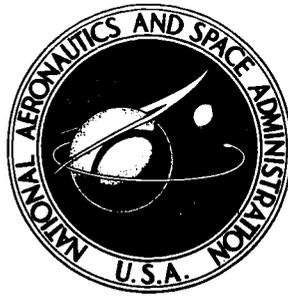


NASA TECHNICAL NOTE



NASA TN D-7714

NASA TN D-7714

# OPTIMAL AND SUBOPTIMAL CONTROL TECHNIQUE FOR AIRCRAFT SPIN RECOVERY

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1974

1. Report No. NASA TN D-7714	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle OPTIMAL AND SUBOPTIMAL CONTROL TECHNIQUE FOR AIRCRAFT SPIN RECOVERY		5. Report Date October 1974	6. Performing Organization Code
		8. Performing Organization Report No. L-9625	10. Work Unit No. 760-67-08-05
7. Author(s) John W. Young	9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23665		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546			13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code	
15. Supplementary Notes			
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17. Key Words (Suggested by Author(s)) Aircraft spin recovery Optimization		18. Distribution Statement Unclassified - Unlimited  STAR Category 02	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 27	22. Price* \$ 3.25

# OPTIMAL AND SUBOPTIMAL CONTROL TECHNIQUE FOR AIRCRAFT SPIN RECOVERY

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## SUMMARY

An analytic investigation has been made of procedures for effecting recovery from equilibrium spin conditions for three assumed aircraft configurations. Three approaches which utilize conventional aerodynamic controls are investigated. Included are a constant control recovery mode, optimal recoveries, and a suboptimal control logic patterned after optimal recovery results. The optimal and suboptimal techniques are shown to yield a significant improvement in recovery performance over that attained by using a constant control recovery procedure.

## INTRODUCTION

Many modern fighter aircraft have inertia and aerodynamic characteristics which produce unsatisfactory handling qualities during maneuvers at angles of attack near the stall region (refs. 1 and 2). Thus, inadvertent entries into the poststall-spin region occur, and, unless correct recovery techniques are promptly initiated, steady developed spins may take place. Should a fast, flat spin occur, recovery using conventional methods is difficult, if not impossible.

The objective of the present study is to evaluate analytically the effectiveness of various procedures for achieving recovery from equilibrium spin conditions. Three separate techniques which utilize conventional aerodynamic controls are investigated. Included are recoveries for which constant control surface deflections are used, recoveries obtained by using a six-degree-of-freedom optimization program, and recoveries using a suboptimal control logic patterned after optimal recovery results. Comparisons between the performance of each recovery procedure are given for three different assumed aircraft configurations. Recovery performance is evaluated with respect to turns, altitude loss, and time required to achieve recovery.

## SYMBOLS

All aerodynamic data and flight motions are referenced to the body system of axes shown in figure 1.

b	wing span
$C_l$	rolling-moment coefficient
$C_m$	pitching-moment coefficient
$C_n$	yawing-moment coefficient
$C_X$	longitudinal-force coefficient
$C_Y$	side-force coefficient
$C_Z$	vertical-force coefficient
$\bar{c}$	mean aerodynamic chord
g	acceleration due to gravity
h	altitude
$\Delta h$	altitude loss during recovery
$I_X, I_Y, I_Z, I_{XZ}$	body-axis moments and product of inertia about the center of mass
m	mass of airplane
p, q, r	body-axis rolling, pitching, and yawing angular rates (fig. 1)
$\bar{q}$	dynamic pressure, $\frac{1}{2}\rho V^2$
$r_L$	yaw rate limit; constant used in stability augmentation logic (table III)

S	wing area
u, v, w	components of airplane resultant velocity along $X_b$ , $Y_b$ , and $Z_b$ body axes (fig. 1)
V	resultant velocity of airplane
$X_b, Y_b, Z_b$	body-axis reference system
$\alpha$	angle of attack
$\alpha_L$	constant used in stability augmentation logic (table III)
$\alpha_T$	trim angle of attack; used as a constant in stability augmentation logic (table III)
$\beta$	angle of sideslip
$\delta_a$	aileron deflection, positive when right aileron trailing edge is down (fig. 1)
$\delta_e$	elevator deflection, positive when trailing edge is down (fig. 1)
$\delta_r$	rudder deflection, positive when trailing edge is left (fig. 1)
$\theta, \phi, \psi$	Euler angles
$\rho$	air density
$\omega$	angular rate about center of mass, $\sqrt{p^2 + q^2 + r^2}$

Stability derivatives:

$$C_{l_p} = \frac{\partial C_l}{\partial \left( \frac{pb}{2V} \right)}$$

$$C_{n_{\delta_e}} = \frac{\partial C_n}{\partial \delta_e}$$

$$C_{l_r} = \frac{\partial C_l}{\partial \left( \frac{rb}{2V} \right)}$$

$$C_{n_{\delta_r}} = \frac{\partial C_n}{\partial \delta_r}$$

$$C_{l\delta_a} = \frac{\partial C_l}{\partial \delta_a}$$

$$C_{X\delta_e} = \frac{\partial C_X}{\partial \delta_e}$$

$$C_{l\delta_e} = \frac{\partial C_l}{\partial \delta_e}$$

$$C_{Yp} = \frac{\partial C_Y}{\partial \left(\frac{pb}{2V}\right)}$$

$$C_{l\delta_r} = \frac{\partial C_l}{\partial \delta_r}$$

$$C_{Yr} = \frac{\partial C_Y}{\partial \left(\frac{rb}{2V}\right)}$$

$$C_{m\delta_e} = \frac{\partial C_m}{\partial \delta_e}$$

$$C_{Y\delta_a} = \frac{\partial C_Y}{\partial \delta_a}$$

$$C_{mq} = \frac{\partial C_m}{\partial \left(\frac{q\bar{c}}{2V}\right)}$$

$$C_{Y\delta_e} = \frac{\partial C_Y}{\partial \delta_e}$$

$$C_{np} = \frac{\partial C_n}{\partial \left(\frac{pb}{2V}\right)}$$

$$C_{Y\delta_r} = \frac{\partial C_Y}{\partial \delta_r}$$

$$C_{nr} = \frac{\partial C_n}{\partial \left(\frac{rb}{2V}\right)}$$

$$C_{Z\delta_e} = \frac{\partial C_Z}{\partial \delta_e}$$

$$C_{n\delta_a} = \frac{\partial C_n}{\partial \delta_a}$$

A dot over a symbol indicates a derivative with respect to time.

## METHOD

The effectiveness of various spin recovery techniques was evaluated analytically for three different aircraft configurations. The nonlinear aircraft dynamics was represented by use of a six-degree-of-freedom model, and the associated equations of motion are given in the appendix.

## Description of Aircraft Configurations

Mass and dimensional characteristics for the assumed aircraft, herein referred to as configurations A, B, and C, are shown in table I. Also presented in table I are the control surface deflection limits for the respective aircraft. These data were taken from references 3 and 4. Configuration A represents a twin-jet swept-wing fighter, and configurations B and C represent two different delta-wing fighters.

Aerodynamic data for configuration A were taken from references 5 and 6. Two assumptions were made relating to these data. As shown in reference 5, the rudder and pitch-down elevator effectiveness at high angles of attack is so small in magnitude and so uncertain in sign that it is practically indistinguishable from zero. Therefore, for angles of attack greater than  $50^\circ$ , rudder effectiveness and pitch-down elevator effectiveness were assumed to be  $0^\circ$ . Aerodynamic data for configurations B and C were taken from references 4 and 7, respectively.

## Spin Characteristics of Aircraft Configurations

Previous analytical studies (for example, ref. 3) have shown that several spin modes may exist for aircraft of the type given in table I. These spin modes can be characterized as steep, intermediate, or flat, depending on their angle of attack (the higher the angle of attack, the flatter the spin). Spins may be either oscillatory or steady in nature. By a steady spin it is meant that the aircraft is in the poststall region of its flight envelope and is descending along a helical path, the axis of which is vertical. Also, the axis of the helix, its radius, and the descent speed are constant, atmospheric density variations with altitude and other external disturbances being neglected. Under these conditions the linear and angular velocities are constant in the body set of axes ( $\dot{\alpha} = \dot{\beta} = \dot{V} = \dot{p} = \dot{q} = \dot{r} = 0$ ).

Since recoveries from flat, steady spins are generally the most difficult to achieve, only the flat mode is considered in the analysis. Those spin modes considered are summarized in table II. As shown therein, two flat spins exist for configuration A, whereas only one flat spin exists for configurations B and C. The spin characteristics given are either taken from reference 3 or are calculated by using the analytic spin prediction method of reference 3. As also shown in table II, only erect spins (positive angle of attack) are considered.

