COMPUTER EXPERIMENTS ON PERIODIC SYSTEMS IDENTIFICATION USING ROTOR BLADE TRANSIENT FLAPPING-TORSION RESPONSES AT HIGH ADVANCE RATIO

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Abstract

Systems identification methods have recently been applied to rotorcraft to estimate stability derivatives from transient flight control response data. While these applications assumed a linear constant coefficient representation of the rotorcraft, the computer experiments described in this paper used transient responses in flap-bending and torsion of a rotor blade at high advance ratio which is a rapidly time varying periodic system. It was found that a simple system identification method applying a linear sequential estimator also called equation of motion estimator, is suitable for this periodic system and can be used directly if only the acceleration data are noise polluted. In the case of noise being present also in the state variable data the direct application of the estimator gave poor results, however after prefiltering the data with a digital Graham filter having a cut-off frequency above the natural blade torsion frequency, the linear sequential estimator successfully recovered the parameters of the periodic coefficient analytical model.

Notation+

B	Blade tip loss factor
$F = (I_1/16I_f)(c/R)^2$	First blade torsional inertia number
F(x,t)	State matrix
G(t)	Process noise modulating matrix
$H(\omega)$	Fourier transform of weighting function
H(x,t)	Measurement matrix
$H(\xi,a)$	State matrix = measurement matrix
I_1	Blade flapping moment of inertia.
I_f	Blade feathering moment of inertia.
J	Quadratic cost function.
P(t) or P	Covariance matrix of conditional state vector probability distribution given measurements.
P	Blade flapping natural frequency.
$Q= (I_1/4I_f)c/R$	Second blade torsional inertia number.
R	Measurement noise covariance matrix.
R	Blade radius
a,b,c	Unknown parameters to be estimated in flapping-torsion problem.
a	Parameter vector.
c	Blade chord.
f	Blade torsional natural frequency.
t	Non-dimensional time.
v	Measurement noise vector
$w(j\Delta t)$	Smoothing weights.
w	Process noise vector
x	State vector
z	Measurement vector.
β	Flapping angle.
γ	Blade Lock number.
δ	Blade torsion deflection
ζ	Acceleration vector.
η	Rate of displacement vector.
θ	Blade pitch angle.
λ	Rotor inflow ratio, constant over disk.
μ	Rotor advance ratio.
ξ	Displacement vector.

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+In order to retain the conventional symbols in helicopter aerodynamics (Reference 7) and in systems analysis (Reference 9) some symbols are used in two different meanings.

Notation
(cont')

σ	Standard deviation.
ω	Circular frequency.

Subscripts

o	Initial or mean value.
c,t	Beginning and end of filter cut-off frequencies.

Superscripts

$\dot{}$	Time differentiation.
$\bar{}$	Smoothed data after filtering.
$\hat{}$	Estimate
T	Matrix transpose

The question often arises, how to best select some parameters of a given analytical model of a dynamic system on the basis of transient responses to certain inputs either obtained analytically with a more complete math model or obtained experimentally. In rotorcraft flight dynamics one may want to use a linear constant coefficient math model and select the state matrix in an optimal way from the measured data obtained in a number of transient flight maneuvers. One also may have a more sophisticated non-linear analytical model of the rotorcraft. The problem then is how can the simpler linear math model be selected to best represent the responses of the more complete analytical model; or one may have the dynamic equations of a rotorcraft without the effects of dynamic inflow and one desires to modify some of the parameters in such a way that dynamic inflow effects are best approximated. It is known from theoretical studies, for example Reference 1, that a reduction in blade Lock number can approximately account for rotor inflow effects in steady conditions. The question then is whether changes in parameters can also account for inflow effects during transient conditions.

The idea of using transient response data to determine parameters of an analytical model is certainly not new. Recently, however, considerable interest in this area has been developed and a number of approaches have been studied which are unified under the title of "system identification". There is a considerable and rapidly growing literature in this field. System identification methods generally fall into two classes: (1) deterministic methods - usually some variation of the classical least squares technique and (2) probabilistic methods which determine the parameters as maximum likelihood estimates of random variables. Some methods can also be interpreted either on a deterministic or on a probabilistic basis. References 2 and 3 are typical of recent work using deterministic methods. Both of these studies illustrate the feasibility of determining coefficients in time invariant linear systems from transient response data. Reference 4 describes many of the probabilistic techniques. Reference 5 gives a detailed discussion of the various methods in their application to V/STOL aircraft and Reference 6 presents an identification method suitable for obtaining stability derivatives for a helicopter from flight test data in transient maneuvers. The studies of References 5 and 6 assume a linear constant coefficient representation of the system. A rotorcraft blade is, however, a dynamic system with rapidly changing periodic coefficients. It appeared, therefore, desirable to try out methods of system identification for a periodic dynamic system.

