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ON CONDITIONAL SAMPLING FOR TURBULENT FLOW STUDIES

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16. ABSTRACT

The conditional sampling technique is analyzed as a weighted time average. The various conditional averages are obtained by using different types of weighting functions. A "second averaging" relation is obtained between the conventional averages and the conditional averages. A few examples are given in which simplified expressions are used.

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ON CONDITIONAL SAMPLING FOR TURBULENT FLOW STUDIES

I. INTRODUCTION

A conditional sampling technique has been used recently by many researchers in studying turbulent shear flows [1-9]. Under the conditional sampling, a discriminating criterion is chosen and flow measurements are made using two or more probes. One of the probes is the triggering probe and the rest are sampling probes. Averaging at the sampling probe is made only when the discriminating criterion is satisfied at the triggering probe. By so doing, the flow field is divided into different regions and average flow quantities in each region are obtained. A tremendous amount of data may be generated and by using the various forms of correlations, the detailed structure of the turbulent flow field is obtained. This is usually shielded from us in a conventional measurement.

Any detectable flow quantity may be used to provide the discriminating criterion. The location of the turbulent-nonturbulent interface at the outer edge of a turbulent boundary layer has been used by Kovasznay et al. [1], by Imaki [2], and by Antonia [4]. The same has been used by Wagnanski and Fielder [5] in a two-dimensional mixing layer, and by Thomas [6] in a two-dimensional wake. The location of the viscous sublayer was used in studying turbulent "bursting" phenomena by Narahari et al. [7] and by Blackwelder and Kovasznay [8]. Other quantities that may be used include the flow temperature used by Dean [9], the species concentration, and the location of a fluctuation shock front.

Mathematically speaking, the conditional average is just a form of weighted average, with the weighting function generated from the reading at the triggering probe. By using different types of weighting functions, different types of conditional averages can be obtained. The conventional average, for which no discriminating criterion is used, becomes a special case.

In the following sections, the concept of conditional sampling is demonstrated by considering the turbulent boundary layer over a flat plate. The flow field consists of two distinctive regions: the turbulent region near the flat plate, and the nonturbulent region of the ambient flow. These regions are separated by a thin, sharp interface that is highly contoured, and its position fluctuates in time. By using the location of the interface as a discriminating criterion, the following types of conditional averages can be obtained: the turbulent zone average, the nonturbulent zone average, and the point conditional average.

A "second averaging" relation is then obtained which relates the conditional averages to the conventional average through the statistical distribution of the interface position. With this relation it is thus possible to incorporate the experimental data of conditional measurement into the theoretical modeling of turbulent shear flow. Simplified relations for the "second averaging" are obtained under special assumptions.

II. THE WEIGHTED TIME AVERAGES

Consider the turbulent boundary layer over a flat plate as shown in Figure 1.

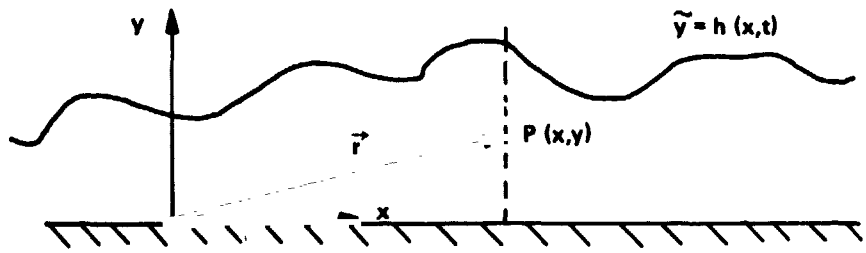

Figure 1. Turbulent boundary layer near the wall.

Let the instantaneous location of the turbulent interface, assumed to be single-valued, be given by $\tilde{y} = h(x,t)$. Let $Q(\vec{r},t)$ be a flow quantity measured at a point $P(x,y)$. Then at any time t; one of the following conditions must be satisfied:

$$h(x,t) > y \quad , \tag{1a}$$

$$h(x,t) < y \quad , \tag{1b}$$

or

$$h(x,t) = y \quad , \tag{1c}$$

where the point P is said to be either inside the turbulent region, inside the nonturbulent region, or at the interface, depending on whether condition (1a), (1b), or (1c) is satisfied.

