_1 X + \mu_1 Y + \nu_1 Z$$

$$q = \lambda_2 X + \mu_2 Y + \nu_2 Z$$

$$r = \lambda_3 X + \mu_3 Y + \nu_3 Z$$

then

$$\frac{\partial F}{\partial \sigma_1} = \frac{c \left( \frac{\partial p}{\partial \sigma_1} r - \frac{\partial r}{\partial \sigma_1} p \right)}{r^2}$$

$$\frac{\partial F}{\partial \sigma_2} = \frac{c \left( \frac{\partial p}{\partial \sigma_2} r - \frac{\partial r}{\partial \sigma_2} p \right)}{r^2}$$
\[ \frac{\partial F}{\partial \sigma_3} = \frac{c\left( \frac{\partial p}{\partial \sigma_3} r - \frac{\partial r}{\partial \sigma_3} p \right)}{r^2} \]

\[ \frac{\partial F}{\partial \sigma_1} = \frac{c\left( \frac{\partial q}{\partial \sigma_1} r - \frac{\partial r}{\partial \sigma_1} q \right)}{r^2} \]

\[ \frac{\partial F}{\partial \sigma_2} = \frac{c\left( \frac{\partial q}{\partial \sigma_2} r - \frac{\partial r}{\partial \sigma_2} q \right)}{r^2} \]

\[ \frac{\partial F}{\partial \sigma_3} = \frac{c\left( \frac{\partial q}{\partial \sigma_3} r - \frac{\partial r}{\partial \sigma_3} q \right)}{r^2} \]

\[ \frac{\partial p}{\partial \sigma_1} = \mu_1 X - \lambda_1 Y \]

\[ \frac{\partial q}{\partial \sigma_1} = \mu_2 X - \lambda_2 Y \]

\[ \frac{\partial r}{\partial \sigma_1} = \mu_3 X - \lambda_3 Y \]

\[ \frac{\partial p}{\partial \sigma_2} = -\nu_1 (X \sin \sigma_1 + Y \cos \sigma_1) - (\sin \sigma_2 \sin \sigma_3) Z \]

\[ \frac{\partial q}{\partial \sigma_2} = -\nu_2 (X \sin \sigma_1 + Y \cos \sigma_1) - (\sin \sigma_2 \cos \sigma_3) Z \]

\[ \frac{\partial r}{\partial \sigma_2} = -\nu_3 (X \sin \sigma_1 + Y \cos \sigma_1) + Z \cos \sigma_2 \]

\[ \frac{\partial p}{\partial \sigma_3} = \lambda_2 X + \mu_2 Y + \nu_2 Z \]

\[ \frac{\partial q}{\partial \sigma_3} = -(\lambda_1 X + \mu_1 Y + \nu_1 Z) \]

\[ \frac{\partial r}{\partial \sigma_3} = 0 \]
APPENDIX A – Concluded

Coordinate Transformation

Object space points and the position of the camera are initially identified in terms of geodetic latitude, longitude, and altitude above sea level. These data are obtained from maps of the photographed area and radar observations of the vehicle trajectory. In order to reference the data relative to the camera as described previously, the Earth-centered geocentric coordinates of both the object space points and the vehicle position are computed by

\[
X_i = (R_i + h_i) \cos \phi_i \cos \lambda_i \\
Y_i = (R_i + h_i) \cos \phi_i \sin \lambda_i \\
Z_i = (R_i + h_i) \sin \phi_i
\]

and

\[
\Phi_i' = \Phi_i - 11'35.6635'' \sin 2\Phi_i + 1.1731'' \sin 4\Phi_i - 0.0025'' \sin 6\Phi_i \\
R_i = 6378.388(0.998320047 + 0.001683494 \cos 2\Phi_i - 0.000003549 \cos 4\Phi_i)
\]

where

\[i = 1 \text{ for the vehicle position} \]
\[i = 2 \ldots n + 1 \text{ for the object space points} \]

In the preceding discussion the \( \overline{X} \) axis is in the equatorial plane pointing toward the Greenwich meridian, the \( \overline{Y} \) axis 90° east in the equatorial plane, and the \( \overline{Z} \) axis toward the north pole. A slight error is introduced by adding altitude to the Earth's radius vector but this error is negligible for the accuracy desired of this system.

The following transformation maps the geocentric coordinates into object space coordinates relative to the camera where the \( X \) axis points east, the \( Y \) axis points north, and the \( Z \) axis points toward the zenith.

\[
\begin{bmatrix}
X_{i+1} \\
Y_{i+1} \\
Z_{i+1}
\end{bmatrix} = \begin{bmatrix}
-sin \lambda_1 & cos \lambda_1 & 0 \\
-cos \lambda_1 sin \phi_1 & -sin \lambda_1 sin \phi_1 & cos \phi_1 \\
-cos \lambda_1 cos \phi_1 & sin \lambda_1 cos \phi_1 & sin \phi_1
\end{bmatrix} \begin{bmatrix}
\overline{X}_{i+1} - \overline{X}_1 \\
\overline{Y}_{i+1} - \overline{Y}_1 \\
\overline{Z}_{i+1} - \overline{Z}_1
\end{bmatrix}
\]

This transformation is obtained by a positive rotation about the \( \overline{Z} \) axis through an angle of \( 90^o + \lambda_1 \) followed by a position rotation about the new \( X \) axis through an angle of \( 90^o - \phi_1 \).
APPENDIX B

PROGRAM LISTING

The program listing follows:

```
PROGRAM VOPD(INPUT, OUTPUT, TAPE5=INPUT, TAPE10=OUTPUT, TAPE10=INPUT)
DIMENSION KRR(2S), ISAVE(JO), ASAVEOUT, YSAVE(30), XRR(25), RR(20)
1) » OXY(20) A 2
DIMENSION XLAM(20), XNU(20), XNU(20), H(2), S(6), AN(6), EP A 3
DIMENSION XMM(20), XWM(20), OO(20), NG(20), X(20), Y(20), S
DIMENSION PH180(100), PH18M(100), PH18S(100), XLOHD(100), XLOHM(100)
DIMENSION XLOBS(100), TXYT(20), TGTAB(20), XZAH(3), XX(20)
1) » YF(20), XTEMP(6), ANUNIT(6) A 4
DIMENSION THJ(90), TTHJ(6), TPHJ(6), TPHJ(90), TKJ(90)
DIMENSION GENOA(6,3), GENUO(6,3) A 5
DIMENSION NPT(200), IFHM(100), TIMX(100), XXX(50), YYY(50), ZZZ(50)
1) » KFXM(100) A 6
DIMENSION OUT(100), IFM(100), JOUT(6,100)
COMMON /A5/ II, IIVV, TX(50), VXX(50), VY(50), VVZ(50), H(50), ALT(50), V
COMMON /VVO/ ,50, YVO(50)
COMMON /VRS/ XLAM, XNU, ANU
DATA (KAD)=57.29577951, (PI)=3.14159265, (FTPKM)=3280.8336, (FACTOR)=
1713154.00251, (KAD)=637.0165, (V=0.0)
DATA (FOCAL)=.0015, (SLAT)=.00333333333333, (SXYZ)=200., (SRJ)=200.,
1(SXY=0.01, (SXY=0.01), (FTPKM)=.0003040066, (RN=0.1)
1X0(51), YO(50)
COMMON /XIVSC/ X usually
DATA (KAD)=57.29577951, (PI)=3.14159265, (FTPKM)=3280.8336, (FACTOR)=
1713154.00251, (KAD)=637.0165, (V=0.0)
DATA (FOCAL)=.0015, (SLAT)=.00333333333333, (SXYZ)=200., (SRJ)=200.,
1(SXY=0.01, (SXY=0.01), (FTPKM)=.0003040066, (RN=0.1)
1X0(51), YO(50)
COMMON /VRS/ XLAM, XNU, ANU
```

