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(NASA-TM-X-70342) MATHEMATICAL CREW MOTION DISTURBANCE MODELS FOR SPACECRAFT CONTROL SYSTEM DESIGN M.S. Thesis - George Washington Univ. (NASA) 63 p HC $6.25 CSCL 22B

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ABSTRACT

Several techniques for modeling the disturbances to a spacecraft's attitude caused by moving crew members are presented. These disturbances can be the largest moments acting on a manned spacecraft, and knowledge of their effect is important in the sizing, design, and analysis/simulation of spacecraft attitude control systems.

The modeling techniques are identified as two principal types: deterministic and stochastic. Three techniques of each type are presented. The deterministic models include point-mass motion derivatives and a discussion on dynamic models of moving crew members. The stochastic techniques are highlighted by a Fourier transform method and the representation of long-term crew disturbance activities as outputs from appropriately designed filters. A z-transform technique is developed to obtain a difference-equation form of stochastic models for use on digital computers.

An appendix derives spacecraft equations of motion which can be used with many of the models discussed.

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TABLE OF CONTENTS

LIST OF SYMBOLS

A_x, A_y, A_z	linear accelerations of point mass
A_n	Fourier Coefficient (cosine series)
A	filter parameter function
a, b_i, c_i	filter parameters
B_n	Fourier coefficient (sine series)
B_i	filter parameter function
e	exponential
F	force
f	frequency
g	acceleration due to gravity
G_x, G_y, G_z	control torque
H	angular momentum
$H(i\omega), H(s), H_1(s), H(z)$	transfer functions
I	moment of inertia
i	imaginary $(\sqrt{-1})$
i, j, k, ℓ	indices
ℓ	length
k_1, k_2, k_3	load cell centroid location
m	mass
M	moment

n	harmonic; number of masses; number of time increments
o	origin
p	integration variable (Cauchy's theorem)
Q	mass factor
R,r	radius
s	Laplace transform variable
S	power spectral density
t	time
T	period
W	weight
Z	z-tranform operator
X,Y,Z	coordinate axes
z	z-transform variable
x,y,z	coordinates with respect to axes
α_i, β_i	z-transfer function coefficients
Δ	function of ω
Δt	time increment
ξ, ξ	damping ratio
$\omega_x, \omega_y, \omega_z$	angular velocity components
ω	circular frequency; angular velocity
ρ	radius of convergence of z-transform series
ϕ, θ, ψ	Euler angles
$\theta_x, \theta_y, \theta_z$	angles
τ	autocorrelation lag

Subscripts

a	arm
f	final
c.g.	center of gravity
c.m.	center of mass
0	origin
s	spacecraft
m	man
rb	rigid body

Superscripts

$\overline{(\quad)}$	vector quantity

Miscellaneous

$\frac{d(\ )}{dt}$	total derivative with respect to time

LIST OF FIGURES

INTRODUCTION

Advances in manned space flight have led to the participation of man as a useful part of missions involving the carrying out of sophisticated experiment programs. He can now contribute to the success of missions requiring maximum use of his capabilities as an observer, operator, repairman, or subject. His presence in space, however, is not without its penalties (which should be more than offset by its advantages). Life support systems must be provided, at significant costs in weight and spacecraft complexity. Other spacecraft systems must be made highly reliable, and redundancy provided, in order to insure that man will return from his missions. Again, cost in weight and complexity are incurred. Finally, man by his presence imposes a disturbance environment on the spacecraft which could hinder the carrying out of certain experiment tasks. The expected dynamic environment is of primary importance in the design and analysis of any spacecraft stabilizaion and control system. For an orbital spacecraft, external disturbances can be caused by aerodynamic forces, gravity gradient torques, or meteoroid impacts, for example. The internally-generated disturbances are due to, for example, rotating component imbalance, mass (cargo or equipment) shifts or transfers, or as mentioned above, by motions and actions of the crew members.

Crew motions can produce some of the largest disturbances acting on a manned spacecraft. These motions can include restrained activities (where an astronaut is performing tasks at a console, for example), or translation, where the astronaut may be moving [illegible] point to another within the spacecraft. As motion occurs while a crew member is in contact with the spacecraft, disturbance moments are generated. This dynamic interaction of man with his spacecraft requires that the attitude control system be designed to take into account the disturbance torques arising from the man's motion within the vehicle.

Early work on modeling crew motion disturbances was, in general, confined to simple representations of motions within a spacecraft (see refs 1-5) as instantaneous or (usually) constant velocity translations. This type of modeling is best suited to rigid-body spacecraft considerations, as there is no discrete frequency content expressed in the movements. Also, no treatment of the restrained crew activities is afforded.

Subsequent investigations have resulted in the modeling of crew disturbance forces and moments as stochastic processes (see refs 6-8) with finite band-width, so that frequency domain control system response problems can be explored.

More recently, determininstic modeling of astronaut body motions, and the resultant disturbance forces and moments, has been carried out by Kurzhals and Reynolds (ref. 9, Appendix B) using models of the human body developed by Hanavan (ref. 10) and modified by Woolley (ref. 9, Appendix A).

This thesis focuses on some of the modeling techniques developed to date, and indicates some logical extensions to some of these. In particular, since much analysis and simulation work is carried out by high-speed digital computers, the modeling of stochastic processes in this domain is deemed important; development of difference-equation descriptions of a stochastic crew motion model is presented as a potentially powerful technique for computer applications.

TYPES OF CREW MOTION DISTURBANCE MODELS

Two types of astronaut crew disturbance models will be considered herein: deterministic and stochastic. The deterministic models are completely specified as functions of time, and may range from representations of simple point-mass motions to sophisticated dynamic human body models. The stochastic models will generally be represented by their statistics (e.g. statistical moments) and frequency content or bandwidth.

Deterministic models will be considered first.

DETERMINISTIC MODELS

Crew Motions Represented by Moving Point Masses

As pointed out in the Introduction, one method of representing crew motion disturbances is to treat one or more crew members as point masses, and allow this mass (masses) to translate within the spacecraft. The simplest means of implementing this method in a dynamic analysis is to consider only a single mass (which could represent any number of crew) and examine the effects of its motion about the spacecraft. Appendix A describes the derivation of this approach. To include the effects of separate masses in motion (either simultaneously or in series), modification of the basic equations given in the Appendix must be made. For i masses, $i=1,\ldots$ total number of moving masses to be considered (n), define

$$Q_i = \frac{m_i\,(m_s - m_i)}{m_s} \qquad (i = 1,2,\ldots n) \tag{1}$$

Equations (A36) of the Appendix would then be written as

$$\left.\begin{aligned}
M_x = \sum_{i=1}^{n} & Q_i\left\{\left(y_i\ddot{z}_i - z_i\ddot{y}_i\right) + \left[\omega_y\left(x_i\dot{y}_i - y_i\dot{x}_i\right) - \omega_z\left(z_i\dot{x}_i - x_i\dot{z}_i\right)\right]\right\} \\
& + \left(I_x\dot{\omega}_x + \omega_x\dot{I}_x - I_{xy}\dot{\omega}_y - \omega_y\dot{I}_{xy} - I_{xz}\dot{\omega}_z - \omega_z\dot{I}_{xz}\right) \\
& + \left[\omega_y\left(I_z\omega_z - I_{xz}\omega_x - I_{xz}\omega_y\right) - \omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{xy}\omega_x\right)\right] \\
M_y = \sum_{i=1}^{n} & Q_i\left\{\left(z_i\ddot{x}_i - x_i\ddot{z}_i\right) + \left[\omega_z\left(y_i\dot{z}_i - z_i\dot{y}_i\right) - \omega_x\left(x_i\dot{y}_i - y_i\dot{x}_i\right)\right]\right\} \\
& + \left(I_y\dot{\omega}_y + \omega_y\dot{I}_y - \dot{I}_{yz}\omega_z - \dot{\omega}_z I_{yz} - I_{xy}\dot{\omega}_x - \omega_x\dot{I}_{xy}\right)
\end{aligned}\right\} \tag{2}$$

$$
\left.\begin{aligned}
&+\left[\omega_{\dot{z}}\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) - \omega_x\left(I_z\omega_z - I_{xz}\omega_x - I_{yz}\omega_y\right)\right] \\
M_z &= \sum_{i=1}^{n} Q_i\left\{\left(x_i\ddot{y}_i - y_i\ddot{x}_i\right) + \left[\omega_x\left(z_i\dot{x}_i - x_i\dot{z}_i\right) - \omega_y\left(y_i\dot{z}_i - z_i\dot{y}_i\right)\right]\right\} \\
&+\left(I_z\dot{\omega}_z + \omega_z\dot{I}_z - I_{xz}\dot{\omega}_x - \omega_x\dot{I}_{xz} - I_{yz}\dot{\omega}_y - \omega_y\dot{I}_{yz}\right) \\
&+\left[\omega_x\left(I_y\omega_y - I_{yz}\omega_z - I_{xy}\omega_x\right) - \omega_y\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\right]
\end{aligned}\right\} \quad (2)
$$

Moments and products of inertia in the above equations are given by

$$
\left.\begin{aligned}
I_x &= I_{x,o} + \sum_{i=1}^{n} Q_i\left(y_i^2 + z_i^2\right) \\
I_y &= I_{y,o} + \sum_{i=1}^{n} Q_i\left(x_i^2 + z_i^2\right) \\
I_z &= I_{z,o} + \sum_{i=1}^{n} Q_i\left(x_i^2 + y_i^2\right) \\
I_{xy} &= I_{xy,o} + \sum_{i=1}^{n} Q_i\, x_i y_i
\end{aligned}\right\} \quad (3)
$$

$$\left.\begin{aligned} I_{xz} &= I_{xz,o} + \sum_{i=1}^{n} Q_i\, x_i z_i \\ I_{yz} &= I_{yz,o} + \sum_{i=1}^{n} Q_i\, y_i z_i \end{aligned}\right\} \quad (3)$$

The inertia time derivatives are given by

$$\left.\begin{aligned} \dot{I}_x &= 2 \sum_{i=1}^{n} Q_i \left(y_i \dot{y}_i + z_i \dot{z}_i \right) \\ \dot{I}_y &= 2 \sum_{i=1}^{n} Q_i \left(x_i \dot{x}_i + z_i \dot{z}_i \right) \\ \dot{I}_z &= 2 \sum_{i=1}^{n} Q_i \left(x_i \dot{x}_i + y_i \dot{y}_i \right) \\ \dot{I}_{xy} &= \sum_{i=1}^{n} Q_i \left(x_i \dot{y}_i + y_i \dot{x}_i \right) \\ \dot{I}_{xz} &= \sum_{i=1}^{n} Q_i \left(x_i \dot{z}_i + z_i \dot{x}_i \right) \\ \dot{I}_{yz} &= \sum_{i=1}^{n} Q_i \left(y_i \dot{z}_i + z_i \dot{y}_i \right) \end{aligned}\right\} \quad (4)$$

For these equations, the position time histories for the n separate masses must be specified.