If a weighting function $w(\vec{r},t)$ can be defined that takes different values for P in different regions, then a weighted average of Q is defined as

$$\overline{Q}_w(\vec{r}) = \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)\, w(\vec{r},t)\, dt}{\int_0^T w(\vec{r},t)\, dt} \tag{2}$$

According to the weighting function $w(\vec{r},t)$, the types of averages listed as follows can be obtained.

1. Conventional Averages. If a constant weighting function is used for P in all the three regions in (1), then the conventional average, or the "long time" average, is obtained,

$$\overline{Q}(\vec{r}) = \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)dt}{\int_0^T dt} \tag{3}$$

The fluctuation of $Q(\vec{r},t)$ about its average $\overline{Q}(\vec{r})$ is given as

$$q(\vec{r},t) = Q(\vec{r},t) - \overline{Q}(\vec{r}) \tag{4}$$

which has a "long time" average of zero,

$$\overline{q}(\vec{r}) = 0 \tag{5}$$

The standard deviation of $Q(r,t)$ is given by

$$\sigma(\vec{r}) = \left[\lim_{T\to\infty} \frac{\int_0^T q(\vec{r},t)^2\, dt}{\int_0^T dt}\right]^{1/2} \tag{6}$$

which is a measure of the intensity of the fluctuation.

2. Zone Average. If the functional for the weighting function to be used is the turbulent signal $I(\vec{r},t)$, which is unity in the turbulent zone and zero elsewhere, i.e.

$$I(\vec{r},t) = \begin{cases} 1 \quad , \quad h(x,t) > y \\ \\ 0 \quad , \quad h(x,t) < y \end{cases} \quad , \tag{7}$$

then the following turbulent zone average is obtained:

$$\bar{\bar{Q}}(\vec{r}) \equiv \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)\, I(\vec{r},t)\, dt}{\int_0^T I(\vec{r},t)\, dt} \quad . \tag{8}$$

A nonturbulent zone average can be obtained similarly by using $1\text{-}I(\vec{r},t)$ as the weighting function, giving

$$\tilde{\bar{Q}}(\vec{r}) \equiv \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)\, [1 - I(\vec{r},t)]\, dt}{\int_0^T [1 - I(\vec{r},t)]\, dt} \quad . \tag{9}$$

Introducing an intermittency factor, $\gamma(r)$, as the conventional average of the turbulent signal,

$$\gamma(\vec{r}) \equiv \bar{I}(\vec{r}) \equiv \lim_{T\to\infty} \frac{\int_0^T I(\vec{r},t)\, dt}{\int_0^T dt} \quad , \tag{10}$$

then the turbulent and nonturbulent zone averages can be rewritten as

$$\overset{\approx}{Q}(\vec{r}) = \frac{1}{\gamma} \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)\, I(\vec{r},t)\, dt}{\int_0^T dt} \tag{11}$$

and

$$\overset{\approx}{Q}(\vec{r}) = \frac{1}{1-\gamma} \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)\, [1 - I(\vec{r},t)]\, dt}{\int_0^T dt} \tag{12}$$

Obviously, the conventional average can be obtained from the zone averages using the relation

$$\overline{Q}(\vec{r}) = \gamma(\vec{r})\, \overset{\approx}{Q}(\vec{r}) + [1 - \gamma(\vec{r})]\, \overset{\approx}{Q}(\vec{r}) \tag{13}$$

which was first obtained by Townsend [10].