## Recovery Techniques

Three separate recovery techniques which utilize conventional aerodynamic controls were analyzed. These techniques are referred to as (1) constant control recoveries, (2) optimal recoveries, and (3) suboptimal recoveries. A brief description of these recovery techniques is given in this section.

The assumed aircraft configurations of table I are all loaded heavily along the fuselage ( $I_x - I_y$  negative). The generally recommended recovery technique (ref. 2) for this type of aircraft is to apply aileron with the spin (bank right in a right spin), rudder against the spin, and pitch-up elevator. The constant control recovery mode of the current analysis is similar to the generally recommended recovery procedure and was included in order to establish a standard for comparison of other recovery methods. Recoveries were initiated by using full aileron with the spin and full rudder against the spin. Elevator position was determined by computing recoveries at various elevator settings and then using that setting which gave the best recovery performance with regard to recovery turns, altitude loss, and time. These control deflections were maintained until angle of attack and spin rate were reduced sufficiently to return the vehicle to a controllable condition. A stability augmentation or rate-damper control logic was then used to damp out oscillations in pitch, roll, and yaw and to trim the aircraft at a desired angle of attack below the stall region. The augmentation logic used in the analysis is outlined in table III. The significance of various parameters in the table is discussed in later sections of the report.

The second recovery technique involved an application of optimal control theory. A computer program of three-degree-of-freedom trajectory optimization (ref. 8) was modified to include a full six-degree-of-freedom capability. The computer program was used to determine optimal aileron, rudder, and elevator control histories for effecting recovery. The program utilized an iterative, gradient approach to determine optimal control histories so that some functions of the state variables were minimized while constraints on other state and control variables were met.

The third recovery technique employed a suboptimal control logic in conjunction with the stability augmentation system of table III. During the high angle-of-attack phase of a recovery, control histories were patterned after optimal recovery results, whereas, during the terminal phase of a recovery, the damper logic of table III was used.

It should be noticed that the stability augmentation logic of table III was used only for constant control and suboptimal recoveries. For optimal recoveries, the optimization technique determined all the control histories. Also, as shown in table III, the augmentation system provided damping only about one aircraft axis at a time with no control cross connects; that is, the elevator was used only for pitch damping, the aileron for roll damping, and the rudder for yaw damping.

## RESULTS

Recovery performance from the equilibrium spin conditions of table II is evaluated for the three aircraft configurations of table I. The results obtained by using a constant

control, optimal, and suboptimal recovery logic are compared with respect to recovery time, turns, and altitude loss.

### Configuration A

Constant control recovery.- A constant control recovery time history for configuration A is shown in figure 2. The initial conditions for this recovery are the same as those of run 1, table II. As previously outlined, constant control recoveries are made by using those combinations of constant control deflections which give the best recovery performance. These control settings are maintained until the angle of attack and spin rate are reduced sufficiently for a stability augmentation system to damp out pitch, roll, and yaw rates and trim the aircraft at a desired angle of attack. (Recovery turns, time, and altitude loss refer to the values attained at this point and not that required to regain straight and level flight.)

For the recovery data of figure 2, the best results were obtained by using full rudder opposing the spin, full aileron with the spin, and a pitch-up elevator of  $-10^\circ$ . As shown in figure 2, these control settings were maintained for about 18 sec, at which time the stability augmentation or rate-damper logic of table III was activated.

The augmentation logic is activated for  $\alpha < \alpha_L$ . The analysis showed that values for  $\alpha_L$  between  $40^\circ$  and  $50^\circ$  were adequate for all configurations of the study. Although this parameter is not overly critical, it must be low enough for the controls to be effective. A value of  $\alpha_L = 50^\circ$  was used for all configurations since this is the angle of attack at which the rudder generally becomes effective.

As is shown in figure 2, for  $\alpha > \alpha_L$ ,  $\delta_r = 0^\circ$ . This occurs since the rudder was assumed to be ineffective for  $\alpha > 50^\circ$  and was set at zero for computational purposes. In actual practice, the rudder could have been maintained at  $\delta_r = 30^\circ$  until the yaw rate was reduced to zero.

As is shown in table III, two additional parameters,  $r_L$  and  $\alpha_T$ , are required to define the stability augmentation system. The yaw-rate limit  $r_L$  was used to insure that the aircraft was out of a spin before an attempt was made to trim the vehicle at a desired angle of attack  $\alpha_T$ . As was the case with  $\alpha_L$ , recovery performance was not found to be overly sensitive to the assumed value for  $r_L$ , so that values of approximately 0.2 to 0.6 rad/sec were adequate for all aircraft configurations considered. The value chosen for  $r_L$  was 0.4 rad/sec for all configurations except configuration B. Better recovery performance was achieved with configuration B by removing the  $r_L$  requirement; that is, for  $\alpha < 50^\circ$ , the pitch damper was not used and the trim angle of attack was commanded.

In figure 2 the pitch-down elevator deflection at approximately 25 sec is a result of the switching of the damper logic to the trim angle-of-attack phase. The desired trim

angle of attack  $\alpha_T$  was set at  $10^\circ$  in the current study, as this value is well below the stall for all aircraft considered. This angle of attack appears to be a reasonable point from which to initiate the final pullout following recovery since it resulted in a low-g pullup maneuver for all assumed configurations (0.5g to 1.5g). In actual practice, a trim angle of attack near the maximum lift condition might be desirable because it would yield a more rapid pullup.

It should be emphasized that the previously described recovery control logic and damper logic were not "tuned" to each configuration, since the objective was to investigate basic principles of recovery control which were not sensitive to configurations. In any real control system design, variables such as  $\alpha_L$ ,  $\alpha_T$ , and  $r_L$  would probably be adjusted for each configuration. In addition, the damper logic for a practical system would have to include realistic limits on control surface deflection rates. The near "bang-bang" switches in the control histories shown in figure 2 were the result of high gains used in the damper logic. In addition, control surfaces were not rate-limited. This idealized damper system was used to avoid having to design a damper system for each configuration and to allow the same damper logic to be used for all configurations.

Optimal recovery. - A six-degree-of-freedom optimization computer program was employed to investigate spin recovery. The objective was to obtain optimal control motions for rapid recovery from a spin and thereby to improve on the constant control results of figure 2. The computer program had a capability of iteratively determining optimal control histories so that some function of the state variables was minimized. Moreover, this minimization was achieved while constraints on specified state and control variables were met.

In the current application, the same spin conditions as those in figure 2 were used to initiate the recovery. Control variables were taken to be  $\delta_e$ ,  $\delta_a$ , and  $\delta_r$ . Constraints were placed on these so that they could not exceed the limits given in table I. In addition, pitch-down elevator effectiveness and rudder effectiveness were set at zero for  $\alpha > 50^\circ$ . Terminal constraints were placed on  $\alpha$ ,  $\psi$ ,  $p$ ,  $q$ , and  $r$  and were as follows:  $\alpha = 10^\circ$ ,  $\psi = 0$ ,  $p = q = r = 0$ . Thus, the aircraft was required to meet the same terminal conditions as in the recovery data of figure 2. Time was chosen as the performance index since, by minimizing time, recovery turns and altitude loss are also minimized.

Optimization programs of this type require a nominal control history to initiate the iterative process. This nominal history is used to compute an initial time history after which the program adjusts the control variables on succeeding iterations in an attempt to obtain the optimal control motions. The nominal control history was one for which all control deflections were maintained at zero throughout the initial computation.

One additional quantity, a cut-off or stopping variable, is required by the optimization program. The cut-off variable is normally taken to be one of the state variables such as angle of attack, yaw rate, or time. However, in the present application none of the state variables can be used to terminate an iterative cycle. Time cannot be used since it is the minimization function. Likewise, other variables such as  $\alpha$  and  $r$  cannot be used since there is no assurance that they will reach the desired value ( $\alpha = 10^\circ$ ,  $r = 0$ ) during an iterative cycle. Hence, a "dummy" cut-off variable was adopted to terminate each cycle. This dummy variable corresponds to time on the nominal computation, but on succeeding iterations the program adjusts the cut-off variable to achieve optimal performance. A cut-off value of 16 sec was used for the nominal calculation.

After multiple iterative cycles, the final recovery results shown in figure 3 were obtained. As is shown, an impressive improvement in recovery time, altitude loss, and turns is achieved in that the constant control results were all reduced by greater than 50 percent.