Crew Motions Represented by a Dynamic Model of Man

In the previous section, where a crewman was treated simply as a point mass, no condideration was given to the fact that man possesses local moments of inertia, and articulated body segments which, by their motion, result in the generation of local forces and moments. A more sophisticated means of modeling crew motions is through the use of mathematical equations describing the motion of these articulated body members. A simple example will be used to demonstrate this concept.

If it is assumed that an astronaut is in contact with the spacecraft at his feet (see figure 1), then any forces and moments he generates by moving various parts of his body will be transmitted to the spacecraft through this point of contact. Assume that the man depicted in figure 1 is rigid, with the capability of moving his right arm in the yz-plane. Considering the right arm to be rigid, and allowing it to move through the angle θ, force and moment components required to produce the movement are given by

$$F_{y_m} = m_a \ddot{y}_{cm} \qquad (5)$$

$$F_{z_m} = m_a \ddot{z}_{cm} \qquad (6)$$

$$M_{x_m} = -I_{a_s} \ddot{\theta} \qquad (7)$$

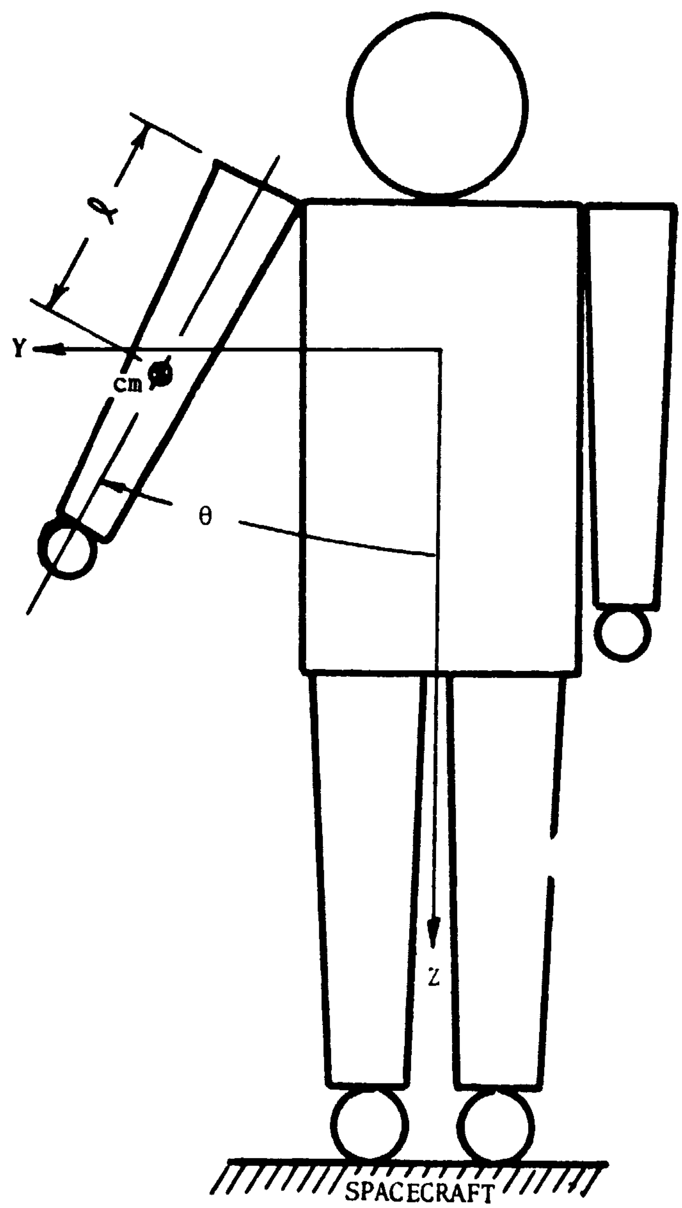

Figure 1 - Schematic of dynamic model of man in contact with spacecraft.

where

$$Y_{cm} = \ell \sin \theta \tag{8}$$

$$Z_{cm} = -\ell(1 - \cos \theta) \tag{9}$$

The quantities $\ddot{Y}_{cm}$ and $\ddot{Z}_{cm}$ can be expressed as functions of θ and its derivatives as

$$\ddot{Y}_{cm} = \ell\ (\ddot{\theta} \cos\theta - \dot{\theta}^2 \sin\theta) \tag{10}$$

$$\ddot{Z}_{cm} = -\ \ell(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \tag{11}$$

It can be seen that expressing the angle θ as a function of time allows the computation of the forces and moments required to produce this pendulum-like motion, and hence (since the astronaut is in contact with the spacecraft) the reaction forces and moments acting on the spacecraft.

$$F_{y_s} = m_a\ \ell\ (\dot{\theta}^2 \sin\theta - \ddot{\theta} \cos \theta) \tag{12}$$

$$F_{z_s} = m_a\ \ell\ (\dot{\theta}^2 \cos\theta + \ddot{\theta} \sin\theta) \tag{13}$$

$$M_{x_s} = I_{a_s} \ddot{\theta} \tag{14}$$

Similar expressions can be obtained for movements of other body members. For move complex motions ("double-pendulum" arm movements, for example), transformations to a single reference axis system are required. Appendices A and B of reference 9 present the development of equations for describing the motion of a nine-segment model of man. Inputs required for the use of this model are time histories of Euler angles describing the orientations of the nine body segments; from these inputs applied forces and moments are obtained.

The dynamic model developed in reference 9 may also be used for an astronaut moving with respect to a spacecraft (as in the performance of translational "soaring" maneuvers, for example). For this type of application, equations describing the motion of his center-of -mass with respect to the spacecraft are required, in addition to the Euler angle histories for the body segments.

One method of defining the motions of the model segments is to record the segment motions of an actual subject performing typical activities. This can be done by taking motion pictures and extracting the required information from the resulting film data (see, for example, reference 11.) More directly, use of an exoskeletal device such as described in references 12 and 13, can be made. Potentiometers at the joints between principal body segments permit the recording of relative segment motions on magnetic tape or punched cards. These motion histories, with appropriate transformations to reference co-ordination systems, can then be used to generate applied force and moment histories for input to spacecraft control system analyses.

Crew Motions Represented by Their Force and Moment Histories

For control systems analyses and simulations, it is generally most convenient to represent crew disturbance inputs as forces and moments along or about the spacecrafts axes. This can be accomplished (for restrained activities) by attaching a subject to force and moment measuring transducers, and recording the outputs of the transducers as the subject performs typical tasks. Load cells or a strain gage balance are devices which can be used to obtain the force and moment histories. A load cell arrangement which has been used successfully in both laboratory and flight experiments is shown in Figure 2. This array features essentially equal sensitivity for both forces and moments with respect to all axes. The resolved forces and moments are given by the following equations (taking load cell compression as a positive force):

$$\left.\begin{aligned}
F_x &= \sin 45^\circ\, [F_1 - F_6 + \sin 30^\circ\, (F_2 - F_3 + F_4 - F_5)] \\
F_y &= \sin 45^\circ \sin 60^\circ\, (-F_2 + F_3 + F_4 - F_5) \\
F_z &= \sin 45^\circ\, (F_1 + F_2 + F_3 + F_4 + F_5 + F_6) \\
M_x &= \sin 45^\circ\, (k_1)\, [-F_1 - F_6] + \sin 45^\circ\, (k_2)\, [F_2 + F_3 + F_4 + F_5] \\
M_y &= \sin 45^\circ\, (k_3)\, (-F_2 - F_3 + F_4 + F_5) \\
M_z &= \sin 45^\circ\, (k_1)\, (F_1 - F_2 + F_3 - F_4 + F_5 - F_6)
\end{aligned}\right\} \quad (15)$$

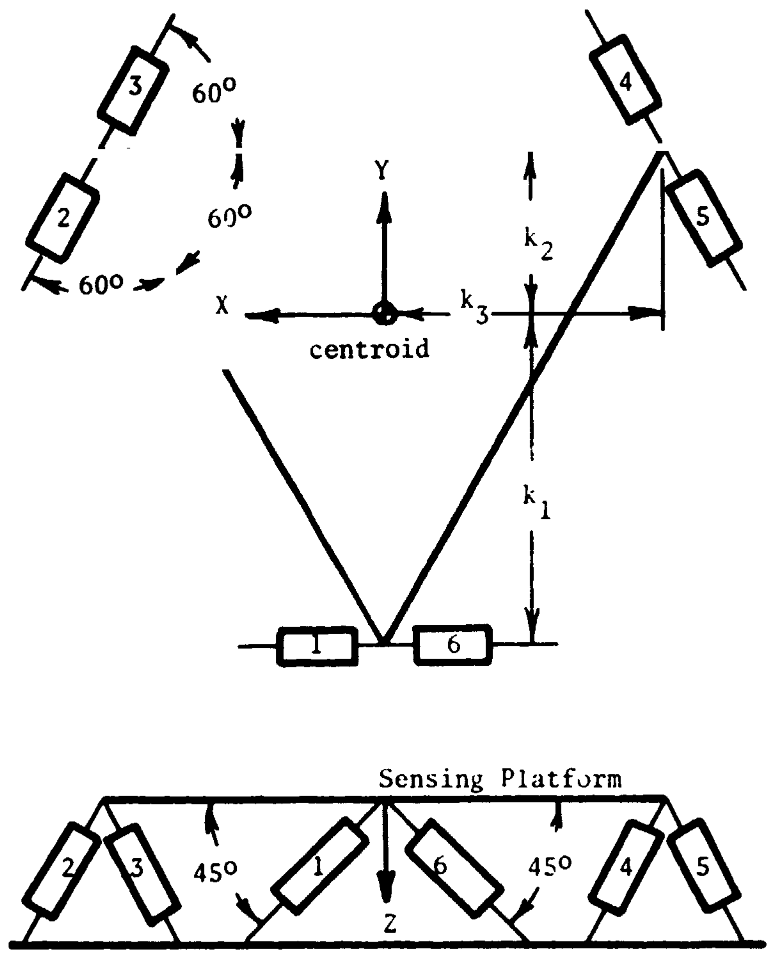

Figure 2 - Load cell array for experimental force and moment measurement.

In simulating (on earth) the forces and moments caused by a moving crew member, the introduction of certain moment contributions from a subject's center of gravity shift may occur. Consider figure 3 representing a subject whose disturbance forces and moments (from typical limb movements) are being measured.