Selection of Identification Method

If one assumes that only the state variables have been measured but not the accelerations, one must use a non-linear estimator since the estimate of a system parameter and the estimate of a state variable appear as a product of two unknowns. A non-linear sequential estimator was tried on the simplest linear periodic system described by the Mathieu Equation. It was found that the non-linear estimating process diverged in most cases, unless the initial estimate and its standard deviation were selected within rather narrow limits. Reference 6 uses a sequential non-linear estimator but initializes the process by first applying a least square estimator, which needs in addition to the state variable measurements also measurements of the accelerations. In the case of the problem of Reference 6 the least square estimator yielded a rather good set of derivatives and the improvement from the much more involved non-linear estimation was not very pronounced. From this experience it would appear that one needs to apply the least square or an equivalent linear estimator any way and that in some cases it is doubtful whether or not the subsequent application of a non-linear estimator is worth the considerable effort.

After conducting the rather unsatisfactory computer experiments to identify a simple periodic system with the non-linear estimator, all subsequent work was done with a linear sequential estimator. This estimator is equivalent to least square estimation but has the advantage of being usable for "on-line" system identification. The inversion of large matrices is avoided and replaced by numerical integration of a number of ordinary differential equations. The computer experiments were conducted with the system equations of Reference 7 for the flapping - torsion dynamics of a rotorblade operating at advance ratio 1.6. Reference 7 assumes a straight blade elastically hinged at the rotor center and stipulates linear elastic blade twist. The system used here for the computer experiments represents only a relatively crude approximation, since at 1.6 advance ratio blade bending flexibility is of importance, see for example Reference 8. The coefficients in the system equations are non-analytic periodic functions which include the effects of reversed flow.

The identification algorithm used in this report is easily derived using the extended Kalman filter discussed in the next section. Although the algorithm does not provide for noise in the state variables, one can nevertheless use it also for noisy data if one interprets the estimate, which normally is a deterministic variable, as a sample of a random variable. The effects on system identification of computer generated noise in both the acceleration data and in the state variable data were studied. However, no errors in modeling were introduced since their effects can only be evaluated on a case by case basis.

Extended Kalman Filter

The extended Kalman filter is an algorithm for obtaining an estimate $\hat{x}$ of a state vector x satisfying

$$\dot{x} = F(x,t) + G(t)w \qquad \text{Process Equation (1)}$$

given noisy measurements z related to x via

$$z = H(x,t) + v \qquad \text{Measurement Equation (2)}$$

In the above equations w represents zero mean white Gaussian process noise with covariance matrix Q, v represents zero mean white Gaussian measurement noise with covariance matrix R. An optimum estimate $\hat{x}$ of x can be obtained by solving the extended Kalman filter equations (see Reference 9)

$$\dot{\hat{x}} = F(x,t) + P\left(\frac{\partial H}{\partial x}\right)^T R^{-1}\left(z - H(\hat{x},t)\right) \qquad \text{Filter Equation (3)}$$

$$\dot{P} = \frac{\partial F}{\partial x} P + P\left(\frac{\partial F}{\partial x}\right)^T + GQG^T - P\left(\frac{\partial H}{\partial x}\right)^T R^{-1} \frac{\partial H}{\partial x} P \qquad \text{Covariance Equation (4)}$$

$$\hat{x}(o) = x_o \quad , \quad P(o) = P_o \qquad \text{Initial Conditions (5)}$$

$\hat{x}$ and P can be interpreted as vector mean and covariance matrix of a conditional probability distribution of the state vector x, given the measurement vector z.

However, since the extended Kalman filter is a biased estimator (see Reference 5) and since the correct value of P_o is not known, P cannot be used as a measure of the quality of the estimate. Rather, the rate of decrease of P is an indication of the amount of information being obtained from the data. When P approaches a constant value then no further information is being obtained.

The extended Kalman filter may also be interpreted as an algorithm for obtaining a least square estimate recursively. The estimate is such as to minimize the following quadratic cost function

$$J = 1/2 \left\{ (x_o - \hat{x}_o)^T P_o^{-1} (x_o - \hat{x}_o) + \int_o^t w^T Q^{-1} w + \left(z - H(x,t)\right)^T R^{-1} \left(z - H(x,t)\right) dt \right\} \qquad \text{Cost Function (6)}$$

where now P_o, R and Q are arbitrary weighting matrices, which may be selected for good convergence of the algorithm. Since 1.) numerical methods for solving ordinary differential equations are well developed and 2.) R is usually a diagonal matrix so that R^{-1} is easy to obtain, this algorithm is computationally very efficient.