24
APPENDIX B - Continued

9  DO 100 J=1,NT
  READ (5,93) ITGT,THJ,THJMI,PHIJ,PHIJMI,RJ
  KJ=KJ*1000./FTPKM
  THJ(I1) = THJ
  THJMI(I1) = THJMI
  PHIJ(I1) = PHIJ
  PHIJMI(I1) = PHIJMI
100  THJ(I1) = RJ
10  READ (5,90) NOUT
  IF (NOUT.LT.1) GO TO 12
11  READ (5,94) (IFM(I),JOUT(J,I),J=1,6),I=1,NOUT
  DO 13 J=1,NOUT
    DO 13 J=1,6
      IF (JOUT(J,I).LT.1) JOUT(J,I)=1
13  CONTINUE
12  READ (5,90) NGO,(IOUT(I),I=1,NT)
  DO 15 LL=1,NT
    IF (IOUT(LL).LT.101) IOUT(LL)=101
15  CONTINUE
13  IF (JUMP.GT.0) GO TO 20
  DO 19 N=1,NFR
    JUMP=NPT(N)*3+12
    READ (5,95) (ISAVE(I),XSAVE(I),YSAVE(I),IFRAM,J,I=1,1UP)
    YPC=0.
    XPC=0.
    DO 16 I=1,12
      XPC=XPC*XSAVE(I)/12.
      YPC=YPC*YSAVE(I)/12.
      IUP=NPT(N)
      11=12
      DO 18 J=1,1UP
        XSAVE(I)=0.
        YSAVE(I)=0.
        DO 17 J=1,3
          XSAVE(I)=XSAVE(I)+XSAVE(I)/3.
          YSAVE(I)=YSAVE(I)+YSAVE(I)/3.
        17  ISAVE(I)=ISAVE(I)
        JUMP=1
20  IF (IFCFG).NE.0) GO TO 22
  REAOFG=0
  IFCFG=1
  KK=0
  ALPHAP=ALPHAI
  ALPHAM=ALPHAM
  KK=KK+1
  IF (KK.LE.NFR).GO TO 22
  KK=0
  ENO FILE 10
  WRITE (6,77)
  GO TO (12,2,3,4,5,6,7,8,9,10,11,12,13) NGO
22  NU=NPT(KK)
  READ (6,77) IFRAM,XPC,YPC
  XFRAM=IFRAM
  CALL FILUP (XFRAM,TI,11,NFK,XFRM,TIMX)
  CALL FILUP (TI,XYZ,11,111,TX,XXX)
  XYZTA6(1)=XYZ
  CALL FILUP (TI,XYZ,11,111,TX,YYY)
  XYZTA6(2)=XYZ
  CALL FILUP (TI,XYZ,11,111,TX,ZZZ)
  XYZTA6(3)=XYZ

25
APPENDIX B – Continued

IF (NOUT.LT.1) GO TO 25
DO 24 I=1+NOUT
IF ([FKAM].NE.[IFM(I)]) GO TO 24
IF (JOUT(I+1).NE.100) GO TO 25
DO 23 L=1+NJ
READ (9*N7) ITGT,XCO,YCO
WRITE (6,N9) IFRAM
GO TO 21
CONTINUE
24 INJ=0
NK=0
WRITE (6,77)
WRITE (6,89)
DO 31 J=1+NJ
READ (9*N7) ITGT,XCO,YCO
IF (NOUT.LT.1) GO TO 25
DO 27 I=1+NOUT
IF ([FKAM].NE.[IFM(I)]) GO TO 27
DO 26 M=1+6
IF ([ITGT].NE.[JOUT(M+I)]) GO TO 26
NK=NK+1
IF ((NJ-NK),LT.2) GO TO 27
WRITE (6,90) ITGT
GO TO 31
CONTINUE
27 CONTINUE
28 DO 29 I=1+NT
IF ([JOUT(I)].NE.[ITGT]) GO TO 29
NK=NK+1
IF ((NJ-NK),LT.2) GO TO 30
WRITE (6,90) ITGT
GO TO 31
CONTINUE
29 INJ=INJ+1
X(INJ)=XCO
Y(INJ)=YCO
THJ=THJ(11G)
TMJ=[TMJ1+(11G1)
PHI=PHI1([11G1)
RM=[RM1+(11G1)
I0EN([11J)=ITGT
TMJ=TMJ1+TMJ1(11G0
GEOOA(INJ,1)=THJ
THJ=THJ+NK*SLAT
GEOOA(INJ,1)=THJ
PHI=PHI1+PHI1(MJ/60
GEOOA(INJ,2)=PHI
PHI=PHI1+PHI1(MJ/60
GEOOA(INJ,2)=PHI
PHI=PHI1
GEOOA(INJ,3)=N
KJ=NK+SN
GEOOA(INJ,3)=N
RJ=RJ+TPKM*ERADKM
PHI=PHI
T(KOH)=RAD
FONPHI=[FON]/RAD
S1AP=SAP99/
PHI=PHI1CM
SINTD=SINT([T(KOH)]
SINF2=SINF([FON])
SINSP=SINF([99])

26
APPENDIX B – Continued

```
F(1,J)=1.0, 193.2398017111110*SINPD+.000032586111111110*SINFPD+.72.222222
A 195
12222222K=07*SINSPD
A 196
TGTIA3(I,N+1)=M1J
A 197
TGTIA3(IN+1)=PNJ
A 198
TGTIA3(IN+3)=1J
A 199
11TGF=J
A 200
NJ=INJ
A 201
N=NJ
A 202
CCSAVE=0.
A 203

00HUNFLG=1.
A 204
IKUNCO=10
A 205
KNSXYZ=NN*SXYZ
A 206
XYZTA+1(I)=XYZTAH(I)+HNSXYZ
A 207
XYZTA+1(I+2)=XYZTAH(I+2)+HNSXYZ
A 208
XYZTA+1(I+3)=XYZTAH(I+3)+HNSXYZ
A 209
TXY(T+1)I=1
A 210
TXY(T+1)2=XYZTAB(I+1)*VFTPKM
A 211
TXY(T+1)3=XYZTAB(I+2)*VFTPKM
A 212
TXY(T+1)4=XYZTAB(I+3)*VFTPKM
A 213
CALL TGTICAM (G1,NJ+RZ*PH1Z+THZ,TXY+TGTABB)
A 214
DO 33 I=1.NJ
A 215
TGTIA3(I,N)=TGTABB(I,N)*VFTPKM/1000.
A 216
00 WRITE (6,91) I,ENT(I),TGTABB(I,J),J=1,3.
A 217
DO 33 I=1,NJ
A 218
TGTIA3(I,3)=TGTABB(I,3)*VFTPKM/1000.
A 219
CC=CC*FACT0R
A 220
CCLAST=CC
A 221
IF (CCSAVE,GT,0.) CC=SAVE
A 222
CC=CC
A 223
ALFSAV=ALPHAP
A 224
GCPSAV=XMEMG
A 225
AXASAV=AXAPP
A 226
WRITE (6,92)
A 227
WHILE (6,93) (XYZTAB(I)+I=1,3)*XPC+YPC*CC
A 228
XPC=XPC+HNSXYZ
A 229
YPC=YPC+HNSXYZ
A 230
XPC=XPC*FACT0R
A 231
YPC=YPC*FACT0R
A 232
APCSTO=XPC
A 233
APCSTO=YPC
A 234
APCSTO=APC
A 235
APCSTO=YPC
A 236
APCSTO=APC
A 237
APCSTO=YPC
A 238
APCSTO=APC
A 239
APCSTO=YPC
A 240
TMCN=2.*N
A 241
DO 34 I=1,N
A 242
AX(I)=AX(I)
A 243
YY(I)=Y(I)
A 244
DO 35 I=1,N
A 245
X(I)=X(I)+RNSXY
A 246
DO 35 I=1,N
A 247
Y(1)=Y(I)+RNSXY
A 248
WRITE (5,95)
A 249
DO 37 I=1,N
A 250
WRITE (6,94) IDENT(I),AX(I),Y(I)
A 251
DO 38 I=1,N
A 252
X(I)=X(I)*FACT0R
A 253
Y(I)=Y(I)*FACT0R
A 254
IF (IRUN) 42,42,39
A 255
GO TO (45,40), IFOCFG
A 256
```
CC=CC+DELFL
XPC=XPCSAV
YPC=YPCSAV
XPC1=XPCSAV
YPC1=YPCSAV
DO 41 I=1,N
X(I)=X(I)
Y(I)=Y(I)
41 CONTINUE
IHUN=IHUN+1
42 IF (IHUN=ICRUN-1) 43,44,45
IF (IHUNFLG) 44,44,32
43 HUNFLG=1.
CC=CC1
DELFL=-DELFL
IRUN=0
GO TO 40
44 CONTINUE
ICASE=1
ICASE=ICASE+1
ALPHA=ALPSAV
OMEG=OMESA
KAP=KASA
ALPHA=ALPHA/RAD
OMEGAR=OMEG/RAD
KAPR=KAP/RAD
LINE=0
SIG1=1.0
SIG2=1.0
NP1=0
NP2=0
EPSC=0.00
ICOUNT=1
IF (ICOUNT) 49,49,48
J=1
SUM=0.0
46 CONTINUE
IF (ICASE=-1) 48,48,47
47 CONTINUE
IF (ITCAL<-22) 50,48
48 SIG1=SIG1*ALPHA+SIG1
SIG2=SIG2*OMEGA+SIG2
SIG3=SIG3*KAP+SIG3
IF (SIG1.LT.0.) SIG1=SIG1+360.
IF (SIG1.LT.0.) SIG1=SIG1+360.
IF (SIG2.LT.-360.) SIG2=SIG2+360.
IF (SIG2.LT.-360.) SIG2=SIG2+360.
IF (SIG2.LT.-360.) SIG2=SIG2+360.
IF (SIG3.LT.360.) SIG3=SIG3-360.
IF (SIG3.LT.360.) SIG3=SIG3-360.
WRITE (5,96) DO 43 I=1,N
43 WRITE (5,97) I,ENT(I)+X(I)+Y(I)+KAP(I)+ORR(I)
CALL ALPBE (TI,ALPHA,BETA,ETA,CVP,SIG2,SIG1,SIG3)
WRITE (6,98)
WRITE (5,99) IFRAM,ITI,ALPHA,OMEGA,KAP,XPC,YPC,SIG1,SIG2,SIG3,ALFA
1=ALFA+BETA
WRITE (10) Ti,SIG1,SIG2,SIG3,ALFA,BETA,ETA
50 CONTINUE
SALPHA=SIN(ALPHA)
CALPHA=COS(ALPHA)
SOMEGA=SIN(OMEGA)
COMEGA=COS(OMEGA)
SKAPPA=SIN(KAP)
CKAPPA=COS(KAP)
APPENDIX B - Continued