3. Multiple Zone Averages. We shall now deduce from the zone averages the point conditional averages. First let us introduce the multiple zone averages. Assume that the range of the possible values of the interface position $h(x,t)$ is given by the open interval (a,b), where a may be minus infinity and b may be plus infinity. Assuming that a sequence of ordered points $p_0 < p_1 < p_2 < \ldots < p_J$ exists such that $p_0 = a$ and $p_J = b$, then the interval (a,b) is divided into J smaller intervals by the sequence of points. If the following weighting function for each interval is defined

$$w_j(\vec{r},t) = \begin{cases} 1 , & \text{when } p_{j-1} \leq h(x,t) < p_j \\ 0 , & \text{otherwise} \end{cases} \tag{14}$$

for $j = 1, 2, 3, \ldots, J$

then a multiple zone average at the jth interval is obtained as

$$\tilde{Q}_j(\vec{r}) \equiv \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)\, w_j(\vec{r},t)\, dt}{\int_0^T w_j(\vec{r},t)\, dt} \tag{15}$$

Similar to equation (13), the conventional average is related to the multiple zone averages by the summation relation

$$\overline{Q}(\vec{r}) = \sum_{j=1}^{J} f_j(\vec{r})\, \tilde{Q}_j(\vec{r}) \tag{16}$$

where

$$f_j(\vec{r}) \equiv \lim_{T\to\infty} \frac{\int_0^T w_j(\vec{r},t)\, dt}{\int_0^T dt} \tag{17}$$

is the probability that the interface position $h(x,t)$ falls in the interval between p_{j-1} and p_j. A normalization condition has to be satisfied by these probability functions, which yields

$$\sum_{j=1}^{J} f_j(\vec{r}) = 1 \tag{18}$$

4. Point Conditional Averages. Let us now take the limit such that the largest interval in the multiple zone average of item 3 becomes infinitesimally small, so that the weighting function in (15) becomes the Dirac delta functions,

$$w_j(\vec{r},t) = \delta(h - p_j) = \begin{cases} 1 & , \quad h = p_j \\ & \\ 0 & , \quad \text{otherwise} \end{cases} \quad . \tag{19}$$

Then the following point condition average is obtained:

$$\hat{Q}_j(\vec{r}) \equiv \lim_{T\to\infty} \frac{\int_0^T Q(\vec{r},t)\, \delta(h - p_j)\, dt}{\int_0^T \delta(h - p_j)\, dt} \quad . \tag{20}$$

In the same limit, as the largest interval goes to zero, a probability density function $P_j(h)$ for the turbulent interface position can be introduced in the expression

$$f_j(r) = P_j\,(h)\, dh \quad . \tag{21}$$

Substituting (21) into equation (16) and taking the same limit, the following integral expression is obtained:

$$\overline{Q}(\vec{r}) = \int_{-\infty}^{\infty} P_j(h)\, \hat{Q}_j(\vec{r})\, dh \quad . \tag{22a}$$

Here integration is taken over all possible values of h, assumed to be from minus infinity to plus infinity. This can be rewritten as

$$\overline{Q}(\vec{r}) = \frac{\int_{-\infty}^{\infty} P_j(h)\,\hat{Q}_j(\vec{r})\,dh}{\int_{-\infty}^{\infty} P_j(h)\,dh} \tag{22b}$$

by using the normalization relation

$$\int_{-\infty}^{\infty} P_j(h)\,dh = 1 \quad . \tag{23}$$

Noticing the analogy between equations (2) and (22), the relation between the conventional averages and the conditional averages can thus be interpreted as another weighted averaging process. Since the conditional average $\hat{Q}(\vec{r})$ is already an averaged quantity, this relation is thus termed a "second averaging" relation. The weighting function for this "second averaging" is the probability density function P (h) of the turbulent interface position, which may take any value from 0 to .

III. CORRELATIONS IN CONDITIONAL AVERAGES

In this section, we shall derive the relations between the fluctuation relations in the conditional and conventional averages using the "second averagin ion, equation (22).

Let the flow quantity to be averaged, $Q(\vec{r},t)$, be the product o o measurable fluctuating quantities, $R(\vec{r},t)$ and $S(\vec{r},t)$; i.e.,

$$Q(\vec{r},t) = R(\vec{r},t)\,S(\vec{r},t) \quad . \tag{24}$$

Here the dependency of Q, R, and S on the space cc dinate $\vec{r}$ is well understood and shall be omitted from the following discussions.