Estimation of Unknown Parameters

If we wish to estimate the vector a of unknown parameters we substitute a for x in the Kalman filter Eq. 3. For constant parameters we have

$$\dot{a} = o \qquad \text{Process Equation (7)}$$

so that F(x,t) = w = o. The system equation is then used as the measurement equation

$$\zeta = H(\xi,a) + v \qquad \text{Measurement Equation = System Equation (8)}$$

ζ is the vector of measured accelerations, ξ is the measured state vector and v can be interpreted as acceleration measurement noise or as system noise (including modeling errors). The Kalman filter equations are then

$$\dot{\hat{a}} = P\left(\frac{\partial H}{\partial a}\right)^T R^{-1} [\zeta - H(\xi,\hat{a})]$$

Filter Equation (9)

$$\dot{P} = -P\left(\frac{\partial H}{\partial a}\right)^T R^{-1} \frac{\partial H}{\partial a} P$$

Covariance Equation (10)

For $P \rightarrow o$ the measurements lose influence on the estimate and one obtains

$$\dot{\hat{a}} = o \qquad \text{Asymptotic Filter Equation (11)}$$

which agrees with the process equation. Again P_o and R may be selected for good convergence. A convenient choice for the initial estimate is $\hat{a}(o) = o$. The elements of R should be large enough to prevent the elements of P from becoming negative due to computation errors in the numerical integration.

Note that ξ, the state vector, is also a measured quantity. If measurement errors are present then this estimation algorithm is biased by an amount approximately proportional to the noise to signal ratio in the state variable measurements, see Reference 5. It is therefore advantageous to reduce the noise ratio before using the estimator. Methods for doing this are discussed in a later section on filtering of the response data.

In practice, one can almost always choose the parameters to be identified in such a way that $H(\xi,a)$ is a linear function of a. The estimator (9), (10) is then linear and problems of nonuniqueness and filter divergence are easily avoided. For this case, we call the algorithm the linear sequential estimator.

The extended Kalman filter assumes that the noise processes w and v are white and Gaussian. This will never be the case in practice especially if w must account for the effects of modeling errors. Because the extended Kalman filter may also be interpreted as yielding a least squares estimate for a given sample of the state ξ and acceleration ζ, we can regard the resulting estimate as a sample from a random variable. Determination of this random variable would necessitate a complete simulation, i.e., mean and variance determined by averaging over many runs. Since this approach is expensive of computing time, efforts here have been directed toward recovering parameters from a single run of computer generated response data.

The above approach to parameter estimation allows the use of high order of accuracy numerical integration (i.e., predictor corrector) schemes to solve the system of ordinary differential equations provided that the response data are sufficiently smooth. The parameter estimation is rapid and requires little computer time. R and P_o can be freely selected to obtain good convergence. The reason for this benign behavior of the estimation method is the linearity of the filter equations in the unknowns. If the accelerations of the system are not measured, one must estimate state variables and parameters simultaneously from a nonlinear filter equation. This nonlinear estimation requires an order of magnitude more computer effort and it is very sensitive to the initializations and to the correct assumptions of process noise and measurement noise. As mentioned before, we began by applying the nonlinear estimator to the identification of parameters in Mathieu's equation for a periodic system. The results were unsatisfactory since filter divergence occured for many choices of P_o and R. However, for the linear sequential estimator divergence could be avoided by following simple rules in selecting $\hat{x}(o)$, P_o and R.

Identifiability of System Parameters

It is obvious from the filter equation (9) that $\hat{a}$ will asymptotically approach a constant value provided that $P \rightarrow o$. The covariance equation (10) can be solved explicitly (see Appendix A) to yield

$$P = \left[P_o^{-1} + \int_o^t \left(\frac{\partial H}{\partial a}\right)^T R^{-1} \frac{\partial H}{\partial a}\, dt\right]^{-1} \qquad (12)$$

If the integral is replaced by a sum, this is the error equation for the standard least square method. If $P_o \neq o$, then $P(t) \rightarrow o$ whenever the integrand in the above equation is positive definite for all t. This is then a sufficient condition for identifiability. Note that $\frac{\partial H}{\partial a}$ is a function of the system response and hence also of the excitation, so that the identifiability depends not only upon the system but also upon the type of excitation. From the measurement equation (8) we see that the matrix $\frac{\partial H}{\partial a}$ is a measure of the sensitivity of acceleration measurements to changes in the parameters. For estimating parameters, a well designed excitation is obviously one which causes the elements of the P matrix to decrease rapidly. If any elements of P are decreasing slowly or not at all, then a different type of excitation is needed. A look at which elements of P are causing the trouble will give a clue as to which modes of the system are not being properly excited.

Filtering the Response Data

In practice, we usually have some knowledge of the character of the response data. For example, because of the damping present in physical systems, the true response will not contain much energy at high frequencies. We also know that the acceleration is the derivative of the velocity which is in turn the derivative of the displacement, etc. so that these responses are not independent.