AAC=CALPHA*CAPPALPHA-SALPHA-SALPHA-SKAPPA A 320
BAC=CALPHA*CAPPALPHA-SALPHA-SALPHA-SKAPPA A 321
CPC=CAPPALPHA*CAPPALPHA A 322
AAC=CALPHA*CAPPALPHA-SALPHA-SALPHA-SKAPPA A 323
BAC=-SALPHA*CAPPALPHA-SALPHA-SALPHA-SKAPPA A 324
CPC=-SALPHA*CAPPALPHA A 325
DLC=CAPPALPHA*CAPPALPHA A 326
ECE=CAPPALPHA*CAPPALPHA A 327
FFC=SALPHA A 328
IF (ICASE=1) 52,52,51 A 329
CONTINUE A 330
IF (ITEX=22) 53,52,52 A 331
CONTINUE A 332
WHITE (0,10O) A 333
CONTINUE A 334
A 335
J=0 A 336
1=1 A 337
IF (ITEX=1) 55,56,55 A 338
CONTINUE A 339
IF (ITEX=22) 57,56,56 A 340
CONTINUE A 341
WHITE (0,100) A 342
CONTINUE A 343
A 344
A 345
A 346
A 347
A 348
A 349
A 350
A 351
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A 381
A 382
A 383
APPENDIX B – Continued

65 JJ=JJ+1
GO TO 70

66 F1=4AC-UDC*XMM(J)
F2=2*AC-EEC*XMM(J)
F3=CCC*FFC*XMM(J)
F01=A3CP-DDC*XNN(J)
F02=B3CP-EEC*XNN(J)
F33=CCCP-FFC*XNN(J)
G1=XMM(J)*CO*EGA*FFC*KAPPA
G2=XNN(J)*COMEGA*FFC*KAPPA
H1=XMM(J)*XALPHA-XNU(J)*CALPHA
S1=-F2*XALPHAXF1*XNU(J)
S2=F1*(H1+G1)*XNU(J)
S3=F3*(H1+G1)*XNU(J)
b(1,1)=S1*QU(J)
H(1,1)=S2*QU(J)
H(1,2)=S3*QU(J)
H(2,1)=Sd*QU(J)
H(2,2)=Sb*QU(J)
H(3,1)=SC*XMM(J)
UU=07 L=1,3
b5(1,1)=b5(1,1)
b5(1,2)=b5(2,1)

67 CONTINUE
DO 69 L=1,3
C(1,L)=C(1,L)+S*(I+L)*-EPS*(L,J))

68 CONTINUE
DO 60 K=1,3
DO 59 L=1,2
AX(1,K)=XN(I,K)+S*(I+L)*=L,K)
XNTMP(I,K)=XN(I,K)+S*(I+L)*=L,K)

59 CONTINUE

70 CONTINUE
J1=3
J2=1
CALL MATINV (XN(1,1)+J1+C(1,1)+J2+UETERM,IPIVOT,INDEX,6,ISCALE)

71 ANUNIT(I,J)=0.
DO 72 L=1,3
DO 71 J=1,3
DO 70 K=1,3
DO 69 J=1,3

72 ANUNIT(I,K)=XN((I,J)+XNUNIT(J,K)+XNUNIT(I,K)
ALPHAK=ALPHAK+C(1,1)
OMEGA=OMEGA+C(2,1)
XKAPK=XKAPK+C(3,1)
IF (XKAPK.LT.12) XKAPK=XKAPK+TWOP
IF (ALPHAK.LT.12) ALPHAK=ALPHAK+TWOP
IF (XKAPK.GT.12) XKAPK=XKAPK-TWOP
IF (ALPHAK.GT.12) ALPHAK=ALPHAK-TWOP

73 SS=30.
DO 74 J=1,N
APPENDIX B – Continued

74 SUM = SUM + EPS (1 + J) * 2 + EPS (2 + J) * 2
75 SUM = SQRT ( SUM / T * ON )
76 WRITE (10, 1044) SUM
77 ALPHAP = ALPHAP
78 XOMEGAP = XOMEGAP
79 XAPP = XAPP
80 IF ( ALPHAP > ALPHAP ) ALPHAP = ALPHAP
81 IF ( XAPP > XAPP ) XAPP = XAPP
82 GO TO ( 21400 , 15 )
83 WRITE ( 10 , 105 )
84 STOP
85 FORMAT ( 2F10.3 )
86 FORMAT ( 4D15.6F5.0 )
87 FORMAT ( 7F10.1 )
88 FORMAT ( 1F45.2E13.7 )
89 FORMAT ( 21X, 9L-MAK LONGITUDE LAT-GCN RAD EN KM LAT-GOTT ALTIT )
90 XH = XH + XH
91 FORMAT ( 115X, 7I )
92 FORMAT ( 7I )
93 FORMAT ( 7I )
94 FORMAT ( 7I )
95 FORMAT ( 7I )
96 FORMAT ( 7I )
97 FORMAT ( 7I )
98 FORMAT ( 7I )
99 FORMAT ( 7I )
100 FORMAT ( 7I )
101 FORMAT ( 7I )
102 FORMAT ( 7I )
103 FORMAT ( 7I )
104 FORMAT ( 7I )
105 FORMAT ( 7I )

SUBROUTINE TGCAM ( XG, YG, ZG, PHIG, THIG, TXYGZ, TGGTA )

DIMENSION A(3,3), XYGZ(3,1), XYSGS(3,1), TGGTA(20,3), B(3,3)

COMMON /TYP/ D(20), AMUTAB(20), VTAB(20)

UPR = 37.29577951
COSPI = COS ( PHIG )
COSPI = COS ( THIG )
SINPI = SIN ( PHIG )
SINPI = SIN ( THIG )
X2 = COS ( XG ) = COS ( ZG )

END

31
APPENDIX B - Continued

\begin{align*}
Y_2 &= r_2 \cdot \cos \theta_2 \cdot \sin \theta_2 \\
Z_2 &= r_2 \cdot \sin \theta_2 \\
A_{\theta_2} &= S \sin \theta_2 \\
A_{\theta_2} &= S \cos \theta_2 \\
A_{\theta_2} &= S \sin \theta_2 \\
A_{\theta_2} &= S \cos \theta_2 \\
A_{\theta_2} &= S \sin \theta_2 \\
INJ &= 0
\end{align*}

1. \( \text{INC} = \text{INC} + 1 \)
   \( T_1 = f \cdot YZT(\text{INC} + 1) \)
   \( YZT(1,1) = TAYZT(\text{INC} + 2) \)
   \( YZT(2,1) = TAYZT(\text{INC} + 3) \)
   \( YZT(3,1) = TAYZT(\text{INC} + 4) \)
   \( \text{TGTFLG} = 0 \).