The applied forces and moments are measured with respect to the point O, which is on the z' axis. When the subject moves a particular body member so as to cause his total c.g. to shift, a moment about the point O will be produced by any motion of the c.g. other than along the z axis. This moment will be equal to the static weight of the subject times the distance through which the c.g. travels in the x-y plane. For example, if the subject raises his arm over his head, his total c.g. will move along the (-x) axis, creating a gravity moment about the y-axis equal to the $-x \cdot W$. Similarly, if he holds his arm at a 135° angle with respect to his torso (in the x-y plane of figure 3), moments about the y- and x-axes will be produced.

There are several methods by which these gravity moments can be mechanically eliminated from the data. These methods consist of supporting various body members to exactly balance out the gravity force. Such arrangements as suspension slings and underwater neutral buoyancy weighting are two examples. However, it is not always practical to provide these mechanical supports (for reasons of available space or cost, for example). It would thus be desirable to analytically remove these gravity-produced moments from resultant

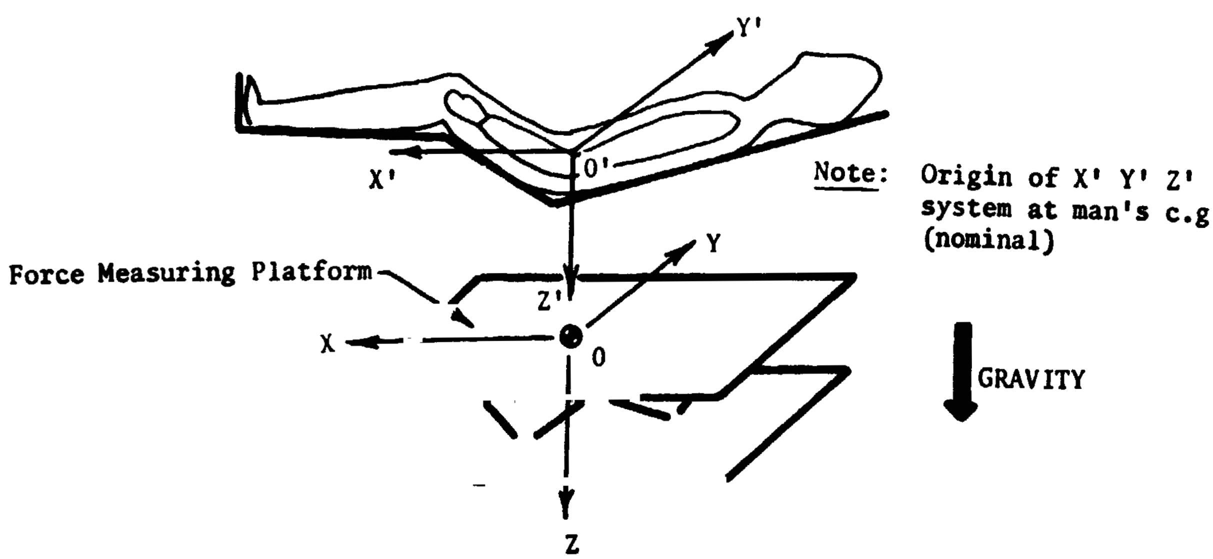

Figure 3 - Schematic of subject force and moment measurement.

disturbance data, keeping in mind, of course, that the presence of gravity cannot be eliminated and will therefore inherently affect the way in which simulated motions are performed. One method of mathematically subtracting out gravity moments consists of instrumenting a subject with an exoskeletal device which measures all limb rotations (as described in a previous section). These rotations can be used, along with mass and center of gravity data for each limb, to compute the amount of overall c.g. travel due to the articulation of one or more limbs. This method has been used (ref. 13) with results ranging from fair to good. In no cases were perfect fits obtained, however, since the limb motion measuring device could not be fitted to measure every conceivable body member motion.

Another method of reducing applied moment and force data to eliminate gravity contributions is described in the following analysis. For this derivation it is necessary to assume an axis system orientation; the orientation shown in figure 3 will be selected, that is, gravity forces are exerted in the +z direction.

As the subject performs various motions, forces and moments are produced at the point 0 in the force measuring unit (which could consist of load cells or a strain-gage balance). These forces and moments are measured, and are as follows:

$$
\left.\begin{aligned}
F_{x_{measured}} &= F_{x_{actual}} \\
F_{y_m} &= F_{y_a} \\
F_{z_m} &= F_{z_a} + W \\
M_{x_m} &= M_{x_a} + y_{c.g.} W \\
M_{y_m} &= M_{y_a} - x_{c.g.} W \\
M_{z_m} &= M_{z_a}
\end{aligned}\right\} \quad (16)
$$

Since the subject is in contact with the force measureing unit, there is exerted on the subject equal and opposite reaction forces which produce accelerations of the body c.g.

$$
\left.\begin{aligned}
F_{x_{subject}} &= -F_{x_m} = m_s \ddot{x}_{c.g.} \\
F_{y_s} &= -F_{y_m} = m_s \ddot{y}_{c.g.} \\
F_{z_s} &= -F_{z_m} = m_s \ddot{z}_{c.g.}
\end{aligned}\right\} \quad (17)
$$

The mass of the subject is simply equal to his weight divided by the acceleration of gravity constant:

$$
m_s = \frac{W_s}{g} \qquad (18)
$$

To compute the c.g. travel then, a double integration of the forces may be used.

$$\left.\begin{aligned}
x_{c.g.} &= \iint \frac{F_{x_s}}{m_s}\, dt\, dt = \frac{g}{W_s}\iint F_{x_s}\, dt\, dt + \dot{x}_{o_{c.g.}}\, t + x_{o_{c.g.}} \\
y_{c.g.} &= \frac{g}{W_s}\iint F_{y_s}\, dt\, dt + \dot{y}_{o_{c.}} \quad t + y_{o_{c.g.}} \\
z_{c.g.} &= \frac{g}{W_s}\iint F_{z_s}\, dt\, dt + \dot{z}_{o_{c.g.}} \qquad {}_{.g.}
\end{aligned}\right\}\quad(19)$$

Since the subject will normally begin all simulation sequences from some calibrated rest position, the initial velocities and desplacements of the c.g. can be taken as 0. Thus

$$\left.\begin{aligned}
x_{c.g.}(t) &= \frac{g}{W_s}\iint F_{x_s}\, dt\, dt = -\frac{g}{W_s}\iint F_{x_m}\, dt\, dt \\
y_{c.g.}(t) &= \frac{g}{W_s}\iint F_{y_s}\, dt\, dt = -\frac{g}{W_s}\iint F_{y_m}\, dt\, dt \\
z_{c.g.}(t) &= \frac{g}{W_s}\iint F_{z_s}\, dt\, dt = -\frac{g}{W_s}\iint F_{z_m}\, dt\, dt
\end{aligned}\right\}\quad(20)$$

The correction for c.g. shift would then be written as follows

$$\left.\begin{aligned}
M_{x_a} &= M_{x_{measured}} + y_{c.g.} W = M_{x_m} - \frac{gW}{W}\iint F_{y_m}\, dt\, dt \\
M_{y_{actual}} &= M_{y_m} - x_{c.g.} W = M_{y_m} + \frac{gW}{W}\iint F_{x_m}\, dt\, dt \\
M_{z_{actual}} &= M_{z_m}
\end{aligned}\right\}\quad(21)$$

Remember that these corrections are for the parti[illegible]ar axis system orientations assumed. It is seen that the math[illegible]cal correction is independent of subject weight (which appears implicitly in the F_m variables) and dependent on the [illegible]ective gravity acceleration constant at the location of the simulation. The correction equations could be used at any level of apparent gravity by proper adjustment of g. Thus, in a zero-g orbital environment, where gravity acceleration is exactly counterbalanced by centrifugal force, the effective g would be zero, and measured moments would be those actually produced by the crew motion.

STOCHASTIC CREW MOTION MODELS

In many areas of spacecraft control system analysis, there is either no requirement for deterministic representations of crew disturbance forces and moments, or their use might be unwieldy. Instead, the disturbance forces and moments can be represented by other characteristics, such as frequency content or statistical moments. The so-called stochastic crew motion disturbance models can be used in the time domain for control and pointing system simulation analyses, or in the frequency domain for system sizing or structural analysis problems.

Fourier Series Representation of Force and Moment Histories

If time histories of simulated crew motion disturbance forces and moments are available (for selected activities), then a Fourier series representation of each force and moment history can sometimes conveniently be obtained.

For a force (or moment) history, $F(t)$ over a time interval $t = 0$ to $t = t_f$ there can be derived Fourier coefficients A_n and B_n $(n = 0, 1, 2, \ldots)$ to obtain the Fourier series

$$f(t) = \frac{A_o}{2} + \sum_{n=1}^{\infty} [A_n \cos n\,\omega_o t + B_n \sin n\omega_o t] \quad (\text{for } 0 \leq t \leq t_f) \tag{22}$$

where

$$A_n = \frac{2}{t_f} \int_0^{t_f} F(t) \cos n\omega_0 t \, dt \qquad \text{for } n = 0, 1, 2, \ldots \infty \quad (23)$$

and

$$B_n = \frac{2}{t_f} \int_0^{t_f} F(t) \sin n\omega_0 t \, dt \qquad \text{for } n = 0, 1, 2, \ldots \infty \quad (24)$$

The quantity ω_0 is the fundamental frequency of the series and n is the harmonic of this fundamental frequency. In actual practice, n would have some finite limit, which would depend, in part, on the error limits desired between the Fourier series and the original data. The fundamental frequency ω_0 is taken to be

$$\omega_0 = \frac{2\pi}{t_f} \qquad (25)$$

A set of six Fourier series can thus be obtained for the force and moment time histories representative of a given period of crew activity. A constraint which should be noted is

$$\int_0^{t_f} f_x(t) \, dt = \int_0^{t_f} f_y(t) \, dt = \ldots = \int_0^{t_f} m_z(t) \, dt = 0 \qquad (26)$$

A non-zero integral would tend to produce a net change in momentum of the man-spacecraft system, if the force and moment models were applied to an attitude control system simulation for example.

In addition, it should be assumed that the activity would be initiated from (and return to) some condition of rest, i.e. net forces and moments equal to zero. This condition requires the additional constraint that

$$\frac{A_o}{2} + \sum_{n=1}^{N} A_n = 0 \qquad \text{for whatever } N \text{ chosen.}$$

In practice, the number of Fourier coefficients required (i.e. the value of N) may be quite large to give a good approximation to measured data, unless the measured wave form is periodic with a relatively small bandwidth. For those activities whose force and moment histories are such that the foregoing Fourier series models are not readily derivable, more formal Fourier transform techniques may be employed.