To remove high frequency noise without effecting the signal a zero phase shift low band pass digital filter was used. This filter completely removes all of the signal and noise above a certain termination frequency ω_t without phase or amplitude distortion below a cutoff frequency ω_c. The digital filter used, due to Graham, Reference 10, generates the smoothed data as a numerical convolution of the raw data and a set of numerical smoothing weights, i.e.,

$$\tilde{f}(t_o + i\Delta t) = \sum_{j=-N}^{N} w(j\Delta t) f(t_o + (i + j)\Delta t) \tag{13}$$

where $f(t_o + (i + j)\Delta t)$ are the sampled values of the signal, $\tilde{f}(t_o + i\Delta t)$ are the smoothed sampled values and where the smoothing weights are given by

$$w(j\Delta t) = \frac{\pi c}{2j\Delta t} \frac{\sin \omega_t j\Delta t + \sin \omega_c j\Delta t}{\pi^2 - (\omega_t - \omega_c)^2 j^2 \Delta t^2} \qquad j = -N, \ldots, +N \quad j \neq o$$

$$w(o) = \frac{c(\omega_t + \omega_c)}{2\pi} \qquad \omega_c < \omega_t \tag{14}$$

where the constant c is chosen to satisfy the constraint

$$\sum_{j=-N}^{+N} w(j\Delta t) = 1 \tag{15}$$

The continuous weighting function $w(t)$, of which $w(j\Delta t)$ is a discretization, has the Fourier transform, i.e., frequency domain representation, shown in Figure 1. Convolution of this function with an arbitrary signal will obviously result in a smoothed signal which has all frequencies above ω_t completely suppressed and all signal components below ω_c undistorted. If ω_c and ω_t are properly selected then response data with low frequency signal and high frequency noise can be improved via digital filtering, that is, signal to noise ratio can be significantly increased.

In using the digital filter, it is tempting to achieve a "sharp" filter by taking $\omega_c \doteq \omega_t$. Graham, Reference 10, has determined empirically that the number of points N needed to achieve a given accuracy is approximately inversely proportional to $|\omega_t - \omega_c|$ at least over a limited frequency range. Since N = 40 points were used to filter the data, we selected $|\omega_t - \omega_c| \geq 1$ which according to Graham is sufficient to yield 2% accuracy.

In this study, the numerical convolution was accomplished by using a moving average, i.e., $\tilde{f}(t_o + i\Delta t)$ was computed separately for each i using Eq. (13). For long data records it is possible to achieve considerable savings in computer time by using the Fast Fourier transform algorithm to do this convolution, see Reference 11.

Improvements in the response data can also be obtained by making use of relationships among the various response signals. For the coupled flapping-torsion system considered in the next section the displacements ξ, velocities η and accelerations ζ are related by

$$\begin{aligned} \dot{\xi} &= \eta \\ \dot{\eta} &= \zeta + v \end{aligned} \tag{16}$$

We can use these equations as process equations in a Kalman filter along with measurement equations

$$\begin{bmatrix} \bar{\xi} \\ \bar{\eta} \end{bmatrix} = \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{17}$$

where $\bar{\xi}$ and $\bar{\eta}$ denote smoothed measured values. In the process equation (16) replace ζ by its smoothed measured value $\bar{\zeta}$ and let R, the process noise covariance matrix account for remaining errors. Then the Kalman Filter is given by

$$\begin{bmatrix} \dot{\hat{\xi}} \\ \dot{\hat{\eta}} \end{bmatrix} = \begin{bmatrix} \hat{\eta} \\ \hat{\zeta} \end{bmatrix} + PR^{-1} \begin{bmatrix} \bar{\xi} - \hat{\xi} \\ \bar{\eta} - \hat{\eta} \end{bmatrix} \tag{18}$$

Note that $\dot{\hat{\eta}}$ is available when solving the above equations and can be used as an improved estimate of $\bar{\zeta}$. Although this technique has not been used in this study, a similar procedure has been used successfully in Reference 6 for helicopter derivative identification.