2. \( \text{INJ} = \text{INJ} + 1 \)
   \( \text{TMJO} = TGTTAB(\text{INJ} + 1) \)
   \( \text{PMJO} = TGTTAB(\text{INJ} + 2) \)
   \( \text{KJ} = TGTTAB(\text{INJ} + 3) \)
   \( \text{TMJ} = \text{KJ} \cdot \text{UPK} \)
   \( \text{PMJ} = \text{PMJO} \cdot \text{UPK} \)
   \( \text{IF} (\text{TGTFLG}) \), 3, 3, 5

3. \( \text{CONTINU} \)
   \( \text{TGTFLG} = 1 \).
   \( \text{DO} \) 4 \( I = 1, 3 \)
   \( \text{DO} \) 4 \( J = 1, 3 \)

4. \( A_{\theta_2} = A_3 \cdot \cos \theta_3 \cdot \sin \theta_3 \)
   \( \text{XSYSZS}(1,1) = A(I,J) \cdot \text{XYSZS}(1,1) \)
   \( \text{X} = \text{XSYSZS}(1,1) \)
   \( \text{Y} = \text{XSYSZS}(2,1) \)
   \( \text{Z} = \text{XSYSZS}(3,1) \)
   \( \text{TEMP} = \text{SINT} \cdot \text{TEMP} \)
   \( \text{AL} = \text{ATAN} \cdot \text{TEMP} \)
   \( \text{ALPT} = \text{ATAN} \cdot \text{TEMP} \)
   \( \text{TPT} = \text{ATAN} \cdot \text{TEMP} \)
   \( \text{TEMP} = \text{XSYSZS}(2,2) \cdot \text{ZSYSZS}(2,2) \)
   \( \text{K} = \text{SINT} \cdot \text{TEM} \)
   \( \text{SINT} = \text{SIN}(\text{T}) \)
   \( \text{COST} = \text{COS}(\text{T}) \)
   \( \text{SINAL} = \text{SIN}(\text{AL}) \)
   \( \text{COSAL} = \text{COS}(\text{AL}) \)
   \( b(1,1) = -\text{SINT} \)
   \( b(1,2) = \text{COST} \)
   \( b(1,3) = \text{SINAL} \)

5. \( \text{CONTINU} \)
   \( \text{COS} = \text{COS}(\text{PMJO}) \)
   \( \text{COST} = \text{COST}(\text{PMJ}) \)
APPENDIX B - Continued

SINP=J=SIGN(PHIJ)
SINTHJ=SIGN(THJ)
XJ=J*COSPJP*CUSTHJ
YJ=J*COSPJP*SINTHJ
ZJ=J*SINPHJ
DELAJ=XJ-XS
DELYJ=YJ-YS
DELZJ=ZJ-ZS
DELPJ(T.INJ+1)=DELXJ
DELPJ(T.INJ+2)=DELYJ
DELPJ(T.INJ+3)=DELZJ
TEMP=DELJ**2+DELYJ**2+DELZJ**2
DELPJ=TEMP
AMV(J+1)=DELAJ/DELPJ
AMV(J+2)=DELYJ/DELPJ
AMV(J+3)=DELZJ/DELPJ
DO 6 J=1,3
AMV(B(J+1))=0.
DO 6 J=1,3
AMV(B(J+1))=A(J+1)*AMV(J+1)*AMV(J+1)

6 AMV(J+1)=AMV(J+1)+AMV(J+1)
DXYJ=DELXJ
DXYJ=DELYJ
DXYJ=DELZJ
DO 7 J=1,3
DXYJ(J+1)=0.
DO 7 J=1,3

7 DXYJ(J+1)=DXYJ(J+1)+DXYJ(J+1)
ATAD(J+1)=AMV(J+1)
AMV(J+1)=AMV(J+1)
VTAD(J+1)=AMV(J+1)
IF (NJ-INJ) 8,9,11

8 CONTINUE
DJPFG=J
DJPFG=J
IF (DJPFR) 9,9,11

9 RETURN
DO 10 INJ=INJ,1

10 RETURN
DO 11 K=1,3
CONTINUE
IF (NC-INC) 12,1,2,1

12 RETURN
FORMAT (1H0,5A,7E15.7)

SUBROUTINE ALPHET (TIME,ALPHAD,BETADETAAD,CVP,THEFTA,PSI,PHI)
COMMON /A/ Y,III,IV,W,TX(50),VX(50),VY(50),VZ(50),H(50),ALT(50),V
X0(50),Y0(50)
DO=0.174532925
MD=57.29577951
CALL FTLUP (TIME,VA1+1,III+1,TX+VX)
CALL FTLUP (TIME,VA1+1,III+1,TX+VY)
CALL FTLUP (TIME,VA1+1,III+1,TX+VZ)
CALL FTLUP (TIME,VA1+1,III+1,TX+H)
CALL FTLUP (Z+VA0+1,IV+ALT+VA0)
CALL FTLUP (Z+VY0+1,IV+ALT+VY0)
VZ0=0,
PSI0=0,PSI0
PHI0=0,PHI0
APPENDIX B — Concluded

THE4=THE4U3
X1=SIN(PSIK)
X2=COS(PSIK)
X3=SIN(PIH)
X4=COS(PIH)
X5=SIN(THETAN)
X6=COS(THETAN)
E1=x6*x2*x6*x1+x6*x1*VY1-x5*VZ1
E2=(x2*x5*x3-1*x4)*x1+(x2*x4+1*x5*x3)*VY1*x6*x3*VZ1
E3=(x2*x5*x4+1*x3)*x1+(1*x5*x4-x2*x3)*VY1*x6*x4*VZ1
E4=x6*x2*x6*x1*x5*x1*VY1-x5*VZ0
E5=(x2*x5*x3-1*x4)*VX0+(x2*x4+1*x5*x3)*VY0*x6*x3*VZ0
E6=(x2*x5*x4+1*x3)*VX0+(1*x5*x4-x2*x3)*VY0*x6*x4*VZ0
U3=+1-U4
V3=+U2
WP=+3-WP
ALPHA=ATAN2(UP*U3)
ETA=ATAN2(VP*V3)
ALPHA=ALPHA*UP
BETA=ETA*V3
ETA=ETA*V3
V2=UP*UP+VP*VP+WP*WP
CVP=SQRT(V2)
RETURN
END
APPENDIX C

LANGLEY LIBRARY SUBROUTINE MATINV

Language: FORTRAN

Purpose: MATINV solves the matrix equation \( AX = B \), where \( A \) is a square coefficient matrix and \( B \) is a matrix of constant vectors. The solution to a set of simultaneous equations, the matrix inverse, and the determinant may be obtained. If the user does not want the inverse, use SIMEQ for savings in time and storage. For the determinant only, use DETEV.