Use of Fourier Transforms in Crew Motion Modeling

A time history (of, say, a force or moment component of a crew motion disturbance), F(t) has a Fourier transform

$$f(\omega) = \int_{-\infty}^{\infty} F(t)e^{-i\omega t}dt \tag{27}$$

and since F(t) can be assumed to be zero for all time except $0 \le t \le t_f$,

$$f(\omega) = \int_{0}^{t_f} F(t)e^{-i\omega t}dt \tag{28}$$

This equation is essentially equivalent to the two equations used to compute the A_n's and B_n's in the preceeding section, since

$$e^{-i\omega t} \equiv \cos \omega t - i \sin \omega t \qquad (29)$$

It is used when the force or moment component possesses a continuous frequency spectrum, rather than a few discrete frequencies as is the case in the preceeding Fourier series discussion. In practical situations involving crew motion disturbance forces and moments, frequency content is limited to a range, say

$$|\omega| \leq |\omega_f| \quad , \text{ where } \omega_f \text{ may be arbitrary.}$$

Thus $f(\omega)$ would be essentially zero for all $|\omega| > |\omega_f|$, and the time functions, $F(t)$, can be expressed as (from ref.6).

$$F(t) = \frac{1}{2\pi} \int_{-\omega_f}^{\omega_f} f(\omega) e^{i\omega t} d\omega \qquad (30)$$

These relations, $f(\omega)$ and $F(t)$, constitute a Fourier transform pair, and apply to continuous wave forms, and thus have somewhat limited use (e.g., in analog computer simulations or analyses).

For most applications involving control system analyses (at least in recent years), the high-speed digital computer is utilized. This in turn requires that discrete versions of the Fourier transform be used, and, of course, all wave forms or functions must be sampled and not continuous.

The discrete Fourier transform pair associated with $f(\omega)$ and $F(t)$ respectively are

$$f(j) = \frac{1}{N} \sum_{k=1}^{N} F(k)e^{-\frac{[2\pi i \ (k-1)\ (j-1)]}{N}} \qquad (j = 1,2,\ldots N) \tag{31}$$

and

$$F(k) = \sum_{j=1}^{N} f(j)e^{\frac{[2\pi i \ (k-1)\ (j-1)]}{N}} \qquad (k = 1,\ 2,\ \ldots N) \tag{32}$$

where N is the number of samples in the waveform. F(k) and f(j) are generally complex sequences defining functions in the time and frequency domains respectively, The index k refers to a sample period number from the time history, and the index j represents a harmonic of the basic frequency in the frequency function. The time and frequency corresponding to these indices can be found from the relations

$$t_k = (k-1) \quad \Delta t \qquad \left(\text{where } \Delta t = \frac{t_f}{N}\right)$$

$$f_j = (j-1) \quad \Delta f = (j-1)\,\frac{\Delta\omega}{2\pi} \left(\text{where } \Delta\omega = \frac{2\pi}{t_f}\right)$$

A note of caution is required. The sequences f(j) and F(k) are periodic over N because of the exponential in equations (31) and (32). In obtaining the response of a control system, for example, this periodicity becomes of concern when the force or moment input is convolved with the transfer function of the system under study. Consider the following application of crew disturbance modeling.

A time history representing disturbance forces and moments will be a real function; hence, the sequence $F(k)$ will be real, and the sequence $f(j)$ of the discrete Fourier transform will be complex and exhibit the properties that

(1) $\mathrm{Re}\ f(j) = \mathrm{Re}\ f(N+2-j) \quad (j=2, 3, \ldots\frac{N}{2})$

and

(2) $\mathrm{Im}\ f(j) = -\mathrm{Im}\ f(N+2-j) \quad (j=2, 3, \ldots\frac{N}{2})$

That is, the real part of $f(j)$ is symmetric about the frequency $\omega_f = \frac{N}{2}\omega_o$, which is referred to as the "folding frequency," and the imaginary part of $f(j)$ is antisymmetric about ω_f. Since the sequence $f(j)$ is periodic, $\mathrm{Re}\ f(j)$ can be considered an even function, and $\mathrm{Im}\ f(j)$ an odd function. The Fourier coefficients $f(\frac{N}{2} + 2)\ldots f(N)$ can be thought of as the negative frequency harmonics for frequencies between $-(\omega_f - \omega_o)$ and $-\omega_o$.
For the time history being used, enough samples, N, must be available to insure that all expected frequency content of the input is at frequencies lower than the folding frequency $\omega_f = \frac{\pi N}{t_f}$. Also, the transfer function of the control system being studied, $H(s)$, $(s = i\omega)$, must be adequately described between $\omega = \omega_o$ and $\omega = \omega_f$. If the input, $F(t)$ is not periodic over the interval $t = o$ to $t = t_f$, then an incorrect system response $r(t)$ will be obtained from convolution

of the input with the transfer function. To obtain the correct response an additional N terms of zero magnitude should be added to the input sequence to obtain a sequence of 2N terms. The discrete Fourier transform, the inverse transform, and hence the convolution involving this 2N - term sequence will then be periodic over 2N points, and an N-point convolution of F(k) and H(s) will yield the aperiodic convolution of two N-point sequences. The transfer function, H(m), is defined as

$$H(m) = H\,[i\,\omega_o\,(m-1)] \qquad m = 1, 2, \ldots N+1$$

over the first (N+1) points, and

$$H\,(N + 1 + j) = \text{conj}\ H(N + 1 - j) \quad j = 1, 2, \ldots N-1$$

over the remaining (N-1) points, for a total of 2N points in the sequence. Convolving H(m) and f(m) over m= 2N points gives the frequency response array

$$R(m) = H(m)\ f(m) \qquad m = 1, 2, \ldots 2N$$

The inverse discrete Fourier transform given by equation 32 can be applied to R(m) and the sampled output response, f(t), can be obtained from

$$r\,(m) = \text{Re}\ R(m) \qquad m = 1, 2, \ldots N$$

by noting that

$$t = (m-1)\,\Delta t = (m - 1)\,\frac{t_f}{N}$$

The system response to the crew motion disturbance input over the interval $0 \leq t \leq t_f$ has then been obtained.

Crew Motion Disturbance Approximation by Filtered Random Noise

A great majority of astronaut crew motions have been, and undoubtedly will continue to be, associated with long-term, essentially continuous, activities such as experiment console monitoring, EVA preparations, meal preparations, or exercising. These activities, or combinations of activities, are composed of more or less random body movements. The resultant forces and moments thus appear somewhat random in nature, and resemble stochastic processes. This section will discuss the modeling of crew motion disturbance forces and moments (for continuous activities) by treating them as stochastic processes.

A stochastic process can be defined as a family of functions of time, $y_i(t)$, the realization of which depends on an "outcome" (which can occur with some probability p) of some "experiment" (ref. 14). A relevant example might be the force applied along the y-axis by an astronaut performing a task such as preparation of a meal or taking a shower. The first time the task is performed (taking some length of time, T) the applied force history could be represented as $F_{y_1}(t)$. The second performance would probably involve slightly different movements, and the force history $F_{y_2}(t)$ would be different, in a deterministic sense, from $F_{y_1}(t)$. Similarly, $F_{y_3}(t)$, $F_{y_4}(t)$, etc. would be expected to be different from each other. For a single performance then (a given i) $F_y(t)$ denotes a single time function. For a specific time t_j for any performance F_y is a random variable, dependent upon the performance i. Finally, for a given performance at a given time, $F_{y_i}(t_j)$ is a single number.

Since a stochastic process can be thought of as a time series of random variables then the statistics of the random variable (such as mean, variance, autocorrelation, etc) can be used to characterize the process.

If the random process under consideration is stationary (i.e., its statistics, such as mean, variance,etc.,are not affected by a shift in the time axis), the statistics will not be affected by a shift in the origin of the time axis.

Further, if the statistics of a random process can be determined from a single function (in the present example $F_{y_i}(t)$) of the process, then the process is said to be ergodic, and the various statistical parameters are expressible as time averages. For a stationary random process (again use $F_y(t)$) the power spectrum is the Fourier transform of its autocorrelation $R(\tau)$ given as

$$S(\omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau} R(\tau)\, d\tau \tag{33}$$

where (from ref. 15)

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} F_y(t) \cdot F_y(t + \tau)\, dt \tag{34}$$

and τ is a time "lag" between two samples of the time function $F_y(t)$. It should be noted that the equation defining $R(\tau)$ assumes that the mean of the function $F_y(t)$ is zero, or

$$\frac{1}{T}\int_0^T F_y(t)\ dt = 0$$

a constraint which is essentially the same as that discussed in a previous section (see eqn 26).

Once the power spectrum has been obtained for a given force or moment history, appropriate mathematical filters can be developed so that a filter output, when excited by some standard input (random or "white" noise is convenient to use), possesses the same power spectrum characteristics as the force or moment history being modeled. In equation form

$$S_{F_y}(\omega) = S_{filter} \cdot S_{input} \qquad (35)$$

using the example of the force F_y.

The next section presents a developme..., using z-transform techniques, of an extension of this model.

A z-TRANSFORM TECHNIQUE FOR CREW MOTION MODELING

As previously noted, many crew motion disturbance modeling techniques will find use in digital computer simulations and analyses of spacecraft stabilization and control systems. Digital computer representations of real-world differential equations used to describe physical systems makes use of difference equations, in contrast to the use of integrator elements in an analog computer. Formulation of these difference equations for use on the digital computer can, in many cases, be most easily accomplished by employing z-transform methods. The purpose of this section is to develop the methodology for generation of difference-equation representations of crew motion disturbance models, through use of z-transform techniques.

If $f(t)$ is a continuous function (in the time domain), then the z-transform of the sequence $f(n\ \Delta t)$, obtained by sampling $f(t)$ every Δt seconds is defined as

$$Z[f(n\ \Delta t)] = F(z) = \sum_{n=0}^{\infty} f(n\ \Delta t) z^{-n} \tag{36}$$

where z is a complex variable and

$$|z| > \frac{1}{\rho} \tag{37}$$

where ρ is the radius of convergence of the series $f(n\ \Delta t)z^{-n}$ (ref. 16)). It is noted that the z-transform can be related to the Laplace transform by

$$z = e^{s\,\Delta t} \tag{38}$$

where s is the Laplace variable.

Reconsider the filtered random noise representation of crew motion disturbances which was outlined in an earlier section of this paper. It is recalled that Fourier transforms were utilized to represent the filter structure, with power spectra computed in the frequency domain. It has been noted (refs. 17, 18, 19) that Laplace (or Fourier) transforms of sampled functions result in infinite series, which are nonalgebraic (containing factors of the form $e^{s\,\Delta t}$), making handling of sampled-data, or discrete, systems more involved and often unwieldy. Use of the z-transform in describing the transfer function of the model filters enables the derivation of difference equations which are readily programable on the digital computer. The following discussion develops the applicable theory for obtaining the requisite difference equations. It should be noted that this development is most amenable for use with activities of an essentially random nature, whose force and moment time histories would be expected to possess relatively smooth power spectra. Short-term, discrete, activities would be more suited to modeling by other, deterministic techniques.