Substituting equation (24) into equation (22) gives the relation.

$$\overline{RS} = \int_{-\infty}^{\infty} P_j(h)\,\widehat{RS}_j\,dh \quad . \tag{25}$$

Separating R(t) and S(t) into their conventional mean and fluctuating parts gives

$$R(t)\,S(t) = [\overline{R} + r(t)]\,[\overline{S} + s(t)] = \overline{R}\,\overline{S} + \overline{R}\,s(t) + \overline{S}\,r(t) + r(t)\,s(t) \quad . \tag{26}$$

Defining fluctuating parts in the point conditional average as

$$r_\wedge = R(t) - \hat{R}$$

$$s_\wedge = S(t) - \hat{S} \tag{27}$$

gives similarly

$$R(t)\,S(t) = \hat{R}\,\hat{S} + \hat{R}\,s_\wedge(t) + \hat{S}\,r_\wedge(t) + r_\wedge(t)\,s_\wedge(t) \quad . \tag{28}$$

Substituting (26) and (28) into (25) and using the relations

$$\overline{r} = \overline{s} = \hat{r}_\wedge = \hat{s}_\wedge = 0 \quad , \tag{29}$$

gives

$$\overline{R}\,\overline{S} + \overline{r\,s} = \int_{-\infty}^{\infty} P_j(h)\,(\hat{R}\,\hat{S} + \widehat{r_\wedge s_\wedge})_j\,dh \quad , \tag{30}$$

which can be rewritten as

$$\overline{r\,s} = \int_{-\infty}^{\infty} P_j(h)\,(\hat{R}\,\hat{S} + \widehat{r_\wedge s_\wedge})_j\,dh - \int_{-\infty}^{\infty} P_j(h)\,\hat{R}_j\,dh \int_{-\infty}^{\infty} P_j(h)\,\hat{S}_j\,dh \quad . \tag{31}$$

Thus the conventional correlation $\overline{rs}$ can be obtained from the point conditional averages $\hat{R}$, $\hat{S}$ and the point conditional correlation $\widehat{r_\wedge s_\wedge}$ once the probability density distribution function $P_j(h)$ is known. Contracting equation (31) gives the fluctuating level, which is

$$\overline{r^2} = \int_{-\infty}^{\infty} P_j(h)\,(\hat{R}^2 + \hat{r^2_\wedge})_j\,dh - \left(\int_{-\infty}^{\infty} P_j(h)\,\hat{R}_j\,dh\right)^2 \tag{32}$$

Higher order correlations can also be obtained in a similar manner. The expression always includes correlation of the same order in the point conditional average and correlations of lower order. A third order correlation is given here as an example:

$$\begin{aligned}\overline{qrs} = & \int_{-\infty}^{\infty} P_j(h)\,(\hat{Q}\hat{R}\hat{S} + \hat{Q}\,\widehat{r_\wedge s_\wedge} + \hat{R}\,\widehat{q_\wedge s_\wedge} + \hat{S}\,\widehat{q_\wedge r_\wedge} + \widehat{q_\wedge r_\wedge s_\wedge})_j\,dh \\ & - \overline{Q}\,\overline{R}\,\overline{S} - \overline{Q}\,\overline{rs} - \overline{R}\,\overline{qs} - \overline{S}\,\overline{qr} \quad .\end{aligned} \tag{33}$$

The above expressions can also be applied to two-point correlations like $\overline{q(x,t)\,r(x + dx,t)}$. Similar relations for time-delayed correlations like $\langle q(x,t)\ r(x,\ t+dt)\rangle$ can be obtained using ensemble averages.

IV. SIMPLE EXAMPLES

Application of the "second averaging" relations, equations (22) and (31) through (33), in a real flow situation may become impractical when extensive conditional measurements are needed. Under certain conditions, however, approximations for the conditional averages can be made, and simplified relations for the second averaging can be obtained. In the following examples, similarity assumptions are used to draw up simplified relations.