Consider the optimal control histories shown in figure 3. The normal recovery mode of the aileron with the spin is used throughout the recovery. The rudder is used against the spin as soon as it becomes effective ( $\alpha < 50^\circ$ ). The elevator history of figure 3 provides the most interesting study. Superimposed on the elevator history is the pitch rate  $\dot{\theta}$  of the aircraft. During the initial part of the recovery the elevator is switched in phase with pitch rate. (It should be emphasized that the switching is based on  $\dot{\theta}$  and not  $q$ .) That is, for negative  $\dot{\theta}$ , pitch-down elevator (normally  $\delta_e$ , but in this case,  $\delta_e = 0^\circ$ ) was used, while for positive  $\dot{\theta}$ , pitch-up elevator was used. This procedure quickly excited an unstable oscillation in angle of attack which effects a rapid recovery as compared with the constant control technique of figure 2. (It is of interest to note that using the elevator to "rock" out of a spin was proposed in 1931 in ref. 9.) The fact that maximum rudder was not used when effective ( $\alpha < 50^\circ$ ) results from the slow terminal convergence characteristics of the optimization procedure. Had the iterative procedure been continued to obtain a closer minimum, maximum rudder deflection would probably have been employed, but little change in overall recovery performance would have resulted.

In addition to the results of figure 3, optimal recoveries were computed for other aircraft configurations with various initial spin conditions. The control history results were in general agreement with those of figure 3; that is, the  $\dot{\theta}$  and  $\delta_e$  histories were similar to those of figure 3.

Based on these optimal results, a suboptimal recovery logic was devised as follows:

For  $\alpha > \alpha_L$ : Rudder against spin  
Aileron with spin  
Elevator: Pitch up for  $\dot{\theta} > 0$   
Pitch down for  $\dot{\theta} \leq 0$

For  $\alpha \leq \alpha_L$ , the stability augmentation logic of table III was used. Typical results obtained using this suboptimal recovery logic, referred to as pitch excitation, are given below.

Pitch-excitation recovery.- A recovery using the suboptimal, pitch-excitation logic is shown in figure 4. Initial conditions for this case are the same spin conditions as were used on the constant control and optimal recoveries of figures 2 and 3. A comparison of figures 3 and 4 shows that the pitch-excitation recovery results compare favorably with those obtained optimally with only slight increases occurring in time, turns, and altitude loss. Thus, for the recoveries of figures 2 and 4, the pitch-excitation mode results in about a 50-percent reduction in turns, altitude loss, and time over that achieved using a constant control procedure.

### Configuration B

Configuration B provides an interesting application of the pitch-excitation procedure. As was concluded in reference 4 and verified in the current study, constant control recovery from the spin conditions of table II is impossible without exceeding the control surface deflection limits of the aircraft.

Constant control recovery.- An attempted constant control recovery history for configuration B is given in figure 5. Initial conditions for this case are those given in table II. Although no recovery was achieved, the control settings used yielded the greatest possible reduction in yaw rate and angle of attack. As shown in figure 5, the attempted recovery caused some reduction in yaw rate and forced the aircraft into a somewhat steeper spin. Extending the history of figure 5 to about 50 sec caused the aircraft to damp into another stable spin condition at an angle of attack and yaw rate of about  $71^\circ$  and 1.35 rad/sec, respectively.

Pitch-excitation recovery.- A recovery from the same equilibrium spin conditions as in figure 5 was attempted by use of the pitch-excitation technique. The results appear in figure 6. As is shown, recovery was achieved after 24 sec and 5 turns. Thus, although no constant control recovery was possible for configuration B, pitch excitation effectively produced an unstable oscillation in angle of attack and returned the aircraft to a nonspinning condition.

The recovery histories previously shown for configurations A and B are typical of those for other configurations and spin conditions investigated. Therefore, no further detailed time histories are included in the report. Results for other spin conditions and aircraft are, however, briefly discussed.

## Summary of Spin Recovery Results

Previously given recovery results for configurations A and B (figs. 2 to 6) are summarized in table IV. Additional recovery results are also provided for configurations A and C.

A comparison is given in table IV between the constant control and pitch-excitation techniques for recovery from the run 2 spin condition of table II for configuration A. Although improvement in recovery performance from this flat spin condition using pitch excitation is not as great as for the intermediate spin, a significant reduction in turns, altitude loss, and time is seen to exist.

An analysis of table II reveals that the flat spin characteristics for configuration C (run 4) are not as severe as for the other configurations; that is, the spin rate and angle of attack for the flat spin are consistently lower than those obtained with other models. These less severe spin characteristics for aircraft C are reflected in the recovery results given in table IV. Recovery is quickly achieved for both techniques, the pitch-excitation mode yielding some improvement over the constant control approach.

## Application of Spin Recovery Results

The constant control, optimal, and pitch-excitation recovery histories presented should not be considered as representing typical or recommended piloting techniques. Although the constant control logic resembles recommended pilot procedures for initiating recovery from a spin during the early constant control portion, the subsequent switching in the damper logic is a highly idealized representation of piloting technique. Moreover, the pitch-excitation mode may not be a feasible pilot input even in the initial phase, since the large coupled oscillations would tend to disorient the pilot and make it difficult for him to respond accurately to pitch rate. Therefore, the suboptimal control logic should be viewed as indicating a possible approach to the design of an automatic aid to the pilot in recovering from a spin.

In a practical system, it seems reasonable to assume that the pilot's role would be to perform the final pullout after yaw rate had decreased sufficiently. However, precise definition of the pilot's role and the form of a practical automatic-control system based on the pitch-excitation logic would require detailed piloted simulation studies and flight tests. Remotely piloted vehicles would seem to be ideally suited for initial studies of this type.

## CONCLUDING REMARKS

Results have been presented from an analytical study of procedures for effecting recovery from steady spin conditions for three assumed aircraft configurations. The

performance of three recovery procedures which utilize conventional aerodynamic controls were analyzed with respect to altitude loss, turns, and time required to achieve recovery. Recovery procedures included a constant mode, an optimal mode, and a suboptimal pitch-excitation logic patterned after optimal recovery results. Recovery performance using the suboptimal technique was shown to approach that obtained optimally and to yield a significant improvement over that achieved with the constant control approach.

Langley Research Center,  
National Aeronautics and Space Administration,  
Hampton, Va., June 27, 1974.

## APPENDIX

### EQUATIONS OF MOTIONS

The six-degree-of-freedom nonlinear equations of motion used in the analysis are listed below. The aerodynamic coefficients are written in a form consistent with the data available for the most extensively tested model.

Normal force:

$$\begin{aligned} \dot{\alpha} = & q + (\tan \alpha \tan \beta) \dot{\beta} - \left( \frac{\tan \alpha}{V} \right) \dot{V} + \frac{\bar{q}S}{mV} \frac{C_Z(\alpha, \beta) + C_{Z\delta_e}(\alpha, \beta) \delta_e}{\cos \alpha \cos \beta} \\ & + \frac{g}{V} \left( \frac{\cos \theta \cos \phi}{\cos \alpha \cos \beta} \right) - p \left( \frac{\tan \beta}{\cos \alpha} \right) \end{aligned}$$

Lateral force:

$$\begin{aligned} \dot{\beta} = & - \left( \frac{\tan \beta}{V} \right) \dot{V} + \frac{\bar{q}S}{mV \cos \beta} \left\{ C_Y(\alpha, \beta) + C_{Y\delta_e}(\alpha, \beta) \delta_e + C_{Y\delta_a}(\alpha, \beta) \delta_a \right. \\ & \left. + C_{Y\delta_r}(\alpha, \beta) \delta_r + \frac{b}{2V} [C_{Y_p}(\alpha) p + C_{Y_r}(\alpha) r] \right\} + \frac{g}{V} \left( \frac{\cos \theta \sin \phi}{\cos \beta} \right) \\ & + p \sin \alpha - r \cos \alpha \end{aligned}$$

Longitudinal force:

$$\begin{aligned} \dot{V} = & (V \tan \alpha) \dot{\alpha} + (V \tan \beta) \dot{\beta} + \frac{\bar{q}S}{m} \frac{C_X(\alpha, \beta) + C_{X\delta_e}(\alpha, \beta) \delta_e}{\cos \alpha \cos \beta} \\ & - \frac{g \sin \theta}{\cos \alpha \cos \beta} - qV \tan \alpha + rV \frac{\tan \beta}{\cos \alpha} \end{aligned}$$

Pitching moment:

$$\dot{q} = \left( \frac{I_Z - I_X}{I_Y} \right) pr + \frac{I_{XZ}}{I_Y} (r^2 - p^2) + \frac{\bar{q}S\bar{c}}{I_Y} \left[ C_m(\alpha, \beta) + C_{m\delta_e}(\alpha, \beta) \delta_e + \frac{\bar{c}}{2V} C_{mq}(\alpha) q \right]$$