Assume a set of N data points as a result of obtaining a force or moment time history (M) from some laboratory or experimental crew motion disturbance measurement activity. The sampling period, which will be assumed to be constant, is Δt. A third assumption which will be made is that the data, represented by the N data points, has zero mean. Following Blackman and Tukey (refs. 15 and 20), the correlation function is first determined as

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} M(t)M(t + \tau)dt \tag{39}$$

which, for the discrete case, can be written as

$$R(\tau) = \lim_{N \to \infty} \frac{1}{N + 1} \sum_{k = 0}^{N} M(k)M(k + \tau) \tag{40}$$

where τ is commonly referred to as the "lag" and is an integer multiple of $\Delta\tau$ ($\tau = n\,\Delta\tau$).

The one-sided power spectral density (PSD) is given as the discrete Fourier transform of the autocorrelation

$$S(\omega) = 2\,\Delta t \sum_{n = -\infty}^{\infty} R(n\Delta\tau)e^{-i\omega n\Delta\tau} \quad 0 \leq \omega \leq \infty \tag{41}$$

The PSD, $S(\omega)$, is a real, positive function of ω. This is the function to be modeled.

The next step is to assume a transfer function, $H(s)$, which will be excited by a random, or "white," noise input to yield an output with the power spectrum characteristics of the time history, m. $H(s)$ is an "open-loop" transfer function, and the output PSD as a function of ω can be determined in a conventional manner as

$$S_{output} = H(i\omega)\, H(-\, i\omega) \cdot S_{input} \tag{42}$$

where $H(i\omega) = H(s)|_{s\,=\,i\omega}$

It is now necessary to define a form for $H(s)$ which will give the appropriate output PSD. In the frequency domain, this filter must yield a zero output at zero frequency, in order that a resultant force or moment history not exhibit a bias (which would result in addition of net momentum to the spacecraft dynamic system). Accordingly, an arbitrary transfer function is assumed; following Hendricks and Johnson (ref. 7) this function might be specified as

$$H(s) = \frac{a(s)}{[(s)^2 - 2b_1(s) + c_1{}^2]\,[(s)^2 - 2b_2\,(s) + c_2{}^2]} \tag{43}$$

If it is assumed that a random (Gau[illegible]se input with an rms value of unity, the model PSD as a function of ω is (from equations (42) and (43))

$$\underset{output}{S(\omega)} = \frac{A\omega^2}{1 + B_1\,\omega^2 + B_2\,\omega^4 + B_3\,\omega^6 + B_4\,\omega^8} \cdot 1 \tag{44}$$

where A and $B_i (i = 1 - 4)$ are functions of the filter parameters a, b_1, b_2, c_1, c_2. A method of curve-fitting must be used to fit the model PSD (equation (44)) to the measured PSD (equation (41)). A form of least-squares curve-fit algorithm could be utilized to determine the factors A and B_i, and hence the filter parameters.

Once the transfer function has been defined in the frequency domain, derivation of an equivalent digital filter is pursured. It should be noted that the filter being developed (for use on digital computers) will essentially receive input from a digital sampler, whose output can be considered a series of impulses. In order to more accurately represent the continuous input to the continuous filter being modeled, the zero-order hold is commonly used (ref. 21). The mathematical formulation for this "element" is given by

$$H_{zoh}(s) = \frac{1 - e^{-s\Delta t}}{s} \tag{45}$$

The factor $\frac{1}{s}$ in $H_{zoh}(s)$ must be included with the filter transfer function $H(s)$, and use is made of equation (38) to obtain the digital filter:

$$H(z) = \frac{z - 1}{z} Z[H_1(s)] \tag{46}$$

where

$$H_1(s) = \frac{H(s)}{s}$$

For the present problem, $H_1(s)$ becomes

$$H_1(s) = \frac{a}{(s^2 - 2b_1 s + c_1^2)(s^2 - 2b_2 s + c_2^2)} \tag{47}$$

The transfer function $H_1(s)$ given in equation (47) posses poles (which will be complex for most cases) at

$$s = b_1 + \sqrt{b_1^2 - c_1^2}$$

$$b_1 - \sqrt{b_1^2 - c_1^2}$$

$$b_2 + \sqrt{b_2^2 - c_2^2}$$

$$b_2 - \sqrt{b_2^2 - c_2^2}$$

To obtain the digital filter, $H(z)$, corresponding to the model filter (with zero-order hold) $H_1(s)$, Cauchy's residue theorem (and eqn. 38) can be used (ref. 19)

$$H(z) = \left(\frac{z - 1}{z}\right) \frac{1}{2\pi i} \oint \left\{ \frac{H_1(p)}{1 - e^{-\Delta t(s - p)}} \right\} dp$$

$$= \left(\frac{z - 1}{z}\right) \sum_{k=1}^{N_p} (p - p_k) H_1(p) \left. \frac{z}{z - e^{\Delta t p}} \right|_{p = p_k} \tag{48}$$

where N_p = number of poles of $H_1(s)$ (in this case $N_p = 4$) and p_k is the value of the kth pole. The path of integration in the z-plane must include all poles p_k. From equation (48), a ratio of polynomials in (z^{-1}) can be obtained:[1]

---

[1]It would have been as easy to obtain a ratio of polynomials in z; however, as will be seen, the given ratio has certain advantages in subsequent developments.

$$H(z) = \frac{F(z)}{G(z)} = \frac{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \ldots + \alpha_\ell z^{-1}}{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} + \ldots + \beta_m z^{-m}} \tag{49}$$

where the α's and β's are functions of a, b_1, b_2, c_1, c_2.

The transfer function input-output relation can now be applied to determine the difference equations to be used on the digital computer. It is first noted from z-transform theory (refs. 16 and 18) that an inverse transform may be expressed as

$$Z^{-1}\left[z^m G(z)\right] = \sum_{n=0}^{\infty} g(n\,\Delta t + m\,\Delta t)\delta(t - n\,\Delta t) \tag{50}$$

for the sample sequence where $t \geq 0$, $g(n\,\Delta t + m\,\Delta t)$ is a discrete sample from some time history $g(t)$ whose z-transform is $G(z)$, and $\delta(t - n\,\Delta t)$ is an impulse function. Equation (49) is rewritten as

$$(\beta_0 + \beta_1 z^{-1} + \ldots + \beta_m z^{-m})F(z) = (\alpha_0 + \alpha_1 z^{-1} + \ldots + \alpha_\ell z^{-\ell})G(z) \tag{51}$$

The inverse z-transform of equation (51) is taken term by term to yield

$$\begin{aligned}\beta_0 \sum_{n=0}^{\infty} f(n\,\Delta t)\delta(t - n\,\Delta t) &+ \beta_1 \sum_{n=0}^{\infty} f\left[(n-1)\Delta t\right] \delta(t - n\,\Delta t)\\ &+ \ldots + \beta_m \sum_{n=0}^{\infty} f\left[(n-m)\Delta t\right] \delta(t - n\,\Delta t)\\ &= \alpha_0 \sum_{n=0}^{\infty} g(n\,\Delta t)\delta(t - n\,\Delta t) + \alpha_1 \sum_{n=0}^{\infty} g\left[(n-1)\Delta t\right]\delta(t - n\,\Delta t)\\ &+ \ldots + \alpha_\ell \sum_{n=0}^{\infty} g\left[(n-\ell)\Delta t\right] \delta(t - n\,\Delta t)\end{aligned} \tag{52}$$

The coefficients (β_i $f[(n - i)\Delta t]$ and α_i $g[(n - i)\Delta t]$ of the impulses $\delta(t - n\,\Delta t)$ at the same sampling instant, $n\,\Delta t$, can be equated to get

$$\beta_0 f(n\,\Delta t) + \beta_1 f[(n - 1)\Delta t] + \ldots + \beta_m f[(n - m)\Delta t]$$
$$= \alpha_0 g(n\,\Delta t) + \alpha_1 g[(n - 1)\Delta t] + \ldots + \alpha_\ell g[(n - \ell)\Delta t] \tag{53}$$

which is the difference-equation description of the process obtained by passing the "white" noise signal through the filter H. Rewriting equation (53) as

$$f(n\,\Delta t) = \frac{\alpha_0}{\beta_0} g(n\,\Delta t) + \frac{\alpha_1}{\beta_0} g[(n - 1)\Delta t] + \ldots$$
$$+ \frac{\alpha_\ell}{\beta_0} g[(n - \ell)\Delta t] - \frac{\beta_1}{\beta_0} f[(n - 1)\Delta t]$$
$$- \frac{\beta_2}{\beta_0} f[(n - 2)\Delta t)] - \ldots - \frac{\beta_m}{\beta_0} f[(n - m)\Delta t] \tag{54}$$

it is seen that the output $f(t)$, $t = n\Delta t$, is a function of the backward differences of input and output or, in other words, the output at a given sample instant is dependent on both output and input at earlier sample instants. The quantities $g(n\Delta t)$ will be values from the "white" noise sequence, and as such, will be merely a set of normally distributed random numbers with a mean of zero and a variance of unity.

Implementation of equation (54) on a digital computer is straightforward. Since the difference equation was developed for $t \geq 0$, the values of input and output prior to $t = 0$ (i.e., $g(-\Delta t)$, $g(-\ell\Delta t), \ldots, f(-\Delta t)$, $f(-\ell\Delta t)$, etc.) can be taken as zero.

Example

As an illustrative example of the utility of the difference-equation description of a filter output, consider a relatively simple filter structure to be given by

$$H(s) = \frac{s}{(s+2)(s+4)} \tag{55}$$

Following equations (45) and (46), to include the requisite zero-order hold, and equation (48), H(z) can be obtained as

$$H(z) = \frac{z-1}{z}\left\{\frac{1}{2}\left[\frac{z}{z-e^{-2\Delta t}} - \frac{z}{z-e^{-4\Delta t}}\right]\right\} \tag{56}$$

Assuming Δt = .005 sec., H(z) becomes (after simplification)

$$H(z) = \frac{.00492558025\ z^{-1} - .00492558025\ z^{-2}}{1-1.970248507\ z^{-1} + .9704455927\ z^{-2}} \tag{57}$$

The difference-equation relation between input, x_i, and output, x_o, with x_i being given as a sampled input, can be written

$$\begin{aligned} x_o(n\Delta t) &= .00492558025\ x_i[(n-1)\Delta t] - .00492558025\ x_i[(n-2)\Delta t] \\ &+ 1.970248507\ x_o[(n-1)\Delta t] - .9704455927\ x_o[(n-2)\Delta t] \end{aligned} \tag{58}$$

To ascertain the accuracy of equation (58) in describing a system, output response to a unit step function[1] through the filter was determined using a small desk-top computer system. Figure 4 shows a plot of the equivalent analog response (solid curve) with the

[1] The unit step function is defined (ref. 17) as having the value 1 for positive time, t, 0 for negative t, and 1/2 for t = 0.