Example 1. Fluctuations in an Infinite Domain. Consider the motion of a fluctuating interface in an infinite domain as shown in Figure 2. This interface may be a material surface which separates two different flow regions, or a nonmaterial surface such as a shock front or a shear front. It is reasonable to assume that the motion of the fluid very near the interface is always in phase with the motion of the interface; and the flow properties at a fixed distance from the interface position always remain constants, so that they can be given a single-valued function F as

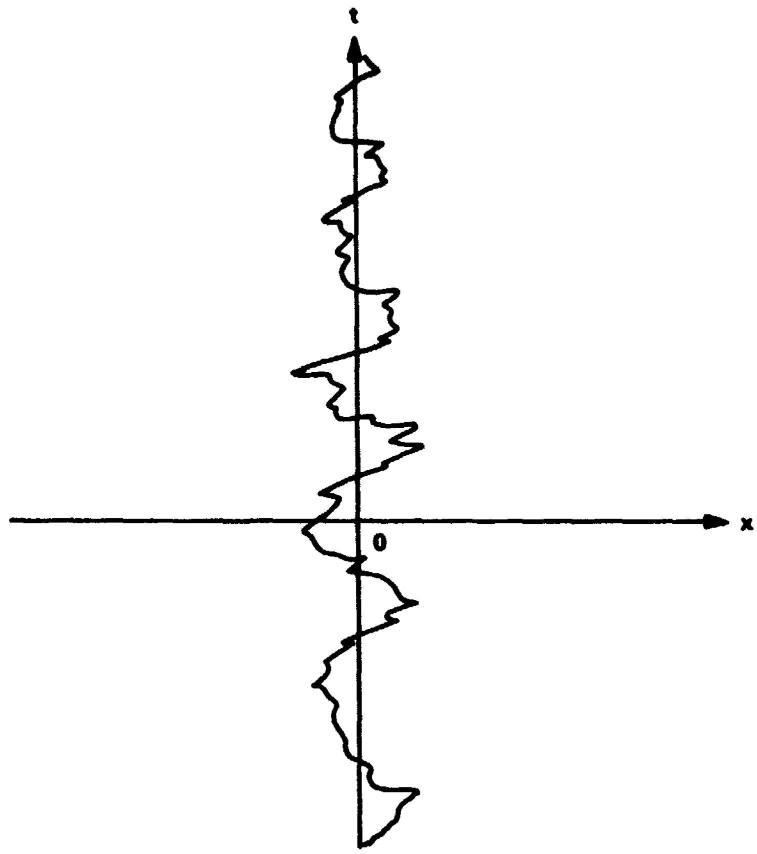

Figure 2. Fluctuating interface in an infinite domain.

$$\hat{Q}_j(x) = F\,[x - h(t)] \quad , \tag{34}$$

where $h(t)$ is the instantaneous position of the interface. Under these assumptions, the second average relation (22) becomes

$$\overline{Q}(x) = \int_{-\infty}^{\infty} P_j(h)\, F(x - h)\, dh \quad , \tag{35a}$$

which reduces to

$$\bar{Q}(x) = \int_{-\infty}^{\infty} P(x - y)\, F(y)\, dy \tag{35b}$$

by using new variables

$$y = x - h \quad .$$

A particular example of this case is the averaging of the turbulent signal, which is

$$F(y) = I(t) = \begin{cases} 0 \quad , & y \geqslant 0 \\ 1 \quad , & y < 0 \end{cases} \quad . \tag{36}$$

The average from (35b) gives

$$\bar{I}(x) = \int_{-\infty}^{0} P(x - y)\, dy = \int_{x}^{\infty} P(h)\, dh = \gamma \quad , \tag{37}$$

which gives the intermittency factor.

Example 2. Linear Fluctuation Between Nodal Points. Consider two layers of mutually immiscible fluid bounded on both sides by solid walls as shown in Figure 3. Assume a one-dimensional fluctuating motion is produced in the x-direction, normal to the interface, and no reflective waves are produced so that the motion of the fluid is in phase with the motion of the interface. Furthermore, assume each fluid layer is stretched and compressed linearly by the motion of the interface. Then the point conditional averages in the fluid region can be represented by one of the two similar profiles given below:

$$\hat{Q}(x) = \begin{cases} F_1\left(\dfrac{x}{h}\right) \quad , & 0 \leqslant x < h(t) \\ F_2\left(\dfrac{L - x}{L - h}\right) \quad , & h(t) < x \leqslant L \end{cases} \quad . \tag{38}$$

The functions F_1 and F_2 are both smooth and single-valued, and they are normalized with the distance between the interface and the walls.