Use: CALL MATINV(A,N,B,M,DETERM,IPIVOT,INDEX,NMAX,ISCALE)

\( A \) A two-dimensional array of the coefficients. On return to the calling program, \( A^{-1} \) is stored in \( A \)

\( N \) The order of \( A \), \( 1 \leq N \leq NMAX \)

\( B \) A two-dimensional array of the constant vectors \( B \). On return to the calling program, \( X \) is stored in \( B \)

\( M \) The number of column vectors in \( B \). The expression \( M = 0 \) signals that the subroutine is used solely for inversion; however, in the CALL statement an entry corresponding to \( B \) must still be present

DETERM Gives the value of the determinant by the formula
\[
\text{DET}(A) = (10^{100}) \times \text{ISCALE} \times \text{DETERM}
\]

IPIVOT A one-dimensional array of temporary storage used by the routine

INDEX A two-dimensional array of temporary storage used by the routine

NMAX The maximum order of \( A \) as stated in the DIMENSION statement of the calling program

ISCALE A scale factor computed by the subroutine to keep the results of computation within the floating-point word size of the computer
Restrictions: Arrays A, B, IPIVOT, and INDEX have variable dimensions in the subroutine. The maximum size of these arrays must be specified in a DIMENSION statement of the calling program as $A(NMAX,NMAX)$, $B(NMAX,M)$, $IPIVOT(NMAX)$, and $INDEX(NMAX,2)$. The original matrices A and B are destroyed. They must be saved by the user if there is further need for them. The determinant is set to zero for a singular matrix.

Method: Jordan's method is used to reduce a matrix A to the identity matrix I through a succession of elementary transformations $l_n, l_{n-1}, \ldots, l_1$. $A = I$. If these transformations are simultaneously applied to I and to a matrix B of constant vectors, the results are $A^{-1}$ and $X$ where $AX = B$. Each transformation is selected so that the largest element is used in the pivotal position. (See ref. (a).)

Accuracy: Total pivotal strategy is used to minimize the rounding errors; however, the accuracy of the final results depends upon how well-conditioned the original matrix is.


Storage: 5428 locations.

Subroutine date: August 1, 1968.
APPENDIX D

LANGLEY LIBRARY SUBROUTINE FTLUP

Language: FORTRAN

Purpose: Computes \( y = F(x) \) from a table of values using first- or second-order interpolation. An option to give \( y \) a constant value for any \( x \) is also provided.

Use: CALL FTLUP(X, Y, M, N, VARI, VARD)

X  The name of the independent variable \( x \).

Y  The name of the dependent variable \( y = F(x) \).

M  The order of interpolation (an integer)
   \( M = 0 \) for \( y \) a constant. \( VARD(I) \) corresponds to \( VARI(I) \) for
   \( I = 1, 2, \ldots, N \). For \( M = 0 \) or \( N \leq 1 \), \( y = F(VARI(1)) \) for any value
   of \( x \). The program extrapolates.
   \( M = 1 \) or \( 2 \). First or second order if \( VARI \) is strictly increasing (not
   equal).
   \( M = -1 \) or \( -2 \). First or second order if \( VARI \) is strictly decreasing (not
   equal).

N  The number of points in the table (an integer).

VARI  The name of a one-dimensional array which contains the \( N \) values of the
       independent variable.

VARD  The name of a one-dimensional array which contains the \( N \) values of the
       dependent variable.

Restrictions: All the numbers must be floating point. The values of the independent
variable \( x \) in the table must be strictly increasing or strictly decreasing. The fol-
lowing arrays must be dimensioned by the calling program as indicated: \( VARI(N) \),
\( VARD(N) \).

Accuracy: A function of the order of interpolation used.
APPENDIX D — Concluded


Storage: 430₅ locations.

Error condition: If the VARI values are not in order, the subroutine will print TABLE BELOW OUT OF ORDER FOR FTLUP AT POSITION xxx TABLE IS STORED IN LOCATION xxxxxxx (absolute). It then prints the contents of VARI and VARD, and Stops the program.

Subroutine date: September 12, 1969.
### APPENDIX E

**BASIC DATA**

**Computer Card Format**

A typical basic data computer card format is as follows:

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>shows whether the data are for a landmark or fiducial point and, if the latter, which fiducial system is being used: 4 is used for landmarks; 1 for frame corner fiducial points; and 0 for film sprocket hole fiducial points</td>
</tr>
<tr>
<td>2 to 5</td>
<td>vehicle or test identification numbers</td>
</tr>
<tr>
<td>7</td>
<td>onboard camera identification number</td>
</tr>
<tr>
<td>8</td>
<td>blank</td>
</tr>
<tr>
<td>9 to 10</td>
<td>landmark number for landmark readings; 00 for fiducial point readings</td>
</tr>
<tr>
<td>11</td>
<td>film reading system identification number</td>
</tr>
<tr>
<td>12</td>
<td>blank</td>
</tr>
<tr>
<td>13 to 14</td>
<td>1, 2, or 3; indicates which of the required three readings (for averaging purposes) of each fiducial point or landmark is on the card</td>
</tr>
<tr>
<td>15 to 19</td>
<td>accumulated point number, including fiducials</td>
</tr>
<tr>
<td>20 to 40</td>
<td>blank</td>
</tr>
<tr>
<td>41 to 47</td>
<td>(x)-value (in inches (\times 10^{-3})) of the landmark or fiducial point on the frame, measured horizontally on the projected image, with origin at right edge of image or right sprocket holes of film</td>
</tr>
<tr>
<td>48 to 50</td>
<td>blank</td>
</tr>
</tbody>
</table>
APPENDIX E – Continued

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>51 to 57</td>
<td>( \bar{y} )-value (in inches ( \times 10^{-3} )) of the landmark or fiducial point on the frame, measured vertically on the projected image, with origin at bottom edge of projected image or bottom sprocket holes of film</td>
</tr>
<tr>
<td>58 to 63</td>
<td>blank</td>
</tr>
<tr>
<td>64 to 67</td>
<td>frame number based on a convenient zero reference frame</td>
</tr>
</tbody>
</table>

Only the data in columns 9 to 10 and after column 40 serve as inputs to the computer program. The remaining data are used for identification and gross reading or system error analyses only and may be altered at the user's discretion. For the system employed at Langley Research Center, the data through column 11 are input manually on dials located on the digitizer. The remaining data are input automatically and displayed on the digitizer. All data are recorded by the typer as well as on the punched computer cards.

Data Reading

The following discussion is related to the procedures and system currently employed at the NASA Langley Research Center utilizing the Gerber film reading system and focuses on getting the basic data into a form usable by the computer.

First a frame reference time must be established and the landmarks and features to be used must be identified. Next, sketches showing the position of the landmarks (and features) in relation to other distinct features (for example, mountain ranges, rivers, surface discolorations) on the film should be prepared. These sketches are useful in determining the general landmark location on the projected film image.

A typical sequence in reading a frame of film, after the film has been installed in the film reader, is enumerated. These instructions are for the Gerber film reading system at the Langley Research Center and may be modified for other systems.

1. Set the frame counter to the proper value.
2. Establish the image space axes system \((\bar{x}, \bar{y})\) origin (for example, frame corner).
3. Focus the frame (do not change the focus until the next frame).
4. Assure that the \( \bar{x} \) and \( \bar{y} \) values increase in the desired direction by moving the crosshairs.
5. Aline the crosshairs on the first fiducial point (for example, frame corner, film sprocket hole).
(6) Assure that all dials on the digitizer are properly set and reset the $\bar{x}$ and $\bar{y}$ values on the digitizer to zero

(7) Assure the readiness of the typer and card punch

(8) Punch the fiducials (three readings for each of the four fiducials)

(9) Adjust the digitizer dials as necessary for landmark readings and punch the landmarks (three readings each)

(10) Check the printing from the typer for general correctness

(11) Advance to the next frame to be read and repeat these steps.

Finally, the cards should be interpreted and listed. A review of this listing should reveal any errors or blank cards which should be removed prior to processing.
For user's convenience, program VOPD has been divided into distinctive input and output groups. Following is a detailed description in computer terminology of the input for each group of control data.