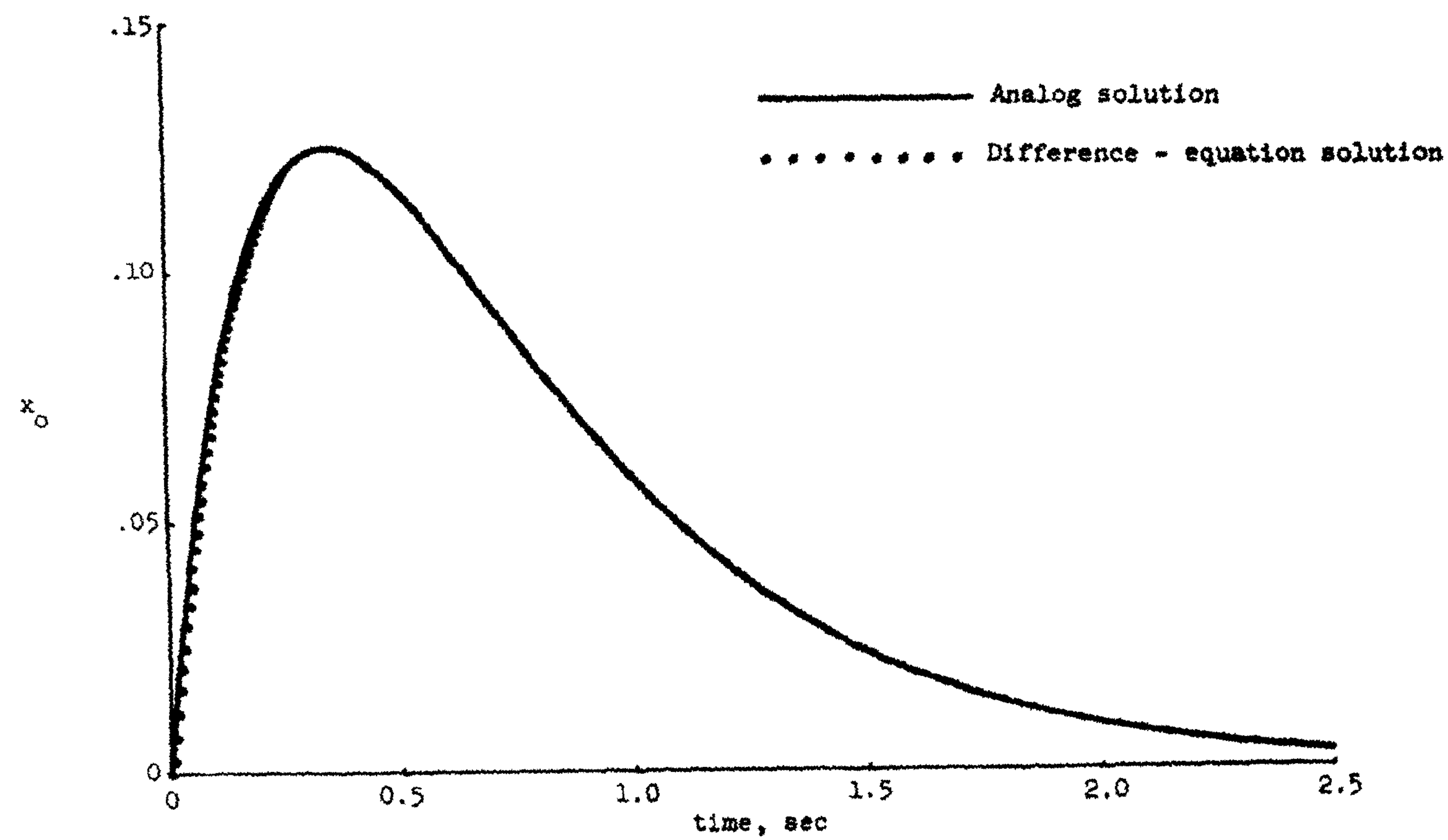

Figure 4.- Response of example filter to unit step input.

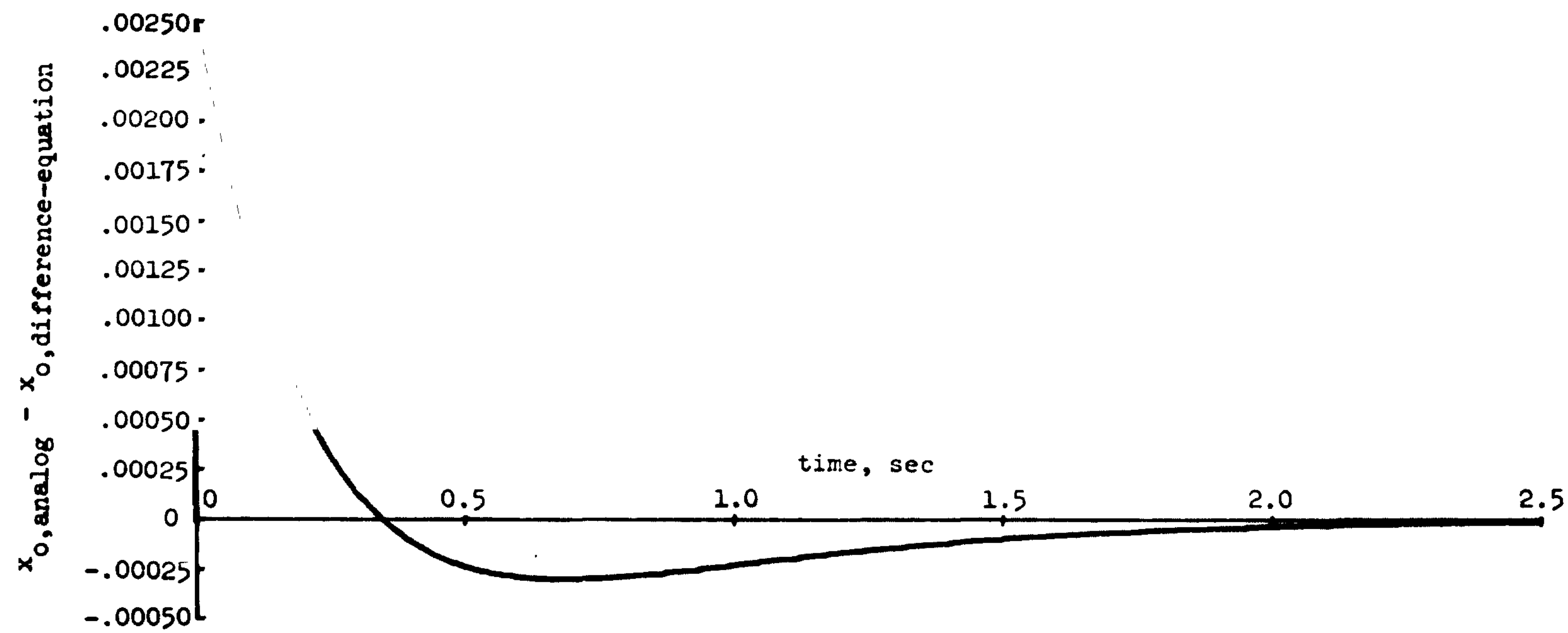

Figure 5.- Plot of difference between analog and difference-equation results.

difference equation output for comparison. As can be noted from figure 4, the difference-equation result is essentially equal to the analog curve after 0.4 seconds. Figure 5 gives the difference (analog minus difference-equation) between the two results for the same 2.5 second period shown in figure 4. Agreement of the difference-equation description with the analog result is within two percent from 0.4 seconds on.

Next, response of the filter (equation (57)) to a random noise input will be examined for its implementation characteristics.

The technique used to digitally generate pseudo-random numbers for input to this problem consisted of first obtaining pseudo-random numbers uniformly distributed on the interval (0, 1) through the relation

$$u_i = \text{Fractional part of } [(\pi + u_{i-1})^8] \tag{59}$$

Normally-distributed pseudo-random numbers, N_i, with mean 0 and unity variance are then generated from (see ref. 22)

$$\begin{aligned} N_i &= (-2 \ln u_i)^{1/2} \cos (2\pi\, u_{i+1}) \\ N_{i+1} &= (-2 \ln u_i)^{1/2} \sin (2\pi\, u_{i+1}) \end{aligned} \tag{60}$$

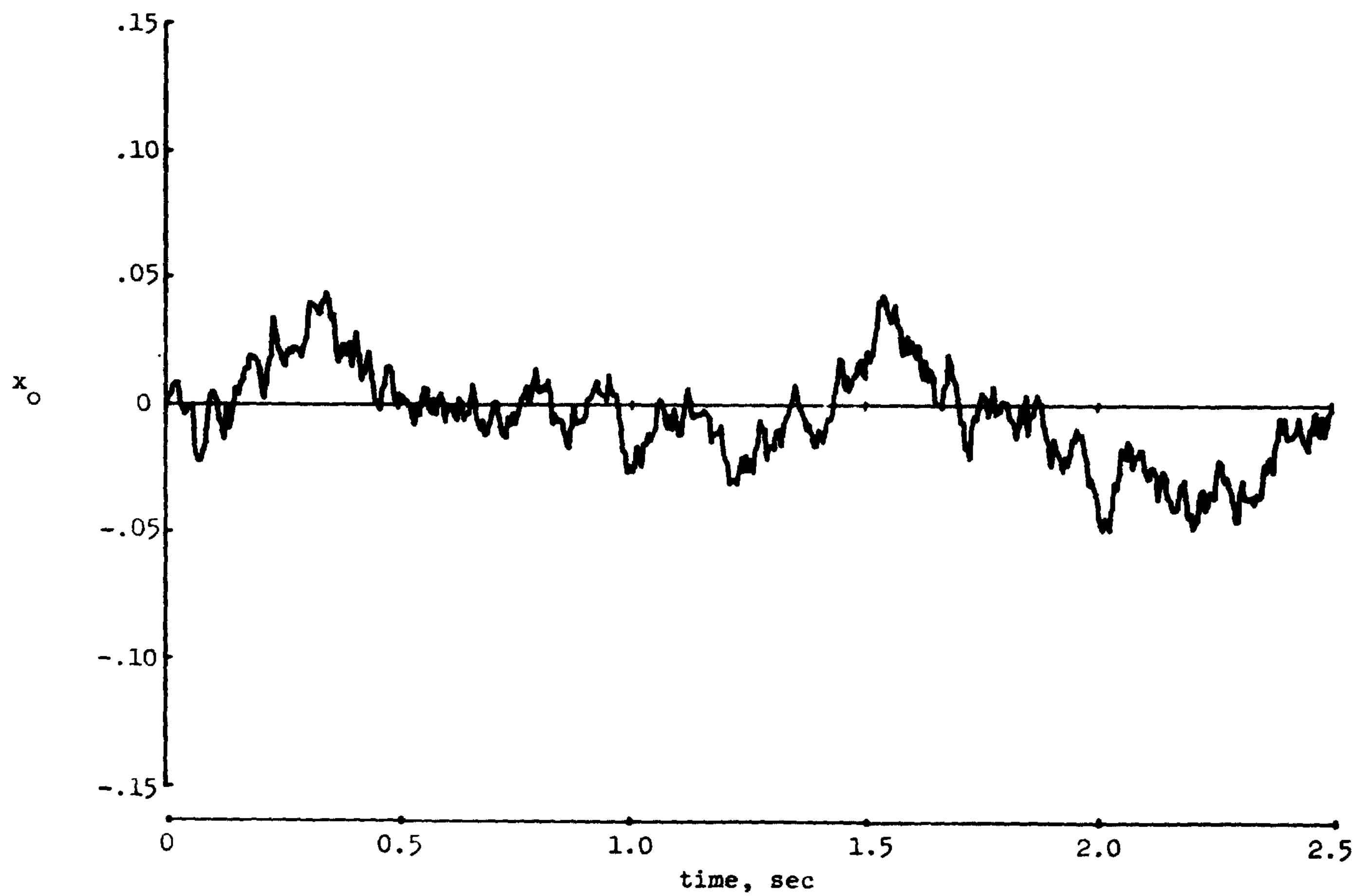

Figure 6.- Response of example filter to random noise input.

where u_i and u_{i+1} must be independent uniform variables. This condition necessitates the use of two sequences of the form given by equation (59), with separate initializing uniform random numbers.