Computer Experiments

Coupled flapping-torsion vibrations of a rotor blade at high advance ratio are governed by the equations

$$\ddot{\beta} + P^2\beta = \frac{\gamma}{2}\,[M_{\theta_1}(t)\delta + M_\lambda(t)\lambda + M_{\theta_o}(t)\theta_o - C(t)\dot{\beta} - K(t)\beta]$$

$$\ddot{\delta} + f^2\delta = -\frac{3}{2}\,\ddot{\theta}_o - 3\gamma F\,[C_{\theta_o}(t)\dot{\theta}_o + C_\delta(t)\dot{\delta}]$$

$$- 3\gamma Q[\ell_{r\dot{\beta}}(t)\dot{\beta} + \ell_{r\beta}(t)\beta + \ell_{r\lambda}(t)\lambda + \ell_{r\theta_o}(t)\theta_o + K_\delta(t)\delta] \tag{20}$$

where the periodic coefficients are defined in Reference 7. Responses to the gust excitations shown in Figure 2 were generated by solving Eq. (20) numerically using a fourth order Adams Moulton method with a time step of .05 and the following parameter values:

$$p^2 = 1.69 \qquad \gamma = 4.0 \qquad Q = 15.$$
$$f^2 = 64. \qquad \mu = 1.6 \qquad \theta_o = 0. \tag{21}$$
$$B = .97 \qquad F = .24$$

Simulated noisy measurements were obtained by adding samples from zero mean computer generated Gaussian random sequences to the computer generated responses. First the noise was added only to accelerations using the standard deviations

$$\sigma_{\ddot{\beta}} = 1.0 \qquad \sigma_{\ddot{\delta}} = 10 \tag{22}$$

The following three parameters with the values

$$a = \gamma/2 = 2.0$$
$$b = -3\gamma F = -2.88 \tag{23}$$
$$c = -3\gamma Q = -180$$

were assumed to be unknown.

They represent blade flapping and torsional inertia numbers. Unsteady aerodynamic inflow effects may possibly be considered by modifications of these inertia numbers from transient rotor model wind tunnel tests. The linear sequential estimator was started with the initial values of the estimates and errors of the estimates

$$\begin{bmatrix} \hat{a}(o) \\ \hat{b}(o) \\ \hat{c}(o) \end{bmatrix} = o \qquad P(o) = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 55 & 0 \\ 0 & 0 & 4000 \end{bmatrix} \tag{24}$$

The linear sequential estimator is, as mentioned before, quite insensitive to the initial standard deviations which could have been selected still much larger. The values for R used are the following

$$R = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \tag{25}$$

The method allows wide variations in the assumptions of the noise covariance matrix R. The integration scheme for solving filter and covariance Eqs. (9) and (10) was again a 4th order Adams Moulton method with a time step of .05. Fig. 3, shows the estimates $\hat{a}$, $\hat{b}$, $\hat{c}$ normalized with the true values and the 3 diagonal terms of the error covariance matrix P normalized with the initial values vs. non-dimensional time t. The excitation for this case was a unit step gust at time t = o, as indicated in Fig. 2 by the dash line. In about one revolution ($t = 2\pi$) the diagonal components of the covariance matrix P_a P_b P_c are approximately zero and further improvements of the parameter estimates $\hat{a}$ $\hat{b}$ $\hat{c}$ are not obtained. There is a small bias error (deviation from the value 1) in two of the parameters, which have been recovered within about 5% error.

The next case assumes that not only the accelerations but also the state variables are noisy. The following standard deviations were used

$\sigma_\beta = .2 \qquad \sigma_\delta = .5$

$$\sigma_{\dot\beta} = .6 \qquad \sigma_{\dot\delta} = 3.0 \qquad (22a)$$

$\sigma_{\ddot\beta} = 1.0 \qquad \sigma_{\ddot\delta} = 10$

The linear sequential estimator was first applied to the raw data. In this case the responses are far from smooth so that the use of a high order numerical integration scheme was unjustified. A first order Euler's method was used for the integration of the estimator equations. The initial values were

$$\begin{bmatrix} \hat a(o) \\ \hat b(o) \\ \hat c(o) \end{bmatrix} = o \qquad P(o) = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \qquad (24a)$$

The values for the R used in the estimator were

$$R = \begin{bmatrix} 16 & 0 \\ 0 & 225 \end{bmatrix} \qquad (25a)$$

The excitation consisted of a upward unit step gust at $t = 2.0$ followed by a down step gust to $\lambda = -1$ at $t = 6.0$, as indicated in Fig. 2. The second gust was added in order to provide to the system another transient useful for the estimator process. Fig. 4 shows that though two of the diagonal covariance terms go to zero after the second gust, the associated parameter estimates remain quite erroneous. The linear sequential estimator cannot be used if noise is present in accelerations as well as in the state variables.

Next the same data were passed through a digital filter with cut-off frequencies $\omega_c = 12$, $\omega_t = 13$, see Fig. 1. These cut-off frequencies are about 50% higher than the torsional frequency of $f = 8$. Applying now the linear sequential estimator to the filtered data, the initial values were the same as before, Eq. (24a), however R was reduced:

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix} \qquad (25b)$$

The results of the estimation are shown in Fig. 5. All diagonal terms of the covariance matrix go to zero soon after the second gust, the estimates stabilize in less than 2 rotor revolutions and have only a small bias error of about 5%; same as for the case with zero noise in the state variables. Digitally filtering the data to remove high frequency noise has thus appreciably extended the range of applicability of the linear sequential estimator. It might be argued that the success of the digital filter is due to the "white" character of the computer generated noise whereas real data will contain energy only at finite frequencies. It should be noted that the digital filter removes all of the signal above the truncation frequency and hence would be equally successful for any other distribution of the energy above ω_t.