Rolling moment:

$$\begin{aligned} \dot{p} = & \left( \frac{I_{XZ}}{I_X} \right) \dot{r} + \left( \frac{I_Y - I_Z}{I_X} \right) qr + \left( \frac{I_{XZ}}{I_X} \right) pq + \frac{\bar{q}Sb}{I_X} \left\{ C_l(\alpha, \beta) + C_{l\delta_e}(\alpha, \beta) \delta_e + C_{l\delta_a}(\alpha, \beta) \delta_a \right. \\ & \left. + C_{l\delta_r}(\alpha, \beta) \delta_r + \frac{b}{2V} [C_{l_p}(\alpha) p + C_{l_r}(\alpha) r] \right\} \end{aligned}$$

APPENDIX - Concluded

Yawing moment:

$$\dot{r} = \left( \frac{I_{XZ}}{I_Z} \right) \dot{p} - \left( \frac{I_{XZ}}{I_Z} \right) q r - \left( \frac{I_Y - I_X}{I_Z} \right) p q + \frac{\bar{q} S b}{I_Z} \left\{ C_n(\alpha, \beta) + C_{n_{\delta_e}}(\alpha, \beta) \delta_e + C_{n_{\delta_a}}(\alpha, \beta) \delta_a + C_{n_{\delta_r}}(\alpha, \beta) \delta_r + \frac{b}{2V} [C_{n_p}(\alpha) p + C_{n_r}(\alpha) r] \right\}$$

In addition, the following formulas were used:

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi)$$

$$\dot{\psi} = \frac{q \sin \phi + r \cos \phi}{\cos \theta}$$

$$\text{Turns} = \frac{1}{2\pi} \int \dot{\psi} dt$$

$$u = V \cos \alpha \cos \beta$$

$$v = V \sin \beta$$

$$w = V \sin \alpha \cos \beta$$

$$\dot{h} = u \sin \theta - v \sin \phi \cos \theta - w \cos \phi \cos \theta$$

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TABLE I.- WEIGHT, INERTIA, AND GEOMETRIC CHARACTERISTICS

Configuration	Weight, N	I <sub>x</sub> , 2 kg-m <sup>2</sup>	I <sub>y</sub> , 2 kg-m <sup>2</sup>	I <sub>z</sub> , 2 kg-m <sup>2</sup>	I <sub>xz</sub> , 2 kg-m <sup>2</sup>	S <sub>y</sub> , m <sup>2</sup>	b, m	c̄, m	Center of gravity, percent c̄	Control deflection limits		
										δ <sub>e</sub> , deg	δ <sub>r</sub> , deg	δ <sub>a</sub> , deg
A	160 968	35 398	157 576	178 460	0	50.01	11.71	4.89	33.3	-21 to 9	±30	±30
B	76 568	5 911	57 785	62 937	0	34.0	8.23	5.27	25	-20 to 6	±12	±4δ <sub>e</sub> ≠ 0
C	110 365	18 438	173 539	187 096	5844	64.57	11.62	7.24	30	-25 to 10	±25	±8δ <sub>e</sub> = 0 ±7.5

TABLE II.- SPIN CHARACTERISTICS OF AIRCRAFT

Configuration	Run	α, deg	β, deg	θ, deg	φ, deg	V, m/sec	ω, rev/sec	h, m
A	1	73.106	0.495	-16.928	1.252	122.577	0.365	12 192
A	2	82.210	.153	-7.941	.355	118.304	.584	12 192
B	3	76.247	4.465	-13.963	3.513	86.992	.271	9 144
C	4	73.210	-2.383	-16.810	-.365	78.550	.188	9 144

TABLE III.- STABILITY AUGMENTATION LOGIC

$$\alpha \leq \alpha_L: \delta_r = 1000r$$

$$\delta_a = 1000p$$

$$r > r_L: \delta_e = 1000q$$

$$r \leq r_L: \delta_e = 5(\alpha - \alpha_T) + 100\dot{\alpha}$$

TABLE IV.- SUMMARY OF SPIN RECOVERY RESULTS

Configuration	Run	Recovery technique	Turns	Altitude loss, m	Time, sec
A	1	Constant controls	8.3	3250	28
		Pitch excitation	4.0	1750	14
		Optimal	3.8	1550	13
A	2	Constant controls	12.9	4000	36
		Pitch excitation	7.3	2220	19
B	3	Constant controls	---	No recovery	---
		Pitch excitation	5.0	2050	24
C	4	Constant controls	3.3	1800	22
		Pitch excitation	2.6	1400	17

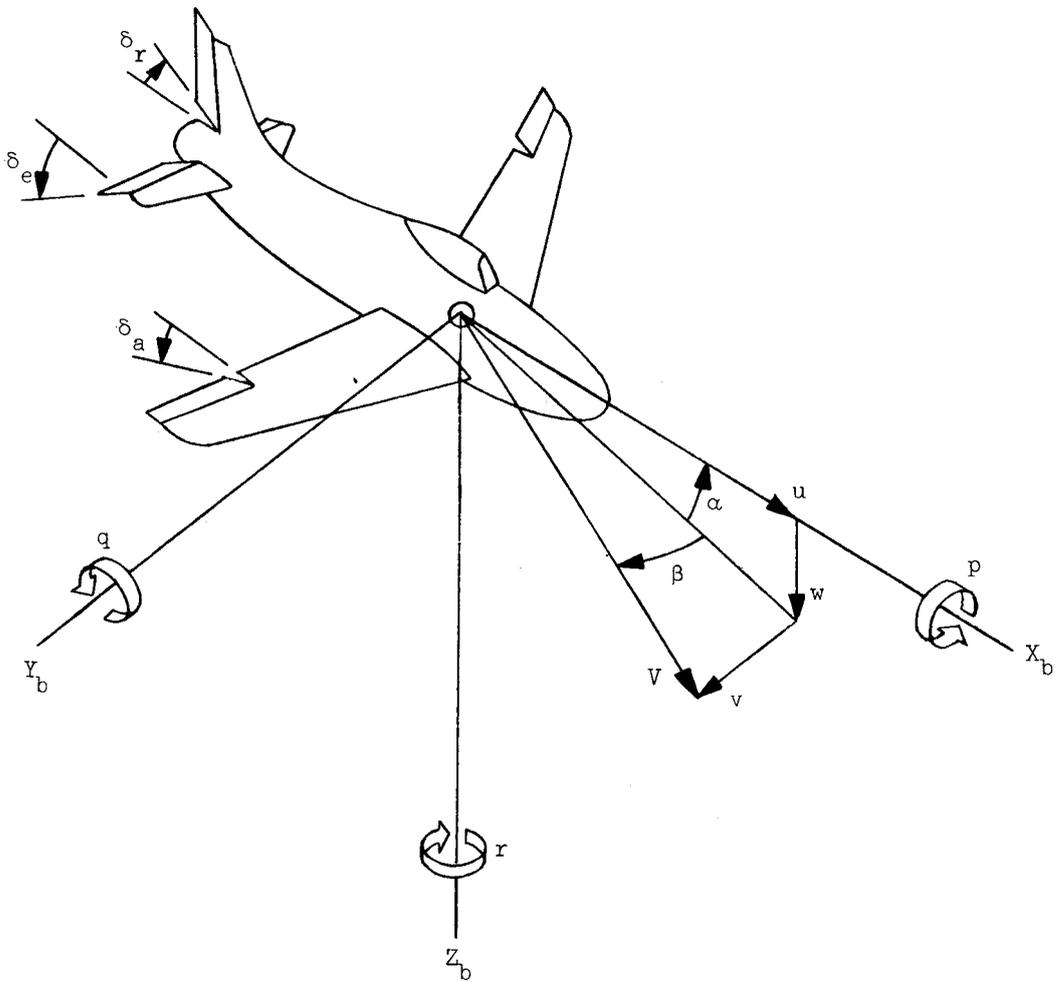


Figure 1.- Body system of axes. Arrows indicate positive direction of quantities.