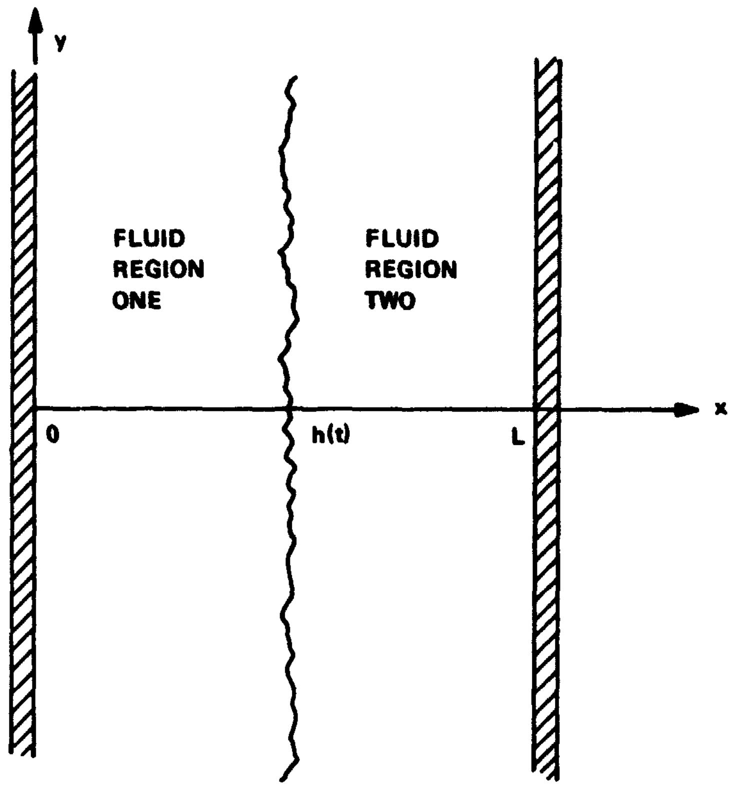

Figure 3. Linear stretching of two fluid regions between two solid boundaries.

Substituting equation (38) into equation (22) gives

$$\overline{Q}(x) = \int_0^x P(h)\, F_2\left(\frac{L-x}{L-h}\right) dh + \int_x^L P(h)\, F_1\left(\frac{x}{h}\right) dh \quad , \tag{39}$$

which, after changing variables, becomes

$$\overline{Q}(x) = (L-x) \int_{1-\frac{x}{L}}^{1} \frac{F_2(y)}{y^2}\, P\left(L - \frac{L-x}{y}\right) dy + x \int_{\frac{x}{L}}^{1} \frac{F_1(y)}{y^2}\, P\left(\frac{x}{y}\right) dy \ . \tag{40}$$

Thus, by knowing the distributions of P, F_1, and F_2, the simple integration will give the conventional averages.

V. CONCLUSION

The conditional sampling averages have been analyzed as weighted time averages. The various forms of conditional averages are obtained by using different types of weighting functions. A relation is derived which relates the conventional averages to the conditional averages through a "second averaging" process. This "second averaging" process is also a weighted averaging over possible values of the discriminating quantity. A few examples are given in which simple expressions for the point conditional averages can be used.

The "second averaging" relations, (22b) and (31) through (33), relate the conditional averages to the conventional averages. Application of relation (22b) in the mathematical modeling of turbulent shear flow has been done by Nee and Kovasznay [11] and by Wang [12]. Using a simplified form, encouraging improvements in the mean flow fields were obtained. The justification for these relations lies in the assumptions made on the conditional averages. Detailed conditional measurements are thus needed for this purpose. Recently, Paizis and Schwarz [13] used a rack of 20 probes in studying the structure of the turbulent interface. From the simultaneous measurements at the 20 locations, detailed structure and motion of the interface were obtained. This information is valuable in the constructing of a better model for the turbulent shear flows.

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APPROVAL

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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