In selecting the parameters for the digital filter it is important to keep ω_c large enough so that the responses are not significantly distorted. Initially, the noisy data was processed using different digital filters for the torsion and flapping responses. A digital filter with high cut-off frequency i.e., $\omega_c = 12.$ and $\omega_t = 13.$ was used for torsion responses while a lower bandpass filter with $\omega_c = 2.$ and $\omega_t = 3.$ was used to filter flapping responses. This resulted in poor identification of the parameter **a** in the flapping equation. When the same filter with high cut-off frequency was used for all of the data, adequate identification of all parameters was obtained. Although $\omega_c = 2.$ is above the natural frequency of flapping vibration, the flapping response obviously contains higher frequency components because of the coupling with torsion. This can easily be seen by inspection of the flapping response in Figure 6. For a good identification it is necessary that these higher frequency components not be removed from the signal. Fig. 6 compares the response without noise to the response with noise but after filtering. Also indicated are the standard deviations for flapping and torsion before filtering. It is seen that the filter was very effective in removing the noise corruption from the data.

Conclusions

1. The linear sequential estimator, also called equation of motion estimator, has been successfully applied to recover the system parameters of a periodic system representing rotor blade flapping-torsion dynamics at high rotor advance ratio with noise contaminated accelerations. Filtering of the noisy acceleration data was found to be not necessary.

2. If noise is present in the state variables as well as in the accelerations, the linear sequential

estimator performed very poorly.

3. Filtering both state variables and accelerations with a Graham digital filter with a low cut-off frequency for flapping and a high cut-off for torsion before estimation lead to a poor estimate for the flapping parameter.

4. Filtering both flapping and torsion response with a high cut-off frequency digital filter before estimation resulted in an adequate parameter recovery both in flapping and in torsion.

5. As compared to non-linear estimation methods which are applicable also if acceleration information is not available, the linear sequential estimator has the great advantage of being insensitive to the assumption of initial values for the estimate and for the error of the estimate. No matter what the actual measurement noise is, the assumed noise covariance matrix should be over-rather than underestimated.

6. As compared to the usual form of the least square estimation the linear sequential estimator does not require the inversion of large matrices but merely the numerical solution of a system of ordinary differential equations, thus allowing on-line application. The digital filter smoothes the data sufficiently so that high order of accuracy predictor corrector methods can be used for the integration.

7. The computer studies were performed assuming rather large measuring errors with standard deviations for the deflections of about 10% of the maximum measured values. The foregoing conclusions assume the absence of modeling errors, which would require special investigations.

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Appendix A

Solution of the Covariance Equation

The covariance equation of the linear sequential estimator

$$\dot{P} = -P\left(\frac{\partial H}{\partial a}\right)^T R^{-1} \frac{\partial H}{\partial a} P \qquad \text{(A-1)}$$

is a matrix Ricatti differential equation. It is well known that the general matrix Ricatti Equation with all matrices being time functions

$$\dot{P} = -PA - DP - PBP + C \qquad \text{(A-2)}$$

of which (A-1) is a special case, has the solution

$$P = VU^{-1} \qquad (A-3)$$

where U and V satisfy

$$\begin{aligned} \dot{V} &= CU - DV \\ \dot{U} &= AU + BV \end{aligned} \qquad (A-4)$$

This and other aspects of matrix Ricatti equations are discussed in Reference 12.

By comparing Eqs. (A-1) and (A-2) we see that Eq. (A-1) is of the form of Eq. (A-2) with A=C=D=0 and $B = \left(\frac{\partial H}{\partial a}\right)^T R^{-1} \frac{\partial H}{\partial a}$

Therefore, from Eq. (A-4) $V = V_o$, a constant matrix and

$$\dot{U} = BV_o \qquad (A-5)$$

Integrating yields

$$U = U_o + \int_o^t B\, dt V_o \qquad (A-6)$$

Now since from (A-3)

$$P_o = V_o U_o^{-1} \qquad (A-7)$$

we can satisfy the initial condition by taking $V_o = I$ and $U_o = P_o^{-1}$. Hence

$$U = P_o^{-1} + \int_o^t B\, dt \qquad (A-8)$$

and

$$P = \left[P_o^{-1} + \int_o^t \left(\frac{\partial H}{\partial a}\right)^T R^{-1} \frac{\partial H}{\partial a}\, dt\right]^{-1} \qquad (A-9)$$

Minimizing the cost function Eq. (6) with w = o, x = a and z = ζ, one obtains the least square estimate

$$\hat{a} = [P_o^{-1} + \int_o^t \left(\frac{\partial H}{\partial a}\right)^T R^{-1} \frac{\partial H}{\partial a}\, dt]^{-1}[P_o^{-1} a_o + \int_o^t \left(\frac{\partial H}{\partial a}\right)^T R^{-1} \zeta dt] \qquad (A-10)$$

where the first factor is the covariance P from Eq. (A-9). Eq. (A-10) is the equivalent of solving Eqs. (9) and (10) and has been used in Ref. 6 with $P_o^{-1} = o$ after replacing the integrals by sums. In this case the result is independent of R which cancels out.