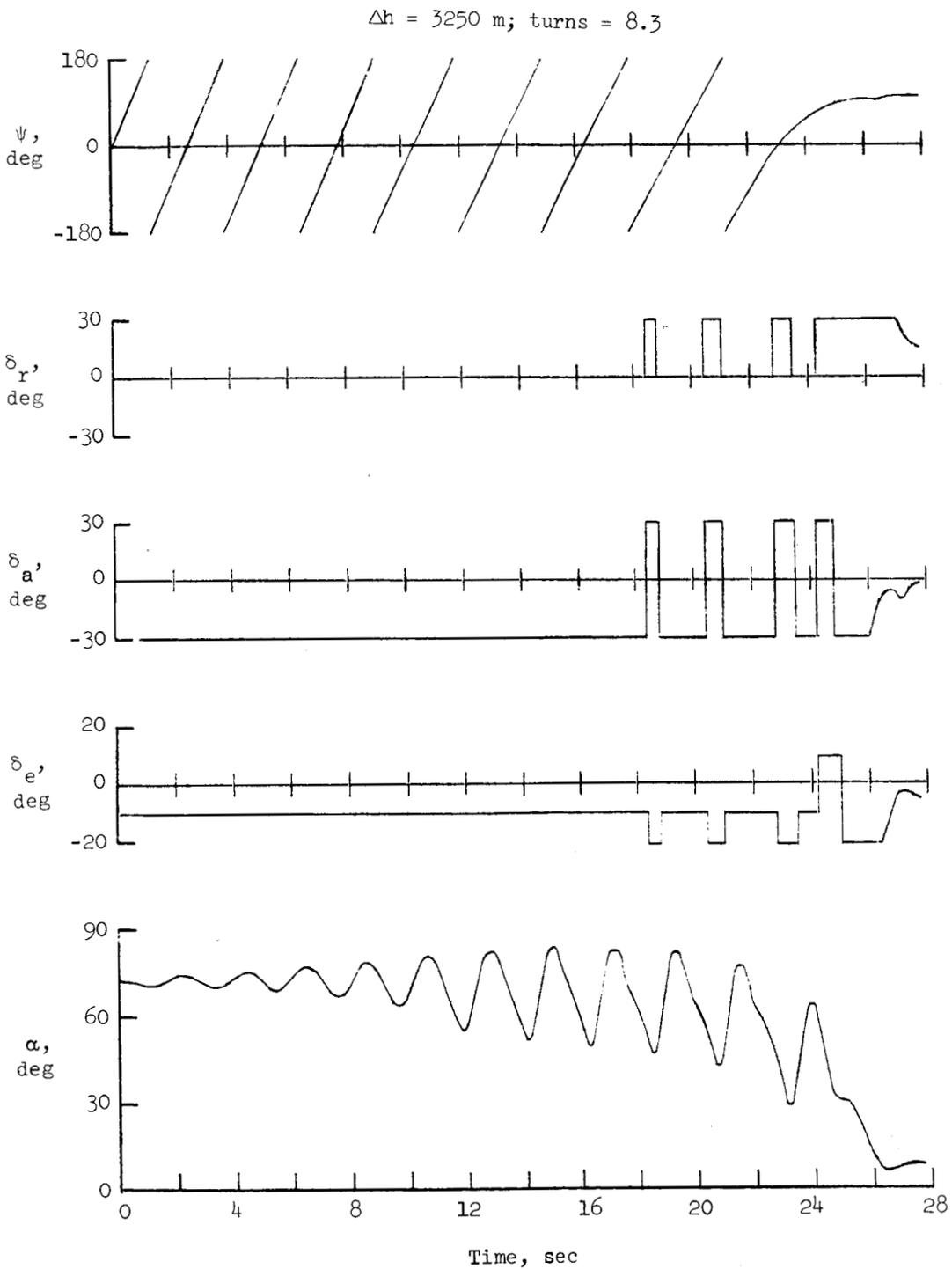


Figure 2.- Constant control spin recovery for configuration A.  
 (Initial conditions: run 1, table II.)

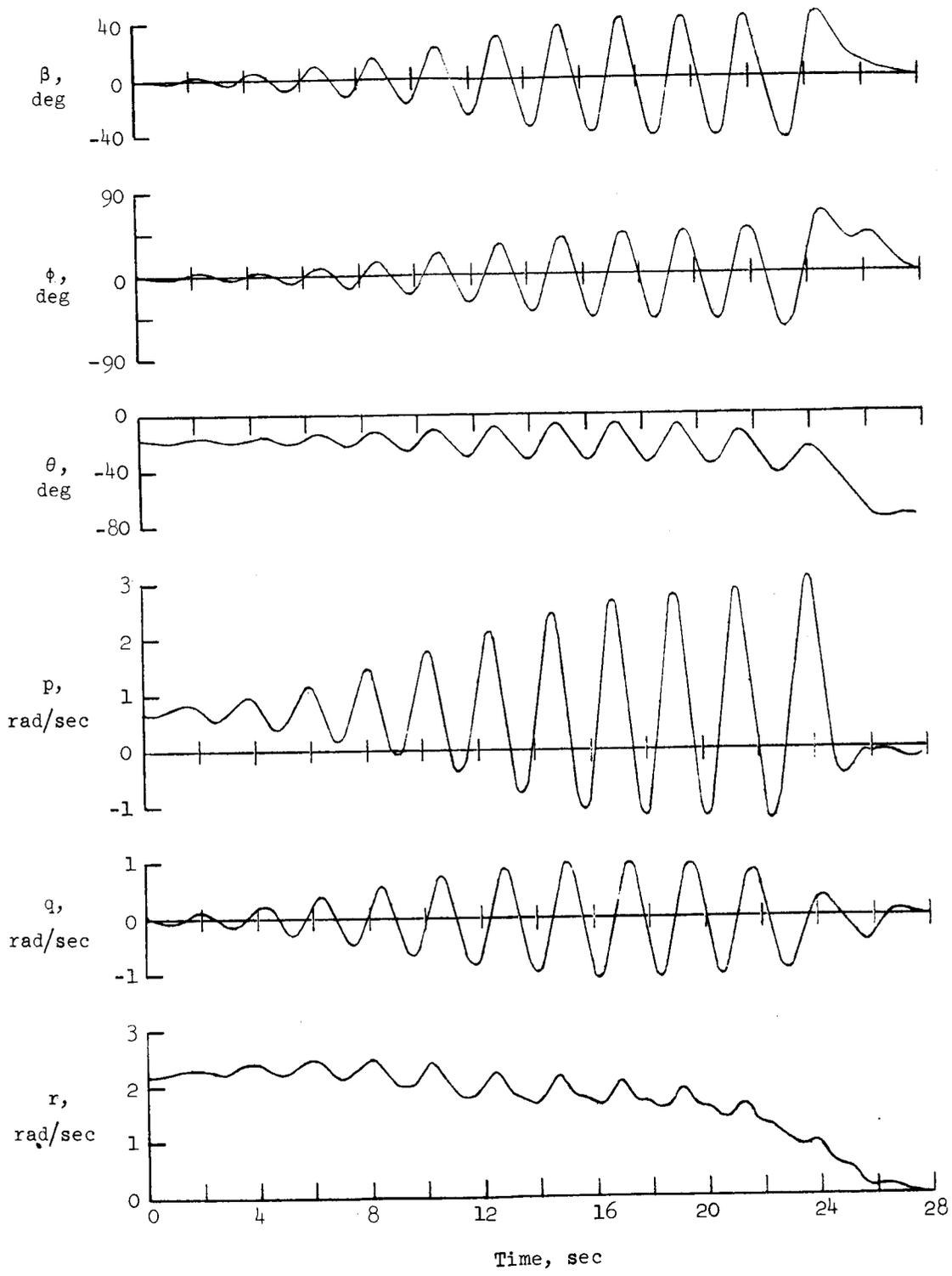
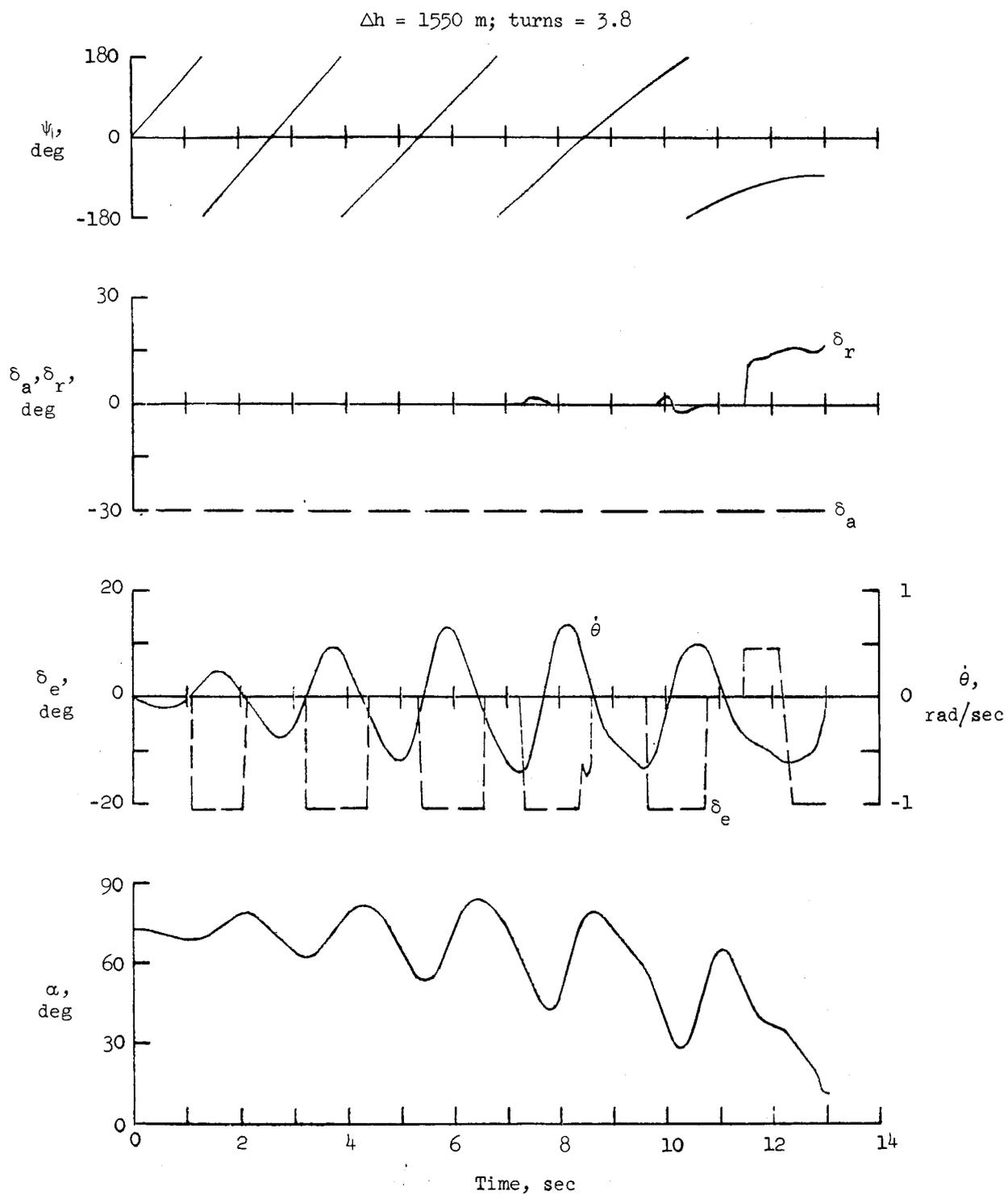