<table>
<thead>
<tr>
<th>Column</th>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Group 1</td>
<td>Format (2F10.3)</td>
<td>25 cards</td>
<td></td>
</tr>
<tr>
<td>1 to 10</td>
<td>XRR</td>
<td>Radial distance from frame center</td>
<td>mm</td>
</tr>
<tr>
<td>11 to 20</td>
<td>RRR</td>
<td>Lens radial distortion</td>
<td>µm</td>
</tr>
<tr>
<td>Input Group 2</td>
<td>Format (80H)</td>
<td>1 card</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Punch a 1 in column 1</td>
<td></td>
</tr>
<tr>
<td>2 to 80</td>
<td>TITLE</td>
<td>Descriptive title, printed at top</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>of each output page</td>
<td></td>
</tr>
<tr>
<td>Input Group 3</td>
<td>Format (2I5, 6F5.1)</td>
<td>1 card</td>
<td></td>
</tr>
<tr>
<td>1 to 5</td>
<td>III</td>
<td>Number of data values in Input Group 7</td>
<td></td>
</tr>
<tr>
<td>6 to 10</td>
<td>IWV</td>
<td>Number of data values in Input Group 4</td>
<td></td>
</tr>
<tr>
<td>11 to 15</td>
<td>SIM</td>
<td>Multiplying factor to convert camera yaw angle (σ1) to vehicle yaw angle (ψ)</td>
<td>deg</td>
</tr>
<tr>
<td>16 to 20</td>
<td>S1A</td>
<td>Addition factor to convert camera yaw angle (σ1) to vehicle yaw angle (ψ)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>PSI = SIM * SIG1 + S1A, SIM = +1 or -1</td>
<td></td>
</tr>
</tbody>
</table>
## APPENDIX F – Continued

<table>
<thead>
<tr>
<th>Column</th>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 to 25</td>
<td>S2M</td>
<td>Multiplying factor to convert camera pitch angle ($\sigma_2$) to vehicle pitch angle ($\theta$)</td>
<td></td>
</tr>
<tr>
<td>26 to 30</td>
<td>S2A</td>
<td>Addition factor to convert camera pitch angle ($\sigma_2$) to vehicle pitch angle ($\theta$)</td>
<td>deg</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{THETA} = S2M \times \text{SIG2} + S2A, S2M = +1 or -1$</td>
<td></td>
</tr>
<tr>
<td>31 to 35</td>
<td>S3M</td>
<td>Multiplying factor to convert camera roll angle ($\sigma_3$) to vehicle roll angle ($\phi$)</td>
<td></td>
</tr>
<tr>
<td>36 to 40</td>
<td>S3A</td>
<td>Addition factor to convert camera roll angle ($\sigma_3$) to vehicle roll angle ($\phi$)</td>
<td>deg</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{PHI} = S3M \times \text{SIG3} + S3A, S3M = +1 or -1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input Group 4</td>
<td></td>
<td>Format (3F10.1)</td>
<td>IWV cards</td>
</tr>
<tr>
<td>1 to 10</td>
<td>ALT</td>
<td>Altitude table, mean sea level</td>
<td>ft</td>
</tr>
<tr>
<td>11 to 20</td>
<td>VXO</td>
<td>North-South wind velocity table, + from South</td>
<td>fps</td>
</tr>
<tr>
<td>21 to 30</td>
<td>VYO</td>
<td>East-West wind velocity table, + from West</td>
<td>fps</td>
</tr>
<tr>
<td>Input Group 5</td>
<td></td>
<td>Format (16I5)</td>
<td>(NFR + 3)/16 cards</td>
</tr>
<tr>
<td>1 to 5</td>
<td>NT</td>
<td>Number of landmarks read on this film, also number of Input Group 9 data values</td>
<td></td>
</tr>
<tr>
<td>Column</td>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>------------</td>
<td>-----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>6 to 10</td>
<td>NFR</td>
<td>Number of frames read from this film</td>
<td></td>
</tr>
<tr>
<td>11 to 15</td>
<td>NTFR</td>
<td>Number of Input Group 6 data values</td>
<td></td>
</tr>
<tr>
<td>16 to 20 (etc.)</td>
<td>NPT(I)</td>
<td>Number of landmarks in each frame, etc., I = 1 to NFR</td>
<td></td>
</tr>
<tr>
<td>Input Group 6</td>
<td>Format (1I5, 1F10.1)</td>
<td>NTFR cards</td>
<td></td>
</tr>
<tr>
<td>1 to 5</td>
<td>IFRM</td>
<td>Frame number, does not have to correspond to frames read, but must encompass those of the basic data deck</td>
<td></td>
</tr>
<tr>
<td>6 to 15</td>
<td>TIMX</td>
<td>Flight time of this frame</td>
<td>sec</td>
</tr>
<tr>
<td>Input Group 7</td>
<td>Format (8F10.1)</td>
<td>III cards</td>
<td></td>
</tr>
<tr>
<td>1 to 10</td>
<td>TX</td>
<td>Time table</td>
<td>sec</td>
</tr>
<tr>
<td>11 to 20</td>
<td>XXX</td>
<td>X-location of the vehicle from origin, + East</td>
<td>ft</td>
</tr>
<tr>
<td>21 to 30</td>
<td>YYY</td>
<td>Y-location of the vehicle from origin, + North</td>
<td>ft</td>
</tr>
<tr>
<td>31 to 40</td>
<td>ZZZ</td>
<td>Z-location of the vehicle from origin, + zenith</td>
<td>ft</td>
</tr>
<tr>
<td>41 to 50</td>
<td>VVX</td>
<td>Vehicle velocity, + toward North</td>
<td>fps</td>
</tr>
<tr>
<td>51 to 60</td>
<td>VVY</td>
<td>Vehicle velocity, + toward East</td>
<td>fps</td>
</tr>
<tr>
<td>Column</td>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-----------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>61 to 70</td>
<td>VVZ</td>
<td>Vehicle velocity, + toward Earth</td>
<td>fps</td>
</tr>
<tr>
<td>71 to 80</td>
<td>H</td>
<td>Altitude of vehicle, mean sea level</td>
<td>ft</td>
</tr>
<tr>
<td>Input Group 8</td>
<td>Format (7F10.0)</td>
<td>1 card</td>
<td></td>
</tr>
<tr>
<td>1 to 10</td>
<td>CC</td>
<td>Camera focal length</td>
<td>mm</td>
</tr>
<tr>
<td>11 to 20</td>
<td>PHI</td>
<td>Latitude of the origin of Group 7 coordinate system, geodetic</td>
<td>deg</td>
</tr>
<tr>
<td>21 to 30</td>
<td>THZ</td>
<td>Longitude of the origin of Group 7 coordinate system, geodetic</td>
<td>deg</td>
</tr>
<tr>
<td>31 to 40</td>
<td>RZ</td>
<td>Distance from center of Earth to origin of Group 7 coordinate system</td>
<td>$10^3$ ft</td>
</tr>
<tr>
<td>41 to 50</td>
<td>SIG1</td>
<td>Initial estimate of camera Euler yaw angle</td>
<td>deg</td>
</tr>
<tr>
<td>51 to 60</td>
<td>SIG2</td>
<td>Initial estimate of camera Euler pitch angle</td>
<td>deg</td>
</tr>
<tr>
<td>61 to 70</td>
<td>SIG3</td>
<td>Initial estimate of camera Euler roll angle</td>
<td>deg</td>
</tr>
<tr>
<td>Input Group 9</td>
<td>Format (11I10, 5F10.1)</td>
<td>NT cards</td>
<td></td>
</tr>
<tr>
<td>1 to 10</td>
<td>ITGT</td>
<td>Landmark ID number</td>
<td></td>
</tr>
<tr>
<td>11 to 20</td>
<td>THJ</td>
<td>Longitude of this landmark, degree part only</td>
<td>deg</td>
</tr>
<tr>
<td>Column</td>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>------------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>21 to 30</td>
<td>THJMI</td>
<td>Longitude of this landmark, minute part only</td>
<td>min</td>
</tr>
<tr>
<td>31 to 40</td>
<td>PHLJ</td>
<td>Geodetic latitude of this landmark, degree part only</td>
<td>deg</td>
</tr>
<tr>
<td>41 to 50</td>
<td>PHIJMI</td>
<td>Geodetic latitude of this landmark, minute part only</td>
<td>min</td>
</tr>
<tr>
<td>51 to 60</td>
<td>RJ</td>
<td>Landmark altitude, mean sea level</td>
<td>ft</td>
</tr>
</tbody>
</table>

**Input Group 10**

| Format (1I5) | 1 card |

**Input Group 11**

<table>
<thead>
<tr>
<th>Format (7I5)</th>
<th>NOUT cards</th>
</tr>
</thead>
</table>

| IFM | Frame number as read from film |

| JOUT(I) | Landmarks to be ignored for this frame only, I = 1 to 6 for as many as required |

If JOUT(I) = 100 this entire frame is ignored; do not remove its data from the basic data deck.
APPENDIX F – Concluded

<table>
<thead>
<tr>
<th>Column</th>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Group 12</td>
<td>Format (1615)</td>
<td>NT/16 cards</td>
</tr>
<tr>
<td>1 to 5</td>
<td>NGO</td>
<td>Input group number to which program returns after processing all the frames of this job = 13 to stop further calculations</td>
</tr>
<tr>
<td>6 to 10 (etc.)</td>
<td>IOUT</td>
<td>Landmarks to be ignored in computations for all frames</td>
</tr>
</tbody>
</table>

**Basic Data**

The basic data deck as punched from the film reader is input after the control data deck. It consists of three cards for each of four fiducial frame corners plus three cards for each of the NPT landmarks for a total of 12 + 3 * NPT cards. Following is a list of the essential information from the basic data deck required for program operation. For an explanation of inputs for all columns of the basic data cards see appendix E.