Even in the general case of finite P(o) the error covariance matrix R need not be considered as a separate input. If R is a diagonal constant matrix it is evident that Eqs. (9) and (10) can be written in the form

$$\dot{\hat{a}} = P_r \left(\frac{\partial H}{\partial a}\right)^T [\zeta - H(\xi,\hat{a})] \qquad (A-10)$$

$$\dot{P}_r = - P_r \left(\frac{\partial H}{\partial a}\right)^T \frac{\partial H}{\partial a} P_r \qquad (A-11)$$

where $P_r = P R^{-1}$. This was pointed out to the authors by John A. Molusis.

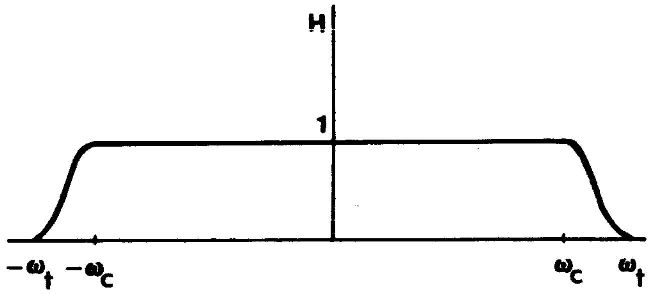


Fig. 1. Fourier Transform of Weighting Function

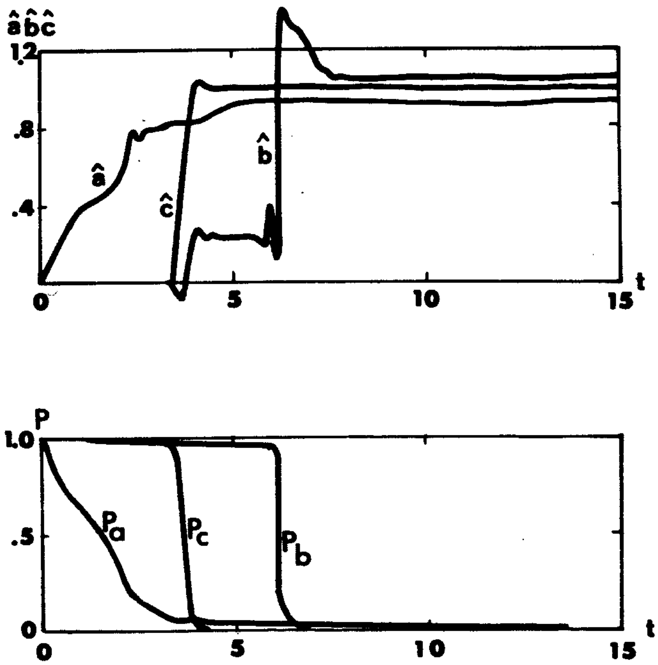


Fig. 3. Estimates & Covariances vs. Time, Acceleration Noise Only

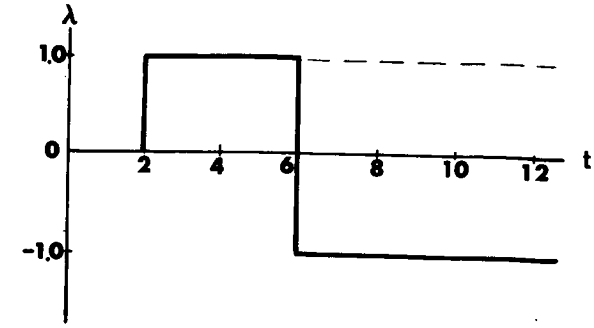


Fig. 2. Gust Excitations

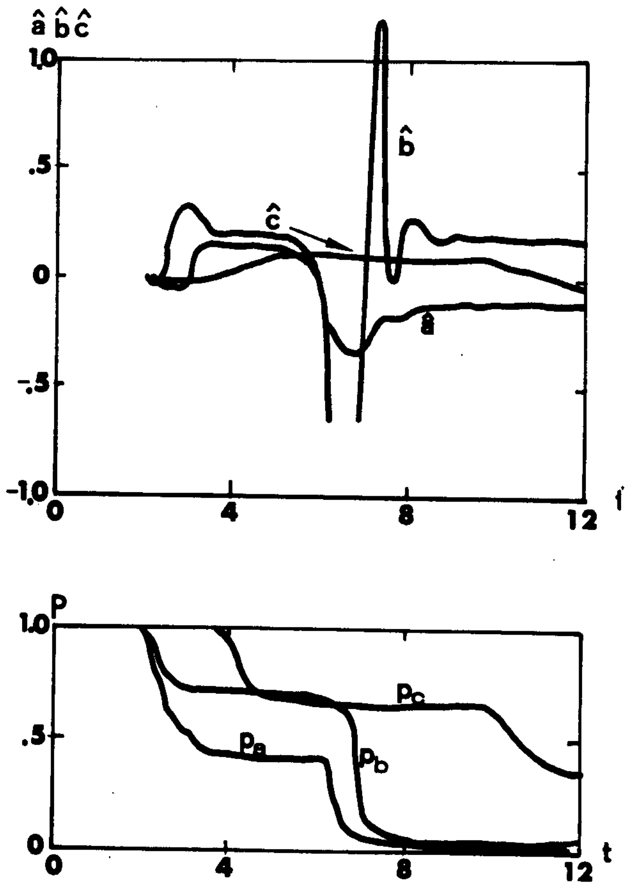
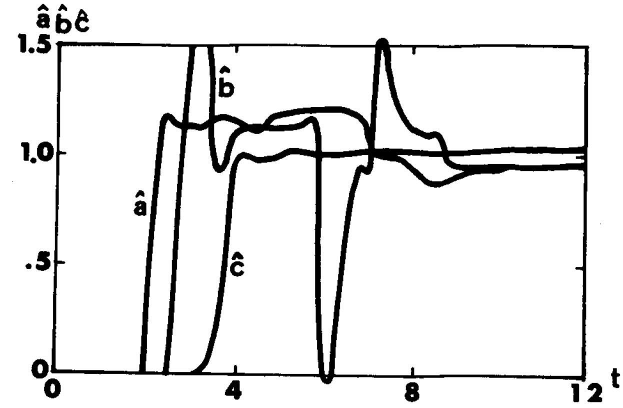


Fig. 4. Estimates & Covariances vs. Time, Acceleration and State Variable Noise



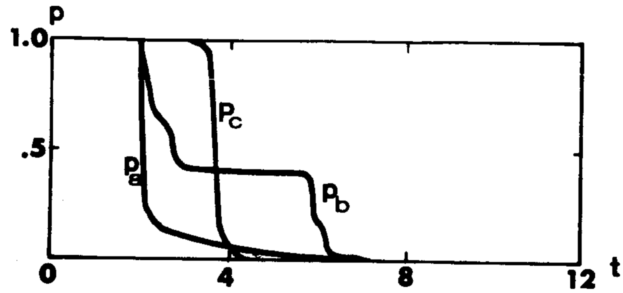


Fig. 5. Estimates & Covariances, Filtered Data

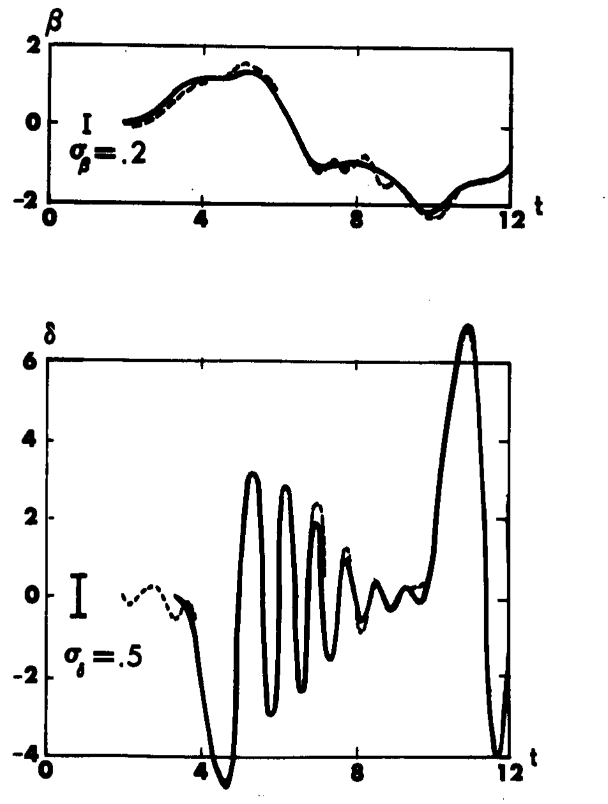


Fig. 6. Exact and Filtered Noisy Responses (Solid & dash line respectively)