Figure 2.- Concluded.



**Figure 3.- Optimal spin recovery for configuration A.  
(Initial conditions: run 1, table II.)**

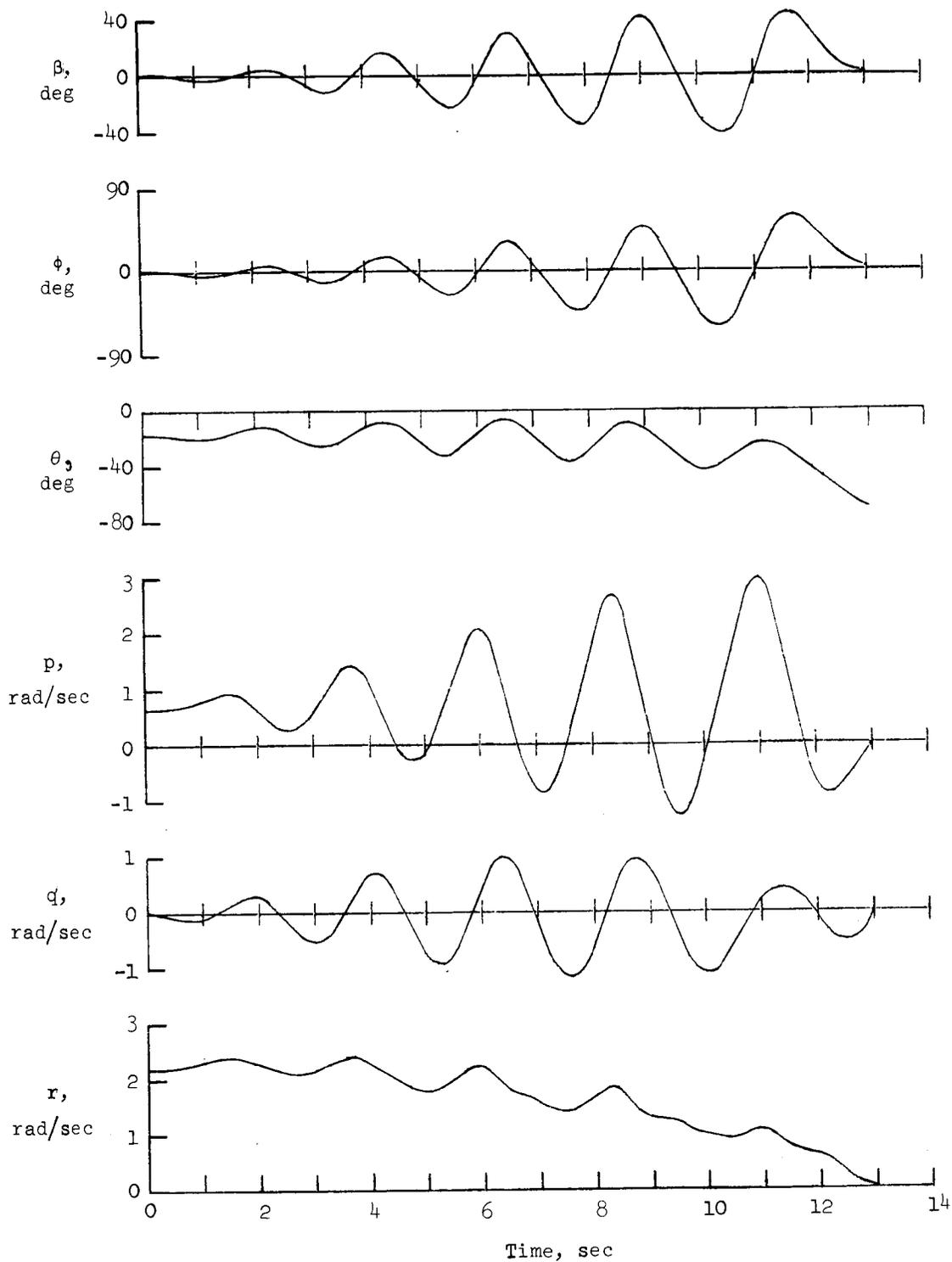
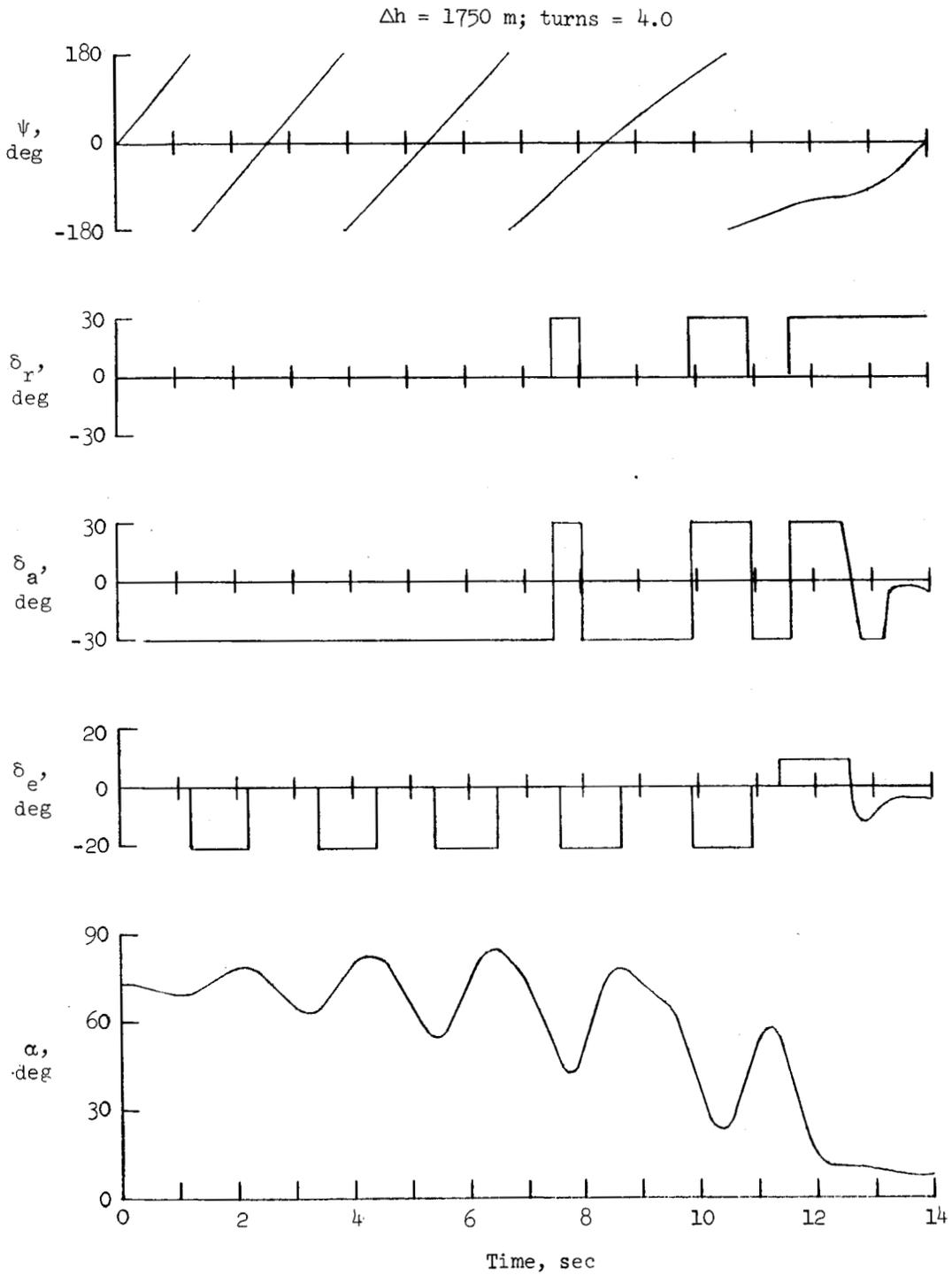