The N_i generated are used as step inputs to the difference-equation filter description (equation (58)). A sample result of applying a pseudo-random noise sequence to the present filter is given as figure 6. This figure indicates the types of "random" time histories that can be developed from filters of the form (or similar form) used.

CONCLUDING REMARKS

The objective of this thesis has been to present an outline of the methods or techniques which have become available to represent crew motion disturbances in a manned spacecraft. The wide varieties of payloads, crew sizes, and control systems contemplated for future space missions require that updated methods of control system sizing, design, and analysis be utilized. The techniques included herein highlight both analytical and experimental approaches. Of perhaps greatest utility of these techniques is the z-transform application, developed here specifically for time-domain digital computer applications requiring a variety of possible crew motion disturbance inputs. An example illustrates the ease of implementation of this method, which ease should enhance its utility in any future studies.

Several other crew motion disturbance modeling techniques have been presented. The three deterministic methods discussed have been applied to ground-based simulations by a few investigators; in addition, the dynamic model of man has been investigated in a flight experiment. Of the thre. so-called stochastic methods presented, the filtered random noise technique has been perhaps the most useful, in frequency domain studies of the Skylab experiment pointing control system. Some time domain applications of this technique, in conjunction with pure Fourier transform methods have also been utilized.

An appendix is included in this thesis; it derives a set of basic equations of motion for a spacecraft with applied disturbance which, although requiring numerical solutions, can be used with many of the crew motion disturbance models (as well as with other types of defined disturbances).

It is anticipated that results of the research efforts reported herein would be most useful in initial sizing studies of attitude control systems; of almost equal utility would be their incorporation into existing simulation programs (both all computer and computer/hardware hybrid). Finally, it should be noted that the results contained in this thesis are not intended to be all-inclusive; no tables of parameters to use with various models are given, nor are specific inplementation schemes suggested. It is felt that a more diverse treatment of the subject of crew motion disturbance modeling has resulted, with enough detail to facilitate applications to a variety of control system analyses.

Appendix

Development of Spacecraft Dynamics Equations

In an investigation of astronaut crew motion disturbance effects, relations between disturbance force and moment inputs and spacecraft response outputs are needed. Spacecraft attitude cnn be expressed in terms of Euler angles or direction cosines which in turn are expressible as integral functions of angular velocities about spacecraft body-fixed axes. In order to obtain attitude response information, then, it is first necessary to determine these angular velocities, or body rates. This appendix presents a derivation of the spacecraft dynamic equations, with one arbitrary moving mass, based on momentum considerations (refs. 23 and 24). A vector approach will be used for its economy in notation. Figure 7 illustrates the vector relationship between the spacecraft body-fixed axis X_b Y_b Z_b and inertial axes X_I Y_I Z_I.

The angular momentum of a spacecraft, defined in coordinates of the body-fixed axis system (origin at 0 in figure 7) is the sum of the angular momenta of all of its mass elements, or

$$H_o = \sum \bar{r}_{mj} \times m_j \dot{\bar{R}}_{mj} \qquad \text{(A1)}$$

and the momentum of its mass center is

$$\bar{H}_{cm} = \bar{r}_{ms} \times m_s \dot{\bar{R}}_{ms} \qquad \text{(A2)}$$

where $m_s = \sum m_j$

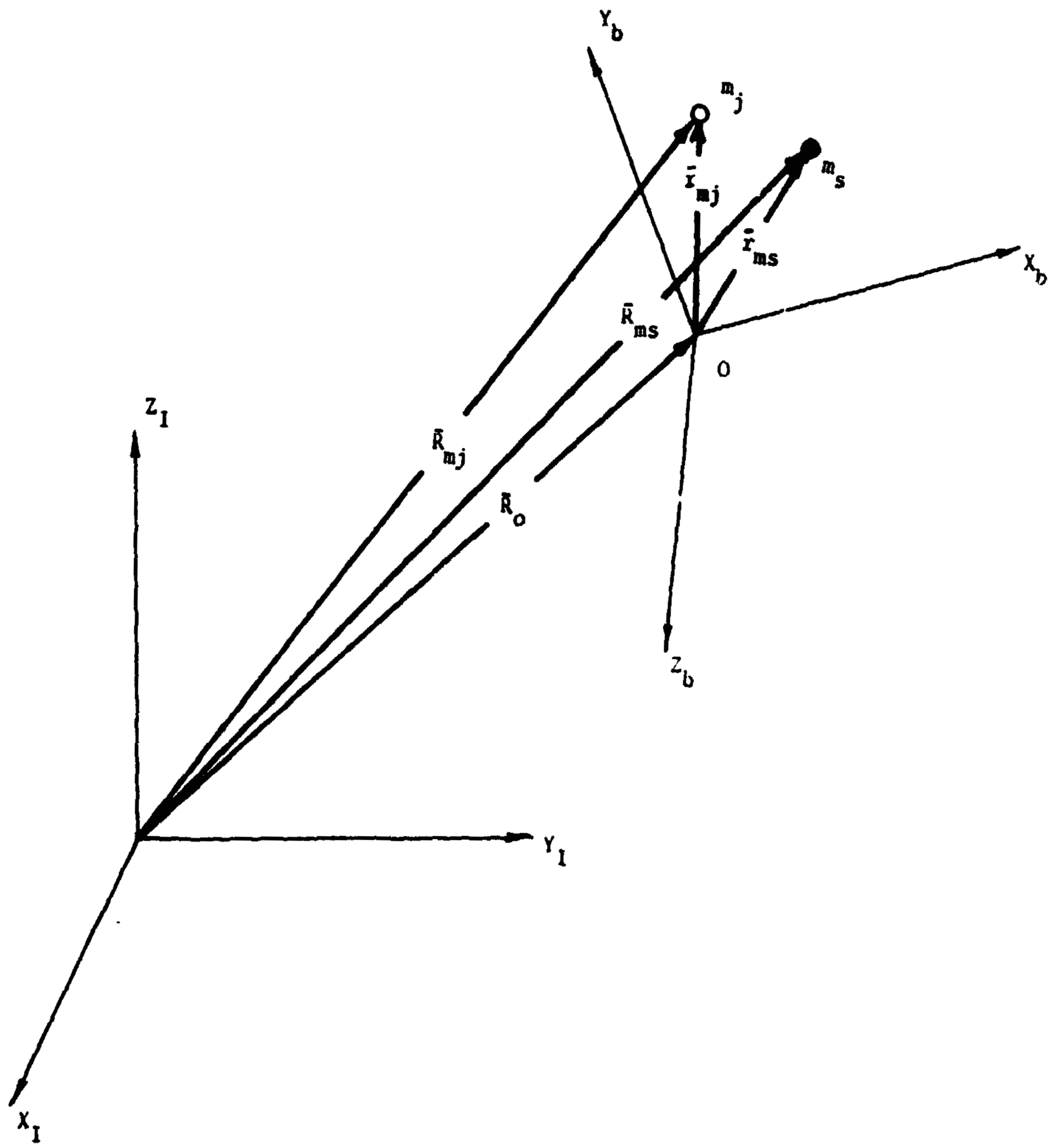

Figure 7 - Relationship between body-fixed and inertial axes.

The total angular momentum about the spacecraft mass center is then

$$\bar{H}_{cm} = \bar{H}_o - \bar{H}_s \tag{A3}$$

or

$$\bar{H}_{cm} = \sum \bar{r}_{mj} \times m_j (\dot{\bar{R}}_o + \dot{\bar{r}}_{mj} + \bar{\omega} \times \bar{r}_{mj}) - \bar{r}_{ms} \times m_s (\dot{\bar{R}}_o + \dot{\bar{r}}_{ms} + \bar{\omega} \times \bar{r}_{ms}) \tag{A4}$$

where $\bar{\omega}$ is the angular velocity of the $X_b Y_b Z_b$ axis system.

The coordinates of the spacecraft mass center, $\bar{r}_{ms}$, are given by

$$\bar{r}_{ms} = \frac{\sum m_j \bar{r}_{mj}}{\sum m_j} = \frac{\sum m_j \bar{r}_{mj}}{m_s} \tag{A5}$$

Equation (A4) can be rewritten usint the relation (A5) as

$$\bar{H}_{cm} = \sum \bar{r}_{mj} m_j \times \dot{\bar{R}}_o - \frac{\sum m_j \bar{r}_{mj}}{m_s} \times m_s \dot{\bar{R}}_o + \sum \bar{r}_{mj} m_j \dot{\bar{r}}_{mj} - \bar{r}_{ms} \times$$

$$m_s \dot{\bar{r}}_{ms} + \sum \bar{r}_{mj} \times m_j (\bar{\omega} \times \bar{r}_{mj}) - \bar{r}_{ms} \times m_s (\bar{\omega} \times \bar{r}_{ms}) \tag{A6}$$

or

$$\bar{H}_{cm} = \sum \bar{r}_{mj} \times m_j \dot{\bar{r}}_{mj} - \bar{r}_{ms} \times m_s \dot{\bar{r}}_{ms} + \sum \bar{r}_{mj} \times m_j (\bar{\omega} \times \bar{r}_{mj}) - \bar{r}_{ms} \times$$

$$m_s (\bar{\omega} \times \bar{r}_{ms}) \tag{A7}$$

For convenience, first consider the spacecraft as a rigid body; $\dot{\bar{r}}_{mj} = \dot{\bar{r}}_{ms} = 0$, and a rigid-body angular momentum can be expressed using equation (A7) as

$$\bar{H}_{rb} = \sum \bar{r}_{mj} \times m_j \, (\bar{\omega} \times \bar{r}_{mj}) - \bar{r}_{ms} \times m_s \, (\bar{\omega} \times \bar{r}_{ms}) \tag{A8}$$

or by using elementary vector analysis

$$\bar{H}_{rb} = \sum m_j \left[(\bar{r}_{mj} \cdot \bar{r}_{mj})\bar{\omega} - (\bar{r}_{mj} \cdot \bar{\omega})\bar{r}_{mj} \right] - m_s \left[(\bar{r}_{ms} \cdot \bar{r}_{ms})\bar{\omega} - (\bar{r}_{ms} \cdot \bar{\omega})\bar{r}_{ms} \right] \tag{A9}$$

Equation (A9) can be expanded into Cartesian coordinates, by recognizing certain quantities defined as moments of inertia, the rigid-body momentum can be written in matrix form as

$$\bar{H}_{rb} = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \tag{A10}$$

It should be noted that the moments of inertia are about axes through the center of mass of the spacecraft.