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 to 10</td>
<td>Landmark number</td>
<td></td>
</tr>
<tr>
<td>41 to 47</td>
<td>$\bar{x}$-value of landmark on the film</td>
<td>in.</td>
</tr>
<tr>
<td>51 to 57</td>
<td>$\bar{y}$-value of landmark on the film</td>
<td>in.</td>
</tr>
<tr>
<td>64 to 67</td>
<td>Frame number</td>
<td></td>
</tr>
</tbody>
</table>

Only one copy of this basic data deck is required. For additional calculations with these data only these input groups, including and following the one specified by NGO, must be inserted behind this basic data deck.
APPENDIX G

PROGRAM VOPD OUTPUT DESCRIPTION

The output of this program is listed one page to a film frame and includes the following groups of data:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Group 1</td>
<td>NPT lines</td>
<td></td>
</tr>
<tr>
<td>L-MARK</td>
<td>Landmark number</td>
<td>deg</td>
</tr>
<tr>
<td>LONGITUDE</td>
<td>Landmark longitude</td>
<td>deg</td>
</tr>
<tr>
<td>LAT-GOCN</td>
<td>Landmark geocentric latitude</td>
<td>deg</td>
</tr>
<tr>
<td>RAD-ER-KFT</td>
<td>Earth center to landmark distance</td>
<td>$10^3$ ft</td>
</tr>
<tr>
<td>LAT-GODT</td>
<td>Landmark geodetic latitude</td>
<td>deg</td>
</tr>
<tr>
<td>ALTITUDE-FT</td>
<td>Landmark altitude, mean sea level</td>
<td>ft</td>
</tr>
<tr>
<td>XLAM</td>
<td>Direction cosines of image space coordinate system relative to object space coordinate system (see appendix A)</td>
<td></td>
</tr>
<tr>
<td>XMU</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XNU</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output Group 2</td>
<td>1 line</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>X-location of vehicle, same as Input Group 8 XXX</td>
<td>ft</td>
</tr>
<tr>
<td>Y</td>
<td>Y-location of vehicle, same as Input Group 8 YYY</td>
<td>ft</td>
</tr>
<tr>
<td>Z</td>
<td>Z-location of vehicle, same as Input Group 8 ZZZ</td>
<td>ft</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-------------------</td>
<td>------------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>XPC</td>
<td>X-location of film frame center, average of fiducials</td>
<td>in.</td>
</tr>
<tr>
<td>YPC</td>
<td>Y-location of film frame center, average of fiducials</td>
<td>in.</td>
</tr>
<tr>
<td>CC</td>
<td>Camera focal length</td>
<td>mm</td>
</tr>
<tr>
<td>Output Group 3</td>
<td>NPT lines</td>
<td></td>
</tr>
<tr>
<td>XCO FINAL</td>
<td>X-location of landmark in image space (on film)</td>
<td>in.</td>
</tr>
<tr>
<td>YCO FINAL</td>
<td>Y-location of landmark in image space (on film)</td>
<td>in.</td>
</tr>
<tr>
<td>Output Group 4</td>
<td>NPT lines</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>X-location of landmark in object space</td>
<td>in.</td>
</tr>
<tr>
<td>Y</td>
<td>Y-location of landmark in object space</td>
<td>in.</td>
</tr>
<tr>
<td>R DIST</td>
<td>Landmark location from frame center, image space</td>
<td>in.</td>
</tr>
<tr>
<td>DELTA R</td>
<td>Radial distortion of landmark due to camera lens</td>
<td>in.</td>
</tr>
<tr>
<td>Output Group 5</td>
<td>1 line</td>
<td></td>
</tr>
<tr>
<td>FRAME</td>
<td>Frame number</td>
<td></td>
</tr>
<tr>
<td>TIME</td>
<td>Flight time</td>
<td>sec</td>
</tr>
<tr>
<td>SIG1</td>
<td>Camera Euler angle, yaw</td>
<td>deg</td>
</tr>
<tr>
<td>SIG2</td>
<td>Camera Euler angle, pitch</td>
<td>deg</td>
</tr>
<tr>
<td>SIG3</td>
<td>Camera Euler angle, roll</td>
<td>deg</td>
</tr>
<tr>
<td>XP</td>
<td>X-location of frame center in object space</td>
<td>in.</td>
</tr>
<tr>
<td>YP</td>
<td>Y-location of frame center in object space</td>
<td>in.</td>
</tr>
<tr>
<td>L-9461</td>
<td></td>
<td>49</td>
</tr>
</tbody>
</table>
### APPENDIX G – Concluded

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSI</td>
<td>Vehicle Euler angle, yaw</td>
<td>deg</td>
</tr>
<tr>
<td>THETA</td>
<td>Vehicle Euler angle, pitch</td>
<td>deg</td>
</tr>
<tr>
<td>PHI</td>
<td>Vehicle Euler angle, roll</td>
<td>deg</td>
</tr>
<tr>
<td>ALPHA</td>
<td>Vehicle pitch angle of attack</td>
<td>deg</td>
</tr>
<tr>
<td>BETA</td>
<td>Vehicle yaw angle of attack</td>
<td>deg</td>
</tr>
<tr>
<td>ETA</td>
<td>Vehicle total angle of attack</td>
<td>deg</td>
</tr>
<tr>
<td>Output Group 6</td>
<td>NPT lines</td>
<td></td>
</tr>
<tr>
<td>RX(I)</td>
<td>Error in X-coordinate from frame center</td>
<td>in.</td>
</tr>
<tr>
<td>RY(I)</td>
<td>Error in Y-coordinate from frame center</td>
<td>in.</td>
</tr>
<tr>
<td>L-MARK</td>
<td>Landmark number</td>
<td></td>
</tr>
<tr>
<td>Output Group 7</td>
<td>1 line</td>
<td></td>
</tr>
</tbody>
</table>

The standard deviation of the residuals is listed here.
APPENDIX H

EXAMPLE DATA

Example input and output data for several sample cases are presented in this appendix.

Input

<table>
<thead>
<tr>
<th>COLUMN</th>
<th>111111111122222222333333333344444444444555555555555666666666667777777777</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12345678901234567890123456789012345678901234567890</td>
</tr>
</tbody>
</table>