Figure 3.- Concluded.



**Figure 4.- Pitch-excitation spin recovery for configuration A.  
(Initial conditions: run 1, table II.)**

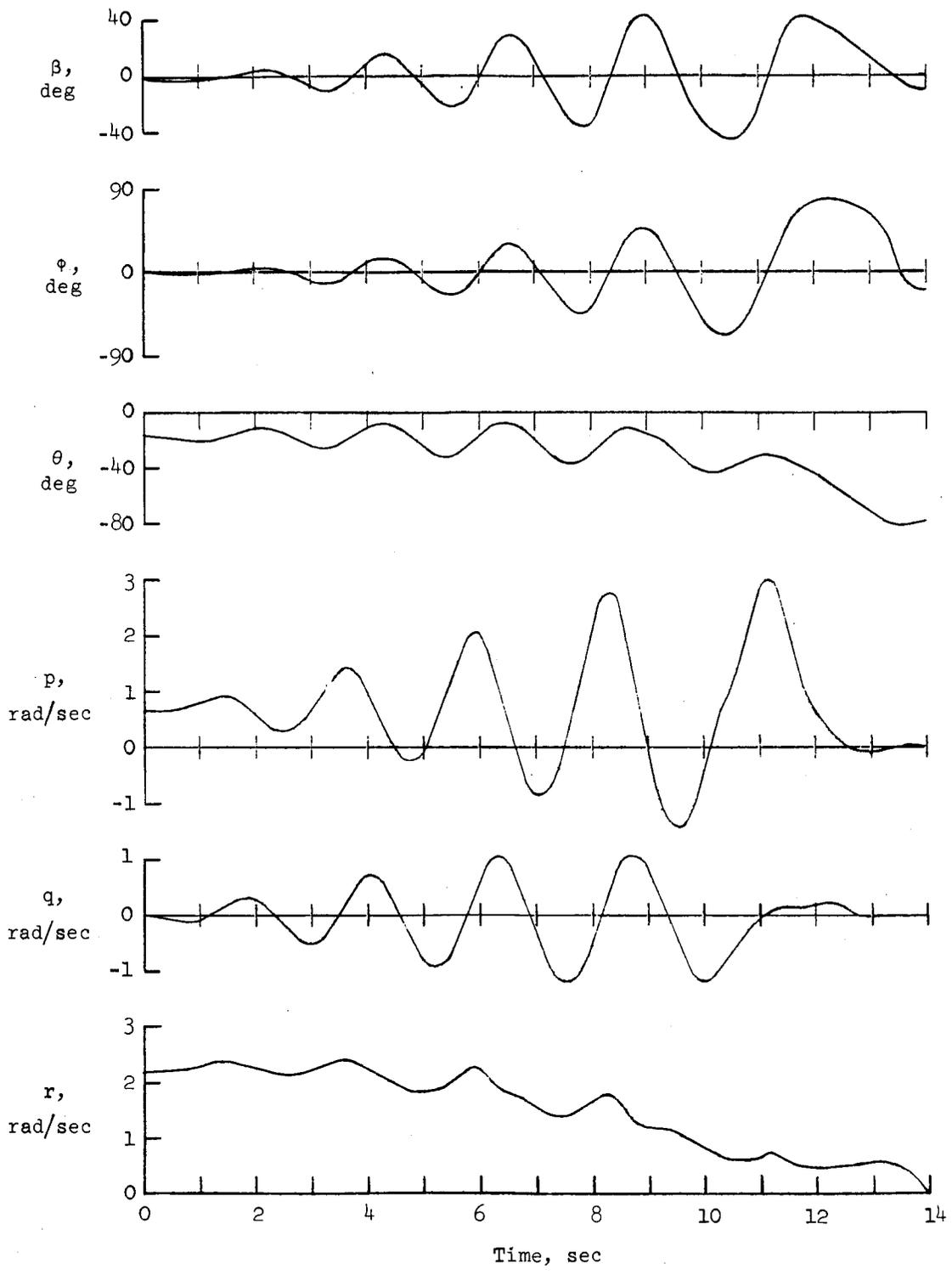


Figure 4.- Concluded.

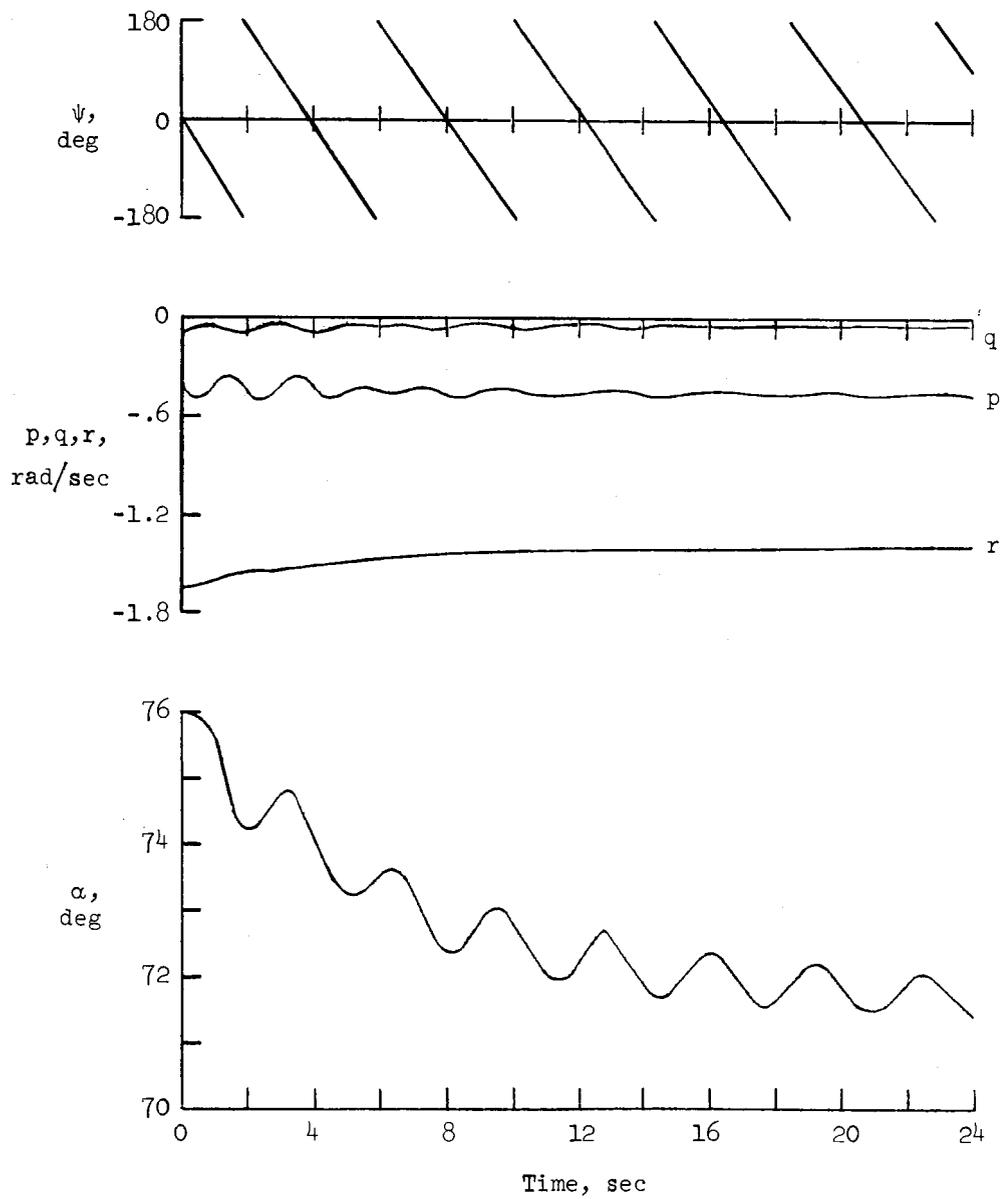


Figure 5.- Attempted constant control recovery for configuration B  
 $(\delta_e = 0^\circ; \delta_a = 8^\circ; \delta_r = -12^\circ)$ . (Initial conditions: run 3, table II.)