Consider the origin 0 to be at the center of mass of the spacecraft (taken without any moving masses); then the consideration of some number of moving masses, k, permits the expression of (say) the x centroidal axis moment of inertia as

$$I_x = \sum_{j=1}^{n-k} m_j \, (y_j^2 + z_j^2) + \sum_{j=n-k+1}^{n} m_j \, (y_j^2 + z_j^2) - m_s \, (y_s^2 + z_s^2) \tag{A11}$$

Here the first term on the right-hand side of equation (A11) represents the spacecraft moment of inertia, exclusive of any moving masses, and can be assumed constant for many analyses. The second term is the moment of inertia due to K moving masses, referred to the origin O. The third term is the inertia shift to the overall mass center of the spacecraft and moving masses (crew, for example).

For convenience in subsequent development, assume only one moving mass m located at a distance r from the origin O. The moment of inertia about the x-axis (centroidal) can now be written as

$$I_x = I_{x,o} + m\ (y^2 + z^2) - m_s\ (y_s^2 + z_s^2) \tag{A12}$$

where $I_{x,o}$ is the first term on the right side of equation (A11). The distance from the origin O to the mass center, r_s, is now

$$r_s = \frac{m\ r}{m_s} \tag{A13}$$

and

$$I_x = I_{x,o} + (m - \frac{m^2}{m_s})\ (y^2 + z^2) \tag{A14}$$

Defining a mass factor Q as

$$Q = \frac{m(m_s - m)}{m_s} \tag{A15}$$

the x-axis moment of inertia can be written

$$I_x = I_{x,o} + (y^2 + z^2) \tag{A16}$$

All moments and products of inertia can be expressed in a similar manner:

$$
\begin{aligned}
I_x &= I_{x',o} + Q(y^2 + z^2) \\
I_y &= I_{y',o} + Q(x^2 + z^2) \\
I_z &= I_{z',o} + Q(x^2 + y^2) \\
I_{xy} &= I_{xy',o} + Q(xy) \\
I_{xz} &= I_{xz',o} + Q(xz) \\
Iyz &= I_{yz',o} + Q(yz)
\end{aligned}
\qquad (A17)
$$

Again considering all mass elements, from equation (A1) is obtained

$$\frac{d\,\bar{H}_o}{dt} = \frac{d}{dt} \sum \bar{r}_{mj} \times m_j \dot{\bar{R}}_{mj} \qquad (A18)$$

$$\frac{d\,\bar{H}_o}{dt} = \sum \frac{d\bar{r}_{mj}}{dt} \times m_j \dot{\bar{R}}_{mj} + \sum \bar{r}_{mj} \times m_s \ddot{\bar{R}}_{mj} \qquad (A19)$$

$$\frac{d\,\bar{H}_o}{dt} = \sum \frac{d\bar{r}_{mj}}{dt} \times m_j \left(\dot{\bar{R}}_o + \frac{d\bar{r}_{mj}}{dt}\right) + \sum \bar{r}_{mj} \times m_j \dot{\bar{R}}_{mj} \qquad (A20)$$

$$\frac{d\,\bar{H}_o}{dt} = \sum m_j \frac{d\bar{r}_{mj}}{dt} \times \dot{\bar{R}}_o + \sum \bar{r}_{mj} \times m_j \ddot{\bar{R}}_{mj} \qquad (A21)$$

$$\frac{d\,\bar{H}_o}{dt} = m_s \frac{d\bar{r}_{ms}}{dt} \times \dot{\bar{R}}_o + \sum \bar{r}_{mj} \times m_j \ddot{\bar{R}}_{mj} \qquad (A22)$$

Similarly, from equation (A2)

$$\frac{d\,\bar{H}_s}{dt} = m_s \frac{d\bar{r}_{ms}}{dt} \times \dot{\bar{R}}_o + \bar{r}_{ms} \times m_s \ddot{\bar{R}}_{ms} \tag{A23}$$

From equation (A3)

$$\frac{d\,\bar{H}_{cm}}{dt} = \frac{d\,\bar{H}_o}{dt} - \frac{d\,\bar{H}_s}{dt} \tag{A24}$$

$$\frac{d\,\bar{H}_{cm}}{dt} = \sum \bar{r}_{mj} \times m_j \ddot{\bar{R}}_{mj} - \bar{r}_{ms} \times m_s \ddot{\bar{R}}_{ms} \tag{A25}$$

Newton's force equation, through the mass center is

$$\bar{F}_s = m_s \ddot{\bar{R}}_{ms} \tag{A26}$$

Equation (A25) can now be written

$$\frac{d\,\bar{H}_{cm}}{dt} = \sum \bar{r}_{mj} \times m_j \ddot{\bar{R}}_{mj} - \bar{r}_{ms} \times \bar{F}_s \tag{A27}$$

Further, the total moment about the origin 0 is

$$\bar{M}_o = \sum \bar{r}_{mj} \times m_j \ddot{\bar{R}}_{mj} \tag{A28}$$

Equations (A27) and (A28) can be combined to yield

$$\bar{M}_o = \frac{d\,\bar{H}_{cm}}{dt} + \bar{r}_{ms} \times \bar{F}_s \tag{A29}$$

For most problems involving spacecraft rotations (of principal concern in this development) the external force applications linked with mass center shifts can be neglected. Thus, the applied moment about O will equal the derivative of the angular momentum about the overall mass center:

$$\bar{M}_o = \frac{d\,\bar{H}_{cm}}{dt} \qquad \text{(A30)}$$

Combining and rearranging equations (A7) and (A8) yields

$$\bar{H}_{cm} = \sum \bar{r}_{mj} \times m_j \, \dot{\bar{r}}_{mj} - \bar{r}_{ms} \times m_s \, \dot{\bar{r}}_{ms} + \bar{H}_{rb} \qquad \text{(A31)}$$

and (considering only one moving mass m at distance r from O) equation (A30) can be expanded as

$$\bar{M}_o = \frac{d\,\bar{H}_{cm}}{dt} = \frac{d}{dt}\left(\bar{r} \times m\,\dot{\bar{r}} - \bar{r}_{ms} \times m_s \, \dot{\bar{r}}_{ms} + \bar{H}_{rb}\right) \qquad \text{(A32)}$$

Using equation (A13) to substitute for $\bar{r}_{ms}$ and $\dot{\bar{r}}_{ms}$, equation (A32) becomes

$$\bar{M}_o = \frac{d}{dt}\left[\bar{r} \times m\,\dot{\bar{r}} - \frac{m\bar{r}}{m_s} \times m_s \frac{m}{m_s}\,\dot{\bar{r}} + \bar{H}_{rb}\right] \qquad \text{(A33)}$$

Also

$$\bar{M}_o = \frac{d}{dt}\left[Q(\bar{r} \times \dot{\bar{r}}) + \bar{H}_{rb}\right] \qquad \text{(A34)}$$

by making use of equation (A15).

Expanding equation (A34) results in

$$\bar{M}_o = Q\left[\bar{r} \times \ddot{\bar{r}} - \bar{\omega} \times (\bar{r} \times \dot{\bar{r}})\right] + \dot{\bar{H}}_{rb} + \bar{\omega} \times \bar{H}_{rb} \qquad \text{(A35)}$$

Now, by substituting for $\overline{H}_{rb}$ (and $\dot{\overline{H}}_{rb}$) by making use of equation (A12), equation (A35) can be expanded to yield the components of $\overline{M}_o$ about the X_b -, Y_b -, and Z_b- axes:

$$\left.\begin{aligned}
M_x = {} & Q\left[(y\ddot{z} - z\ddot{y}) + \omega_y(x\dot{y} - y\dot{x}) - \omega_z(z\dot{x} - x\dot{z})\right] + (I_x\dot{\omega}_x + \omega_x\dot{I}_x - \\
& I_{xy}\dot{\omega}_y - \omega_y\dot{I}_{xy} - I_{xz}\dot{\omega}_z - \omega_z\dot{I}_{xz}) + \omega_y(I_z\omega_x - I_{xz}\omega_x - I_{yz}\omega_y) - \\
& \omega_z(I_y\omega_y - I_{yz}\omega_z - I_{xy}\omega_x) \\[1ex]
M_y = {} & Q\left[(z\ddot{x} - x\ddot{z}) + \omega_z(y\dot{z} - z\dot{y}) - \omega_x(x\dot{y} - y\dot{x})\right] + (I_y\dot{\omega}_y + \omega_y\dot{I}_y - \\
& I_{yz}\dot{\omega}_z - \omega_z\dot{I}_{yz} - I_{xy}\dot{\omega}_x - \omega_x\dot{I}_{xy}) + \omega_z(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \\
& \omega_x(I_z\omega_z - I_{xz}\omega_x - I_{yz}\omega_y) \\[1ex]
M_z = {} & Q\left[(x\ddot{y} - y\ddot{x}) + \omega_x(z\dot{x} - x\dot{z}) - \omega_y(y\dot{z} - z\dot{y})\right] + (I_z\dot{\omega}_z + \omega_z\dot{I}_z - \\
& I_{xz}\dot{\omega}_x - \omega_x\dot{I}_{xz} - I_{yz}\dot{\omega}_y - \omega_y\dot{I}_{yz}) + \omega_x(I_y\omega_y - I_{yz}\omega_z - I_{xy}\omega_x) - \\
& \omega_y(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z)
\end{aligned}\right\} \quad \text{(A36)}$$

If the moments M_x, M_y, and M_z and coordinates for the moving mass x, y, z are expressed as functions of time, then equations (A36) can be numerically integrated to find the body rates ω_x, ω_y, and ω_z.

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