Sample case 1

<table>
<thead>
<tr>
<th>0.</th>
<th>0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.0</td>
</tr>
<tr>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>1.5</td>
<td>15.0</td>
</tr>
<tr>
<td>2.0</td>
<td>20.0</td>
</tr>
<tr>
<td>2.5</td>
<td>30.0</td>
</tr>
<tr>
<td>3.0</td>
<td>45.0</td>
</tr>
<tr>
<td>3.25</td>
<td>45.0</td>
</tr>
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<td>3.5</td>
<td>55.0</td>
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<tr>
<td>3.75</td>
<td>65.0</td>
</tr>
<tr>
<td>4.0</td>
<td>70.0</td>
</tr>
<tr>
<td>4.2</td>
<td>75.0</td>
</tr>
<tr>
<td>4.4</td>
<td>76.5</td>
</tr>
<tr>
<td>4.6</td>
<td>77.5</td>
</tr>
<tr>
<td>4.8</td>
<td>77.5</td>
</tr>
<tr>
<td>5.0</td>
<td>76.5</td>
</tr>
<tr>
<td>5.2</td>
<td>76.5</td>
</tr>
<tr>
<td>5.4</td>
<td>75.0</td>
</tr>
<tr>
<td>5.6</td>
<td>70.0</td>
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<tr>
<td>5.8</td>
<td>65.0</td>
</tr>
<tr>
<td>6.0</td>
<td>60.0</td>
</tr>
<tr>
<td>6.2</td>
<td>55.0</td>
</tr>
<tr>
<td>6.4</td>
<td>50.0</td>
</tr>
<tr>
<td>6.6</td>
<td>40.0</td>
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<tr>
<td>6.8</td>
<td>30.0</td>
</tr>
<tr>
<td>7.0</td>
<td>22.2</td>
</tr>
<tr>
<td>COLUMN 1</td>
<td>VIKING BLT AV-4 FOWARD MILLIKEN CAMERA</td>
</tr>
<tr>
<td>----------</td>
<td>---------------------------------------</td>
</tr>
<tr>
<td>4</td>
<td>31.0 0.0 1.0 0.0 1.0 -210.0</td>
</tr>
<tr>
<td>136000.</td>
<td>32.0 -90.0</td>
</tr>
<tr>
<td>140000.</td>
<td>-4.0 -144.0</td>
</tr>
<tr>
<td>150000.</td>
<td>-2.0 -156.0</td>
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<td>0</td>
<td>48.073</td>
</tr>
<tr>
<td>2520</td>
<td>126.774</td>
</tr>
<tr>
<td>10.4</td>
<td>33.083333 -106.3333 20925.698 198.0  -36.0 285.0</td>
</tr>
<tr>
<td>65</td>
<td>-106.0 -36.05 32.0 50.3 10000.0</td>
</tr>
<tr>
<td>66</td>
<td>-106.0 -32.7 32.0 50.3 10000.0</td>
</tr>
<tr>
<td>57</td>
<td>-106.0 -24.5 32.0 49.75 4000.0</td>
</tr>
<tr>
<td>COLUMN</td>
<td>11111111122222222333333344444455555555556666666667777777778</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------</td>
</tr>
<tr>
<td>12</td>
<td>112402004010100100  000003  000003  000730  000002  000002  000003  000730</td>
</tr>
<tr>
<td></td>
<td>112402004010201000  -000004  -000004  000730</td>
</tr>
<tr>
<td>0</td>
<td>112402004010301000  014401  000002  000730</td>
</tr>
<tr>
<td></td>
<td>11240200401010101   014477  -000001  000730</td>
</tr>
<tr>
<td>COLUMN</td>
<td>11111111112222222233333344444445555555555666666666677777778</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Sample case 2

<table>
<thead>
<tr>
<th>9</th>
<th>65</th>
</tr>
</thead>
</table>

Sample case 3

| 65 -106. | -36.65 | 32. | 50.3 | 10000. |
| 66 -106. | -32.7 | 32. | 50.3 | 10000. |
| 67 -106. | -24.5 | 32. | 49.75 | 4000. |

GROUP 9

GROUP 10

GROUP 12

Sample case 4

<table>
<thead>
<tr>
<th>1</th>
<th>VIKING</th>
<th>BLOT</th>
<th>LOAD</th>
<th>HAR</th>
<th>CAMERA</th>
<th>AV-4</th>
</tr>
</thead>
</table>

| 3 | 21. | 0. | 1. | 0. | 1. | 0. |

GROUP 2

GROUP 3

GROUP 4

<p>| 120000. | -22. | -70. |
| 123000. | -25. | -94. |</p>
<table>
<thead>
<tr>
<th>COLUMN</th>
<th>1111111112222222223333333444444445555555555555555555566666666677777777778</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12345678901234567890123456789012345678901234567890123456789012345678901234567890</td>
</tr>
<tr>
<td>3 3 3 3 3</td>
<td>GROUP 5</td>
</tr>
<tr>
<td>55  -0.782</td>
<td></td>
</tr>
<tr>
<td>101  0.</td>
<td>GROUP 6</td>
</tr>
<tr>
<td>-0.8</td>
<td>31160.44</td>
</tr>
<tr>
<td>-0.4</td>
<td>31124.71</td>
</tr>
<tr>
<td>0.</td>
<td>31088.2</td>
</tr>
<tr>
<td>10.4</td>
<td>33.083333</td>
</tr>
<tr>
<td>28  -106.</td>
<td>-14.2</td>
</tr>
<tr>
<td>62  -106.</td>
<td>-18.3</td>
</tr>
<tr>
<td>69  -106.</td>
<td>-20.7</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>55  100</td>
<td>GROUP 10</td>
</tr>
<tr>
<td>60  69</td>
<td></td>
</tr>
<tr>
<td>65  69</td>
<td>GROUP 11</td>
</tr>
<tr>
<td>13</td>
<td>GROUP 12</td>
</tr>
<tr>
<td>COlumn</td>
<td>1122402000410100001</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------</td>
</tr>
<tr>
<td></td>
<td>-0.0002 0.00003 0.00055</td>
</tr>
<tr>
<td></td>
<td>0.0004 -0.00001 0.00055</td>
</tr>
<tr>
<td></td>
<td>0.14392 0.00499 0.00055</td>
</tr>
<tr>
<td></td>
<td>0.00000 0.10497 0.00055</td>
</tr>
<tr>
<td></td>
<td>0.00002 0.10493 0.00055</td>
</tr>
<tr>
<td>4122402023410100005</td>
<td>0.10255 0.08847 0.00055</td>
</tr>
<tr>
<td></td>
<td>0.08298 0.03371 0.00055</td>
</tr>
<tr>
<td></td>
<td>0.01059 0.04780 0.00055</td>
</tr>
<tr>
<td></td>
<td>0.01055 0.04783 0.00055</td>
</tr>
<tr>
<td>4122402023410100007</td>
<td>-0.00002 -0.00010 0.00060</td>
</tr>
<tr>
<td></td>
<td>0.14386 0.00002 0.00060</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>0.14386 0.00485 0.00060</td>
</tr>
<tr>
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<td>1122402000410300008</td>
<td>-0.00002 0.10480 0.00060</td>
</tr>
<tr>
<td>1122402000410100009</td>
<td>-0.00004 0.00004 0.00060</td>
</tr>
<tr>
<td>1122402000410100009</td>
<td>-0.00004 0.00004 0.00060</td>
</tr>
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<td>-0.00004 0.00004 0.00060</td>
</tr>
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<td>-0.00002 0.10480 0.00060</td>
</tr>
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**BASIC DATA**
<table>
<thead>
<tr>
<th>COLUMN</th>
<th>012340202401000012</th>
<th>010287 008795 000060</th>
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<tr>
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<td>010284 008794 000060</td>
<td></td>
</tr>
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APPENDIX H - Continued
Output

Sample case 1

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<tr>
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<td>11.13700000</td>
<td>2.97966670</td>
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<td>4.45540</td>
<td>4.08156</td>
<td>-.07204</td>
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<th>YP</th>
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<th>BETA</th>
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STANDARD DEVIATION OF RESIDUALS .11718
Sample case 2

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<td>L-MARK LONGITUDE  LAT-GOCN RAD ER KM  LAT-Goldt ALTITUDE-FT  XLM  XMU  XMU</td>
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</tr>
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<td>66  -106.54500 32.66227 20935.698 32.83833 10000.000 -0.227328 -0.5279743 -0.8195323</td>
</tr>
<tr>
<td>67  -106.40833 32.65313 20529.698 32.82917 4000.000 0.0349096 -0.5390263 -0.8415652</td>
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<thead>
<tr>
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<td>-28971.56 -3396.84 143179.38 7.23925 5.22825 10.40000</td>
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APPENDIX H – Continued
### Sample Case 3

**VIKING BLDT AV-4 FOWARD MILLIKEN CAMERA**

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**L-MARK XCO FINAL YCO FINAL**

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**L-MARK**

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<tr>
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<th>PHI</th>
<th>ALPHA</th>
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**RESIDUALS RX(II) RY(II) L-MARK**

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**STANDARD DEVIATION OF RESIDUALS**

-.06593
Sample case 4

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<td>-.01100635 -.03052913 62</td>
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### VIKING BLDT LOAD BAR CAMERA AV-4

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<th>SIG3</th>
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<th>PSI</th>
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**Standard Deviation of Residuals** | .02806
REFERENCES


"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."
—National Aeronautics and Space Act of 1958

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