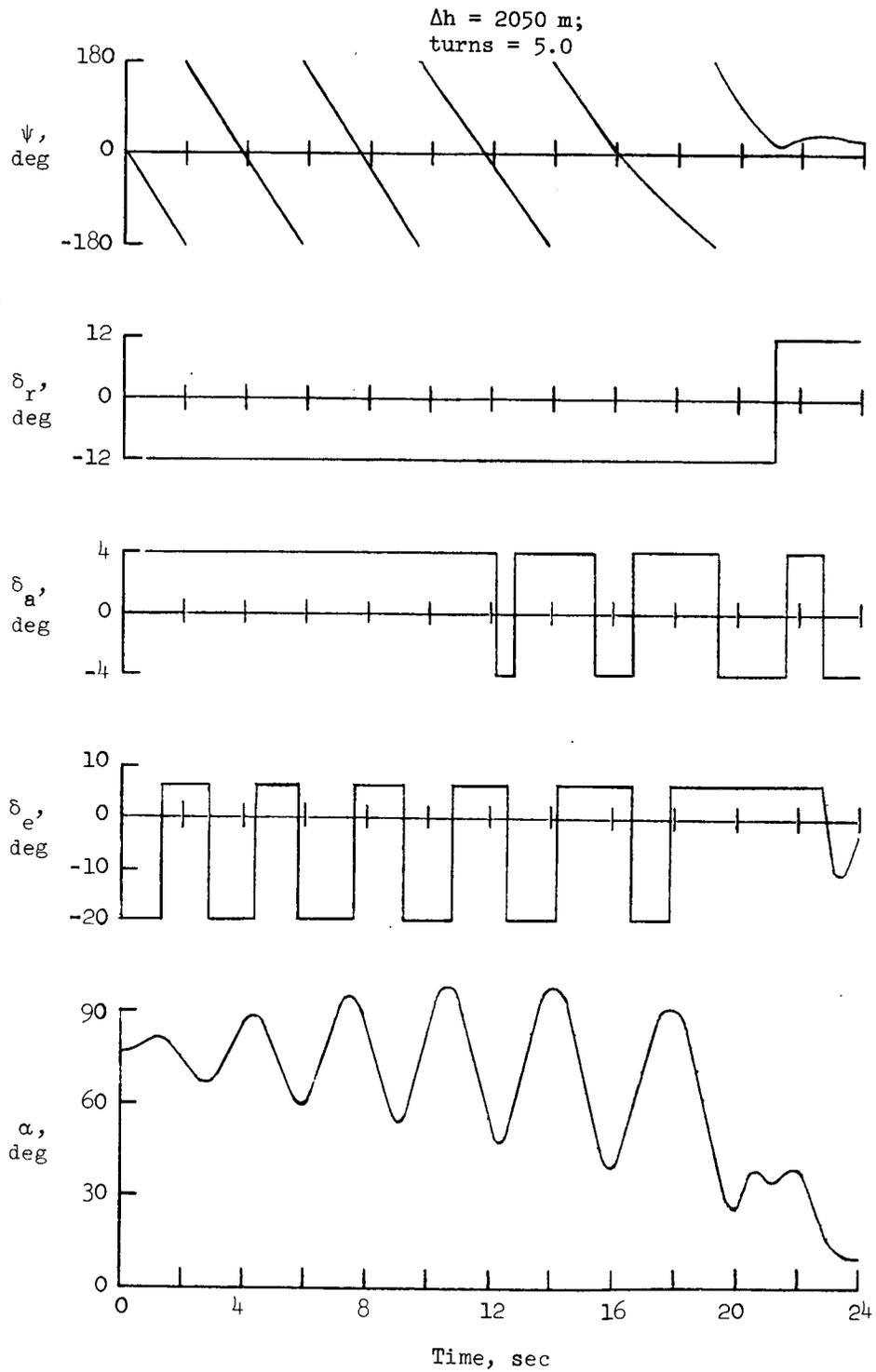


Figure 6.- Pitch-excitation spin recovery for configuration B.  
(Initial conditions: run 3, table II.)

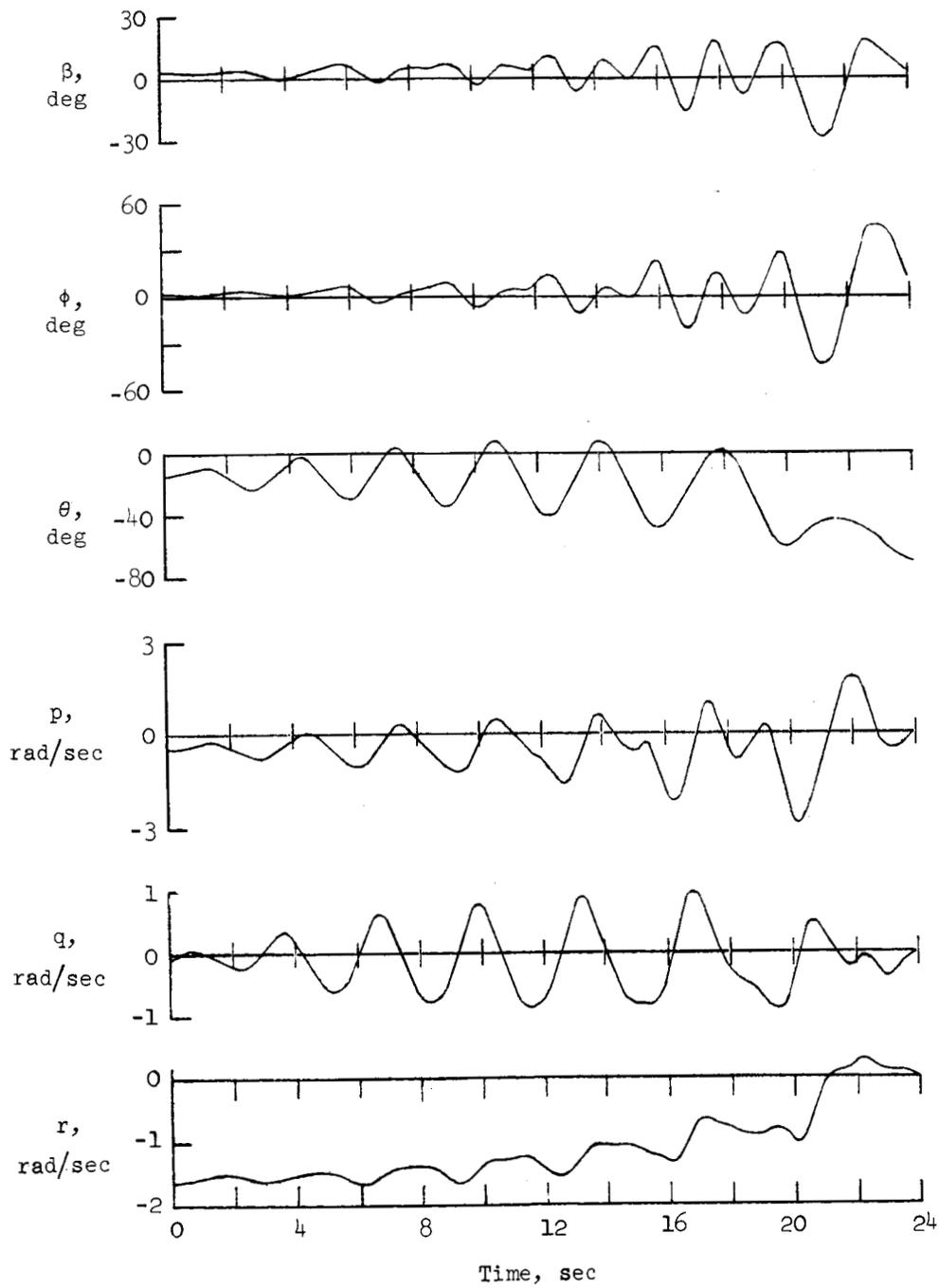


Figure 6.- Concluded.